

## Golomb code ( $G_m(n)$ )

- It is used to encode integer value.
- It is used for lossless compression.
- Determining Golomb code or codeword.

Assumption:

larger the integer lesser the probability to occur.

Algorithm:

Step 1: unary code of  $q = \left\lfloor \frac{n}{m} \right\rfloor$   $m = \text{divisor}$

$\swarrow$  integer  
 $\searrow$  divisors

Step 2: let  $k = \lceil \log_2 m \rceil$

$$c = 2^k - m \text{ and } r = n \bmod m$$

$$r' = \begin{cases} r \text{ truncated to } k-1 \text{ bits, } 0 \leq r < c \\ r+c \text{ truncated to } k \text{ bits, otherwise} \end{cases}$$

Step 3: concatenate result of step 1 & step 2  
// concatenate  $(q, r')$

Example: Golomb code

Q: Design Golomb code for  $q$  with divisor 4,  
 $n=9, m=4, G_4(9)=?$

$$q = \left\lfloor \frac{9}{4} \right\rfloor = 2$$

Unary code of  $q = 110 \rightarrow \textcircled{1}$

unary code

$$2 = 110$$

$$3 = 1110$$

$$4 = 11110$$

$$5 = 111110$$

n of 1's followed by 0

not 0's followed by 1

$$k = \lceil \log_2 4 \rceil = 2$$

$$\begin{aligned} c &= 2^k - m \\ &= 2^2 - 4 \\ &= 0 \end{aligned}$$

$$\begin{aligned} r &= n \bmod m \\ &= 9 \bmod 4 \\ &= 1 \end{aligned}$$

$$r=1, c=0 \quad \boxed{r > c}$$

represented

$$\begin{aligned} r' &= r + c \text{ truncated to } k \text{ bits} \\ &= 0 + 1 \text{ truncated to } 2 \text{ bits} \\ &= 1 \text{ truncated to } 2 \text{ bits} \\ &= \underline{\underline{01}} \end{aligned}$$

Step: 3      cancel  $\boxed{11001}$  ✓

$$G_4(g) = 11001$$



**Entropy:** It shows the shortest possible average length of a lossless compression of data

$$\text{Probability} = \frac{\text{no. of occurrence}}{\text{total length}}$$

**Q: Find Entropy:**

$$P(a_1) = 1/2, \quad P(a_2) = 1/4, \quad P(a_3) = P(a_4) = 1/8$$

**Entropy =** 1 for  $(1/2)$  occurrence

for  $a_1$

$$-1 \left( \frac{1}{2} \right) \left( \log_2 \frac{1}{2} \right)$$

$$-1 \left( \frac{1}{4} \right) \left( \log_2 \frac{1}{4} \right)$$

for  $a_2$

$$-2 \left( \frac{1}{8} \right) \left( \log_2 \frac{1}{8} \right)$$

$$= \left( -\frac{1}{2} \right) (-1) - \frac{1}{4} - \frac{1}{4} (-2)$$

$$= \left( -\frac{1}{2} \right) (-1) - \left( \frac{1}{4} \right) (-2) - \left( \frac{1}{4} \right) (-3)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = 1.75 \text{ bits/symbols}$$

\* Find entropy if the given sequence is

1 2 1 2 3 3 3 3 1 2 3 3 3 3 1 2 3 3 1 2

Unique characters = 1, 2, 3

$$P(1) = \frac{5}{20} = \frac{1}{4} \quad P(2) = \frac{5}{20} = \frac{1}{4}$$

$$P(3) = \frac{10}{20} = \frac{1}{2}$$

$$\text{Entropy} = -2 \left( \frac{1}{4} \right) \log_2 \frac{1}{4} - (1) \frac{1}{2} \log_2 \frac{1}{2}$$

$$= -\frac{1}{2} (1 + 0.5) = 1.5 \text{ bits/symbols}$$

A 0.221

0.220