3. Any character that either overlaps or is outside a window boundary is clipped.

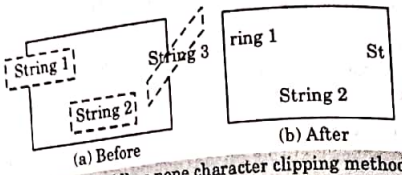

Fig. 2.25.2. All-or-none character clipping method.

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3 UNIT

Three Dimensional

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3D Object Representation

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PART-1

Three Dimensional : 3D Geometric Primitive, 3D Object Representation.

CONCEPT OUTLINE

- Geometric primitives are the basic geometrical shapes used to construct graphics scenes and the resulting final images.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **What do you mean by geometric primitives ? Discuss.**

Answer

Geometric primitives are the basic geometrical shapes used to construct computer graphics scenes and the resulting final images.

Commonly used 3D geometric primitives include :

1. **Points :** A point is an exact position or location on a plane surface.
2. **Line :** A line is a straight curve. When geometry is used to model the real world, lines are used to represent straight objects with negligible width and height.
3. **Line segments :** A line segment is a part of a line that is bounded by two end points, and contains every point on the line between its end points.
4. **Planes :** A plane is the two dimensional analogue of a point (zero-dimensions), a line (one-dimension) and a space (three-dimensions).
5. **Circles :** A circle is a simple shape of Euclidean geometry consisting of those points in a plane which is equidistant from a given point called the centre.
6. **Ellipses :** Ellipses are closed curves and are the bounded case of the conic sections, the curves that result from tne intersection of a circular cone and a plane that does not pass through its apex.
7. **Triangles :** A polygon with three corners or vertices and three sides or edges which are line segments is called a triangle.
8. **Polygons :** A polygon is traditionally a plane figure that is bounded by a closed path or circuit, composed of a finite sequence of straight line segments.

9. **Spline :** A spline is a special function defined piecewise by polynomials and preferred to polynomial interpolation.
10. **Spheres :** A sphere is a perfectly round geometrical object in three dimensional space, such as the shape of a round ball.

Que 3.2. **Discuss 3D representation of objects.**

Answer

To represent a 3D object we have categorised the object into some model representation :

1. Polygon and quadric surfaces provide precise descriptions for simple Euclidean objects such as polyhedrons and ellipsoids.
2. Spline surfaces end construction techniques are useful for designing aircraft wings, gears, and other engineering structures with curved surfaces.
3. Procedural methods, such as fractal constructions and particle systems, allow us to give accurate representations for clouds, clumps of grass, and other natural objects.
4. Physically based modeling methods using systems of interacting forces can be used to describe the non-rigid behavior of a piece of cloth.
5. Visualization techniques are applied to three dimensional discrete data sets to obtain visual representations of the data.
6. Representation schemes for solid objects are often divided into two broad categories :
   a. Boundary representations (B-reps) describe a three dimensional object as a set of surfaces that separate the object interior from the environment.
   a. Space partitioning representations are used to describe interior properties, by partitioning the spatial region containing an object into a set of small, non-overlapping, contiguous solids (usually cubes).

PART-2

3D Transformation, 3D Viewing.

CONCEPT OUTLINE

- In 3D transformation we can change the shape, position and direction of any object with respect to any coordinates system.
- A 3D viewing in general referred as projection.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

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Que 3.3. **What is 3D transformation ? Write the matrix of each transformation.**

OR

Write short note on 3D transformation.

Answer

3D transformation :

1. In a three dimensional homogeneous coordinate representation, a point is translated from position $P(x, y, z)$ to position $P'(x', y', z')$ with matrix operation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = T.P$$

$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

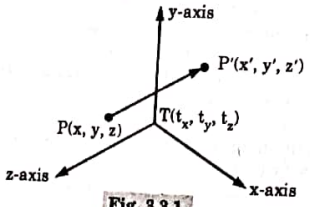

Fig. 3.3.1.

2. An object is translated in three dimensions by transforming each of the defining points of the object.

Rotation :

1. In 3D rotation transformation a point is rotated from $P(x, y, z)$ to $P'(x', y', z')$ and require an angle of rotation and an axis of rotation.
2. Matrix representation of rotation about z-axis is given as :

$$x' = x \cos\theta - y \sin\theta$$

$$y' = x \sin\theta + y \cos\theta$$

$$z' = z$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling : The matrix expression for the scaling transformation of a position $P(x, y, z)$ relative to the coordinate origin can be written as :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = SP$$

where

$$x' = x.s_x$$

$$y' = y.s_y$$

$$z' = z.s_z$$

Que 3.4. **Derive rotation about x-axis, y-axis and z-axis matrices in 3D. Also, prove that for any rotation matrix $R^{-1}(\theta) = R(-\theta)$.**

AKTU 2012-13, Marks 10

AKTU 2013-14, Marks 06

Answer

i. **Rotation about x-axis (*i.e.* axis of rotation is x) :**

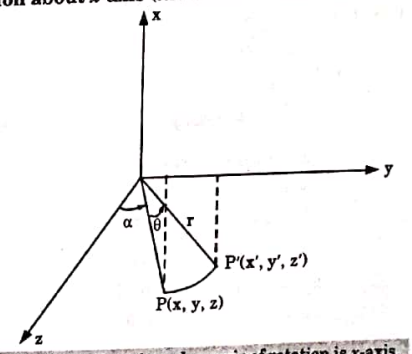

Fig. 3.4.1. Rotation when axis of rotation is x-axis.

$$x' = x$$

$$y' = r\cos(\alpha + \theta)$$

$$y' = r\cos\alpha\cos\theta - r\sin\alpha\sin\theta \quad \text{...(3.4.1)}$$

Since $r\cos\alpha = y$ and $r\sin\alpha = z$

Put the value in equation (3.4.1) we get,

$$y' = y\cos\theta - z\sin\theta$$

$$z' = r\sin(\alpha + \theta)$$

$$= r\sin\alpha\cos\theta + r\cos\alpha\sin\theta$$

$$= z\cos\theta + y\sin\theta$$

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or, $$R_{\theta,i} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

The points after rotation can be calculated by the following equation:

$$P'(x', y', z') = R_{\theta,i} P(x, y, z)$$

Equation in the matrix form is given as :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ii. Rotation about *y*-axis (*i.e.*, *y*-axis is axis of rotation) :

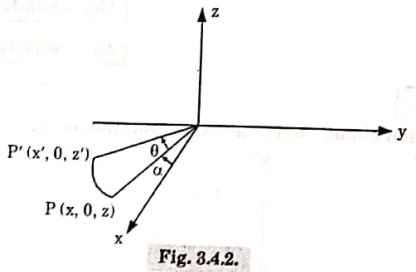

Fig. 3.4.2.

$$x' = x\cos\theta + z\sin\theta$$
$$y' = y$$
$$z' = -x\sin\theta + z\cos\theta$$

or, $$R_{\theta,i} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Equation in the matrix form is given as :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

iii. Rotation about the *z*-axis (*i.e.*, *z*-axis is axis of rotation)

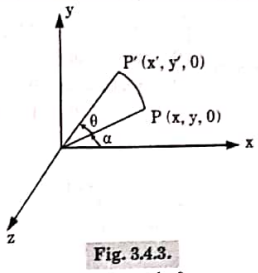

Fig. 3.4.3.

$$x' = x\cos\theta - y\sin\theta$$
$$y' = z\cos\theta + y\sin\theta$$
$$z' = z$$

or, $$R_{\theta,i} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation in the matrix form is given as :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Numerical :

Let $$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Put $\theta = (-\theta)$, we get

$$R(-\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) & 0 \\ 0 & \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since $\cos(-\theta) = \cos(\theta)$, $\sin(-\theta) = -\sin\theta$, we have

$$R(-\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Now,

$$R(-\theta) * R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

We have,

$$R(-\theta) \,.\, R(\theta) = I$$

Multiply $R^{-1}(\theta)$ both sides,

$$R(-\theta).R(\theta).R^{-1}(\theta) = I.R^{-1}(\theta)$$

$$R(-\theta) = R^{-1}(\theta)$$

Hence proved.

Que 3.5. **Write the steps involved in constructing 3D viewing transformation.**

Answer

The steps involved in constructing 3D viewing transformations :
The complete three dimensional viewing process (without hidden surface removal) is described by the following steps :

1. Transform coordinates from world coordinates to normalized viewing coordinates by applying the transformation N_{par} or N_{per}.
2. Clip in normalized viewing coordinates against the canonical clipping volumes.
3. Project onto the screen projection plane using the projections *Par* or *Per*.
4. Apply the appropriate (two dimensional) viewing transformation.

In terms of transformations, we can describe the above process in terms of a viewing transformation V_T, where

$$V_T = V_2\, Par\, CL\, N_{par} \text{ or } V_T = V_2\, Per\, CL\, N_{per}$$

Here *CL* and V_2 refer to the appropriate clipping operations and two dimensional viewing transformations.

PART-3

Projection.

CONCEPT OUTLINE

- Projection is a process of representing a three dimensional object or scene into two dimensional medium *i.e.*, projection is a shadow of the object.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.6. **What do you mean by projection ? Explain the types of projection differentiate between parallel and perspective projection.**

Answer

Projection :

1. Projection is a process of representing a three dimensional object or scene into two dimensional medium *i.e.*, projection is a shadow of the object.
2. Projections transform points in a coordinate system of dimension '*n*' into points in a coordinate system of dimension less than *n*.

Types of projections :

1. **Parallel projection :**
   a. In a parallel projection, coordinate positions are transformed to the view plane along parallel lines.
   b. These are linear transforms that are useful in blueprints to produce scale drawings of three dimensional objects.
2. **Perspective projection :**
   a. In a perspective projection, object positions are transformed to the view plane along lines that converge to a point called the center of projection.
   b. The projected view of an object is determined by calculating the intersection of the projection lines with the view plane.

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Difference between parallel and perspective projection :

S. No.	Parallel projection	Perspective projection
1.	In parallel projection, parallel lines from each vertex on the object are extended until they intersect the view plane.	Perspective projection produces realistic view.
2.	Lines of projection are parallel.	Lines of projection are not parallel.
3.	These are linear transform.	These are non-linear transform.
4.	In parallel projection, the center of projection is at infinity.	In perspective projections, the center of projection is at a point.

Que 3.7. **What do you understand by parallel projection ? Also describe orthographic and oblique projection.**

Answer

Parallel projection : Refer Q. 3.6, Page 3–9C, Unit-3.

