Greedy Algorithms

- It is used to solve optimization problem.

→ Optimization problem : (maximization or minimization Problem)
It is the problem of finding the best solution from all feasible solutions.

- Feasible solution:
Solution that satisfy the constraint.

- Greedy algorithm always makes the choice that look best at the moment.

- Greedy ~~al~~ algorithms do not always yield optimal solutions, but for many problem they do.

* Elements of Greedy Algorithm

Greedy choice property : Globally optimal solution can be obtained by making a locally optimal choice.

Optimal substructure :
A problem exhibits optimal substructure, if optimal solution to the problem ~~is~~ contains is composed of optimal solutions to subproblems.

→ If we can demonstrate that the problem has these properties, then we can develop greedy algorithm.

Greedy vs Dynamic

→ In Greedy subproblems are independent.

→ It is top down approach

→ After making choice we can't undone.

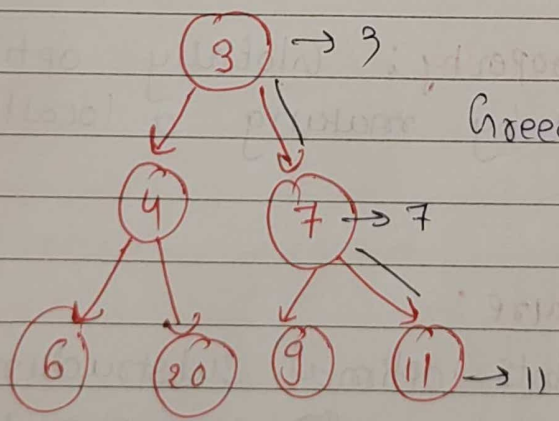
→ It will not depend on future choice

→ Finding local optimal solution at each step.

→ It will not give always optimal solution.

Example

Problem: To find largest sum from root to leaf value.



Greedy answer = 3+7+11
= 21

This is not optimal

3 + 4 + 20 = 27 → optimal solution

Making locally optimal choices ending up with non-optimal solution.

Applications of Greedy strategy

optimal solution:

- making change Problem
- minimum spanning tree (MST): Prim's and Kruskal's
- single-source shortest path: Dijkstra's algo
- Activity selection Problem
- Huffman Code

Approximations / heuristics:

- Travelling salesman Problem (TSP)
- Knapsack Problem

Activity selection problem (Greedy approach) O(nlogn)

→ It is a optimization problem.

Problem is to select the maximum number of activities that can be performed by a single person or machine, assuming that a person can only work on a single activity at a time

constraints : activities should be compatible or non overlapping or non conflicting.

objective : maximum set of nonconflicting activities

i/p → set of activities, start time and finish time

Activities a_i and a_j are compatible if the interval $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap

$0 \leq s < f < \infty$

Greedy Approach of activity selection

O(nlogn) sort the activities according to their finish time

- Select first activity from sorted array.
- The greedy choice is to always pick the next activity whose start time is greater or equal to finish time of priviously selected activity

Greedy-Activity-Selector (s, f) Iterative algorithm

```
n = length[s]
A = {a1}
i = 1
for m = 2 to n
    do if s[m] ≥ f[i]
        then A = A ∪ {am}
             i = m
return A
```

O(n)

Activity selection Problem

→ sort activity with it's increasing Finish time (O(nlogn))

Initially R-A-S(s,f,0,n)

Recursive-Activity-selector (s,f,k,n)

Initially $R_i = 0$

```
Recursive-Activity-selector (s,f,k,n)
    m = k+1
    while m ≤ n and s[m] < f[k]
        m = m+1
    if m ≤ n
        return {am} ∪ Recursive-activity-selector(s,f,m,n)
    else Return ∅
```

Example :

s	1	3	0	5	8	5
f	2	4	6	7	9	9

R-A-S(s,f,0,6)

m = 0+1 = 1

while 1 ≤ 6 and 1 < 0 ×

1 ≤ 6 ✓

{a1} ∪ R-A-S-(sf, 1, 6)

k=1

s	1	3	0	5	8	5
f	2	4	6	7	9	9

m

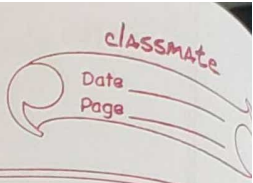
s	1	3	0	5	8	5
f	2	4	6	7	9	9

k m m

3 < 2 ~~0 < 2~~

m

Fractional Knapsack Problem



Knapsack problem

- Fractional
  - Items should be divisible
  - Greedy
- 0-1
  - not divisible
  - Dynamic

Fractional Knapsack Problem

- n items, each have w_i and Profit P_i, and given knapsack capacity C.
- The problem is to ~~find~~ fill the knapsack with object/ items without exceeding knapsack's capacity in order to maximum the total Profit.

maximize $\sum_{i=1}^{n} P_i x_i$

subject to $\sum_{i=1}^{n} w_i x_i <= C$

x_i denote fraction of object i

$0 \le x_i \le 1$

$1 \le i <= n$

Different approaches

1. Maximum Profit
2. minimum weight
3. max value/weight Ratio

Example:

$m=5$ objects and knapsack capacity $W=100$ as in Table 1.

	1	2	3	4	5
W	10	20	50	40	50
V	20	30	66	40	60
V/W	2.0	1.5	2.2	1.0	1.2

Select	x_i					Profit/value
max v_i	0	0	1	0.5	1	146
~~max~~ min w_i	1	1	1	1	0	156
max v_i/w_i	1	1	1	0	0.8	164

Optimal

Greedy approach (W, v, w)

$O(n \log n)$ Sort n objects from large to small base on v_i/w_i
- Assume array $w[1,..n]$ and $v[1...n]$ store the respective weights and values after sorting
- Initialize array $x[1...n]$ to zeros

weight = 0, i = 1, value = 0
while (i <= n and weight < W)
    if weight + w[i] ≤ W
        x[i] = 1
    else
        x[i] = (W - weight)/w[i]
    weight = weight + x[i] * w[i]
    value = value + x[i] * v[i]
    i++
Return value

$O(n \log n)$

Spanning Tree → Does not generate cycle.

Spanning Tree: It is subset of edges of connected undirected graph, that connects all the vertices togather without any cycle

Total No. of possible spanning tree = $E C_{V-1}$ − no of cycle (include those cycle which contain not all vertices)

If $G = (V, E)$, spanning tree $G' = (V', E')$

$V' = |V|$

$E' = |V| - 1$

- Minimum Spanning Tree:
  - Unidirected weighted connected graph.

Spanning tree which has total cost minimum

two methods to find MST

1) Prims
2) Khushkal

Best = $O((V+E)\log v)$ or $O(E \log v)$

worst $O(v^2 \log v)$

* Prims Algorithms (Greedy approach)

Remove all loops and parrallel adges

Prim (G, w, r)

for each u in V[G] } O(v)
  do key[u] = ∞
  π[u] = NIL π for parent

key[r] = 0

Buildheap Q = V[G] Queue → Q → O(v)

while Q ≠ ϕ → O(v) (extract minheap algo)
  do
  u = Extract-MIN(Q) → minheap (extract vertex which has min key value)
  for each v in Adj[u]
    if v in Q and w(u,v) < key[v]
      π[v] = u
      → key[v] = w(u,v) → Decreaskey of minheap

O(v) * log v
v * (v-1)
v * (v-1)
v * (v-1)
v * log v * (v-1) → Dense

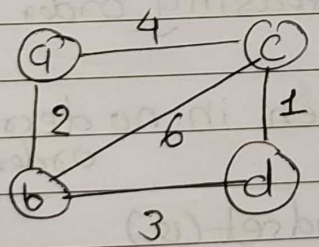
worst case = $v^2 \log v$ (complete graph)

(Q is min Priority Queue)

in worst case for one node adjsane node will be v-1 (in complete graph)

