

# SORTING ALGORITHM

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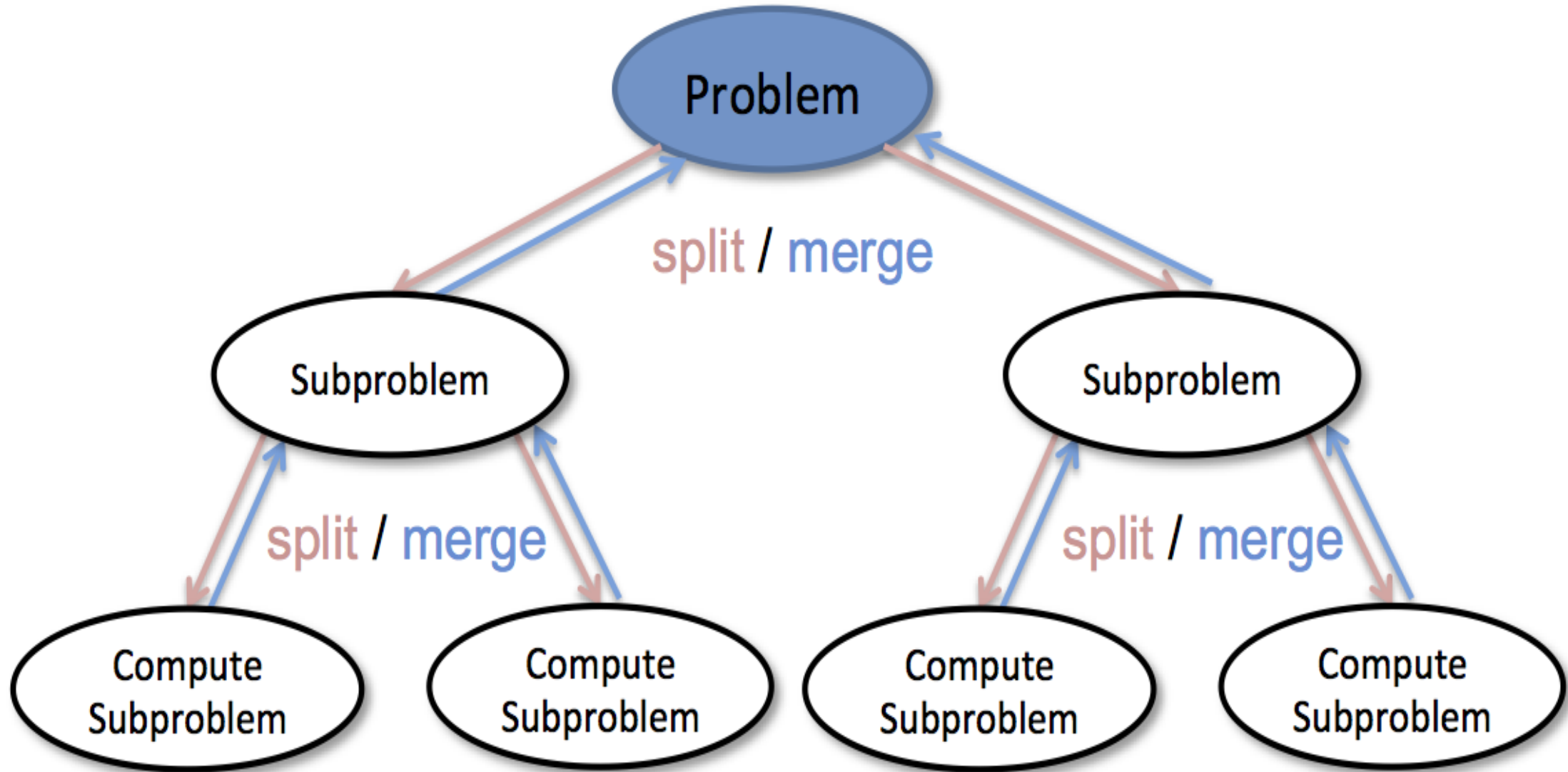
# **Sorting**

- **Sorting is a process that organizes a collection of data into either ascending or descending order.**
- **In-place sorting vs not In-place sorting.**
- **Quick sort and Merge sort use divide and conquer approach.**

# Why Sorting is important ?

- Contact numbers in the mobile phone
- List of exam scores
- Words of dictionary in alphabetical order
- Students names listed alphabetically
- Student records sorted by ID#
- **Searching**(enhance the performance of an algorithm)

# DIVIDE AND CONQUER



# Divide and Conquer

- **Basic Idea**
- Divide instance of problem into two or more smaller instances
- Solve smaller instances recursively
- Obtain solution to original (larger) instance by combining these solutions
- Divide and Conquer algorithms consist of two parts:
  - **Divide**: Smaller problems are solved recursively (except, of course, the base cases).
  - **Conquer**: The solution to the original problem is then formed from the solutions to the subproblems.

# QUICK SORT

- **Quicksort** (sometimes called **partition-exchange sort**) is an efficient sorting algorithm, serving as a systematic method for placing the elements of an array in order.
- Quicksort can operate in-place on an array, requiring small additional amounts of memory to perform the sorting.
- It is very similar to selection sort, except that it does not always choose worst-case partition.
- On average, the algorithm takes  $O(n \log n)$  comparisons to sort  $n$  items.
- In the worst case, it makes  $O(n^2)$  comparisons, though this behaviour is rare.

# Quicksort Procedure

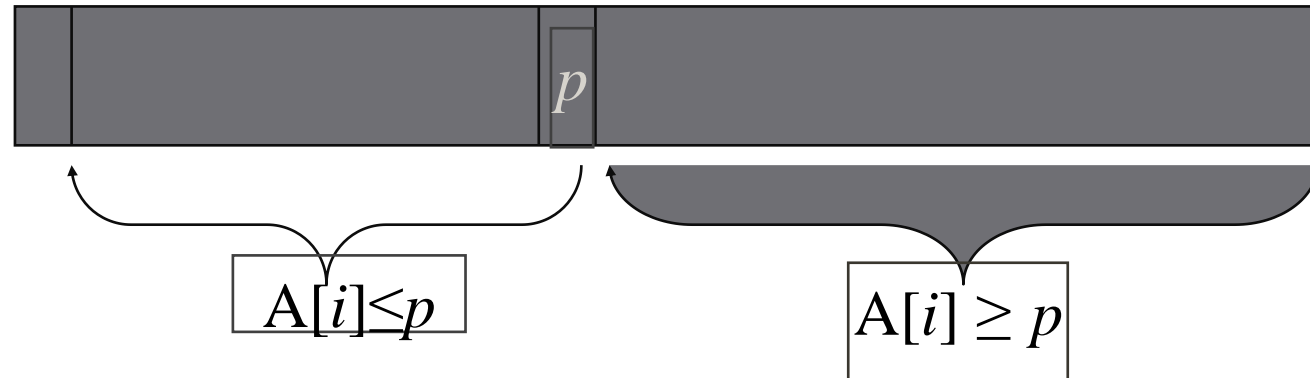
- Select a pivot which divide or partition the list in two part.
  - e.g., pivot =  $A[n-1]$  or  $A[0]$  or any random element

**Step 1** – Make the left-most index value pivot

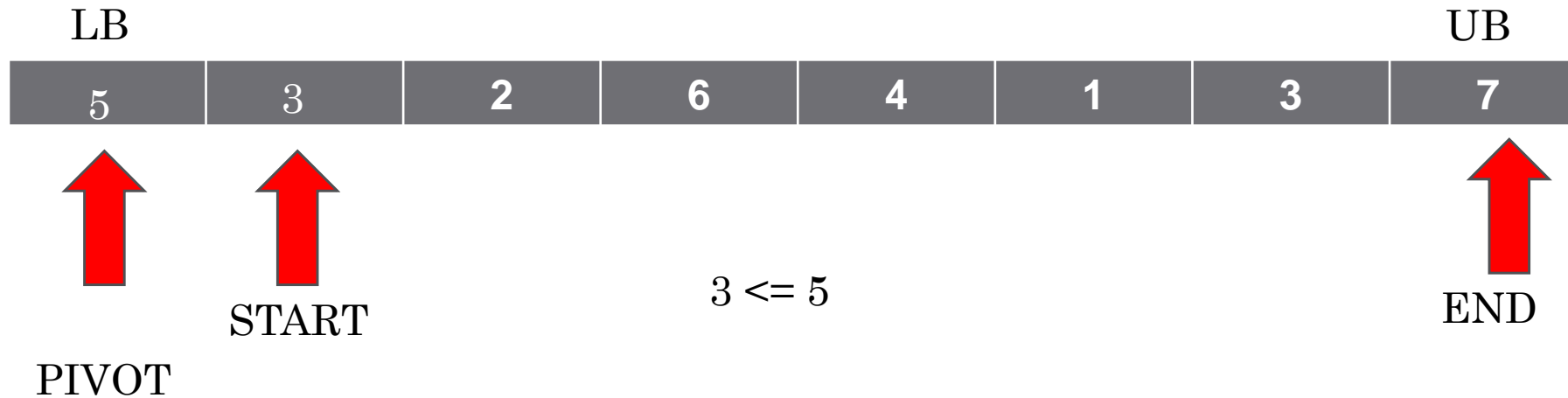
**Step 2** – partition the array using pivot value

