Tutorial 1 :-

1. List the ordered pairs in the relation R from $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a,b) \in R$ if and only if (a) $gcd(a,b) = 1$

a) $\{(1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)\}$

(b) $LCM(A,B) = 2 \; \{(2,1), (1,2), (2,2)\}$

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2) Determine ~~w~~ whether the relation R on the set ~~of~~ all real number is reflexive, symmetric, antisymmetric, and transitive, where $(x, y) \in R$ if only if

(a) $x + y = 0$

(i) Not Reflexive because two same ~~N~~ Number addition can't be 0.

(ii) Symmetric $x, y \in R$, $x + y = 0$ $y + x = 0$

(iii) Not transitive

(b) $x = \pm y$

Reflexive, symmetric, transitive.

(c) ~~x y~~ $R = \{(x, y) \mid x - y$ is a rational number $\}$

→ reflexive, symmetric, transitive.

(3) Let R and S be relation on a set A represented by matrix $M_R = \begin{bmatrix} 0 & 1 & \cancel{0} \\ \cancel{0} & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

and $M_S = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find the matrix

representing the following relations.

(a) $R \cup S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ ~~$(R \cup S) \Rightarrow \{(1,1), (1,2), (1,3)\}$~~

$\downarrow$ or

(b) $R \cap S = \begin{bmatrix} 0 & \cancel{1} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\downarrow$ And

~~(c) $R \circ S = R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$~~

$\times$

~~$R \circ S = \begin{bmatrix} 0 & 1 & \cancel{1} \\ 1 & 1 & 1 \\ 1 & \cancel{1} & 1 \end{bmatrix}$~~

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~~$S \circ R$~~ $= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

~~② S o R =~~

③ $S \circ R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$(0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0)$

$0 \vee (1) \vee 0$

$(0 \wedge 1)\ \vee\ 1$

④ $R \circ S$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

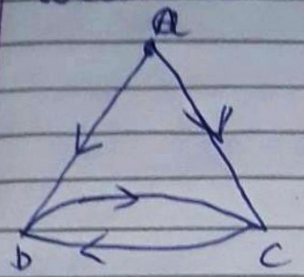
$R \circ S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

⑤ $R \oplus S$

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & \phi \end{bmatrix}$

Q. Write the relation represented by the following digraph and also write the matrix representing this relation.

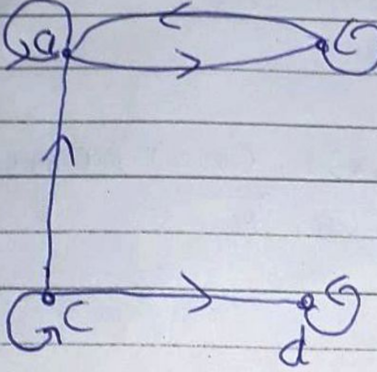
(a)



R = {(a, b), (a, c) (b, c), (c, b)}

$$mR = \begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & 1 \\ b & 0 & 0 & 1 \\ c & 0 & 1 & 0 \end{array}$$

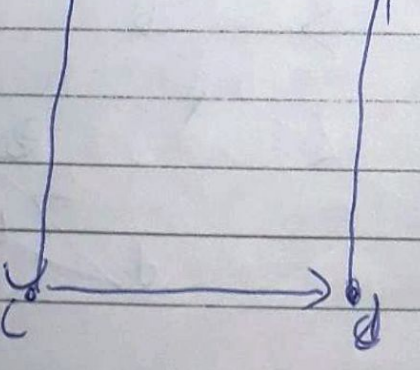
(b)



R = {(a, a) (b b), (c, c), (d, d), (a, b) (b, a), (c, a) (c, d)}

$$mR = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 0 \\ b & 1 & 1 & 0 & 0 \\ c & 1 & 0 & 1 & 1 \\ d & 0 & 0 & 0 & 1 \end{array}$$

(c)



~~{(b, a)}~~, (d, b), (c, d), (a, c)}

R = {(b, a), (d, b), (c, d), (a, c)}

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ a & 0 & 0 & 1 & \\ b & 1 & 0 & 0 & \\ c & 0 & 0 & & \\ d & & & & \end{array}$$

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ a & 0 & 0 & 1 & 0 \\ b & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 1 & 0 & 0 \end{array}$$