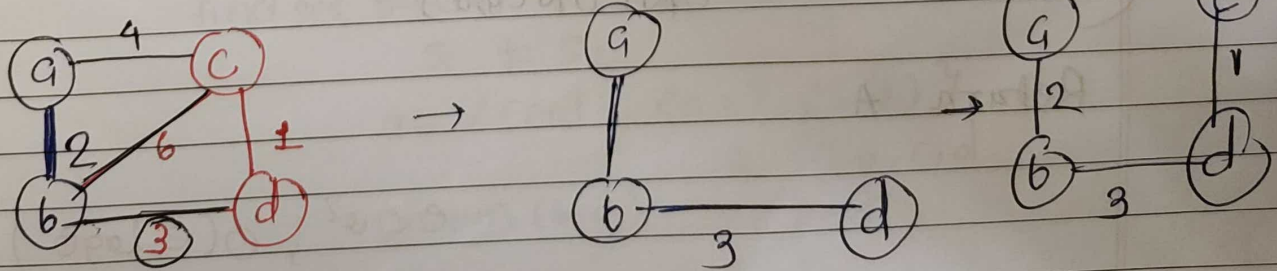
no. of edges in worst case $E = v^2$
in Best case = v
Sparse $E = v$

Example:



- select arbitary vertex

- find minimum adjucancy edges and select in MST (if it create no cycle)



total minimum cost = 6

Krushaka's Algorithm

- Greedy approach
- Sort the edges in non-decreasing order of lengths (weights)
- Build forest of mst by growing tree one edge at a time.
- At each iteration, add the next edge from sorted list unless this would create a cycle. (If it would, skip the edge)
- Repeatedly connect the trees to create a subset of a mst, until all nodes are covered.

Forest tree → more than one

* Kruskal(G, w)

A = ∅ set A = { } O(v)

for each vertex v in V[G] { }

do makeset(v)

O(eloge) sort edges of E in nondecreasing order by weight w

O(v+e)logv for each edge (u,v) in E, taken in no decreasing order by weight

do if Findset(u) ≠ findset(v)

A = A ∪ {(u,v)}

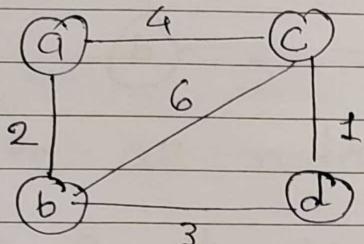
UNION(u,v)

Return A

$e < v^2$; O(e log v)

Example

with Algo;



1 → A = { }

2 → {a}, {b}, {c}, {d}
S1 S2 S3 S4

3 E = {(c,d), (a,b), (b,d), (a,c)}
1 2 3 4

take (c, d)

$S_3 \neq S_4$

if Find set (c) ≠ Find set (d) ✓

both are in different set

A = {(c, d)}

Union (c,d)

{a} {b} {c,d}

S_1	a
S_2	b
S_3	c,d
S_4	~~d~~

take (a,b)

Find set (a) ≠ find set (b) ✓
S_1 S_2

A = {(c,d), (a,b)}

Union {a,b}

{a,b} {c,d}

S_1	a,b
~~S_2~~	~~b~~
S_3	c,d

take (b, d)

find set (b) ≠ find set (d) ✓

$S_1 \neq S_2$

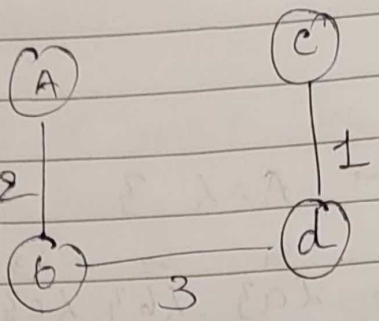
A = {(c,d), (a,b), (b,d)}

Union (b,d)

S_1	a,b,c,d
S_2	~~c,d~~

now all u & v are in one set so we can't go further,

Final MST

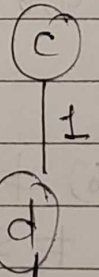


→ ~~Non~~ kruskal ~~os~~ without Algo

step: 1 - short edges in non decreasing order by ω

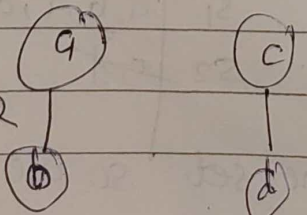
(c,d) → 1
(a,b) → 2
(b,d) → 3
(a,c) → 4
(b,c) → 6

Select 1st edge and include in MST if It will not generate cycle
Repeat till all node are added (Repeat till |V|-1 edges in in MST

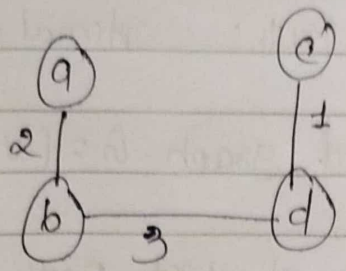


(a,b) will be selected

Forest trees are generated

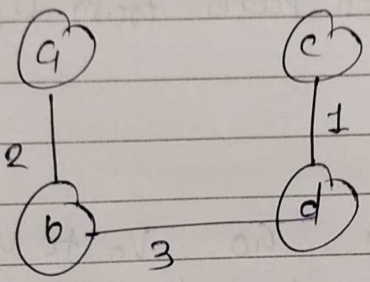


select (b,d) → 3



- Here, all nodes are connected so, stop

∴ Result is



Single source Shortest Path

* lemma → subpaths of shortest path are shortest Paths (optimal substructure)

Given a weighted directed graph $G = (V, E)$ with weight function $w : E \to R$,
Let $p = \langle v_0, v_1 \dots v_k \rangle$ be a shortest path from vertex v_0 to vertex v_k and, for any i and j such that $0 \le i \le j \le k$, let $p_{ij} = \langle v_i, v_{i+1}, \dots v_j \rangle$ be the subpath of p from vertex v_i to v_j.
Then p_{ij} is a shortest path from v_i to v_j

i, j vertex

→ Proof :

If we want to go v_0 to v_k via i and j and if it will give shortest path. then subpath of p_{ij} will be shortest subpath

$$p = p_{0i} + p_{ij} + p_{jk}$$

$$w(p) = w(p_{0i}) + w(p_{ij}) + w(p_{jk}) \quad \text{— (1)}$$

for contradiction, let's assume that p_{ij} is not shortest path. and another p'_{ij} is shortest path.

$$\therefore w(p'_{ij}) < w(p_{ij})$$

Instead of taking $w(p_{ij})$ we will take another path weight $(w(p'_{ij})$ so we will replace $w(p_{ij})$ with $w(p'_{ij})$

$$w(p) = w(p_{0i}) + w(p'_{ij}) + w(p_{jk})$$

So Now our assumption is incorrect
P'ij will always be shortest path between i to j

Shortest Path Problem :

Single Destination shortest path:

- Find a shortest path to a given destination vertex t from each vertex.

Single Pair Shortest path

- Find a shortest path from u to for given vertices u and v (Dijkstra , Bellman-ford)
  +ve edge , both +ve and -ve edge)

All pairs shortest-Paths problem (Floyd warshall)

find a shortest path from u to for every pair of vertices u and v.

* All Single source Shortest Path
→ Greedy approach
- For nonnegative edge weight

→ Dijkstra Algorithm ! (Greedy algo)

we assume that $w(u,v) \geq 0$ for each edge $(u,v) \in E$

directed/undirected weighted graph

~~Int~~
DIJKSTRA(G, w, s)
{
Initialize-single-source(G,s)
S ← ∅ ⟶ Shortest path tree set
Q ← V[G] → Min priority Queue
while Q ≠ ∅
do u ← Extract-MIN(Q) (extract vertex which has minimum distance value)
S ← S ∪ {u}
for each vertex v ∈ Adj[u]
do RELAX(u,v,w)
}

→ Initialize-single-source(G,S)
└ source vertex
{
for each vertex v ∈ V[G]
do d[v] ← ∞
π[v] ← NIL
d[s] = 0
}

```
RELAX(u,v,w)
{
- If d[u]+w(u,v) < d[v]
      then   d[v] ← d[u]+w(u,v)
             π[v] ← u
}
```