**Step 3** – quicksort left partition recursively

**Step 4** – quicksort right partition recursively



# Example of partition

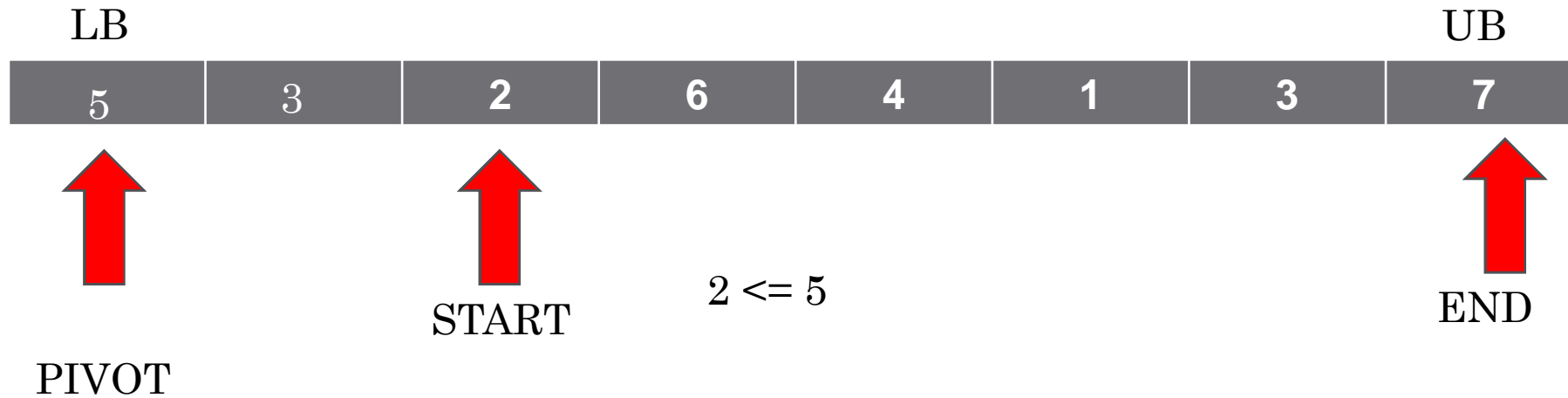


WHILE A[START] <= PIVOT

START++

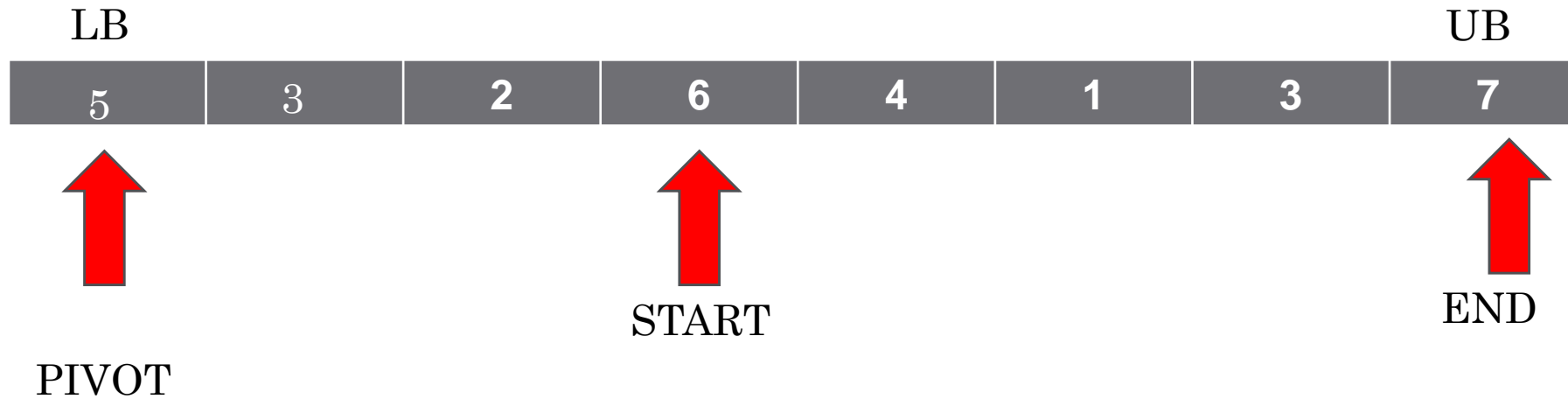
Initially START=LB+1 And  
END=UB





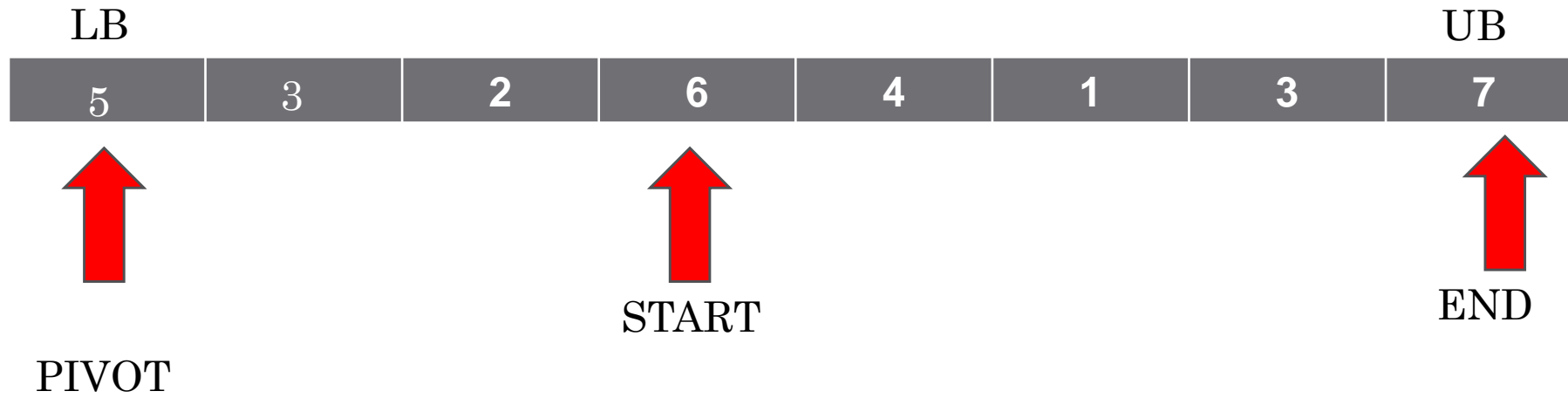
WHILE  $A[\text{START}] \leq \text{PIVOT}$

START++



$6 \leq 5$                       FALSE HENCE STOP

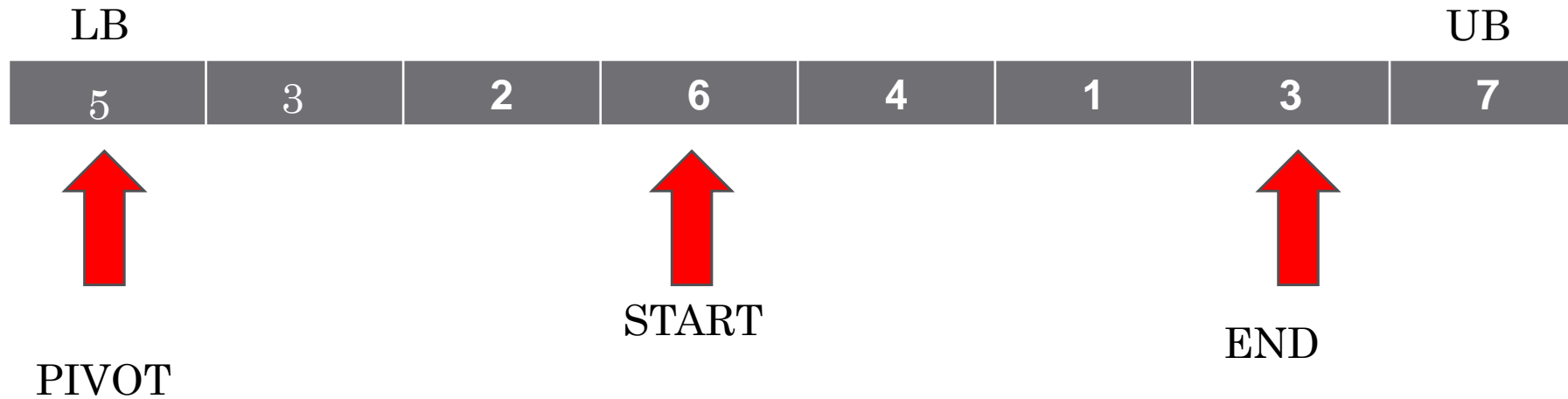
WHILE  $A[\text{START}] \leq \text{PIVOT}$



