Intro: Fibonacci Numbers I

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Data Structures and Algorithms Algorithmic Toolbox

Learning Objectives

- Understand the definition of the Fibonacci numbers.
- Show that Fibonacci numbers become very large.

Definition

$$F_n = \begin{cases} 0, & n = 0\,, \\ 1, & n = 1\,, \\ F_{n-1} + F_{n-2}, & n > 1\,. \end{cases}$$

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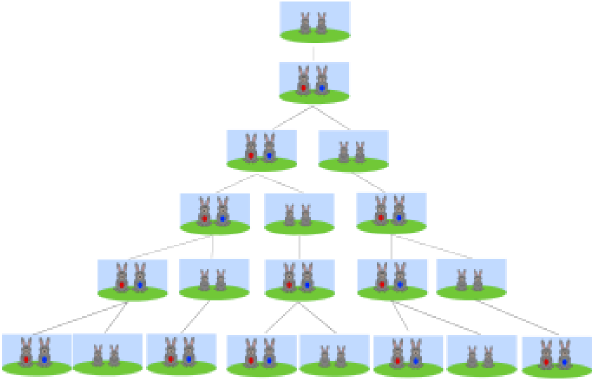
$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

Developed to Study Rabbit Populations



Rapid Growth

Lemma

$F_n \geq 2^{n/2}$ for $n \geq 6$.

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Inductive step:

$$F_n = F_{n-1} + F_{n-2} \geq 2^{(n-1)/2} + 2^{(n-2)/2} \geq 2 \cdot 2^{(n-2)/2} = 2^{n/2}. \quad \square$$

Formula

Theorem

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right).$$

Example

$$F_{20} = 6765$$

Example

$$
\begin{aligned}
F_{20} &= 6765 \\
F_{50} &= 12586269025
\end{aligned}
$$

Example

$$\begin{aligned} F_{20} &= 6765 \\ F_{50} &= 12586269025 \\ F_{100} &= 354224848179261915075 \end{aligned}$$

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F_{20} &= 6765 \\
F_{50} &= 12586269025 \\
F_{100} &= 354224848179261915075 \\
F_{500} &= 139423224561697880139724382 \\
&\quad 870407283950070256587697307264 \\
&\quad 108962948325571622863290691 \\
&\quad 557658876222521294125
\end{aligned}
$$

Computing Fibonacci numbers

Compute F_n

Input: An integer $n \geq 0$.

Output: F_n.