Name: Henil Chovatiya
200303108003.
(3ITA)

DM - Assignment-1

Q.1) i) let R be relation {(1,2), (1,3), (2,3), (2,4), (3,1)} & let S be relation {(2,1), (3,1), (3,2), (4,2)} find $S \circ R$.

A. ~~Not Possible~~

~~$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}$~~

$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}$

R	S
1,2	2,1
1,3	3,1
1,3	3,2
2,3	3,1
2,3	3,2
2,4	4,2

ii) $R = \{(a,b) \in R^2 \mid a > b\}$ & $S = \{(a,b) \in R^2 \mid a \leq b\}$ find $S \circ R$, $R \circ S$.

$\left.\begin{matrix} S \circ R \\ R \circ S \end{matrix}\right\} \rightarrow \{(a,b) \mid a \neq b\}$.

Q.2) let S be Relation on set {2,3,4,5} containing ordered pairs (2,3), (3,3), (3,4), (4,2), (4,4) & (5,2) find

a) Reflexive closure of S (b) Symmetric closure of S.

PAGE:
DATE:

A. a). $A = \{2, 3, 4, 5\}$.
$R = \{(2,3), (3,3), (3,4), (4,2), (4,4), (5,2)\}$.

Diagonal Relation on A is $\Delta = \{(2,2), (3,3), (4,4), (5,5)\}$.

Reflexive closure of S is $S = R \cup \Delta$.
$= \{(2,2), (2,3), (3,3), (3,4), (4,2), (4,4), (5,2), (5,5)\}$

b) Symmetric closure of S.

$R^{-1} = \{(3,2), (3,3), (4,3), (2,4), (4,4), (2,5)\}$.

Symmetric closure of R is S.

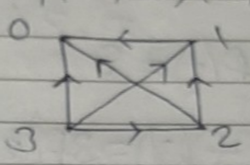
$S = R \cup R^{-1}$

$S = \{(2,3), (3,2), (3,4), (4,3), (3,3), (4,2), (2,4), (4,4), (5,2), (2,5)\}$

Q.3) Draw directed graph & find matrix relation of $S = \{(a,b) \in S \mid a > b\}$ on set $\{0, 1, 2, 3\}$.

A. $S = \{(a, b \in S \mid a > b\}$ on set $\{0, 1, 2, 3\}$

$S = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2)\}$



$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q.4) find $f(2)$, $f(3)$, $f(4)$ & $f(5)$ if f is defined recursively by $f(0) = -1$, $f(1) = 2$ & for $n = 1, 2, 3 \ldots$

$\Rightarrow f(n+1) = f(n)^2 f(n-1)$

A. $n = 1 \Rightarrow f(2) = (f(1))^2 * f(0)$

$= 2^2 (-1)$

$= -4$

$n = 2 \Rightarrow f(3) = f(2)^2 \cdot f(1)$

$= (-4)^2 \cdot (2)$

$f(3) = 32$

$n = 3 \Rightarrow f(4) = f(3)^2 \cdot f(2)$

$= (1024) \cdot (-4)$

$f(4) = -4096$

$n = 4 \Rightarrow f(5) = f(4)^2 \cdot f(3)$

$= (-4096)^2 (32)$

$f(5) = 536870912$

PAGE:
DATE:

(b) $f(n+1) = 3f(n)^2 - 4nf(n-1)^2$

A. $n=1 \Rightarrow f(2) = 3f(1)^2 - 4(f(0)^2)$
$= 3(4) - 4(-1)^2$
$f(2) = 8$

$n=2 \Rightarrow f(3) = 3f(2)^2 - 4(f(1)^2)$
$= 3(8)^2 - 4(2)^2$
$= 192 - 16$
$f(3) = 176$

$n=3 \Rightarrow f(4) = 3f(3)^2 - 4f(2)^2$
$= 3(176)^2 - 4(8)^2$
$f(4) = 92672$

$n=4 \Rightarrow f(5) = 3f(4)^2 - 4f(3)^2$
$= 3(92672)^2 - 4(176)^2$
$= 3(92672)^2 - 123904$
$f(5) = 2576 \times 10^{10}$

Q.5) Let R be relation represented by matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ find matrix representing

(a) R^{-1}

$$M_{R'} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) R^{-1}

$$M_{R^T} = M_{R^{-1}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

c) R^2

$$M_{R^2} = M_R \circ M_R$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

d) $R \circ R$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M_{R \circ R} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q.6) Check whether the following relations are Equivalence relation or not.

on the set of all integers where aRb. if & only if i) $a \neq b$ ii) $a \geq b$.