Orthographic projection :

1. It is a part of the parallel projection in which the center of projection lies at infinity.
2. In orthographic projection, the projectors are perpendicular to the projection plane and hence true size and shape of a single plane face of object is obtained.
3. Orthographic projections do not change the length of line segments which are parallel to projection plane and other lines are projected with reduced length.
4. It is the projection on one of the coordinate planes *i.e.*,

$$x = 0, \quad y = 0 \quad \text{or} \quad z = 0$$

5. The matrix for projection onto the $x = 0$, $y = 0$ and $z = 0$ plane is

$$P_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } P_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oblique projection :

1. An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane.
2. In this projection angle between the projectors and the plane of projection is not equal to 90° *i.e.*, projection are non-perpendicular to view plane.
3. This type of projection is seen in form of shadow of any object due to sunlight.
4. Transformation matrix for oblique projection is given as

$$M = \begin{bmatrix} 1 & 0 & L_1 \cos\theta & 0 \\ 0 & 1 & L_1 \sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, L_1 is the length of line.

Que 3.8. **Derive perspective projection transformation matrix. Also define anomalies of perspective projection.**

Answer

Transformation matrix of perspective projection :

1. A perspective transformation is determined by prescribing a center of projection and a view plane.
2. The view plane is determined by its view reference point R_0 and view plane normal N.
3. The object point p is located in world coordinates at (x, y, z). The problem is to determine the image point coordinates $p'(x', y', z')$.
4. The standard perspective projection is shown in Fig. 3.8.1. Here the view plane is the xy-plane and center of projection is taken as the point $C(0, 0, -d)$ on negative z-axis.

$$x' = \frac{d.x}{z+d}, y' = \frac{d.y}{z+d}, z' = 0$$

5. The perspective projection transformation between object and image point is non-linear and so cannot be represented as 3 × 3 matrix transformation.
6. If we use homogeneous coordinate, the perspective transformation can be represented as 4 × 4 matrix :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} d.x \\ d.y \\ 0 \\ z+d \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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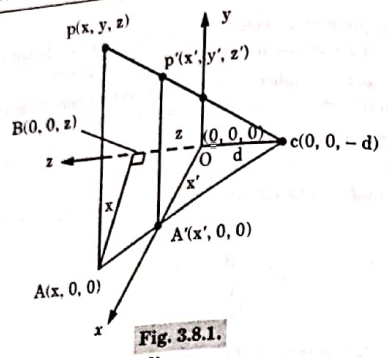

Fig. 3.8.1.

Perspective projection anomalies :

1. **Perspective foreshortening :** The farther an object is from the center of projection, the smaller it appears.

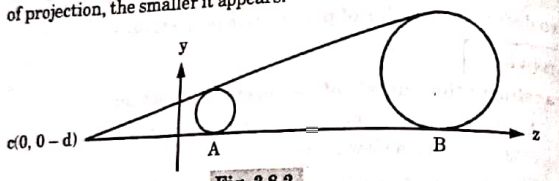

Fig. 3.8.2.

2. **Vanishing point :** Projections of lines that are not parallel to the view plane appear to meet at some point on the view plane.
3. **View confusion :** Objects behind the center of projection are projected upside down and backward onto the view plane.

Que 3.9. **Derive oblique parallel projection and perspective projection matrices.** **AKTU 2012-13, 2013-14; Marks 06**

Answer

Derivation of oblique parallel projection :

1. An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane.
2. An oblique projection of a point $P(x, y, z)$ on the $z = 0$ plane is shown in Fig. 3.9.1.
3. $P_{ob}(x_p, y_p)$ is oblique projection of the point P and $P_{or}(x, y)$ is the orthographic projection of P on the $z = 0$ plane.
4. Let L be the length of the line joining points $P_{ob}(x_p, y_p)$ and $P_{or}(x, y)$, θ be the angle made by the line L with x-axis.

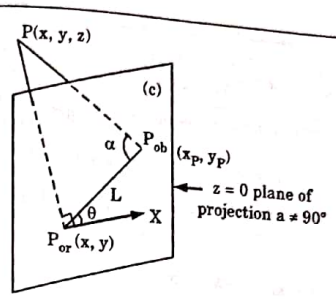

Fig. 3.9.1.

5. We can express the projection coordinates in terms of x, y, L and θ as
$$x_p = x + L\cos\theta$$
$$y_p = y + L\sin\theta$$
6. Length L depends on the angle α and the Z coordinate of the point to be projected
$$\tan\alpha = \frac{Z}{L} \Rightarrow L = \frac{Z}{\tan\alpha} = ZL_1 \quad \left(\text{where } L_1 = \frac{1}{\tan\alpha}\right)$$
when $Z = 1, L = L_1$
7. Now, we can write the oblique projection as
$$x_p = x + Z(L_1\cos\theta)$$
$$y_p = x + Z(L_1\sin\theta)$$
8. Considering $Z_p = 0$, we get
$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1\cos\theta & 0 \\ 0 & 1 & L_1\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Derivation of perspective projection matrix :

1. Let us consider the center of projection is at (x_c, y_c, z_c) and the point on object is (x_1, y_1, z_1), then the parametric equation for the line containing these points can be given as
$$x_2 = x_c + (x_1 - x_c)u$$
$$y_2 = y_c + (y_1 - y_c)u$$
$$z_2 = z_c + (z_1 - z_c)u$$
where u is the parameter
2. For projected point z_2 is 0, therefore, the third equation can be written as
$$0 = z_c + (z_1 - z_c)u$$

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$$\therefore \quad u = -\frac{z_c}{(z_1 - z_c)}$$

3. Put this value of u in first two equations, we get

$$x_2 = x_c - \frac{z_c}{(z_1 - z_c)}(x_1 - x_c)$$

$$= \frac{x_c z_1 - x_c z_c - x_1 z_c + x_c z_c}{(z_1 - z_c)}$$

$$\therefore \quad x_2 = \frac{x_c z_1 - x_1 z_c}{(z_1 - z_c)}$$

and

$$y_2 = y_c - \frac{z_c}{(z_1 - z_c)}(y_1 - y_c)$$

$$= \frac{y_c z_1 - y_c z_c - y_1 z_c + y_c z_c}{(z_1 - z_c)}$$

$$y_2 = \frac{y_c z_1 - y_1 z_c}{(z_1 - z_c)}$$

4. So, the homogeneous matrix can be written as

$$[x_2 \quad y_2 \quad z_2 \quad 1] = [x_1 \quad y_1 \quad z_1 \quad 1]\begin{bmatrix} -z_c & 0 & 0 & 0 \\ 0 & -z_c & 0 & 0 \\ x_c & y_c & 0 & 1 \\ 0 & 0 & 0 & -z_c \end{bmatrix}$$

Que 3.10. **Explain parallel and perspective projection. Justify the depth cueing projection for 3D display methods.**

AKTU 2015-16, Marks 10

Answer

Parallel projection : Refer Q. 3.6, Page 3–9C, Unit-3.

Types of parallel projection :

i. **Orthographic projection :** Refer Q. 3.7, Page 3–10C, Unit-3.

ii. **Oblique projection :** Refer Q. 3.7, Page 3–10C, Unit-3.

Perspective projection : Refer Q. 3.6, Page 3–9C, Unit-3.

Different types of perspective projection are :

1. **Single-point perspective transformation :** One-point perspective projection occurs when only one principal axis intersects the plane of projection.
2. **Two-point perspective transformation :** Two-point or two principal vanishing point perspective projection occurs when the plane of projection intersects exactly two of the principal axis.

3. **Three-point perspective transformation :** Three-point projection is obtained when the projection plane intersects all three of the principal axis *i.e.*, none of the principal axis is parallel to the projection plane.

Depth cueing projection :

1. In depth cueing, we include the depth information in the displayed object.
2. The depth of an object can be represented by the intensity of the image.
3. Depth cueing is applied by choosing maximum and minimum intensity values.
4. The parts of the objects closest to the viewing position are displayed with the highest intensities and objects farther away are displayed with decreasing intensities.
5. Reason to use depth cueing projection for 3D display method:
   a. To create realistic view of an object.
   b. To easily identify the particular viewing direction of displayed object.
6. A simple method for indicating depth is using wireframe display.

Que 3.11. **Explain the matrix for perspective projection for three vanishing points.**

Answer

Matrix for perspective projection for three vanishing point :

1. Three vanishing point projection occurs when the projection plane intersects all three of the principal axis *i.e.*, none of the principal axis is parallel to the projection plane.
2. The need of three vanishing point perspective transformation is to reconstruct the shape of a 3D object.
3. The matrix representation of three vanishing point perspective transformation is

$$\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[x' \quad y' \quad z' \quad 1] = [x \quad y \quad z \quad 1]\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \quad y \quad z \quad (px + qy + rz + 1)]$$

$$= \left[\frac{x}{(px + qy + rz + 1)} \quad \frac{y}{(px + qy + rz + 1)} \quad \frac{z}{(px + qy + rz + 1)} \quad 1\right]$$

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4. It has three center of projections, one on x-axis at $[-1/p, 0, 0, 1]$, second on y-axis at $[0, -1/q, 0, 1]$ and third on z-axis at $[0, 0, -1/r, 1]$.
5. It also has three vanishing points, one on x-axis $[1/p, 0, 0, 1]$, one on y-axis at $[0, 1/q, 0, 1]$ and third on z-axis at $[0, 0, 1/r, 1]$.

Que 3.12. **Find the perspective projection onto the view plane $z = d$ where the centre of projection is the origin (0, 0, 0).**

Answer

The plane $z = d$ is parallel to the xy plane. Thus the view plane normal vector N is the same as the normal vector K to the xy plane, *i.e.*, $N = K$. Select view reference point as $R_0(0, 0, d)$, then

$$N(n_1, n_2, n_3) = (0, 0, 1)$$
$$R_0(x_0, y_0, z_0) = (0, 0, d)$$
$$d_0 = n_1x_0 + n_2y_0 + n_3z_0 = d$$

then projection matrix is

$$\text{Per}_{k,R_0} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Que 3.13. **Find the general form of an oblique projection onto the xy plane.**

Answer

1. Oblique projection can be specified by a number f and an angle θ. The number f prescribes the ratio that any line L perpendicular to the xy plane will be foreshortened after projection.
2. The angle θ is the angle that the projection of any line perpendicular to the xy plane makes with the x-axis.

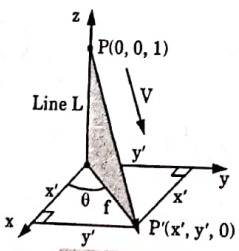

Fig. 3.13.1.

3. To determine the projection transformation, we need to find the direction vector V. From above figure we have line L of length 1, we see that vector $\overline{P'P}$ has the same direction on V.

$$V = \overline{P'P} = x'\hat{i} + y'\hat{j} - \hat{k} = a\hat{i} + b\hat{j} + c\hat{k}$$
$$a = x' = f\cos\theta,\ b = y' = f\sin\theta$$
$$c = -1$$

$$\text{Par}_V = \begin{bmatrix} 1 & 0 & f\cos\theta & 0 \\ 0 & 1 & f\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Que 3.14. **Find the transformation for (*a*) cavalier with $\theta = 45°$ and (*b*) cabinet projection with $\theta = 30°$.**

Answer

a. A cavalier projection is an oblique projection where there is no foreshortening of line perpendicular to the xy plane. We then see for $f = 1$, with $\theta = 45°$, we have

$$\text{Par}_{V1} = \begin{bmatrix} 1 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. A cabinet projection is an oblique projection with $f = \frac{1}{2}$, with $\theta = 30°$, we have

$$\text{Par}_{V2} = \begin{bmatrix} 1 & 0 & \frac{\sqrt{3}}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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PART-4
3D Clipping

CONCEPT OUTLINE

- Clipping in three dimensions can be accomplished using extensions of two dimensional clipping methods.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.15. **What do you mean by 3D clipping ? Discuss in detail.**

OR

Write a short note on 3D clipping.

