A New Technique for Image Compression Using PCA

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Abstract

*PCA is a statistic approach which widely used in many fields of study. Recently, this technique is used in image processing as a powerful tool especially in face recognition and face database compression. By working on this method in the recently researches, this method has been improved and kernel PCA, 2D-PCA and some other methods has been introduced. Generally the methods have been used in face recognition and processing **a group of images** which belongs to similar objects, such as face image databases. In this paper a new method is introduced to compress **one** coloured image using PCA technique.*

1. Introduction

PCA method for images is called HOTELLING or KL transform. Common KL transform focuses on similarities between different images in the database. In this paper, we will show that by applying some changes in KL transform, we can use this technique for compressing just one coloured image.

Keywords: Image compression, HOTELLING, KL transform, database compression, PCA.

2. PCA method

PCA is a statistical tool which has many applications especially in database processing [1],[2]. Suppose there are M square $N \times P$ monochrome images. This obligation doesn't make any restriction for colored images, since colored images can be supposed as 3 monochrome images that are red, green, and blue color components for each individual pixel. $N \times P$ monochrome images are equivalent to $N \times P$ matrixes that the values of the components of the matrixes are the light intensities of the corresponding pixel's location. Suppose $N \times P = Q$. By reshaping the matrixes, the image can be expressed as $1 \times Q$ vectors $\underline{F}_i$ in equation 1. In proposed approach, images are transferred to another field. All images are put in X matrix that its elements are the intensity values of images.

$$X = \begin{bmatrix} \underline{F}_1 \\ \vdots \\ \underline{F}_M \end{bmatrix}_{M \times Q}, \underline{F}_i = \left(x_{i1}, x_{i2}, \dots, x_{iQ}\right)_{1 \times Q} \qquad \text{Eq. 1}$$

The term $\underline{F}_i$ indicates the i^{th} image that converted to a vector. Now, in order to applying PCA method, we make some definitions;

The mean vector, $\underline{\overline{M}}_x$: that contains mean values of each image and expressed as:

$$\underline{\overline{M}}_x = \frac{1}{Q} \begin{bmatrix} \sum_{k=1}^{Q} x_{1k} \\ \sum_{k=1}^{Q} x_{2k} \\ \cdot \\ \cdot \\ \sum_{k=1}^{Q} x_{Mk} \end{bmatrix}_{M \times 1} = \begin{bmatrix} m_1 \\ m_2 \\ \cdot \\ \cdot \\ \cdot \\ m_M \end{bmatrix} \qquad \text{Eq. 2}$$

$\widetilde{M}_x$ matrix, that contains the values of $\underline{\overline{M}}_x$ for M times and expressed as:

$$\widetilde{M}_x = [\underline{\overline{M}}_x, \underline{\overline{M}}_x, \dots, \underline{\overline{M}}_x]_{M \times Q} \qquad \text{Eq. 3}$$

Covariance matrix C_x for M row of X matrix[1]:

$$C_x = \left[c_{i,j}\right]_{M \times M}$$

That: $c_{i,j} = \frac{1}{Q-1} \times \sum_{k=1}^{Q}\left[(x_{ik} - \underline{\overline{M}}_x(i,1)) \times (x_{jk} - \underline{\overline{M}}_x(j,1))\right]$ Eq. 4

For applying KL transform, M eigenvectors $\underline{v}_i, i = 1,2,\dots,M$ and M eigenvalues $\lambda_i, i = 1,2,\dots,M$ can be found, which satisfy equation 5:

$$\forall i \in \{1,2,\dots,M\} \quad C_x.\underline{v}_i = \lambda_i.\underline{v}_i$$

$$\underline{v}_i = \begin{bmatrix} v_1(i) \\ v_2(i) \\ \vdots \\ v_M(i) \end{bmatrix} \qquad \text{Eq. 5}$$

If we put all eigenvectors in a matrix, the modal matrix "Λ" will be obtained that its columns are the eigenvectors of C_x as shown below[2]:

$$\Lambda = [\underline{v}_1, \underline{v}_2, \dots, \underline{v}_M] \qquad \text{Eq. 6}$$

Now we can define V matrix:

$$V = \Lambda^{-1} \qquad \text{Eq. 7}$$

Λ is a unitary matrix. So:

$$\Lambda^{-1} = \Lambda^T \Rightarrow V = \Lambda^T \qquad \text{Eq. 8}$$

$$\Rightarrow V = [\underline{v}_1, \underline{v}_2, \dots, \underline{v}_M]^T{}_{M\times M} \qquad \text{Eq. 9}$$