Time complexity

Depends upon how we will implement priority queue

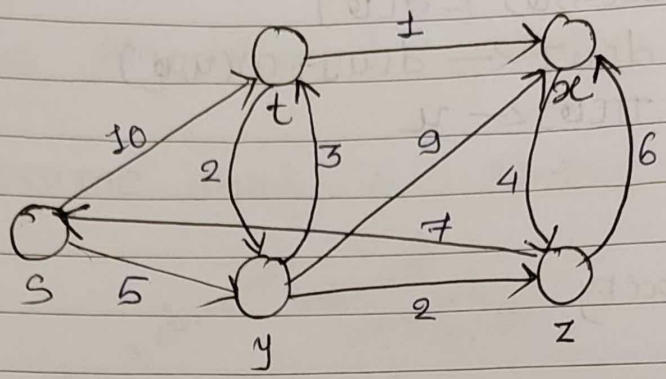
Time complexity
Priority queue

- graph represented by weigh matrix & array for queue
  ↓
  $O(|V|^2)$

- graph represent by adjacency list and minheap for queue
  ↓
  $O(E \log V)$

Example :

Through ~~without~~ Algorithm :



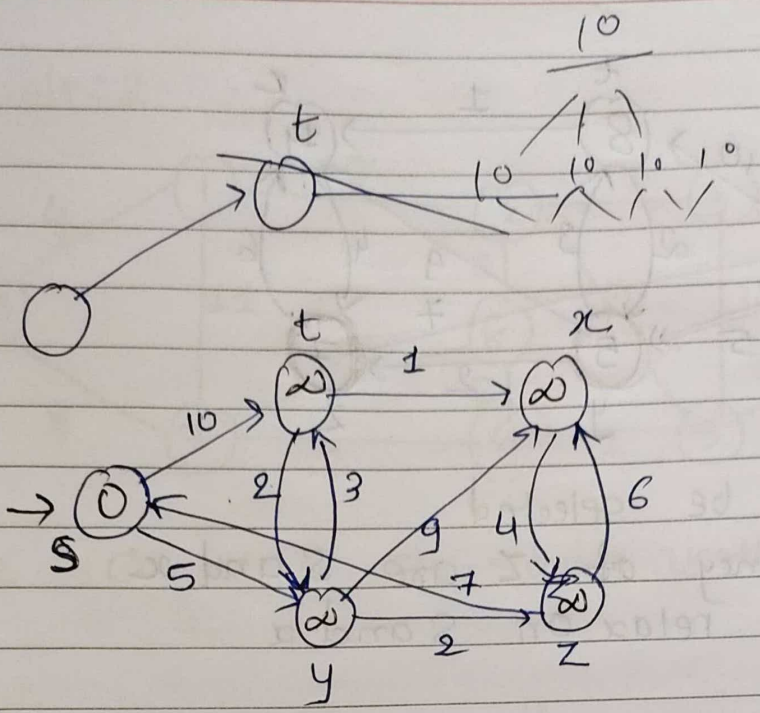
Source vertex = S

Initialization

→ other than source vertex distance of each vertex is ∞ and π value will be NIL

→ distance of source vertex will be 0 Parent π value will be NIL

selected vertex	S	d[v] t	x	y	z
~~S~~	(0)	∞	∞	∞	∞
~~S~~ y		10	∞	(5)	∞
z		~~8~~	14		(7)
x		(8)	13		
z			(9)		



adjacancy vertex of s are t and y

apply Relaxation on t and y

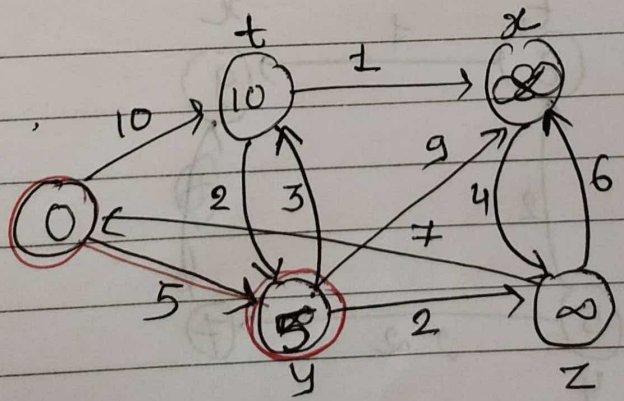
Relax (S, t, 10)

if (d[u] + w(u,v) < d[v])

0 + 10 < ∞ ✓

d[v] = 10 d[t] = 10

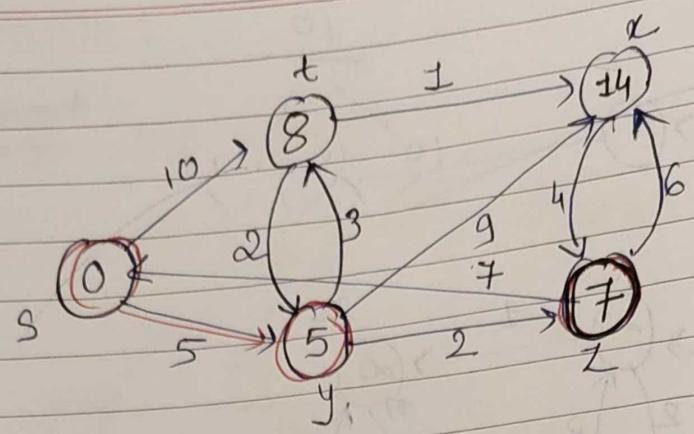
same way d[y] = 5



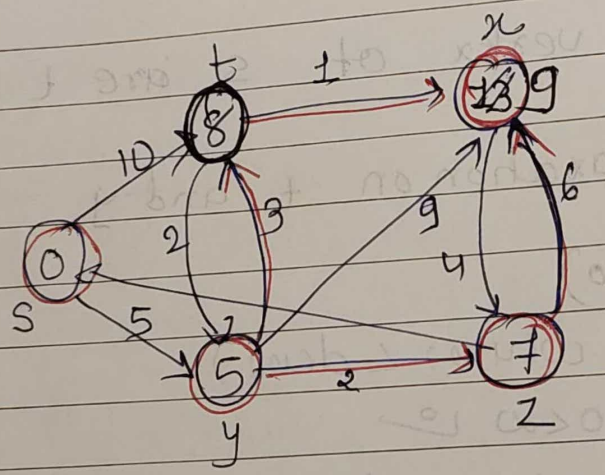
Now y has minimum distance so y will be selected.

