Theory of Computation

Automata :- is a Theoritical branch of Computer Science and mathematics, which mainly deals with the logic of computation with respect to simple machines.

Basic terminologies :-

Symbol :- is the smallest building block which can be any alphabet, letter or picture

$$\{ a, - b, - z, 0, 1 \cdots 9 \}$$

Alphabet :- Set of symbols, which are always finite.

Eg :- $\Sigma = \{0, 1\}$ is an alphabet of binary digit

$\Sigma = \{0, 1 \ldots 9\}$

$\Sigma = \{a, b, c\}$

String :- A string is a finite sequence of symbols from some alphabet. A string is generally denoted as w and length of a string is denoted as $|w|$

$\varepsilon \Rightarrow$ Empty string

Closure Representation of TOC :—

L^+ → "Positive closure" does not contain ε

L^* → "Kleene closure" contain ε

$L^* = \varepsilon L^+$

Language →

A language is a set of strings, chosen from some Σ^* or we can say —

A language is a subset of Σ^*

language can be finite or infinite

Eg: $L = \{$ set of string of 2 $\}$
$\Sigma = \{a, b\}$

$L = \{ab, aa, ba, bb\}$ finite

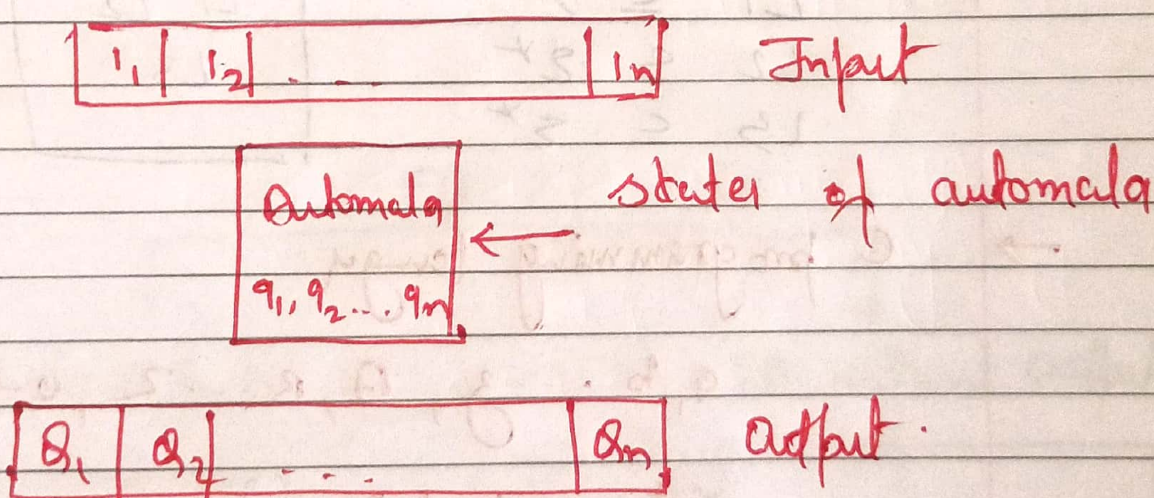
$L = \{$ starts with b $\}$

$L = \{ba, b, bbb \ldots\}$ infinite

Theory of Computation :- is a theoretical branch of Computer Science and mathematics, which mainly deals with the logic of computation with respect simple machines.

Automata enables scientists to understand how machines compute the functions and solve problem.

FA :-> finite automata is the simplest machine to recognize pattern. The finite automata or finite state machine is an abstract machine that have five elements



$\{ Q, \Sigma, q, F, \delta \}$

- Q : final set of states
- Σ : set of input symbols
- q : initial state
- F : set of final state
- δ : Transition function.