So:

$$V = \begin{bmatrix} v_1(1) & \cdots & v_M(1) \\ \vdots & \ddots & \vdots \\ v_1(M) & \cdots & v_M(M) \end{bmatrix}_{M\times M} \qquad \text{Eq.10}$$

Applying KL transform makes Y matrix:

$$Y = V.\left(X - \widetilde{M}_x\right) \qquad \text{Eq. 11}$$

Now we can obtain X with the inverse process of equation 11:

$$Y = V.\left(X - \widetilde{M}_x\right) \Rightarrow V^{-1}.Y = X - \widetilde{M}_x$$
$$\Rightarrow X = V^{-1}.Y + \widetilde{M}_x \qquad \text{Eq. 12}$$

According to equation 8 we can write:

$$V^{-1} = (\Lambda^T)^{-1} = (\Lambda^{-1})^{-1} = \Lambda = V^T \qquad \text{Eq. 13}$$

Now, X matrix can be retrieved using equation 14:

$$X = V^T.Y + \widetilde{M}_x \qquad \text{Eq. 14}$$

In order to compress X some definitions are made as below[3]:

$$V_k \triangleq [v_1, v_2, \dots, v_k]^T{}_{M\times k}, \ k \in \{1,2,\dots,M\} \qquad \text{Eq. 15}$$

$$Y_k \triangleq \begin{bmatrix} Y(1,1) & \cdots & Y(1,Q) \\ \vdots & \ddots & \vdots \\ Y(k,1) & \cdots & Y(k,Q) \end{bmatrix}_{k\times Q} \qquad \text{Eq.16}$$

$$\widehat{X} \triangleq V_k{}^T.Y_k + \widetilde{M}_x \qquad \text{Eq. 17}$$

Considering the first k Eigen vectors from the M Eigen vectors (K<M), the $\widehat{X}$ matrix that is slightly different from X matrix, can be retrieved. The values of $\widehat{X}$ are near to values of X.

The $\hat{X}$ matrix is compressed matrix which obtained from X matrix. So the compression ratio - the ratio of the required memory to save $\hat{X}$ to the required memory to save X - can be calculated as equation 18 shows:

$$Memory\ ratio = \frac{required\ memory\ to\ save\ \hat{X}}{required\ memory\ to\ save\ X}$$
$$\triangleq \frac{\text{mem 1}}{\text{mem 2}} \qquad \text{Eq. 18}$$

- mem1 is the required memory to save $V_k{}^T, Y_k, \widetilde{M}_x$.
- mem2 is the required memory to save X.

$$\Rightarrow \text{Memory ratio} = \frac{kM + kQ + M}{MQ} \qquad \text{Eq. 19}$$

3. PCA method for compressing one image

Commonly, PCA method is applied to group of images as we done for M images. Now we shall apply PCA to an image, so the image -which is generally a coloured image-, is divided to some rows. This division is based on variations in the image. Each row or section acts as an input for ordinary PCA and if the rows have high similarity, the HOTELLING transform will show better results. We should find an algorithm to sense the variations in rows and divide the image in rows that have the maximum variation in respect of the next row. After applying some statistical parameters such as variance, covariance, correlation and etc. the best results reached when covariance values between the rows of image matrix have been used. Suppose a MbyN image, since covariance of two random variables, such as a and b is defined as:

$$\text{cov}(a, b) = E\left[\left(a - \mu_a\right)\left(b - \mu_b\right)\right] \qquad \text{Eq. 20}$$

in equation 20:

- $\mu_a = E[a]$
- $\mu_b = E[b]$

Now, we define a CRITERION as below:

$$\text{similarity}(i) \triangleq \text{cov}(i'\text{th row}, (i+1)'\text{th row})$$

$$i \epsilon \{1,2,\dots,N-1\} \quad \text{(for a MbyN image)} \qquad \text{Eq. 21}$$

So, let two neighbour rows of image matrix as a and b, as figure 1 shows and calculate the similarity values. As it seen, for a MbyN image, there are $N-1$ adjacent rows so $N-1$ similarity values will be calculated.