Answer

1. Clipping in three dimensions can be accomplished using extensions of two dimensional clipping methods.
2. In 3D clipping we clip the objects against the boundary planes of the view volume.
3. To clip a line segment against the view volume, we would need to test the relative position of the line using the view volume's boundary plane equations.
4. By substituting the line endpoint coordinates into the plane equation of each boundary which determines whether the endpoint is inside or outside that boundary.
   a. An endpoint (x, y, z) of a line segment is outside a boundary plane if $Ax + By + Cz + D > 0$
   b. Similarly, the point is inside the boundary if $Ax + By + Cz + D < 0$.
   c. Lines with both endpoints outside a boundary plane are discarded, and those with both endpoints inside all boundary planes are saved.
5. The intersection of a line with a boundary is found using the line equations along with the plane equation.

6. Intersection coordinates (x_I, y_I, z_I) are values that are on the line and that satisfy the plane equation $Ax_I + By_I + Cz_I + D = 0$, where A, B, C, and D are the plane parameters for that boundary.

Que 3.16. **Write 3D line clipping algorithm for Cohen-Sutherland region code method.**

OR

Establish and write Cohen and Sutherland 3D line clipping algorithm.

AKTU 2012-13, Marks 10

Answer

1. The Cohen Sutherland algorithm extends easily to 3D. It extends the 3D clipping window boundaries to define 27 regions. Assign a 6 bit code to each region, that is, for each point (x, y, z).
2. Suppose the bands for cuboid view volume are (X_{vmin}, X_{vmax}), (Y_{vmin}, Y_{vmax}) and (Z_{vmin}, Z_{vmax}) where the subscript vmin and vmax stands for view volume minlimit and view volume maxlimit (front and back end planes).
3. We assign bit codes to any point (x, y, z) as :

   Bit1 = 1 if $x < X_{min}$

   Bit 2 = 1 if $x > X_{max}$

   Bit 3 = 1 if $y < Y_{min}$

   Bit 4 =1 if $y > Y_{max}$

   Bit 5 = 1 if $z < Z_{min}$

   Bit 6 =1 if $z > Z_{max}$.

   Otherwise bits are set to 0 (zero).
4. A point is inside if the region code is 000000. Therefore a line is visible if both ends are 000000.
5. A line is invisible if it lies entirely to one side of the clipping region, that is, if logical AND operation on the bit codes of the end points is non-zero.

Que 3.17. **Establish and write Cyrus-Beck 3D line clipping algorithm.**

AKTU 2013-14, Marks 06

Answer

1. Cyrus-Beck algorithm is applicable to an arbitrary convex region.
2. This algorithm uses a parametric equation of line segment to find the intersection points of a line with the clipping edges.

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3. The parametric equation of a line segment from P_1 to P_2 is

$$P(t) = P_1 + (P_2 - P_1)\,t,\ 0 \le t \le 1$$

where t is a parameter, $t = 0$ at P_1 and $t = 1$ at P_2.

4. Consider a convex clipping region R, f is a boundary point of the convex region R and n is an inner normal for one of its boundaries, as shown in Fig. 3.17.1.

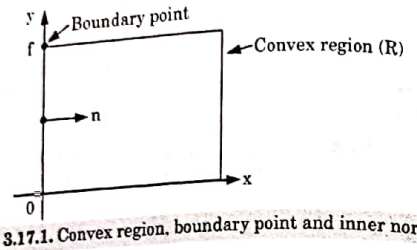

Fig. 3.17.1. Convex region, boundary point and inner normal.

5. We evaluate the dot product $n \cdot [P(t) - f]$, and check the following condition as :

a. If dot product is negative, *i.e.*,

$$n \cdot [P(t) - f] < 0$$

Then the vector $P(t) - f$ is pointed away from the interior of R.

b. If dot product is zero, *i.e.*,

$$n \cdot [P(t) - f] = 0$$

Then $P(t) - f$ is pointed parallel to the plane containing f and perpendicular to the normal.

c. If dot product is positive, *i.e.*,

$$n \cdot [P(t) - f] > 0$$

Then the vector $P(t) - f$ is pointed towards the interior of R, as shown in Fig. 3.17.2.

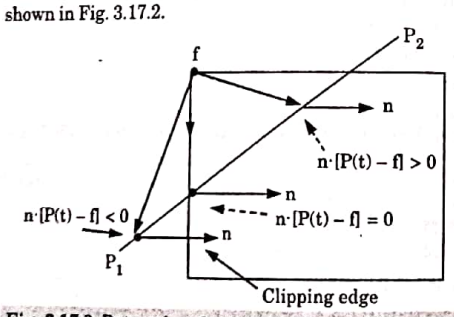

Fig. 3.17.2. Dot product for three inside, outside and on the boundary of the clipping region.

Cyrus-Beck line clipping algorithm :

1. Read two endpoints of the line, P_1 and P_2.
2. Read vertex coordinates of the clipping window.
3. Calculate $D = P_2 - P_1$.
4. Assign boundary point (f) with particular edge.
5. Find inner normal vector for corresponding edge.
6. Calculate $D \cdot n$ and $W = P_1 - f$.
7. If $D \cdot n > 0$.

$$t_L = -\frac{W \cdot n}{D.n}$$

else

$$t_U = -\frac{W \cdot n}{D.n}$$

end if

8. Repeat steps 4 to 7 for each edge of the clipping window.
9. Find maximum lower limit and minimum upper limit.
10. If maximum lower limit and minimum upper limit do not satisfy condition $0 \le t \le 1$ then ignore the line.
11. Calculate the intersection points by substituting values of maximum lower limit and minimum upper limit in the parametric equation of the line $P_1 P_2$.
12. Draw the line segment $P(t_L)$ to $P(t_U)$.
13. Stop.

Que 3.18. **Consider the line from $P_1(-2, 1)$ to $P_2(8, 4)$ clipped to the rectangular region R as shown in the Fig. 3.18.1. The line P_1P_2 intersects the window. Calculate the intersection points.**

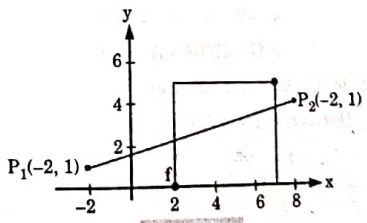

Fig. 3.18.1.

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Answer

1. The parametric representation of the line P_1P_2 is

$$P_{(t)} = P_1 + (P_2 - P_2)t = [-2\ 1] + [10\ 3]\ t$$
$$= (10t - 2)\ i + (3t + 1)\ j \qquad 0 \le t \le 1$$

where i and j are the unit vectors in the x and y directions, respectively.

2. The four inner normal's are given as

Left : $n_L = i$

Right : $n_R = -i$

Bottom : $n_B = j$

Top : $n_T = -j$

3. Choosing $f(2, 0)$ for the left edge gives

$$P(t) - f = (10t - 4)\ i + (3t + 1)\ j \quad \text{and}$$
$$n_L \cdot [P(t) - f] = 10t - 4 = 0$$
$$\therefore \qquad t = 2/5$$

4. Substituting value of t in parametric equation we get,

$$P(2/5) = [-2 \quad 1] + [10 \quad 3]\ (2/5)$$
$$= [-2 \quad 1] + [4 \quad 6/5]$$
$$= [2 \quad 2.2]$$

5. Choosing $f(7, 5)$ for the right edge gives

$$P(t) - f = (10t - 9)\ i + (3t - 4)\ j \text{ and}$$
$$n_R \cdot [P(t) - f] = -(10t - 9) = 0$$
$$t = 9/10$$

6. Substituting value of t in parametric equation we get,

$$P(9/10) = [-2 \quad 1] + [10 \quad 3]\ (9/10)$$
$$= [-2 \quad 1] + [9 \quad 27/10]$$
$$= [7 \quad 37/70] = [7 \quad 3.7]$$

7. Using $f(2, 0)$ for the bottom edge gives

$$n_B \cdot [P(t) - f] = 3t + 1 = 0$$
$$\therefore \qquad t = -1/3$$

8. This value of t is outside the range of $0 \le t \le 1$ and hence it is rejected. Using $f(7, 5)$ for the top edge gives

$$n_T \cdot [P(t) - f] = -(3t - 4) = 0$$
$$\therefore \qquad t = 4/3$$

9. This value of t is outside the range of $0 \le t \le 1$ and hence it is rejected. Thus, we get two intersection points (2, 2.2) and (7, 3.7) with left edge and right edge, respectively.

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4 UNIT

Curves and Surfaces

CONTENTS

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Quadric Surface
Spheres
Ellipsoid
Blobby Objects

Part-2 : Introductory Concepts of Spline ..... 4–4C – 4–9C

Part-3 : B-spline and Bezier ..... 4–9C – 4–17C
Curve and Surfaces

PART-1

Curves and Surfaces : Quadric Surfaces, Spheres, Ellipsoid, Blobby Objects.

CONCEPT OUTLINE

- Curves are the graphical representation of any quantity between x and y-axis.
- Ellipsoid surface is the extension of a spherical surface.
- Quadric surface include spheres, ellipsoids, tori, paraboloids, etc.
- Blobby object are non-rigid object that do not have fixed shape and change their surface characteristics in certain motions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Explain various types of quadratic surfaces with their equation and blobby objects.** **AKTU 2014-15, Marks 06**

Answer

Various types of quadratic surfaces :

1. **Sphere :**

a. A sphere with a radius r is defined as the set of points (x, y, z) that satisfies the following equations :

$$x^2 + y^2 + z^2 = r^2$$

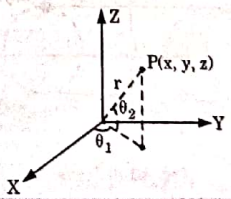

Fig. 4.1.1. Parametric coordinate position (r, θ_1, θ_2) on the surface of a sphere.

b. The parametric equations for spherical surfaces are :

$$x = r \cos \theta_1 \cos \theta_2$$

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$$y = r \sin \theta_1 \cos \theta_2$$
$$z = r \sin \theta_2$$

where, $-\frac{\pi}{2} \le \theta_2 \le \frac{\pi}{2}$ and $-\pi \le \theta_1 \le \pi$

2. **Ellipsoid :**

a. An ellipsoid surface is the extension of a spherical surface. There are three mutually perpendicular radii that have different values as shown in Fig. 4.1.2.

