CS 2050 Homework 2

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Section B1
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1.2 #20

A says The two of us are both knights.
B says A is a knave.
p: A is knight
q: B is a knight

$(p \to (p \land q)) \land (\neg p \to \neg(p \land q)) \land (q \to \neg p) \land (\neg q \to \neg\neg p)$	given
$(p \to (p \land q)) \land (\neg p \to (\neg p \lor \neg q)) \land (q \to \neg p) \land (\neg q \to \neg\neg p)$	demorgans
$(p \to (p \land q)) \land (\neg p \to (\neg p \lor \neg q)) \land (q \to \neg p) \land (\neg q \to p)$	double negation
$(\neg p \lor (p \land q)) \land (\neg p \to (\neg p \lor \neg q)) \land (q \to \neg p) \land (\neg q \to p)$	material implication
$(\neg p \lor (p \land q)) \land (\neg\neg p \lor (\neg p \lor \neg q)) \land (q \to \neg p) \land (\neg q \to p)$	material implication
$(\neg p \lor (p \land q)) \land (p \lor (\neg p \lor \neg q)) \land (q \to \neg p) \land (\neg q \to p)$	double negation
$(\neg p \lor (p \land q)) \land (p \lor (\neg p \lor \neg q)) \land (\neg q \lor \neg p) \land (\neg q \to p)$	material implication
$(\neg p \lor (p \land q)) \land (p \lor (\neg p \lor \neg q)) \land (\neg q \lor \neg p) \land (\neg\neg q \lor p)$	material implication
$(\neg p \lor (p \land q)) \land (p \lor (\neg p \lor \neg q)) \land (\neg q \lor \neg p) \land (q \lor p)$	double negation
$(\neg p \lor (p \land q)) \land ((p \lor \neg p) \lor \neg q) \land (\neg q \lor \neg p) \land (q \lor p)$	associative
$(\neg p \lor (p \land q)) \land (T \lor \neg q) \land (\neg q \lor \neg p) \land (q \lor p)$	negation law
$(\neg p \lor (p \land q)) \land (T) \land (\neg q \lor \neg p) \land (q \lor p)$	domination
$(\neg p \lor (p \land q)) \land (\neg q \lor \neg p) \land (q \lor p)$	identity
$((\neg p \lor p)\land (\neg p \lor q)) \land (\neg q \lor \neg p) \land (q \lor p)$	distributive
$((p \lor \neg p)\land (\neg p \lor q)) \land (\neg q \lor \neg p) \land (q \lor p)$	commutative
$((T)\land (\neg p \lor q)) \land (\neg q \lor \neg p) \land (q \lor p)$	domination
$((\neg p \lor q)\land (T)) \land (\neg q \lor \neg p) \land (q \lor p)$	commutative
$(\neg p \lor q) \land (\neg q \lor \neg p) \land (q \lor p)$	identity
$(\neg p \lor q) \land (\neg p \lor \neg q) \land (q \lor p)$	commutiative
$(\neg p \land (q \lor \neg q)) \land (q \lor p)$	distributive
$(\neg p \land T) \land (q \lor p)$	negation law
$\neg p \land (q \lor p)$	identity
$(\neg p \land q) \lor(\neg p \land p)$	distributive
$(\neg p \land q) \lor(p \land \neg p)$	commutative
$(\neg p \land q) \lor F$	negation law
$\neg p \land q$	identity

LADBAP - A is knave B is knight

1.2 #22

A says A is a knight
B says B is a knight
p: A is a knight
q: B is a knight

$(p \to p) \wedge (\neg p \to \neg p) \wedge (q \to q) \wedge (\neg q \to \neg q)$	given
$(\neg p \vee p) \wedge (\neg p \to \neg p) \wedge (q \to q) \wedge (\neg q \to \neg q)$	material implication
$(\neg p \vee p) \wedge (\neg\neg p \vee \neg p) \wedge (q \to q) \wedge (\neg q \to \neg q)$	material implication
$(\neg p \vee p) \wedge (\neg\neg p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \to \neg q)$	material implication
$(\neg p \vee p) \wedge (\neg\neg p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg\neg q \vee \neg q)$	material implication
$(\neg p \vee p) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg\neg q \vee \neg q)$	double negation
$(\neg p \vee p) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (q \vee \neg q)$	double negation
$(p \vee \neg p) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (q \vee \neg q)$	commutative
$(p \vee \neg p) \wedge (p \vee \neg p) \wedge (q \vee \neg q) \wedge (q \vee \neg q)$	commutative
$T \wedge (p \vee \neg p) \wedge (q \vee \neg q) \wedge (q \vee \neg q)$	negation law
$T \wedge T \wedge (q \vee \neg q) \wedge (q \vee \neg q)$	negation law
$T \wedge T \wedge T \wedge (q \vee \neg q)$	negation law
$T \wedge T \wedge T \wedge T$	negation law
$T \wedge T \wedge T$	idempotent
$T \wedge T$	idempotent
T	idempotent