adjacancy of y are t, x, z

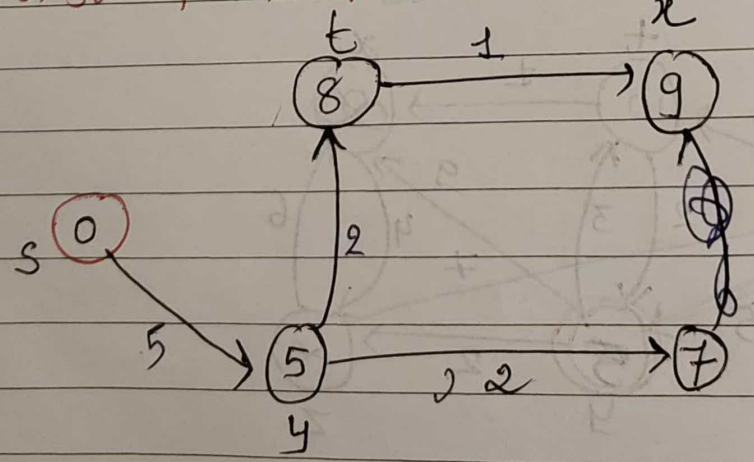
apply Relax (u, v, w) on t, x, z



z will be selected
Adjacancy of z are s and x
apply relax on s and x



Shortest path from s is



classmate
Date
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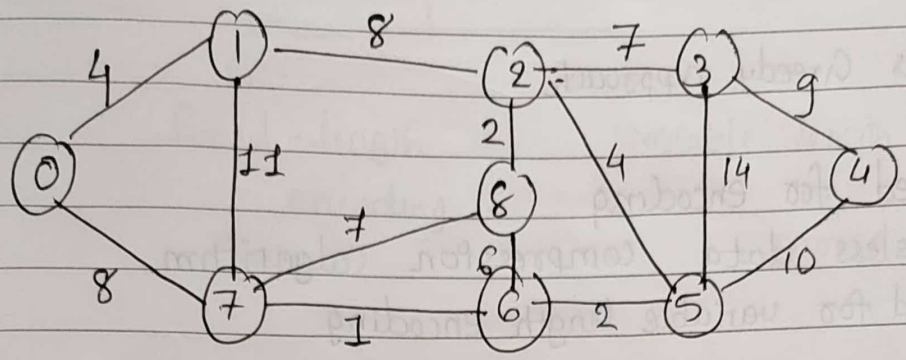
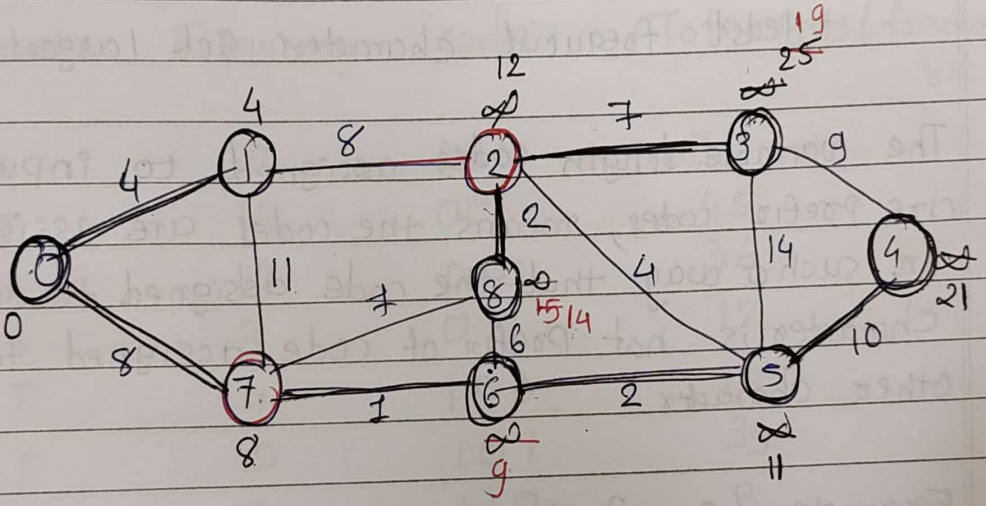
* Example: 2



take 0 as a ~~shor~~ source vertex



* Huffman coding

- Uses Greedy approach

- Used for encoding
- Lossless data compression algorithm.
- Used for variable length encoding

Idea of variable length encoding
- most frequent characters get smallest code
- least frequent characters get largest code.

The variable length codes assigned to input characters are prefix codes, means the codes are assigned in such a way that the code assigned to one character is not prefix of code assigned to any other character.

Example {0, 11} ✓

{0, 1, 11} ✗ 1 is prefix of 11

{a:00, b:01, c:0, d:1} ✗

code assign to c is prefix of a and b

- Huffman coding makes sure that there is no ambiguity when decoding the generated bit stream.

Two-Types of codes

- fixed-length encoding (ASCII)
- variable length encoding morse code

* Fixed length encoding

character	Frequency	code	Total Bits (frequency × bit length)
A	10	000	30
E	15	001	45
I	12	010	36
S	3	011	12
T	4	100	12
P	13	101	39
Newline	1	110	3
~~8×1=56~~		~~16×3=48~~ 24	total bit used : 174

Here total ~~8~~ 7 characters are there so we required (2^3) 3 bits.

Can we reduce no. of bits ?
Yes using variable length encoding

- when we will send the msg we have to send character & code table along with encoded msg. so d. we can do decoding.
  (~~56~~ + ~~48~~ 21) + 174 bits will be received
  ~~248~~ 252

→ ~~A~~ .

If we will use ASCII for sending msg than $58 \times 8 = 464$ bits are required

Huffman coding procedure

i/p → array of unique characters along with frequency count

o/p → Huffman tree

1) Build a huffman tree
2) Traverse the Huffman tree and assign codes to characters.

Build a huffman tree

1 - Create a leaf node for each character and a build min heap of all leaf nodes

2 - Extract two nodes with minimum frequency from min heap

3.1 - Create a new internal node with frequency equal to sum of two node frequenc.

3.2 - Make first extracted node as its left child and other as it's right child. Add this node to the min heap

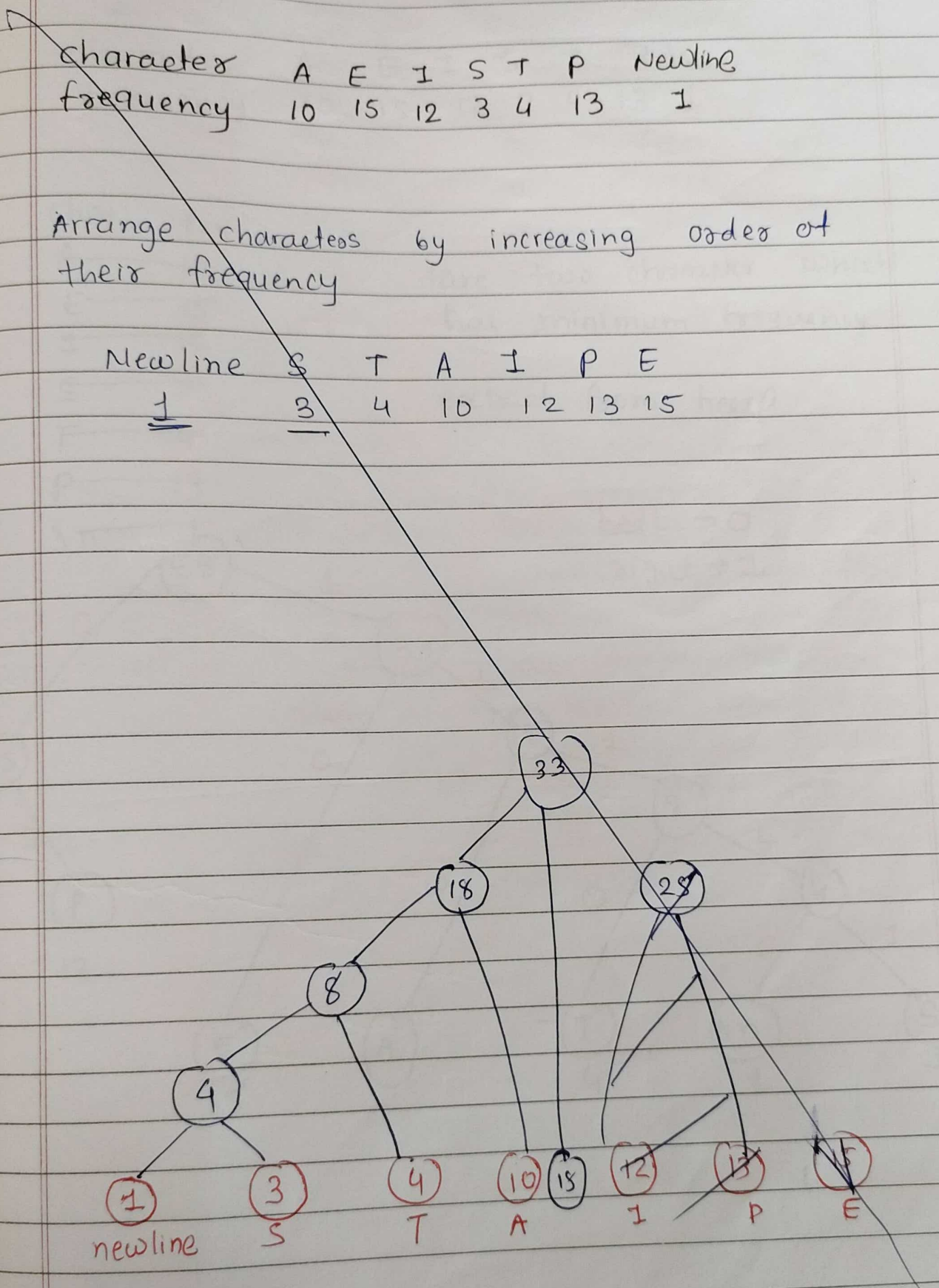
4 - Repeat step 2 & 3 until heap contain only one node. The remaining node is root node and tree is complete.