$7 > 5$

WHILE A[END] > PIVOT

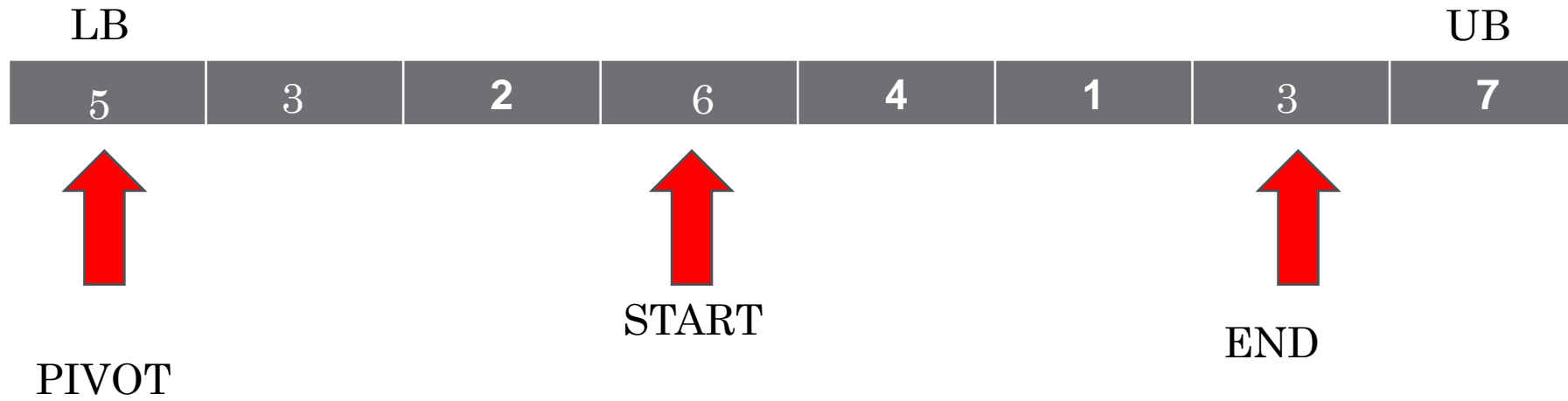
END--



$3 > 5$

FALSE HENCE STOP

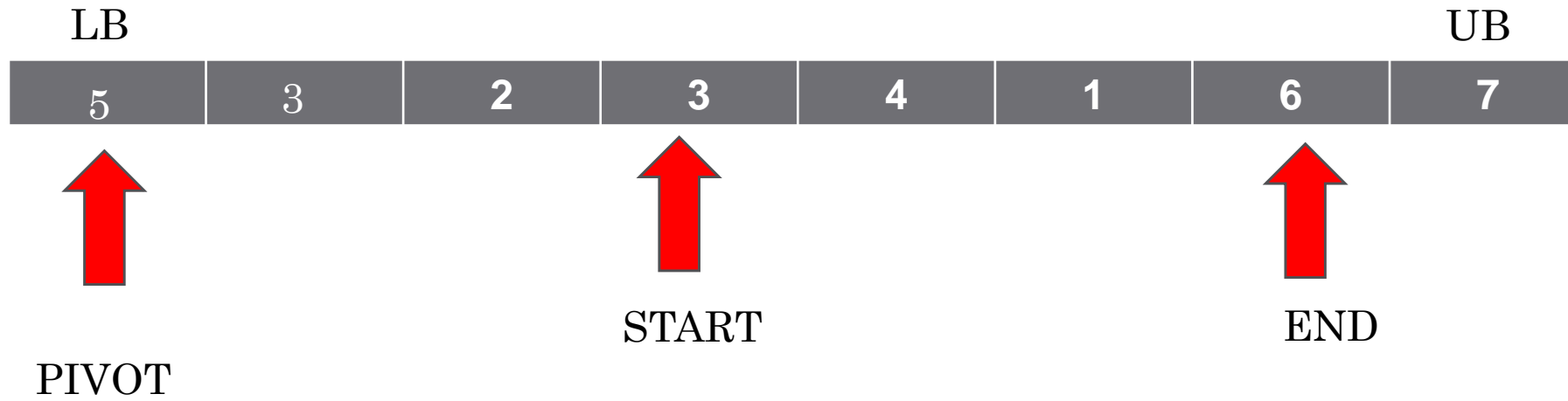
WHILE A[END] > PIVOT



$3 < 6$

IF  $START < END$

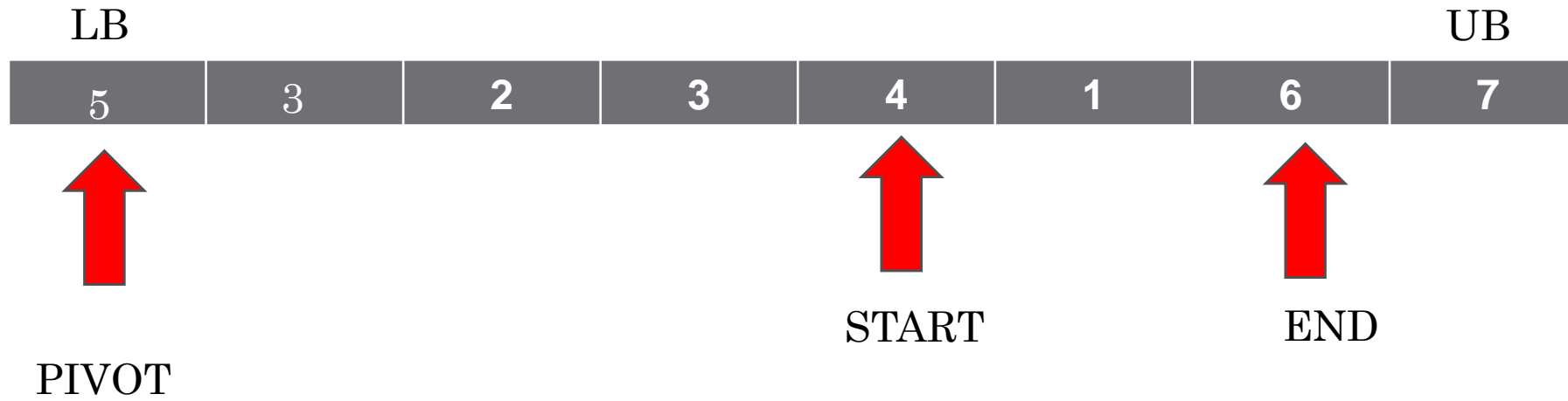
SWAP ( $A[START], A[END]$ )



