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NUMERICAL ASSIGNMENT STATISTICAL STRUCTURES IN DATA

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Univariate Analysis Dataset 1

1. Data Overview

The mtcars dataset contains data about car models and their attributes. It comprises 32 observations and 11 variables.:

- Numerical: mpg (miles per gallon), hp (horsepower), wt (weight), etc.
- Categorical: cyl (number of cylinders), gear (number of gears).

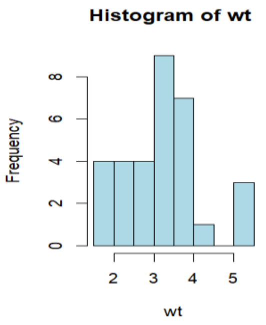
Car	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21	6	160	110	3.9	2.62	16.46	0	1	4	4
Mazda RX4 Wag	21	6	160	110	3.9	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.32	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.44	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.46	20.22	1	0	3	1

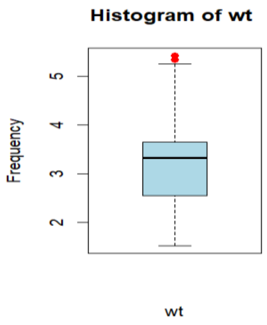
2. Summary Statistics

vars	n	mean	sd	median	skew	kurtosis	se
mpg	1	32	20.09062	6.0269481	23.5	0.610655	-0.372766
hp	4	32	146.6875	68.562869	283	0.7260237	-0.135551
wt	6	32	3.21725	0.9784574	3.911	0.4231465	-0.022711

Mean: 20.09 Median: 19.20 Standard Deviation: 6.03 Minimum: 10.40 Maximum: 33.90

3. Distribution Visualization





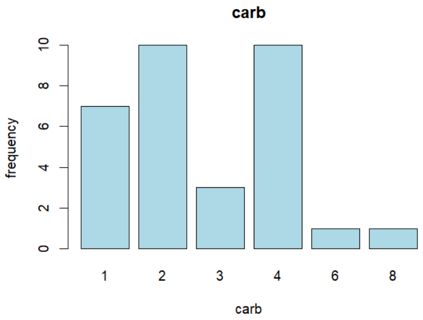
Shape of the Distribution: The distribution of the variable "wt" appears to be **right-skewed**. This means that the tail of the distribution extends more to the right side, indicating that there are more data points with lower values of "wt" compared to higher values.

Potential Outliers: The boxplot suggests the presence of potential outliers approximately **2 to 5**.

4. Categorical Variable Analysis

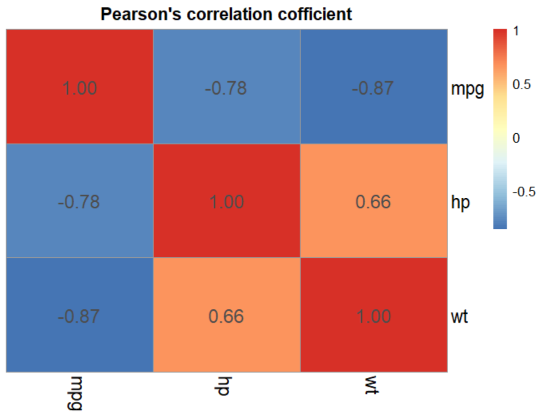
Distribution:

- The distribution of the "carb" variable appears to be **multimodal**, with two distinct peaks.
- One peak is around the value of 1, and the other is around the value of 4.
- This suggests that there are two clusters of data



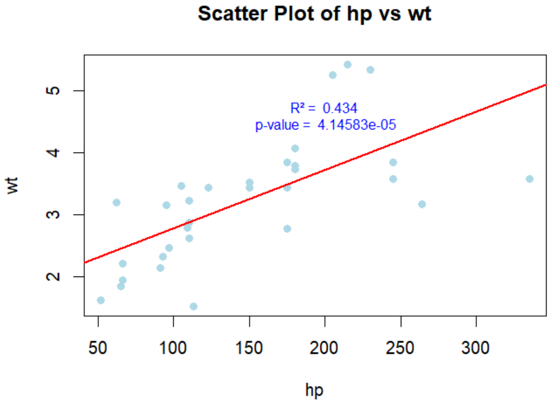
5. Correlation Analysis

Pearson Correlation Coefficient between mpg and hp: -0.7761684, mpg and wt-0.87.
Summary: The Pearson correlation suggests a **negative relationship** between mpg and hp. A **strong negative correlation** between mpg and wt.



6. Scatter Plot Visualization

The scatter plot shows a **positive, linear relationship** between the variables "hp" (horsepower) and "wt" (weight). As the value of "hp" increases, the value of "wt" also tends to increase. The p-value (p-value = 4.14583e-05) is very small, indicating that the observed relationship between "hp" and "wt" is statistically significant.