LADBAP - We don't know anything about either of them. Their being a knight or a knave does not change the truth value

1.3 #4

a.

p	q	r	(p ∨ q)	(p ∨ q) ∨ r	(q ∨ r)	p ∨ (q ∨ r)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

The columns for $(p \vee q) \vee r$ and $p \vee (q \vee r)$ match exactly and so are equivalent

1.3 #8

a. Kwame will take a job in industry or go to graduate school
i: Kwame will take a job in industry
g: Kwame will go to graduate school
$\neg(i \vee g)$
$\neg i \wedge \neg g$
Kwame will not take a job in industry and he will not go to graduate school

b. Yoshiko knows Java and calculus
j: Yoshiko knows Java
c: Yoshiko knows calculus
$\neg(j \wedge c)$
$\neg j \vee \neg c$
Yoshiko does not know either Java or calculus or both.

c. James is young and strong
y: James is young
s: James is strong
$\neg(y \wedge s)$
$\neg y \vee \neg s$
James is not young or not strong or not both

d. Rita will move to Oregon or Washington
o: Rita will move to Oregon

w: Rita will move to Washington
$\neg(o \lor w)$
$\neg o \land \neg w$
Rita will not move to Oregon and Washington

1.3 #10

Tautology by truth tables
$[p \land (p \to q)] \to q$

p	q	$p \to q$	$p \land (p \to q)$	$[p \land (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$

p	q	r	$(p \lor q)$	$(p \to r)$	$(q \to r)$	$(p \lor q) \land (p \to r)$	$(p \lor q) \land (p \to r) \land (q \to r)$	$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

1.3 #12

Prove tautology by logical equivalence
$[p \land (p \to q)] \to q$

$[p \land (p \to q)] \to q$	given
$[p \land (\neg p \lor q)] \to q$	material implication
$\neg[p \land (\neg p \lor q)] \lor q$	material implication
$[\neg p \lor \neg(\neg p \lor q)] \lor q$	demorgans
$[\neg p \lor (\neg\neg p \land \neg q)] \lor q$	demorgans
$[\neg p \lor (p \land \neg q)] \lor q$	double negation
$[(\neg p \lor p) \land (\neg p \lor \neg q)] \lor q$	distributive
$[(T) \land (\neg p \lor \neg q)] \lor q$	negation law
$[(\neg p \lor \neg q) \land T] \lor q$	commutative
$[\neg p \lor \neg q] \lor q$	identity
$\neg p \lor [\neg q \lor q]$	associative
$\neg p \lor T$	negation law
T	domination

LADBAP - This thing even looks a little like an arrow looking down. That is how BA this P is. True means that this is a tautology

$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$	given
$[(p \vee q) \wedge (\neg p \vee r) \wedge (q \to r)] \to r$	material implication
$[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \to r$	material implication
$\neg[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \vee r$	material implication
$[\neg(p \vee q) \vee \neg(\neg p \vee r) \vee \neg(\neg q \vee r)] \vee r$	demorgans
$[(\neg p \wedge \neg q) \vee (\neg\neg p \wedge \neg r) \vee \neg(\neg q \vee r)] \vee r$	demorgans
$[(\neg p \wedge \neg q) \vee (\neg\neg p \wedge \neg r) \vee (\neg\neg q \wedge \neg r)] \vee r$	demorgans
$[(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg\neg q \wedge \neg r)] \vee r$	double negation
$[(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r)] \vee r$	double negation
$[(\neg p \wedge \neg q) \vee (\neg r \wedge p) \vee (q \wedge \neg r)] \vee r$	commutative
$[(\neg p \wedge \neg q) \vee (\neg r \wedge p) \vee (\neg r \wedge q)] \vee r$	commutative
$[(\neg p \wedge \neg q) \vee (\neg r \wedge (p \vee q))] \vee r$	distributive
$r \vee [(\neg p \wedge \neg q) \vee (\neg r \wedge (p \vee q))]$	commutative
$[r \vee (\neg p \wedge \neg q)] \vee (\neg r \wedge (p \vee q))$	associative
$[r \vee (\neg p \wedge \neg q)] \vee \neg[r \vee \neg(p \vee q)]$	demorgans
$[r \vee (\neg p \wedge \neg q)] \vee \neg[r \vee (\neg p \wedge \neg q)]$	demorgans
T	negation law