$3 \leq 5$

WHILE  $A[\text{START}] \leq \text{PIVOT}$

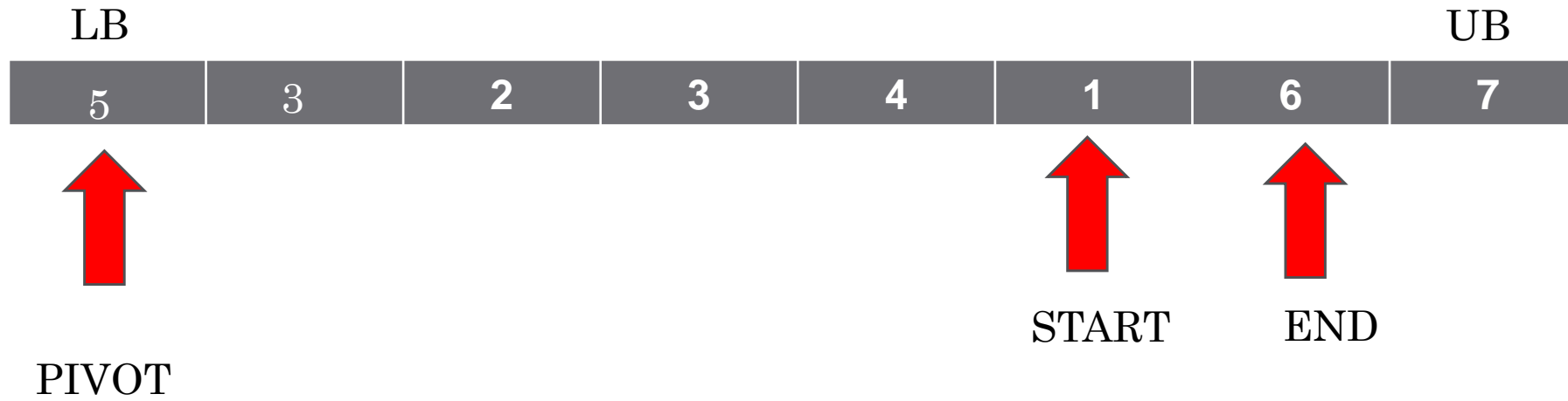
$\text{START}++$



$4 \leq 5$

WHILE  $A[\text{START}] \leq \text{PIVOT}$

$\text{START}++$

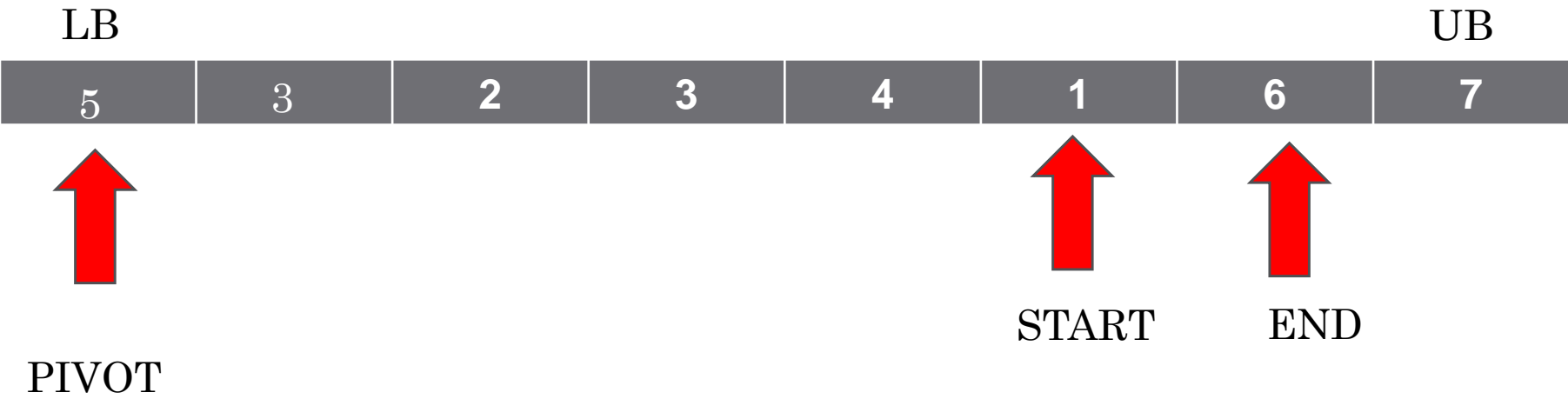


$1 \leq 5$

WHILE  $A[\text{START}] \leq \text{PIVOT}$

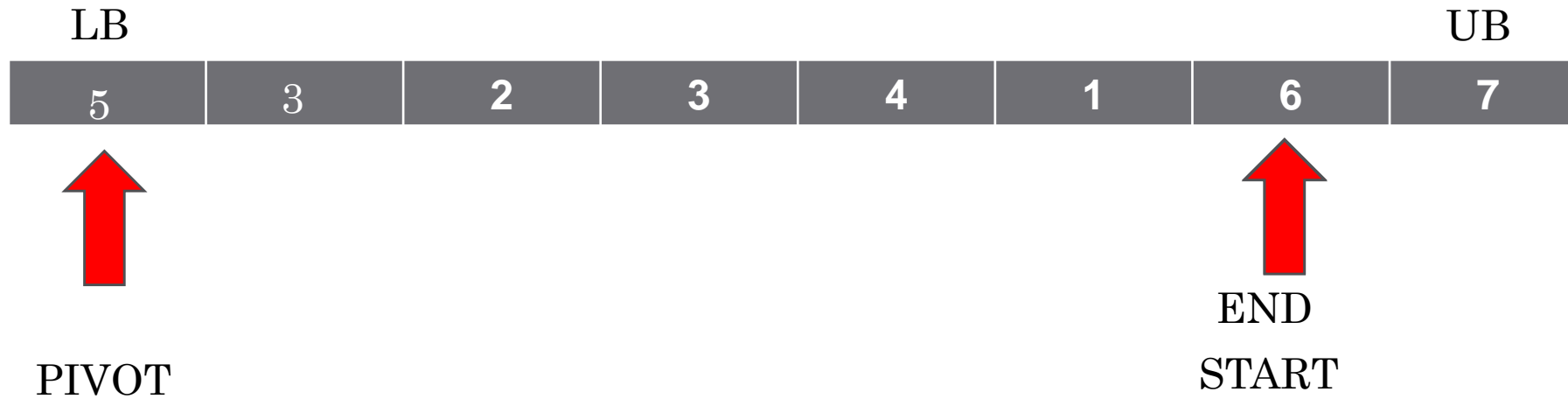
$\text{START}++$





6 <= 5            FALSE    HENCE STOP

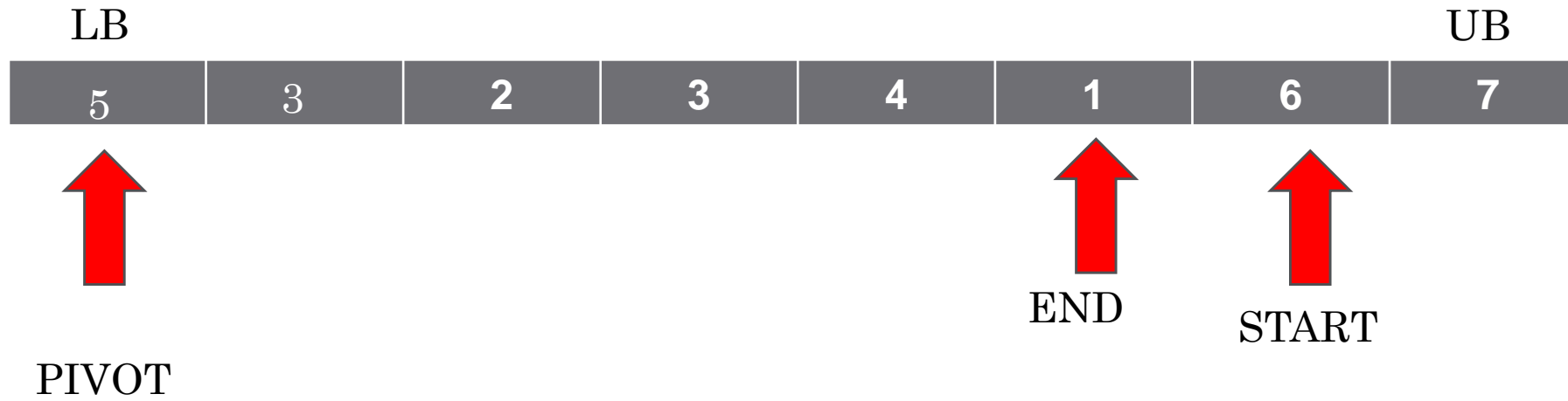
WHILE A[START] <= PIVOT



$$6 > 5$$

WHILE A[END] > PIVOT

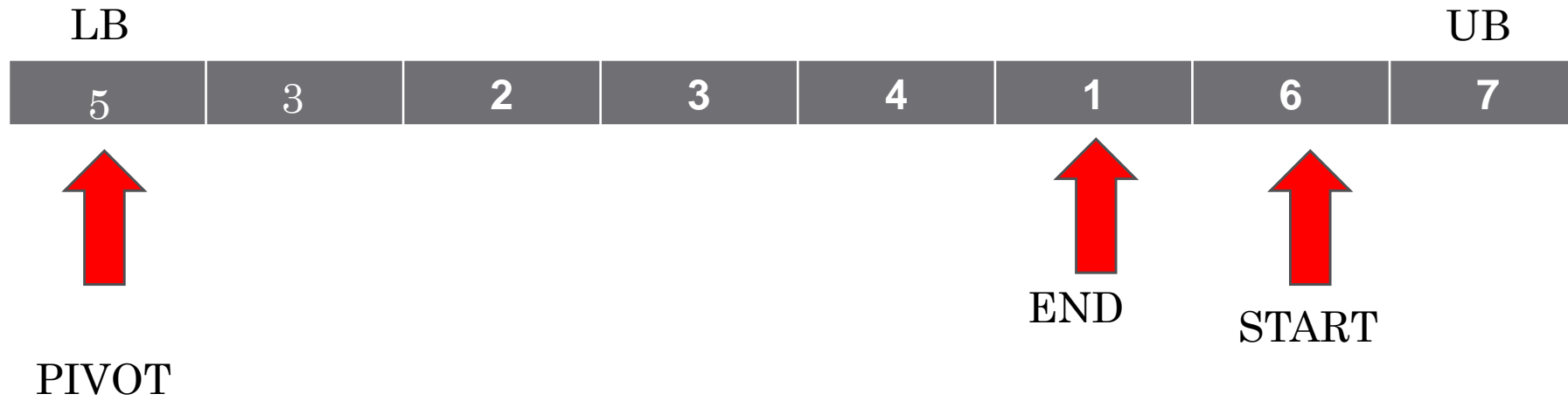
END--



$1 > 5$

FALSE HENCE STOP

WHILE  $A[END] > PIVOT$

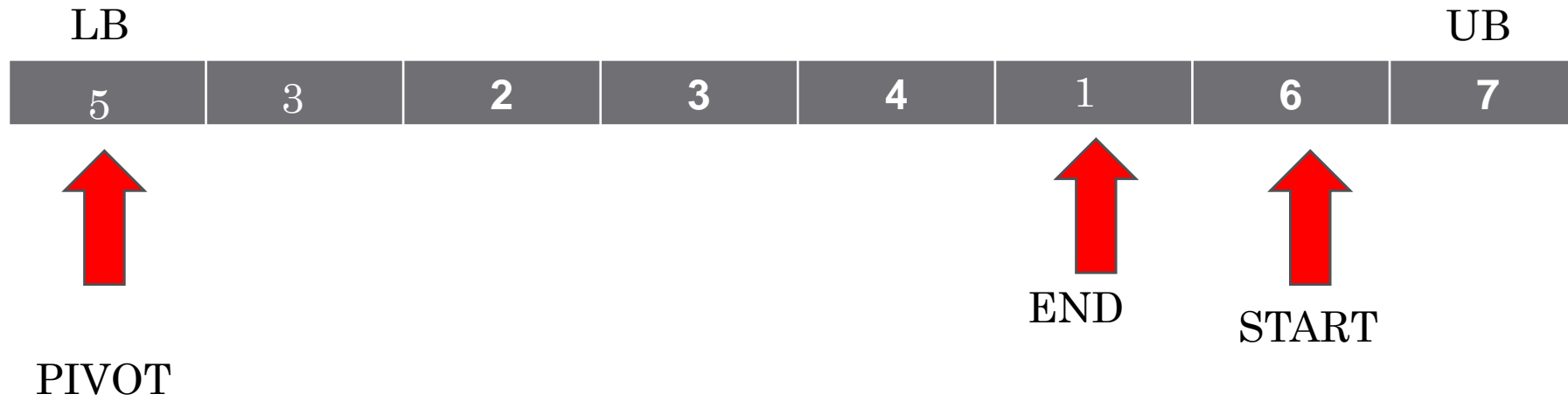


$$6 < 5$$

FALSE HENCE NO SWAP

IF  $START < END$

PIVOT'S  
CORRECT  
POSITION



NOW (SWAP  $A[LB]$  , $A[END]$ )

LB

1	3	2	3	4	5	6	7
---	---	---	---	---	---	---	---

UB

PARTITION

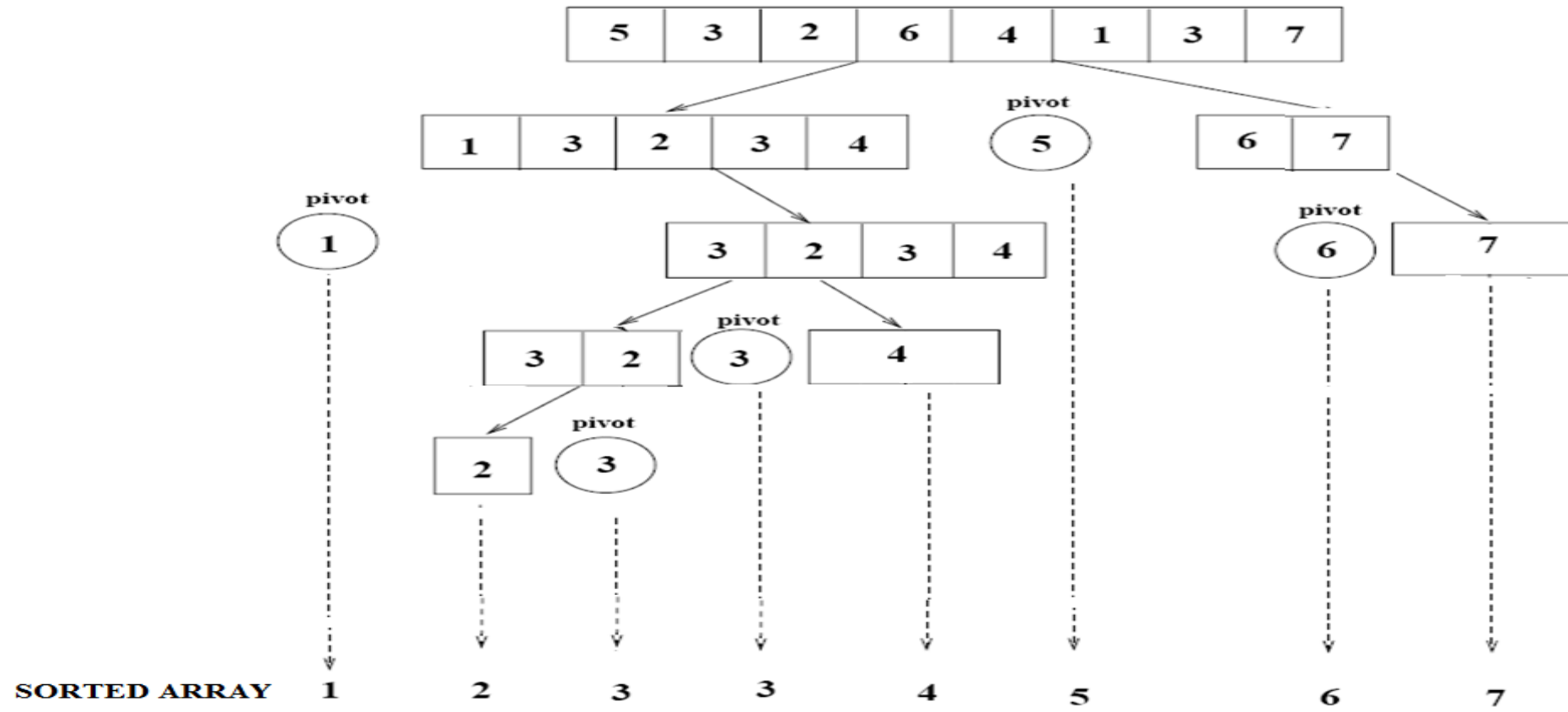
PIVOT

1	3	2	3	4
---	---	---	---	---

5
---

6	7
---	---

# FINAL EXAMPLE



# Quicksort Algorithm

---

```
quicksort ( A , lb , ub )
{
    If ( lb < ub )
    {
        {
            q = Partition ( A , lb , ub );
            quicksort ( A , lb , q-1 );
            quicksort ( A , q+1 , ub );
        }
    }
    Partition ( A , lb , ub )
    {
        pivot = A[lb];
        start = lb+1;
        end = ub;
        while ( start < end )
        {
            while ( A[start] <= pivot )
            {
                start ++;
            }
            while ( A[end] > pivot )
            {
                end --;
            }
            if ( start < end )
            {
                swap( A[start] , A[end] );
            }
        }
        swap( A[lb] , A[end] );
        return end;
    }
}
```



# Merge sort: Motivation

If I have two helpers, I'd...