Power of $\Sigma = \{a, b\}$

$\Sigma^1 = \{a, b\}$

$\Sigma^2 = \{a, b\}\{a, b\} = (aa, ab, ba, bb).$

$\Sigma^3 = \Sigma . \Sigma . \Sigma$

$\Sigma^n = \Sigma^1 . \Sigma^2 . \Sigma^3 \ldots \Sigma^n$

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \ldots \Sigma^n$

$= \{\epsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \ldots$

Σ^*

$\boxed{L1 \quad L2 \quad L3}$

$L1 \subseteq \Sigma^*$

$L2 \subseteq \Sigma^*$

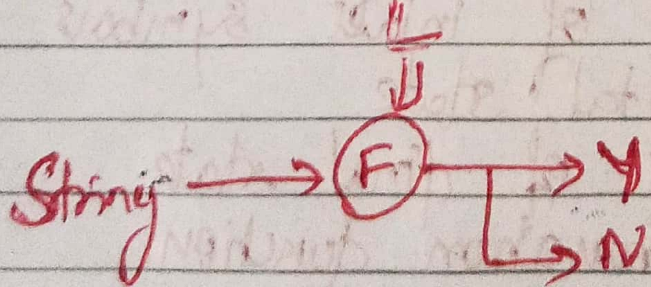
$L3 \subseteq \Sigma^*$

→ C programming langu

$\{a, b \ldots z, A, B \ldots Z, 0\text{-}9, + * \ldots\}$

```
void main ()
{
    int a, b;
}
```

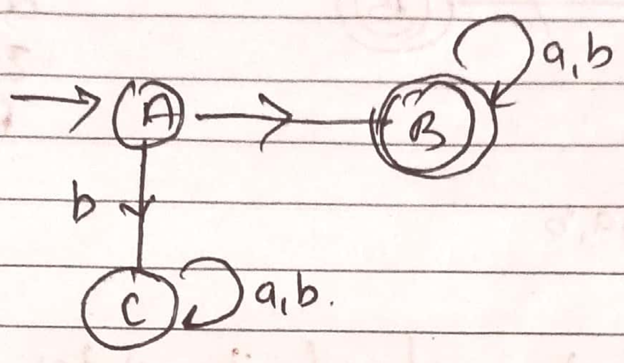
in C it is prog.
in TOC it is string



L_1 = Set of all strings which starts with a

$= \{a, aa, aaa \dots ab \dots\}$

FA representation



aab is present or not

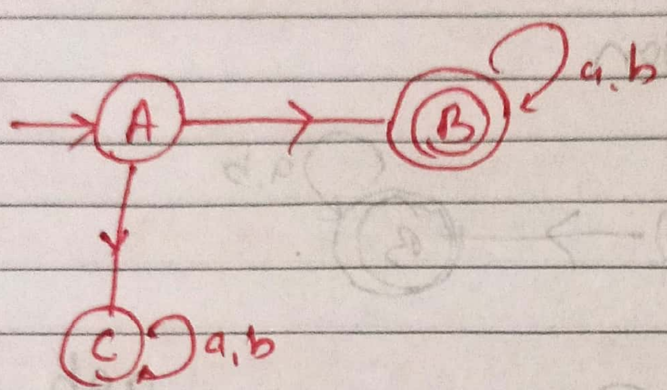
$A \xrightarrow{a} B \xrightarrow{a} B \xrightarrow{b} B$ | Accepted.

Finite automata

- FA
  - FA with output
    - moore
    - mealy
  - FA without output
    - DFA
    - NFA
    - ϵ-NFA

