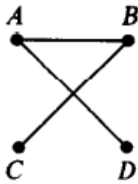
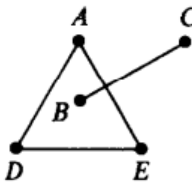
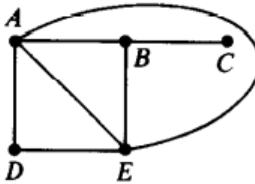
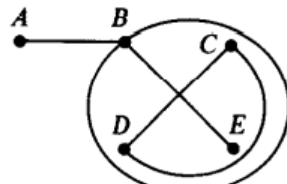
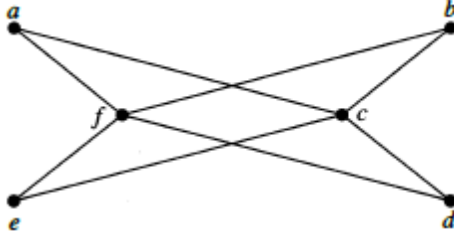
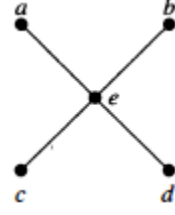
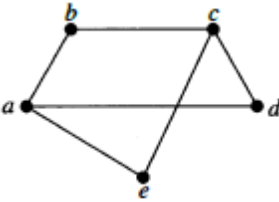
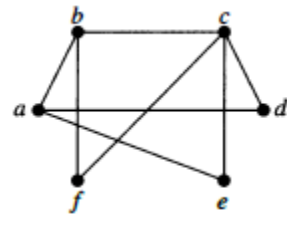
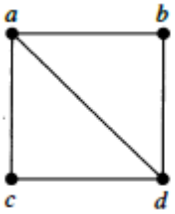
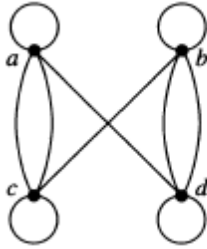
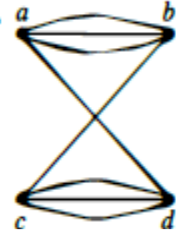
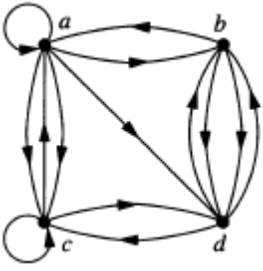
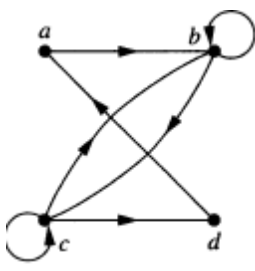
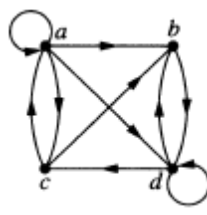
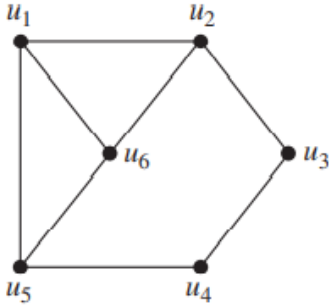
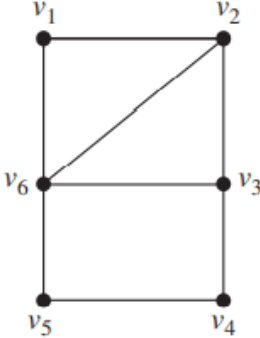
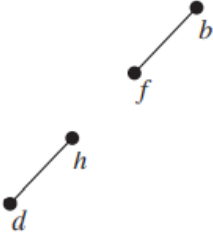
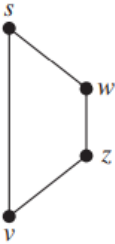
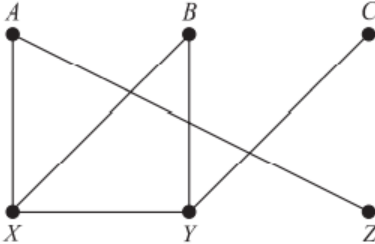