A. i) $R = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2), \ldots\}$.

→ Not Reflexive ∵ $(a,b) \notin R$.
→ Symmetric ∵ $(a,b) \in R$
→ Transitive ∵ $(a,b); (b,c) \in R$ then $(a,c) \in R$
→ Not ~~transitive~~ Antisymmetric.

∴ The given relation is not an Equivalence Relation.

ii) $a \geq b$, aRb.

$R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3), \ldots\}$

→ Reflexive ∵ $(a,a) \in R$
→ Not Symmetric $(a,b) \in R$ but $(b,a) \notin R$
→ Transitive.
→ Anti Symmetric.

∴ The given Relation is not an Equivalence Relation.

Q.7) Check whether from the following relation sets, which are satisfying the transitive, reflexive & symmetric property which relation is an Empty Equivalence relation & partially ordered relation.

1) $R_1 = \{(a,a), (b,b), (c,c)\}$
2) $R_2 = \{(a,a), (b,b), (a,b), (b,a), (c,a)\}$
3) $R_3 = \{(a,b), (b,a), (b,c), (c,b), (c,a), (a,c), (a,a), (b,b)\}$

A. 1) R_1 is Reflexive, symmetric, Transitive, Antisymmetric, Equivalence, Partially ordered

3) Reflexive - No
Symmetric - Yes
Transitive - No
Antisymmetric - No
Equivalence - No
Partially Ordered - No

2) $R_2 = \{(a,a), (b,b), (a,b), (b,a), (c,a)\}$

R_2 is Not Reflexive,
Not Transitive,
Not Symmetric,
Not Antisymmetric,
Not Equivalence, Not Partially ordered.

Q.8) Construct ~~tt~~ truth table

a) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

b) $(p \leftrightarrow q) \oplus (\sim p \leftrightarrow q)$

p	q	$p \leftrightarrow q$	$\sim p$	$\sim p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\sim p \leftrightarrow q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

c) $(p \to q) \vee (\sim p \to r)$

~~p q ~p r~~

p	q	r	$\sim p$	$\overset{a}{p \to q}$	$\overset{b}{\sim p \to r}$	$a \vee b$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Q.9) Show that $\sim(p \vee (\sim p \wedge q))$ & $\sim p \wedge \sim q$ are logically Equivalent by using law of logical Expn.

A. LHS :- $(\sim(p \vee (\sim p \wedge q)))$

$\sim p \wedge \sim(\sim p \wedge q)$

$\sim p \wedge (p \vee \sim q)$

$(\sim p \wedge p) \vee (\sim p \wedge \sim q)$.

$\phi \vee (\sim p \wedge \sim q)$

$(\sim p \wedge \sim q)$

LHS = RHS

Q.10) State converse, contrapositive & Inverse of proposition of each of following Statement.

p q

A. a) If it rains today, I will travel tommorow.

b) Converse:- If I will travel tommorow then it rain today. $(q \rightarrow p)$

Contrapositive:- If I will not travel tommorow then it did not rain today. $(\sim q \rightarrow \sim p)$.

Inverse:- If it not rain today then I will not travel tommorow.

p

b) I come to class whenever there is going to be quiz.

q

Converse: There is going to be quiz whenever I come to class. $(q \rightarrow p)$.

Contrapositive: There is not going to be quiz whenever I do not come to class. $(\sim q \rightarrow \sim p)$.

Inverse:- ~~There is not going to be a quiz~~ I do not come to class whenever there is not going to be quiz $(\sim p \rightarrow \sim q)$

c) A positive Integer is a prime only if it has no division other than 1 & Itself.

Converse:- $(q \to p)$
A positive Integer has no divisor other than 1 & Itself only if it is a prime.

Contrapositive $(\sim q \to \sim p)$
A positive integer has to divisors others than 1 and itself only if it is not prime.

Inverse $(\sim p \to \sim q)$:
A positive integer is not a prime only if it has divisor other than 1 & itself.

Q.11) Rewrite Statement without using Conditional.

a) If it is cold, he wears a hat.
$p \to q \equiv \sim p \vee q$
It is not cold or he wears a hat.

b) If productivity ~~does not~~ increases, then wages rise.

A. productivity does not increases or wage rise.

Q.12) Construct truth table.

a) $p \Rightarrow \sim p$.

p	$\sim p$	$p \Rightarrow \sim p$
T	F	F
F	T	T

b) $p \Rightarrow \sim q$.

p	q	$\sim q$	$p \Rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d) $(p \vee q) \rightarrow (p \wedge q)$.

