

Signal & System

DATE-09/10/14

Sagar Sen  
8871453536

Syllabus →

(1) Signal definitions & its classifications.

(2) Different operation on Signal.

(a.) Shifting (d.) Differentiation

(b.) Scaling (e.) Integration

(c.) Reversal (f.) Convolution.

(3) Basic system operations.

(a.) Static/dynamic

(b.) Linear/non-linear

(c.) Causal/Non-causal

(d.) Time invariant/time-variant

(e.) Stable/Unstable.

(4) Continuous time Fourier series

(5) Continuous time Fourier X-form

continuous time  
sig & sys:

(6) Laplace X-form.

(7) Sampling theorem

(8) Discrete time sys.

(9) Z-transform

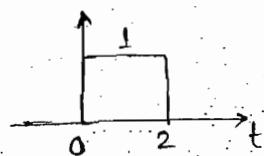
discrete time  
sig & sys.

### \* Different operations on signal →

- \* Amplitude shifting
- \* Time shifting
- \* Time scaling
- \* Time reversal
- \* Amplitude Reversal
- \*

### (1) Time shifting →

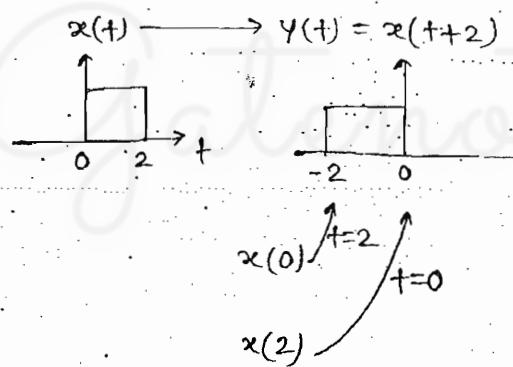
$$x(t) \longrightarrow y(t) = x(t+k)$$



#### Case(1)

when  $k > 0$

Eg:-  $k=2$

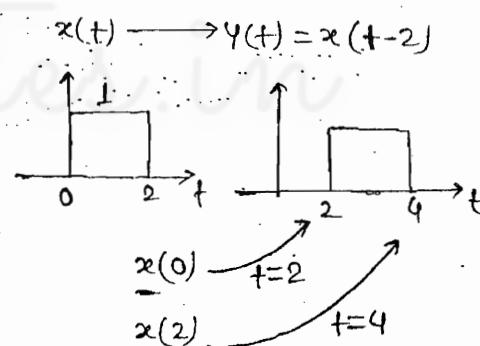


\* It is a case of left shifting.

#### Case(2)

when  $k < 0$

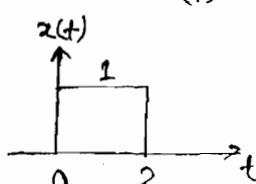
Eg:-  $k=-2$



\* It is a case of right shifting.

### (2) Amplitude Shifting →

$$x(t) \longrightarrow y(t) = k + x(t)$$



$$x(t) = \begin{cases} 0 & , t < 0 \\ \end{cases}$$

Case(1) → When  $k < 0$

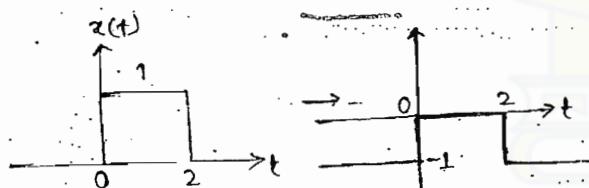
$$\text{Eg: } k = -1$$

$$x(t) \longrightarrow y(t) = -1 + x(t)$$

$$y(t) = -1 + x(t)$$

$$= \begin{cases} -1+0 & , t < 0 \\ -1+1 & ; 0 \leq t \leq 2 \\ -1+0 & ; t > 2 \end{cases}$$

$$= \begin{cases} -1 & , t < 0 \\ 0 & ; 0 \leq t \leq 2 \\ -1 & ; t > 2 \end{cases}$$



Case(2) → When  $k > 0$

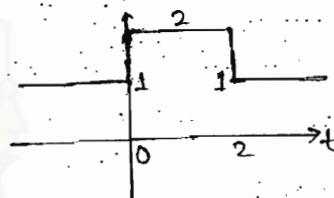
$$\text{Eg: } k = +1$$

$$x(t) \longrightarrow y(t) = 1 + x(t)$$

$$y(t) = 1 + x(t)$$

$$= \begin{cases} 1+0 & , t < 0 \\ 1+1 & ; 0 \leq t \leq 2 \\ 1+0 & ; t > 2 \end{cases}$$

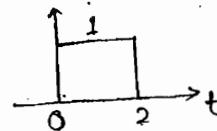
$$= \begin{cases} 1 & ; t < 0 \\ 2 & ; 0 \leq t \leq 2 \\ 1 & ; t > 2 \end{cases}$$



\* It is a case of downward shifting      \* It is a case of upward shifting

(3) Time Scaling →

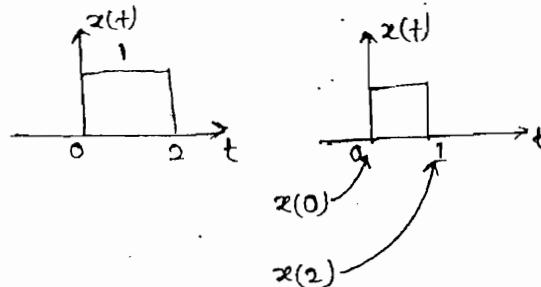
$$x(t) \longrightarrow y(t) = x(at)$$



Case(1) → When  $a > 1$

$$\text{Ex: } a = 2$$

$$x(t) = y(t) = x(2t)$$

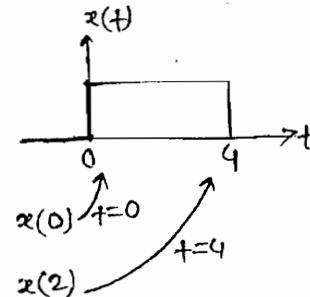


Time compression

Case(2) → when  $a < 1$

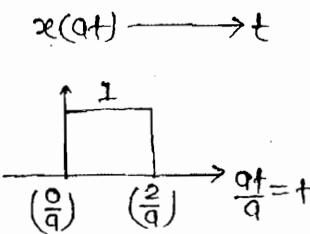
$$\text{Eg: } a = 0.5$$

$$x(t) = y(t) = x(0.5t)$$

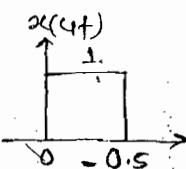
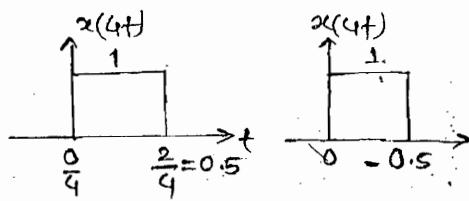
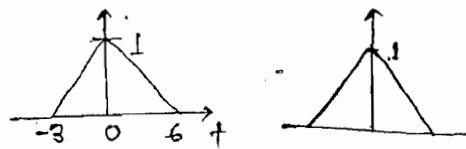


Time expansion

### Rule General $\rightarrow$



Ex:-  $x(t) \longrightarrow x(-3t)$

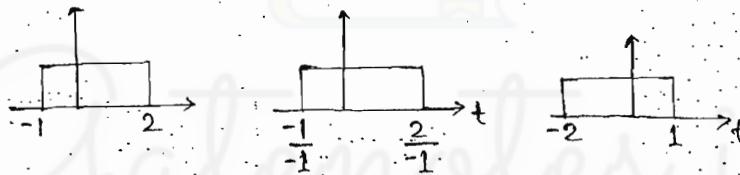


### (4.) Time-reversal $\rightarrow$

$$x(t) = y(t) = x(-t)$$

\* Time reversal is a special case of time scaling in which signal folding will take place around  $y$ -axis.

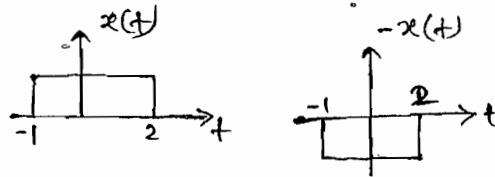
$$x(-t) = a(-1)$$



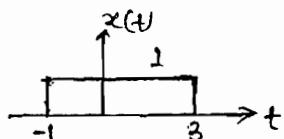
### (5.) Amplitude Reversal $\rightarrow$

$$x(t) \longrightarrow y(t) = -x(t)$$

\* In this case, signal folding will take place about  $x$ -axis.

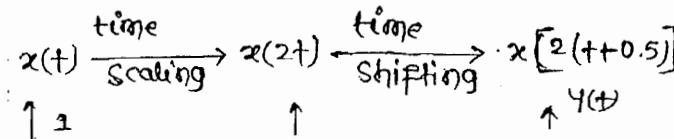


Q.  $\rightarrow$

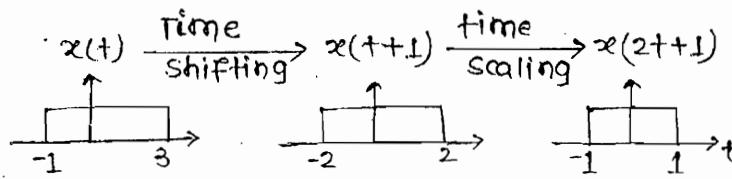


Draw signal  $y(t)$  if  $y(t) = 2x(2t+1)$

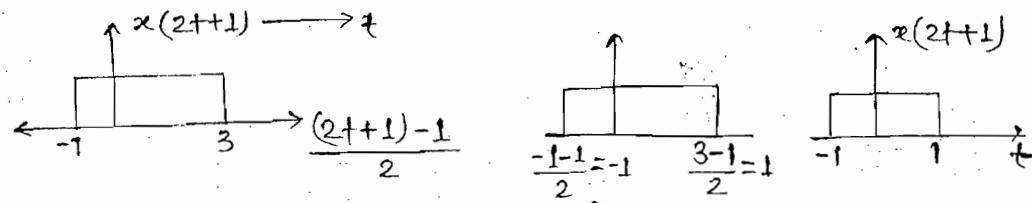
Soln  $\rightarrow$  1<sup>st</sup> method  $\rightarrow$



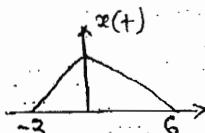
2<sup>nd</sup> method →



3<sup>rd</sup> method → (Trick)



Q. →

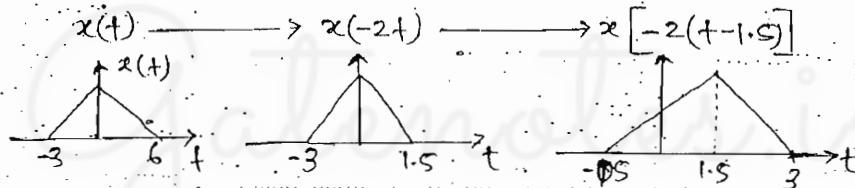


draw sig 'y(t)' if  $y(t) = x(-2t+3)$

SOLN →

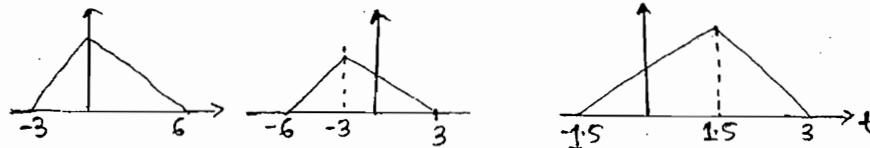
2<sup>nd</sup> method →

$$y(t) = x[-2(t-1.5)]$$

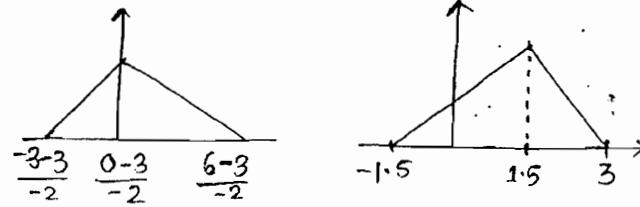


2<sup>nd</sup> method →

$$x(t) \rightarrow x(t+3) \rightarrow x(2t+3) \rightarrow$$



3<sup>rd</sup> method →



Chapter-01  
Signal definition & classifications

Signal → A signal is a fn which contains some information.

System → A sys. is interconnection of devices or components which converts signal from one form to another form.

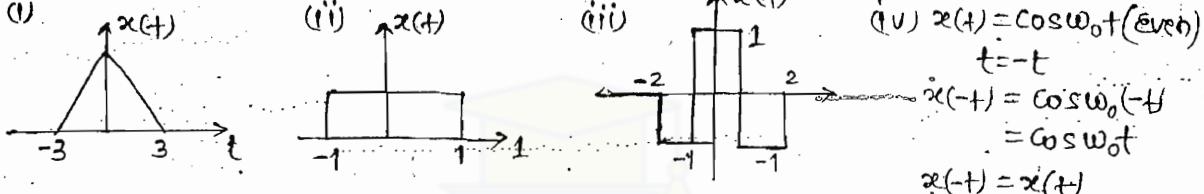
Classification of signals →

1) Even & odd signals →

\* Even → This are symmetrical (or) mirror image about y-axis.

i.e.  $x(t) = x(-t)$  → time reversal

Eg:-

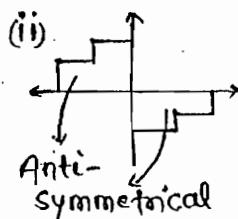
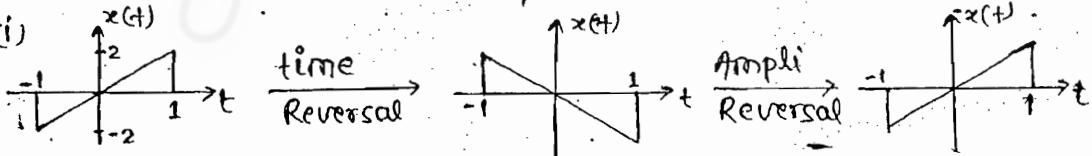


\* Odd → This are antisymmetrical about y-axis.

i.e.  $x(-t) = -x(t)$   
 (or)  
 $x(t) = -x(-t)$

time reversal  
 amplitude reversal

Eg:-



(iii)  $x(t) = \sin \omega_0 t \rightarrow$  odd signal.  
 $(t = -t)$

$x(-t) = \sin \omega_0 (-t)$

$x(-t) = -\sin \omega_0 t$

$x(-t) = -x(t)$

\* The avg. value of an odd signal is 0; but converse of this statement is not true.

Important points →

### Important points →

(1.) Even  $\times$  Even = Even ;  $t^2 \times t^4 = t^6$

(2.) Even  $\times$  odd = odd ;  $t^2 \times t^3 = t^5$

(3.) Odd  $\times$  odd = Even ;  $t^3 \times t^5 = t^8$

(4.) Even  $\pm$  Even = Even

$$x(t) = t^2 + \cos t$$

$$x(-t) = t^2 + \cos t = x(t)$$

(5.) Odd + Odd = Odd

$$x(t) = \sin t + t^3$$

$$x(-t) = -\sin t - t^3$$

$$x(t) = -x(-t)$$

(6.) Even + odd = Neither even nor odd.

$$x(t) = t^2 + \sin t$$

$$x(-t) = t^2 - \sin t$$

$$x(-t) \neq x(t)$$

\* Any signal can be divided into 2 part in which one part will be even & the other part will be odd.

i.e.  $x(t) = x_E(t) + x_O(t)$

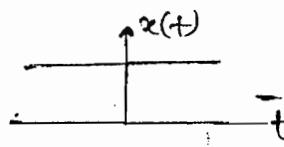
Where;

$$x_E(t) = \text{even part of } x(t) = \frac{x(t) + x(-t)}{2}$$

$$x_O(t) = \text{odd part of } x(t) = \frac{x(t) - x(-t)}{2}$$

Eg.  $\rightarrow x(t) = 2 = \text{dc signal}$

$$\begin{array}{l} \downarrow \\ x = -t \\ x(-t) = 2 = x(t) \quad [\text{Even signal}] \end{array}$$



dc signal is a Even signal.

(2.)  $f(k) = \sin(k^2)$

$$\downarrow k = -k$$

$$f(-k) = \sin(k^2) = f(k) \quad [\text{Even signal}]$$

(3.)  $f(\sigma) = \sin \pi/2$

$$= 1$$

$$f(\sigma) = f(-\sigma) \quad [\text{Even signal}]$$

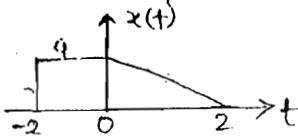
(4) Find  $x_E(t)$  &  $x_O(t)$  of the signal.