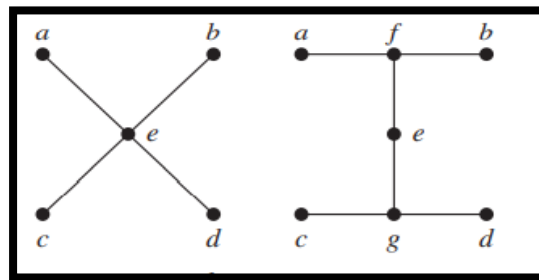
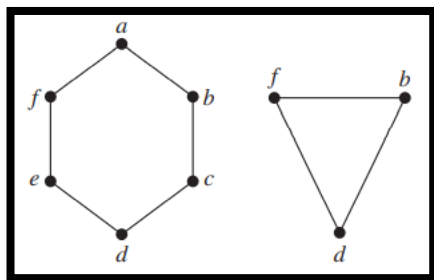


TUTORIAL 5(a): Graphs and Trees

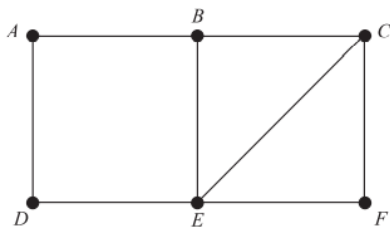
1	State the difference between the following: (a) Simple graph and Pseudo-graph (b) Isolated vertex and Pendant vertex (c) Path and Circuit (d) Euler Path and Hamiltonian Path (e) Euler Circuit and Hamiltonian Circuit
2	Prove undirected graph has an even number of vertices of odd degree.
3	Find the number of vertices, number of edges, and degree of each vertex in the following Undirected graphs. Identify all isolated and pendant vertices. Verify Handshaking Theorem. <div style="display: flex; flex-wrap: wrap; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div>
4	Find the number of vertices, number of edges, in-degree and out-degree of each vertex in the following directed graphs and hence, verify that the sum of in-degrees, the sum of out-degrees of the vertices and the number of edges in the following graphs are equal. <div style="display: flex; flex-wrap: wrap; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div>
5	Draw the graphs: K_6 , $K_{2,5}$, $K_{3,3}$, C_9 , W_9 , Q_3
6	Consider the multigraphs

	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p>(1)</p> </div> <div style="text-align: center;">  <p>(2)</p> </div> <div style="text-align: center;">  <p>(3)</p> </div> <div style="text-align: center;">  <p>(4)</p> </div> </div> <p>(a) Which of them are connected? If a graph is not connected, find its connected components. (b) Which are cycle-free (without cycles)? (c) Which are loop-free (without loops)? (d) Which are (simple) graphs?</p>
7	<p>Check whether the following graphs are bipartite or not</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>
8	<p>Represent the following graphs using adjacent and incidence matrices:</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>
9	<p>Draw the undirected graphs with the given adjacency matrix:</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$ </div> <div style="text-align: center;"> $\begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$ </div> </div>
10	<p>Draw the directed graphs with the given adjacency matrix:</p>

	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$
11	<p>Determine whether the given graphs are isomorphic or not? Justify by giving appropriate reasons.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p style="text-align: center;">(a)</p>
	<div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p style="text-align: center;">(b)</p>
12	<p>Let G be the graph</p> <div style="text-align: center;">  </div> <p>Find:</p> <ul style="list-style-type: none"> (a) all simple paths from A to C; (b) $G - Y$; (c) all cycles; (d) all cut points; (e) subgraph H generated by $V = \{B, C, X, Y \}$; (f) all bridges.
13	<p>Determine whether the given graphs are connected or not? Also find the connected components for each of the graphs.</p> <div style="display: flex; justify-content: space-around; align-items: center;">    </div> <p style="display: flex; justify-content: space-around;"> (a) (b) (c) </p>
14	<p>Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)</p>



