* Divide and conquer

→ Matrix multiplication

It we will use iterative algo then time complexity $O(n^3)$

can we achieve better ?

Yes, using ~~dei~~ devide & conquer

what will be smaller problem size ?
→ 2×2

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$C_{11} = a_{11} * B_{11} + a_{12} * B_{21}$
$C_{12} = a_{11} * B_{12} + a_{12} * B_{22}$
$C_{21} = a_{21} * B_{11} + a_{22} B_{21}$
$C_{22} = a_{21} * B_{12} + a_{22} B_{22}$

Let's take 4x4 size
Here we are assuming that size each matrix is in $2^n \times 2^n$

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Example using Divide and Conquer

$$A = \begin{bmatrix} a_{11} & \cancel{a_{12}} & a_{13} & a_{14} \\ \cancel{a_{12}} & a_{22} & & \\ \cancel{a_{13}} & & & \\ a_{14} & & & \end{bmatrix}$$

$$\begin{array}{cc} & A_{11} \quad\quad\quad A_{12} \\ A = & \left[\begin{array}{cc|cc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right] \\ & A_{21} \quad\quad\quad A_{22} \end{array} \quad B = \left[\begin{array}{cc|cc} B_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ \hline b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{array}\right]$$

divide into $\frac{n}{2} \times \frac{n}{2}$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

matrix

$C = A * B$ $C_{11} = A_{11} * B_{11} + A_{12} * B_{21}$

$C_{12} = A_{11} * B_{12} + A_{12} * B_{22}$

$C_{21} = A_{21} * B_{11} + A_{22} * B_{21}$

$C_{22} = A_{21} * B_{12} + A_{22} * B_{22}$

total 8 matrix multiplication and
4 matrix Addition $\rightarrow n^2$

Recurrence Relation $= \begin{cases} 1, & n \le 2 \\ 8T(n/2) + n^2, & n > 2 \end{cases}$ $O(n^3)$

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Strassen's Matrix Multiplication using Divide and Conquer

$$\begin{array}{ccc} & \xrightarrow{T\ +} & \\ S\uparrow - & \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} & \downarrow R - \\ & \xrightarrow{Q\ +} & \end{bmatrix}$$

$B_{11}\ A_{11}\ A_{22}\ B_{22}$

$P = (A_{11} + A_{22})(B_{11} + B_{22})$

Add diagonal value and then multiply

$Q = B_{11}(A_{21} + A_{22})$

$R = A_{11}(B_{12} - B_{22})$

$S = A_{22}(B_{21} - B_{11})$

$T = B_{22}(A_{11} + A_{12})$

for QRST put $B_{11}\ A_{11}\ A_{22}\ B_{22}$ if B then formulla will be in form of A

$U = (A_{21} - A_{11})(B_{11} + B_{12})$

Q will using S & T take bracket value chang variable

$V = (B_{21} + B_{22})(A_{12} - A_{22})$

same as U

$C_{11} = P + S - T + V$

$C_{12} = R + T$

RAT (R addition with T)

$C_{21} = Q + S$

for C_{2i} decrement value of C_{12}

$C_{22} = P + R - Q + U$

Total 7 matrix multiplication of size n/2

Recurrence Relation

$$T(n) = \begin{cases} 1, & n=1 \\ 7T(n/2) + n^2, & n>1 \end{cases}$$

* Example:

$$A = \begin{bmatrix} 5 & 6 \\ -4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -7 & 6 \\ 5 & 9 \end{bmatrix}$$

T + , - S, R -, Q +

$P = (A_{11} + A_{22}) * (B_{11} + B_{22})$
$= (5+3)(-7+9)$
$= 8 * 2 = 16$

$Q = B_{11}(A_{21} + A_{22})$
$= -7(-4+3)$
$= 7$

$R = \cancel{A_{11}(6-9)}\ A_{11}(B_{12} - B_{22})$
$= 5(6-9)$
$= -15$

$S = A_{22}(B_{21} - B_{11})$
$= 3(5+7)$
$= 36$

$$T = B_{22}(A_{11} + A_{12})$$
$$= 9(5+6)$$
$$= 99$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12})$$
$$= (-4 - 5)(-7 + 6)$$
$$= (-9)(-1) = 9$$

$$V = (B_{21} + B_{22})(A_{12} - A_{22})$$
$$= (5+9)(6-3)$$
$$= (14)(3) = 42$$

$$C_{11} = P + S - T + V = 16 + 36 - 99 + 42 = -5$$

$$C_{12} = R + T = -15 + 99 = 84$$

$$C_{21} = Q + S = 7 + 36 = 43$$

$$C_{22} = P + R - Q + U = 16 + (-15) - 7 + 9 = 3$$

$$A \times B = C = \begin{bmatrix} -5 & 84 \\ 42 & 3 \end{bmatrix}$$