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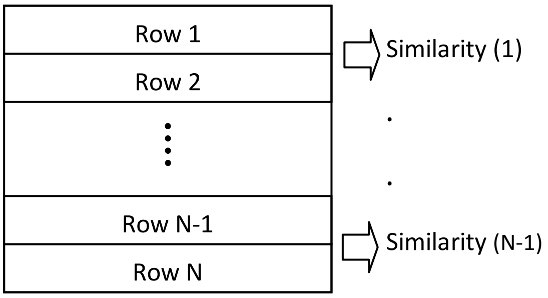


Figure 1. The rows of a MbyN image, and the similarity values.

In order to show the efficiency of mentioned algorithm to divide image into some rows, figure 2 shows an image and the curve of similarity values between its rows. There is a punctual relationship between the curve and the image. The large variations in the rows of the image that have most variations rather than their neighbours is equivalent to the sharp changes in the plot of the similarity values. This points of the curve has been marked with small red squares in figure 2 and the equivalent rows of image has been shown with blue lines in Figure 2 .

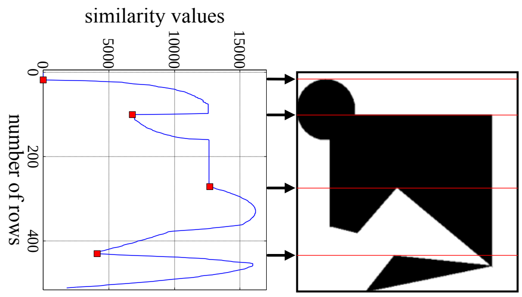


Figure 2. A simple test image and the similarity value curve of the image.

Mentioned algorithm has been applied to the Lena image. The similarity values of this image are shown in figure 3. As it seen, the similarity curve is noisy and has many points that have sharp increases. The algorithm of finding the sharp increases can find too many points which are more than needed and this can cause the algorithm to not work properly. In order to cancel the noise, the similarity curve has been crossed from a Median Filter with length 10. The filtered curve is shown in figure 4.

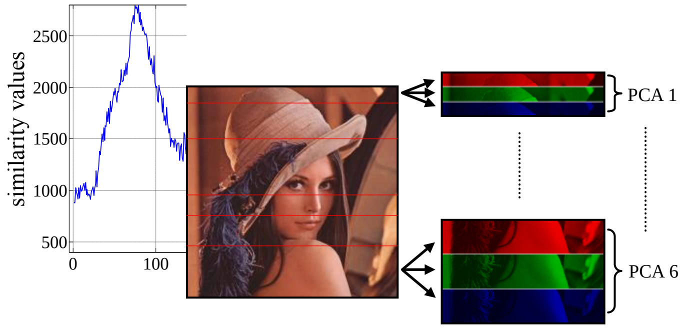


Figure 3. The similarity values for 512by512 image of Lena.

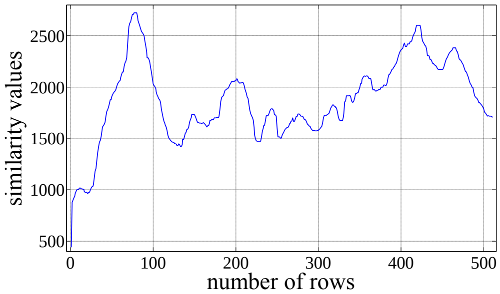
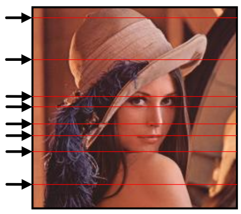


Figure 4. The filtered curve of the similarity values for 512by512 image of Lena.

The relationship between the points that have been obtained with mentioned algorithm and the efficiency of the algorithm is shown in figure 5. As



it seen, the rows that have most variations respect to their neighbors have been chosen correctly and reasonably.

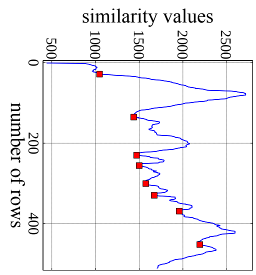


Figure 5. Lena and the similarity values and the result of applying mentioned algorithm in order to split image to some bands.

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Now, the HOTELLING transform can be applied to these bands. Inputs for PCA are the RGB values of generated bands. Figure 6, shows the steps of applying PCA to these bands. As shown, the Lena image has been partitioned to 6 bands.

Figure 6. The process of applying PCA on the bands of Lena

Note that, because the number of bands is 6, so the PCA transform will be applied for 6 times and since these transforms are independent each other, so the parallel processing can be applied to decrease the required time.

By joining 3 bands of red, green and blue components together, the X matrix is generated for PCA. According to equation 16, as k increases the difference between X and $\hat{X}$ will be decreased. The amount of k to participate in the procedure is based on minimum required SSIM value.