7. Multiple Regression

Interpretations

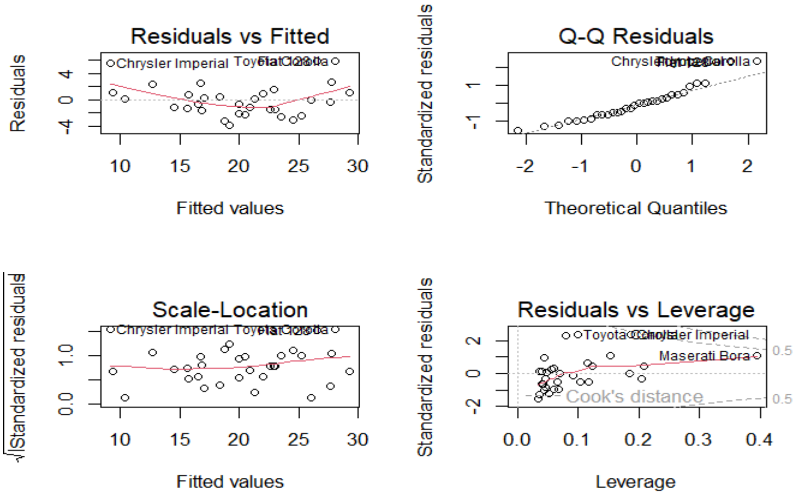
- **Intercept (37.23 mpg):** This represents the expected miles per gallon (mpg) when both horsepower (hp) and weight (wt) are zero. However, this is an unrealistic scenario for cars, so the value itself doesn't hold much practical meaning.
- **hp :**A 1 unit increase in horsepower is associated with a decrease of 0.032 mpg (interpreted with caution due to reasons below).
- **wt :** A 1 unit increase in weight is associated with a decrease of 3.88 mpg.
- **R-squared :** 82.68% of the variation in mpg is explained by hp and wt.
- **Adjusted R-squared :**More reliable estimate of explained variance.
- **F-statistic :** The model is statistically significant (p-value < 0.001).
- **p-value (F-statistic) :** The model is highly significant.

Statistic	**Value**
Coefficients	
Intercept	37.23 mpg
hp	-0.032 mpg
wt	-3.88 mpg
Residuals	
Minimum	-3.94 mpg
Q1	-1.60 mpg
Median	-0.18 mpg
Q3	1.05 mpg
Maximum	5.85 mpg
Model Fit	
R-squared	0.8268
Adjusted R-squared	0.8148
F-statistic	69.21
p-value (F-statistic)	9.11E-12

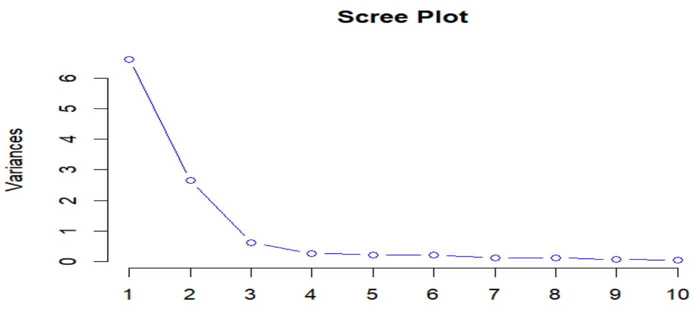
8. Model Diagnostics

The provided diagnostic plots are crucial for evaluating the assumptions of linear regression and the overall fit of the model. Let's analyze each plot:

1. Residuals vs. Fitted Values: The plot show some non-constant variance, especially at higher fitted values. This indicates **heteroscedasticity**, which violates the assumption of constant variance.
2. Q-Q Plot of Residuals: The points deviate from the straight line, especially in the tails. This suggests that the residuals are **not normally distributed**.
3. Scale-Location Plot: Similar to the first plot, there seems to be some non-constant variance, especially at higher fitted values. This further confirms the presence of **heteroscedasticity**.
4. Residuals vs. Leverage: This plot identifies influential points that might have a significant impact on the regression model. Points with high leverage can exert undue influence on the regression coefficients. There are a few points with high leverage, particularly the "Maserati Bora" point, which might be influential. However, without further analysis, it's difficult to determine their exact impact.



9. Principal Component Analysis (PCA)

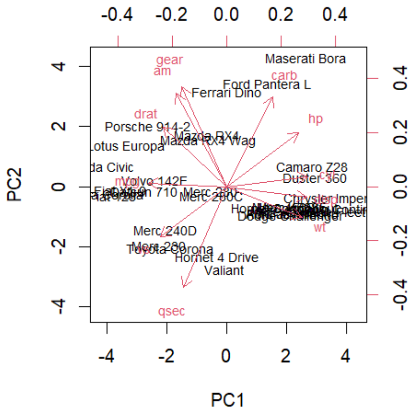


Based on the scree plot, we can choose the **first three components** as they capture a significant proportion of the variance. The elbow in the plot is visible after the third component, indicating that adding more components would not significantly improve the explanation of the variance. By selecting the first three components, we balance the need for parsimony with the desire to capture most of the underlying patterns in the data.

10. PCA Interpretation

Loadings:

- **PC1:** The variables "hp" (horsepower) and "wt" (weight) have positive loadings on PC1, indicating that these variables are positively correlated with each other and contribute positively to the variation explained by this component. This suggests that cars with higher horsepower tend to be heavier.
- **PC2:** The variables "qsec" (quarter mile time) and "drat" (rear axle ratio) have negative loadings on PC2, suggesting a negative correlation between these two variables. This implies that cars with faster quarter mile times tend to have lower rear axle ratios.



Patterns and Groupings:

- **Car Groups:** We can observe some interesting groupings of cars based on their positions in the biplot:
  - **Sporty Cars:** Cars like the "Maserati Bora," "Ford Pantera L," and "Ferrari Dino" are positioned towards the top-right quadrant, indicating high values on both PC1 and PC2. This suggests that these cars have high horsepower, weight, and relatively slower quarter mile times.
  - **Fuel-Efficient Cars:** Cars like the "Toyota Corolla," "Fiat X1-9," and "Honda Civic" are positioned towards the bottom-left quadrant, indicating low values on both PC1 and PC2. This suggests that these cars have lower horsepower, weight, and faster quarter mile times, which are characteristics associated with fuel efficiency.
  - **Other Groups:** Other cars, such as the "Porsche 914-2" and "Lotus Europa," appear to form their own distinct groups, potentially based on specific characteristics that are not captured by the first two principal components.