Example

character	A	E	I	S	T	P	Newline
frequency	10	15	12	3	4	13	1

Arrange characters by increasing order of their frequency

Newline	S	T	A	I	P	E
1	3	4	10	12	13	15



Example

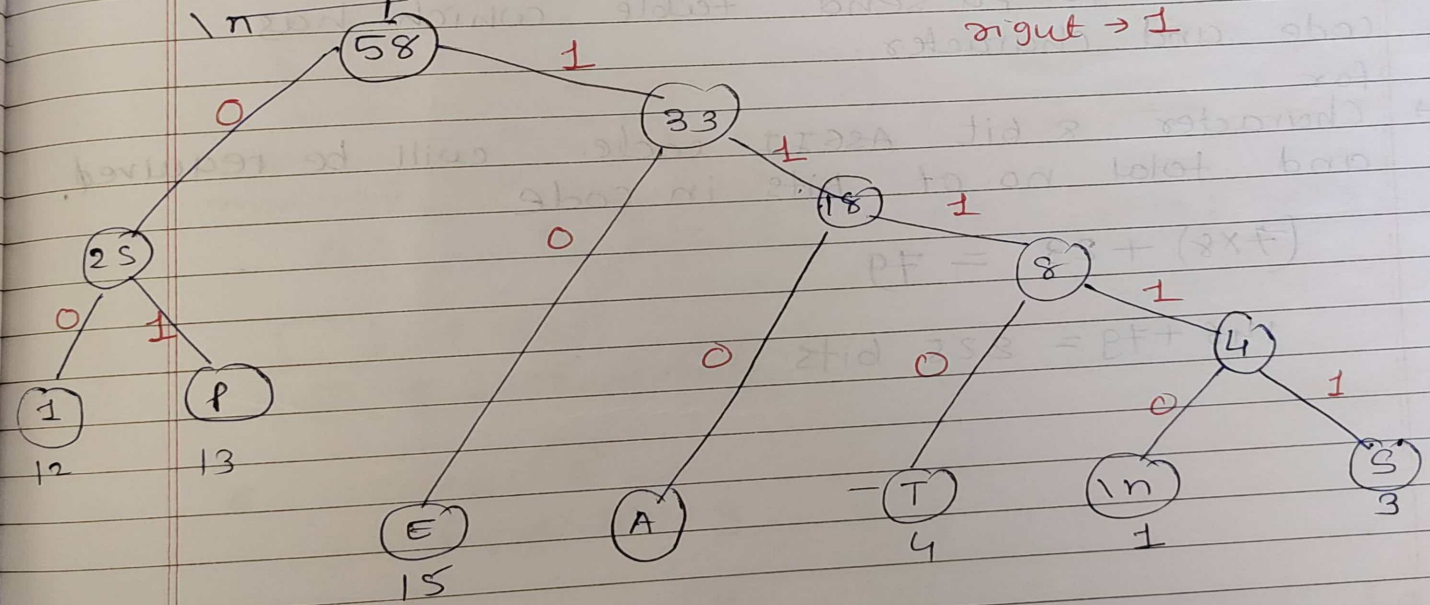
character	A	E	I	S	T	P	\n
freequency	10	15	12	3	4	13	1

chas	fre
A	10
E	15
I	12
S	3
F	4
P	13
\n	1

take two character which has minimum frequency

extract from heap

left → 0
right → 1



Char.	code	freq	Total bits
A	110	10	30
E	10	15	30
I	00	12	24
S	11111	3	15
T	1110	4	16
P	01	13	26
\n	11110	1	5
			146

Total bits for sending massage

we have to ~~pa~~ send table which has code and character.

~~for~~

→ Character 8 bit ASCII code will be required, and ~~total~~ no. of bits in code

$(7 \times 8) + 23 = 79$

$146 + 79 = 225$ bits

Algorithm:

Huffman (C) → always generate full binary tree
{ C is set of n characters and each character has it's frequency

$n = |C|$

min priority queue $Q = C$ → $O(n)$

for $i = 1$ to $n-1$ $(n-1)$

allocate a new node z

z.left = x = Extract_MIN(Q) } $\log n$ $(n-1)$ min heap

z.right = y = Extract_MIN(Q)

z.freq = x.freq + y.freq.

Insert (Q, z)

return Extract_MIN(Q) // return root of the tree

→ $O(n \log n)$

We can reduce running time to $O(n \log\log n)$ by replacing min heap with Van Emde Boas tree.

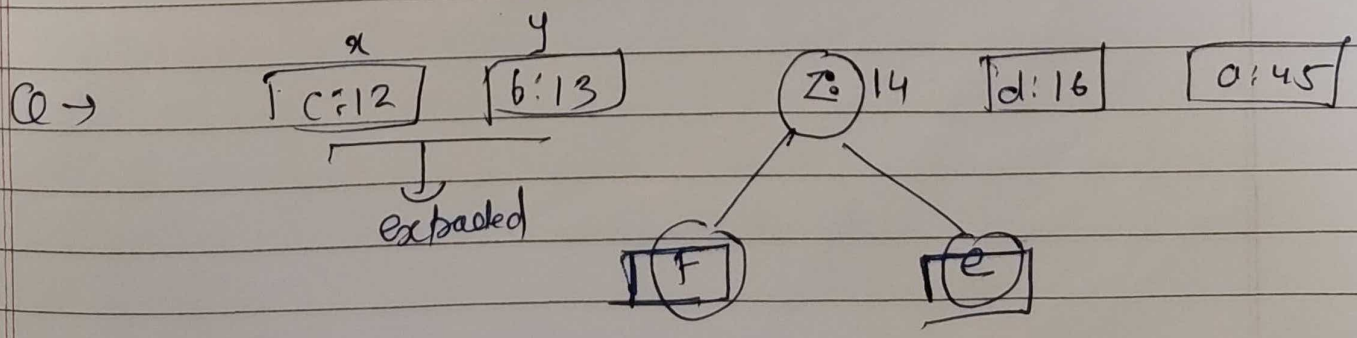
Q: | F:5 | e:9 | c:12 | b:13 | d:16 | a:45 |

