Tutorial - 4 (Master's Method)

1. $T(n) = 3T(n/2) + n^2$

$a=3$, $b=2$, $f(n) = n^2$; $a \geq 1$, $b \geq 1$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$\because f(n) > n^c$

$\therefore T(n) = \theta(f(n)) = \theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a=4$, $b=2$, $f(n^2)$

$c = \log_2 2^2 = 2$

$n^c = n^2$

$\Rightarrow f(n) = n^c$

$\therefore T(n) = \theta(n^c \log n) = \theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$, $f(n) = 2^n$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$

$1 < 2^n \Rightarrow f(n) > n^c$

$\therefore T.C = \cancel{\theta(n^2 \log n)}$ $\theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$ $f(n) = n^n$

$c = \log_2 2^n = n$

$n^c = n^n$, $f(n) = n^c$

$\therefore \Theta(n^n \log n)$

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16$, $b = 2$ $f(n) = n$

$c = \log_4 16 = 2$

$n^c = n^2$

$n^c > f(n)$

$\therefore$ $T.C = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$a = 2$, $b = 2$ $f(n) = n\log n$

$= n^1 \log^1 n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1$. $\Rightarrow$ $\log_b a = k$

and $p > -1$

$\therefore$ $T.C = \Theta(n^k \log^{p+1} n)$

$= \Theta(n \log^2 n)$

7. $T(n) = 2T(n/2) + n/\log n$

$a = 2$, $b = 2$, $f(n) = \Theta\left(\frac{n}{\log n}\right) \Rightarrow k = 1$, $p = -1$

$\log_a b = \log_2 2 = 1 = k$

and $p = -1$

$T.C = \Theta(n^k \log\log n)$ $= \Theta(n^k \log\log n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \ b = 4, \qquad k = 0.51, \ p = 0$

$\log_b a = \log_4 2 = 0.5 < k$

and $p = 0$

$\therefore \ T.C = \Theta(n^{0.51} \log^0 n) = \Theta(n^{0.51})$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5, \quad b = 2, \quad k = -1$

$\log_b a = \log_2 0.5 = -1 = k$

$p = 0$

$\therefore \ T.C = \Theta(n^k \log^{p+1} n)$

$= \Theta(n^{-1} \log n)$

10. $T(n) = 16T(n/4) = n!$

$a = 16, \quad b = 4, \quad f(n) = n! = n^n, \ \exists \ k = n, \ p = 0$

$\log_b a = \log_4 16 = 2 < k$

$\therefore \ T.C = \Theta(n^n \log^0 n) = \Theta(n^n)$

11. $T(n) = 4T(n/2) + \log n$

$a = 4, \ b = 2 \qquad k = 0, \ p = 1$

$\log_b a = \log_2 4 = 2 > k$

$\therefore \ T.C = \Theta(n^2)$

12. $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{n}, \quad b = 2 \qquad k = \cancel{\log}\, 0, \quad p = 1$

$\log_b a = \log_2 n^{1/2} = \frac{1}{2} > k$

$T \cdot C = \theta(n^{1/2})$

13. $T(n) = 3T(n/2) + n$

$a = 3, \quad b = 2, \quad k = 1, \quad p = 0$

$\log_b a = \log_2 3 = 1.58 > k$

$\therefore \quad T \cdot C = \theta(n^{1.58})$

14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 \quad b = 3, \quad f(n) = \sqrt{n}, \quad k = 1/2, \quad p = 0$

$\log_b a = \log_3 3 = 1 > k$

$\therefore \quad T \cdot C = \theta(n)$

15. $T(n) = 4T(n/2) + cn$

$a = 4, \quad b = 2, \quad f(n) = cn, \quad k = 1, \quad p = 0$

$\log_b a = \log_2 4 = 2 > k$

$\therefore \quad \theta(n^2)$

16. $T(n) = 3T(n/4) + n \log n$

$a = 3 \quad b = 4 \quad \cancel{n} \quad k = 1 \quad p = 1$

$\log_b a = \log_4 3 = 0.79, \quad \cancel{xx} < k$

$\therefore \quad \theta(n^k \log^p n) = \theta(n \log n)$

17. $T(n) = 3T(n/3) + n/2$

$a=3 \quad b=3 \quad , f(n)=n/2 \quad , p=0, k=1$

$c = \log_3 3 = 1 = k \quad ,$

$\therefore T.C = \theta(n^k \log^{p+1} n) = \theta(n \log n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 1 \quad , b=3 \quad k=2 \quad , p=1$

$\log_b a = \log_3 6 = 1.63 < k \quad$ and $p > -1$

$\therefore T.C = \theta(n^k \log^{p+1} n) = \theta(n^2 \log^2 n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a=4 \quad , b=2 \quad , \quad k=1 \quad , p=-1$

$\log_b a = \log_2 4 = 2 > k$

$\therefore T.C = \theta(n^2)$

20. $T(n) = 64\, T(n/8) + (-n^2 \log n)$

$a = 64 \quad , \quad b=8 \quad , \quad k=2 \quad p=1$

$\log_b a = \log_8 64 = 2 = k \quad$ and $p > -1$

$\therefore T.C = \theta(n^k \log^{p+1} n) = \theta(n^2 \log^2 n)$

21. $T(n) = 7T(n/3) + n^2$

$a=7 \quad , b=3 \quad k=2 \quad p=0$

$\log_b a = 1.77 < k$

$\therefore T.C = \theta(n^k \log^p n) = \theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1$, $b = 2$, $\cancel{f(n)}$ $= k = 1$, $p = 0$

$\log_b a = \log_2 1 = 0$, $< k$

$\therefore$ T.C $= \theta(n^k \log^p n) = \theta(n)$.