Univariate Analysis Dataset 2

1. Data Overview

The iris dataset consists of **150 observations** (rows) and **5 variables** (columns), with the first four columns containing numerical measurements and the fifth column representing the species of the iris flower. The dataset contains both continuous (sepal and petal measurements) and categorical (species) data.

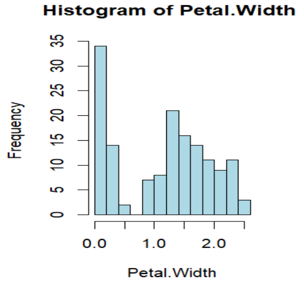
Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa

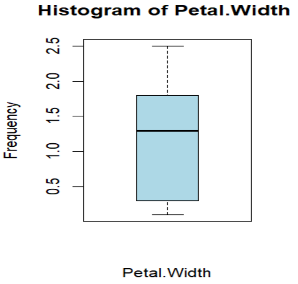
2. Summary Statistics

vars	n	mean	sd	range	skew	kurtosis	se
Sepal.Length	**1**	**150**	**5.843333**	**3.6**	**0.3086407**	**-0.6058125**	**0.06761132**
Sepal.Width	**2**	**150**	**3.057333**	**2.4**	**0.3126147**	**0.1387047**	**0.03558833**
Petal.Length	**3**	**150**	**3.758000**	**5.9**	**-0.2694109**	**-1.4168574**	**0.14413600**
Petal.Width	**4**	**150**	**1.199333**	**2.4**	**-0.1009166**	**-1.3581792**	**0.06223645**

Mean: 5.843333 Median: 5.8 Standard Deviation: 0.8280661 Minimum: 4.3 Maximum: 7.9

3. Distribution Visualization





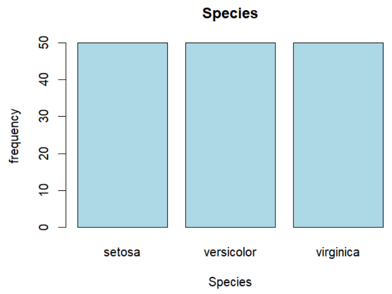
Shape of the Distribution: The distribution appears to be **unimodal** with a single peak around 0.2-0.3. The distribution is **right-skewed. This means the tail extends more to the right side, indicating that there are more data points.**
Potential Outliers: The absence of outliers in the boxplot suggests that there are no extreme values

4. Categorical Variable Analysis

Distribution:

- The plot shows that the three species (setosa, versicolor, and virginica) are equally distributed in the dataset. Each species has the same frequency, indicating a balanced representation.One peak is around the value of 1, and the other is around the value of 4.

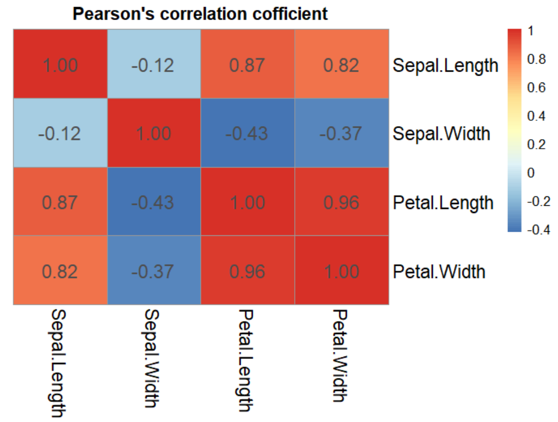
There is no dominant species in the dataset, as all three species have the same frequency. This suggests that the dataset is not



5. Correlation Analysis

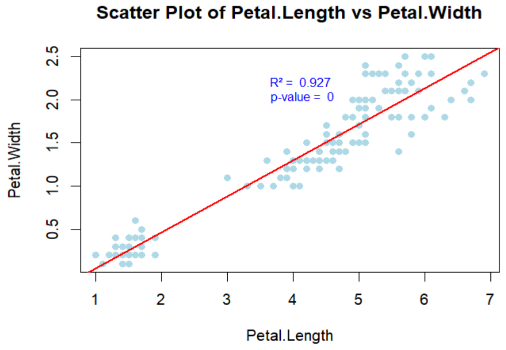
Pearson Correlation Coefficient between mpg and hp: -0.1175698

Summary: The Pearson correlation suggests strong positive correlation between "Sepal.Length" and "Petal.Length." As "Sepal.Length" increases, "Petal.Length"



6. Scatter Plot Visualization

This value indicates that approximately 92.7% of the variation in Petal.Width can be explained by the variation in Petal.Length. This is a very high R-squared value, suggesting that the linear model fits the data quite well. A p-value of 0 indicates that the relationship between Petal.Length and Petal.Width is statistically significant. In other words, it is highly unlikely that the observed



7. Multiple Regression

Interpretations

- **Intercept (-6.782343):** When all predictor variables are 0, the predicted crime rate is -6.78. However, this intercept might not have a practical interpretation in this context.
- **Age (0.008646):** A one-unit increase in age is associated with a 0.0086 increase in the crime rate, holding other variables constant. However, this effect is not statistically significant.
- **dis (-0.312893):** A one-unit increase in Petal.Length is associated with a **0.71 unit increase** in Sepal.Length, on average, holding Sepal.Width and Petal.Width constant. This coefficient is statistically significant (p-value < 2e-16).
- **R-squared :** 0.3465 indicates that approximately 34.65% of the variation in crime rate can be explained by the three predictor variables.
- **Adjusted R-squared :** 0.3426 is slightly lower than the R-squared, accounting for the number of predictors in the model.
- **F-statistic :** 502 (highly significant, p-value < 2.2e-16)
- **p-value (F-statistic) :** The model is highly significant.