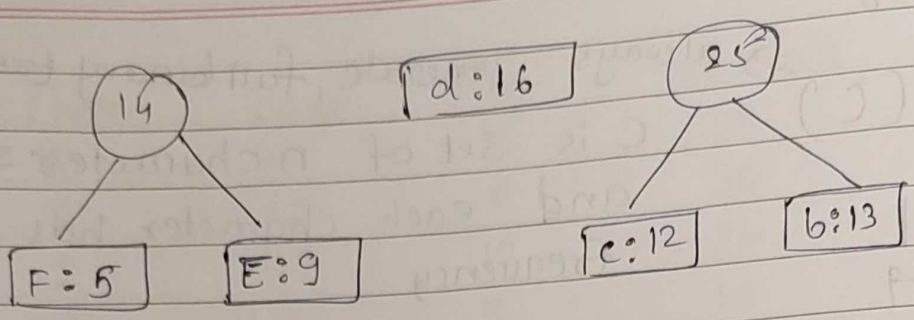
□ → leaf

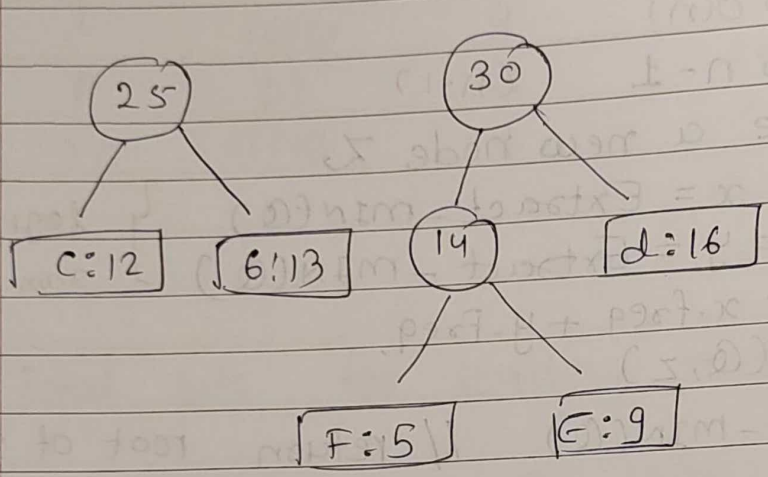
○ → Internal node,

z.left = F = x

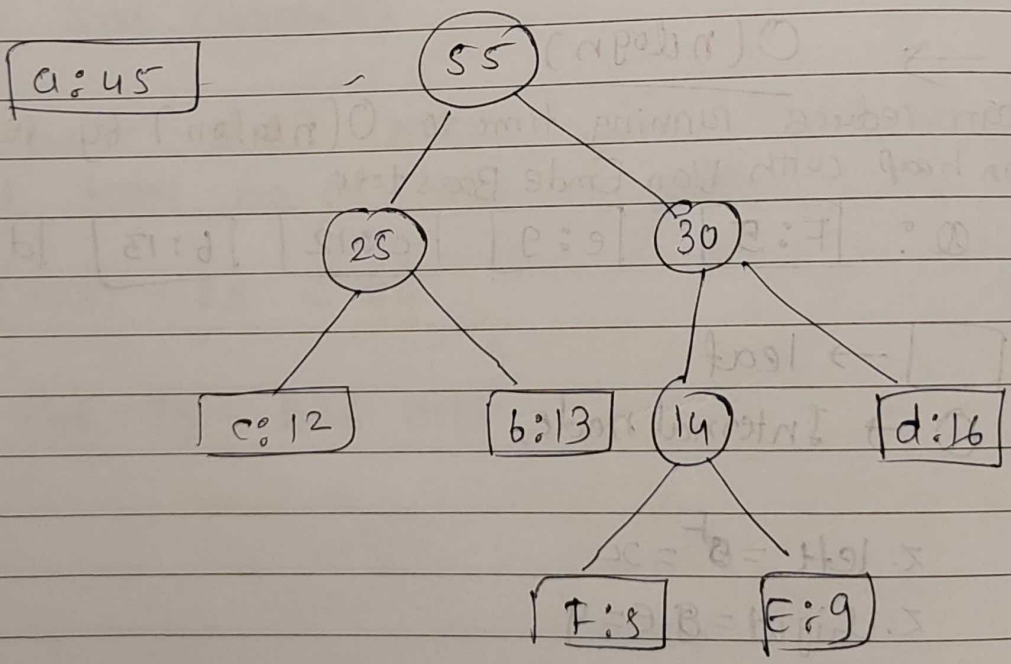
z.right = e = y

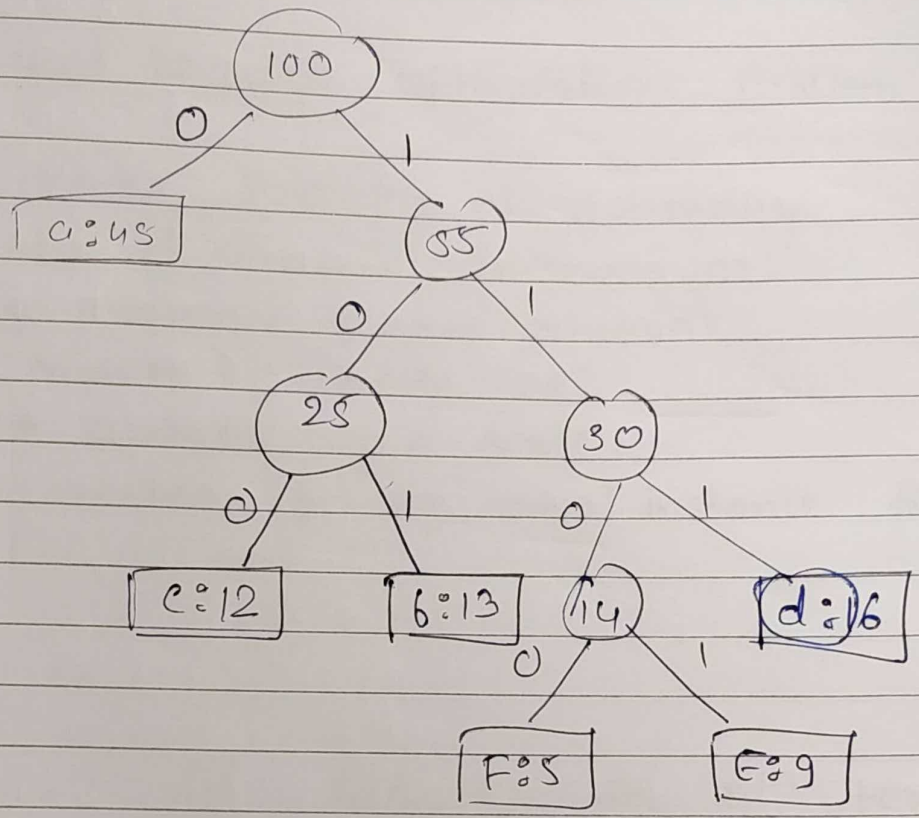






Extract 2 chara
which has
minimum fre





Ch: 4

Dynamic Programming

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+ It is used to solve optimization Problem.

- It is follow Principle of optimally

- Main Idea of Dynamic Programming:
- set up a recurrence relation solution
- solve smaller Instances once
- Record solutions in a table.
- Extract solution to the initial instance from table.

→ Problem of fibbonacci of n^{th} term

Algorithm

```
fib(n)
{
  in (n<=1)
     return n;
  else
     return fib(n-2) + fib(n-1);
}
```

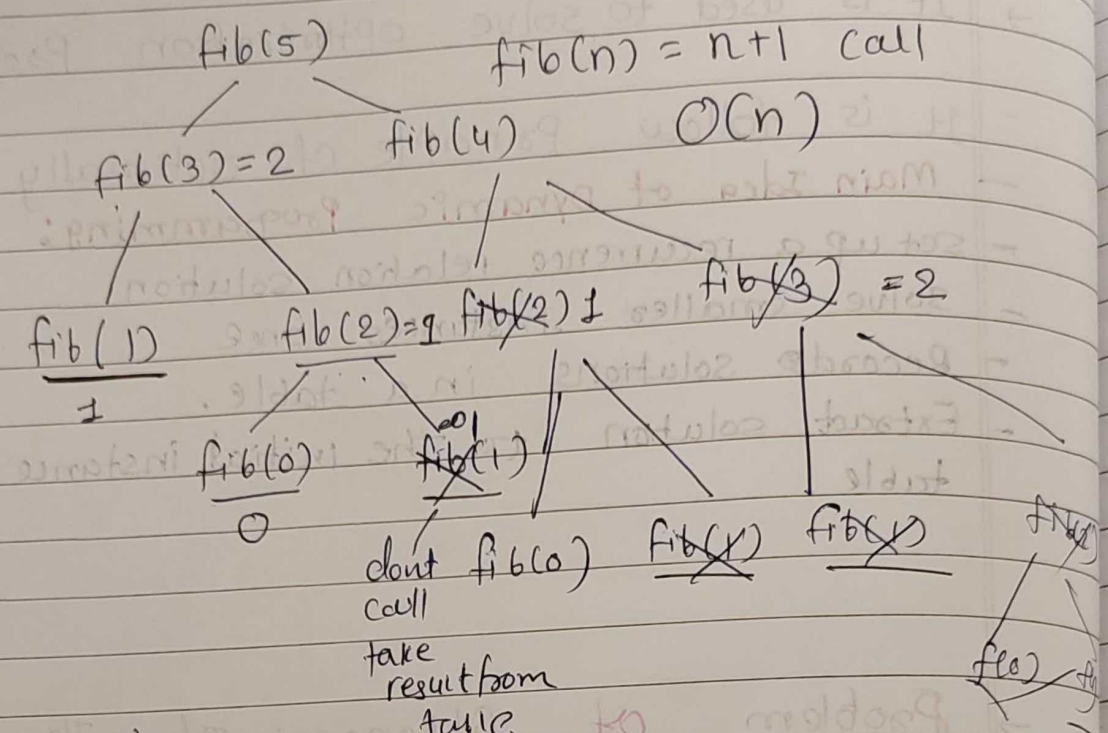
Recurrence Relation

$$fib(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ fib(n-1) + fib(n-2) + 1 & n>1 \end{cases}$$

total = 6 calls

fib(n) = n+1 call

O(n)



Time complexity

let us assume $T(n-2) \approx T(n-1)$

$T(n) = 2T(n-1) + 1$

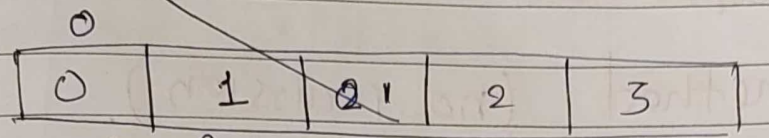
Time complexity $\approx O(2^n)$

Can we reduce ?

