

Greedy - Fractional - knapsack (c, u, w)

1. for $i = 0$ to n .
2. do $x[i] = 0$
3. weight = 0
4. value = 0. }
5. while weight $\leq W$.
6. do $i =$ best remaining item
7. if weight + $w[i] \leq W$
8. then $x[i] = 1$.
9. weight = weight + $w[i]$
10. value = value + $u[i]$
11. else $x[i] = (W - \text{weight}) / w[i]$
12. { weight = weight + $w[i] * x[i]$
13. value = value + $u[i] * x[i]$.
14. return value.

eg. $n=3$,
 $W=50$

$(v_1, v_2, v_3) = (60, 100, 120)$

$(w_1, w_2, w_3) = (10, 20, 30)$

Apply Greedy Strategy,

- 1) decreasing values.
- 2) Increasing weights
- 3) decreasing order of v_i/w_i

Item	weight	val.
1	10	60
2	20	100
3	30	120

case:1 decreasing values.

$W = 50$.

Item	weight	value.
3	30	120
2	20	100
1	10	60

20	100
30	120

So, $\sum v_i = 120 + 100 = \underline{\underline{220}}$

case:2 Increasing weights.

apply fraction.

Item	weight	value
1	10	60
2	20	100
3	30	120

30 * $\frac{2}{3}$	120 * $\frac{2}{3}$
20	100
10	60

$60 + 100 + 80 = \underline{\underline{240}}$

$\Rightarrow 40 * 2 \Rightarrow 80$

$w = 30$
 $v = 120$

find out fractional. $x_i = \frac{v_i - w}{w}$ entered.
 $x_i = \frac{50 - 30}{80} = \frac{20}{80}$

~~Case 2 weighted~~ $x_i = 20/80 = 1/4$

case: 3 decreasing order of v_i/w_i

Item	weight	val	v_i/w_i
1	10	60	$60/10 = 6$
2	20	100	$100/20 = 5$
3	30	120	$\frac{120}{30} = 4$

So, ~~100 + 120~~

$60 + 100 + 80 = \underline{\underline{240}}$
✓

30 $\frac{2}{3}$	120 $\frac{2}{3}$
20	100
20	100
10	60
30	120

case-3 is best.

Items.
Weight
Value

(5, 10, 20, 30, 40)


(30, 20, 100, 90, 160)

$W = 60$

(Knapsack capacity).

- ① ~~decreasing~~ decreasing value. $\rightarrow 260$
- ② Increasing weights $\rightarrow 225$
- ③ decreasing order of w_i/v_i $\rightarrow \underline{\underline{270}}$

5/6.


$$x_i = \left[\frac{W - w_i}{w_i} \right]$$

$$= 60 - \frac{35}{30}$$

$$= \frac{25}{30} = \frac{5}{6}$$