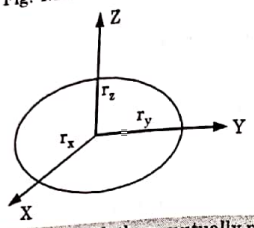

Fig. 4.1.2. An ellipsoid with three mutually perpendicular radii r_x, r_y and r_z, centered on the coordinate origin.

b. In the cartesian coordinate system, the ellipsoid centered at the origin can be defined as :

$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

c. The parametric equations are :

$$x = r_x \cos \theta_1 \cos \theta_2$$
$$y = r_y \sin \theta_1 \cos \theta_2$$
$$z = r_z \sin \theta_2$$

where, $-\pi/2 \le \theta_1 \le \pi/2$ and $-\pi \le \theta_2 \le \pi$

3. **Torus :**

a. A doughnut-shaped object as shown in Fig. 4.1.3 is called a torus.

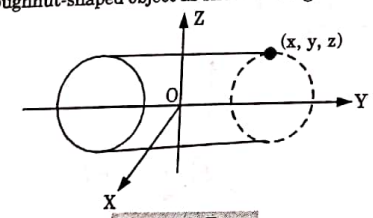

Fig. 4.1.3. Torus

This can be generated by rotating a conic object like a circle about a specified axis.

b. In cartesian coordinates, equations for points over the surface of a torus is given as

$$\left[r - \sqrt{\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)}\right]^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

c. The parametric representations for a torus can be given as

$$x = r_x(r + \cos\phi) \cos \theta, \qquad -\pi \le \phi \le \pi$$
$$y = r_y(r + \cos\phi) \sin \theta, \qquad -\pi \le \theta \le \pi$$
$$z = r_z \sin \phi$$

Blobby objects :

1. Blobby objects do not maintain a fixed shape and change their surface characteristics in certain motions or when they come in contact with other objects.
2. Molecular structures, melting objects, water droplets, muscle shapes in the human body, etc., are examples of blobby objects.
3. As objects show a certain degree of fluidity and blobbiness, these objects are simply known as blobby objects.

Fig. 4.1.4. Molecular bonding as two molecules move away from each other, the surface shape stretch, snap and finally contract into spheres.

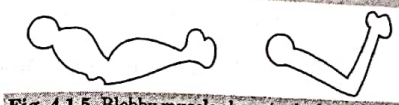

Fig. 4.1.5. Blobby muscle shape in the human arm.

PART-2

Introductory Concepts of Spline.

CONCEPT OUTLINE

- B-spline is basis spline function which contains a set of control points.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.2. **Explain the B-spline curves. What are the properties of B-spline curve ?**

OR

Write the properties of B-spline curves. Also write advantages of B-spline curves over Bezier curves.

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Answer

B-spline curves :

1. B-spline is basis spline function which contains a set of control points.
2. B-spline function is a combination of flexible band that passes through the number of control points and creates a smooth curve.
3. Each vertex affects the shape of the curve only over a range of parameter values where its associated basis function is non-zero.

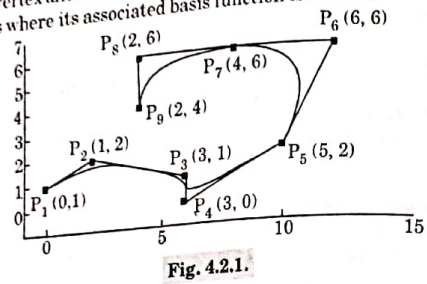

Fig. 4.2.1.

4. The B-spline is non-global because each vertex B_i is associated with a unique basis function.
5. If $P(u)$ be the position vectors along the curve as a function of the parameter u, a B-spline curve is given by

$$P(u) = \sum_{k=0}^{n} P_k B_{k,d}(u), \; u_{min} \le u \le u_{max}$$

$$2 \le d \le n + 1$$

where the p_k is an input set of $n + 1$ control points.

Example : The unweighted spline set of 10 control points, 10 splines, 14 knots, and but only 7 intervals are shown is Fig. 4.2.2.

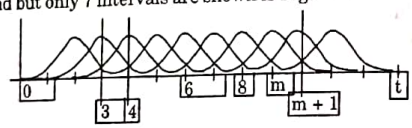

Fig. 4.2.2.

Properties of B-splines curves :

1. The polynomial curve has degree d–1 and C^{d-2} continuity over the range of u.
2. For $n + 1$ control points, the curve is described with $n + 1$ blending functions.
3. Each blending function $B_{k,d}$ is defined over d subintervals of the total range of u, starting at knot value u_k.

4. The range of parameter u is divided into $n + d$ subinterval by the $n + d + 1$ values specified in the knot vector.
5. Each section of spline curve is influenced by d control points.
6. Any one control point can affect the shape of atmost d curve sections.
7. For any value of u in the interval from knot value u_{d-1} to u_{n+1} the sum over all basic function is 1 :

$$\sum_{k=0}^{n} B_{k,d}(u) = 1$$

Advantages of B-spline curves over Bezier curves :

1. The degree of B-spline polynomial can be set independently of the number of control points.
2. B-spline allows local control over the shape of a spline curve or surface.
3. The trade-off is that B-splines are more complex than Bezier curves.

Que 4.3. What are the parametric continuity conditions in a spline representation ?

Answer

We imposed various continuity conditions at the connection points to ensure smooth transition from one section to the next section of the piecewise parametric curve. If the spline curve is described in terms of parametric coordinate form like

$$x = x(v), \; y = y(v), \; z = z(v)$$

Spline curve can be represented as :

1. **Zero-order parametric continuity :** When two pieces of the curve meet at a point, this is known as zero-order parametric continuity as shown in Fig. 4.3.1, and is represented by C^0.

Fig. 4.3.1. Zero-order.

The values of x, y, z can be evaluated at v, for the first piece of curve and will be equal to the values of x, y, z evaluated at v, for the second curve.

2. **First-order parametric continuity :** The first-order parametric continuity shows that the first derivatives of the coordinate function $x = x(v)$, $y = y(v)$, $z = z(v)$ for two successive pieces are equal at their joining point as shown in Fig. 4.3.2. This is represented by C^1.

Fig. 4.3.2. First-order.

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3. **Second-order parametric continuity :** The second-order parametric continuity shows that both the first and second-order derivatives of the two curve pieces are the same at intersection as shown in Fig. 4.3.3. This is represented by C^2.

Fig. 4.3.3. Second-order.

Que 4.4. Explain the characteristics of B-spline curve. How is it useful for interpolation ?

Answer

Characteristics of B-spline curve :

1. The sum of the B-spline basis functions for any parameter value is 1.
2. Each basis function is positive or zero for all parameter values.
3. Each basis function has precisely one maximum value, except for $k = 1$.
4. The maximum order of the curve is equal to the number of vertices of defining polygon.
5. The degree of B-spline polynomial is independent on the number of vertices of defining polygon.
6. The curve exhibits the variation diminishing property.
7. The curve generally follows the shape of defining polygon.

B-spline curve is useful for interpolation because :

1. The degree of B-spline polynomial is independent of the number of control points of defining polygon.
2. B-spline allows local control over the curve surface because each control point affects the shape of a curve only over a range of parameters values where its associated basis function is non-zero.

Que 4.5. Describe parametric representation of surfaces.

Answer

Parametric representation of surfaces means a continuous, vector-valued function $P(s, t)$ of two variables, or parameters s and t where the variables are allowed to range over connected region of the uv plane.

Parametric description :

1. The three equations for determining the coordinates of any point on the surface S are described in terms of parameters, say, s and f, and in parameter ranges $[a, b]$ and $[c, d]$, which may be infinite :

$$S : \begin{aligned} x &= f(s, t), \\ y &= g(s, t) \\ z &= h(s, t) \end{aligned}$$

2. The coordinates of any point P on the surface have the form $[f(s, f), g(s, t), h(s, t)]$.
3. Spherical surface in parametric form can be defined in terms of the angular parameters ϕ and θ :

$$x = r \cos \phi \cos \theta, \text{ for } -\pi/2 \le \phi \le \pi/2$$
$$y = r \cos \phi \sin \theta, \text{ for } -\pi \le \theta \le \pi$$
$$z = r \sin \phi$$

4. Parametric representation for a torus is similar to those for an ellipse, except that the angle ϕ extends over 360°. Using latitude and longitude angles ϕ and θ, we can describe the torus surface as the set of points that satisfy

$$X = r_x \cos \phi \cos \theta, \text{ for } -\pi/2 \le \phi \le \pi/2$$
$$Y = r_y \cos \phi \sin \theta, \text{ for } -\pi \le \theta \le \pi$$
$$Z = r_z \sin \phi$$

Que 4.6. Write note on interpolation.

Answer

1. Interpolation is a method of constructing new data points within the range of a discrete set of known data points.
2. The number of data points obtained by sampling or experimentation represents the values of a function for a limited number of values of the independent variable.
3. It is often required to interpolate, *i.e.*, estimate the value of that function for an intermediate value of the independent variable.
4. A few data points from the original function can be interpolated to produce a simpler function which is still fairly close to the original. The resulting gain in simplicity may outweigh the loss from interpolation error.
5. The main task in this process is to find the suitable mathematical expression for the known curve.
6. Interpolation technique is used when we have to draw the curve by determining intermediate points between the known sample points.

Que 4.7. Explain the types of interpolation method.

Answer

Type of interpolation methods are :

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1. **IDW :**
   a. The IDW (Inverse Distance Weighted) tool uses a method of interpolation that estimates cell values by averaging the values of sample data points in the neighbourhood of each processing cell.
   b. The closer a point is to the center of the cell being estimated the more influence or weight it has in the averaging process.
   c. A specified number of points or all points within a specified radius can be used to determine the output value of each location.
2. **Kriging :**
   a. Kriging is a geostatistical procedure that considers both the distance and the degree of variation between known data points when estimating values in unknown areas.
   b. This procedure generates an estimated surface from a scattered set of points with z-values.
   c. It should be done before we select the best estimation method for generating the output surface.
3. **Natural neighbour :**
   a. Natural neighbour interpolation finds the closest subset of input samples to a query point and applies weights to them based on proportionate areas to interpolate a value.
   b. It is also known as Sibson or "area-stealing" interpolation.
4. **Spline :** The spline tool uses an interpolation method that estimates values using a mathematical function that minimizes overall surface curvature, resulting in a smooth surface that passes exactly through the input points.
5. **Spline with barriers :** The spline with barriers tool uses a method similar to the technique used in the Spline tool, with the major difference being that this tool honors discontinuities encoded in both the input barriers and the input point data.
6. **Topo to raster :** It use an interpolation technique specifically designed to create a surface that more closely represents a natural drainage surface and better preserves stream networks from input contour data.
7. **Trend :**
   a. Trend is a global polynomial interpolation that fits a smooth surface defined by a mathematical function (a polynomial) to the input sample points.
   b. The trend surface changes gradually and captures coarse-scale patterns in the data.

PART-3

B-Spline and Bezier Curve and Surfaces.

