Tutorial 3B

1. Use a direct proof to show that if x is an even integer then x^2 is an even integer

* Proof:- $x = 2k$

$$x^2 = (2k)^2$$
$$= 4k^2$$

even ↓ ~~something~~

$$= 2\underbrace{(2k^2)}_{S} = 2S$$

$\Rightarrow$ Multiplying two with any number is always even so x^2 is an even integer.

2. Prove that if n is an integer and $3n+2$ is even, then n is even using

a) a proof by contraposition.

$$p \rightarrow q \Rightarrow \sim q \rightarrow \sim p$$

~~$n = 2k$~~

contraposition ~~mean~~ $\rightarrow$ If n is odd then $3n+2$ is odd.

$$n = 2k+1$$

$$3n+2 = 3(2k+1)+2$$
$$\cancel{6k+1+2} \quad 6k+3+2$$
$$\cancel{6k+3} \quad 6k+4+1$$
$$\cancel{2(3k+2)} \quad 2(3k+2)+1$$
$$2S+1$$

$3n+2$ is odd

(b) Proof by contradiction :- and n is ~~contradiction~~ even

~~$n=2$~~ $3n+2$ is even

~~$3n+2$ is odd.~~ $n = 2k+1$

n is odd.

$3(2k+1)+2 = 6k+3+2$

$6k+4+1 = 2(3k+2)+1$

$= 3n+2 = \underbrace{2(S)+1}_{\text{odd}}$

$P \to q$		
P q		$P \to q$
T F		F
T T		T
F F		T
F T		T

So it is odd.

(3) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

suppose $\sqrt{2}$ is rational ~~false~~ $\to$ false

$\sqrt{2} = \frac{p}{q}$, $q \neq 0$

$2 = \frac{p^2}{q^2}$

$2q^2 = p^2$

p^2 is even.

~~$2q^2$ is even~~.

$2q^2 = (2k)^2 = 2q^2 = 4k^2$

$q^2 = 2k^2$

q is even

So $\sqrt{2}$ is irrational.

(4) Show that these statements about the integer n are equivalent.

P_1: n is even.

P_2: $n-1$ is odd.

P_3: n^2 is even.

sol^n:-

P_1	P_2	P_3
n is even	$n-1$ is odd	n^2 is even
$n=2k$ is even	$2k-1$ is odd	$n=2k$ = $n^2=4k^2 = 2\times\underbrace{2k^2}_{m}$ $=2m$
		so even

so all sentence are equivalent.

⑤ Use a direct proof to show that the ~~product~~ product of two odd, numbers is odd

→ ~~n = 2a+1~~ $A = 2K+1$

$B = 2L+1$

$$A \cdot B = 2K+1 \times (2L+1)$$

$$= 4KL + 2K + 2L + 1$$

$$A \cdot B = 2(\underbrace{2KL + K + L}_{S}) + 1$$

$$A \cdot B = \underbrace{2S+1}_{\text{odd}}$$

so product of odd is also odd.

⑦ Prove that the sum of 5 consecutive is always divisible by 5.

→ $1^{st} = n$

$2^{nd} = n+1$

$$n + n+1 + n+2 + n+3 + n+4 = 5n + 10$$

$\div 5$

~~2n~~ ~~5t~~

so consecutive addition of 5 is always divisible by 5.

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(6) Prove that there are no positive perfect cubes less than 1000 that are the sum of the cubes of two +ve integers.

=> supp x, y are integer

$z^3 = x^3 + y^3$

$p : z^3 = x^3 + y^3$

$q : z^3 \geq 1000$

$p \rightarrow q$

$\sim p \rightarrow \sim q$

$z^3 < 1000$

$z^3 < 10^3$

$z < 10$ $\Rightarrow$ 1 to 9

$\sim p \Rightarrow$ $z^3 \neq x^3 + y^3$

$x^3 + y^3 = 1000$