

Ch-6 Backtracking & Branch and Bound

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→ Backtracking

- It uses brute force approach.
(In this approach we will find / solve all possible solutions)

- It is used to find all possible solution

State space tree → used

- It uses DFS like search.

- Backtracking is also called as DFS of a tree

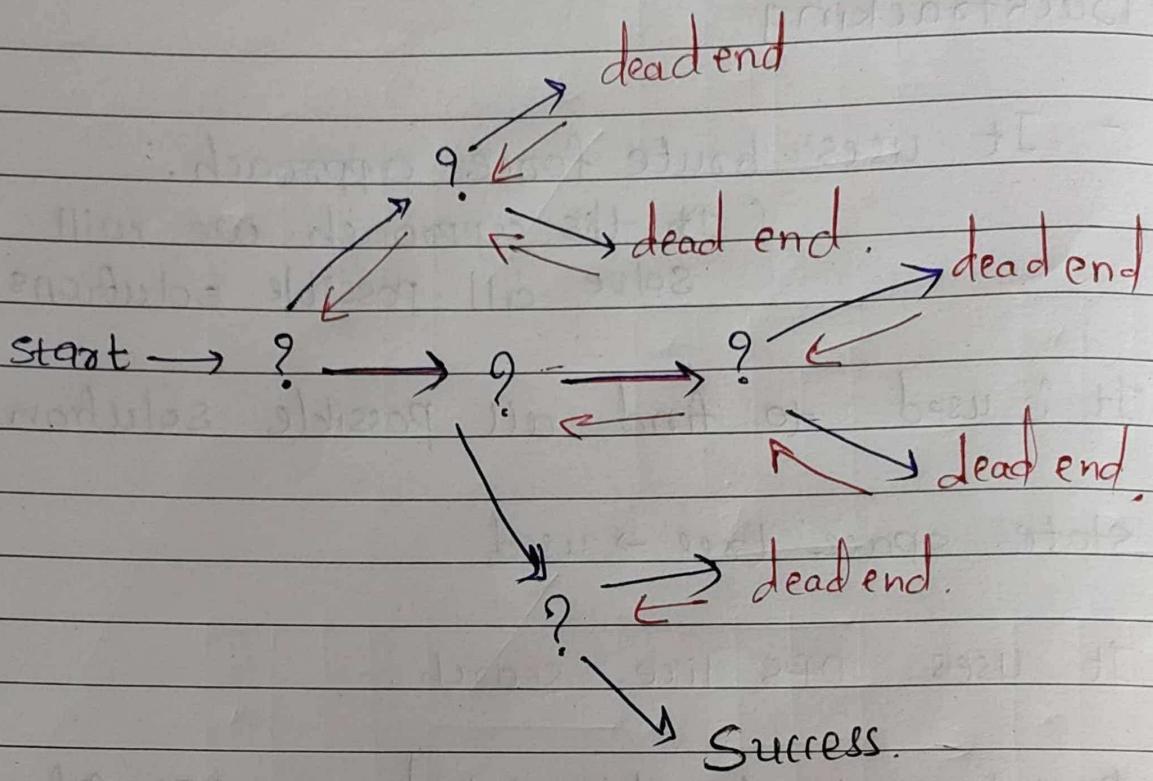
- Backtracking is the procedure whereby, after determining that a node can lead to nothing but dead node, we go back ("backtrack") to the node's parent and proceed with search on the next child.

- Node is nonpromising if when visiting the node we determine that it cannot possibly lead to a solution. otherwise, we call it promising.

→ Summary,

- Doing DFS

- check whether node is promising, and, if it is nonpromising, backtrack to its parent node.



* Backtracking Algorithm.

→ We Explore each node, as follows:

To explore node N :

1. If N is goal node, return "success"
2. If N is leaf node, return "Failure"
3. For each child C of N,

3.1 Explore C

- 3.1.1 If C was successful, return "succ"
4. Return "failure"

* Application of Backtracking

- N-queen Problem
- coloring map
- ~~subset~~ subset
- Solving maze.
- Solving puzzle.

* N - Queen Problem

Problem:- try to place N queens on a $N \times N$ board such that none of the queens can attack another queen.