CONCEPT OUTLINE

- A Bezier curve section can be filled to any number of control points.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.8. **Explain Bezier curves and properties of Bezier curves.**

Answer

Bezier curves :

1. A Bezier curve section can be filled to any number of control points.
2. The number of control points to be approximated and their relative position determine the degree of Bezier polynomial.
3. Bezier curve can be specified with boundary conditions with a characterizing matrix or with blending functions.
4. For general Bezier curves, the blending function specification is most convenient.
5. Suppose we are given $n + 1$ control points position : $P_k = (x_k, y_k, z_k)$ with k varying from 0 to n.

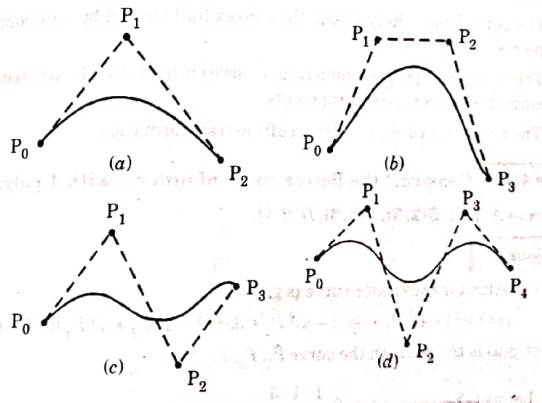

Fig. 4.8.1. Example of Bezier curves generated from three, four, five control points.

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6. These coordinate point can be blended to produce the following position vector $P(u)$, which describes the path of an approximation Bezier polynomial function between P_0 and P_n.

$$P(u) = \sum_{k=0}^{n} P_k BEZ_{k,n}(u) \quad 0 \le u \le 1 \qquad \text{...(4.8.1)}$$

Equation (4.8.1) represents a set of three parametric equations for individual coordinates :

$$x(u) = \sum_{k=0}^{n} x_k BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k BEZ_{k,n}(u)$$

where, $BEZ_{k,n}(u) = c(n, k) \,.\, u^k . (1-u)^{n-k}$

Properties of Bezier curves :

1. Bezier curve always passes through the first and last control points *i.e.*, curve has same end points as the guiding polygon.
2. The degree of the polynomial defining the curve segment is one less than the number of defining polygon point. Therefore, for four control points, the degree of the polynomial is three, *i.e.*, cubic polynomial.
3. The direction of the tangent vector at the end points is the same as that of the vector determined by first and last segments.
4. The curve lies entirely within the convex hull formed by four control points.
5. The convex hull property for a Bezier curve ensures that the polynomial smoothly follows the control points.
6. The curve is invariant under an affine transformation.

Que 4.9. **Construct the Beizer curve of order 3 with 4 polygon vertices $A(1, 1), B(2, 3), C(4, 3), D(6, 4)$.**

Answer

The equation for the Bezier curve is given as :

$$p(u) = (1-u)^3 P_1 + 3u(1-u)^2 P_2 + 3u^2 (1-u) P_3 + u^3 P_4 \text{ for } 0 \le u \le 1$$

where $p(u)$ is the point on the curve P_1, P_2, P_3, P_4.

Let us take $u = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

$\therefore$ $P(0) = (1, 1)$

$$\therefore \quad P\left(\frac{1}{4}\right) = \left(1-\frac{1}{4}\right)^3 P_1 + 3\frac{1}{4}\left(1-\frac{1}{4}\right)^2 P_2 + 3\left(\frac{1}{4}\right)^2\left(1-\frac{1}{4}\right) P_3 + \left(\frac{1}{4}\right)^3 P_4$$

$$= \frac{27}{64}(1, 1) + \frac{27}{64}(2, 3) + \frac{9}{64}(4, 3) + \frac{1}{64}(6, 4)$$

$$= \left[\frac{27}{64}\times 1 + \frac{27}{64}\times 2 + \frac{9}{64}\times 4 + \frac{1}{64}\times 6, \frac{27}{64}\times 1 + \frac{27}{64}\times 3 + \frac{9}{64}\times 3 + \frac{1}{64}\times 4\right]$$

$$= \left[\frac{123}{64}, \frac{139}{64}\right] = (1.9218, 2.1718)$$

$$\therefore \quad P\left(\frac{1}{2}\right) = \left(1-\frac{1}{2}\right)^3 P_1 + 3\frac{1}{2}\left(1-\frac{1}{2}\right)^2 P_2 + 3\left(\frac{1}{2}\right)^2\left(1-\frac{1}{2}\right) P_3 + \left(\frac{1}{2}\right)^3 P_4$$

$$= \frac{1}{8}(1, 1) + \frac{3}{8}(2, 3) + \frac{3}{8}(4, 3) + \frac{1}{8}(6, 4)$$

$$= \left[\frac{1}{8}\times 1 + \frac{3}{8}\times 2 + \frac{3}{8}\times 4 + \frac{1}{8}\times 6, \frac{1}{8}\times 1 + \frac{3}{8}\times 3 + \frac{3}{8}\times 3 + \frac{1}{8}\times 4\right]$$

$$= \left[\frac{25}{8}, \frac{23}{8}\right] = (3.125, 2.875)$$

$$\therefore \quad P\left(\frac{3}{4}\right) = \left(1-\frac{3}{4}\right)^3 P_1 + 3\frac{3}{4}\left(1-\frac{3}{4}\right)^2 P_2 + 3\left(\frac{3}{4}\right)^2\left(1-\frac{3}{4}\right) P_3 + \left(\frac{3}{4}\right)^3 P_4$$

$$= \frac{1}{64}P_1 + \frac{9}{64}P_2 + \frac{27}{64}P_3 + \frac{27}{64}P_4$$

$$= \frac{1}{64}(1, 1) + \frac{9}{64}(2, 3) + \frac{27}{64}(4, 3) + \frac{27}{64}(6, 4)$$

$$= \left[\frac{1}{64}\times 1 + \frac{9}{64}\times 2 + \frac{27}{64}\times 4 + \frac{27}{64}\times 6, \frac{1}{64}\times 1 + \frac{9}{64}\times 3 + \frac{27}{64}\times 3 + \frac{27}{64}\times 4\right]$$

$$= \left[\frac{289}{64}, \frac{217}{64}\right] = (4.5156, 3.3906)$$

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$$P(1) = P_3 = (6, 4)$$

Fig. 4.9.1 shows the calculated points of the Bezier curve and curve passing through it.

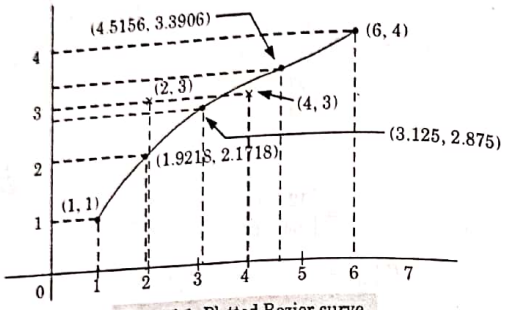

Fig. 4.9.1. Plotted Bezier curve.

Que 4.10. **Construct enough points on the Bezier curve whose control points are $p_0(4, 2)$, $p_1(8, 8)$ and $p_2(16, 4)$ to draw an accurate sketch.**

1. **What is the degree of the curve ?**
2. **What are coordinates at $u = 0.5$.**

Answer

1. Here $n = 2$ *i.e.*, order = 2

$$\text{Degree} = \text{order} - 1 = 2 - 1 = 1$$

2. Blending function of the curve :

$$B_{0,2}(u) = {}^2C_0\, u^0 (1-u)^{2-0} = (1-u)^2$$

$$B_{1,2}(u) = {}^2C_1\, u^1 (1-u)^{2-1} = 2u(1-u)$$

$$B_{2,2}(u) = {}^2C_2\, u^2 (1-u)^{2-2} = u^2$$

then Bezier curve for given control points :

$$C(u) = p_0 B_{0,2}(u) + p_1 B_{1,2}(u) + p_2 B_{2,2}(u)$$

Bezier curve in x and y coordinate is given as :

$$x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$$

$$y(u) = y_0 B_{0,2}(u) + y_1 B_{1,2}(u) + y_2 B_{2,2}(u)$$

Put

$$x_0 = 4, y_0 = 2$$

$$x_1 = 8, y_1 = 8$$

$$x_2 = 16, y_2 = 4$$

$$x(u) = 4(1-u)^2 + 8 \times 2u(1-u) + 16u^2$$

$$= 4u^2 + 8u + 4$$

at $u = 0.5$

$$x(u) = 9$$

$$y(u) = 2(1-u)^2 + 8 \times 2u(1-u) + 4u^2$$

$$= -10u^2 + 12u + 2$$

at $u = 0.5$

$$y(u) = 5.5$$

Therefore, the coordinates at $u = 0.5$ are (9, 5.5).

Que 4.11. **Write at least four properties of Bezier curves. Calculate and also, roughly trace the Bezier curve for three control points (1, 1), (2, 2) and (3, 1).** **AKTU 2012-13, 2013-14; Marks 06**

Answer

Properties of Bezier curves : Refer Q. 4.8, Page 4–10C, Unit-4.

Numerical :

Control functions for point (1, 1), (2, 2), (3, 1) are :

$$B_{0,2}(u) = {}^2C_0\, u^0 (1-u)^{2-0}$$

$$= (1-u)^2$$

$$B_{1,2}(u) = {}^2C_1\, u^1 (1-u)^{2-1}$$

$$= 2u(1-u)$$

$$B_{2,2}(u) = {}^2C_2\, u^2 (1-u)^{2-2}$$

$$= u^2$$

Bezier curve, $C(u) = p_0 B_{0,2}(u) + p_1 B_{1,2}(u) + p_2 B_{2,2}(u)$

Dissolving curve in the x, y coordinate :

$$x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$$

$$y(u) = y_0 B_{0,2}(u) + y_1 B_{1,2}(u) + y_2 B_{2,2}(u)$$

where, $x_0 = 1$, $y_1 = 1$

$x_1 = 2$, $y_2 = 2$

$x_2 = 3$, $y_3 = 1$

$$x(u) = 1(1-u)^2 + 2[2u(1-u)] + 3u^2$$

$$= 1 + u^2 - 2u + 4u - 4u^2 + 3u^2 = 1 + 2u$$

$$y(u) = 1(1-u)^2 + 2 \times [2u(1-u)] + 1u^2$$

$$= 1 + u^2 - 2u + 4u - 4u^2 + u^2 = 1 + 2u - 2u^2$$

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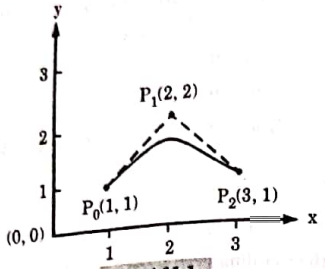

Fig. 4.11.1.