The SSIM metric is calculated on various windows of an image. The measure between two windows x and y of common size N×N is:

$$\mathrm{SSIM}(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(2\mu_x^2\mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad \text{Eq. 22}$$

With:

- μ_x the average of x;
- μ_y the average of y;
- σ_x^2 the variance of x;
- σ_y^2 the variance of y;
- σ_{xy} the covariance of x and y;
- $c_1 = (k_1L)^2$, $c_2 = (k_2L)^2$ two variables to stabilize the division with weak denominator;
- L the dynamic range of the pixel-values (typically this is $2^{\#\, bits\ per\ pixell} - 1$)
- $k_1 = 0.01$ and $k_2 = 0.03$ by default[8].

For SSIM, value 1 is only reachable in the case of two identical sets of data. In other words the SSIM value of two identical images will be 1.

After production of Y and V matrixes, let $\mathrm{k} = \mathrm{k}_0$. k_0 is an initial value that depends on the minimum required SSIM value. Now $\hat{X}$ can be produced with $\mathrm{k} = \mathrm{k}_0$. Then reconstructed bands are compared with original ones. If the SSIM for these bands is less than the required value, let k=k+1.

In Figure 7, the process of applying PCA and decision to how many eigenvectors used to produce $\hat{X}$ has been showed.

Not that t_0 is the minimum required value for SSIM.

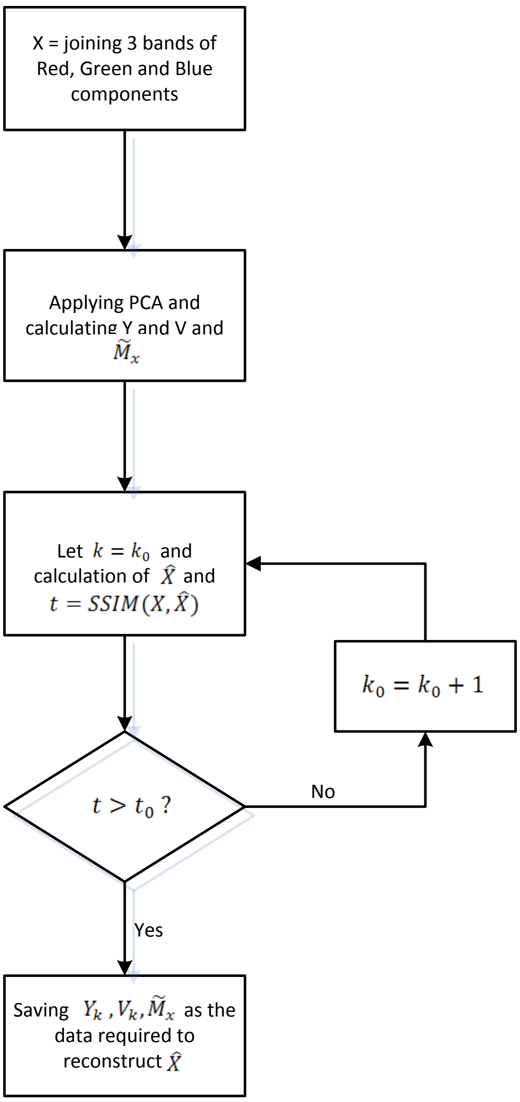

Figure 7. The diagram of applying PCA to the bands of the colored image.

4. Simulation results

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In addition to Lena image, for more simulations, mentioned algorithm is applied to two other famous images, Manfishing and airplane.

Figure 8. Manfishing image

Figure 9. Airplane image.

The result of applying the mentioned algorithm to split image of Lena to some bands has been shown in figure 5. In figures 10 and 11, the results of the algorithm for manfishing and airplane images have been shown:

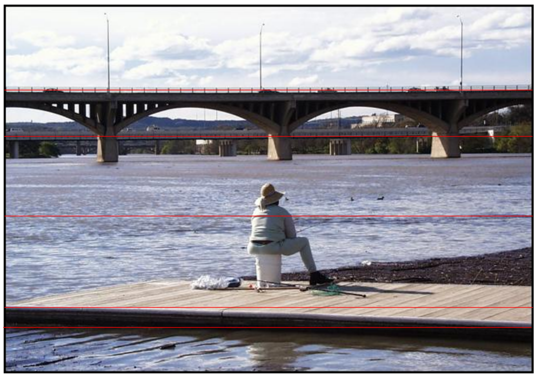

Figure 10. The result of applying the algorithm of splitting images on manfishing image.