- Give each helper half the array to sort
- Then I get back the sorted subarrays and **merge** them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And...

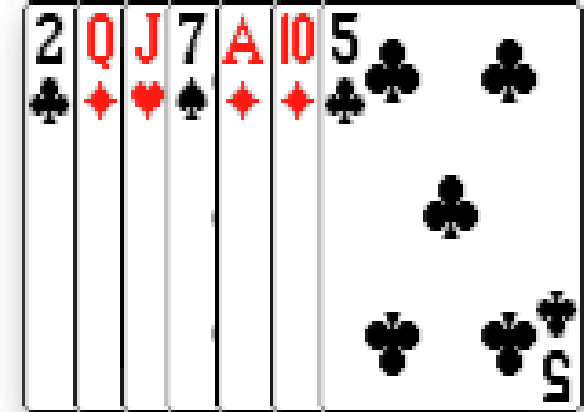
# Mergesort

- Mergesort algorithm is one of two important divide-and-conquer sorting algorithms (the other one is quicksort).
- The merge sort algorithm uses “Bottom up” approach
  - start by solving the smallest pieces of original problem
  - keep combining their results into larger solutions
  - eventually the original problem will be solved
- It is a recursive algorithm.
  - Divides the list into halves,
  - Sort each half separately, and
  - Then merge the sorted halves into one sorted array.

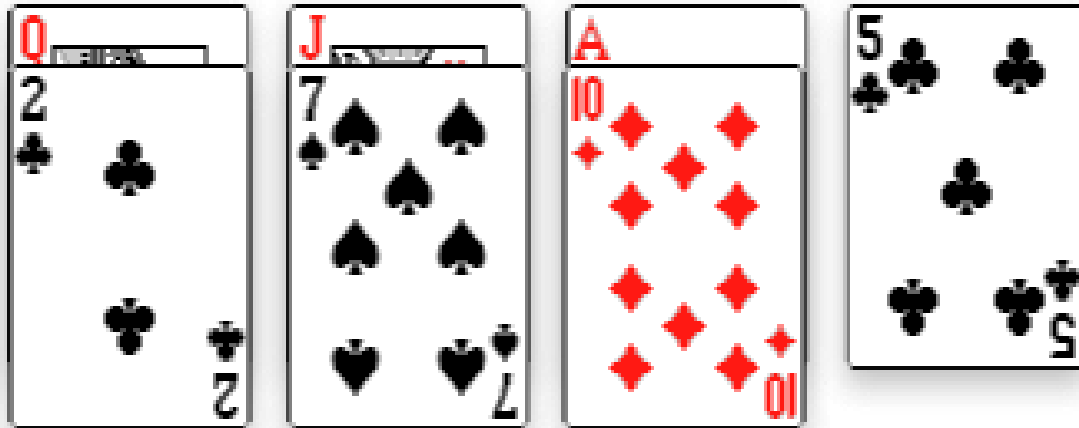
# Example: sorting playing cards

- divide the cards into groups of two
- sort each group -- put the smaller of the two on the top
- merge groups of two into groups of four
- merge groups of four into groups of eight
- ...

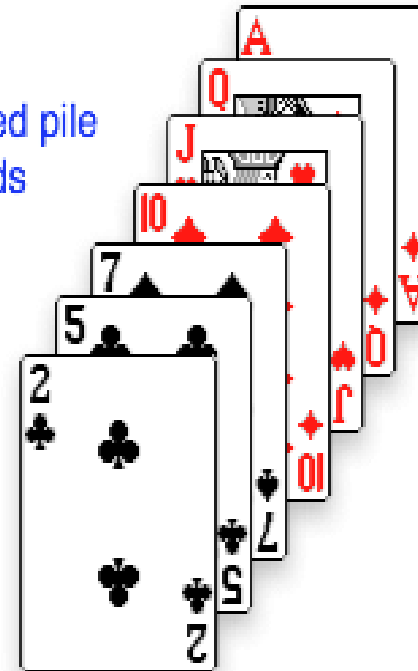
initial hand



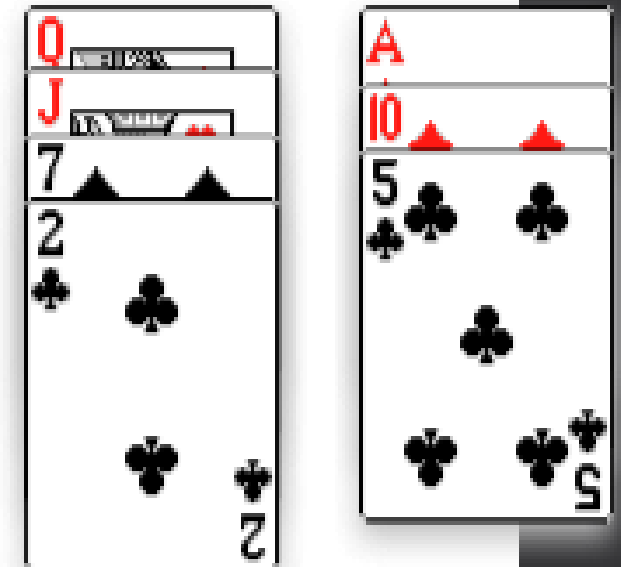
sorted piles of size two



final sorted pile  
of all cards



sorted piles of size four



In this example:

compare 2 with 5, pick up the 2  
compare 5 with 7, pick up the 5  
compare 7 with 10, pick up the 7

....

# Merge Sort Procedure

**Step 1** – if it is only one element in the list it is already sorted, return.

**Step 2** – divide the list recursively into two halves until it can no more be divided.

**Step 3** – merge the smaller lists into new list in sorted order.

# Another Example

- Subdivide the sorting task

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

# Subdivide again



H	E	M	G	B	K	A	Q
---	---	---	---	---	---	---	---

F	L	P	D	R	C	J	N
---	---	---	---	---	---	---	---

H	E	M	G
---	---	---	---

B	K	A	Q
---	---	---	---

F	L	P	D
---	---	---	---

R	C	J	N
---	---	---	---

And again



H E M G

B K A Q

F L P D

R C J N

H E

M G

B K

A Q

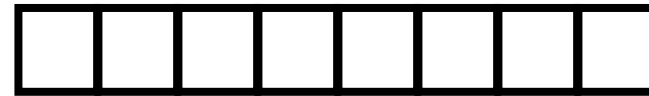
F L

P D

R C

J N

And one last time



H E

M G

B K

A Q

F L

P D

R C

J N



Now merge



E H

G M

B K

A Q

F L

D P

C R

J N

H E

M G

B K

A Q

F L

P D

R C

J N

# And merge again



E G H M

A B K Q

D F L P

C J N R

E H

G M

B K

A Q

F L

D P

C R

J N

# And again



# And one last time

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

A	B	E	G	H	K	M	Q
---	---	---	---	---	---	---	---

C	D	F	J	L	N	P	R
---	---	---	---	---	---	---	---

# Done!

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

# Merging two sorted Array : Example

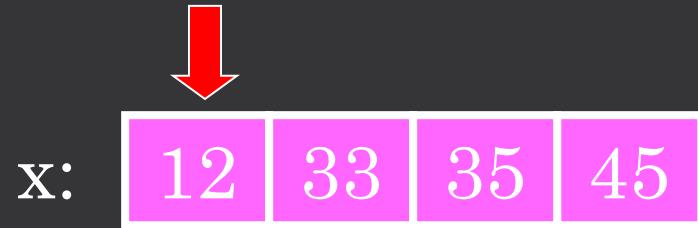
- The central sub-problem is the **merging** of two sorted arrays into one single sorted array

12	33	35	45
----	----	----	----

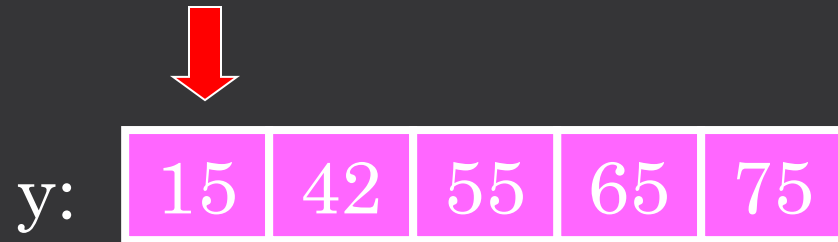
15	42	55	65	75
----	----	----	----	----

12	15	33	35	42	45	55	65	75
----	----	----	----	----	----	----	----	----

# Merge



ix: 



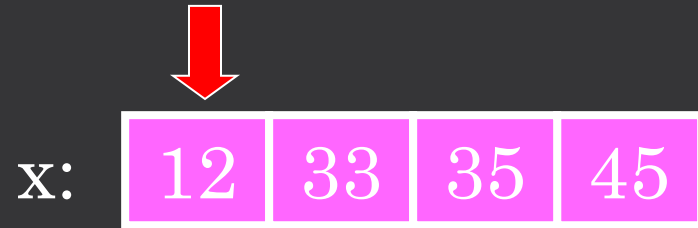
iy: 