DFA → Deterministic finite automata

$\{ Q, \Sigma, \delta, q_0, F \}$

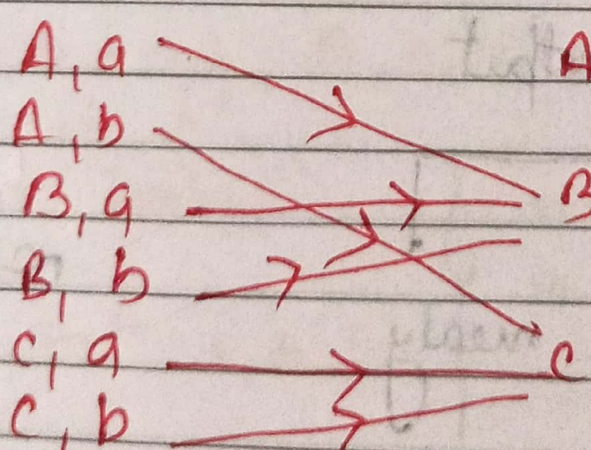


$q_0 = A$
$Q = \{A, B, C\}$
$\Sigma = \{a, b\}$
$F = \{B\}$

Q :- finite set of i/p states
Σ = input alphabet
q_0 = start state

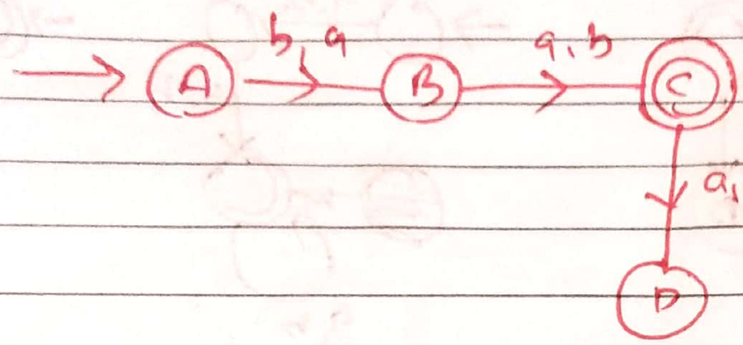
F = Set of final state. $Q \supseteq F$

$\delta = Q \times \Sigma \rightarrow Q$
$\{A, B, C\}$ $\{a, b\}$.



Construct a DFA, that accepts set of all string over $\Sigma = \{a, b\}$ of length 2

$L = \{aa, ab, ba, bb\}$



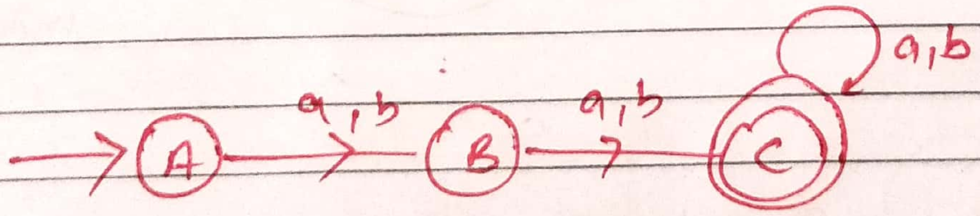
string accept : Scan the entire string, if we reach final state from intial state.

Language accepted : A FA said to accept the language is accepted is all the string in the language is accepted ~~start~~ and all the string is not in language is "rejected"

DFA questions

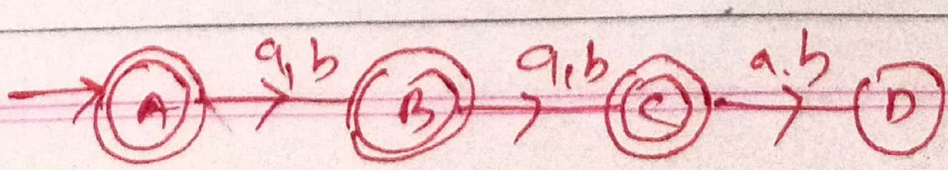
DFA $w \in \{a, b\}$ $|w| \geq 2$.