Statistic	**Value**
Coefficients	
Intercept	-6.782343
age	0.008646
dis	-0.312893
Residuals	
Minimum	-12.523
Q1	-2.772
Median	-0.170
Q3	1.038
Maximum	77.477
Model Fit	
R-squared	0.3465
Adjusted R-squared	0.3426
F-statistic	502
p-value (F-statistic)	2.2e-16

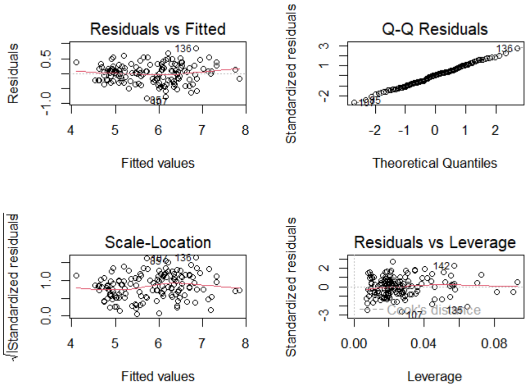
8. Model Diagnostics

The provided diagnostic plots are crucial for evaluating the assumptions of linear regression and the overall fit of the model. Let's analyze each plot:

1. Residuals vs. Fitted Values:

- **Homoscedasticity:** This plot helps assess the assumption of constant variance of errors (homoscedasticity). Ideally, the points should be randomly scattered around a horizontal line. If the spread of points increases or decreases with fitted values, it suggests heteroscedasticity.

- **Interpretation:** The plot appears to show some slight evidence of heteroscedasticity. The spread of points seems to increase slightly as the fitted values increase. This suggests that the model's error variance might be larger for larger fitted values.



2. Q-Q Plot of Residuals:

- **Normality of residuals:** This plot compares the quantiles of the standardized residuals to the quantiles of a standard normal distribution. Ideally, the points should fall along a straight line. Deviations from the line indicate departures from normality.
- **Interpretation:** The Q-Q plot shows a roughly linear pattern, suggesting that the residuals are approximately normally distributed. However, there are some deviations at the tails, which might indicate slight departures from normality.

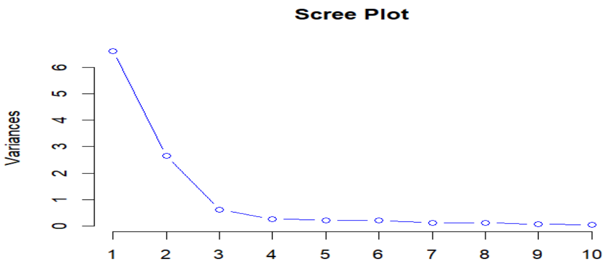
3. Scale-Location Plot:

- **Homoscedasticity:** This plot is similar to the Residuals vs. Fitted plot, but it uses the square root of the standardized residuals to potentially highlight patterns in the variance.
- **Interpretation:** The plot shows a similar pattern to the Residuals vs. Fitted plot, suggesting some slight evidence of heteroscedasticity.

4. Residuals vs. Leverage:

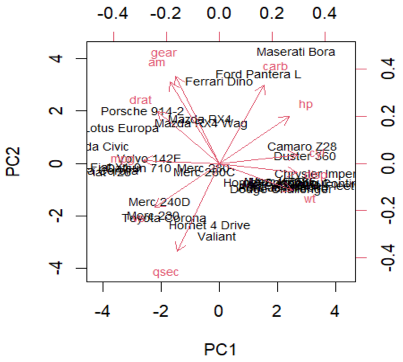
- **Outliers and influential points:** This plot helps identify potential outliers and influential points that might be affecting the model fit. Points with high leverage can have a significant impact on the model's coefficients.
- **Interpretation:** The plot shows some points with high leverage, particularly in the upper right corner. These points might be influential and could be worth investigating further.

9. Principal Component Analysis (PCA)



The "elbow" in the plot is a common heuristic for selecting the number of components.Based on the scree plot, we can choose the **first three components** as they capture a significant proportion of the variance. The elbow in the plot is visible after the third component, indicating that adding more components would not significantly improve the explanation of the variance.

10. PCA Interpretation



Loadings of the First Two Principal Components:

1. **PC1:** This component appears to be primarily related to **Sepal.Length** and **Petal.Length**. Points with positive PC1 scores have larger values for these variables, while points with negative scores have smaller values.
2. **PC2:** This component seems to be more associated with **Sepal.Width** and **Petal.Width**. Points with positive PC2 scores tend to have larger values for these variables, and those with negative scores have smaller values.

Patterns and Groupings:

The biplot reveals distinct groupings of the Iris species:

1. **Setosa:** These points are clustered together in the bottom left corner. They have low values for all four variables, particularly Sepal.Length and Petal.Length.
2. **Versicolor:** These points form a cluster in the middle of the plot. They have intermediate values for all four variables.
3. **Virginica:** These points are located in the top right corner. They have high values for all four variables, especially Sepal.Length and Petal.Length.

Overall, the biplot visually confirms the well-known separation of the three Iris species based on their morphological characteristics. The first two principal components capture the most significant variation in the data, allowing for a clear visualization of the species groups.

Univariate Analysis Dataset 3

1. Data Overview

The **Boston Housing dataset** contains **506 observations** (data points) and **14 variables** in total, including 13 numerical attributes and 1 target variable. These attributes describe various socio-economic and housing-related factors for different areas in the Boston region.