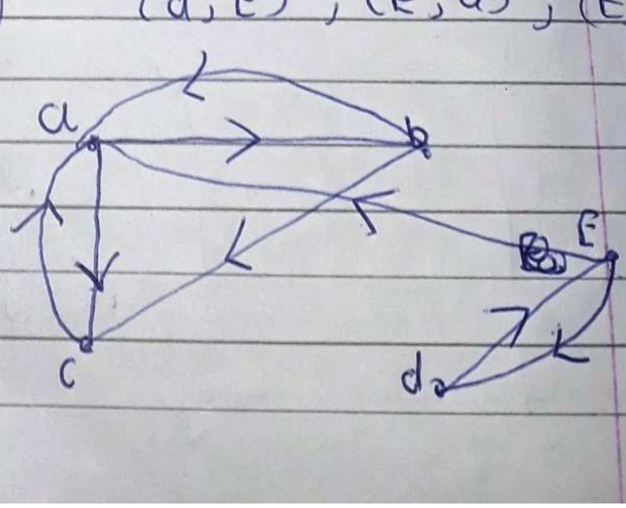
(5) write the relation represented by the following matrices and also draw the corresponding diagraph.

(a)
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

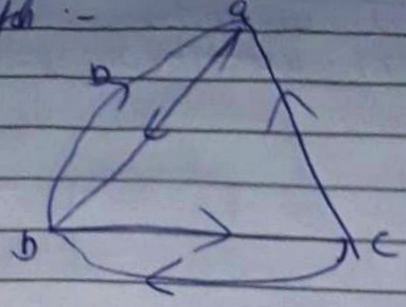
$R = \{(a,b), (b,a), (b,c), (c,a), (c,b)\}$.

(b)
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{(a,b), (a,c), (b,a)$
$(b,c), (c,a), (c,b),$
$(d,E), (E,a), (E,d)\}$



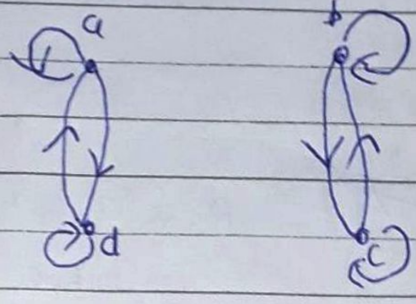
(a) di a graph :-



(c)

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 1 & 0 \\ d & 1 & 0 & 0 & 1 \end{array}$$

$R = \{(a,a), (a,d), (b,b), (b,c), (c,b), (c,c), (d,a), (d,d)\}$



6. check if the relations given by the following are reflexive, symmetric, antisymmetric, and/or transitive:-

(a) $R = \{(x,y) \mid x = 2y\}$

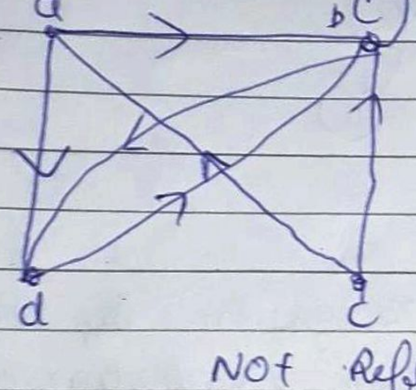
-> The relation R is not reflexive, because $x = 2x$ is only true when $x = 0$. and thus is not true for all real numbers

→ It is not symmetric and transitive.
It is antisymmetric because $x = 2y$, y
~~$x = 2y = 2(2z) = 4z$~~ tol $x = y = 0$.

(D) $M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 1 & 0 \\ d & 1 & 0 & 0 & 1 \end{array}$

$R = \{(a,a)\ (a,d), (b,b), (b,c),$
$(c,b), (c,c), (d,a)\ (d,d)\}$

Reflexive, symmetric, Transitive.

(C) 

$R = \{(a,b), (b,b),$
$(b,d), (d,b)\ (c,a)$
$(c,b)\},$
$(a,d)\}$

Not Reflexive, symmetric,
Anti symmetric, Transitive

~~(7)(a)~~

(7) (a) Airline → Nadir, Nadir, Nadir
Flight-number → 122, 199, 322
Gate → 34, 13, 34
Departure-time → 08:10, 08:47, 09:44

P.2.4.

(b)

Airline	Flight-Number	Destination
Nadir	122	Detroit
Acme	221	Denver
Acme	122	Anchorage
Acme	323	Honolulu
Nadir	199	Detroit
Acme	222	Denver
Nadir	322	Detroit.

(c) P.1.4.

Airline	Destination
Nadir	Detroit
Acme	Denver
Acme	Anchorage
Acme	Honolulu
Nadir	Detroit
Acme	Denver
Nadir	Detroit