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^3} + t^3 \sin^3 t$$

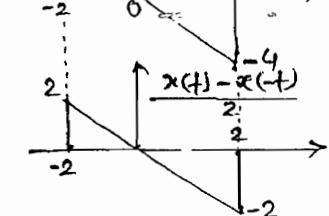
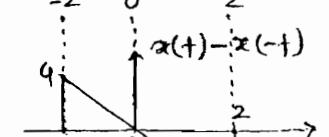
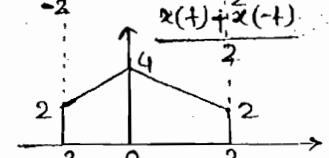
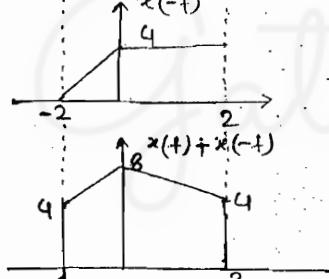
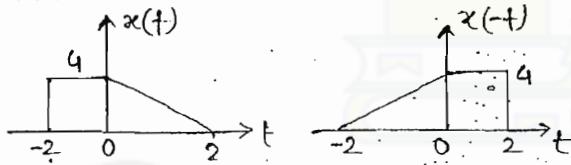
$$\begin{array}{c} E - \frac{E}{0} + \frac{E}{0} - \frac{0}{E} + 0 \times 0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ E \quad 0 \quad 0 \quad E \quad 0 \end{array}$$

$$x_E(t) = 3 - \frac{\sin^2 t}{t^3} + t^3 \sin^3 t, \quad x_O(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Que. → Draw  $x_E(t)$  &  $x_O(t)$  of



Soln → for even part of  $x(t)$



## (2.1) Conjugate Symmetric (CS) & Conjugate Antisymmetric (CAS) signal →

### \* Conjugate symmetric (CS)

$$x(t) = x^*(-t)$$

$$x(t) = a(t) + j b(t) \quad (i)$$

$(t = -t)$

$$x(-t) = a(-t) + j b(-t)$$

$$x^*(-t) = a(-t) - j b(-t) \quad (ii)$$

From eqn (i) & (ii)

$$a(t) = a(-t) \rightarrow \text{Even}$$

$$b(t) = -b(-t) \rightarrow \text{Odd}$$

Eg:-  $x(t) = t^2 + \sin t$

$$\begin{array}{c} \downarrow \\ E \\ \downarrow \\ O \end{array}$$

### \* Conjugate antisymmetric (CAS)

$$x(t) = -x^*(-t)$$

$$x(t) = a(t) + j b(t)$$

$$a(t) = -a(-t) \rightarrow \text{Odd}$$

$$b(t) = b(-t) \rightarrow \text{Even}$$

Eg:-  $x(t) = 8\sin t + jt^2$

$$\begin{array}{c} \downarrow \\ O \\ \downarrow \\ E \end{array}$$

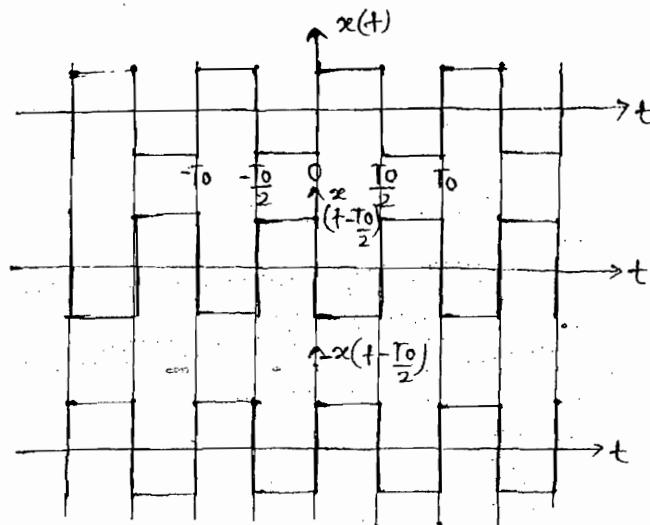
## (3.) Halfwave symmetric signal (HWS) →

for Half-wave symmetry (HWS)

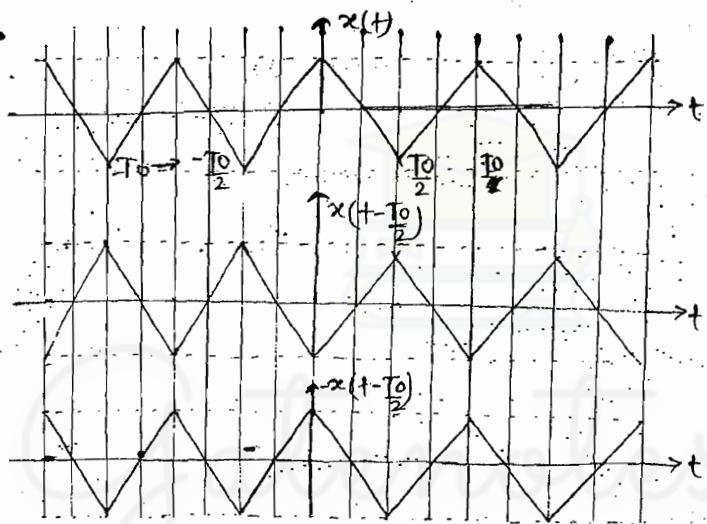
$$x(t) = -x\left(t + \frac{T_0}{2}\right)$$

time shifting  
amp. reversal

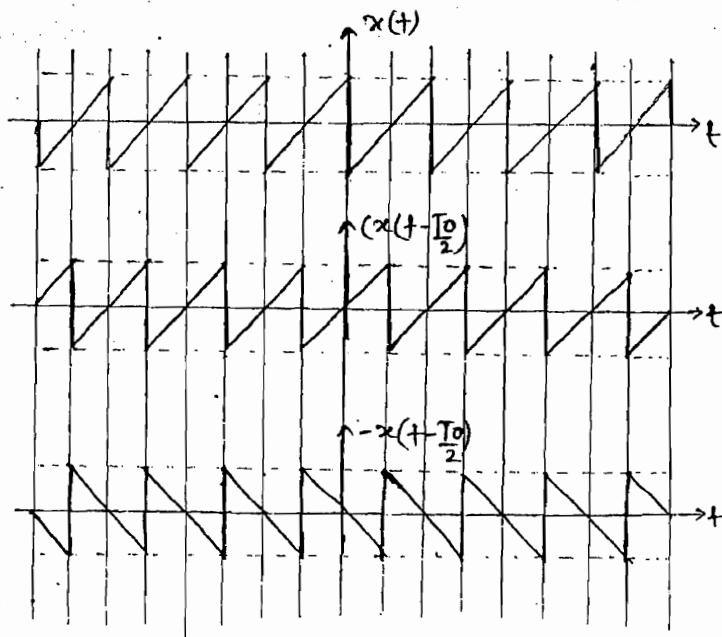
Eg  $\rightarrow$   
(1)



(2)



(3.)



so; sawtooth  
wave doesn't follow  
the HWS.

\* The avg. value of a HWS is 0, but converse of this statement is not true.

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#### (4) Periodic & non-periodic Signal →

Periodic → A signal repeats itself after some time period, the signal is said to be periodic.

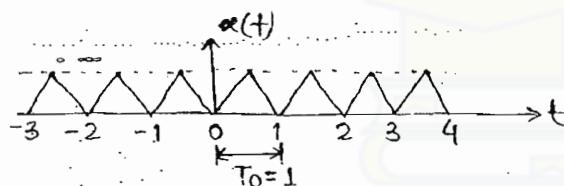
i.e.  $x(t) = x(t \pm nT_0)$

where,  $n = \text{an integer}$

$T_0$  = Fundamental time period.  $\begin{cases} T_0 \neq 0 \\ T_0 \neq \infty \end{cases}$

FTP → It is the smallest, +ve & fixed value of the time for which signal is periodic.

Eg →



FTP = 1

Q. → Find FTP of signal  $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Sol → Let ' $T_0$ ' be the FTP of the signal

i.e.

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = A_0 e^{j\omega_0 (t + T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0 (t + T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0 t} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi k} \quad (\text{where } k = \text{an integer})$$

$$j\omega_0 T_0 = j2\pi k$$

$$\frac{T_0}{(\text{smallest})} = \frac{2\pi k}{\omega_0} \rightarrow 1 \text{ (least integer)}$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Q. Find FTP of following signal  $\rightarrow$

$$(i) x_1(t) = A_0 \sin(2\pi t) \quad (ii) x_2(t) = A_0 \sin(2\pi t + 30^\circ) \quad (iii) x_3(t) = -x_1(t)$$

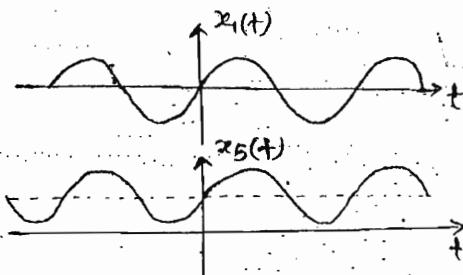
$$\omega_0 = 2\pi \quad \omega_0 = 2\pi \quad \omega_0 = 2\pi$$

$$T_0 = \frac{2\pi}{2\pi} = 1 \quad T_0 = 1 \quad T_0 = 1$$

$$(iv) x_4(t) = x_1(-t) = -A_0 \sin 2\pi t$$

$$\omega_0 = 2\pi, T_0 = 2\pi$$

$$(v) x_5(t) = A_0 + x_1(t) = A_0 + A_0 \sin(2\pi t)$$



$$(vi) x_6(t) = x_1(t-t_0) = A_0 \sin[2\pi(t-t_0)]$$

$$\omega_0 = 2\pi \quad T_0 = 1$$

\* Time period of signal is unaffected by time shifting, time reversal, amp. reversal, amp. shifting & change in phase of signal.

$$(vii) f(t) = \sin^2(4\pi t)$$

$$= \frac{1 - \cos 8\pi t}{2}$$

$$\omega_0 = 8\pi$$

$$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$$

\* The sum of 2 (or) more than 2 periodic signal will be periodic if ratios of their fundamental time period (or) freq. are rational.

$$\text{i.e. } x(t) = x_1(t) + x_2(t)$$

$$\downarrow \quad \downarrow \\ T_1, f_1, \omega_1 \quad T_2, f_2, \omega_2$$

$$\rightarrow \frac{T_1}{T_2} \text{ (or) } \frac{\omega_1}{\omega_2} \text{ (or) } \frac{f_1}{f_2} \text{ (Rational no.)}$$

$$\rightarrow T_0 = \text{LCM}[T_1, T_2]$$

$$\rightarrow f_0 = \text{HCF}[f_1, f_2]$$

Q → Find FTF of signal if it is periodic :-

$$(i) x(t) = \sin 2\pi t + \cos 3\pi t$$

$$\text{Soln} \Rightarrow \omega_1 = 2 \quad \frac{\omega_1}{\omega_2} = \frac{2}{3\pi} \text{ (Irrational no)}$$

Hence it is non-periodic

$$(ii) x(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$$

$$\text{Soln} \Rightarrow \omega_1 = 2\pi, \omega_2 = \sqrt{2}\pi$$

$$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} \text{ (Irrational no.)}$$

Hence it is Non-periodic

$$(iii) x(t) = \sin 4\pi t + \sin 7\pi t$$

$$\text{Soln} \Rightarrow \omega_1 = 4\pi, \omega_2 = 7\pi$$

$$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7} \text{ (Rational no.)}$$

Hence it is periodic. Then calculate  $T_0$ .

1st method :-

$$\omega_0 = 2\pi \text{ HCF}[\omega_1, \omega_2] = \text{HCF}[4\pi, 7\pi]$$

$$\omega_0 = \pi$$

$$T_0 = \frac{2\pi}{\omega_0} = 2$$

$$*** \quad \text{HCF} \left[ \frac{P_1}{q_1}, \frac{P_2}{q_2} \right] = \frac{\text{HCF}[P_1, P_2]}{\text{LCM}[q_1, q_2]} \quad \text{LCM} \left[ \frac{P_1}{q_1}, \frac{P_2}{q_2} \right] = \frac{\text{LCM}[P_1, P_2]}{\text{HCF}[q_1, q_2]}$$

2nd method →

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2} \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$$

$$T_0 = \text{LCM}[T_1, T_2] = \text{LCM} \left[ \frac{1}{2}, \frac{2}{7} \right]$$

$$= \frac{\text{LCM}[1, 2]}{\text{HCF}[2, 7]} = \frac{2}{1} = 2$$

\* Area & Avg. value of signal →

Area of  $x(t)$  :-

$$\text{Area} = \int_{-\infty}^{\infty} x(z) dz$$

Area of  $x(t)$  over Range  $(t_1, t_2)$

$$\text{Area} = \int_{t_1}^{t_2} x(z) dz$$

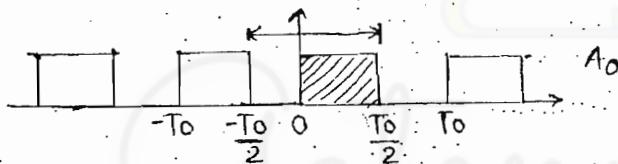
Avg. value of  $x(t)$  :

$$\text{Avg.} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(z) dz, \text{ For periodic sig.}$$

$$\left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(z) dz, \text{ for Non-periodic sig.} \right.$$

Que → Find the avg. value of sig.

(i)



Soln →

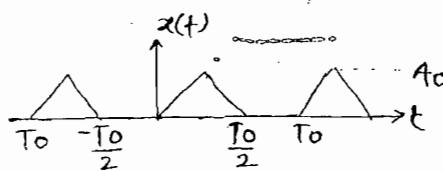
$$\text{Avg.} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(z) dz$$

$$= \frac{\text{Area of } x(t) \text{ over } T_0}{T_0}$$

$$= \frac{A_0 \times \frac{T_0}{2}}{T_0}$$

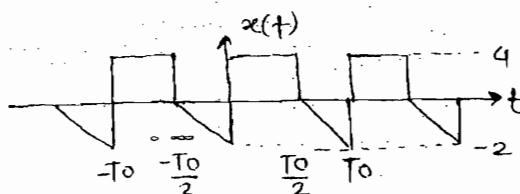
$$= \frac{A_0}{2}$$

(2.)



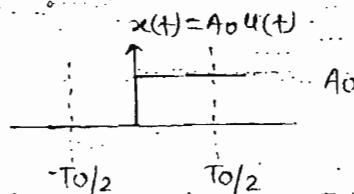
$$\text{Soln} \rightarrow \text{Avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{1/2 \times A_0 \times T_0/2}{T_0} = \frac{A_0}{4}$$

(3.)

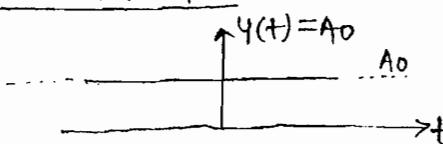


$$\text{Soln} \rightarrow \text{Avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{-1/2 \times \frac{T_0}{2} \times 2 + 4 \times \frac{T_0}{2}}{T_0} = \frac{3}{2}$$

(iv)



$$\begin{aligned} \text{Soln} \rightarrow \text{Avg.} &= \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(z) dz \\ &= \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_0^{T_0/2} A_0 dz \\ &= \lim_{T_0 \rightarrow \infty} \frac{A_0 \times T_0/2}{T_0} \\ &= \frac{A_0}{2} \end{aligned}$$

2nd method  $\rightarrow$ 

$$\text{avg } y(t) = A_0$$

$$\text{avg } x(t) = \frac{\text{avg } y(t)}{2}$$

$$= \frac{A_0}{2}$$

### (5.) Energy & power signal →

\* Energy of  $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

\* Power of  $x(t)$

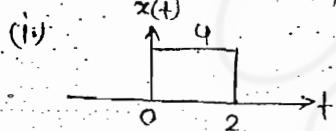
$$P = \begin{cases} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{For periodic sig.} \\ \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{Non periodic sig.} \end{cases}$$

\* For an energy sig., energy should be finite & power should be zero.