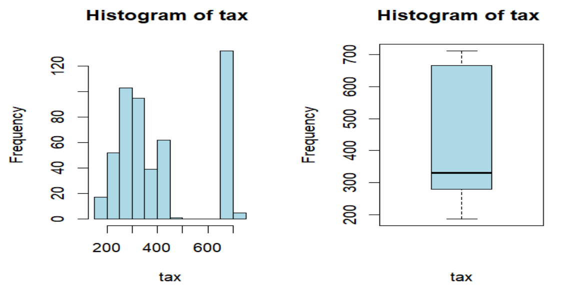
```
     crim zn indus chas   nox    rm  age    dis rad tax ptratio      b lstat medv
1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90  4.98 24.0
2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90  9.14 21.6
3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83  4.03 34.7
4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63  2.94 33.4
5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90  5.33 36.2
6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12  5.21 28.7
```

2. Summary Statistics

vars		n	mean	sd	median	kurtosis	se
crim	1	506	3.613524	8.601545	0.25651	36.5958159	0.38238532
age	7	506	68.574901	28.148861	77.50000	-0.9780297	1.25136953
dis	8	506	3.795043	2.105710	3.20745	0.4575916	0.09361023
tax	10	506	408.237154	168.537116	330.00000	-1.1503176	7.49238869

Mean: 3.61 Median: 8.8 Standard Deviation: 0.2580661 Minimum: 6.3 Maximum: 42

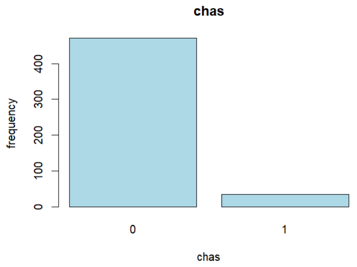
3. Distribution Visualization



The histogram appears to be **right-skewed**. This means that there is a longer tail on the right side, indicating that there are more values at the lower end of the tax range compared to the higher. The boxplot also indicates a potential outlier, represented by the dot above the upper whisker. This corresponds to the very high tax value

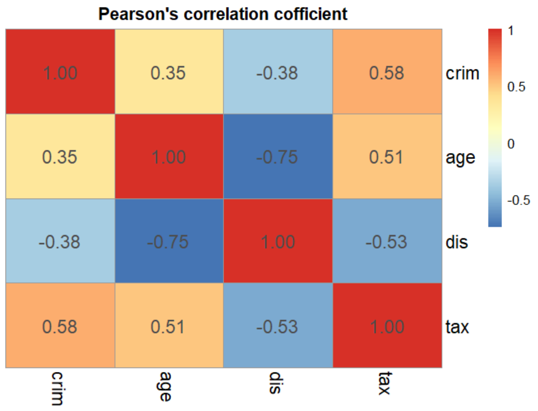
4. Categorical Variable Analysis

Distribution: It appears that the variable is categorical and has two distinct categories: 0 and 1.
Category Imbalance: The plot clearly shows an imbalance in the distribution of the two categories. The majority of observations belong to category 0, while a much smaller proportion falls into category 1.**Dominant Category:** Category 0 is the dominant category, with the frequency being significantly higher compared to category 1.



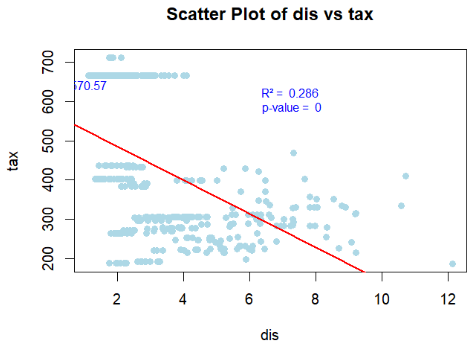
5. Correlation Analysis

The correlation coefficient between "crime" and "dis" is **-0.38**. This indicates a **moderate negative correlation** between the two variables. However, this relationship is not very strong, and there are likely other factors influencing crime rates besides distance to



6. Scatter Plot Visualization

Moderate Correlation: The R-squared value of 0.286 suggests a moderate correlation between "dis" and "tax". This means that approximately 28.6% of the variation in "tax" can be explained by the variation in "dis". The scatter plot suggests that there is a moderate negative relationship between distance to employment centers and property tax rates. However, the relationship is not strong, and



7. Multiple Regression

Interpretations

- **Intercept (1.85600):** This represents the predicted Sepal.Length when all other predictor variables (Sepal.Width, Petal.Length, Petal.Width) are zero. However, it's not meaningful in this context as Sepal measurements wouldn't be zero.
- **Sepal.Width (0.65084):** A one-unit increase in Sepal.Width is associated with a **0.65 unit increase** in Sepal.Length, on average, holding Petal.Length and Petal.Width constant. This coefficient is statistically significant (p-value < 2e-16).
- **Petal.Length (0.70913):** A one-unit increase in Petal.Length is associated with a **0.71 unit increase** in Sepal.Length, on average, holding Sepal.Width and Petal.Width constant. This coefficient is statistically significant (p-value < 2e-16).
- **R-squared :** 0.8586 (85.86% of the variance in Sepal.Length is explained by the model).
- **Adjusted R-squared :** 0.8557 (adjusted for the number of predictors) variance.
- **F-statistic :** 295.5 (highly significant, p-value < 2.2e-16)
- **p-value (F-statistic) :** The model is highly significant.