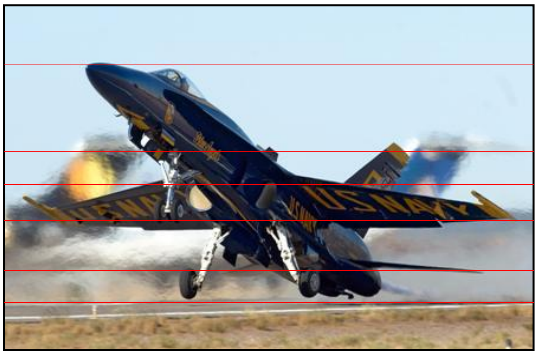

Figure 11. The result of applying the algorithm of splitting images on airplane image.

SSIM values and memory ratios for Green, Red and Blue components of these images are as follows:

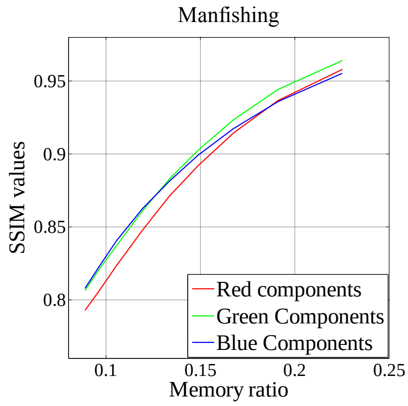

Figure 12. Simulation results of mentioned method for manfishing image.

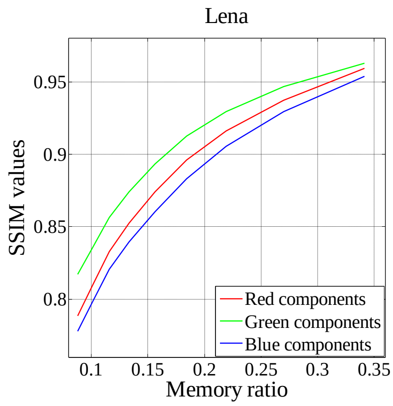

Figure 13. Simulation results of mentioned method for Lena image.

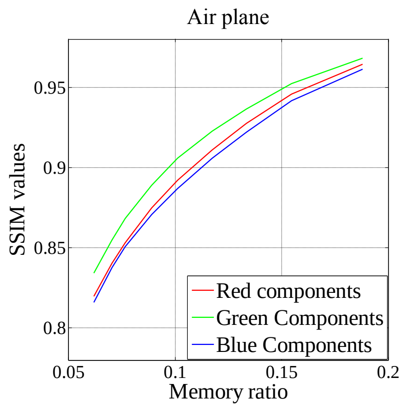

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Figure 14. Simulation results of mentioned method for airplane image.

A. Dwivedi and A. Tolambiya have introduced a method that uses 2-Dimentional PCA for compressing color images. Table 1 shows the results for 2DPCA in comparison with mentioned method in this paper. As it seen, the PSNR - Peak Signal to Noise Ratio - values of mentioned method shows better results than 2DPCA[4],[5],[6].

Table 1. The PSNR values for mentioned method and 2-Dimentional method

Memory ratio	PSNR (Mentioned method)	PSNR (2-DPCA)
0.0996	29.6820	20.0006
0.1432	31.8028	24.1034
0.1660	32.7964	24.7595
0.1897	33.4023	25.6243
0.2371	35.0669	26.8820
0.3174	37.6156	28.6876

$$MSE = \frac{1}{MQ} \sum_{i=1}^{M} \sum_{j=1}^{Q} [X(i,j) - \hat{X}(i,j)] \quad \text{Eq. 23}$$

$$PSNR = 20 \log\left(\frac{255}{\sqrt{MSE}}\right) \quad \text{Eq. 24}$$

Figure 15. The compressed image of Lena, SSIM =0.7945, PSNR= 28.9760, Memory Ratio=0.0878

5. Conclusion

The results of mentioned method show that using PCA with the mentioned algorithm can be a suitable choice to compress colored images. The speed of the algorithm can be increased with parallel programming. Because the compression process of the various bands of the image are independent with each other. As it seen, the mentioned method has better results toward 2D-PCA. The mentioned method can applied for images and a new format for images can be produced. One other advantage of the mentioned method is the short time of reconstruction of compressed images in comparison with other formats such as JPEG.

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