Energy signals are absolutely integrable signal.

i.e.  $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

Q. → Calculate energy of sig.

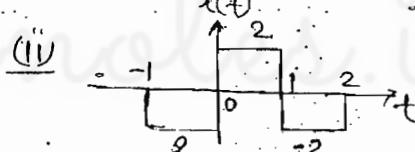


Soln →  $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$   
 $= \int_0^2 4^2 dt = 32$

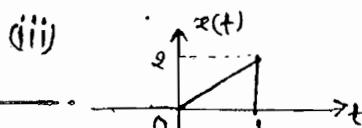
2nd method →

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

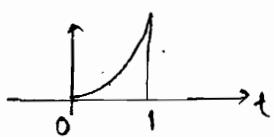
$$= 16 \times 2 = 32$$



Soln →  $E x(t) = \text{Area of } |x(t)|^2$   
 $= 4 \times 3 = 12$



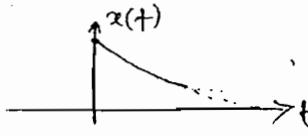
Soln →



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt  
= \int_0^1 (2t)^2 dt = \frac{4}{3}$$

Q. → Cal. area & energy of signal:-

(i)  $x(t) = e^{-at} u(t), a > 0$



Soln

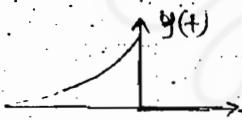
$$\begin{aligned} \text{Area} &= \int_{-\infty}^{\infty} x(t) dt \\ &= \int_0^{\infty} e^{-at} dt \\ &= \left( \frac{e^{-at}}{-a} \right)_0^{\infty} = \frac{e^{-a\infty} - e^0}{-a} \\ &= \frac{0 - 1}{-a} = \frac{1}{a} \end{aligned}$$

$$\begin{aligned} \therefore e^{-a\infty} &= 0, a > 0 \quad (a=2) \\ e^{-2\infty} &= e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0 \end{aligned}$$

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

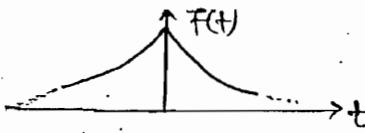
$$= \int_0^{\infty} e^{-2at} dt = \left( \frac{e^{-2at}}{-2a} \right)_0^{\infty} = \frac{e^{-2a\infty} - e^0}{-2a} = \frac{1}{2a}$$

(ii)  $y(t) = x(-t) = e^{at} u(-t), a > 0$



Soln  $\text{Area} = \frac{1}{a}, \text{ Energy} = \frac{1}{2a}$

(iii)  $f(t) = x(t) + y(t) = e^{-|t|}, a > 0$



Soln

$$f(t) = e^{-|t|}, a > 0$$

$$= \begin{cases} e^{at}, & t < 0 \\ e^{-at}, & t > 0 \end{cases}$$

$$\text{Area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$$

$$\text{Energy} = 1 \cdot 1 \cdot 1$$

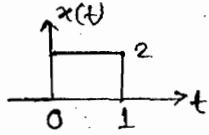
\*  $|t| = \begin{cases} -t, & t < 0 \\ t, & t > 0 \end{cases}$

$$\text{Q.} \rightarrow x(t) \longrightarrow E$$

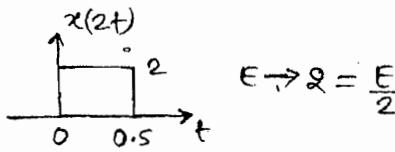
$$x(2t) \longrightarrow ?$$

- (a.)  $\frac{E}{4}$  (b.)  $\frac{E}{2}$  (c.)  $2E$  (d.)  $E$

Soln →



$$E \rightarrow 4$$



$$E \rightarrow 2 = \frac{E}{2}$$

$$x(t) \longrightarrow E$$

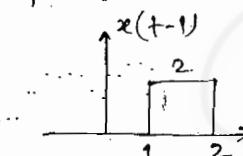
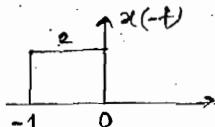
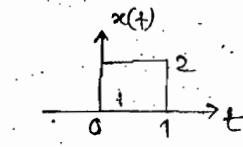
$$x(2t) \longrightarrow \frac{E}{2}$$

$$x(-2t) \longrightarrow \frac{E}{2}$$

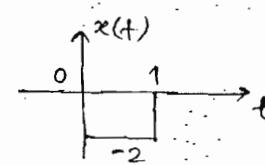
\*\*\*

$$x(qt)_{q \neq 0} \longrightarrow \frac{E}{|q|}$$

\*



$$\text{Energy} = 4$$



\* Energy of signal is independent of amp. reversal, time reversal, time shifting.

\* Power Signal → \* for this signal power should be finite & energy should be  $\infty$ .

\* Periodic power signals are absolutely integrable over their time period.

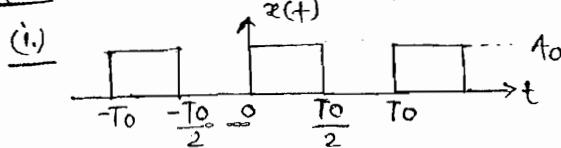
i.e.

$$\int_{T_0}^{\infty} |x(t)|^2 dt < \infty \quad \text{periodic power sig.}$$

$$P = \left\{ \begin{array}{l} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \\ \end{array} ; \text{ for periodic signal.} \right.$$

$$\lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt ; \text{ for Non-periodic}$$

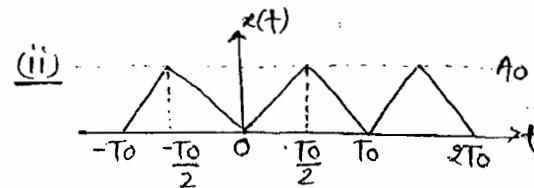
Q → Calculate power of signal :-



Soln →

$$\begin{aligned} P &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \\ &= \frac{1}{T_0} \int_0^{T_0/2} A_0^2 dt \end{aligned}$$

$$P = \frac{A_0^2}{2}$$



Soln →

$$\begin{aligned} P &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (A_0 t)^2 dt \\ &= \frac{2}{T_0} \int_0^{T_0/2} |x(t)|^2 dt \end{aligned}$$

$$x(t) = mt = \left(\frac{2A_0}{T_0}\right)t \quad (\because m = \frac{A_0}{T_0/2})$$

$$\begin{aligned} P &= \frac{2}{T_0} \int_0^{T_0/2} \left(\frac{2A_0}{T_0} t\right)^2 dt \\ &= \frac{8 \times A_0^2}{T_0^3} \int_0^{T_0/2} t^2 dt \\ &= \frac{8A_0^2}{T_0^3} \times \frac{T_0^3}{8 \times 3} \end{aligned}$$

$$P = \frac{A_0^2}{3}$$

(iii)  $x(t) = A_0 \sin \omega_0 t$

Soln →

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (A_0 \sin \omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0} \int_{-T_0/2}^{T_0/2} \frac{(1 - \cos 2\omega_0 t)}{2} dt$$

$$P = \frac{2A_0^2}{T_0} \int_0^{T_0/2} (1 - \cos 2\omega_0 t) dt$$

$$= \frac{A_0^2}{T_0} \left[ \frac{T_0}{2} - \left( \frac{\sin 2\omega_0 t}{2\omega_0} \right) \Big|_0^{T_0/2} \right]$$

$$= \frac{A_0^2}{T_0} \left[ \frac{T_0}{2} - \frac{\sin \omega_0 T_0}{2\omega_0} \right]$$

$$= \frac{A_0^2}{T_0} \left[ \frac{T_0}{2} - \frac{\sin \omega_0 T_0}{2\omega_0} \right]$$

$$= \frac{A_0^2}{T_0} \times \frac{T_0}{2}$$

$$P = \frac{A_0^2}{2}$$

$$(\because \omega_0 T_0 = 2\pi)$$

∴ RMS of the Given signal is  $\frac{A_0}{\sqrt{2}}$

$$\star \star \star \quad \boxed{\text{RMS}^2 = \frac{A_0^2}{2} = P}$$

\* Power is also known as mean square value of signal.

Q. → Calculate power of signal

$$(i) x_1(t) = A_0 \sin \omega_0 t$$

$$(ii) x_2(t) = x_1(t-t_0) = A_0 \sin [\omega_0(t-t_0)]$$

$$(iii) x_3(t) = x_1(2t) = A_0 \sin 2\omega_0 t$$

$$(iv) x_4(t) = A_0 \sin (\omega_0 t + \phi)$$

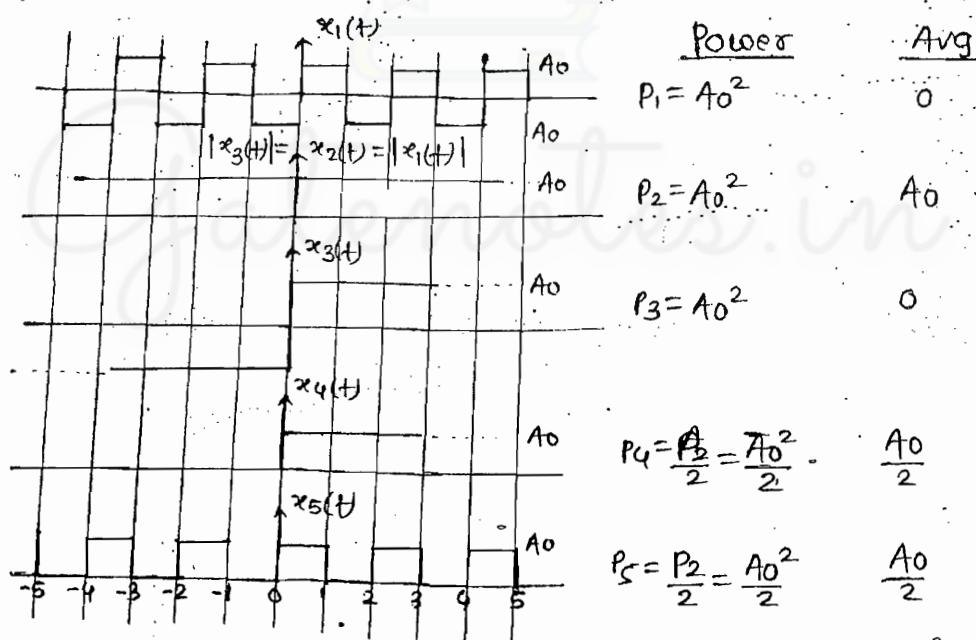
Soln → for above all signals

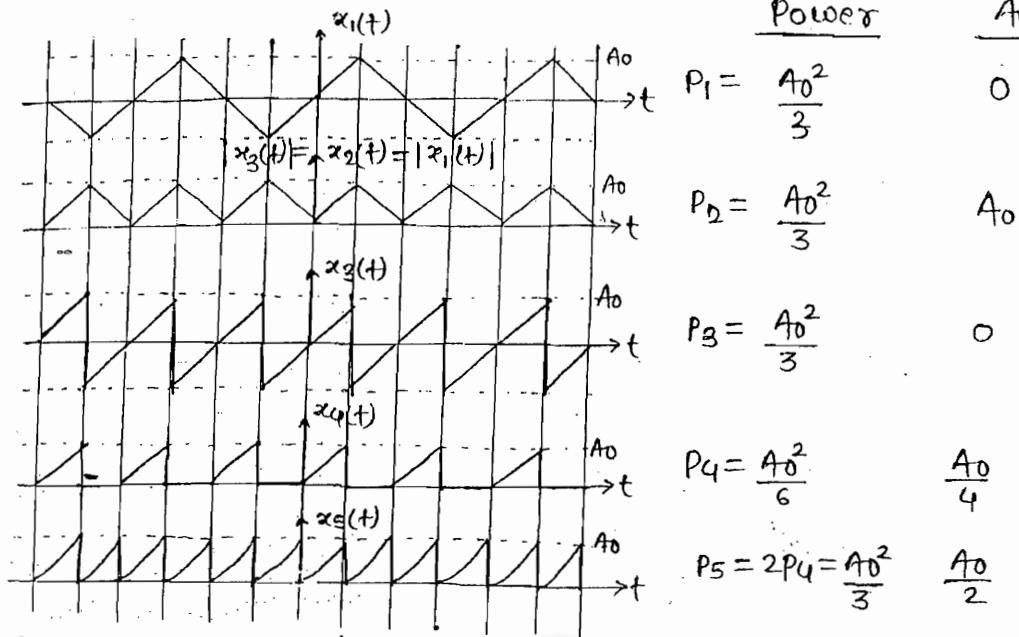
$$RMS = \frac{A_0}{\sqrt{2}}$$

$$\text{Power} = \frac{A_0^2}{2}$$

\* Power calculation is independant of time shifting, time scaling, change in freq. (or) time period & change in phase of signals.

Q. →



Q.DATE-13/10/14

\* Concept of Orthogonality → 2 signals  $x_1(t)$  &  $x_2(t)$  are said to be orthogonal if

$$* \int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) dt = 0, \text{ for non-periodic sig.}$$

$$* \int_{T_0}^{\infty} x_1(t) \cdot x_2(t) dt = 0; \text{ For periodic sig.}$$

Use of orthogonality for energy & power calculation →

If  $x_1(t)$  &  $x_2(t)$  are orthogonal &  $z(t) = x_1(t) \pm x_2(t)$  then;

$$P_z = P_{x_1} + P_{x_2} \quad \{ \text{If } x_1 \text{ & } x_2 \text{ are power signal} \}$$

(or)

$$E_z = E_{x_1} + E_{x_2} \quad \{ \text{If } x_1 \text{ & } x_2 \text{ are energy signals} \}$$

Important trigonometrical results →

$$(1) \int_{T_0} \sin(m\omega_0 t + \phi) dt = 0, \quad (m = \text{an integer}, T_0 = \frac{2\pi}{\omega_0})$$

$$(2) \int_{T_0} \cos(m\omega_0 t + \phi) dt = 0$$

$$(3) \int_{T_0} \sin^2(m\omega_0 t + \phi) dt = \frac{T_0}{2}$$

$$\star (4) \int_0 T_0 \cos^2(m\omega_0 t + \phi) dt = \frac{T_0}{2}$$

$$\star (5) \int_0 T_0 \sin(m\omega_0 t + \phi) \cdot \sin(n\omega_0 t + \phi_2) dt = 0; \quad (m \neq n \text{ & both are integer})$$

$$Q \rightarrow z(t) = 2\sin(3\pi t + 30^\circ) - 4\sin(7\pi t + 40^\circ)$$

Soln In the above sig. the freq. of the signals are diff. ( $m \neq n$ ). So that they are orthogonal.

$$P_z = P_{x_1} + P_{x_2}$$

$$P_{x_1} = \frac{2^2}{2} = 2 \quad P_{x_2} = \frac{4^2}{2} = 8$$

$$P_z = 10$$

$$Q \rightarrow z(t) = 2\sin 3\pi t + 4\sin(7\pi t + 30^\circ) + 5\sin(10\pi t + 45^\circ)$$

$$SOLN \rightarrow P_z = P_1 + P_2 + P_3$$

$$= \frac{2^2}{2} + \frac{4^2}{2} + \frac{5^2}{2}$$

$$\star (6) \int_0 T_0 \cos(m\omega_0 t + \phi_1) \cdot \cos(n\omega_0 t + \phi_2) dt = 0 \quad \{m \neq n\}$$

$$Q \rightarrow z(t) = 3\cos(3\pi t + 70^\circ) + 4\cos(7\pi t + 85^\circ)$$

$$SOLN \rightarrow P_z = \frac{3^2}{2} + \frac{4^2}{2}$$

$$\star (7) \int_0 T_0 \cos(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2) dt = 0 \quad \begin{cases} \rightarrow (m \neq n) \\ \rightarrow (m = n, \phi_1 = \phi_2) \end{cases}$$

$$Q \rightarrow z(t) = 2\sin(3\pi t + 40^\circ) + 3\cos(7\pi t)$$

$$SOLN \rightarrow P = \frac{2^2}{2} + \frac{3^2}{2}$$

$$Q \rightarrow z(t) = 2\sin(2\pi t + 45^\circ) + 3\cos(2\pi t + 45^\circ)$$

$$SOLN \rightarrow P = \frac{2^2}{2} + \frac{3^2}{2}$$

$$\star (8) \int_0 T_0 A_0 \sin(m\omega_0 t + \phi) dt = 0$$

To

↓  
AC      ↓  
sinusoidal ( $\sin, \cos$ )

$$Q \rightarrow z(t) = 2 + 4 \sin(3\pi t + 45^\circ)$$

$$\text{Soln} \rightarrow P_z = P_1 + P_2$$

$$= 2^2 + \frac{4^2}{2}$$

\* Harmonics of diff. freq. are orthogonal.