Statistic	**Value**
Coefficients	
Intercept	1.85600
Sepal.Width	0.65084
Petal.Length	0.70913
Residuals	
Minimum	-0.82816
Q1	-0.21989
Median	0.01875
Q3	0.19709
Maximum	0.84570
Model Fit	
R-squared	0.8586
Adjusted R-squared	0.8557
F-statistic	295.5
p-value (F-statistic)	0.9349

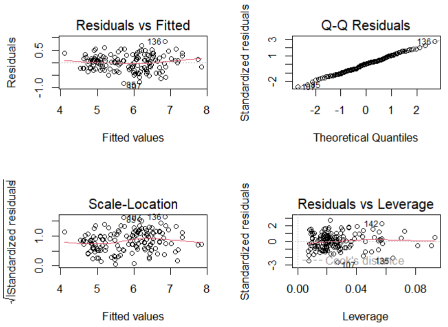
8. Model Diagnostics

The provided diagnostic plots are crucial for evaluating the assumptions of linear regression and the overall fit of the model. Let's analyze each plot:

Homoscedasticity

- **Residuals vs. Fitted Plot:** This plot checks if the variance of the residuals is constant across different fitted values. Ideally, the points should be randomly scattered around a horizontal line. In this case, the points seem to be fairly evenly distributed, suggesting that the assumption of homoscedasticity is reasonably met.

- **Scale-Location Plot:** This plot is another way to check homoscedasticity. Here, the square root of the standardized residuals is plotted against the fitted values. Ideally, the points should be randomly scattered around a horizontal line. Again, the points appear to be randomly scattered, supporting the assumption of homoscedasticity.



Normality of Residuals

- **Q-Q Plot:** This plot compares the quantiles of the standardized residuals to the quantiles of a standard normal distribution. If the residuals are normally distributed, the points should fall along a straight line. In this case, the points roughly follow a straight line, indicating that the normality assumption is reasonably met. However, there are some deviations from the line, particularly in the tails, which might suggest some departure from normality.

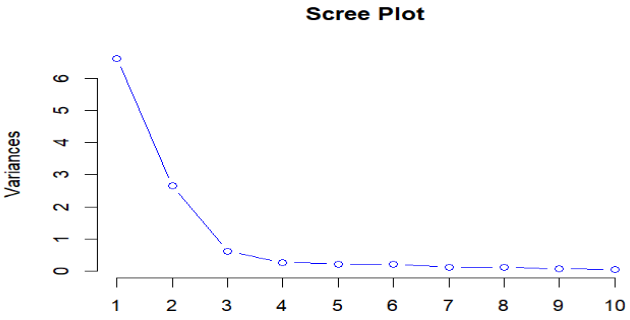
Overall Model Fit

Based on these diagnostic plots, the model appears to have a reasonable fit. The assumptions of homoscedasticity and normality of residuals are generally met, although there are some minor deviations. These deviations might not be severe enough to significantly impact the model's validity, but it is always good practice to further investigate potential issues, especially if the model is being used for critical decision-making.

Additional Considerations

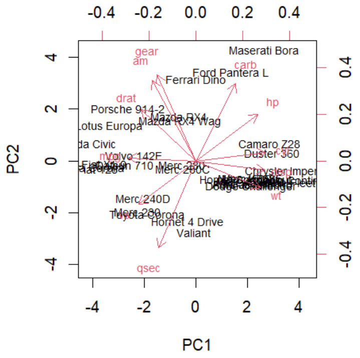
- **Outliers:** The "Residuals vs. Leverage" plot can help identify influential points or outliers that might be affecting the model's fit. In this case, there are a few points with high leverage, but they don't seem to be excessively influential.
- **Model Complexity:** The model's complexity (number of predictors) can also affect its fit. More complex models may be more prone to overfitting, which can lead to violations of model assumptions.

9. Principal Component Analysis (PCA)



Rationale for Choosing 3 Components: **Significant Variance Explained:** The first three components capture a significant portion of the total variance in the data. **Diminishing Returns:** Beyond the third component, the variance explained by each subsequent component decreases rapidly. This suggests that adding more components would not significantly improve the explanation of the data. **Practical Considerations:** Using too many

10. PCA Interpretation



Loadings of the First Two Principal Components

The loadings of the principal components (PCs) indicate the correlation between the original variables and the new components. In this biplot, the vectors represent the variables, and their direction and length show the relationship with the PCs.

Observations:

1. **PC1: Positive Loadings: Variables like "hp" (horsepower) and "wt" (weight) have positive loadings on PC1. This suggests that cars with higher horsepower and weight tend to have higher scores on PC1.Negative Loadings: Variables like "qsec" (quarter mile time) and "disp" (displacement) have negative loadings on PC1. This indicates that cars with faster quarter mile times and smaller displacements tend to have lower scores on PC1.**
2. **PC2:Positive Loadings: Variables like "gear" (number of gears) and "carb" (number of carburetors) have positive loadings on PC2. This suggests that cars with more gears and carburetors tend to have higher scores on PC2. Negative Loadings: Variables like "drat" (rear axle ratio) have negative loadings on PC2. This indicates that cars with higher rear axle ratios tend to have lower scores on PC2.**

Patterns and Groupings:

- **Performance-Oriented Cars: Cars like the Maserati Bora, Ferrari Dino, and Ford Pantera L have high scores on PC1, suggesting they are performance-oriented cars with high horsepower and weight.**
- **Fuel-Efficient Cars: Cars like the Toyota Corolla and Honda Civic have low scores on PC1, indicating they are more fuel-efficient with lower horsepower and weight.**
- **Complex Engines: Cars with higher scores on PC2 tend to have more complex engines with multiple gears and carburetors.**

Univariate Analysis Dataset 4

1. Data Overview

The **swiss** dataset in R is a dataset that contains various socio-economic indicators for Switzerland's 47 French-speaking provinces in 1888. It includes both continuous and categorical variables that describe aspects such as education, fertility rate, and socio-economic conditions.