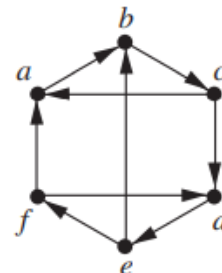
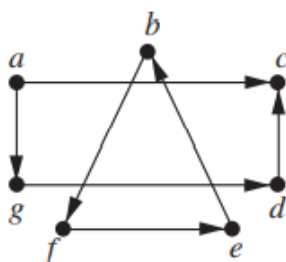
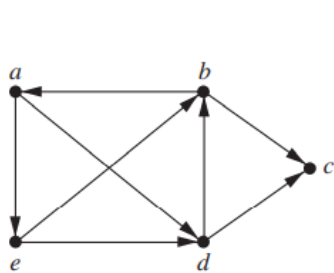
15 Consider the graph G



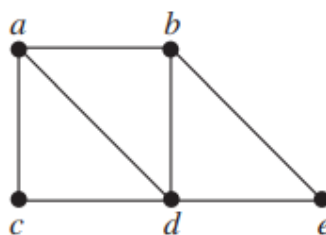
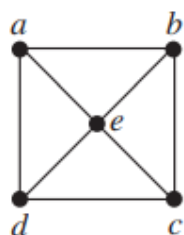
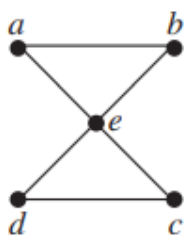
Find

- (a) all simple paths from A to F;
- (b) $\text{diam}(G)$, the diameter of G;
- (c) all trails from A to F;
- (d) all cycles which include vertex A;
- (e) $d(A, F)$, the distance from A to F;
- (f) all cycles in G

16 Determine whether each of these graphs is strongly connected or weakly connected or unilaterally connected?

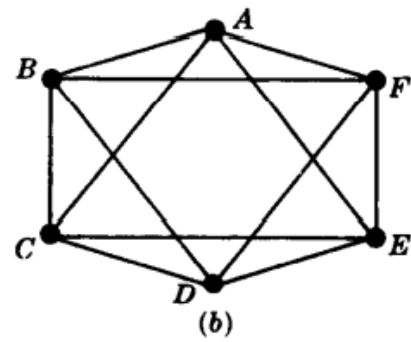
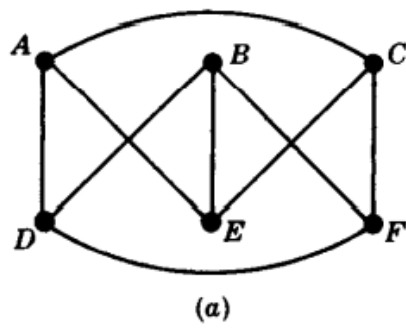


17 Determine whether the given graph has an Euler circuit or an Euler path if it exists, construct it.

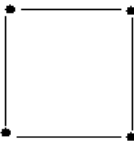
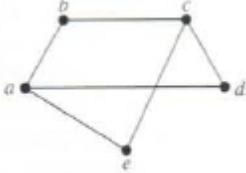
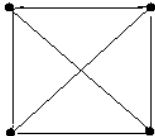
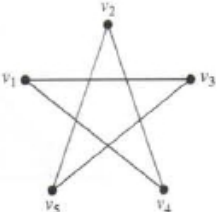
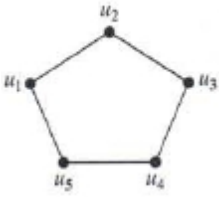
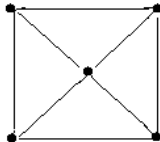


18 Determine whether given graphs have a Hamiltonian Circuit or a Hamiltonian path. If it exists, construct it. Justify your answer by giving appropriate reason.

19	<p>Find the length of a shortest path between a and z in the following weighted graphs:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>(a)</p> </div> <div style="text-align: center;"> <p>(b)</p> </div> </div>
20	<p>In Exercises find the chromatic number of the given graph.</p>
21	<p>Use paths either to show that these graphs are not isomorphic or to find an isomorphism between them.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p><i>G</i></p> </div> <div style="text-align: center;"> <p><i>H</i></p> </div> </div>
22	<p>Find all the cut vertices of the given graph.</p>
23	<p>Draw a planar representation, if possible, of the graphs (a), (b)</p>



24 Math the Following:

Types of Graph	Graphs
K_4	
2-regular	
C_5	
W_4	
2-cube	
$K_{2,2}$	
Bipartite	