As you can see in tree Same function call are happen again and again.

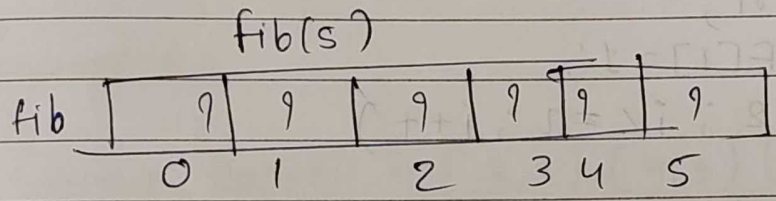
So ~~ist~~ instead of executing all time if we will store result of each and every unique function call.

lets take array



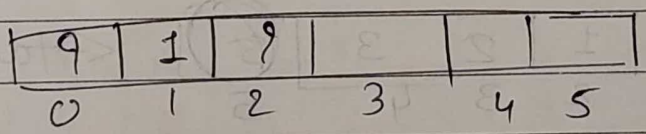
Tab Memoization

→ Top down approach



fib(4) fib(3)

fib(1) → return 1.

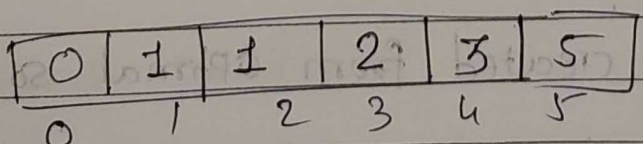


fib(2)

fib(1) fib(0)

↓ ↓

dont call return 0



→ Recursion call will be made

only problems that satisfy the Principle of optimality are suitable for DP.

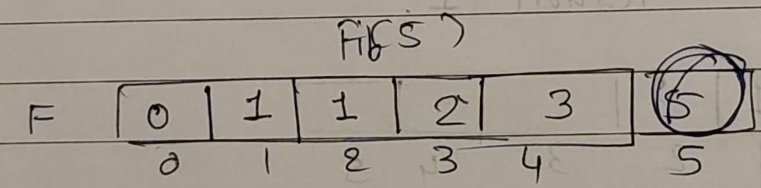
→ Principle of optimality applies if the optimal solution to a problem always contains optimal soln to all subproblem

Tabulation Method

→ bottom up approach

- Iterative method (no recursion).

```
int fib(int n)
{
   if (n<=1)
      return n;
   F[0]=0 ; F[1]=1;
   for( i=2 ; i<=n ; i++ )
      {
         F[i] = F[i-2] + F[i-1]
      }
   return F[n];
}
```



time compleocity : O(n)

* Elements of Dynamic Programming

- Optimal substructur:
  optimal solution is created from optimal solution of subproblems

- overlapping subproblem.
- optimal solution to a subproblem is reused in solving more than one subproblem.

0-1 Knapsack Problem

→ Optimization Problem (maximization Problem)

→ Items are not divisible.

Optimal substructure:

$$B[k,W] = \begin{cases} B[k-1, w], & \text{if } w_k > w \\ \max\{ B[k-1, w], v_k + B[k-1, w-w_k] \} \end{cases}$$

* Example:

(weight, profit) = {(3,2) (4,3) (6,1) (5,4)}

W = 8, n = 4

size of table = ~~9~~ 5 × 9

(w, v)	i \ w →	0	1	2	3	4	5	6	7	8
(3,2)	0	0	0	0	0	0	0	0	0	0
(4,3)	1	0	0	0	2	2	2	2	2	2
(6,1)	2	0	0	0	2	3	3	3	5	5
(5,4)	3	0	0	0	2	3	3	3	5	5
	4	0	0	0	2	3	4	4	5	6

↓ max Profit

Maximum Profit = 6

Now, we will find which items are selected.

V

i \ w_0 →	0	1	2	3	4	5	6	7	8
0	0	0	0	0 ←	0	0	0	0	0
1	0	0	0	2 ←	2	2	2	2	2
2	0	0	0	2 ←	3	3	3	5	5
3	0	0	0	2 ←	3	3	3	5	5
4	0	0	0	2	3	4	4	5	6 ←

2 - 2 = 0

6 - 4 = 2

start from V[4, 8], check value of above cell.
if value is different then select
4th item. (means 4th item was in knapsack)
so $x_4 = 1$
else $x_4 = 0$

Next check for Repeat same step for V[3,8]
Now Total Profit = 6
Repeat until we get
4th items profit = 4
∴ Remaining profit = 6 - 4 = 2
Now go to previous row and find 2 from right side and check it's above cell's value,
if same then move upwards in same colums

$x = [1, 0, 0, 1]$

0-1 Knapsack Algorithm

for $w = 0$ to W
$V[0, w] = 0$ } bag capacity 0 so Profit will be 0

for $i = 1$ to n
$V[i, 0] = 0$ } no item is selected so Profit 0

```
for i=1 to n
    for w=1 to n

        if w_i <= w // item i can be part of soln.
            if b_i + V[i-1, w-w_i] > V[i-1, w]
                V[i,w] = b_i + V[i-1, w-w_i]
            else
                V[i,w] = V[i-1, w]

        else
            V[i,w] = V[i-1;w]  →  w_i > w
```

→ Time complexity $O(n*W)$

How to find actual knapsack items.

V[n, w] is maximum profit of items that can be placed in knapsack.

let i=n and k=w

if V[i, k] ≠ V[i-1, k] then
mark i^{th} item as in the knapsack
i = i-1, k = k-w_i

else
i = i-1

* Another Example.

W=5

item(i)	1	2	3	4
value(v)	100	20	60	40
Weight(w)	3	2	4	1

Ans. max Profit=140
item selected
4^{th} and 1st

→ w

		0	1	2	3	4	5
v, w	0	0	0	0	0	0	0
(100, 3)	1	0	0	0	100	100	100
(20, 2)	2	0	0	20	100	100	120
(60, 4)	3	0	0	20	100	100	120
(40, 1)	4	0	40	40	60	140	140

Change-Making Problem

* It is optimization problem

→ Problem Statement:

i/p → Given values, we want to make a change for v cents.
we have infinite supply of each coin (denomination)

o/p → Minimum number of coins to make the change.

Examples:

i/p → coins[] = {25, 10, 5}, V = 30
ouptut : Two coins {25, 5}

i/p → coins[] = {9, 6, 5, 1}, V = 12
o/p → Two coins {6, 6}

Optimal Sub Structure.

a c[i, j] = c[i-1, j] if j < di;

1. if i = 1, c[i, j] = c[i, j - di] + 1, if i = 1

2. if j < di then c[i, j] = c[i-1, j]

3. otherwise c[i, j] = min (c[i-1, j], 1 + c[i, j - di]).

Find out min of no. of coins to make change of given amount using given coins.

Coins (denominations) = {1, 5, 6, 9}

Change W = 10

i \ w →	0	1	2	3	4	5	6	7	8	9	10
0 (1)	0	1	2	3	4	5	6	7	8	9	10
1 (5)	0	1	2	3	4	1	2	3	4	5	2
2 (6)	0	1	2	3	4	1	1	2	3	4	2
3 (9)	0	1	2	3	4	1	1	2	3	1	2

(w → i, j →)

look in same row

if coins[i] > w → copy value from above cell.

for W[1, 7] → min(W[0, 7), 1 + W[1, w-di]

= min(7, 1 + W[1, 7-5])

= min(7, 1 + W[1, 2])

= min(7, 1 + 2) = 3

→ minimum no. of coins = 2

which denomination

(5,

C

i \ j →	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	0	0	1	2	3	4	5	6	7	8	9
5	1	0	1	2	3	4	1	2	3	4	2
6	2	0	1	2	3	4	1	1	2	3	2
9	3	0	1	2	3	4	1	1	2	3	1	2

10
2 } diff so 5 will be selected.

now W - di = 10 - 5 = 5

check 5 in same row find out 5th colum

5
1 } diff means 5th coin is selected.