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Q.13) find bitwise OR, bitwise AND and bitwise XOR.

a) 1111010, 1100011

```
 1111010
 1100011
--------
 1111011  OR
 1100010  AND
 0011001  XOR
```

b) 11110000, 11101000

```
 11110000
 11101000
---------
 11111000  OR
 11100000  AND
 00011000  XOR
```

PAGE:
DATE:

Q.14) State the following laws for mathematical logic & prove them using truth table:-

a) Associative laws:-

① $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

p	q	r	$p \vee q$ (a)	$q \vee r$ (b)	$a \vee r$	$p \vee b$	$a \vee r \leftrightarrow p \vee b$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

$(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$ is tautology.

∴ It is a logical Equivalent.

$(p \vee q) \vee r \equiv p \vee q \vee r$.

② $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

p	q	r	$\overset{a}{p \wedge q}$	$\overset{b}{q \wedge r}$	$a \wedge r$	$p \wedge b$	$a \wedge r \leftrightarrow p \wedge b$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T.

$(p \wedge q) \wedge r \leftrightarrow p \wedge (q \wedge r)$ is a tautology.

So, it is a logically Equivalent.

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

b) Distributive laws

① $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

② $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

PAGE:
DATE:

① $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$\overset{a}{p \vee q}$	$\overset{b}{p \vee r}$	$\overset{c}{q \wedge r}$	$p \vee c$	$a \wedge b$	$p \vee c \leftrightarrow a \wedge b$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F	T
F	F	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F	T

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ is tautology.
So it is logically Equivalent.

② $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$\overset{a}{q \vee r}$	$\overset{b}{p \wedge q}$	$\overset{c}{p \wedge r}$	$\overset{d}{p \wedge a}$	$\overset{e}{b \vee c}$	$d \Leftrightarrow e$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	F	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
is a tautology.
$\therefore$ It is Equivalent.

c) De Morgan law.

(1) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\overset{a}{\sim(p \wedge q)}$	$\overset{b}{\sim p \vee \sim q}$	$a \leftrightarrow b$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

Tautolog $\therefore$ it is Equivalent

(2) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim p$	$\sim q$	$\overline{\sim(p \vee q)}^{a}$	$\overline{\sim p \wedge \sim q}^{b}$	$a \leftrightarrow b$
T	T	T	F	F	F	F	T
T	F	T	F	T	F	F	T
F	T	T	T	F	F	F	T
F	F	F	T	T	T	T	T

∴ Tautology, So it is logical Equivalent.

Q.15) Show that each of these conditional statement is a tautology by using truth tables.

a) $(\sim p \wedge (p \vee q)) \rightarrow q$.

p	q	$\sim p$	$p \vee q$	$\overline{\sim p \wedge (p \vee q)}^{a}$	$a \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Tautology it is Equivalent.

b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$\overset{a}{p \to q}$	$\overset{b}{q \to r}$	$a \wedge b$	$p \to r$	$a \wedge b \to p \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

It is tautology.
$\therefore$ It is Equivalent.

Q.16) Determine whether the given compound proposition is satisfiable.

$(p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim s) \wedge (p \vee \sim r \vee \sim s)$
$\wedge (\sim p \vee \sim q \vee \sim s) \wedge (p \vee q \vee \sim s)$

								a	b
p	q	r	s	$\sim p$	$\sim q$	$\sim r$	$\sim s$	$p \vee q \vee \sim r$	$p \vee \sim q \vee \sim s$
T	T	T	T	F	F	F	F	T	T
T	T	T	F	F	F	F	T	T	T
T	T	F	T	F	F	T	F	T	T
T	T	F	F	F	F	T	T	T	T
T	F	T	T	F	T	F	F	T	T
T	F	T	F	F	T	F	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	T	T	T
F	T	T	T	T	F	F	F	T	F
F	T	T	F	T	F	F	T	T	T
F	T	F	T	T	F	T	F	T	F
F	T	F	F	T	F	T	T	T	T
F	F	T	T	T	T	F	F	F	T
F	F	T	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F	T	T
F	F	F	F	T	T	T	T	T	T

c	d	e	
$(p \vee \sim r \vee \sim s)$	$(\sim p \vee \sim q \vee \sim s)$	$(p \vee q \vee \sim s)$	$a \wedge b \wedge c \wedge d \wedge e$
T	F	T	F
T	T	T	T
T	F	T	F
T	T	T	T
T	T	T	T
T	T	T	T
T	T	T	T
T	T	T	T
F	T	T	F
T	T	T	T
T	T	T	F
T	T	T	T
F	T	F	F
T	T	T	F
T	T	F	F
T	T	T	T

→ Compound preposition is Satisfiable where p, q, r are true & s is false.
& where q is true & p, r, s is false
& where p, q, r, s are false.