\* Sine & cosine  $f^n$  of same freq. & same phase are also orthogonal.

\* DC & sinusoidal  $f^n$  are also orthogonal.

$$Q \rightarrow z(t) = A_1 \sin(\omega_0 t + \phi_1) + A_2 \sin(\omega_0 t + \phi_2) \text{ where } \phi_1 - \phi_2 \neq \frac{n\pi}{2}; (n=\text{integer})$$

$$\text{Soln} \rightarrow P = \frac{1}{T_0} \int_{T_0} z^2(t) dt$$

$$P = \frac{1}{T_0} \int_{T_0} [A_1 \sin(\omega_0 t + \phi_1) + A_2 \sin(\omega_0 t + \phi_2)]^2 dt$$

$$= \frac{1}{T_0} \int_{T_0} [A_1^2 \sin^2(\omega_0 t + \phi_1) + \frac{A_2^2}{2} \sin^2(\omega_0 t + \phi_2) + 2A_1 A_2 \sin(\omega_0 t + \phi_1) \cdot \sin(\omega_0 t + \phi_2)] dt$$

$$= \frac{1}{T_0} \int_{T_0} [A_1^2 (1 - \cos(2\omega_0 t)) + \frac{A_2^2}{2} (1 + \cos(2\omega_0 t)) + 2A_1 A_2 \cos(\phi_1 - \phi_2)] dt$$

$$P = \frac{A_0^2}{2}, \quad \text{RMS} = \frac{A_0}{\sqrt{2}}$$

$$A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2)}$$

$$Q \rightarrow z(t) = 2 \sin 3\pi t + 3 \cos(3\pi t + \frac{\pi}{3})$$

$$\text{Soln} \rightarrow$$

$$A_0 = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \cos(0 - \pi/3)}$$

$$= \sqrt{13 + 12 \times \frac{1}{2}}$$

$$= \frac{\sqrt{19}}{\sqrt{2}}$$

Above calculation is wrong because sin & cos is present.

$$z(t) = 2 \sin 3\pi t + 3 \sin(3\pi t + \frac{\pi}{3} + \frac{\pi}{2})$$

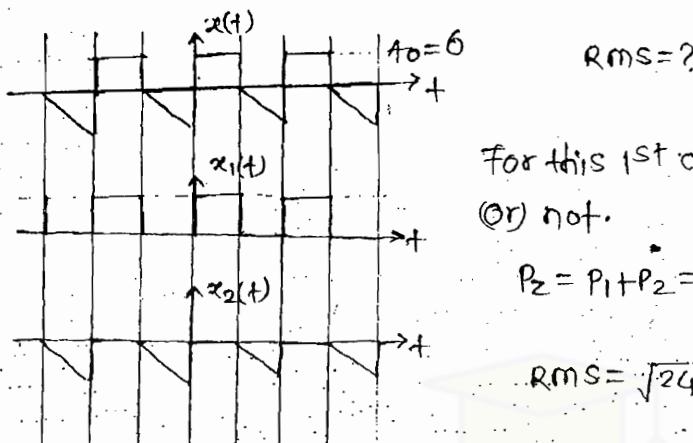
$$= 2 \sin 3\pi t + 3 \sin(3\pi t + \frac{5\pi}{6})$$

$$\phi_1 - \phi_2 = 150^\circ$$

$$A_0 = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \cos(150^\circ)}$$

$$RMS = \frac{A_0}{\sqrt{2}} = 1.14$$

Q. →



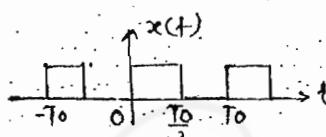
SOL ↴

For this 1st check that are they orthogonal  
(Or) not.

$$P_2 = P_1 + P_2 = \frac{A_0^2}{2} + \frac{A_0^2}{6} = \frac{6^2}{2} + \frac{6^2}{6} = 24$$

$$RMS = \sqrt{24} = 2\sqrt{6}$$

Q. →



SOL ↴

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

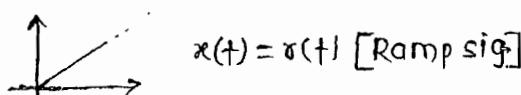
$$= \text{no. of pulses} \times \int_{T_0}^{\infty} |x(t)|^2 dt = \infty$$

Note ↴

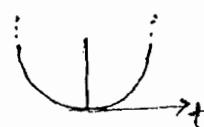
\* Periodic signals are not energy signals because their energy content is  $\infty$ .

\* (1) If magnitude of sig. is  $\infty$  at any instant of time then signal will be neither energy nor power.

Eg. → (a)



(b.)

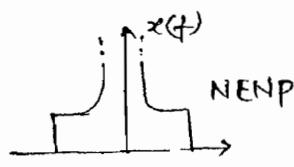
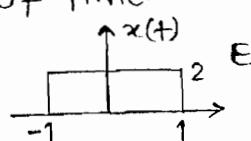


$$(c) \quad x(t) = t^2$$

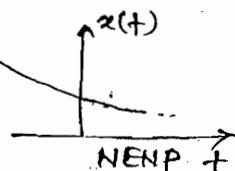
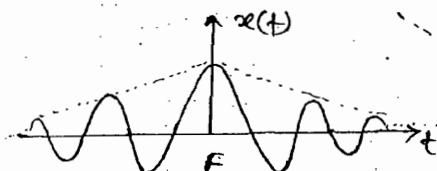
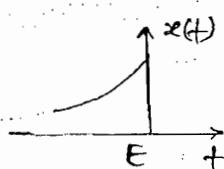
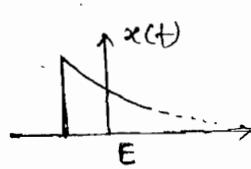
$$(d) \quad x(t) = \frac{1}{t} \quad (\text{because } t=0, x(t)=\infty)$$

\* (2.) Energy signals are:-

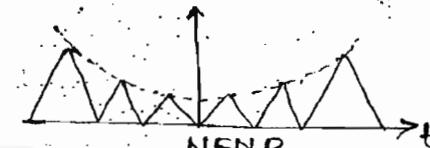
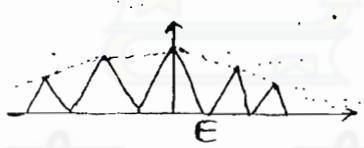
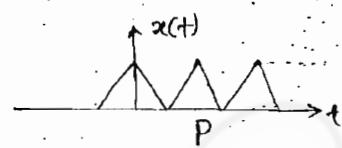
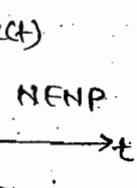
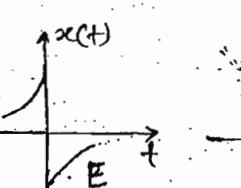
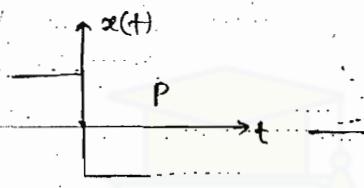
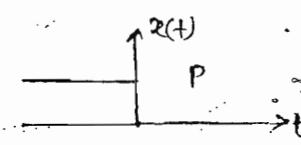
- (i) finite duration signals having finite amp. for each & every instant of time.



- (ii)  $\infty$  extension signals with amp. or peak amp decreasing in nature.



Peak amp. decreasing



\* Periodic Signals

$$\rightarrow P \rightarrow \sin t$$

$$\rightarrow \text{NENP} \rightarrow \tan t$$

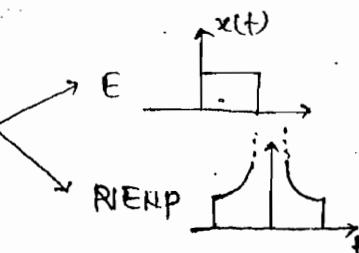
\* Non-Periodic

$$\rightarrow E \rightarrow u(t)$$

$$\rightarrow P \rightarrow u(t)$$

$$\rightarrow \text{NENP} \rightarrow r(t) = tu(t)$$

\* Finite duration



$$\rightarrow \text{NENP}$$

\*  $\infty$  extension

$$\rightarrow E \rightarrow u(t)$$

$$\rightarrow P \rightarrow u(t)$$

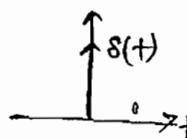
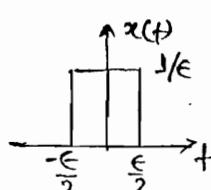
$$\rightarrow \text{NENP} \rightarrow r(t)$$

\* Basic Signals →

(1.) Unit-impulse :-  $\delta(t)$

$$\delta(t) = \lim_{\epsilon \rightarrow 0} x(t)$$

$$= \begin{cases} \infty, t=0 \\ 0, t \neq 0 \end{cases}$$



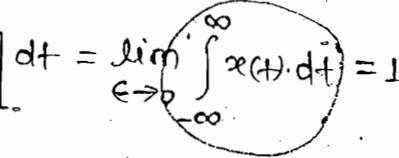
### Properties →

\* (1.)  $\delta(t)$  is an even signal.

\* (2.) It is a NENP signal.

\* (3.) Area under impulse :-

$$= \int_{-\infty}^{\infty} \delta(t) dt = \int_{-\infty}^{\infty} \left[ \lim_{\epsilon \rightarrow 0} x(t) \right] dt = \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} x(t) dt = 1$$

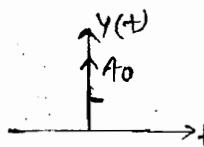


\* (4.) Weight/ strength of impulse :-

$$y(t) = A_0 \delta_0(t)$$

Area of weighted impulse y(t)

$$= \int_{-\infty}^{\infty} y(t) dt = A_0 \int_{-\infty}^{\infty} \delta(t) dt = A_0 = \text{weight of impulse.}$$



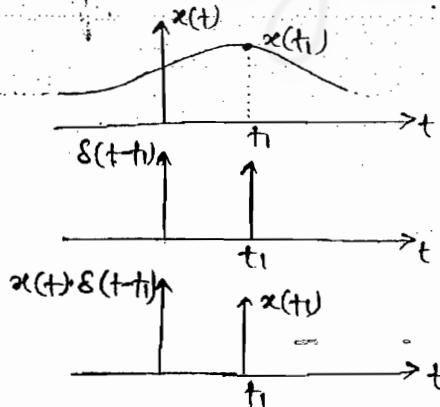
\* (5.) Scaling property of impulse:-

$$\delta[a(t-t_1)] \quad a \neq 0 = \frac{1}{|a|} \delta(t)$$

$$\text{eg. } (1) \delta(-2t) = \frac{1}{2} \delta(t)$$

$$(2) \delta(2t-3) = \delta[2(t-\frac{3}{2})] = \frac{1}{2} \delta(t-\frac{3}{2})$$

$$* (6.) x(t) * \delta(t-t_1) = ? = x(t_1) \cdot \delta(t-t_1)$$



$$\text{eg. } (1) y(t) = 2\sin t \cdot \delta(t - \frac{\pi}{2})$$

$$= 2\sin(\frac{\pi}{2}) \delta(t - \frac{\pi}{2})$$

$$= 2\delta(t - \pi/2)$$

$$(2) y(t) = e^{-2t^2} \cdot \delta(2t-1)$$

$$= e^{-2t^2} \delta[2(t - \frac{1}{2})]$$

$$= e^{-2t^2} \cdot \frac{1}{2} \delta(t - \frac{1}{2})$$

$$= \frac{1}{2} \cdot e^{-2 \times \frac{1}{4}} \delta(t - \frac{1}{2})$$

$$= 1^{-1/2} \dots 11$$

$$\star(7) \int_{-\infty}^{\infty} x(t) \cdot s(t-t_1) dt = ?$$

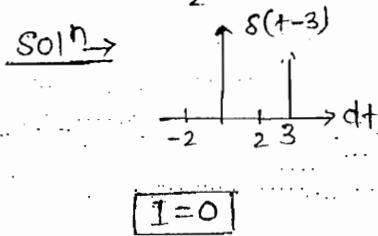
$$= \int_{-\infty}^{\infty} x(t_1) s(t-t_1) dt$$

$$= x(t_1) \int_{-\infty}^{\infty} s(t-t_1) dt$$

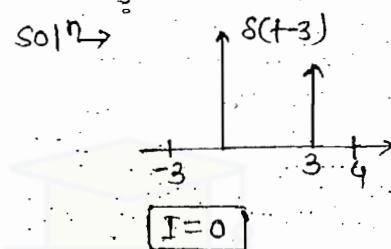
$$= x(t_1)$$

Q. → calculate the value of

$$(i) I = \int_{-2}^2 s(t-3) dt$$



$$(ii) I = \int_{-3}^4 s(t-3) dt$$



$$(iii) I = \int_{-\infty}^{\infty} [2\cos(\frac{t}{2}) + t^2] s(t-\pi) dt$$

Soln →

$$I = \int_{-\infty}^{\infty} \frac{[2\cos(\frac{t}{2}) + t^2]}{x(t)} s(t-\pi) dt$$

$$= x(t_1)$$

$$= \left[ 2\cos \frac{\pi}{2} + \pi^2 \right]$$

$$= \pi^2$$

$$\star(8) \int_{-\infty}^{\infty} x(t) \cdot \frac{d^n s(t-t_1)}{dt^n} dt = (-1)^n \frac{d^n x(t)}{dt^n} \Big|_{t=t_1}$$

Soln →

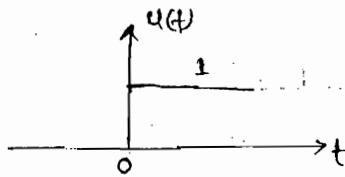
$$Eq \rightarrow \int_{-\infty}^{\infty} (t^2 + 3t) \delta'(t-2) dt$$

$$= (-1)^1 \frac{d}{dt} (t^2 + 3t) \Big|_{t=2}$$

$$= -(t+3) \Big|_{t=2}$$

$$= -(4+3) = -7$$

(2) Unit-step signal  $\rightarrow u(t)$



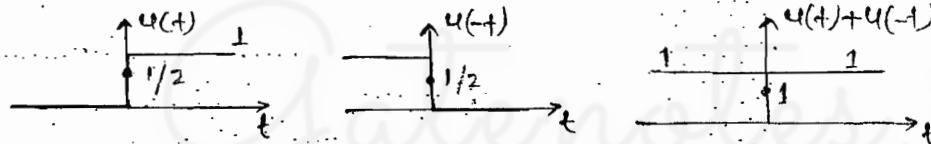
\*  $u(t)$  is discontinuous at  $t=0$ .

Gibb's phenomenon  $\rightarrow$  At the point of discontinuity signal value is given by the avg. of signal value taking just before & after the point of discontinuity.

$$\begin{aligned} u(0) &= \frac{u(0^-) + u(0^+)}{2} \\ &= \frac{0+1}{2} \\ u(0) &= \frac{1}{2} \end{aligned}$$

\* Properties  $\rightarrow$

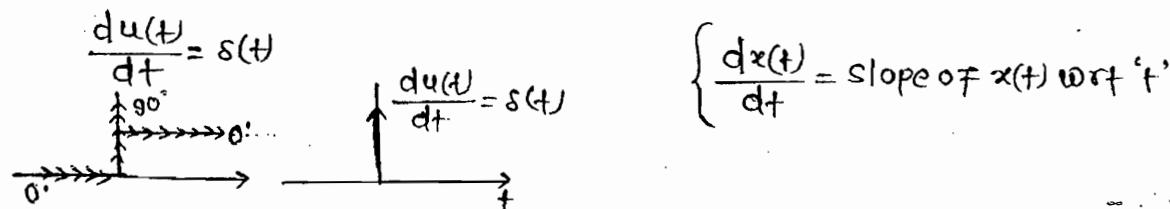
(1)  $u(t) + u(-t) = 1$



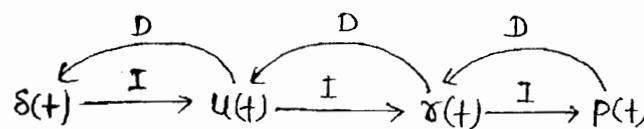
(2)  $u(t)$  is a power signal.