- Queen can move horizontally, vertically or diagonally any distance.
- A queen can attack another queen, if two of them are on same row, column ,or diagonal.

1-queen \rightarrow solution exist

2-queen - No solution exist

3-queen - No solution exist

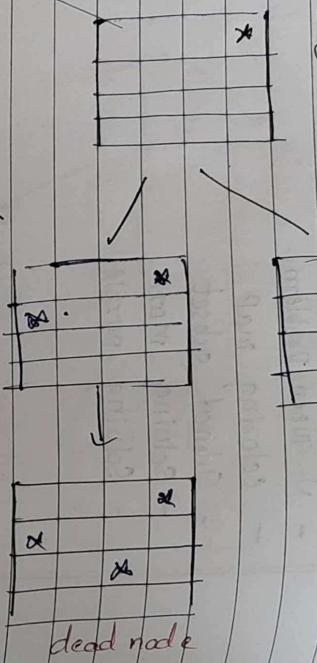
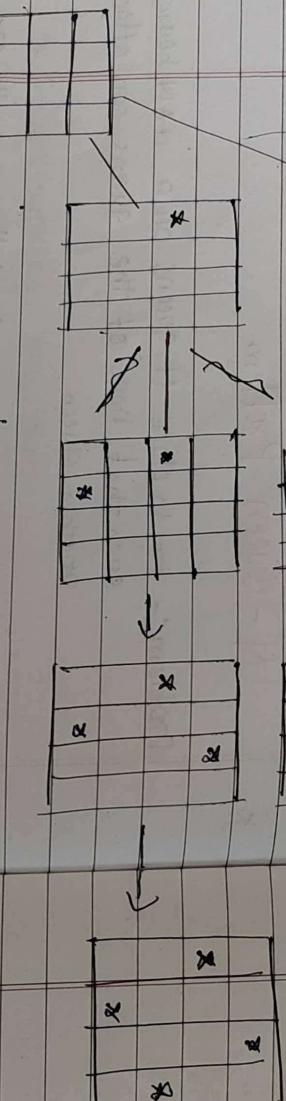
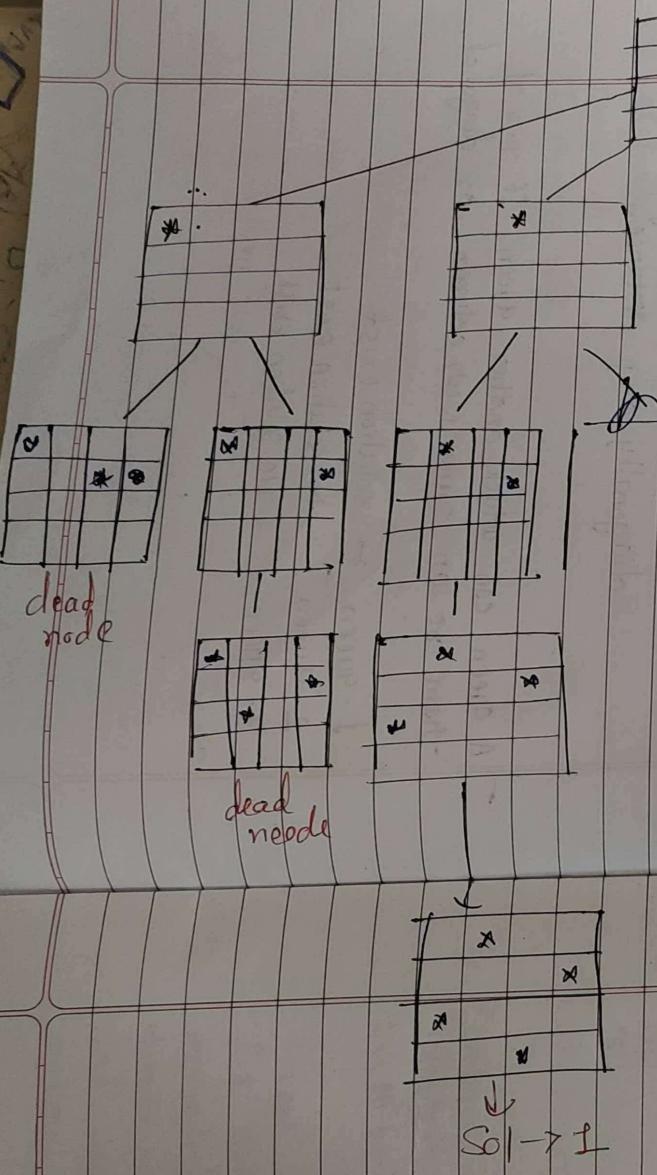
* 4- Queen problem

State Space tree

dead node

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Branch and Bound.