W - di = 5 - 5 = 0 how stop.

·coins are (5, 5)

Algorithm

n → total no. of denomination
w → change

```
for i = 1 to n
    c[i,0] = 0

for i = 1 to n
{
  for j = 1 to w
  {
    if (i == 1 && j < d[i])
        c[i][j] = ∞

    else if (i == 1)
        c[i][j] = 1 + c[1, j - d[i]];

    else if (j < d[i]) then
        c[i][j] = c[i-1, j]

    else
        c[i][j] = min(c[i-1, j], 1 + c[i, j-d[i]])

  }
}

return c[n, w]
```

time complexity $O(n \times w)$

Matrix chain Multiplication

Matrix multiplication

→ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$

3×3 3×2

$$C = A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} \end{bmatrix}$$

3×2

→ total 18 matrix multiplication

$3 \times 3 \times 2 = 18$

Condition for matrix multiplication is

no. of column in first matrix = no. of row in 2nd matrix

⇒ Dimension of Resultant matrix = no. of row in 1st matrix × no. of column is 2nd matrix

Matrix chain multiplication

- It is minimization problem

$A_1 \times A_2 \times A_3$

2 3 3 4 4 2 we can multiply

d_0 d_1 d_1 d_2 d_2 d_3.

we can multiply two matrix at a time

$(A_1 \times A_2) \times A_3$ $A_1 \times (A_2 \times A_3)$ associative.

Ans is same

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$2 \times 3 \times 4$

$(A_1 \times A_2) \times A_3$

2 3 3 4 4 2

$2 \times 3 \times 4 = 12$

Result matrix 2×4 $\quad$ 4×2

$2 \times 4 \times 2 = 16$

$24 + 16 = 40$

$A_1 \times (A_2 \times A_3)$

2 3 3 4 4 2

$3 \times 4 \times 2 = 24$

2 3 3 2

$2 \times 3 \times 2 = 12$

$12 + 24 = 36$ multiplicat

Order is matter

Im mcm we find order in which we have to perform minimum multiplication

→ Here, we find out how we will multiply means in which order. Best possible paranthesizatio

DP approach :

Possible find out all possible parenthesization and pick best one.

Formula :

$C[1,3] = A_1 \times A_2 \times A_3$

2 3 3 4 4 2

d_0 d_1 d_1 d_2 d_2 d_3

$(A_1 \times A_2) \times A_3$

2 3 3 4 4 2

$C[1,2] = 2 \times 3 \times 4$ $\quad C[3,3] = 0$

[2 4] [4, c]

d_0 d_2 d_2 d_3

$C[1,1] = 0$

$C[\underset{i}{1}, \underset{j}{3}] = C[\underset{i}{1}, \underset{k}{2}] + C[\underset{k+1}{3}, \underset{j}{3}] + d_{\overset{i-1}{0}} \times d_{\overset{k}{2}} \times d_{\overset{j}{3}}$

$= 40$

$A_1 \times (A_2 \times A_3)$

$c[1,1]$ $c[2,3] = 24$

$c[1,3] = c[1,1] + c[2,3] + d_0 d_1 d_3$

$i \; k \quad k+1 \; j \qquad i-1 \; k \; j$

$$c[i,j] = \min_{i \le k < j} \{ c[i,k] + c[k+1,j] + d_{i-1} \times d_k \times d_j \}$$

k value will be $i \le k < j$

Possible parenthesization will be if no. of matrices is are n

$$= \frac{2(n-1) C_{(n-1)}}{n}$$

$c[1,3]$

$k = 1, 2$

Example $n = 4$

$$\text{count} = \frac{2*3 C_3}{4} = \frac{6 C_3}{4} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 5$$

$n = 5$ then ans = 14

$$c[i,j] = \min_{i \le k < j} \{ c[i,k] + c[k+1,j] + d_{i-1} d_k d_j \}$$

* $A_1 \times A_2 \times A_3 \times A_4$

$d_0\ d_1 \quad d_1\ d_2 \quad d_2\ d_3 \quad d_3\ d_4$

$$c[1,4] = \min_{1 \le k < 4} \begin{cases} k=1 & c[1,1] + c[2,4] + d_0 d_1 d_4 \quad (A_1)(A_2 A_3 A_4) \\ & \quad 0 \\ k=2 & c[1,2] + c[3,4] + d_0 d_2 d_4 \quad (A_1 A_2)(A_3 A_4) \\ k=3 & c[1,3] + c[4,4] + d_0 d_3 d_4 \quad (A_1 A_2 A_3) A_4 \\ & \quad\quad 0 \end{cases}$$

$$c[2,4] = \min_{2 \le k < 4} \begin{cases} k=2 & c[2,2] + c[3,4] + d_1 d_2 d_4 \quad A_2 (A_3 A_4) \\ k=3 & c[2,3] + c[4,4] + d_1 d_3 d_4 \quad (A_2 A_3) A_4 \end{cases}$$

$$c[3,4] = \min_{3 \le k < 4} \begin{cases} k=3 & c[3,3] + c[4,4] + d_2 d_3 d_4 \end{cases}$$

we will solve smaller value first . using that we will find larger problem.

Example

$A_1 \times A_2 \times A_3 \times A_4$

3 2 2 4 4 2 2 5

$d_0\ d_1\ d_1\ d_2\ d_3\ d_4$

C

i \ j →	1	2	3	4
1	0	24	28	58
2		0	16	36
3			0	40
4				0

K ↓

	1	2	3	4
1	1	1	1	3
2			2	3
3				3
4				

$$C[1,2] = \min_{1 \le k < 2} \begin{cases} k=1 \\ C[1,1] + C[2,2] + d_0 d_1 d_2 \end{cases}$$

Using which value of k we got min value, that value of k we will put in array K.

$$= \min [\ 0 + d_0 \times 2 \times 4\ ]$$

$$= 24$$

$$C[2,3] = \min_{2 \le k < 3} \begin{cases} k=2 \\ C[2,2] + C[3,3] + d_1 d_2 d_3 \end{cases}$$

$$= 0 + 0 + 2 \times 4 \times 2 = 16$$

$$C[3,4] = \min_{3 \le k < 4} \begin{cases} k=3 \\ C[3,3] + C[4,4] + d_2 d_3 d_4 \end{cases}$$

$$= 0 + 0 + 4 \times 2 \times 5 = 40$$

$$C[1,3] = \min_{1 \le k < 3} \begin{cases} k=1 \\ C[1,1] + C[2,3] + d_0 d_1 d_3 \\ k=2 \\ C[1,2] + C[3,3] + d_0 d_2 d_3 \end{cases}$$

$$= \min \begin{pmatrix} k=1 \\ 0 + 16 + 3 \times 2 \times 2 \\ 12 \\ 28 \end{pmatrix}, \begin{pmatrix} 24 + 0 + 3 * 4 * 2 \\ 48 \end{pmatrix}$$

$$= \min\ 28$$

$$c[2,4] = \min_{2 \le k < 4} \begin{cases} k=2 \\ c[2,2] + c[3,4] + d_1 d_2 d_4, \\ k=3 \\ c[2,3] + c[4,4] + d_1 d_3 d_4 \end{cases}$$

$$= \min \left(0 + 40 + 2 \times 4 \times 5 \;,\; 16 + 0 + \underset{k=3}{2 \times 2 \times 5} \right)$$

$$= \min(80, 36)$$

$$= 36$$

$$c[1,4] = \min_{1 \le k < 4} \begin{cases} k=1 \\ c[1,1] + c[2,4] + d_0 d_1 d_4, \\ k=2 \\ c[1,2] + c[3,4] + d_0 d_2 d_4, \\ k=3 \\ c[1,3] + c[4,4] + d_0 d_3 d_4 \end{cases}$$

$$c(1,4) = \min \left(0 + 36 + 3 \times 2 \times 5,\; 24 + 40 + 3 \times 4 \times 5,\; 28 + 0 + 3 \times 2 \times 5 \right)$$

$$= \min(66, 124, 58)$$

$$c(1,4) = \underset{k=3}{58}$$

minimum cost is 58

Para

Order (parentesization) using K table

((A1) A2 A3) A4 58 came from 3

now K[1,3] = 1

((A1)((A2 A3))(A4)) → 58 mul