Que 4.12. **Compare and contrast among spline, B-spline and Bezier algorithms for curve generation and write the algorithm for Bezier curve generation.** **AKTU 2015-16, Marks 15**

Answer

Algorithm for Bezier curve :

1. Get four control points say A (x_A, y_A), B(x_B, y_B), C(x_C, y_C), D(x_D, y_D).
2. Divide the curve represented by points A, B, C and D in two sections

$$x_{AB} = (x_A + x_B) / 2$$
$$y_{AB} = (y_A + y_B) / 2$$
$$x_{BC} = (x_B + x_C) / 2$$
$$y_{BC} = (y_B + y_C) / 2$$
$$x_{CD} = (x_C + x_D) / 2$$
$$y_{CD} = (y_C + y_D) / 2$$
$$x_{ABC} = (x_{AB} + x_{BC}) / 2$$
$$y_{ABC} = (y_{AB} + y_{BC}) / 2$$
$$x_{BCD} = (x_{BC} + x_{CD}) / 2$$
$$y_{BCD} = (y_{BC} + y_{CD}) / 2$$
$$x_{ABCD} = (x_{ABC} + x_{BCD}) / 2$$
$$y_{ABCD} = (y_{ABC} + y_{BCD}) / 2$$

3. Repeat the step 2 for section A, AB, ABC and $ABCD$ and section $ABCD$, BCD, CD and D.
4. Repeat step 3 until we have sections so short that they can be replaced by straight lines.
5. Replace small sections by straight lines.
6. Stop.

Comparison :

S. No.	Spline	B-spline	Bezier
1.	A spline curve can be specified by giving a set of coordinate positions, called control points which indicates the general shape of the curve.	The B-spline curves are specified by a Bernstein basis function that has a limited flexibility.	The Bezier curves can be specified with boundary conditions, with a characterizing matrix or with blending functions.
2.	The curve follows the general shape of the curve.	The curves resulted by the use of open uniform basis function are nearly like Bezier curves.	The curve generally follows the shape of defining polygon.
3.	Typical CAD application for splines include the design of automobile bodies, aircraft and space craft surfaces and ship hulls.	These curves can used to construct blending curves.	Bezier curves are found in painting & drawing packages as well as CAD system.
4.	It possess a high degree of smoothness at the places where the polynomial pieces connect.	The B-spline allows the order of the basis function and hence the degree of the resulting curve is independent on the number of vertices.	The degree of the polynomial defining the curve segment is one less than the number of defining polygon point.

Que 4.13. **Let $P_0(0, 0)$, $P_1(1, 2)$, $P_2(2, 1)$, $P_3(3, 1)$, $P_4(4, 10)$ and $P_5(5, 5)$ be given data control points. If interpolation based on Bezier curve is used to find a curve interpolating these data points. Find parametric midpoint of the gradient and also calculate coordinate of parametric quartiles of the curve.** **AKTU 2014-15, Marks 06**

Answer

Given points : P_0 (0, 0), P_1 (1, 2), P_2 (2, 1), P_3 (3, 1), P_4 (4, 10) and P_5 (5, 5)

$$u = 0.5$$

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$n + 1 = 6$

$n = 5$

$$P(u) = \sum_{k=0}^{5} P_k \, BEZ_{k,n}(u)$$

$$BEZ_{k,5}(u) = C(5, k)\, u^k (1-u)^{5-k}$$

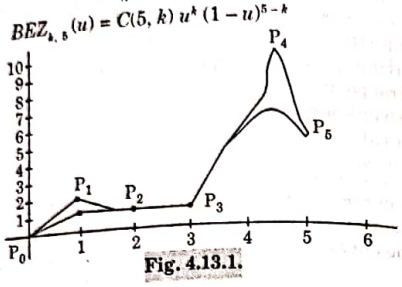

Fig. 4.13.1.

$$C_{5,k} = \frac{\lfloor 5}{\lfloor k \, \lfloor 5-k}$$

$$B_{0,5}(u) = C_{5,0}\, u^0 (1-u)^{5-0} = (1-u)^5$$

$$B_{1,5}(u) = C_{5,1}\, u^1 (1-u)^4 = 5(u)\,(1-u)^4$$

$$B_{2,5}(u) = C_{5,2}\, u^2 (1-u)^3 = 10u^2\,(1-u)^3$$

$$B_{3,5}(u) = C_{5,3}\, u^3 (1-u)^2 = 10u^3\,(1-u)^2$$

$$B_{4,5}(u) = C_{5,4}\, u^4 (1-u)^1 = 5u^4\,(1-u)$$

$$B_{5,5}(u) = C_{5,5}\, u^5 (1-u)^0 = u^5$$

$$C_{5,1} = \frac{\lfloor 5}{\lfloor 1 \lfloor 4} = \frac{5 \times \lfloor 4}{\lfloor 1 \lfloor 4} = 5$$

$$C_{5,2} = \frac{\lfloor 5}{\lfloor 2 \lfloor 3} = \frac{5 \times \lfloor 4}{\lfloor 2 \lfloor 3} = 5 \times 2 = 10$$

$$C_{5,3} = \frac{\lfloor 5}{\lfloor 3 \lfloor 2} = 10$$

$$C_{5,4} = \frac{\lfloor 5}{\lfloor 4 \lfloor 1} = 5$$

at $u = 0.5$

$$X(0.5) = 0 + 1 \times 5(0.5)\,(0.5)^4 + 2 \times 10 \times (0.5)^2\,(0.5)^3 + 3 \times 10(0.5)^5 + 4 \times 5(0.5)^5 + 5 \times (0.5)^5$$

$$= (0.5)^5\,(5 + 20 + 30 + 25) = 2.5$$

$$Y(0.5) = 0 + 2^5 \times (0.5)^5 + 1 \times 10(0.5)^5 + 1 \times 10(0.5)^5 + 10 \times 5(0.5)^5 + 5 \times (0.5)^5$$

$$= (0.5)^5\,(10 + 10 + 10 + 50 + 5)$$

$$= 85 \times (0.5)^5 = 2.6562$$

at $u = 0.5$, $(X, Y) = (2.5, 2.6562)$.

☺☺☺

5 UNIT

Hidden Lines and Surfaces

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PART-1

Hidden Lines and Surfaces : Back-Face Detection Algorithm, Depth Buffer Method, A-Buffer Method, Scanline Method.

CONCEPT OUTLINE

- Surfaces that are not seen by the naked eyes are known as hidden surfaces.
- Back-face detection algorithm is used to detect hidden surface.
- Depth buffer is also known as z-buffer which is used to manage coordinates of image depth in 3D.
- A-buffer is known as anti-aliased.
- Scanline rendering is an algorithm for visible surface determination.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Discuss back-face detection method.**

OR

Write back-face detection algorithm of hidden lines.

AKTU 2012-13, Marks 06

Answer

Back-face detection method :

1. Object surfaces that are orientated away from the viewer are called back-faces.
2. Algorithm used to detect the back-faces is known as back-face detection algorithm.
3. If any three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) on any plane surface are known, the unknown parameters A, B, C and D of the plane surface equation $Ax + By + Cz + D = 0$ can be found as follows :

   i. All the three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) should satisfy the equation $Ax + By + Cz + D = 0$ as it lies on the surface.

   Hence,
$$Ax_1 + By_1 + Cz_1 + D = 0$$
$$Ax_2 + By_2 + Cz_2 + D = 0$$
$$Ax_3 + By_3 + Cz_3 + D = 0$$

   ii. Also, any arbitrary point (x, y, z) lying on the surface should satisfy the equation of the desired surface
$$Ax + By + Cz + D = 0$$

   iii. A unique solution of equations for A, B, C and D can only be obtained if

$$\text{or} \quad \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x & y & z & 1 \end{vmatrix} = 0 \qquad \text{(By Cramer's rule)}$$

$$\text{or} \quad \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 & 0 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 & 0 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 & 0 \\ x & y & z & 1 \end{vmatrix} = 0$$

$$\text{or} \quad \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix} = 0$$

$$\text{or} \quad (x-x_1)\begin{vmatrix} y_1-y_2 & z_1-z_2 \\ y_2-y_3 & z_2-z_3 \end{vmatrix} - (y-y_1)\begin{vmatrix} x_1-x_2 & z_1-z_2 \\ x_2-x_3 & z_2-z_3 \end{vmatrix} + (z-z_1)\begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix} = 0$$

$$\text{or} \quad \begin{vmatrix} y_1-y_2 & z_1-z_2 \\ y_2-y_3 & z_2-z_3 \end{vmatrix} x + \begin{vmatrix} z_1-z_2 & x_1-x_2 \\ z_1-z_3 & x_2-x_3 \end{vmatrix} y + \begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix} z - \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

which is in the form of $Ax + By + Cz + D = 0$, giving the equation of the plane passing through the three points

where,
$$A = \begin{vmatrix} y_1-y_2 & z_1-z_2 \\ y_2-y_3 & z_2-z_3 \end{vmatrix}$$
$$B = \begin{vmatrix} z_1-z_2 & x_1-x_2 \\ z_2-z_3 & x_2-x_3 \end{vmatrix}$$
$$C = \begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}$$

and
$$D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

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iv. Any plane with its inside and outside surfaces can be defined by three points and its normal vector.

v. This normal vector can be calculated with three position vectors of three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

vi. Using right hand thumb rule, the three points are considered in counter clockwise manner for calculating the unit normal directing outside the surface. Thus, the surface normal and equation of the plane are calculated.

vii. Now, if any point (x, y, or z) lies inside the polygon surface, $Ax + By + Cz + D < 0$, and if it is along the line of sight to the surface, the polygon must be back-face.

viii. If L is the line of sight or viewing vector (light vector in case of shadowing), perpendicular to the viewing plane originating from the eye or camera, N is the unit surface normal vector as shown in Fig. 5.1.1(*a*) and (*b*) the polygon is the back-face if $L.N > 0$

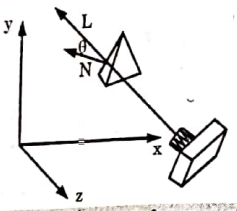

Fig. 5.1.1. (a) Geometry of camera vision.

ix. On aligning the light vector to that of the view vector which is normally along negative z-axis for right-handed viewing system, that is

$$L = 0i + 0j + L_z k$$
$$L.N = L_z K$$

x. Hence, only z-component of normal vector N is required to be considered, and the sign of K is checked for negative value.

xi. Thus, any polygon is a back-face if it has a surface normal with negative z-component value.

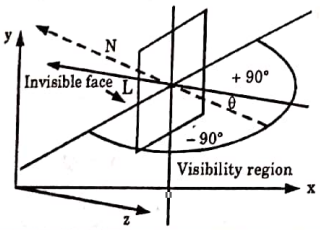

Fig. 5.1.1. (b) Invisible and visible vision.

Que 5.2. **Discuss z-buffer algorithm.**

OR

Write Depth-buffer method algorithm of hidden lines.