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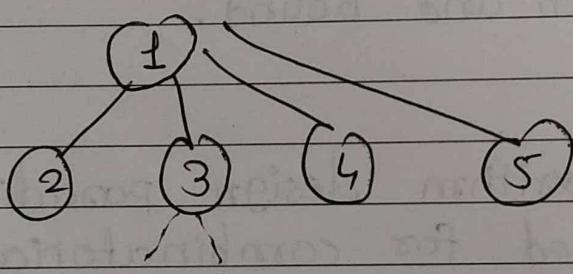
- B&B is used to find optimal solution (Only minimization problem)
- It uses BFS like search.
- It uses State space tree for solution.
- Methods For solving LPP using B&B
or B&B strategies
 - FIFO Branch and bound.
 - LIFO Branch and bound
 - Least-cost Branch and bound.
- BB is an algorithm design paradigm which is generally used for combinatorial optimization problems.
- Combinatorial optimization Problem is an Optimization Problem, where an optimal solution has to be identified from a finite set of solutions
- General idea of B&B is a BFS-like search for optimal solution, but not all node get expanded. Rather, a carefully selected criterion determines which node to expand and when, and another criterion tells the algorithm when an optimal solution has been found.

Live node : is a node that has been generated but whose children have not yet been generated.

E-node (expanded node) : is a live node whose children are currently being explored (or expanded).

Dead Node : is a generated node that is not to be expanded or explored any further.

All children of a dead node have already been expanded.



live node \rightarrow 2, 4, 5

e-node \rightarrow 3

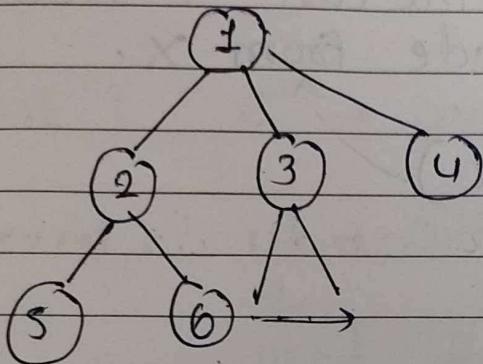
dead node \rightarrow 1

Bounding function is used to kill live node without generating all their children.

B&B strategies

1) FIFO Branch and Bound (or BFS)

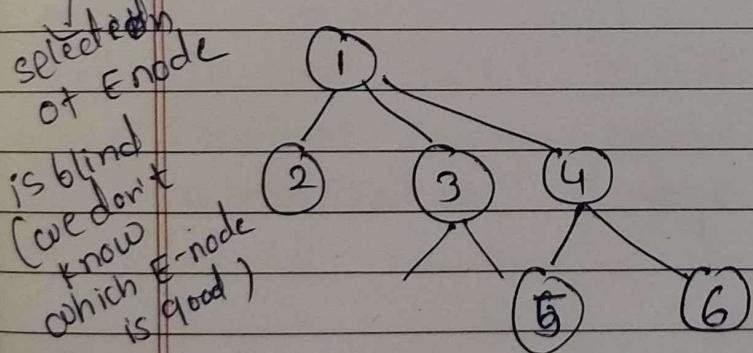
- It is an FIFO Search in terms of live nodes.
- List of live nodes in queue.



children of E-node are inserted in queue

2) LIFO Branch and Bound (or D-search)

- List of live nodes is a stack



children of E-node are inserted in stack

3) Least cost Branch and Bound: (LC-search)

uses approximation cost function

- LC uses Approximate cost Function for selecting E-node (using this we can improve search)

Cost function.

Each node x ,

$c(x) = \text{cost of reaching node } x (\text{E node})$
from root + the cost of reaching
an answer node from x .

$$c(x) = g(x) + h(x)$$

0/1 knapsack problem (Binary knapsack)

I/p : Weight and profit of N items.
Knapsack capacity S .