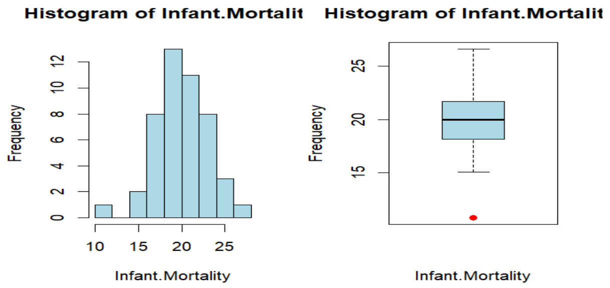
```
             Fertility Agriculture Examination Education Catholic Infant.Mortality
Courtelary      80.2      17.0        15        12     9.96          22.2
Delemont         83.1      45.1         6         9    84.84          22.2
Franches-Mnt     92.5      39.7         5         5    93.40           20.2
Moutier         85.8      36.5        12         7    33.77          20.3
Neuveville      76.9      43.5        17        15     5.16           20.6
Porrentruy      76.1      35.3         9         7    90.57          26.6
```

2. Summary Statistics

vars	n	mean	sd	median	skew	kurtosis
Fertility	**1**	**47**	**70.14255**	**12.491697**	**57.50**	**-0.4556871**
Agriculture	**2**	**47**	**50.65957**	**22.711218**	**88.50**	**-0.3203637**
Catholic	**5**	**47**	**41.14383**	**41.704850**	**97.85**	**0.4789257**
Infant.Mortality	**6**	**47**	**19.94255**	**2.912697**	**15.80**	**-0.3314326**

Mean: 47 Median: 33.80 Standard Deviation: 0.8280661 Minimum: 6 Maximum: 5759

3. Distribution Visualization



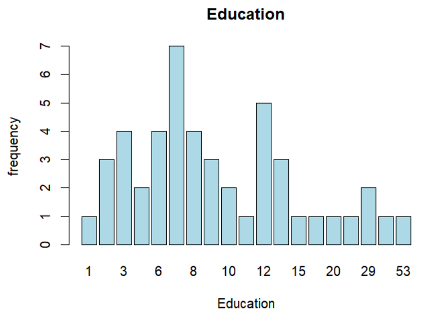
Shape of the Distribution: The histogram appears to be **approximately normally distributed**. This means that the data is roughly symmetric around the center, with most of the values clustered around the mean, and the tails tapering off on both sides.**Potential Outliers:** The boxplot shows a single outlier, represented by the red dot below the lower whisker. This outlier indicates a data point with an unusually low infant mortality rate values

4. Categorical Variable Analysis

Distribution:

- The plot shows a wide range of education levels among the individuals in the dataset. There are individuals with very low levels of education (e.g., 1 or 3 years) as well as those with higher levels (e.g., 12, 20, or 53).

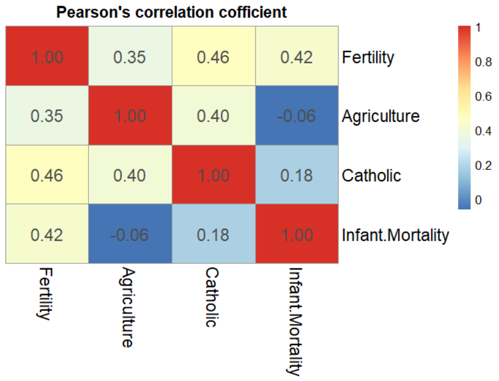
The distribution appears to be slightly right-skewed, meaning there are more individuals with lower levels of education compared to higher levels.



5. Correlation Analysis

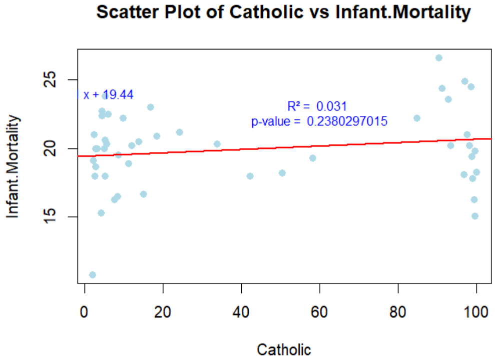
Pearson Correlation Coefficient between fertility and catholic: 0.46112

Summary: As the percentage of the population that is Catholic increases, the fertility rate tends to increase as well. However, this relationship is not very strong,



6. Scatter Plot Visualization

Low Correlation: The R-squared value of 0.031 indicates a very low correlation between "Catholic" and "Infant.Mortality". This means that only 3.1% of the variation in infant mortality can be explained by the variation in the percentage of the population that is Catholic. the scatter plot suggests that there is a very weak positive relationship between the percentage of the population that is Catholic and infant mortality rates. However, the relationship is very weak, and other



7. Multiple Regression

Interpretations

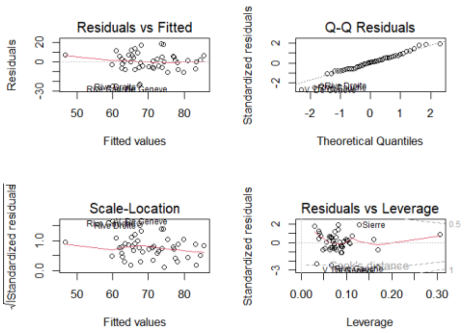
- **Intercept (26.74755):**, represents the estimated fertility rate when all predictor variables are zero. However, this interpretation might not be meaningful in this context, as it's unlikely to have countries with zero agriculture, Catholicism, and infant mortality..
- **Agriculture (0.14229):** This means that, on average, a one-unit increase in the percentage of the population engaged in agriculture is associated with a 0.14229 unit increase in the fertility rate, holding other variables constant.
- **Catholic (0.08778):** This indicates that, on average, a one-unit increase in the percentage of the population that is Catholic is associated with a 0.08778 unit increase in the fertility rate, holding other variables constant.
- **R-squared :** 0.3859 indicates that approximately 38.59% of the variation in fertility can be explained by the three predictor variables.
- **Adjusted R-squared :** 0.343 is slightly lower than the R-squared, accounting for the number of predictors in the modelvariance.
- **F-statistic :** 9.007 (highly significant, p-value < 9.578e-05)
- **p-value (F-statistic) :** The model is highly significant.

