Name: Liza Lad Enrollment Number: 200303108087
Subject: Discrete Mathematics Code: 203191202

Assignment - 1

Q-1) (i) $R = \{(1,2), (1,3), (2,3), (2,4), (3,1)\}$

$S = \{(2,1), (3,1), (3,2), (4,2)\}$

To Find: $S \circ R$

$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}.$

R	S
1, 2	2, 1
1, 3	3, 1
1, 3	3, 2
2, 3	4, 2 3, 1
3, 1 2, 3	3, 2
2, 4	4, 2

(ii) $R = \{(a,b) \in R^2 \mid a > b\}$

$S = \{(a,b) \in R^2 \mid a \leq b\}$

To Find: $S \circ R$, $R \circ S$

$\left.\begin{matrix} S \circ R \\ R \circ S \end{matrix}\right\} \rightarrow \{(a,b) \mid a \neq b\}$

Q2) (a) Reflexive closure of S Q-4)

$A = \{2,3,4,5\}$

$R = \{(2,3), (3,3), (3,4), (4,2), (4,4), (5,2)\}$

Diagonal relation on A is $\Delta = \{(2,2), (3,3), (4,4), (5,5)\}$ (o

Reflexive closure of R is $S = R \cup \Delta$

$\therefore S = \cancel{R\times} R \cup \Delta$

$= \{(2,2), (2,3), (3,3), (3,4), (4,2), (4,4), (5,2), (5,5)\}$

(b) Symmetric closure of S

$R^{-1} = \{(3,2), (3,3), (4,3), (2,4), (4,4), (2,5)\}$

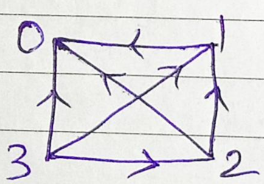
~~Symmetric~~ Symmetric closure of R is S

$S = R \cup R^{-1}$

$= \{(2,3), (3,2), (3,4), (4,3), (3,3), (4,2), (2,4), (4,4), (5,2), (5,2)\}$

Q-3) $S = \{(a,b \in S \mid a > b\}$ on the set $\{0,1,2,3\}$

$S = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2)\}$



Q-4) $f(0) = (-1)$

$f(1) = 2$ for $n = 1, 2, 3, \ldots$

To, find $f(2)$, $f(3)$, $f(4)$ and $f(5)$ ~~when~~ if f is defind recursively

(a) $f(n+1) = f(n)^2 \, f(n-1)$

$n = 1 \Rightarrow f(2) = (f(1))^2 \cdot f(0)$

$= 2^2 \cdot (-1)$

$\boxed{f(2) = -4}$

$n = 2 \Rightarrow f(3) = f(2)^2 \cdot f(1)$

$= (-4)^2 \cdot (2)$

$\boxed{f(3) = 32}$

$n = 3 \Rightarrow f(4) = f(3)^2 \cdot f(2)$

$= 1024 \cdot (-4)$

$\boxed{f(4) = -4096}$

$n = 4 \Rightarrow f(5) = f(4)^2 \cdot f(3)$

$= (-4096)^2 \cdot 32$

$\boxed{f(5) = 536870912}$

(b) $f(n+1) = 3f(n)^2 - 4f(n-1)^2$

$\Rightarrow n=1 \Rightarrow f(2) = 3f(1)^2 - 4[f(0)^2]$

$= 3(4) - 4(-1)^2$

$\boxed{f(2) = 8}$

$\Rightarrow n=2 \Rightarrow f(3) = f(2)^2 - 4(f(1)^2)$

$= 3(8)^2 - 4(2)^2$

$= 192 - 16$

$\boxed{f(3) = 176}$

$\Rightarrow n=3 \Rightarrow f(4) = 3(f(3)^2) - 4f(2)^2$

$= 3(176)^2 - 4(8)^2$

$\boxed{f(4) = 92672}$

$\Rightarrow n=4 \Rightarrow f(5) = 3f(4)^2 - 4f(3)^2$

$= 3(92672)^2 - 4(176)^2$

$= 3(92672)^2 - 123904$

$f(5) = 2.576 \times 10^{10}$

Q-5) $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

find: (a) R' (b) R^{-1} (c) R^2 (d) $R \circ R$

(a) $M_{R'} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(b) R^{-1}

$M_{R^{-1}} = M_{R^T} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) R^2

$R^2 = R \times R = M_R \circ M_R$

$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $R \circ R$

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$M_{R \circ R} = M_R \circ M_R$