Output : Selection for knapsack : x_1, x_2, \dots, x_n
where $x_i \in \{0, 1\}$

Cost : $C \rightarrow$ with Fractional.

Bound : $UB \rightarrow$ without Fractional
 \downarrow
used for branching

- It is an maximization problem, but in branch and bound we can solve only minimization problem
- If we want to solve maximization problem, then convert problem into minimization (change the sign)

Example 2

$n = 4$	1	1	2	3	4
$m = 15$	P	10	10	12	18
	W	2	4	8	9

Initial value of \hat{C} and $UP \rightarrow$

$4 \rightarrow \frac{3}{9}$	$\frac{3}{9} * 18$
$3 \rightarrow 6$	12
$2 \rightarrow 4$	10
$1 \rightarrow 2$	10

$UP = 52$

$\hat{C} = -38$

$x_1 = 1$ means we are taking 1st item into knapsack

we have two live node 2, 3 for branching we select node which has minimum UP .

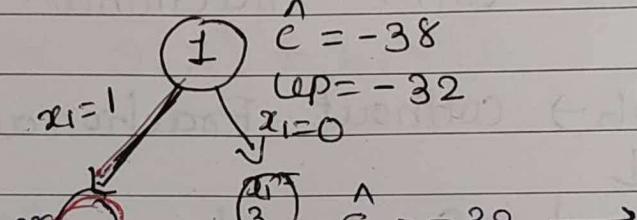
4, 5, 3 live nodes and

4 has minimum UP so expand it

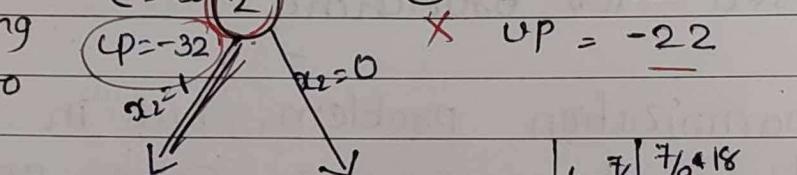
$$\hat{C}(4) = -38$$

maximum profit = 38

items = (1, 1, 0, 1)



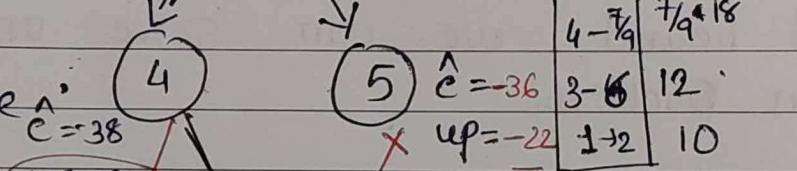
$x_1 = 0$ 1st item is not taken



$4 - \frac{5}{9}$	$\frac{5}{9} * 18$
$3 - 6$	12
$2 - 4$	10

$UP = 22$

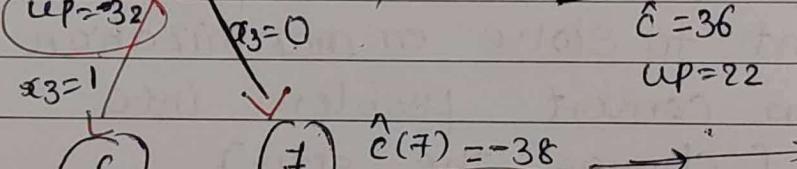
$\hat{C} = 32$



$4 - 9$	18
$2 - 4$	10
$1 - 2$	10

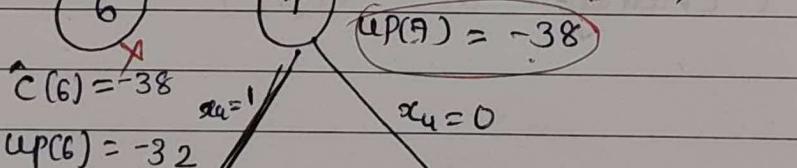
$UP = 38$

$\hat{C} = 38$



$$\hat{C}(7) = -38$$

$$UP(7) = -38$$



$$\hat{C} = -38$$

$$UP = -38$$

X

X

X