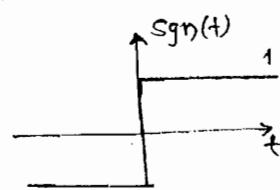
$$\text{Power} = \frac{1}{2}, \text{RMS} = \frac{1}{\sqrt{2}}, \text{avg.} = \frac{1}{2}$$

(3) Derivative of  $u(t)$



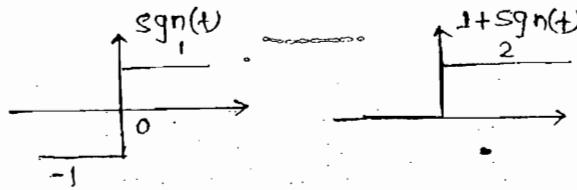
\* And  $\int_{-\infty}^{+\infty} \delta(t) dt = u(t)$



(3.) Signum function  $\rightarrow$ 

\* This is a power signal.

$$P=1, \text{ RMS}=1, \text{ Avg}=0$$

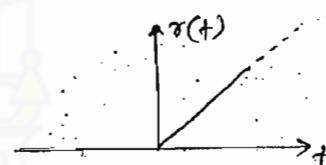


$$1+sgn(t)=2u(t)$$

$$u(t) = \frac{1+sgn(t)}{2}$$

(4.) Ramp Signal  $\rightarrow r(t)$ 

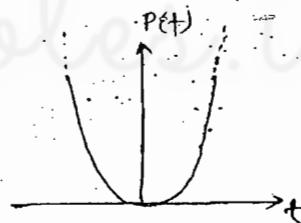
$$r(t) = \int_{-\infty}^t u(\tau) d\tau = t u(t)$$



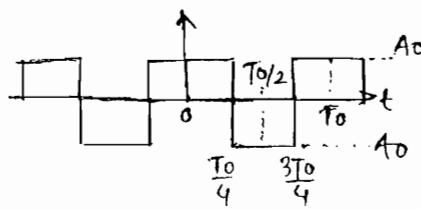
\* This is NENP sig.

(5.) Parabolic signal  $\rightarrow$ 

$$\begin{aligned} P(t) &= \int_{-\infty}^t r(\tau) d\tau \\ &= \int_{-\infty}^t t u(\tau) d\tau \\ &= \frac{t^2}{2} u(t) \end{aligned}$$



\* This is NENP signal.

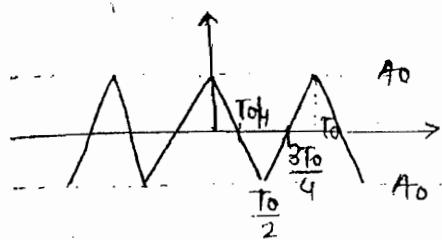
(6.) Square signal  $\rightarrow$ 

$$P = A_0^2$$

$$\text{RMS} = A_0$$

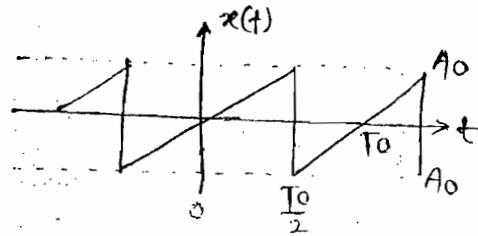
$$\text{Avg} = 0$$

### (7.) Triangular wave $\rightarrow$



$$\begin{aligned} P &= A_0^2/3 \\ \text{RMS} &= A_0/\sqrt{3} \\ \text{Avg.} &= 0 \\ \text{HWS} &= \text{Yes} \end{aligned}$$

### (8.) Sawtooth wave $\rightarrow$



$$\begin{aligned} P &= A_0^2/3 \\ \text{RMS} &= A_0/\sqrt{3} \\ \text{Avg.} &= 0 \\ \text{HWS} &= \text{No.} \end{aligned}$$

### (9.) Sampling Signal $\rightarrow$

$$sq(t) = \frac{\sin t}{t}$$

\*  $sq(0) = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$

\*  $sq(\infty) = \frac{\sin \infty}{\infty} = \frac{(-1, 1)}{\infty} = 0$

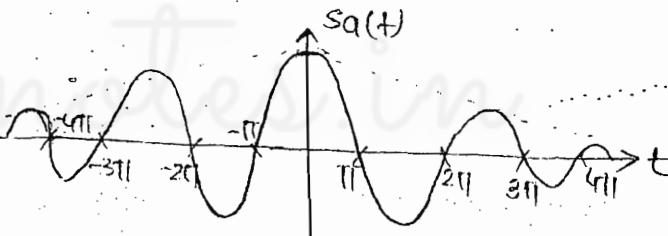
\* If  $sq(t) = 0$ , then  $\frac{\sin t}{t} = 0$

so  $\sin t = 0$ ,  $t = n\pi, n \neq 0$

\* This is a energy signal.

$$\begin{aligned} E = x(t) &= \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt \\ &= \int_{-\infty}^{\infty} \left( \frac{1 - \cos 2t}{2t^2} \right) dt \\ &= \frac{1}{2} \left[ \left( \frac{1}{2t^2} \right)_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{\cos 2t}{t^2} dt \right] \end{aligned}$$

$E = \pi$



### (10.) Sinc function →

$$\text{sinc}t = \frac{\sin \pi t}{\pi t} = \text{sa}(\pi t)$$

\*  $\text{sinc}(0) = \frac{\sin \pi \cdot 0}{\pi \cdot 0} = 1$

\*  $\text{sinc}(\infty) = 0$

\* If  $\text{sinc}(t) = 0$ ,  $\frac{\sin(\pi t)}{\pi t} = 0$

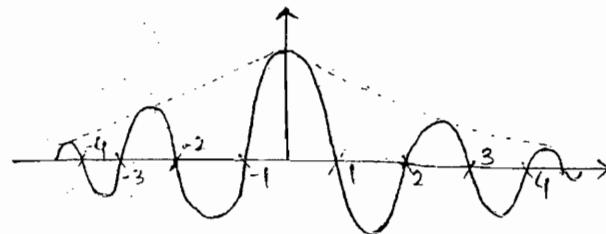
$$\sin \pi t = 0$$

$$\pi t = \eta \pi, \eta \neq 0$$

$$t = \eta, \eta \neq 0$$

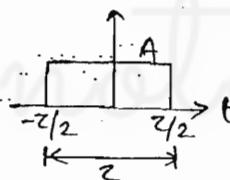
\* Energy = 1

$x(t) \rightarrow E$	$\text{sa}(t) = \frac{1}{\pi}$
$x(\eta t) \rightarrow \frac{E}{ \eta }$	$\text{sinc}(t) = \text{sa}(\pi t) = \frac{1}{\pi}$

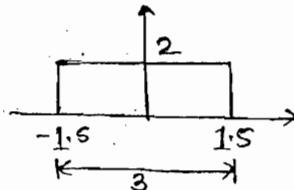


### (11.) Rect function →

$$x(t) = A \text{rect}\left(\frac{t}{2}\right)$$

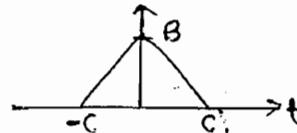


$$x(t) = 2 \text{rect}\left(\frac{t}{3}\right)$$

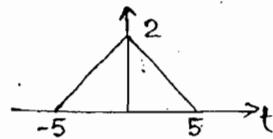


### (12.) Tri-function →

$$x(t) = B \text{tri}\left(\frac{t}{C}\right)$$



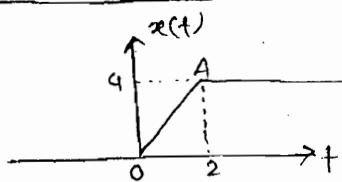
$$x(t) = 2 \text{tri}\left(\frac{t}{5}\right)$$



DATE-14/10/14

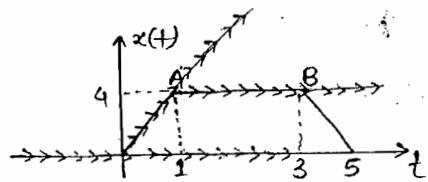
Mathematical representation of waveform →

(1.)



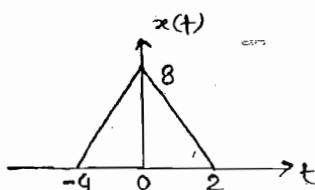
$$\star x(t) = 0 + 2\tau(t-0) - 2\tau(t-2)$$

(2.)



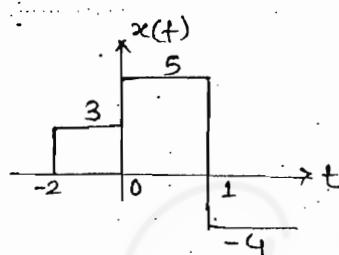
$$\star x(t) = 0 + 4\tau(t-0) + -4\tau(t-1) - 2\tau(t-3) + 2\tau(t-5)$$

(3.)



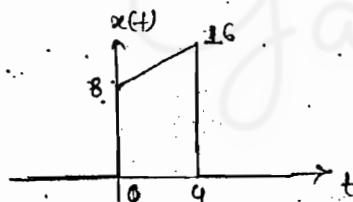
$$\star x(t) = 0 + (+2)\tau(t+4) - 4\tau(t+0) + 4\tau(t-2) - 2\tau(t-0)$$

(4.)



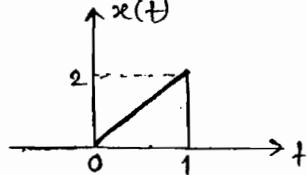
$$\star x(t) = 0 + 3u(t+2) + 2u(t+0) - 9u(t+1)$$

(5.)



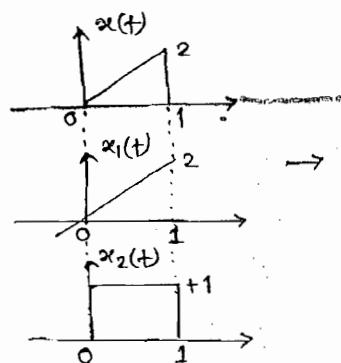
$$\begin{aligned} \star x(t) &= 0 + 8u(t+0) + 2\tau(t+0) - 2\tau(t+4) + -16u(t+4) \\ &= 8u(t) + 2\tau(t) - 2\tau(t+4) - 16u(t+4) \end{aligned}$$

(6.)



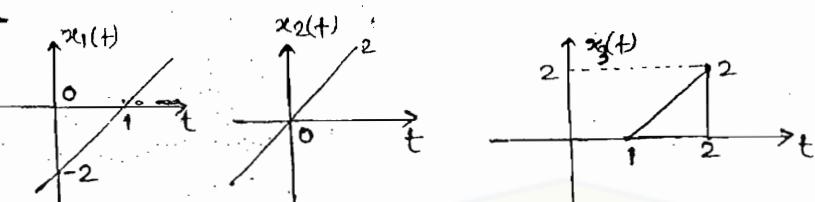
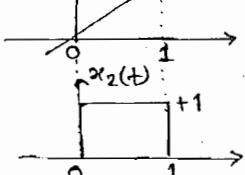
$$\begin{aligned} \star x(t) &= 2\tau(t) - 2\tau(t-1) - 2u(t-1) \\ &= 2+u(t) - 2(t-1)u(t-1) - 2(\cancel{\tau(t)})u(t-1) \\ &= 2+u(t) - 2+u(t-1) + 2u(t-1) - 2u(t-1) \\ &= 2t[u(t) - u(t-1)] \\ &= 2t[u(t) - u(t-1)] \end{aligned}$$

2nd method →



$$\begin{aligned} \therefore x(t) &= x_1(t) + x_2(t) \\ &= 2t[4(t) - 4(t-1)] \end{aligned}$$

$$\rightarrow 2t[4(t) - 4(t-1)]$$



Ans. →

$$x_2(t) = 2t$$

$$x_1(t) = 2(t-1)$$

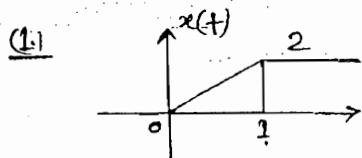
$$x_3(t) = 2(t-1)[4(t-1) - 4(t-2)]$$

Chapter-02:  
Different Operations of  
signal

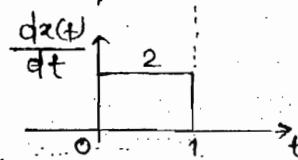
(1) Differentiation →

$$x(t) = \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ w.r.t. } t$$

\* Graphical diff is applicable for triangular & rectangular type signal.

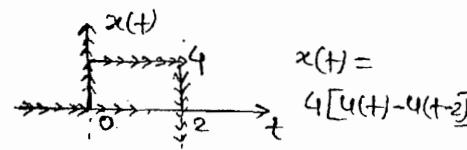


$$x(t) = 2x(t) - 2x(t-1)$$

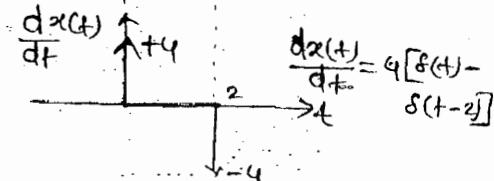


$$\frac{dx(t)}{dt} = 2u(t) - 2u(t-1)$$

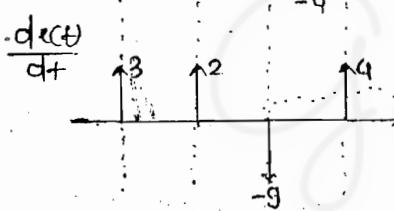
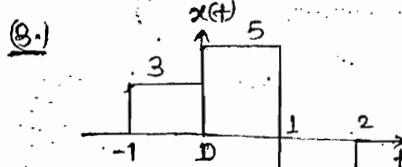
(2)



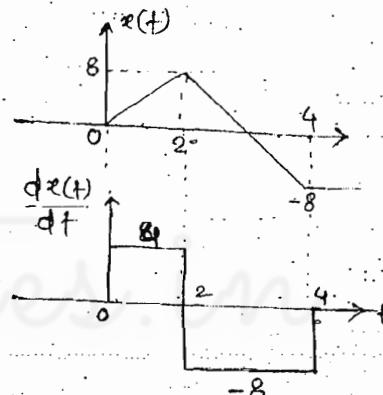
$$x(t) = 4[u(t) - u(t-2)]$$



$$\frac{dx(t)}{dt} = 4[\delta(t) - \delta(t-2)]$$



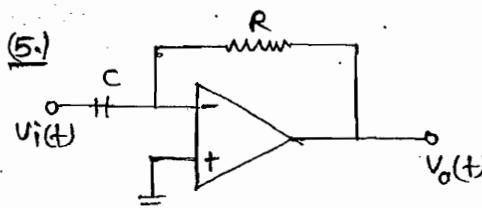
(4.)



$$\frac{dx(t)}{dt}$$

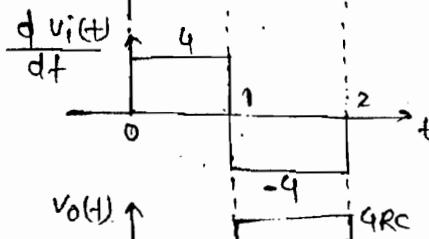
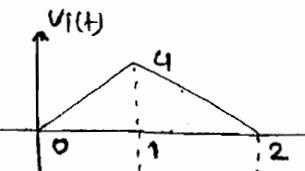
8

-8



$$V_o(t) = -RC \frac{dV_i(t)}{dt}$$

amp reversal



$$V_o(t) \uparrow$$

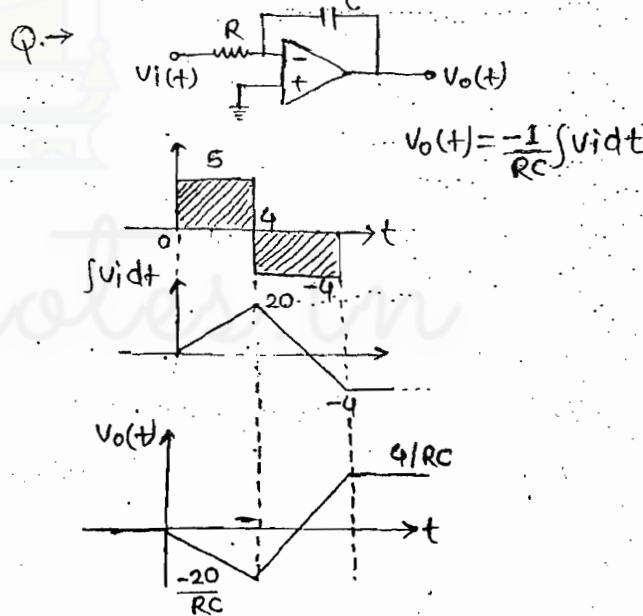
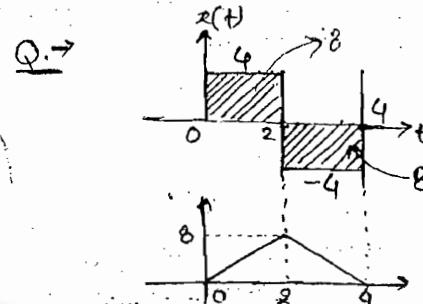
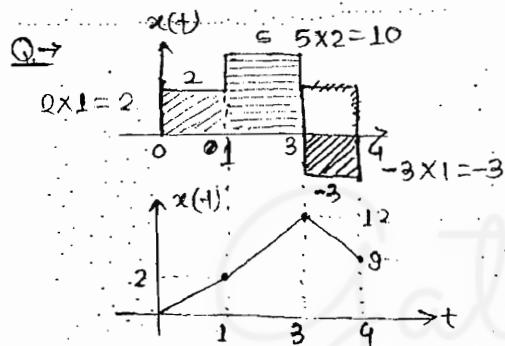
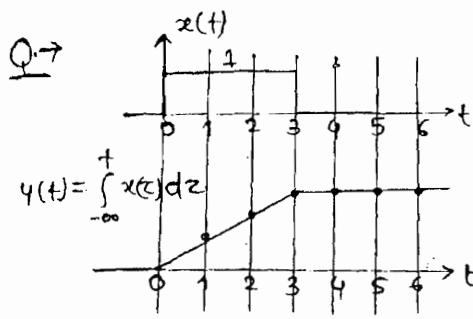
$$4RC$$