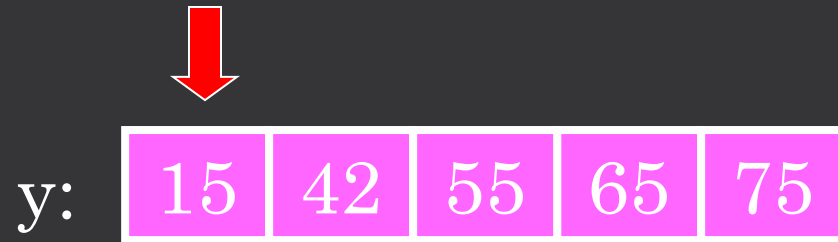
iz: 

$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

## Merge



ix: 



iy: 

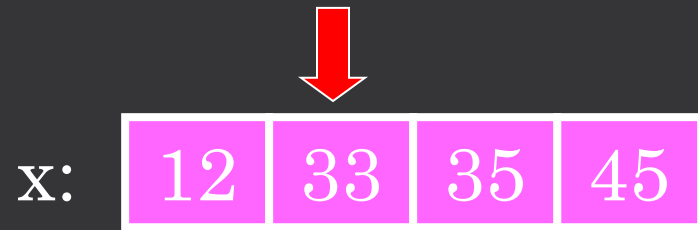


iz: 

$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  YES

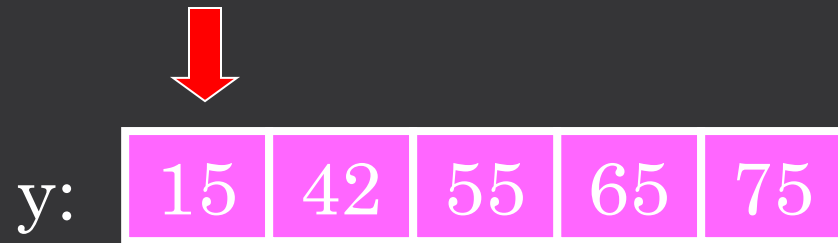


## Merge



ix: 

2
---



iy: 

1
---

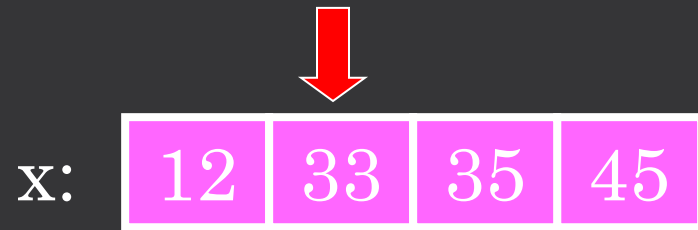


iz: 

2
---

$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

## Merge



ix: 

2
---



iy: 

1
---

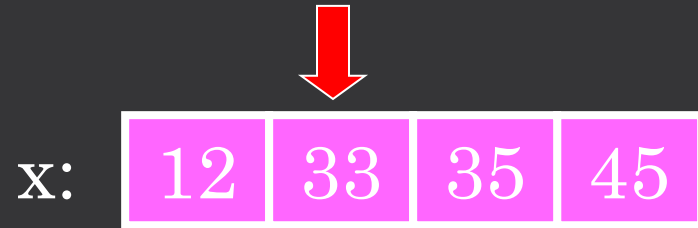


iz: 

2
---

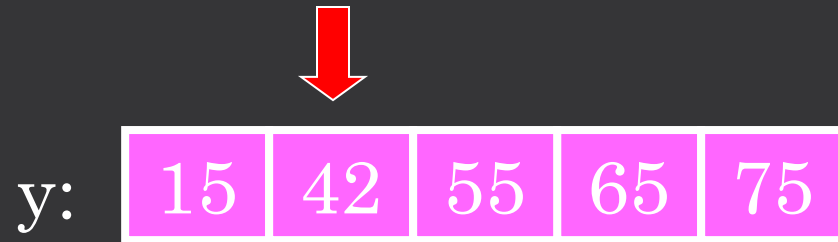
$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  NO

# Merge



ix: 

2
---



iy: 

2
---

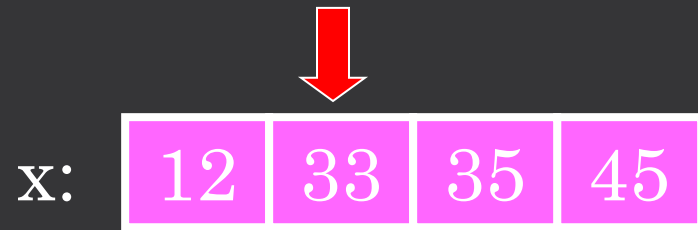


iz: 

3
---

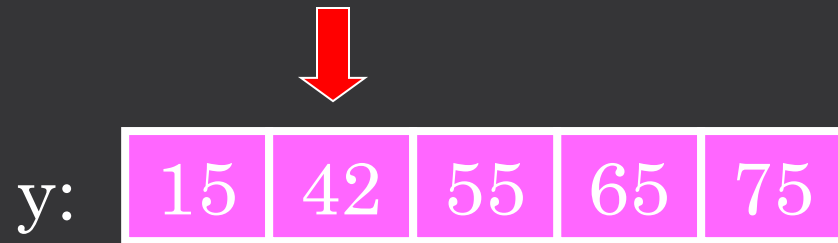
$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

## Merge



ix: 

2
---



iy: 

2
---

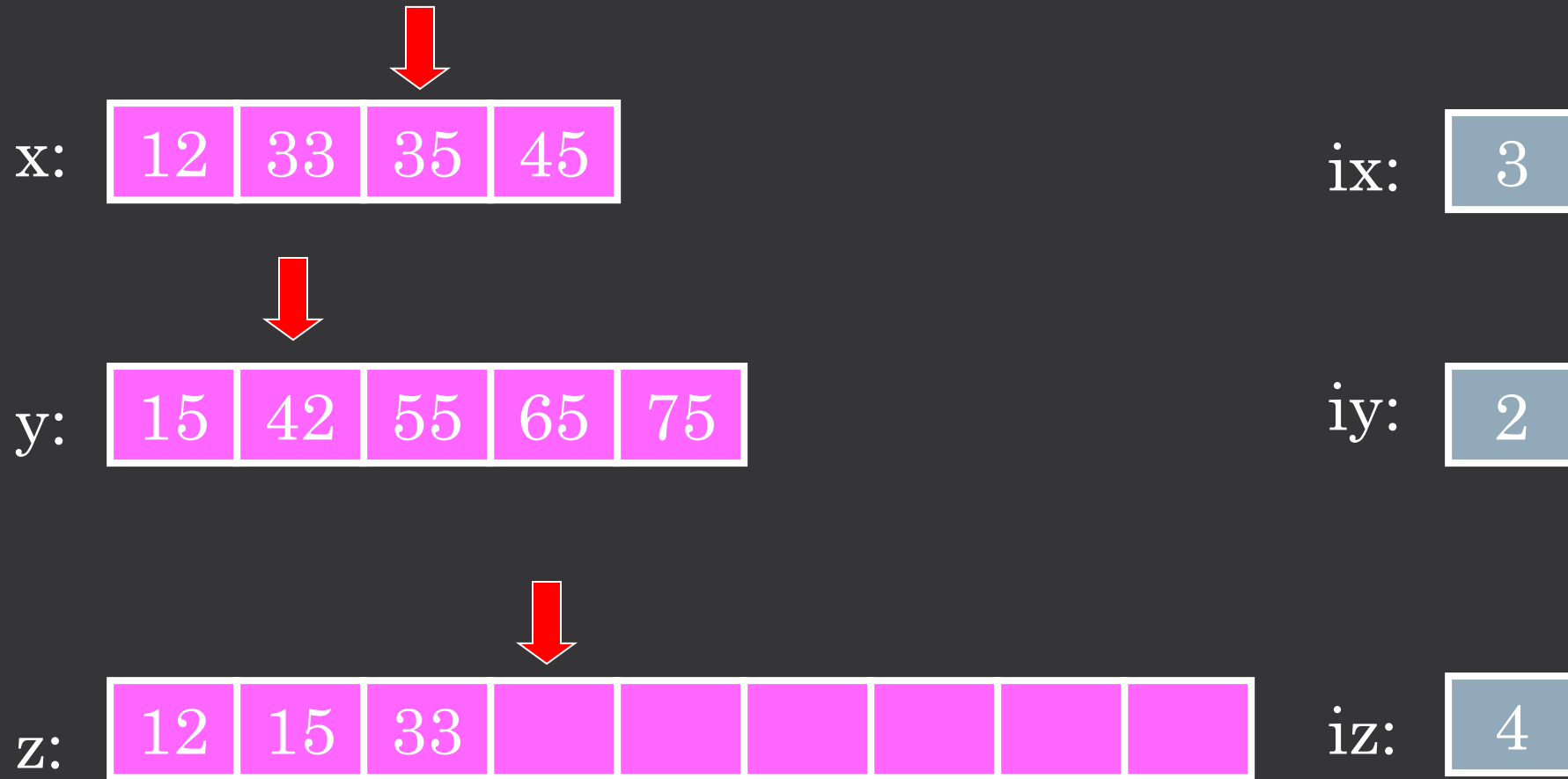


iz: 

3
---

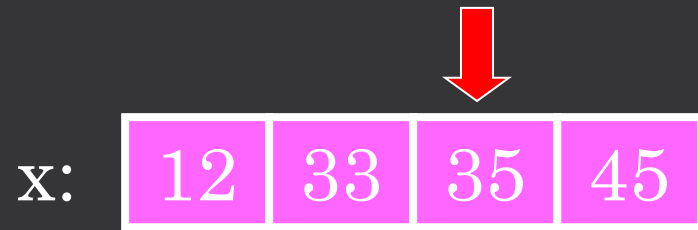
$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  YES