$\rightarrow (1,1), (1,1) \Rightarrow (1,1)$

$(1,1), (1,3) \Rightarrow (1,3)$
$(1,3), (3,2) \Rightarrow (1,2)$
$(1,3), (3,3) \Rightarrow (1,3)$
$(2,1), (1,1) \Rightarrow (2,1)$
$(2,1), (1,3) \Rightarrow (2,3)$
$(2,2), (2,1) \Rightarrow (2,1)$
$(2,2), (2,2) \Rightarrow (2,2)$
$(3,2), (2,1) \Rightarrow (3,1)$
$(3,2), (2,2) \Rightarrow (3,2)$
$(3,3), (3,2) \Rightarrow (3,2)$
$(3,3), (3,3) \Rightarrow (3,3)$

$\rightarrow M_{RoR} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Q.6) Check whether the following relations are equivalence relation or not on the set of all integers where aRb if and only if (i) $a \neq b$
(ii) $a \geq b$

Solⁿ) ① $a \neq b$, aRb

$R = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2), \ldots\}$

→ Not Reflexive
→ symmetric
→ Transitive
→ Not Antisymmetric

⇒ Not Reflexive ⇒ Relation is not equivalence relation.

(ii.) $a \geq b$, aRb

$\therefore$ $R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3), \ldots\}$

→ Reflexive
→ Not symmetric
→ Transitive
→ Anti symmetric

$\therefore$ Not equivalent as it is not symmetric.

Q7) Check transitive, Symmetric, reflexive Equivalence and partially ordered relation

(1) $R_1 = \{(a,a), (b,b), (c,c)\}$

Reflexive - Yes
Transitive - Yes
Symmetric - Yes
Anti symmetric - Yes
Equivalence - Yes
partially ordered - Yes.

(2) $R_2 = \{(a,a), (b,b), (a,b), (b,a), (c,a)\}$

Reflexive - No
Transitive - No
Symmetric - No
Antisymmetric - No
Equivalent - No
Partially ordered - No

(3) $R_3 = \{(a,b), (b,a), (b,c), (c,b), (c,a), (a,c), (a,a), (b,b)\}$

Reflexive - No
Symmetric - Yes
Transitive - No
Anti-symmetric - No
equivalent - No
partially ordered - No

Q-8) Construct Truth table.

(a) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(b) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

(c.) $(p \to q) \vee (\neg p \to r)$

p	q	r	$\neg p$	$\overset{a}{p \to q}$	$\overset{b}{\neg p \to p}$	$a \vee b$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Q9) Show that: $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalat by using laws ~~of~~ logical equivalance

LHS:- $(\neg p \vee (\neg p \wedge q))$

$= \neg p \wedge \neg(\neg p \wedge q)$

$= \neg p \wedge (p \vee \neg q)$

$= (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$= \phi \vee (\neg p \wedge \neg q)$

$= (\neg p \wedge \neg q)$

LHS = RHS

$\therefore$ $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Q10) State converse, contrapositive and Inverse of proposition of each of the following statements.

(a) If it rain today, I will travel ~~Tommorrow~~ Tomorrow
(p above "rain today", q above "I will travel Tomorrow")

Converse: If I will travel ~~tomorrow~~ tomorrow, then it rained today $(q \to p)$

Contrapositive: $(\neg q \to \neg p)$

If I will not travel tomorrow then it ~~to~~ did not rain today

Inverse: $(\neg p \to \neg q)$

If it not rain today then I will not travel tomorrow.

(b) I come to class whenever there is going to be a quiz.
(p above "I come to class", q below "there is going to be a quiz")

Converse: $q \to p$

There is going to be a quiz whenever I come to class

Contrapositive: $(\neg q \to \neg p)$

: There is not going to be a quiz whenever I do not come to class

Inverse: $(\neg p \to \neg q)$

I do not come to class whenever there is not going to be a quiz.

1(c.) A positive integer is a prime only if it has no divisiors other than 1 and itself. (q)
(p)

Converse: $(q \to p)$

A positive integer has no divisor other than 1 and itself only if it is a prime.