Q. (7) find the number of n of distinct permutation that can be formed from all the letters of each word.

a) THOSE

b) UNUSUAL

c) SOCIOLOGICAL

Soln:- a) THOSE

$n = 5$

$$^nP_1 = \frac{n!}{1!} = 5! = 120.$$

b) UNUSUAL

$$= \frac{n!}{(3!)(1!)} = \frac{7!}{3!}$$

$= 7 \times 6 \times 5 \times 4$

$= 840.$

c) SOCIOLOGICAL

$n = 12$

$$\text{Permutation} = \frac{n!}{3!2!2!2!}$$

$$= \frac{12!}{6 \times 2 \times 2 \times 2}$$

$= 99,79,200.$

Q.18) How many different bit strings of length 8 are there?

A. length $= 2^n$, $n = 8$.

$\therefore 2^8 = 256$.

Q.19) A farmer buys 3 cows, 2 pigs & 4 hens from a man ~~visits~~ who has 6 cows, 5 pigs, 8 hens, find number m of choices that farmer has.

A. 3 cows from 6 ~~pigs~~ cows $\Rightarrow {}^6C_3$.
2 pigs from 5 pigs $\Rightarrow {}^5C_2$.
4 hens from 8 hens $\Rightarrow {}^8C_4$.

Total Choices $= {}^6C_3 \times {}^5C_2 \times {}^8C_4$.

$$= \frac{6!}{(6-3)!\,3!} \times \frac{5!}{(5-2)!\,2!} \times \frac{8!}{(8-4)!\,4!}$$

$= 10 \times 60 \times 70$.

$= 14\,000$.

Q.20) find maximum number of students in a class to be sure that three of them are born in same month.

A. Suppose $n = 12$,

$K+1 = 3$

$K = 2$.

$\therefore$ minimum no. of students in a class $= K(12)+1$

$= (12)(2) + 1$

$= 25$.

(Q.21) find the number of mathematics Student at a college taking at least one of language french, german, Russian given the following data: 65 Study French, 20 Study french & German, 45 Study German, 25 study french & Russian, 8 Study all the three languages. 42 Study Russian, 15 Study German & Russian.

A. $n(F) = 65$.

$n(F \cap G) = 20$.

$n(G) = 45$.

$n(F \cap R) = 25$.

$n(F \cap R \cap G) = 8$

$n(R) = 42$.

$n(G \cap R) = 15$

$n(F \cup R \cup G) = n(F) + n(G) + n(R) - n(F \cap R)$

$- n(F \cap R) - n(G \cap R) - n(F \cap R \cap G)$.

$= 65 + 45 + 42 - 20 - 25 - 15 + 8$.

$= 150 - 60 + 2 + 8$.

$n(F \cup R \cup G) = 100$.

Math Student $= U - n(F \cup R \cup G)$

$= 100 - 100$.

$= 0$.

$\therefore$ Zero Mathematical student in class.

Q.22) let L be a list of the 26 letters in English alphabet (which consists of 5 vowels AEIOU & 21 Consonants.)

a) Show that L has sublist consisting of four or more consecutive constants.

A. The five letters partition L into $n=6$ sublists of consecutive consonants.

$\therefore K+1 = 4 \Rightarrow K = 3$.

Hence, $nK+1$

$= 6(3)+1$.

$19 < 21$

Hence, some sublist has at least for consecutive consonants

b) Assuming L begins with a vowel say A show that L has a sub list consisting of five or more consecutive consonants.

A. L begins with vowel, the remainder of vowels partion into $n=5$ sublists.

$$\therefore K+1=5$$
$$K=4$$
$$nK+1$$
$$(5)(4)+1$$
$$=21$$

Thus some sublist has at least five consecutive consonants.

Q.23) let $C(x)$ be statement "x has a cat" let $D(x)$ be statement & let $F(x)$ be statement "x has a ferret" Express each of these statement in terms of $C(x)$, $D(x)$, $F(x)$ Quantifiers & logical connectives.

a) A student in your class has cat, dog, and a ferret.

A. $(\exists x \in X)(C(x) \cap D(x) \cap F(x))$

b) All Students in your class have a cat, a dog and a ferret.

A. $(\forall x \in X)[C(x) \vee D(x) \vee F(x)]$

c) Some Student in your class has a cat & a ferret but not a dog.

A. $(\exists x \in X)(C(x) \cap F(x) \cap (\sim D(x)))$

d) No Student in your class has a cat, a dog and a ferret.

A. $(\sim \exists x \in X)(C(x) \cap D(x) \cap F(x))$

e) For each of these animals, cats, dogs & ferrets there is a Student in your class who has this animal as pets.

A. $((\exists x \in X)(C(x))) \cap ((\exists x \in X)(D(x))) \cap$

$((\exists x \in X)(F(x)))$