# Merge



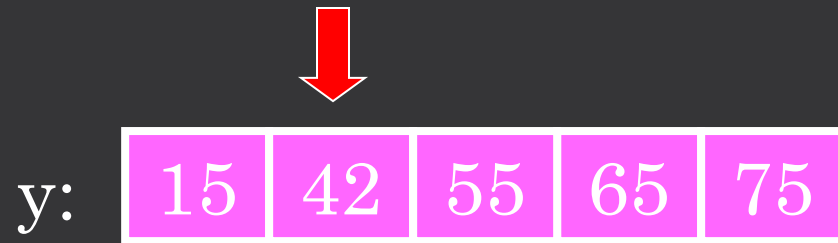
$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

## Merge



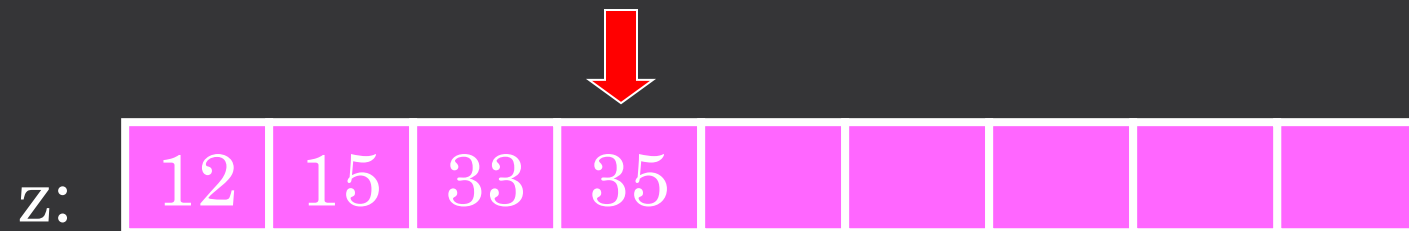
ix: 

3
---



iy: 

2
---

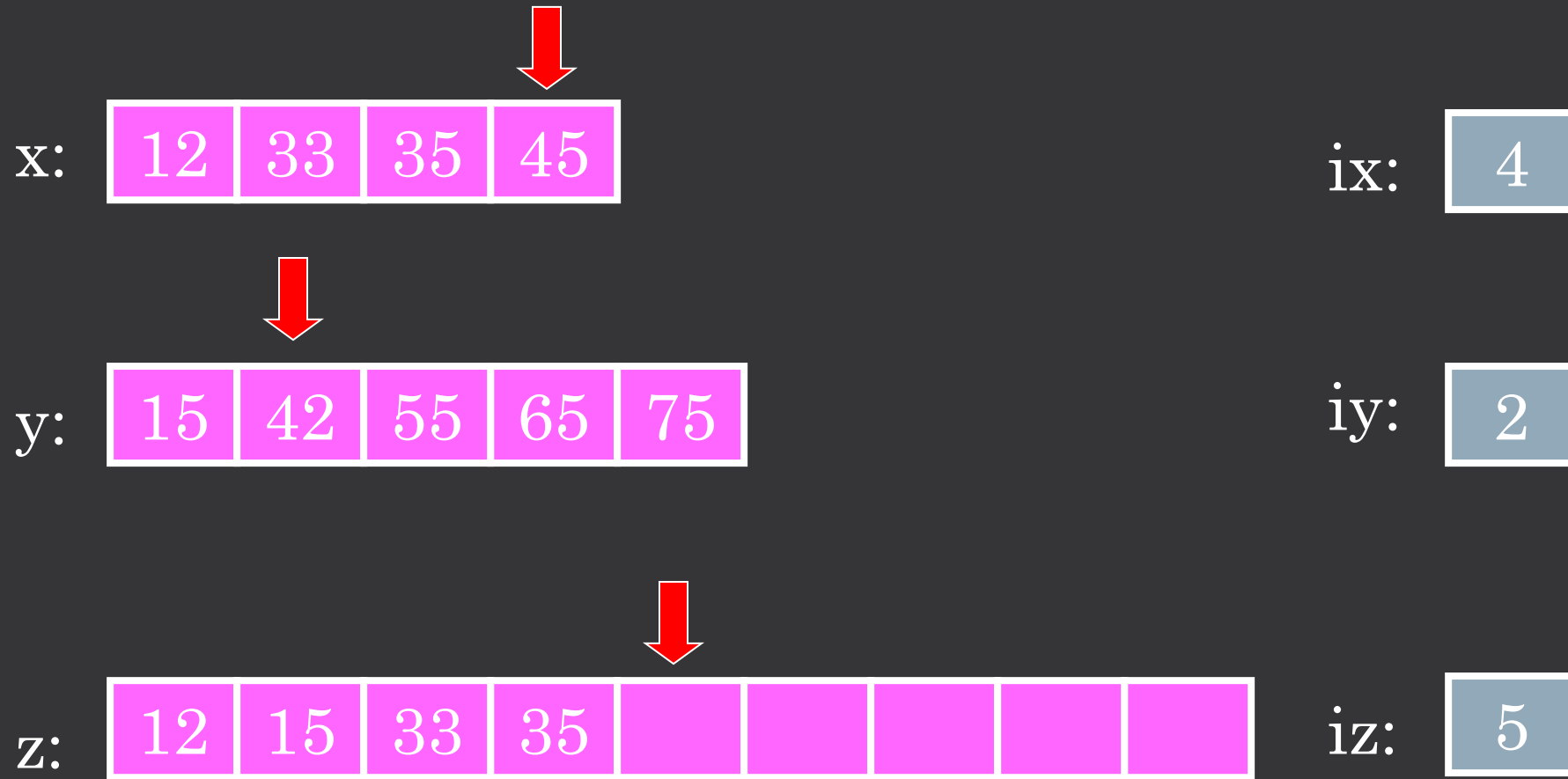


iz: 

4
---

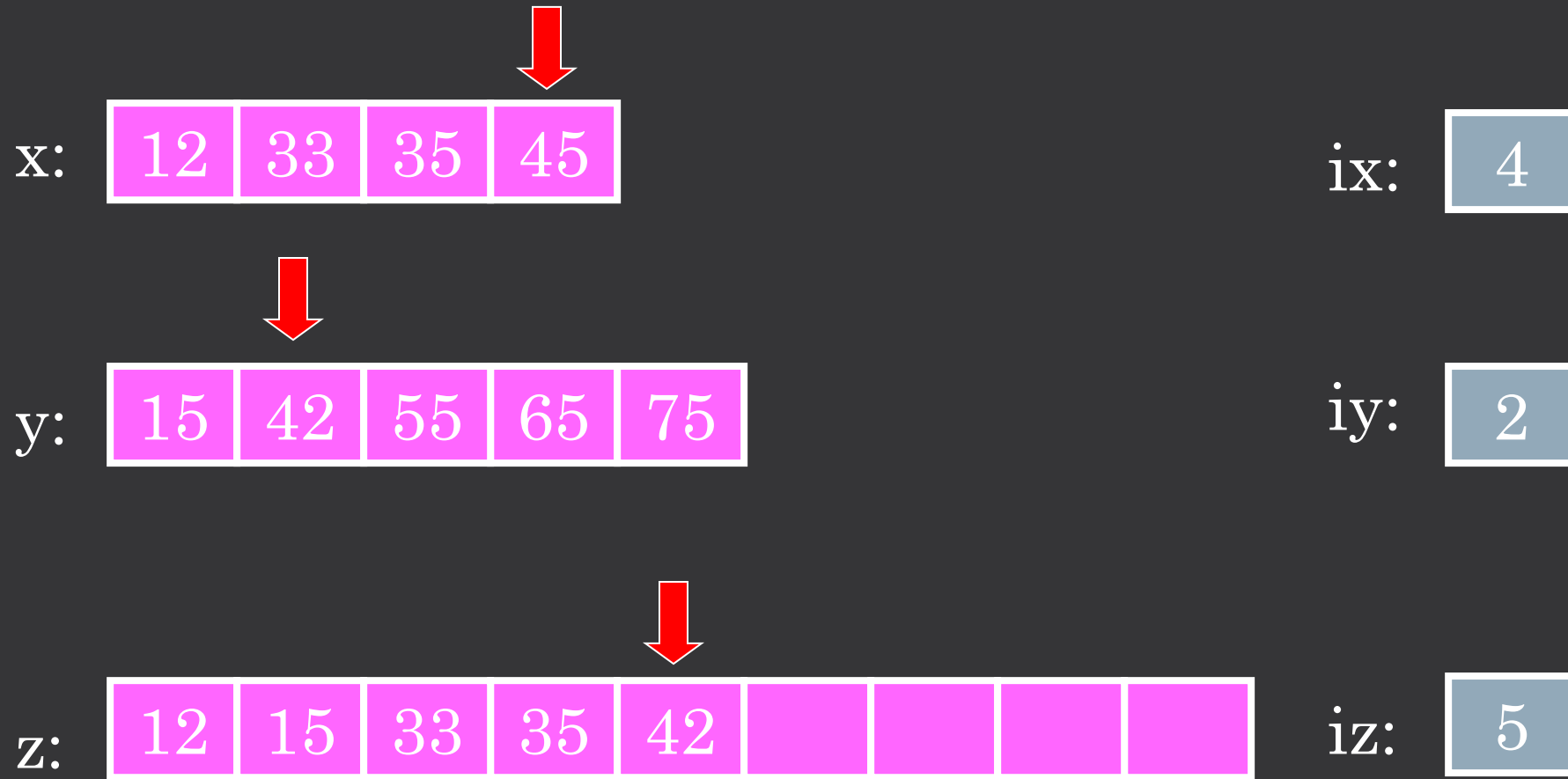
$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  YES