$L = \{aa, ab, bbbaa \ldots\ldots\}$



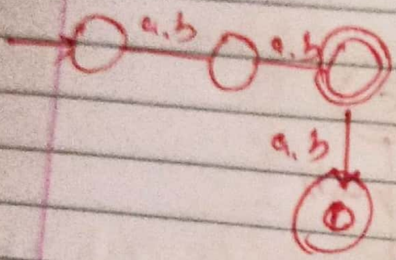
DFA $|w| \leq 2$. at most two

$L = \{\epsilon, a, b, aa, ba \ldots\ldots\}$

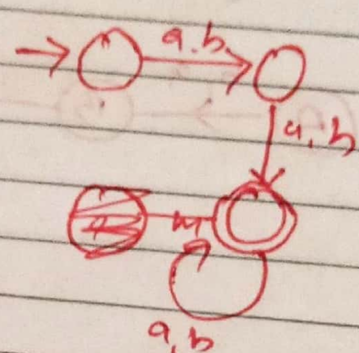


L = have many DFA but only 1 minimal DFA

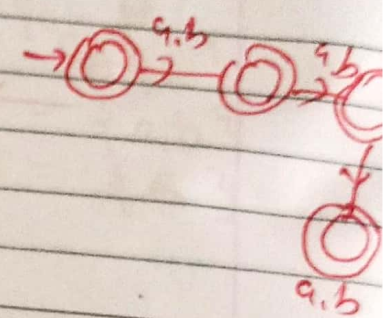
$|w| = 2$



$|w| \geq 2$



$|w| \leq 2$



no. of states: $|w| = n$: $n+2$

$|w| \geq n$: $n+1$

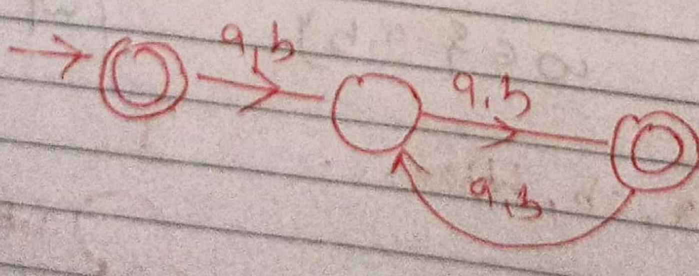
$|w| \leq n$: $n+2$

DFA :-

minimal DFA $w \in \{a,b\}$

$|w| \bmod 2 = 0$

$L = \{ \epsilon, aa, ab, ba, bb, aaaa \ldots \}$

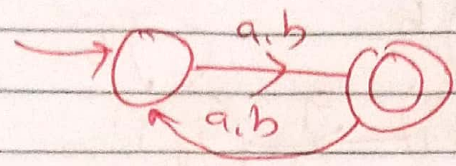


DFA but not minimal

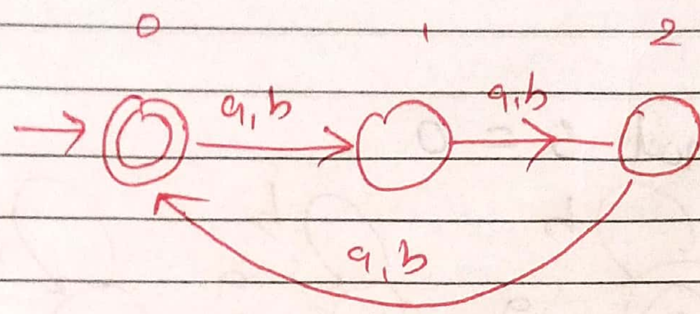
e

o

$|w| \bmod 2 = 1$ $L = \{ a, b, aaa, \ldots \}$



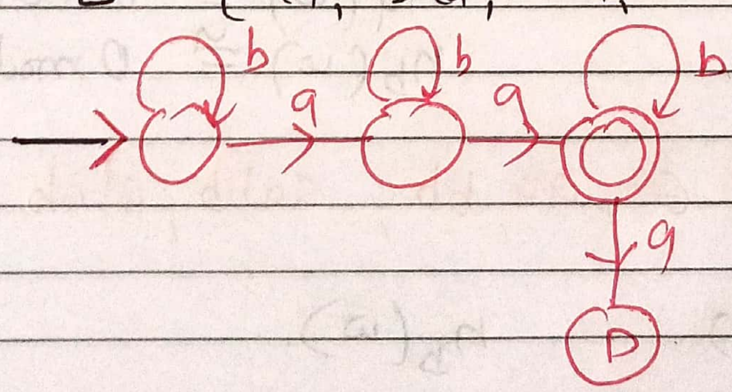
$|w| \bmod 3 = 0$ $L = \{ \epsilon, aaa, aba \ldots \}$