AKTU 2013-14, Marks 06

Answer

z-buffer/Depth-buffer algorithm :

1. z-buffer is a simple extension of the frame buffer idea. A frame is used to store the intensity of each pixel in image space.
2. The z-buffer is a separate depth buffer used to store the z-coordinates or depth or every visible pixel in image space.
3. In this algorithm a buffer of the same size as the frame buffer, is set up which holds depth information.
4. Each elements of the depth buffer corresponds to a pixel in the frame buffer and initially holds the maximum depth in the scene.
5. As each polygon is scan-converted, the depth at each pixel is calculated and compared with the corresponding depth in the depth buffer.
6. If the depth is less than that stored in the depth buffer (*i.e.*, nearer the viewer) then that pixel is set in the frame buffer with the polygon colour at that point and the depth buffer is set to the polygon depth.
7. If the polygon depth is greater (*i.e.*, farther away from the viewer) than the depth buffer at that point is not stored in the frame buffer.
8. The z- buffer algorithm can be stated as :

   Step 1 : Initialize frame buffer to background colour.

   Step 2 : Initialize z-buffer to minimum z value.

   Step 3 : Scan-convert each polygon in arbitrary order.

   Step 4 : For each (x, y) pixel, calculated depth 'z' at that pixel $(z(x, y))$.

   Step 5 : Compare calculated new depth $z(x, y)$ with value previously stored in z-buffer at the location $z(x, y)$.

   Step 6 : If $z(x, y) > z(x, y)$, then write the new depth value to z-buffer and updated frame buffer.

   Step 7 : Otherwise, no action is taken.

Que 5.3. **Discuss A-buffer method.**

Answer

1. A-buffer method is a visibility detection method for the rendering system.
2. The A-buffer expands on the depth buffer method to allow transparency. The key data structure in the A-buffer is the accumulation buffer.

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3. Each position in the A-buffer has two fields :
   a. **Depth field :** It stores a positive or negative real number.
   b. **Intensity field :** It stores surface intensity information or a pointer value.
4. If depth >= 0, the number stored at that position is the depth of a single surface overlapping the corresponding pixel area.
5. The intensity field then stores the RGB components of the surface colour at that point and the percent of pixel coverage.

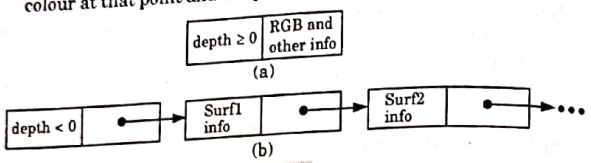

Fig. 5.3.1.

6. If depth < 0, it indicates multiple surface contributions to the pixel intensity.
7. The intensity field then stores a pointer to a linked list of surface data.
8. The surface buffer in the A-buffer includes :
   a. RGB intensity components
   b. Opacity parameter
   c. Depth
   d. Percent of area coverage
   e. Surface identifier
9. The depth and opacity values are used to determine the final colour of a pixel.

Que 5.4. **List the advantages and disadvantages of back-face detection and A-buffer method. Write the algorithm for back-face detection.** **AKTU 2015-16, Marks 15**

Answer

Advantages of back-face detection method :

1. It is fast.
2. It is a simple object space method.

Disadvantages of back-face detection method :

1. It can only be used on solid objects modeled as a polygon mesh.
2. It works fine for convex polyhedra but not necessarily for concave polyhedra.

Advantages of A-buffer method :

1. It is an extension of depth-buffer method.
2. More than one surface intensity can be taken into consideration at each pixel position.
3. It also anti-aliased the object edges.
4. In this, surfaces are sub-divided into a polygon mesh and clipped against the pixel boundaries.
5. The intensity of pixel is determined by considering opacity parameter and percentage of overlaps of the overlapping surface.

Disadvantages of A-buffer method :

1. This algorithm processes multiple objects at a time. The total number of polygons in a picture can be arbitrarily large.

Back-face detection algorithm : Refer Q. 5.1, Page 5–2C, Unit-5.

Que 5.5. **Explain hidden surface removing of scanline method.**

Answer

1. A scanline method of hidden surface removal is another approach of image space method.
2. It is an extension of the scanline algorithm, for filling polygon interiors.
3. This algorithm deals with multiple surfaces.
4. As each scanline is processed, all polygon surfaces intersecting that line are examined to determine which are visible.

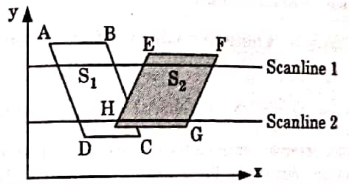

Fig. 5.5.1. Illustration of scanline method of hidden surface removal.

5. Across each scanline, depth calculations are made for each overlapping surface to determine which is nearest to the view plane.
6. When the visible surface has been determined, the intensity value for that position is entered into the frame buffer.
7. This algorithm maintains three table :
   a. **Edge Table (ET) :** It contain the list of edge used in scanline algorithm.

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ET entry	X	y_{max}	Δx	ID	

Fig. 5.5.2.

b. **Active Edge Table (AET) :** It contains only edges that cross the current scanline, stored in order of increasing x.

AET contents

Scanline	Entries
Scanline 1	AD BC EH FG
Scanline 2	AD EH BC FG

c. **Polygon Table (PT) :** Polygon Table (PT) that contains at least the following information for each polygon, in addition to ID :

PT entry	ID	Plane eq.	Shading info.	In-out

Fig. 5.5.3.

i. The coefficient of the plane equation.

ii. Shading or colour information for the polygon.

iii. An in-out Boolean flag, initialized to false and used during scanline processing.

8. In addition, we define a flag for each surface that is set on or off to indicate whether a position along a scanline is inside or outside of the surface. Scanline is processed from left to right.

9. At the leftmost boundary of a surface, the surface flag is turned on, and at the rightmost boundary, it is turned off.

Que 5.6. **Write a short note on visible surface detection or hidden surface elimination.**

Answer

1. Visible surface detection or hidden surface elimination is the procedure that removes any surfaces or lines which are not to be displayed in a 3D scene.

2. A process may be developed which draws all parts of all objects in the scene, starting from the object which is farthest from the viewing plane to the nearest one.

3. When we view a picture containing non transparent objects and surfaces, then we cannot see those objects that are viewed from behind the object. We must remove these hidden surfaces to get realistic screen image.

4. The identification and removal of these surfaces is called the hidden-surface problem.

5. Two approaches for visible surface detection are :

a. **Object space method :** It compares objects and parts of objects to each other within the scene definition to determine which surfaces, as a whole, we should label as visible.

b. **Image space method :** Visibility of objects is decided point by point at each pixel position on the projection plane.

6. Scanline coherence method solves the hidden surface problem one scanline at a time from top to bottom.

7. The simplest scanline algorithm is a one dimensional version of the depth buffer.

8. In each scanline, depth calculations are made for each overlapping surface to determine which is nearest to the view plane.

PART-2

Basic Illumination Models-Ambient Light, Diffuse Reflection, Specular Reflection and Phong Model, Combined Approach, Warn Model.

CONCEPT OUTLINE

- An illumination model is used to calculate the intensity of light that we should see at a given point on the surface of an object.
- Ambient light has no spatial or directional characteristics.
- Diffuse reflection is constant over each surface in a scene.
- Hidden surface methods can be used to locate areas where light sources produce shadows.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.7. **Explain the basic illumination models.**

Answer

1. An illumination model is needed to calculate the intensity of light that we should see at a given point on the surface of an object.

2. An illumination model is also known as lighting model or shading model.

Various illumination models :

1. **Ambient light :**

i. In this model, the intensity of the reflected light for each surface depends on the optical properties of the surface.

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ii. Ambient light has no spatial or directional characteristics.

iii. The amount of ambient light incident on each object surface is constant for all surfaces and over all directions.

iv. The optical properties determine how much of the incident energy is to be reflected and how much absorbed.

2. **Diffuse reflection :**

i. In this model, light coming from all directions is reflected from the walls, floor, ceiling, etc.

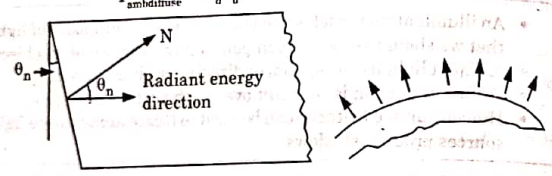
ii. Diffuse reflections are constant over each surface in a scene, independent of the viewing direction.

iii. The amount of diffused reflected light for each surface in a scene can be set with parameter K_d.

iv. For the surfaces which absorbs most of the incident light (black surfaces), the value of the diffuse reflection coefficient (K_d) is close to 0.

v. If a surface is exposed to only ambient light, then the intensity of the diffuse reflection at any point on the surface is given by

$$I_{ambdiffuse} = K_d I_a$$



(a) Radiant energy from surface. (b) Diffuse reflection from surface.

Fig. 5.7.1.

3. **Phong model :**

i. According to the Phong model, the intensity of specular reflection is proportional to $\cos^{n_s}\theta$. The value of n, specular reflection parameter, is determined by the type of surface:

ii. For very shiny surfaces, the value of n_s is large (more than 100) and for dull surfaces the value of n_s is close to 1. For a perfect reflector, the value of n_s is infinite.

iii. The intensity of specular reflection depends on the angle of incidence, on the properties of the surface, colour of incident light and polarization.

iv. The specular reflection coefficient W(θ), for each surface may be used to model the monochromatic specular intensity variation.

v. We can write the Phong specular reflection model as

$$I_{spec} = W(\theta)\, I_l \cos^{n_s}\theta \quad ...(5.7.1)$$

where I_l is intensity of light source, and θ is the viewing angle relative to specular reflection direction R.

vi. In the case of opaque materials, the specular reflection rejection is almost constant for all incidence angles. Therefore, replacing W(θ) with a constant specular reflection coefficient K_s. Since V and R are unit vectors in the viewing and specular reflection directions, $\cos\theta = V.R$. We can rewrite the equation (5.7.1) as

$$I_{spec} = K_s I_l (V.R)^{n_s}$$

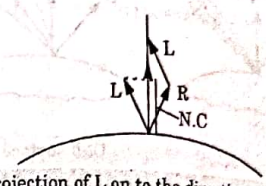


Fig. 5.7.2. Projection of L on to the direction of normal vector N.

Que 5.8. **Explain specular reflection and Phong model.**

AKTU 2012-13, Marks 10

AKTU 2013-14, Marks 06

Answer

Specular reflection :

1. Specular reflection is the result of total or near total reflection of the light in a concentrated region around the specular reflection angle.

2. When we see an illuminated shiny surface, like polished metal, shiny apple, or a pearl, then we observe a bright spot or a highlighted spot at a certain viewing direction.

3. In the Fig. 5.8.1(*a*) the specular reflection angle, denoted by θ is the same as the angle of incident light and N is the unit normal surface vector, R is the unit vector in the direction of ideal specular reflection, L is the unit vector directed towards the point light source, and V is the unit vector pointing towards the viewer from the surface position.

4. Angle θ is the viewing angle relative to the specular reflection direction R.

5. In the case of an ideal reflector or perfect mirror, the incident light is reflected only in the specular reflection direction.