LADBAP - in the second to last line the things in the brackets were the same. This allowed me to get a true using the negation law which means that the statement is tautologous.

1.3 #14

prove that $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology

$(\neg p \wedge (p \to q)) \to \neg q$	given
$(\neg p \wedge (\neg p \vee q)) \to \neg q$	material implication
$\neg(\neg p \wedge (\neg p \vee q)) \vee \neg q$	material implication
$(\neg\neg p \vee \neg(\neg p \vee q)) \vee \neg q$	demorgans
$(\neg\neg p \vee (\neg\neg p \wedge q)) \vee \neg q$	demorgans
$(p \vee (\neg\neg p \wedge q)) \vee \neg q$	double negation
$(p \vee (p \wedge q)) \vee \neg q$	double negation
$((p \vee p) \wedge (p \vee q)) \vee \neg q$	distributive
$((T) \wedge (p \vee q)) \vee \neg q$	idempotent
$((p \vee q) \wedge T) \vee \neg q$	commutative
$(p \vee q) \vee \neg q$	identity
$p \vee (q \vee \neg q)$	associative
$p \vee T$	negation
T	domination

LADBAP - ended up with true so tautology

1.3 #22

show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent

$(p \to q) \wedge (p \to r)$	given
$(\neg p \vee q) \wedge (p \to r)$	material implication
$(\neg p \vee q) \wedge (\neg p \vee r)$	material implication
$\neg p \vee (q \wedge r)$	distributive
$p \to (q \wedge r)$	material implication

LADBAP - the last statement matches the second statement in the question exactly so the two statements are logically equivalent

1.4 #6

Let N(x) be the statement x has visited North Dakota, where the domain consists of the students in your school.
a. There exists a student who has visited North Carolina
b. Every student has visited North Carolina.
e. Not all students have visited North Carolina
f. Every student has not visited North Carolina

1.4 #10

a. $\exists x(C(x) \wedge D(x) \wedge F(x))$

b. $\forall x(C(x) \vee D(x) \vee F(x))$

c. $\exists x(C(x) \wedge \neg D(x) \wedge F(x))$

d. $\forall x \neg(C(x) \wedge D(x) \wedge F(x))$

e. $\exists x C(x) \wedge \exists x D(x) \wedge \exists x F(x)$

1.4 #32

a. $\forall d F(d)$ d $\in$ all dogs and F(d): d has fleas
$\neg(\forall d F(d))$
$\exists d \neg F(d)$ using demorgans
There exists a dog that does not have fleas

b. $\exists h A(h)$ h $\in$ all horses and A(h): h can add
$\neg(\exists h A(h))$
$\forall h \neg A(h)$ using demorgans
All horses can not add

d. $\forall m \neg F(m)$ m $\in$ monkeys and F(m): m can speak French
$\neg(\forall m \neg F(m))$
$\exists m \neg\neg F(m))$ using demorgans
$\exists m F(m))$ using double negation
There exists a monkey that can speak french

1.4 #62

a. $\forall x(P(x) \rightarrow \neg S(x))$
b. $\forall x(R(x) \rightarrow S(x))$
c. $\forall x(Q(x) \rightarrow P(x))$
d. $\forall x(Q(x) \rightarrow \neg R(x))$
e.
$\neg P \vee \neg S$ - material implication from a
$\neg R \vee S$ - material implication from b
$\neg Q \vee P$ - material implication from c
$\neg P \vee \neg R$ - 1 and 2 resolution
$\neg Q \vee \neg R$ - 3 and 4 resolution

it matches the material implcation from d - ($\neg Q \vee \neg R$) so it is a correct conclusion