Tutorial 3

1. which of these sentences are propositions? what are the truth values of those that are propositions

a) good bye → proposition → True
b) what time is it → proposition → True
c) There are no students in the class → True
d) 4 + x = 5 → proposition → false.
e) The moon is made of gold → False
(f) $2^n \geq 100$ → Not proposition

~~Q.2) Negation~~

~~a) If she don't works and she will not earn money.~~
~~b) He will not swim if the water is not warm.~~
~~c)~~

Q.2) Negation

a) If she works and she will not earn money.

b)
$P \to q$ and $q \Rightarrow P$

$\sim(\sim P \vee q) \wedge (\sim q \vee P)$

$(P \wedge \sim q) \vee (q \wedge \sim P)$

~~If He not swims if and~~
Ethier he does not swims or if a water is warm and ~~if water is warm~~

Either he swims if a water is not warm.

(c) If it snows, then they do drive the car.

Q.3) Determine the contrapositive of each statement

$p \to q \Rightarrow \sim q \to \sim p$

(a) If Raja is a poet, then he is poor.

→ If he is not poor then Raja is not poet.

(b) Only if Bharat studies will he pass the test.

→ If ~~Bharat~~ Bharat will not ~~pass the test~~ study then he will ~~did~~ not ~~study~~ pass the test.

Q.4 Let p be "it is cold" and let q be "it is raining". Give a simple verbal sentence which describes each of the following statements:

a) $\sim p \to$ it is not cold.

b) $p \vee q \to$ Either it is cold or it is raining

c) $\sim p \wedge q \to$ It is not cold and it is raining

d) $\sim q \vee (\sim p \wedge q) \to$ If it is raining then it is cold

f) $\sim p \to \sim q \to$ If it is not cold then it is not raining.

~~e) $\sim q \vee (\sim p \wedge q) \to$ It is not raining~~

g) $p \leftrightarrow q \to$ It is cold if and only if it is raining

(h) $\sim q \to \sim p \to$ If it is not raining then it is not cold.

(e) $\sim q \vee (\sim p \wedge q) \to$ Either It is not raining or It is not cold and it is raining.

Q.5 ~~const~~ check that the proposition $P \vee \sim(P \wedge q)$ is a contradiction or tautology.

⇒ $P \vee \sim(P \wedge q)$

P	q	$P \wedge q$	$Q = (\sim P \wedge q)$	$P \vee Q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Tautology because all True

Q.6 construct a truth table for each of the compound.

a) $((P \to q) \to r) \to S$ ⇒ It is same as 12^{th} Q.(a)

P	q	r	S	$p \to q$	$(P \to q) \to r$	$((P \to q) \to r)$
T	T	T	T	T	T	
T	T	T	F	T	T	
T	T	F	T	T	F	
T	T	F	F	T	F	
T	F	T	T	F	T	
T	F	T	F	F	T	
T	F	F	T	F	T	
T	F	F	F	F	T	
F	T	T	T	T	T	
F	T	F	T	T	T	
F	T	F	F	T	F	
F	F	T	T	T	F	
F	F	T	F	T	T	
F	F	F	T	T	T	
F	F	F	F	T	F	
F	F	F	F	T	F	

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(b) $(p \wedge q) \rightarrow (p \vee q)$

p	q	A: $(p \wedge q)$	B = $p \vee q$	$A \rightarrow B$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

∴ compound proposition is tautology.

Q.7 Find the bitwise OR, bitwise AND, and bitwise NOR of each these pairs of bit strings.

→ (a)

000 111 0111
100 100 1011
100 111 1111 bitwise OR
000 100 0011 bitwise AND
100 011 1000 bitwise XOR.

(b)

111111 1001
000011 000 0
111111 1001 bit wise OR
0000 011 0000 bitwise AND
1111 001001 bitwise XOR.

⑧ $A = \{1, 2, 3, 4, 5\}$

=> a $(\exists x \in A)$ $(x + 3 = 10)$

or

$\exists x\, (x+3) = 10$, $x \in A$

or

$P(x)$: "$x + 3 = 10$"

$\therefore \exists x\, P(x)$, $x \in A$

->

$\therefore$ $5 + 3 = 8 \neq 10$

$\therefore (\exists x \in A)\, (x+3 = 10)$ is false

-> ⓑ $(\exists x \in A)$ $(x + 3 < 5)$

-> $1+3 = 4 < 5$

$\therefore$ This is true

-> ⓒ $(\forall x \in A)$ $(x + 3 < 10)$

-> This is true (all every elements condition should be True).