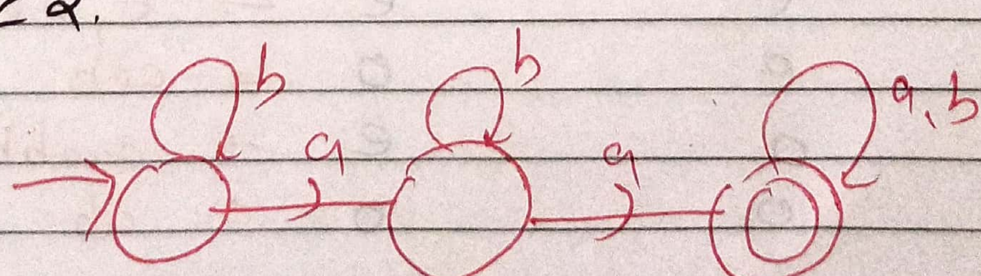
$|w| \bmod n = 0$ have n states.

2) minimal DFA $w \in \{a,b\}^*$ $n_a(w) = 2$

$L = \{ aa, baa, aba, aab, bbaa \ldots \}$

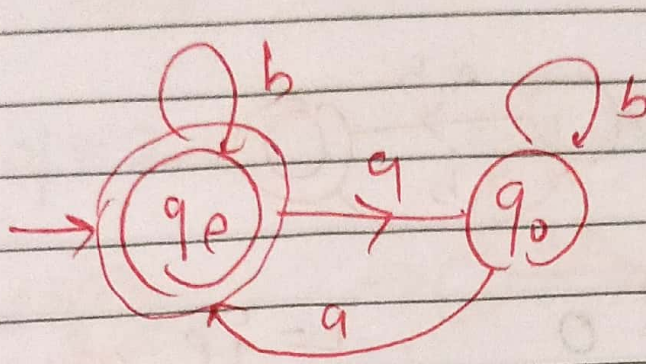


$n_a \geq 2$.

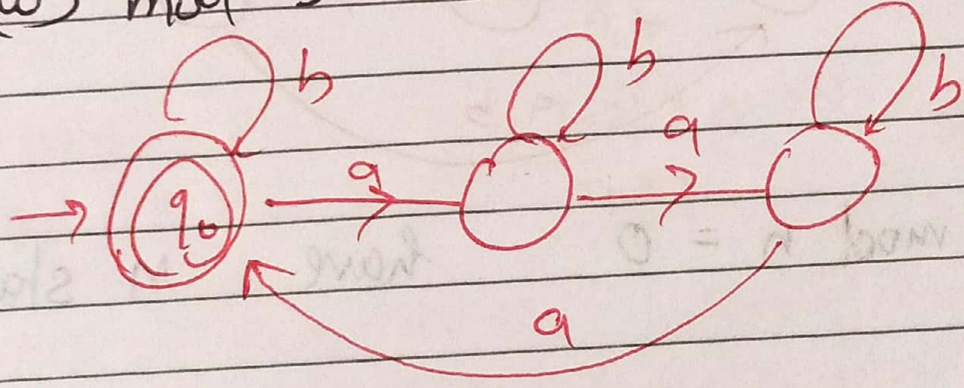


minimal DFA $w \in \{a,b\}^*$ $n_a(w) \bmod 2 = 0$

$n_a(w) \equiv 0 \bmod 2$



$n_a(w) \bmod 3 = 0$



minimal DFA $w \in \{a,b\}^*$

$n_a(w) \equiv 0 \bmod 2$

$n_b(w) \equiv 0 \bmod 2$.

$L = \{ \epsilon, aa, bb, aabb, abab \ldots \}$

$n_a(w)$	$n_b(w)$	
e	e	— ϵ, aa, bb
e	o	→ aab
o	e	→ aaabb
o	o	→ ab