6. In this case, we would only see reflected light when vectors V and R coincide, *i.e.*, θ = 0. The objects other than ideal reflectors have specular reflection aro d vector R.

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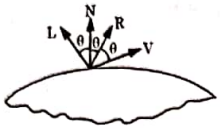

Fig. 5.8.1. (a) Specular reflection angle = Angle of incidence.

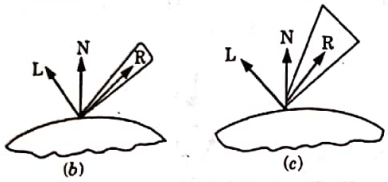

Fig. 5.8.1. (b) Shiny (c) Dull surface.

Phong model : Refer Q. 5.7, Page 5–9C, Unit-5.

Que 5.9. **Explain the different illumination methods and different rendering methods in detail. Write its advantages and disadvantages.** **AKTU 2015-16, Marks 15**

Answer

Various illumination methods : Refer Q. 5.7, Page 5–9C, Unit-5.

Advantages of illumination methods :

1. Fast
2. Acceptable results
3. Hardware support

Disadvantages of illumination methods :

1. Point light source
2. No interaction between objects
3. Adhoc, not based on model of light propagation

Different rendering methods :

1. **Constant-intensity shading :**
   a. In this method, a single intensity is calculated for each polygon and all points in the surface of polygon are displayed with the same intensity value.
   b. It is fast and simplest rendering method.
   c. Light sources illuminating the object and the viewing position is sufficiently far from the surface of polygon.

2. **Gouraud shading :**
   a. It represents a polygon surface by linearly interpolating intensity across the polygon surface.
   b. Intensity values for each polygon are matched with adjacent polygon along common edges.
   c. It eliminates the intensity discontinuity that can occur in constant-intensity shading.
3. **Phong shading :**
   a. Phong shading is method which interpolates normal vectors and then apply the illumination model to each surface point.
   b. This method interpolates the surface normal vector, instead of the intensity.
   c. It is used in 3D system.
4. **Fast Phong shading :**
   a. Fast Phong shading approximates the intensity calculations using a Taylor-series expansion and triangular surface patches.
   b. Surface rendering with Phong shading can be speeded up by using approximations in the illumination model calculations of normal vectors.

Advantages of rendering methods :

i. It removes the intensity discontinuity.
ii. It can be combined with hidden surface algorithm to fill in the visible polygons along each scanline.
iii. It displays more realistic highlights on a surface.
iv. It gives more accurate results.

Disadvantages of rendering methods :

i. It has a problem with specular reflections.
ii. It introduces anomalies known as Mach bands.
iii. It requires more calculations and increases the cost of shading steeply.

Que 5.10. **Discuss combined diffuse and specular reflections with multiple light sources.**

Answer

1. For a single point light source, we can model the combined diffuse and specular reflections from a point on an illuminated surface as

$$I = I_{diff} + I_{spec}$$
$$= K_a I_a + K_d I_l (N.L) + K_s I_l (N.H)^{n_s}$$

2. If we place more than one point source in a scene, we obtain the light reflection at any surface point by summing the contributions from the individual sources :

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$$I = K_a I_a + \sum_{i=1}^{n} I_{li}[K_d(N.L_i) + K_s(N.H_i)^{n_s}]$$

3. To ensure that any pixel intensity does not exceed the maximum allowable value, we can apply some type of normalization procedure.
4. A simple approach is to set a maximum magnitude for each term in the intensity equation.
5. If any calculated term exceeds the maximum, we simply set it to the maximum value.
6. Another way to compensate for intensity overflow is to normalize the individual terms by dividing each by the magnitude of the largest term.
7. A more complicated procedure is first to calculate all pixel intensities for the scene, then the calculated intensities are scaled onto the allowable intensity range.

Que 5.11. **Discuss Warn model.**

OR

Explain Phong and Warn model for illumination.

AKTU 2014-15, Marks 06

Answer

Phong model : Refer Q. 5.7, Page 5–9C, Unit-5.

Warn model :

1. The Warn model provides a method for simulating studio lighting effects by controlling light intensity in different directions.
2. Light sources are modeled as points on a reflecting surface, using the Phong model for the surface points. Then, the intensity in different directions is controlled by selecting values for the Phong exponent.
3. In addition, light controls, such as "barn doors" and spotlighting, used by studio photographers can be simulated in the Warn model.
4. Flaps are used to control the amount of light emitted by a source in various directions.
5. Two flaps are provided for each of the x, y and z-directions.
6. Spotlights are used to control the amount of light emitted within a cone with apex at a point-source position.

PART-3

Intensity Aitenuation. Colour Consideration, Transparency and Shadows.

CONCEPT OUTLINE

- Intensity attenuation refers to any reduction in the strength of a signal.
- A transparent surface produces both reflected and transmitted light.
- A shadowed object is one which is hidden from the light source.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.12. **What do you mean by intensity attenuation ?**

Answer

1. Intensity attenuation refers to any reduction in the strength of a signal.
2. It is relatively compared with the attenuation of peak ground acceleration that is used for engineering design.
3. Knowledge of intensity attenuation is useful in calibrating hazard models against historical experience.
4. Intensity attenuation equation is given by :

$$I = 3.31 + 1.28\,ML - 1.22 \ln R$$

where
I = Intensity
ML = Local Magnitude
R = Hypocentral distance

5. As radiant energy from a point light source travels through space, its amplitude is attenuated by the factor $1/d^2$, where d is the distance that the light has travelled.
6. This means that a surface close to the light source receives higher incident intensity from the source than a distant surface.

Que 5.13. **Write a short note on colour considerations.**

Answer

1. Most graphics displays of realistic scenes are in colour. To incorporate colour, we need to write the intensity equation as a function of the colour properties of the light sources and object surfaces.
2. For an RGB description, each colour in a scene is expressed in terms of red, green, and blue components.
3. We then specify the RGB components of light source intensities and surface colours, and the illumination model calculates the RGB components of the reflected light.

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4. One way to set surface colours is by specifying the reflectivity coefficients as three element vectors.
5. The diffuse reflection coefficient vector would then have RGB components (K_{dR}, K_{dG}, K_{dB}).
6. If we want an object to have a blue surface, we select a non-zero value in the range from 0 to 1 for the blue reflectivity component, K_{dB}, while the red and green reflectivity components are set to zero ($K_{dR} = K_{dG} = 0$).
7. Any non-zero red or green components in the incident light are absorbed, and only the blue component is reflected. The intensity calculation for this example reduces to the single expression

$$I_B = K_{aB} I_{aB} + \sum_{i=1}^{n} f_i(d) I_{iBi} [K_{dB}(N.L_i) + K_{sB}(N.H_i)^{n_s}]$$

8. Surfaces typically are illuminated with white light sources, and in general we can set surface colour so that the reflected light has non-zero values for all three RGB components.
9. Calculated intensity levels for each colour component can be used to adjust the corresponding electron gun in an RGB monitor.

Que 5.14. **Write short note on transparency and shadows.**

Answer

Transparency :

1. A transparent surface produces both reflected and transmitted light.
2. The relative contribution of the transmitted light depends on the degree of transparency of the surface.
3. When a transparent surface is to be modeled, the intensity equations must be modified to include contributions from light passing through the surface.
4. Both diffuse and specular transmission can take place at the surfaces of a transparent object.
5. Realistic transparency effects are modeled by considering light refraction.
6. When light is incident upon a transparent surface, it is reflected and refracted, because the speed of light is different in different mediums so it changes the direction as shown in Fig. 5.14.1.

   where,

   Refraction direction = T

   Reflection direction = R

   Angle of refraction = θ_r

   Angle of incidence θ_i = angle of reflection

   Distance travelled by light in transparent medium = d

   Index of refraction of the refracting material = η_r

Index of refraction of the incidents material = η_i

By Snell's law, we have

$$\sin\theta_r = \frac{\eta_i}{\eta_r} \sin\theta_i$$

From Snell's law and Fig. 5.14.1 we can obtain the unit transmission vector T in the refraction direction θ_r as :

$$T = \left(\frac{\eta_i}{\eta_r}\cos\theta_i - \cos\theta_r\right) N - \frac{\eta_i}{\eta_r} L$$

where N is the unit normal vector on the surface and L is the unit vector in the direction of light source.

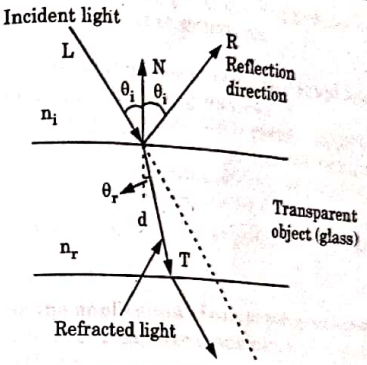

Fig. 5.14.1. Refraction light.

Shadows :

1. A shadowed object is one which is hidden from the light source.
2. It is possible to use hidden surface algorithms to locate the areas where light sources produce shadows.
3. In order to achieve this we have to repeat the hidden surface calculation using light source as the viewpoint.
4. This calculation divides the surfaces into shadowed and unshadowed groups.

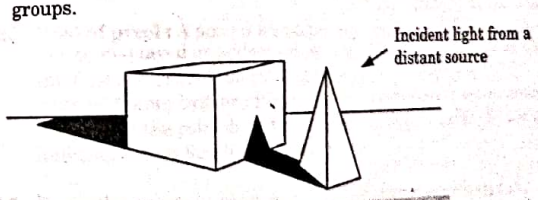

Fig. 5.14.2. Objects modeled with shadow regions.

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5. The surfaces that are visible from the light source are not in shadow; those that are not visible from the light source are in shadow.
6. Surfaces which are visible and which are also visible from the light source are shown with both the background illumination and the light-source illumination.

☺☺☺

1 UNIT Introduction and Line Generation (2 Marks Questions)

1.1. Define computer graphics.

Ans. Computer graphics is an art of drawing pictures on computer screens with the help of programming.

1.2. What are the advantages of computer graphics ?

Ans. **Advantages of computer graphics :**

i. A high quality graphics displays of personal computer provide one of the most natural means of communication with a computer.
ii. It provides tools for producing not only of concrete, real-world objects but also of abstract, synthetic objects.
iii. The computer graphics also provides facility called update dynamics.

1.3. Describe the application of computer graphics.

Ans. **Applications of computer graphics :**

i. User interfaces
ii. Plotting of graphics and chart
iii. Office automation and desktop publishing
iv. Simulation and animation
v. Art and commerce

1.4. Define pixel and frame buffer.

OR

Explain the role of pixel and frame buffer in graphics devices. **AKTU 2015-16, Marks 02**

Ans. **Role of pixel :** A pixel is a smallest size object or colour spot that can be displayed or addressed on a monitor. They represent the smallest controllable element of a picture on the screen.

Role of frame buffer : Picture definition is stored in a memory area called the refresh buffer or frame buffer. It holds the set of intensity values for all the screen points.

1.5. Describe random scan display and raster scan display.

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