-> ⓓ $(\forall x \in A)$ $x + 3 \leq 7)$

-> $5 + 3 = 8 > 7$

-> $\therefore$ This is false

⑨ Determine compound propositions is satisfiable.

-> ⓐ $(P \vee \sim q) \wedge (\sim P \vee q) \wedge (\sim P \vee \sim q)$

~~p q ~p ~q~~ $P \vee \sim q$ $\sim P \vee q$ ~~$\sim P \vee \sim q$~~

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \vee q$	$\sim p \vee \sim q$	$(p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$
T	T	F	F	T	T	F	F
T	F	F	T	T	F	T	F
F	T	T	F	F	T	T	F
F	F	T	T	T	T	T	T

→ ∴ compound preposition is ~~tautology~~ satisfiable where p & q are false.

(5) $(p \vee q \vee r) \wedge (\sim p \sim q \vee \sim r)$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \vee q \vee r$	$\sim p \vee \sim q \vee \sim r$	$(p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	F	T	F

→ compound preposition is satisfiable where p, q is true & r is false.
→ - - - where p, r is true and q is false.
→ - - - where p is true and q, r is false.
→ - - - where q, r is true and p is false
→ - - - where q is True and r, p is false
→ - - - where r is true and p, q is false

(10) use the laws, show that

$\sim(P \wedge q) \vee (\sim P \wedge q) \equiv \sim P \vee \sim q$

$\rightarrow$ L.H.S $\equiv \sim(P \wedge q) \vee (\sim P \wedge q)$

$\rightarrow$ Demorgan's law.

$\equiv (\sim P \vee \sim q) \vee (\sim P \wedge q)$

Distributive law.

$\equiv [(\sim P \vee \sim q) \vee \sim P] \wedge$

$[(\sim P \vee \sim q) \vee q]$

$\equiv [(\sim P \vee \sim P) \vee \sim q] \wedge$

$[(\sim P) \vee (\sim q \vee q)]$

$\equiv [(\sim P) \vee \sim q] \wedge [(\sim P) \vee T]$

T negative law.

$\equiv [(\sim P) \vee (\sim q)] \wedge (P \vee T)$

$\equiv (\sim P \vee \sim q) \wedge (P \vee T)$

$\rightarrow$ Domination law.

$\equiv (\sim P \vee \sim q) \wedge (T)$

$\equiv \boxed{\sim P \vee \sim q}$ $\rightarrow$ Identity law.

L.H.S. $\equiv$ R.H.S.

Hence Proved

11. check logically equivalent or not.

(a) check $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are ~~not~~ logically equivalent or not.

$$\boxed{(p \to q) \to (r \to s)}$$

~~$(p \to q)$~~.

p	q	$p \to q$	r	s	$r \to s$	$(p \to q) \to (r \to s)$
T	T	T	T	T	T	T
T	T	T	T	F	F	F
T	T	T	F	T	T	T
T	T	T	F	F	T	T
T	F	F	T	T	T	T
T	F	F	T	F	F	T
T	F	F	F	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	T	F	F	F
F	T	T	F	T	T	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T
F	F	T	T	F	F	F
F	F	T	F	T	T	T
F	F	T	F	F	T	T

$(p \to r) \to (q \to s)$

p	q	r	s	$p \to r$	$q \to s$	$(p \to r) \to (q \to s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	F	T	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	T	F	T	T	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	T	F	T	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

It is logicaley equivalent.
because ~~they the~~
$(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$
are ~~equt~~ same.

(b) check that $\sim(p \oplus q)$ and $p \to q$ are logically equivalent or not.

p	q	$p \to q$	$p \oplus q$	$\sim(p \oplus q)$	$(p \to q) \Leftrightarrow \sim(p \oplus q)$
T	T	T	F	T	T
T	F	F	T	F	T
F	T	T	T	F	F
F	F	T	F	T	T

→ Not all Val

Not same

→ It is not logically equivalent because its $(p \to q) \Leftrightarrow \sim(p \oplus q)$ are NOT tautology. ~~and~~ and $(p \to q)$ and $\sim(p \oplus q)$ are NOT same.

(2) Construct a truth table for each of these compound propositions.

(a) $((p \to q) \to r) \to s$

p	q	r	s	$p \to q$	$(p \to q) \to r$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

(b) $(\sim p \wedge q) \rightarrow (p \vee q)$

p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(\sim p \wedge q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	T

Tautology. because all True.