(2.) Integration →

$$x(t) = y(t) = \int_{-\infty}^t x(z) dz$$

= area of signal  $x(t)$  w.r.t 't'

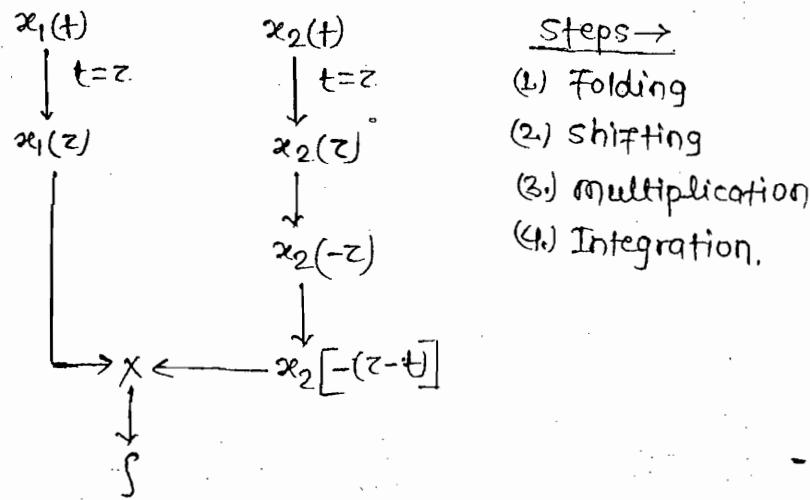
\* Graphical integration is applicable only for rectangular type waveform.



(3.) Convolution → It is a mathematical operator & it is used for calculation of response of LTI system.

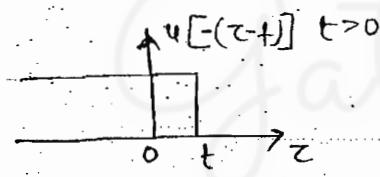
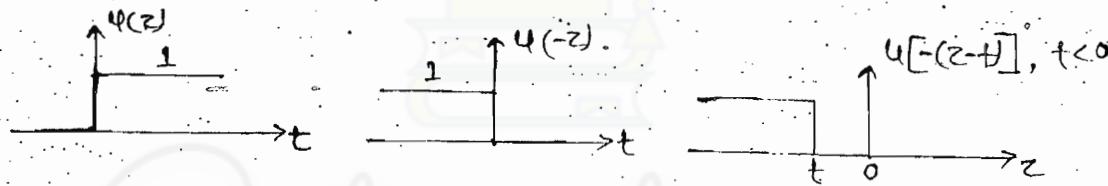
$$\begin{aligned} y(t) &= x_1(t) * x_2(t) \\ &= \int_{-\infty}^{\infty} x_1(z) \cdot x_2(t-z) dz \end{aligned}$$

$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(z) \cdot x_2(t-z) dz$



$$\text{Q.} \rightarrow y(t) = u(t) * u(t)$$

$$\text{SOLN} \rightarrow = \int_{-\infty}^{\infty} u(z) \cdot u(t-z) dz$$



$$y(t) = \int_{-\infty}^{\infty} u(z) \cdot u(t-z) dz$$

$$= \begin{cases} 0 & ; t < 0 \\ \int_0^t dt & ; t > 0 \end{cases}$$

$$= \begin{cases} 0 & , t < 0 \\ t & , t > 0 \end{cases}$$

$$= \tau(t)$$

2nd method →

$$Y(t) = x_1(t) * x_2(t)$$

$$Y(s) = X_1(s) \cdot X_2(s)$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

$$\boxed{Y(t) = \tau(t)}$$

### \* Properties of Convolution →

#### (1.) Commutative →

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

$$\int_{-\infty}^{\infty} x_1(z) \cdot x_2(t-z) dz = \int_{-\infty}^{\infty} x_2(z) x_1(t-z) dz$$

#### (2.) Associative →

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

#### (3.) Distributive →

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

#### (4.) Impulse Response →

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

$$\downarrow t_1=0$$

$$x(t) * \delta(t) = x(t)$$

$$\text{eg:- } (1) u(t-1) * \delta(t+2) = u[(t+2)-1] = u(t+1)$$

#### (5.) Derivative →

$$y(t) = x_1(t) * x_2(t)$$

$$\frac{dy(t)}{dt} = \frac{d}{dt} x_1(t) * x_2(t) = x_1(t) * \frac{dx_2(t)}{dt}$$

eg:- find  $y(t) = ?$

$$y(t) = \frac{d}{dt} [x(t) * u(t)]$$

Soln →

$$y(t) = \frac{d}{dt} r(t) * u(t)$$

$$= u(t) * u(t)$$

$$= \delta(t)$$

$$(OR) y(t) = r(t) * \frac{d u(t)}{dt}$$

$$= r(t) * \delta(t)$$

$$= \delta(t)$$

#### (6.) Step Response →

$$y(t) = x(t) * u(t) = ?$$

$$y(t) = \int_{-\infty}^t \frac{d y(t)}{dt} dt = \int_{-\infty}^t [x(t) * \frac{du(t)}{dt}] dt$$

$$\text{Eq: } (1) \quad u(t) * u(t) = \int_{-\infty}^t u(t) dt = r(t)$$

$$(2) \quad r(t) * u(t) = \int_{-\infty}^t r(t) dt = p(t)$$

### (7.) Time scaling →

If  $x_1(t) * x_2(t) = y(t)$  then;

$$x_1(at) * x_2(at) = \frac{1}{|a|} y(at) \quad (a \neq 0)$$

### (8.) Area →

If  $x_1(t) * x_2(t) = y(t)$  then;

$$\text{Area } y(t) = \text{Area } x_1(t) \times \text{Area } x_2(t)$$

### (9.) Time delay →

$$x_1(t) * x_2(t) = y(t)$$

$$x_1[t-t_1] * x_2[t-t_2] = y[t-(t_1+t_2)]$$

$$\text{Eq: } (1) \quad u(t-1) * u(t-2) = r(t-3)$$

$$= (t-3) u(t-3)$$

$$(2) \quad r(t-1) * u(t+3) = p(t+2)$$

$$= \frac{(t+2)^2}{2} u(t+2)$$

### (10.) Duration →

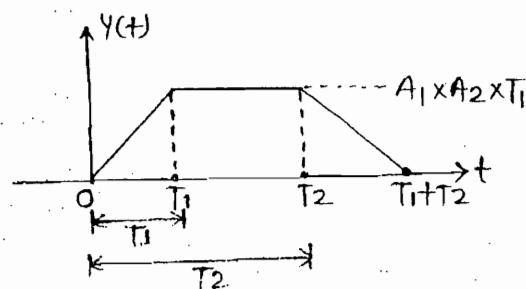
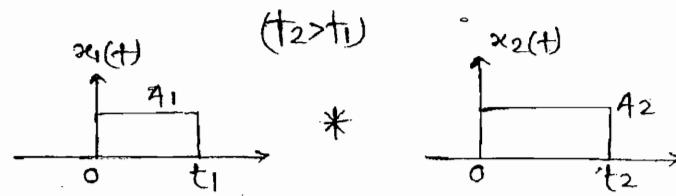
$$y(t) = x_1(t) * x_2(t)$$

signal	duration
$x_1(t)$	$t_1 \leq t \leq t_2$
$x_2(t)$	$t_3 \leq t \leq t_4$
$y(t)$	$t_1+t_3 \leq t \leq t_2+t_4$

\* Convolution of 2 rectangular pulses of equal duration will be a triangle.

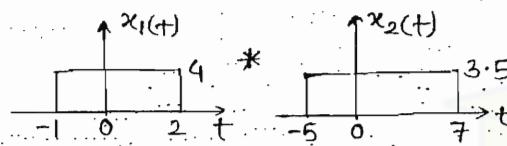
\* Convolution of 2 rectangular pulses of unequal duration will be a trapezoid.

Trapezoid  $\rightarrow$

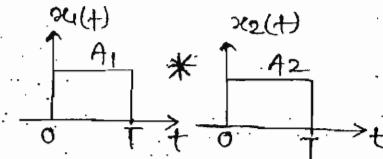


$T_1$  = smaller duration  
 $T_2$  = larger duration

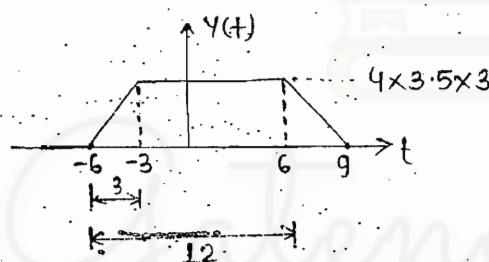
Que.  $\rightarrow$



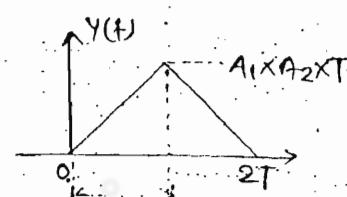
Que.  $\rightarrow$



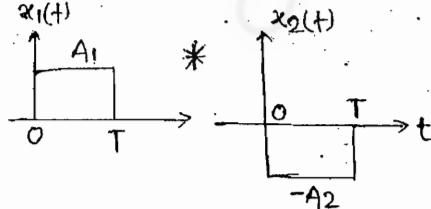
Soln  $\rightarrow$



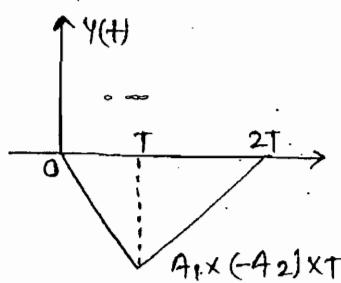
Soln  $\rightarrow$



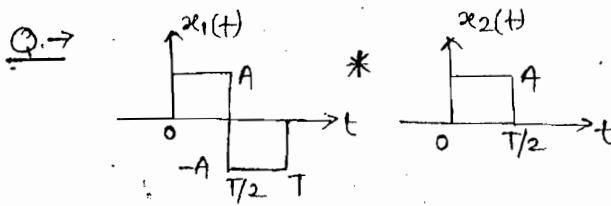
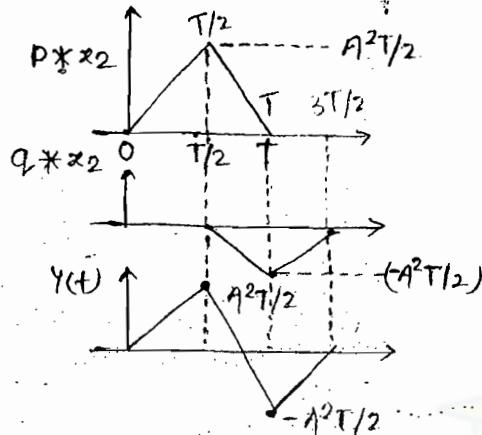
Que.  $\rightarrow$



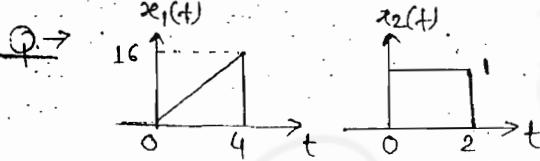
Soln  $\rightarrow$



Because  $(-A_2) \Delta$  will be -ve.

SOL

$$\begin{aligned}
 y(t) &= x_1(t) * x_2(t) \\
 &= [p(t) + q(t)] * x_2(t) \\
 &= [p(t) * x_2(t)] + [q(t) * x_2(t)]
 \end{aligned}$$



$$\begin{aligned}
 y(t) &= x_1(t) * x_2(t) \\
 \text{find value of } y(2) &
 \end{aligned}$$

(a) 4 (b) 8 (c) 16 (d) 32

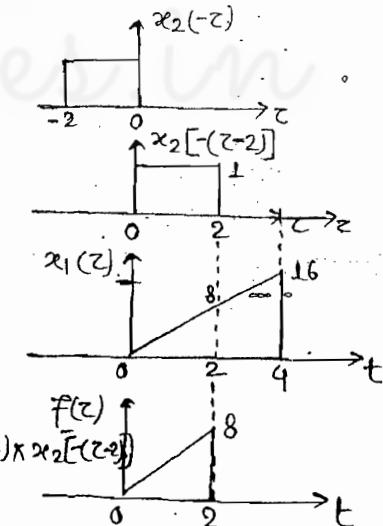
SOL

$$\begin{aligned}
 y(t) &= x_1(t) * x_2(t) \\
 &= \int_{-\infty}^{\infty} x_1(z) x_2(t-z) dz \\
 y(2) &= \int_{-\infty}^{\infty} x_1(z) x_2(2-z) dz \\
 &= \int_{-\infty}^{\infty} f(z) dz = \text{req of } f(z)
 \end{aligned}$$

where  $f(z) = x_1(z) x_2(2-z)$

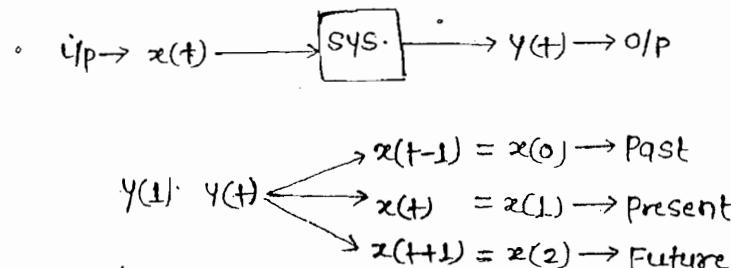
$\text{Area} = \frac{1}{2} \times 2 \times 8 = 8$

$\boxed{\text{Area} = 8}$



DATE-15/10/14

**Chapter-03**  
**Basic system properties**



(1) Static & dynamic sys. →

Static → If O/P of sys. depends only on present values of I/P at each & every instant of time then sys. will be static.

\* These sys. are also known as memoryless system.

Dynamic → \* If O/P of sys. depends on past (or) future values of I/P at any instant of time then sys. will be dynamic.

\* This sys. are also known as sys. with memory.

Q. → Check static/dynamic sys.

$$(1) \quad y(t) = x(t) + x(t-1)$$

$$(5) \quad y(t) = \text{Even}[x(t)]$$

$$(2) \quad y(t) = x(-t)$$

$$(6) \quad y(t) = \text{Real}[x(t)]$$

$$(3) \quad y(t) = x(\sin t)$$

$$(7) \quad y(t) = \int_{-\infty}^{+\infty} x(z) dz$$

$$(4) \quad y(t) = x(t-1)$$

$$(8) \quad y(t) = e^{-(t+1)} x(t)$$

Ans. → (1) Dynamic.

(2) Dynamic.

$$(3) \quad y(t) = x(\sin t)$$

$$y(-\pi) = x(0)$$

$\Rightarrow$  future system is dynamic.

(4) Dynamic

$$(5) \quad g(t) = \frac{x(t) + x(-t)}{2}$$

$$(4=1) \quad y(t) = \frac{x(1) + x(-1)}{2}$$

past

system is dynamic.

$$(6) \quad y(t) = \frac{x(t) + x(t)}{2}$$

\*  $\Rightarrow$  system is static

(7) ....  $\Rightarrow$  system is dynamic

Note →

- (1.) Integral & derivative sys. are dynamic sys.
- (2.) In case of time scaling (or) time shifting system will be dynamic.

