

Tutorial 2

Q1. Find $f(1)$, $f(2)$, $f(3)$ and $f(4)$ and if $f(n)$ is defined recursively as $f(0) = 1$ and $n = 0, 1, 2, \dots$

(a) $f(n+1) = f(n) + 2$ $n=0$

$$f(1) = f(0) + 2$$

$$f(1) = 1 + 2 = 3$$

$$f(2) = f(1) + 2$$

$$f(2) = 3 + 2 = 5$$

$$f(3) = f(2) + 2$$

$$= 5 + 2 = 7$$

$$f(4) = f(3) + 2$$

$$= 7 + 2 = 9$$

(b) $f(n+1) = 2f(n)$

$$f(1) = 2^1$$

$$= 2$$

$$f(2) = 2^2$$

$$= 4$$

$$f(3) = 2^4$$

$$= 16$$

$$f(4) = 2^{16}$$

(c) $f(n+1) = 3f(n)$

$$f(4) = 81$$

$$f(0+1) = 3f(0)$$

$$= 3$$

$$f(2) = 9$$

$$f(3) = 3f(2) = 3 \times 9 = 27$$

Q.2 use the Euclidean algorithm to find.
~~gcd(100, 101)~~

(b)
$$\begin{array}{r} 123 \overline{) 277} \\ \underline{246} \\ 31 \\ 30 \overline{) 31} \\ \underline{30} \\ 1 \end{array}$$

$$\begin{array}{r} 30 \overline{) 30} \\ \underline{30} \\ 0 \end{array}$$

gcd = 1

gcd = 1

(c) gcd(100, 101)

$$\begin{array}{r} 100 \overline{) 101} \\ \underline{100} \\ 1 \end{array}$$

gcd = 1

(c) gcd(1524, 14034)

$$\begin{array}{r}
 9 \\
 1529 \overline{) 148139} \\
 \underline{13761} \\
 00278 \\
 1390 \\
 0139 \overline{) 278} \\
 278 \\
 \underline{000}
 \end{array}$$

gcd

$$\text{gcd} = 139$$

Q-3 Find Prime Factorization

a) ~~88~~

$$\begin{array}{r|l}
 2 & 88 \\
 2 & 44 \\
 2 & 22 \\
 11 & 11 \\
 & 1
 \end{array}$$

$$88 = 2^3 \times 11$$

b) 126

$$\begin{array}{r|l}
 2 & 126 \\
 3 & 63 \\
 3 & 21 \\
 7 & 7 \\
 & 1
 \end{array}$$

$$126 = 2 \times 3^2 \times 7$$

c) 729

$$\begin{array}{r|l}
 3 & 729 \\
 3 & 243 \\
 3 & 81 \\
 3 & 27 \\
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$729 = 3^6$$

d) 1001

$$\begin{array}{r|l} 7 & 1001 \\ 11 & 143 \\ 13 & 11 \\ & 1 \end{array}$$

$$1001 = 7 \times 11 \times 13$$

e) 1111

$$\begin{array}{r|l} 11 & 1111 \\ 101 & 11 \\ & 1 \end{array}$$

$$1111 = 11 \times 101$$

f) 909090

$$\begin{array}{r|l} 2 & 909090 \\ 5 & 454545 \\ 9 & 90909 \\ & 10101 \end{array}$$

$$\begin{array}{r|l} 2 & 909090 \\ 3 & 454545 \\ 3 & 151515 \\ 3 & 50505 \\ 5 & 16835 \\ 7 & 3367 \\ 13 & 481 \\ 37 & 37 \end{array}$$

$$909090 = 2 \times 3^3 \times 5 \times 7 \times 13 \times 37$$

- Q. 4) a) $68 \div 17 = 4$ yes
 b) $84 \div 17 = 4$ Not divided
 c) $357 \div 17$

$$\begin{array}{r} 21 \\ 17 \overline{) 357} \\ \underline{357} \\ 000 \end{array}$$

yes, divided

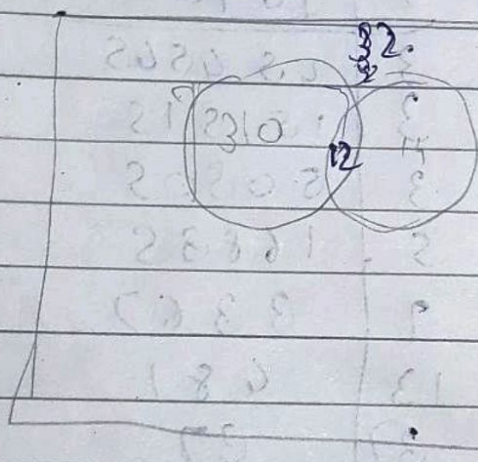
d) 1001

$$\begin{array}{r} 58 \\ 17 \overline{) 1001} \\ \underline{85} \\ 151 \\ \underline{136} \\ 151 \\ \underline{136} \\ 151 \end{array}$$

NOT divisible

- Q. 5) 22 \rightarrow Female
 18 \rightarrow male
 40 \rightarrow total

Q. 6)



$$\begin{array}{r} 44 \\ 32 \\ \hline 12 \end{array}$$

(a) $N(P \cap B) = N(P) + N(B) - N(P \cup B)$

$$12 \times 5 = 30 + 14 - 32$$

$$FE = 12$$

(b) $N(P) - N(P \cap B) = 30 - 12 = 18$

$$7) \quad n=8$$

$$r=3$$

$$\therefore nPr = P(n, r)$$

$$\therefore 8P_3 = P(8, 3)$$

$$= \frac{8!}{(8-3)!}$$

$$= \frac{8!}{5!} = 8 \times 7 \times 6 = 336$$

$$\text{or } 8 \times 7 \times 6 = 8 \times 7 \times 6 = 336$$

$$8) \quad {}^{52}C_5 = \frac{(52!)}{(5!)(47!)} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5!} = \boxed{2598960}$$

$$\Rightarrow {}^{52}C_7 = \boxed{2598960}$$

$$9) \quad \boxed{A} \boxed{B} \boxed{C} \boxed{D} \boxed{E} \boxed{F} \boxed{G} \boxed{H} \\ = 6! = \boxed{720} \quad \leftarrow \text{starting ABC}$$

$$\begin{array}{cccccc} \text{ABC} & , & \text{D} & , & \text{E} & , & \text{F} & , & \text{G} & , & \text{H} \\ \hline 1 & & 2 & & 3 & & 4 & & 5 & & 6 \end{array}$$

$$10) \quad C(30, 6) = {}^{30}C_6$$

$$= \frac{30!}{6! \times 24!} = \frac{30 \times 29 \times 28 \times 27 \times 26 \times 25}{6!} = \boxed{593775}$$