## Merge



$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

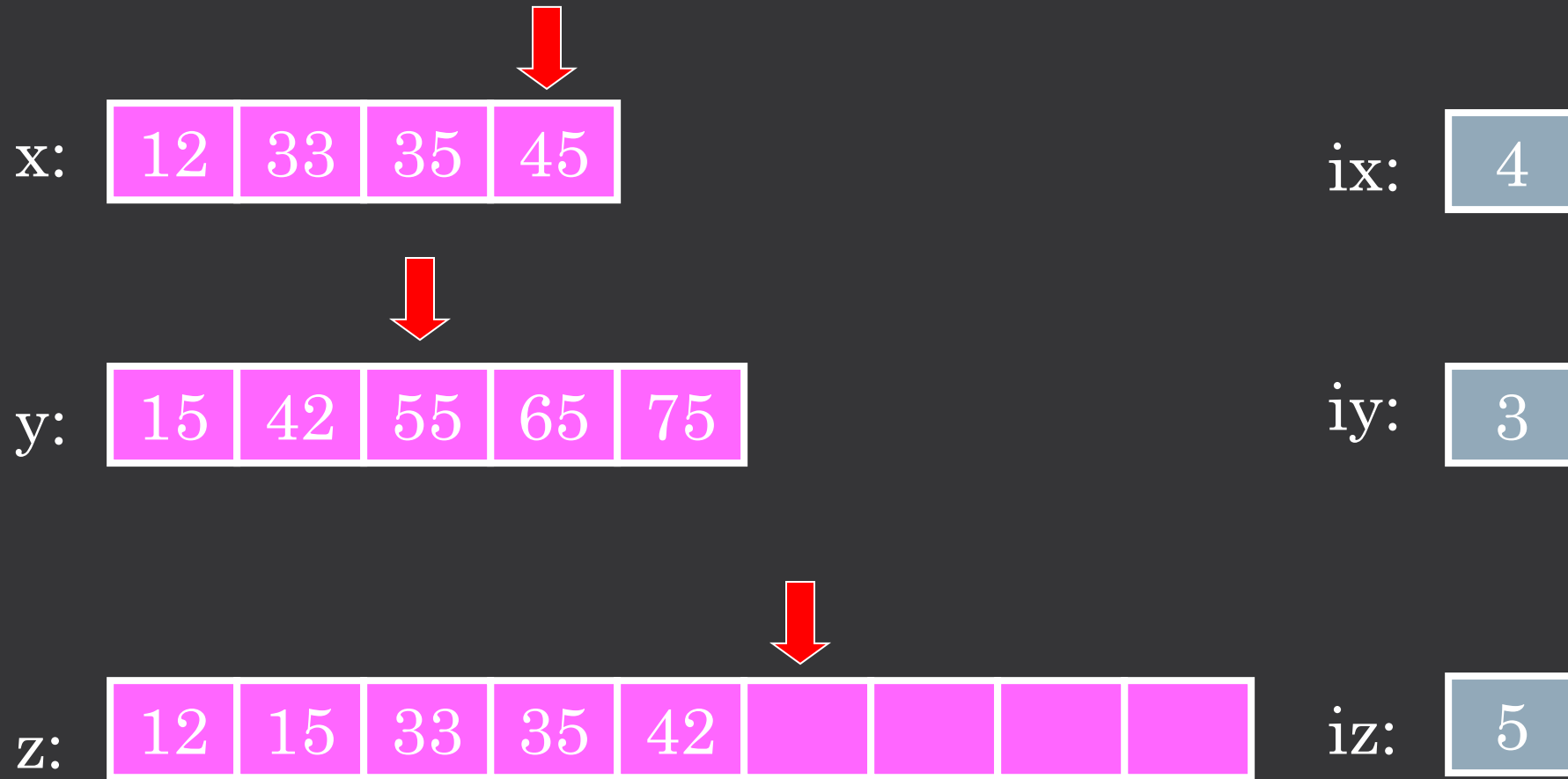
## Merge



$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  NO

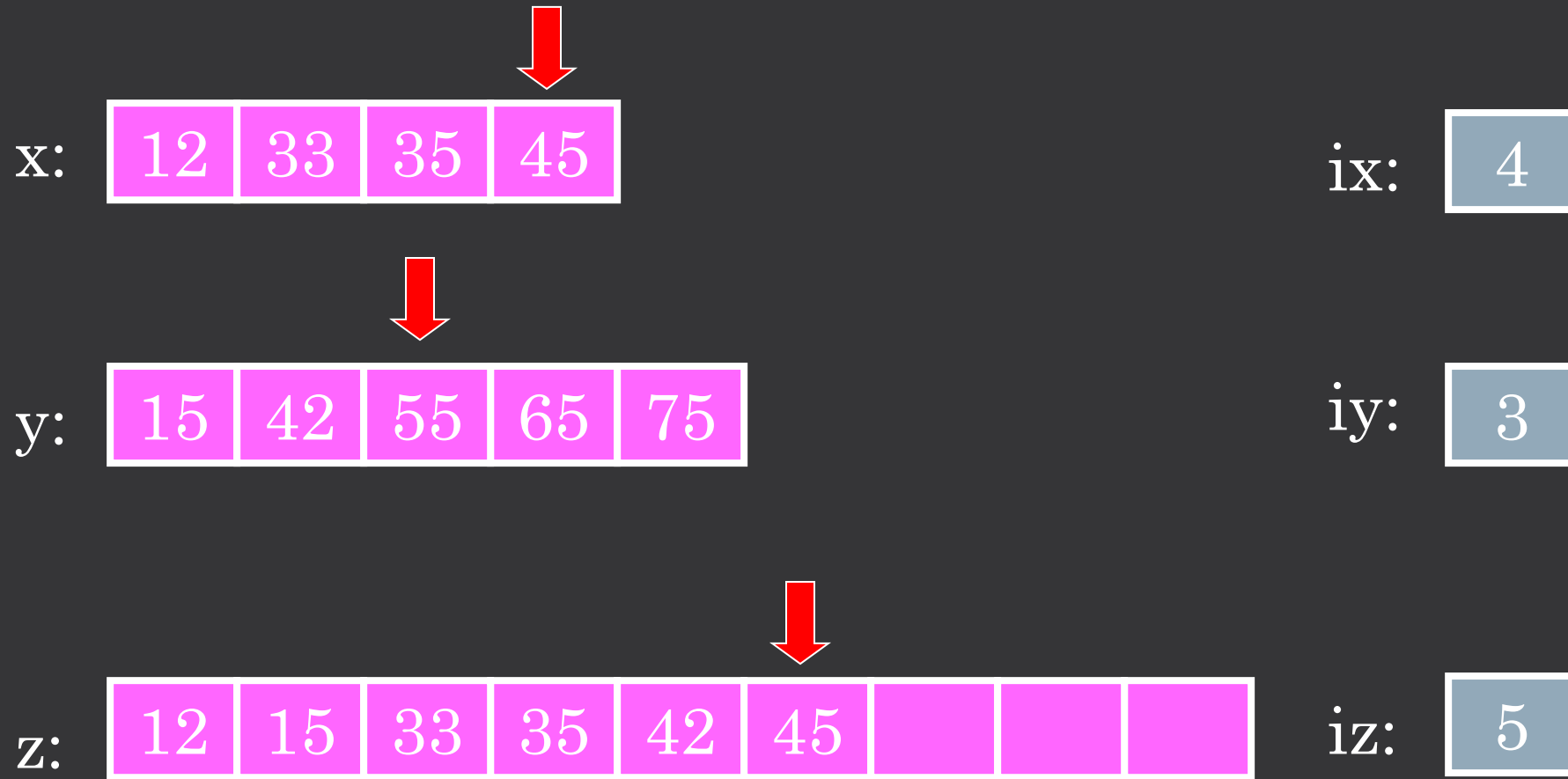


## Merge



$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  ???

## Merge



$ix \leq 4$  and  $iy \leq 5$ :  $x[ix] \leq y[iy]$  YES

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

3
---



z: 

12	15	33	35	42	45			
----	----	----	----	----	----	--	--	--

iz: 

6
---

$ix > 4$

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

3
---



z: 

12	15	33	35	42	45	55		
----	----	----	----	----	----	----	--	--

iz: 

6
---

$ix > 4$ : take  $y(iy)$

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

4
---



z: 

12	15	33	35	42	45	55		
----	----	----	----	----	----	----	--	--

iz: 

8
---

$iy \leq 5$

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

4
---



z: 

12	15	33	35	42	45	55	65	
----	----	----	----	----	----	----	----	--

iz: 

8
---

$iy \leq 5$

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

5
---



z: 

12	15	33	35	42	45	55	65	
----	----	----	----	----	----	----	----	--

iz: 

9
---

$iy \leq 5$

Merge



x: 

12	33	35	45
----	----	----	----

ix: 

5
---



y: 

15	42	55	65	75
----	----	----	----	----

iy: 

5
---



z: 

12	15	33	35	42	45	55	65	75
----	----	----	----	----	----	----	----	----

iz: 

9
---

$iy \leq 5$



# Merge Sort Algorithm

**Alg.:** MERGE-SORT(A, lb, up)

**if** lb < ub

{

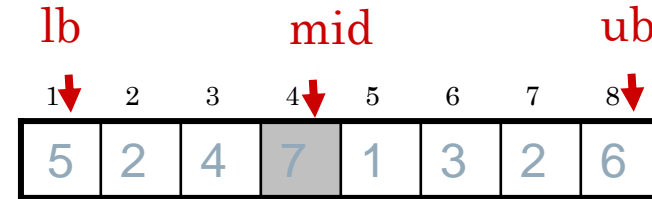
mid  $\leftarrow \lfloor (lb + ub)/2 \rfloor$

MERGE-SORT(A, lb, mid)

MERGE-SORT(A, mid + 1, ub)

MERGE(A, lb, mid, ub)

}



# Merge Algorithm

```
Merge(A, lb, mid, ub)
{
    i=lb; j=mid+1; k=lb;
    while(i<= mid && j <=
ub)
    {
        if(A[i] <= A[j])
        {
            B[k]=A[i];
            i++; k++;
        }
        else
        {
            B[k]=A[j]
            ]; j++;
            k++;
        }
    }
}
```

```
while( i <= mid)
{
    B[k]=A[i];
    i++; k++;
}
while( j <= ub)
{
    B[k]=A[j]; j++;
    k++;
}
for(i=lb; i<=ub; i++)
{A[k]=B[i]; }
```

# Merge Sort - Discussion

- Running time insensitive of the input
- Advantages:
  - Mergesort is extremely efficient algorithm with respect to time.
  - Guaranteed to run in  $\Theta(n \log n)$
- Disadvantage
  - Mergesort requires an extra array whose size equals to the size of the original array.(Not Inplace sorting)
  - So, it requires extra space  $\approx N = O(n)$