Contrapositive $(\neg q \to \neg p)$

A positive integer has ~~the~~ divisors other than I and itself only if it ~~i~~ is not a prime.

Inverse $(\neg p \leftrightarrow \neg q)$

A positive integer is not a prime only if it has divisors other than 1 and itself.

Q11) Rewrite the statement without using the conditional:

(a) If it is cold, he wears a hat

$\Rightarrow$ $p \to q \equiv \neg p \vee q$

It is not cold or he wears a hat.

(b) If productivity increases, then wages rise.

$\to$ productivity does not increases or wages rise.

Q12) Construct truth table for the following.

(a) $p \Rightarrow \sim p$.

p	$\sim p$	$p \Rightarrow \sim p$
T	F	F
F	T	T

(b) $p \Rightarrow \sim q$

p	q	$\sim q$	$p \Rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

(c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

(d) $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Q-13) Find bitwise OR, bitwise AND and bitwise XOR

(a) 1111010, 1100011

```
 1111 0 1 0
 1100 0 1 1
------------
 1111 0 1 1   OR
 1100 0 1 0   AND
 0011 0 0 1   XOR
```

(b) 11110000, 11101000

```
 1111 0000
 1110 1000
-----------
 1111 1000  OR
 1110 0000  AND
 0001 1000  XOR
```

(14) State the following laws for mathematical logic and prove them using truth table:

(a) <u>Associative laws.</u>

(i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$\Rightarrow$ (i) $(p \vee q) \vee r \equiv \cancel{p \wedge (q \wedge r)}\ p \vee (q \vee r)$

p	q	r	$p \vee q$ (a)	$q \vee r$ (b)	$a \vee r$	$p \vee b$	$a \vee r \leftrightarrow p \vee b$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

$(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$ is a tautology.

So, it is a logical equivalent

$\therefore (p \vee q) \vee r \equiv p \vee q \vee r$

② $(p \wedge q) \wedge r = p \wedge (q \wedge r)$:

p	q	r	$\overset{a}{p \wedge q}$	$\overset{b}{q \wedge r}$	$(a \wedge r)$	$(p \wedge b)$	$(a \wedge r) \leftrightarrow p \wedge b$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

$\Rightarrow (p \wedge q) \wedge r \leftrightarrow p \wedge (q \wedge r)$ is a tortology

So, it is a logical equivalent

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) Distributive laws.

① $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

② $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

① $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee q$ (a)	$p \vee r$ (b)	$q \wedge r$ (c)	$p \vee (c)$	$(a) \wedge b$	$p \vee (c) \Leftrightarrow (a) \wedge (b)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F	T
F	F	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F	T

∴ Tautology, so it is logical equivalent

$\Rightarrow p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Prabhat

(b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$\overset{a}{q \vee r}$	$\overset{b}{p \wedge q}$	$\overset{c}{p \wedge r}$	$p \wedge (a)$	$b \vee c$	$p \wedge a \leftrightarrow b \vee c$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	f	f	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

∴ Tortology, So it is logical equivalent

$\Rightarrow p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(c) De Morgan's law.

(1) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(2) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

(1) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\overset{a}{p \wedge q}$	$\neg p \wedge q$	$\neg p$	$\neg q$	$\overset{c}{\neg p \vee \neg q}$	$\neg(a) \leftrightarrow c$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Tortology so logical equivalent

② $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$ (b)	$\neg(p \vee q)$ (a)	$a \leftrightarrow b$
T	T	T	F	F	F	F	T
T	F	T	F	T	F	F	T
F	T	T	T	F	F	F	T
F	F	F	T	T	T	T	T

∴ Tortology, so it is logical equivalent.

Q15) Show that each of these conditional statements is a tautology by using truth tables.

(a) $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

In $\neg p \wedge (p \vee q) \rightarrow q$ all values are true

So, It is a Tortology.

(b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$\overset{a}{p \to q}$	$\overset{b}{q \to r}$	$a \wedge b$	$\overset{c}{p \to r}$	$a \wedge b \to c$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore$ $(p \to q) \wedge (q \to r) \to (p \to r)$ has all values true

$\therefore$ It is a tortology.

Q16) Determine whether the given compound propositions is satisfiable.