(2.) Causal & Non-Causal System →

\* Causal → \* If o/p of sys. is independent of future value of i/p at each & every instant of time then sys. will be causal.

\* This sys. are practical (or) physically reliable sys.

Eg:- (1.)  $y(t) = x(t)$

(2.)  $y(t) = x(t-1)$

(3.)  $y(t) = x(t) + x(t-1)$

\* Non-Causal system → \* If o/p of sys. depends on future values of i/p at any instant of time then sys. will be non-causal.

Eg:- (1.)  $y(t) = x(t+1)$

(2.)  $y(t) = x(t) + x(t+1)$

(3.)  $y(t) = x(t+1) + x(t+2)$

(4.)  $y(t) = x(t) + x(t-1) + x(t+1)$

\* Anti Causal System → \* If o/p of sys. depends only on future values of i/p then sys. will be anticausal.

Eg:-  $y(t) = x(t+1)$

\* All anti-causal systems are non-causal but converse of this statement is not true.

Que. → Check Causal & Non-Causal system.

(1.)  $y(t) = x(2t)$

(7.)  $y(t) = \int_{-\infty}^t x(z) dz$

(2.)  $y(t) = x(-t)$

(8.)  $y(t) = \int_{-\infty}^{t+1} x(z) dz$

(3.)  $y(t) = x(\sin t)$

(9.)  $y(t) = \int_{-\infty}^{2t} x(z) dz$

(4.)  $y(t) = \begin{cases} x(2t); t < 0 \\ x(t-1); t \geq 0 \end{cases}$

(5.)  $y(t) = \text{odd}[x(t)]$

(6.)  $y(t) = \sin(t+2) \cdot x(t-1)$

Soln (i)  $y(t) = x(2t)$   
 $(t=1) \downarrow$   
 $y(t) = x(2)$  (System is non-causal)

(ii)  $y(t) = x(-t)$   
 $(t=-1)$   
 $y(-1) = x(1)$  (System is Non-causal)

(iii)  $y(t) = x(\sin t)$   
 $(t=-\pi)$   
 $y(-\pi) = x(0)$   
 $-3.14 = x(0)$  (System is non-causal)

(iv)  $y(t) = \begin{cases} x(2t), t < 0 \rightarrow \text{past} \\ x(t-1), t \geq 0 \rightarrow \text{past} \end{cases}$   
 (System is causal)

(v.)  $y(t) = \text{odd } x(t)$   
 $= \frac{x(t) - x(-t)}{2}$   
 $(t=-1)$  future  
 $y(-1) = \frac{x(-1) - x(1)}{2}$  (System is non-causal)

(vi)  $y(t) = \sin(t+2) \cdot x(t-1)$   
 (Coefficient)  $\downarrow$  past  
 (System is causal.)

(vii)  $y(t) = \int_{-\infty}^t x(z) dz \rightarrow x(t)$   
 $= \int_{-\infty}^t x(z) dz$  (System is causal)

(viii)  $y(t) = \int_{-\infty}^{(t+1)} x(z) dz \rightarrow x(t+1)$   
 (System is non-causal)

(ix)  $y(t) = \int_{-\infty}^{2t} x(z) dz \rightarrow x(2t)$   
 (System is non-causal)

### (3.) Linear & Non-linear system →

linear → \* A linear sys. follows the law of superposition.

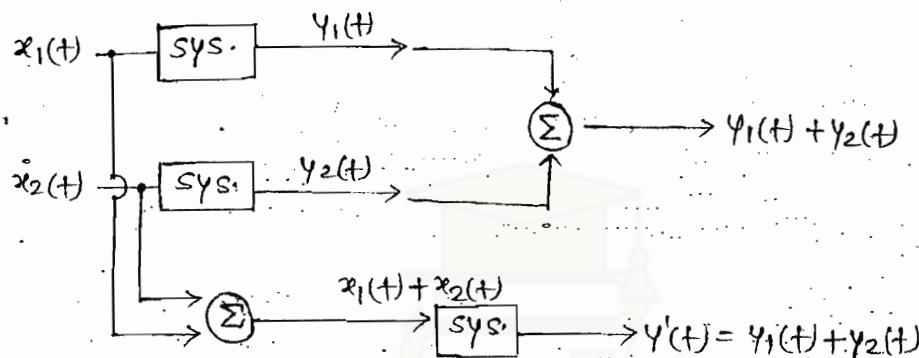
\* This law is necessary & sufficient to prove linearity of system.

\* It is a combination of two laws:-

(i) Law of additivity.

(ii) Law of Homogeneity.

#### (1.) Law of additivity →



$$\text{Eq:- } y(t) = x(t) + 10$$

$$\text{O/p} = \text{I/p} + 10$$

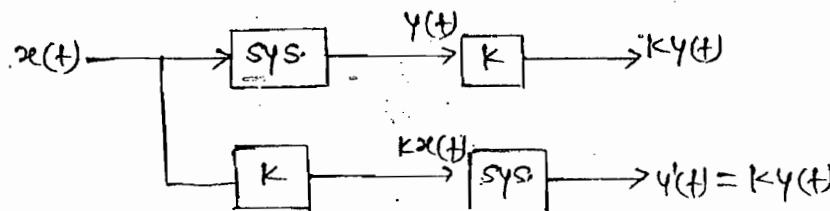
$$y(t) = y_1(t) + 10$$

$$y_2(t) = x_2(t) + 10$$

$$y'(t) = x_1(t) + x_2(t) + 20$$

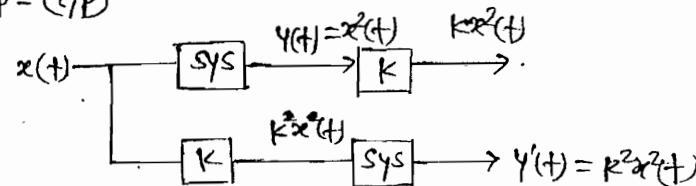
$$y(t) \neq y'(t) \quad [\text{sys. is N}]$$

#### (2.) Law of Homogeneity →



$$\text{Eq:- } y(t) = x^2(t)$$

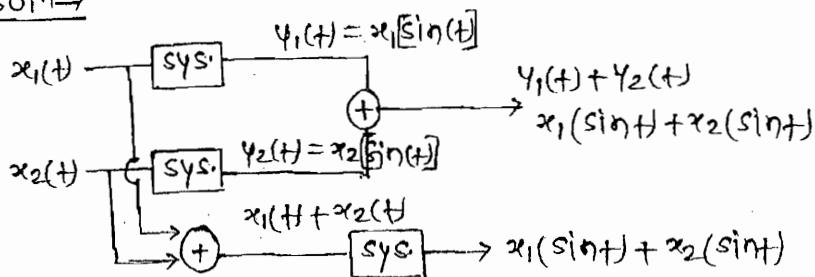
$$\text{O/p} = (\text{I/p})^2$$



Que. → Check linear/Non-linear sys.

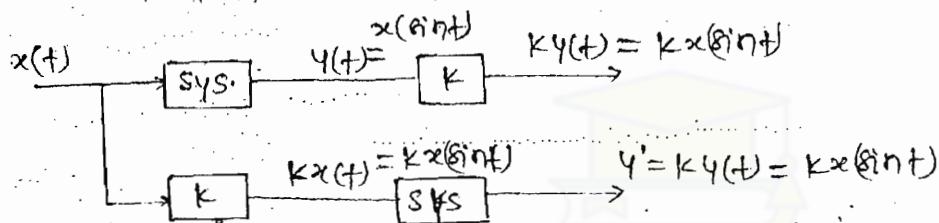
$$(1) y(t) = x(\sin t) \quad (2) y(t) = x(t \sin t) \quad (3) y(t) = x(t^2)$$

SOL →



Note →

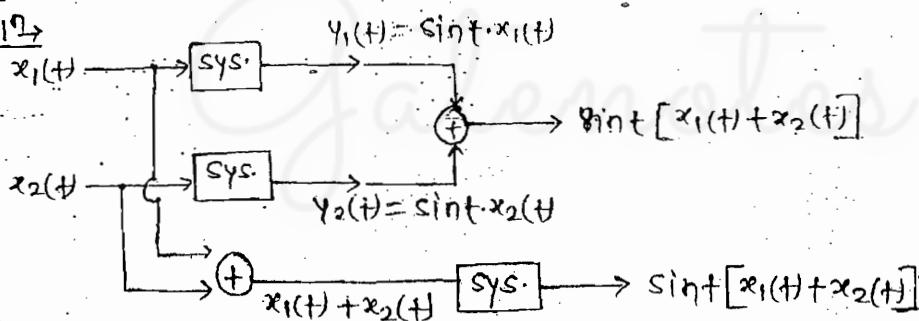
Linearity of sys. is independant of time scaling.



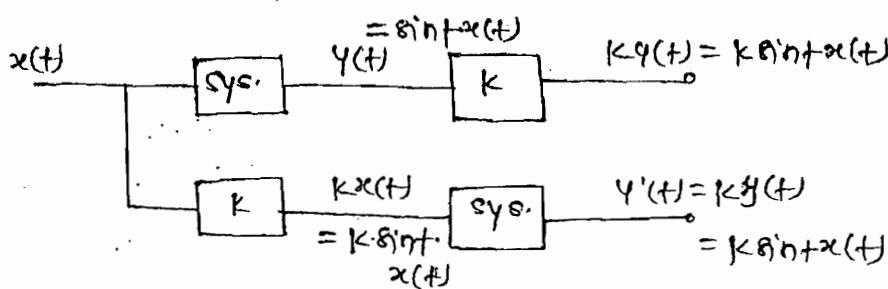
$$(2) y(t) = \sin t \cdot x(t)$$

$$(3) y(t) = \log t \cdot x(t)$$

SOL →



Note:— linearity of sys. is independant of coefficient used in sys. relationship.



## 2nd method →

for linearity :-

- (i) O/p should be 0 for 0 i/p.
- (ii) There should be any 'NL' operation.

e.g:-  $\sin, \cos, \tan, \sec, \csc, \cot, \dots$   
 $\log, \text{exponential}, \text{modulus}, \text{sq}, \text{cube}, \dots$   
 $\dots \text{root}, \dots \text{sq}(), \text{sin}(), \dots \text{sgn}() \text{ etc}$   
either on 'x' or 'y'.

Q → Check linear/NL sys.

- (i)  $y(t) = x(t) + 2 \rightarrow$  put  $t=0$  then  $y(0) \neq x(0) + 2 \rightarrow \text{NL}$
- (ii)  $y(t) = e^{x(t)} \rightarrow$  Because of  $e^{x(t)}$  it is NL & also both condn not satisfying.

- (iii)  $y(t) = x(\tan t) \rightarrow \text{Linear}$

If  $t=0$ , then  $y(0) = x(0)$  means NO i/p no o/p

(ii) above fn is not operating on 'x'; it is operating on the 't'.

- (iv)  $y(t) = \tan[x(t)]$

System is NL

- (v)  $y(t) = x(t-1) + x(t+1)$

$$\text{i/p} \rightarrow \boxed{\text{sys.}} \rightarrow \text{o/p} = \underset{^0}{\text{past i/p}} + \underset{^0}{\text{future i/p}}$$

No any NL fn so this is linear

- (vi)  $y(t) = \text{even}[x(t)]$

$$y(t) = \frac{x(t) + x(-t)}{2}$$

No non-linear operator so linear

- (vii)  $y(t) = \int_{-\infty}^t x(z) dz$

$$y(t) = \int_{-\infty}^t x(z) dz \quad \text{(linear)}$$

- (viii)  $y(t) = \begin{cases} x(t-1), & t < 0 \\ x(t+1), & t \geq 0 \end{cases} = \begin{cases} \underset{^0}{\text{past i/p}}, & t < 0 \\ \underset{^0}{\text{future i/p}}, & t \geq 0 \end{cases} \text{ (linear)}$

Note →

(1) Integral & derivative operators are linear.

(2) Even & odd operators are linear.

$$(ix) \quad y(t) = \int_{-\infty}^t x^2(z) dz$$

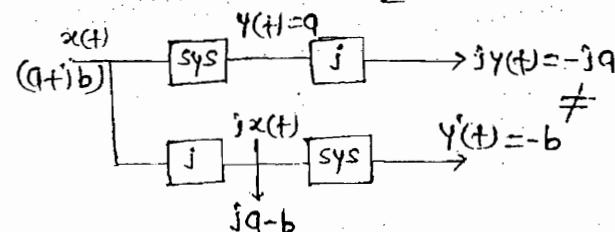
$$y(t) = \int_{-\infty}^t x^2(z) dz \rightarrow \text{NL}$$

$$(x) \quad y(t) = e^t x(t)$$

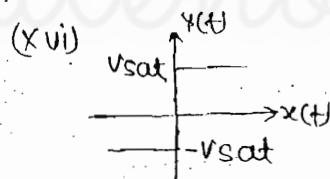
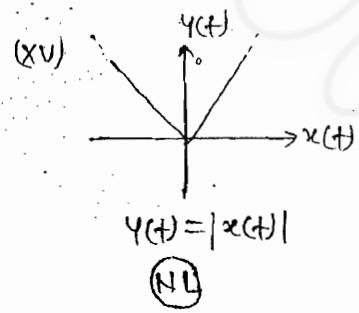
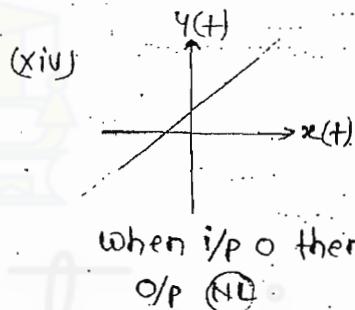
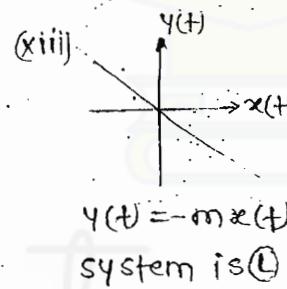
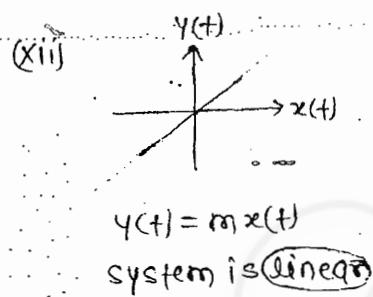
$$y(t) = e^t x(t) \rightarrow \text{Linear}$$

$$(xi) \quad y(t) = \operatorname{Re} q[x(t)]$$

$$y(t) = \frac{x(t) + x^*(t)}{2} \rightarrow \text{NL}$$

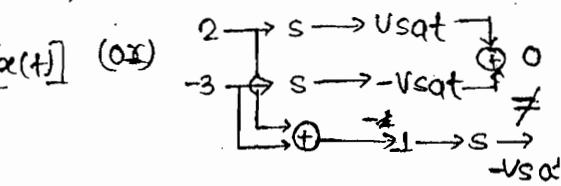


Note → Real & imaginary operators are NL.



