

Variable-wise and Term-wise Recentering

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Variable-wise recentering

- $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$
- Let
 - $\Delta x_i = x_i - \mu_{x_i}$ for $i = 1, 2$, with μ_{x_i} as arbitrary constants (perhaps a mean)
- Then the variable-wise recentered version is
 - $y = \beta_0^\Delta + \beta_1^\Delta \Delta x_1 + \beta_2^\Delta \Delta x_2 + \beta_{12}^\Delta \Delta x_1 \Delta x_2$

Term-wise recentering

- $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$
- Let
 - $\Delta x_i = x_i - \mu_{x_i}$ for $i = 1, 2$ and arbitrary μ_{x_i} .
 - $x_{12} = x_1 x_2$
 - $\Delta x_{12} = x_1 x_2 - \mu_{x_{12}}$, with $\mu_{x_{12}}$ as another arbitrary constant
- Then the term-wise recentered equation is
 - $y = \beta_0^{\text{tw}} + \beta_1^{\text{tw}} \Delta x_1 + \beta_2^{\text{tw}} \Delta x_2 + \beta_{12}^{\text{tw}} \Delta x_{12}$

Deriving variable-wise parameters

- $y = \beta_0^\Delta + \beta_1^\Delta \Delta x_1 + \beta_2^\Delta \Delta x_2 + \beta_{12}^\Delta \Delta x_1 \Delta x_2$
- Starting from the original equation and substituting
 - $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$
 - $y = \beta_0 + \beta_1 (\Delta x_1 + \mu_{x_1}) + \beta_2 (\Delta x_2 + \mu_{x_2}) + \beta_{12} (\Delta x_1 + \mu_{x_1}) (\Delta x_2 + \mu_{x_2})$
 - $y = (\beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} + \beta_{12} \mu_{x_1} \mu_{x_2}) + (\beta_1 + \beta_{12} \mu_{x_2}) \Delta x_1 + (\beta_2 + \beta_{12} \mu_{x_1}) \Delta x_2 + \beta_{12} \Delta x_1 \Delta x_2$
- We get
 - $\beta_{12}^\Delta = \beta_{12}$
 - $\beta_1^\Delta = \beta_1 + \beta_{12} \mu_{x_2}$ and $\beta_2^\Delta = \beta_2 + \beta_{12} \mu_{x_1}$
 - $\beta_0^\Delta = \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} + \beta_{12} \mu_{x_1} \mu_{x_2}$

Deriving term-wise parameters

- $y = \beta_0^{tw} + \beta_1^{tw}\Delta x_1 + \beta_2^{tw}\Delta x_2 + \beta_{12}^{tw}\Delta x_{12}$
- Starting from the original equation and substituting
 - $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$
 - $y = \beta_0 + \beta_1(\Delta x_1 + \mu_{x_1}) + \beta_2(\Delta x_2 + \mu_{x_2}) + \beta_{12}(\Delta x_{12} + \mu_{x_{12}})$
 - $y = (\beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} + \beta_{12} \mu_{x_{12}}) + \beta_1 \Delta x_1 + \beta_2 \Delta x_2 + \beta_{12} \Delta x_{12}$
- We get
 - $\beta_1^{tw} = \beta_1, \beta_2^{tw} = \beta_2, \text{ and } \beta_{12}^{tw} = \beta_{12}$
 - $\beta_0^{tw} = \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} + \beta_{12} \mu_{x_{12}}$
- Which can also be written
 - $y = \beta_0^{tw} + \beta_1 \Delta x_1 + \beta_2 \Delta x_2 + \beta_{12} \Delta x_{12}$

Using the term-wise equation

- In the term-wise centered equation, the value of Δx_{12} depends on the values of Δx_1 and Δx_2 .
 - $\Delta x_{12} = x_1 x_2 - \mu_{x_{12}} = (\Delta x_1 + \mu_{x_1})(\Delta x_2 + \mu_{x_2}) - \mu_{x_{12}}$
 - $\Delta x_{12} = \Delta x_1 \Delta x_2 + \Delta x_1 \mu_{x_2} + \Delta x_2 \mu_{x_1} + \mu_{x_1} \mu_{x_2} - \mu_{x_{12}}$
- Now the term-wise centered equation is equivalent to the variable-wise centered equation with the terms rearranged.
 - $y = \beta_0^{tw} + \beta_1^{tw} \Delta x_1 + \beta_2^{tw} \Delta x_2 + \beta_{12}^{tw} \Delta x_{12}$
 - $y = \beta_0^{tw} + \beta_1 \Delta x_1 + \beta_2 \Delta x_2 + \beta_{12} \Delta x_{12}$
 - $y = \beta_0^{tw} + \beta_1 \Delta x_1 + \beta_2 \Delta x_2 + \beta_{12} (\Delta x_1 \Delta x_2 + \Delta x_1 \mu_{x_2} + \Delta x_2 \mu_{x_1} + \mu_{x_1} \mu_{x_2} - \mu_{x_{12}})$
- We see that the final Δx_{12} term includes adjustments to Δx_1 , Δx_2 , and the constant. If we rearrange and simplify in the usual way, we arrive back at the variable-wise centered equation!
 - $y = (\beta_0^{tw} + \beta_{12} \mu_{x_1} \mu_{x_2} - \beta_{12} \mu_{x_{12}}) + (\beta_1 + \beta_{12} \mu_{x_2}) \Delta x_1 + (\beta_2 + \beta_{12} \mu_{x_1}) \Delta x_2 + \beta_{12} \Delta x_1 \Delta x_2$
 - [Now simplify the constant, and voila!]