$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s)$

$\wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

PTO →

→ p	q	r	s	¬p	¬q	¬r	¬s	a (p∨q∨¬r)	b (p∨¬q∨¬s)
T	T	T	T	F	F	F	F	T	T
T	T	T	F	F	F	F	T	T	T
T	T	F	T	F	F	T	F	T	T
T	T	F	F	F	F	T	T	T	T
T	F	T	T	F	T	F	F	T	T
T	F	T	F	F	T	F	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	T	T	T
F	T	T	T	T	F	F	F	T	F
F	T	T	F	T	F	F	T	T	T
F	T	F	T	T	F	T	F	T	F
F	T	F	F	T	F	T	T	T	T
F	F	T	T	T	T	F	F	F	T
F	F	T	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F	T	T
F	F	F	F	T	T	T	T	T	T

c (p∨¬r∨¬s)	d (¬p∨¬q∨¬s)	e (p∨q∨¬s)	a∧b∧c∧d∧e
T	F	T	F
T	T	T	T
T	F	T	F
T	T	T	T
T	T	T	T
T	T	T	T
T	T	T	T
T	T	T	T
F	T	T	F
T	T	T	T
T	T	T	F
T	T	T	T
F	T	F	F
T	T	T	F
T	T	F	F
T	T	T	T

⇒ compd preposition is satisfiable where p,q,r are true and ~~s~~ s is false.

⇒ " " " " " p,q "
" and r,s " "

⇒ " " " " " p,r,s "
" " q " "

⇒ " " " " " p,r "
" " q,s " "

⇒ " " " " " p,s "
" " q,r " "

⇒ " " " " " p "
" " q,r,s " "

⇒ " " " " " q,r "
" " p,s " "

⇒ The compound preposition is satisfiable where q is true and p,r,s is false

⇒ The compound preposition is satisfiable where p,q,r,s are false.

Q17) Find the number of n of distinct permutations that can be formed from all the letters of each word:

(a) THOSE:

(b) UNUSUAL:

(c) SOCIOLOGICAL

Sol^n) (a) THOSE

$n = 5$

$$\text{permutation} = \frac{n!}{1!} = n! = 5!$$

$$= 120.$$

(b) UNUSUAL

$n = 7$

Permutation $= \dfrac{n!}{(3!)(1!)}$

$= \dfrac{7!}{3!}$

$= 7 \times 6 \times 5 \times 4$

$= 840$

(c) SOCIOLOGICAL

$n = 12$

Permutation $= \dfrac{n!}{3!\ 2!\ 2!\ 2!}$

$= \dfrac{12!}{6(2)(2)(2)}$

$= \dfrac{12!}{48}$

$= \dfrac{\cancel{12} \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4} \times 3 \times 2}{\cancel{48}}$

$= 99{,}79{,}200$

Date

Q18) How many different bit strings ~~of~~ of length 8 are there?

→ length $= 2^n$ $\quad n = 8$

$\therefore 2^8 = 256$

Q19) A farmer ~~buy~~ buys 2 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs, 8 hens. Find the number m of choices that farmer has.

Sol^n)
3 cows from 6 cows ⇒ $^6C_3$
2 pigs from 5 pigs ⇒ $^5C_2$
4 hens from 8 hens ⇒ $^8C_4$

Total choices $= {}^6C_3 \times {}^5C_2 \ {}^8C_4$

$= \cancel{(6 \dots)}$

$= \dfrac{6!}{(6-3)!\,3!} \times \dfrac{5!}{(5-2)!\,2!} \times \dfrac{8!}{(8-4)!\,4!}$

$= \dfrac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 3!} \times \dfrac{5 \times 4 \times 3!}{3!\,2!} \times \dfrac{8 \times 7 \times 6 \times 5 \times 4!}{4!\,4!}$

$= \cancel{60}\ 20 \times 10 \times \dfrac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$

$= 60 \times 10 \times 70$

$= \cancel{42{,}000}\ 14000$

Q-20) Find the minimum number of students in a class to be sure that three of them are born in the same month.

