

# Direct Recentering and Rescaling Solution

## Parameters in Higher-Order Models

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### Project Goal

- Algorithm that accepts a vector of recentering constants (typically means) and rescaling constants (typically standard deviations), and returns a coefficient-recentering matrix and a coefficient-rescaling matrix.
- This should handle continuous, categorical, and polynomial terms.

### Background

Standardized variables have many uses in statistics, in theory, in computation, and in interpretation. In fields of study where data are measured on scales that have no physical or intuitive interpretation (e.g. “intelligence” in education, or “happiness” in psychology) it has long been common practice to express measurements in terms of a “standard score” or a standardized score. This can mean sample standardization, rescaling to a sample mean and sample standard deviation, or standardizing to match some reference or benchmark.

Statistical models based on such standardized measurements can be thought of as having standardized coefficients. Some research journals require papers submitted to include standardized coefficients.

Calculating standardized coefficients is very simple for additive models, and is a topic taught in most regression courses in fields where these variables are commonly used. However, for higher order models – models that include interaction terms or polynomial terms – how to proceed from a model estimated in raw units of measure and calculate standardized coefficients is not obvious.

In practice, two approaches are commonly taken:

- Standardize with the mean and standard deviation of products of variables. While this produces the correct predicted values and residuals, these product terms cannot be interpreted on the same scale as the first order terms (both the means and their standard deviations are, in the general case, different).
- First, standardize all the variables, then re-estimate the model. This not only requires unnecessary computation (an extra pass through the data), but may not be possible where the original data are not available.

## Mathematical Solution

In fact, we can have an easily interpreted model without the extra computation. The key is to understand that the parameter space (the linear space in which the parameter solution is found) is constructed as a tensor product of the basis vectors for the data space.

Standardizing data variables (or more generally, re-centering and rescaling variables) is a change of basis for the data space. This induces a change of basis for the parameter space – although higher-order models are not linear in a geometric sense, they are linear in an algebraic sense. This means that the change of basis for the parameters can be expressed as a linear transformation, a matrix. (And anything else measured in the parameter space, e.g. the parameter variance-covariance matrix, can make use of this transformation as well.)

## Strategy

It is useful to divide this discussion into (1) recentering the independent variables, (2) rescaling the independent variables, and (3) recentering and rescaling the dependent variable in a model.

We can begin with the recentering transformation for a model with one independent variable (and one dependent variable) [[Ref: Searle]]. From this we can construct recentering transformations for higher-order combinations of variables [[Ref: Haberman]], as Kronecker products. Rescaling works in the same manner, it is simply another change of basis.

Models with untransformed variables (e.g. indicator variables) can be further simplified as direct sums.

Polynomial models are typically expressed with like terms collected, so these require an extra aggregating step.

## Computer Implementation

For this project, producing the parameter-recentering and –rescaling matrices is the goal. Eventually these could be used in post-estimation commands for parameter standardization.

### 1. Input Required

For each dimension (variable) in the data space of the model we need to know

- a. Whether the variable is continuous or categorical
- b. A value at which to recenter, for example the mean. This might also be 0 if we are not recentering.
- c. A value by which to rescale, for example the standard deviation. This might also be 1 if we are not rescaling.

### 2. Basic Computational Algorithms

- a. Continuous by Continuous interaction recentering and rescaling

- b. Continuous by Categorical interactions
- c. Polynomial terms