Statistic	**Value**
Coefficients	
Intercept	26.74755
Agriculture	0.14229
Catholic	0.08778
Residuals	
Minimum	-25.0367
Q1	-5.6899
Median	0.3825
Q3	5.9029
Maximum	18.9091
Model Fit	
R-squared	0.3859
Adjusted R-squared	0.343
F-statistic	9.007
p-value (F-statistic)	9.578e-05

8. Model Diagnostics

The provided diagnostic plots are crucial for evaluating the assumptions of linear regression and the overall fit of the model. Let's analyze each plot:

- **Residuals vs. Fitted Plot:** This plot checks if the variance of the residuals is constant across different fitted values. Ideally, the points should be randomly scattered around a horizontal line. In this case, the points seem to be fairly evenly distributed, suggesting that the assumption of homoscedasticity is reasonably met.



- **Scale-Location Plot:** This plot is another way to check homoscedasticity. Here, the square root of the standardized residuals is plotted against the fitted values. Ideally, the points should be randomly scattered around a horizontal line. Again, the points appear to be randomly scattered, supporting the assumption of homoscedasticity.

Normality of Residuals

- **Q-Q Plot:** This plot compares the quantiles of the standardized residuals to the quantiles of a standard normal distribution. If the residuals are normally distributed, the points should fall along a straight line. In this case, the points roughly follow a straight line, indicating that the normality assumption is reasonably met. However, there are some deviations from the line, particularly in the tails, which might suggest some departure from normality.

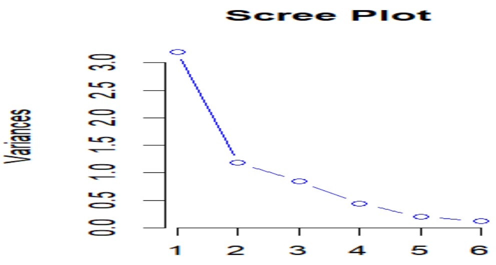
Overall Model Fit

Based on these diagnostic plots, the model appears to have a reasonable fit. The assumptions of homoscedasticity and normality of residuals are generally met, although there are some minor deviations. These deviations might not be severe enough to significantly impact the model's validity, but it is always good practice to further investigate potential issues, especially if the model is being used for critical decision-making.

Additional Considerations

- **Outliers:** The "Residuals vs. Leverage" plot can help identify influential points or outliers that might be affecting the model's fit. In this case, there are a few points with high leverage, but they don't seem to be excessively influential.
- **Model Complexity:** The model's complexity (number of predictors) can also affect its fit. More complex models may be more prone to overfitting, which can lead to violations of model assumptions.

9. Principal Component Analysis (PCA)

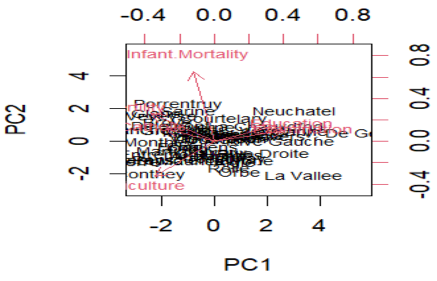


Rationale for Choosing 4 Components:**Significant Variance Explained:** The first three components capture a significant portion of the total variance in the data.**Diminishing Returns:** Beyond the third component, the variance explained by each subsequent component decreases rapidly. This suggests that adding more components would not significantly improve the explanation of the data.**Practical Considerations:** Using too many

10. PCA Interpretation

Loadings of the First Two Principal Components

In this biplot, we can observe the relationships between the original variables (Infant Mortality and Culture) and the principal components (PC1 and PC2).



Observations:

- **PC1: The "Infant Mortality" variable has a positive loading on PC1. This suggests that locations with higher infant mortality rates tend to have higher scores on PC1.**
- **PC2: The "Culture" variable has a negative loading on PC2. This suggests that locations with higher cultural values tend to have lower scores on PC2.**

Patterns and Groupings:

Based on the positions of the points representing different locations, we can observe some patterns and groupings:

- **High Infant Mortality and Low Culture: Locations like "Porrentruy" and "Vallee de Joux" are positioned in the top-right quadrant, indicating high infant mortality and low cultural values.**
- **Low Infant Mortality and High Culture: Locations like "Neuchatel" and "La Chaux-de-Fonds" are positioned in the bottom-left quadrant, indicating low infant mortality and high cultural values.**
- **Intermediate Values: Locations in the middle of the plot have intermediate values for both infant mortality and culture.**
  Overall, the biplot suggests that there is a relationship between infant mortality and cultural factors in these locations. Locations with higher infant mortality rates tend to have lower cultural values, and vice versa.

Additional Considerations:

- **Correlation: The angle between the vectors for "Infant Mortality" and "Culture" indicates the correlation between these two variables. A smaller angle suggests a stronger positive correlation, while a larger angle suggests a weaker or negative correlation.**
- **Outliers: Any points that are far away from the main cluster of points could be considered outliers. These outliers might represent locations with unique characteristics that are not well-explained by the two principal components.**