Sol^n Suppose $n = 12$

$K + 1 = 3$

$k = 2$

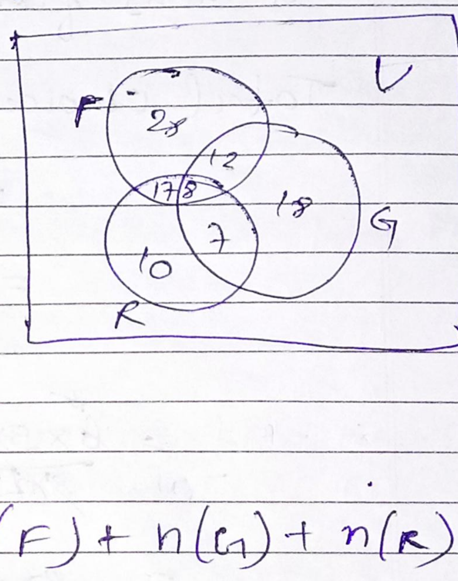
$\therefore$ minimum number of students in a class $= k(12) + 1$

$= (12)(2) + 1$

$= 24 + 1$

$= \underline{25}$

Q-21 $n(F) = 65$



$n(F \cap G) = 20$

$n(G) = 45$

$n(F \cap R) = 25$

$n(F \cap R \cap G) = 8$

$n(R) = 42$

$n(G \cap R) = 15$

$\therefore n(F \cup R \cup G) = n(F) + n(G) + n(R) - n(F \cap G) - n(F \cap R) - n(G \cap R) + n(F \cap R \cap G)$

$n(F \cup R \cup G) = 65 + 45 + 42 - 20 - 25 - 15 + 8$

$= 150 - 60 + 2 + 8$

$n(F \cup R \cup G = \underline{100}$

Page No.
Date

$\therefore$ mathematical students $= \mathcal{U} - n(F \cup R \cup G)$
$= 100 - 100$
$= \underline{\underline{0}}$

$\therefore$ Zero Mathematical students in the class.

Q22 ~~(a)~~ Let L be a list of the 26 letters in the English alphabet (which consists ~~5 vowels~~ of 5 vowels A, E, I, O, U and 21 consonants).

(a.) Show that L has a sublist consisting of four or more consecutive consonants.

Sol^n The five letters partition L into $\underline{\underline{n=6}}$ sublists of consecutive consonants.

$\therefore k+1 = 4$
$k = 3$

Hence, $nk + 1 =$
$= \cancel{66}$
$= 6(3) + 1$
$= 19 < 21$

Hence, some sublist has at least for consecutive consonants.

(b) Assuming L begins with a vowel, say A, show that L has a sub list consisting of five or more ~~conseq~~ consecutive consonants.

Soln) L begins with a vowel, the remainder of the vowels partion ~~or~~ ~~n~~ into $\underline{n = 5}$ sublists

$\therefore \quad K+1 = 5$

$K = 4$

$nK + 1$ ~~obj~~

$(5)(4) + 1$

$= \underline{21}$

Thus some sublist has at least five consecutive consonants.

~~Q-23) let C(x) be a list of the 26 letters~~

Q-23) let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog", and let F(x) be the statement "x has a ferret". Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives.

let the domain ~~const~~ consists of all students in your class.

C(x) – 'x has a cat' D(x) – 'x has a dog' F(x) – 'x has a ferret'

(a) A student in your class has a cat, a dog, and a ferret.

Soln)

$$(\exists x \in X)\big(C(x) \wedge D(x) \wedge F(x)\big)$$

Page No.
Date

(b) All students in your class have a cat, a dog, or a ferret.

Sol^n) $(\forall x \in X)\ C(x) \vee D(x) \vee F(x)$

(c) Some student in your class has a cat and a ferret, but not a dog.

Sol^n) $(\exists x \in X)\ C(x) \wedge F(x) \wedge (\neg D(x))$

~~(c)~~
(d) No student in your class has a cat, a dog, and a ferret.

Sol^n) $\cancel{\left((\exists x \in X)\ C(x) \wedge D(x) \wedge F(x)\right) \Leftrightarrow}$

$\cancel{\neg \exists x\ C(x)}$
$\left((\neg \exists x \in X)\ C(x) \wedge D(x) \wedge F(x)\right)$

(e) For each of the three animals, cats, dogs and ferrets, there is a student in your class who has this animal as a pet.

Sol^n $\left((\exists x \in X)\ C(x)\right) \wedge \left((\exists x \in x)\ D(x)\right) \wedge$

$\left((\exists x \in X)\ F(x)\right)$