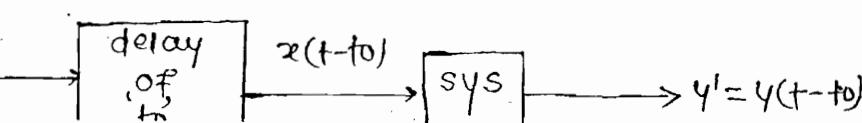
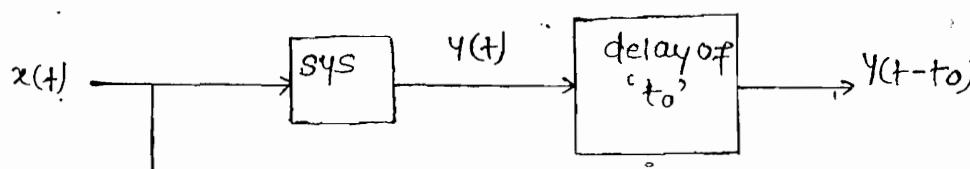
$$y(t) = vsat \operatorname{sgn}[x(t)] \quad (\text{ox})$$

NL



(4) Time invariant & time variant sys. →

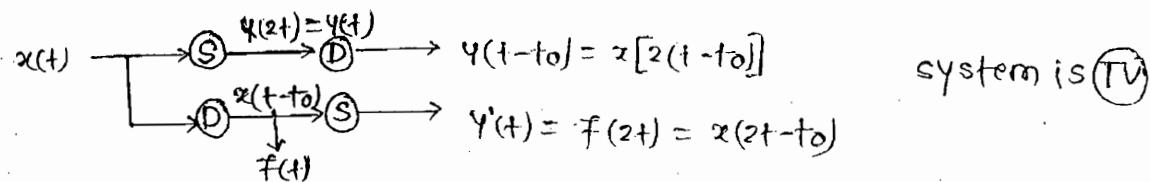
Time invariant →



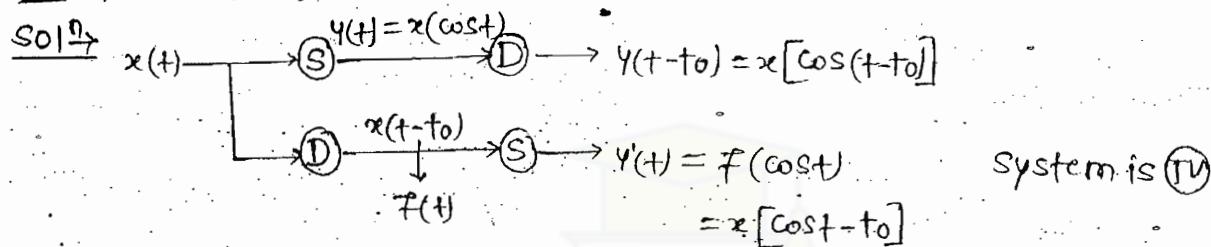
Note:- Any delay provided in i/p must be reflected in o/p for a time invariant system.

Ques. → Check time invariant / variant sys.

$$(1) \quad Y(t) = x(2t)$$



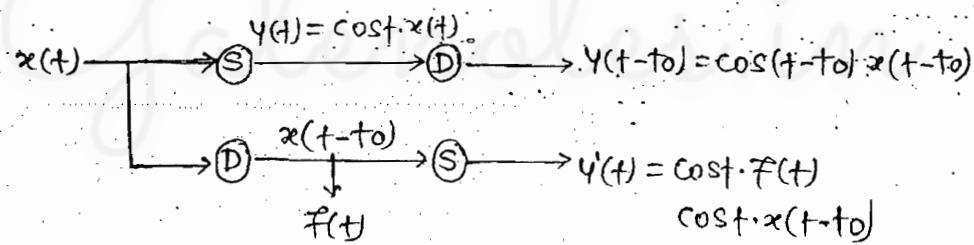
$$(2) \quad Y(t) = x(\cos t)$$



Note:- In case of time scaling sys. will be time variant.

$$(3) \quad Y(t) = \text{cost} \cdot x(t)$$

Soln →



system is (TV)

Note:- If coefficient in sys. relationship is f<sup>th</sup> of time then sys. will be time variant.

$$(5) \quad Y(t) = \text{odd}[x(t)]$$

Soln → 
$$Y(t) = \frac{x(t) - x(-t)}{2}$$

↓  
Time scaling  
(TV)

$$(6) \quad Y(t) = CS[x(t)]$$

$$Y(t) = \frac{x(t) + x^*(t)}{2}$$

↓  
Time scaling  
(TV)

$$(vii) \quad y(t) = x(t-1) + x(t+1)$$

$\text{Soln} \rightarrow y(t) = \cancel{x(t-1)} + \cancel{x(t+1)}$

~~No scaling~~

coefficient are independent  
of time so TIV

$$(viii) \quad y(t) = \int_{-\infty}^{3t} x(z) dz$$

$\text{Soln} \rightarrow y(t) = \int_{-\infty}^{3t} x(z) dz \xrightarrow{\text{2p}} x(3t) \quad (\text{TIV})$

$$(ix) \quad y(t) = \int_{-\infty}^t x(z) dz$$

$\text{Soln} \rightarrow y(t) = \int_{-\infty}^t x(z) dz \xrightarrow{\text{2p}} x(t) \quad (\text{TIV})$

TIV

$$(x) \quad y(t) = \int_{-\infty}^t \cos z \cdot x(z) dz$$

$\text{Soln} \rightarrow y(t) = \int_{-\infty}^t \cos z \cdot x(z) dz \xrightarrow{\text{2p}} \cos t x(t) \quad (\text{TIV})$

$$(xi) \quad y(t) = \begin{cases} x(t-1) & ; t \leq 0 \\ x(t+1) & ; t \geq 0 \end{cases}$$

$\text{Soln} \rightarrow = a(t) \cdot x(t-1) + b(t) \cdot x(t+1)$

split systems are time variant system.

(5.) Stable/Unstable sys.  $\rightarrow$  finite/bounded in amplitude

stable  $\rightarrow$  Bounded i/p bounded o/p (BIBO) criteria.

BIBO  $\rightarrow$  For stable system, o/p should be bounded or finite  
or bounded i/p at each & every instant of time.

Eg:- Bounded i/p are u(t), dc-signal, sint, cost, sgn(t)

Que.  $\rightarrow$  Check stable/unstable sys

$$(1.) \quad y(t) = x(t) + 2$$

$x(t)$	$y(t)$
10	12

(stable)

$$(2.) \quad y(t) = t x(t)$$

$x(t)$	$y(t)$
10	$10t \leftarrow \begin{matrix} \infty = \infty \\ -\infty = \infty \end{matrix}$

(unstable)

$$(3.) \quad y(t) = \frac{x(t)}{\sin t}$$

$x(t)$	$y(t)$
2	$\frac{2}{\sin t} \quad (t=0, \pi)$

(unstable)

$$(4) \cdot y(t) = \sin t \cdot x(t)$$

Soln

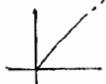
$$y(t) = \sin t \cdot x(t)$$

$$\begin{array}{c|c} x(t) & y(t) \\ \hline -2 & (-2, 2) \\ 2 & (1, 1) \end{array}$$

$\downarrow$   
 $-x(t), x(t)$   
 (stable)

$$(6) \quad y(t) = \int_{-\infty}^{+t} x(z) dz$$

Soln  $y(t) = u(t) \rightarrow$  bounded

$$y(t) = x(t) \rightarrow tu(t)$$


(Unstable)

$$(8) \quad \int_{-\infty}^{+t} \cos z \cdot x(z) dz = y(t)$$

$$\text{Soln} \quad y(t) = \int_{-\infty}^{+t} \cos z \cdot x(z) dz$$

$x(t) \rightarrow \cos t = \text{bounded sig.}$

$$y(t) = \int_{-\infty}^{+t} \cos z \cdot \cos zdz$$

$$= \int_{-\infty}^{+t} \cos^2 z dz$$

$$= \int_{-\infty}^{+t} \frac{1 + \cos 2z}{2} dz$$

$$= \frac{1}{2} \left[ \underset{\infty}{\underbrace{(z)}} + \underset{\infty}{\underbrace{(\sin z)}} \right]$$

finite

$\downarrow$   
 (Unbounded)

(Unstable)

$$(5) \quad y(t) = \sin[x(t)]$$

$$\text{Soln} \quad y(t) = \sin[x(t)]$$

$$\downarrow$$

$(-1, 1)$   
 (stable)

$$(7) \quad y(t) = \frac{dx(t)}{dt}$$

$$\text{Soln} \quad x(t) = u(t) \rightarrow \text{bounded}$$

$y(t) = \delta(t) \rightarrow \text{Unbounded}$   
 (Unstable).

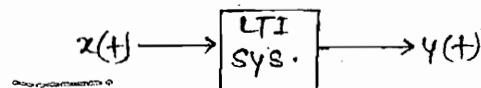
Note:- so the integration & differentiation signals are unstable sys.

Static / Dynamic - D  $\rightarrow$  I, D, TS & TS

linear / NL - L  $\rightarrow$  I, D, E, O

NL  $\rightarrow$  R & I

### \* Linear time invariant (LTI) system →



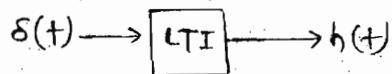
$h(t) \rightarrow$  Impulse Response of sys.

$H(\omega)$  (or)  $H(s) \rightarrow$  TF of sys.

\* Impulse Response & TF terms are used only for LTI system.

\* Impulse Response is used for defining LTI sys. in time domain & TF is used for defining LTI sys. in freq. domain.

### Impulse Response →



\* If i/p to LTI sys. is unit impulse then o/p of sys. is known as impulse Response.

### Transfer function →

\* It is the ratio of Laplace Xform of o/p to Laplace Xform of i/p when all initial condn are assumed to be 0.

$$H(s) = \frac{Y(s)}{X(s)} \quad | \text{zero initial condn}$$

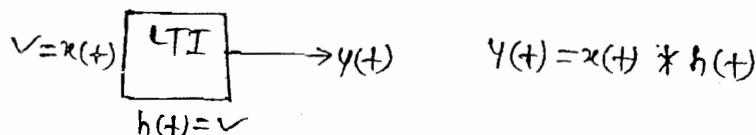
Total o/p = Zero i/p response + zero state response

$$\text{Total o/p} = \overset{\curvearrowleft}{ZIR} + \overset{\curvearrowright}{ZSR}$$

due to  $\begin{matrix} \curvearrowleft \\ \text{initial condn} \end{matrix}$       due to applied  
 $\begin{matrix} \curvearrowright \\ \text{i/p} \end{matrix}$   
states

\* For linearity of sys., initial condn are assumed to be zero, because non-zero initial condn make the sys. non-linear.

### Convolution →



$$y(t) = x(t) * h(t)$$

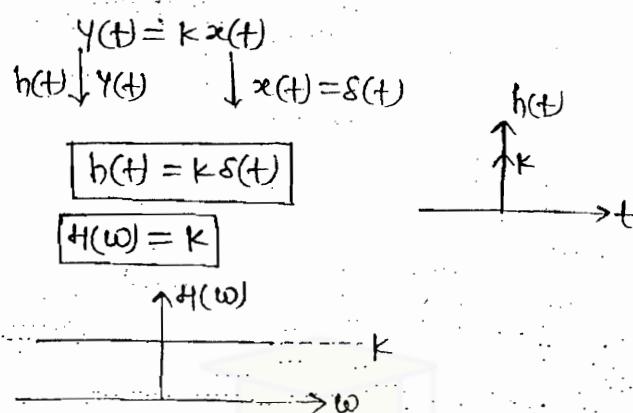
\* Convolution is a linear time invariant operator & it is used only for LTI system.

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^t h(z) \cdot x(t-z) dz$$

\* The above relation is both linear & TIV. So it is LTI system.

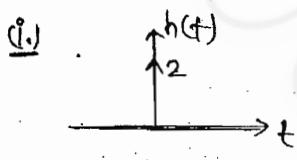
Condition for LTI system to be static  $\rightarrow$



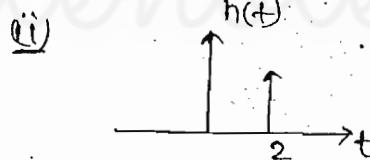
\* Impulse  $\delta(t)$  is the fn whose all xform is one.

\* For static LTI system, impulse response should be impulse at origin & TF should be independant of freq.

Q: Check S/D LTI sys:



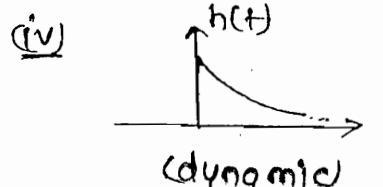
(static)



Impulse is not at origin so dynamic.

(iii)  $h(t) = u(t)$

(Dynamic)



(dynamic)

(v)  $H(s) = 2$

Static (free of freq.)

(vi)  $H(s) = \frac{1}{s+1}$

(dynamic)

\* Filters are dynamic system because their TF depends on freq.

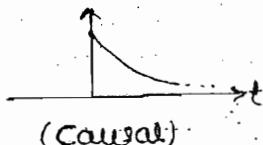
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\* Cond'n for LTI system to be causal  $\rightarrow$

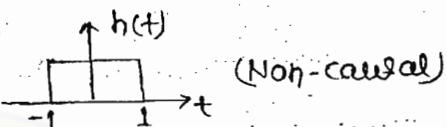
$$\begin{aligned} y(t) &= x(t) * h(t) \quad \text{future if } p \neq 0 \\ y(t) &= \int_{-\infty}^{\infty} h(z) x(t-z) dz \\ h(z) &= 0; z < 0 \\ \downarrow z=t \\ h(t) &= 0, t < 0 \end{aligned}$$

Que.  $\rightarrow$  Check C/NC LTI system.

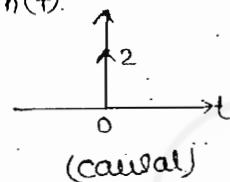
(1.)  $h(t) = e^{-2t} u(t)$



(2.)  $h(t) = u(t+1) - u(t-1)$



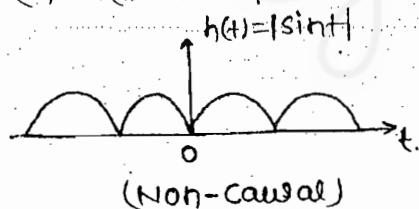
(3.)  $h(t)$



(4.)  $h(t) = e^{-(t+1)} u(t)$

(causal)

(5.)  $h(t) = |\sin t|$



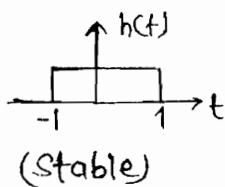
\* Cond'n for LTI sys. to be stable  $\rightarrow$  If impulse Response of LTI sys. is absolutely integrable then sys. will be stable. i.e.

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

\* A sign If impulse response of LTI sys. is represented by energy signal (or) unit impulse fn then sys. will be stable.  
i.e.  $h(t) \rightarrow \text{Energy} / s(t) \rightarrow \text{stable}$

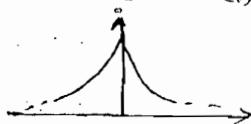
Que → Check S/IUS system.

(1)  $h(t) = u(t+1) - u(t-1)$

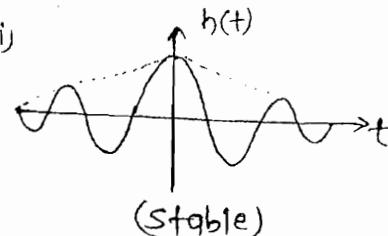


(Stable)

(2)  $h(t) = e^{-2|t|} \text{ (even)}$

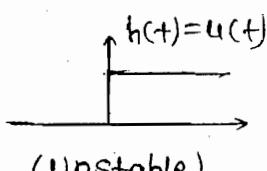


(Stable)



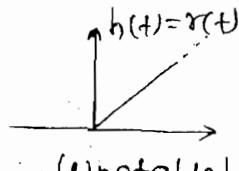
(Stable)

(iv)



(Unstable)

(v)



(Unstable)

(vi)

$$H(s) = \frac{1}{s^2 + 1}$$

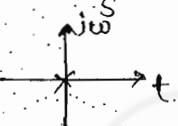
$$\text{Pole} = s = \pm j$$

\* Because of imaginary axis lying so it is marginally stable

$$* h(t) = \sin t + u(t)$$

↓  
(Unstable)

(vii)  $H(s) = \frac{1}{s}$



$$\text{Pole} \rightarrow s = 0$$

\* marginally stable

\*  $h(t) = u(t) \rightarrow \text{Powel sig.}$   
(Unstable)

$$H(s) = \frac{1}{s} = \frac{Y(s)}{X(s)}$$

$$Y(s) = \frac{X(s)}{s}$$

Inverse LT:

$$Y(t) = \left[ \int_{-\infty}^t x(\tau) d\tau \right] \text{ Integrator}$$

According to BIBO criteria:-

$$x(t) = u(t) = \text{bounded sig.}$$

$$y(t) = \int_{-\infty}^t u(\tau) d\tau = x(t)$$

$$x(t) = \text{Unbounded sig.}$$

so it is Unstable.

Note → LTI sys.

\* All marginally stable are BIBO Unstable.

\* Distortions in LTI systems →

Types:- (i) magnitude/Amplitude distortion  
 (ii) Delay/phase distortion.

Note:-

$$(1) \quad x(t) \xrightarrow{\text{NL sys.}} y(t) = x(t) + x^2(t)$$

$$\begin{aligned} \text{If } x(t) = \sin \omega_0 t \text{ then } y(t) &= \sin \omega_0 t + \sin^2 \omega_0 t \\ &= \sin \omega_0 t + \frac{1 - \cos 2\omega_0 t}{2} \end{aligned}$$

$$(2) \quad x(t) \xrightarrow{\text{TV sys.}} y(t) = x(t) + x(2t)$$

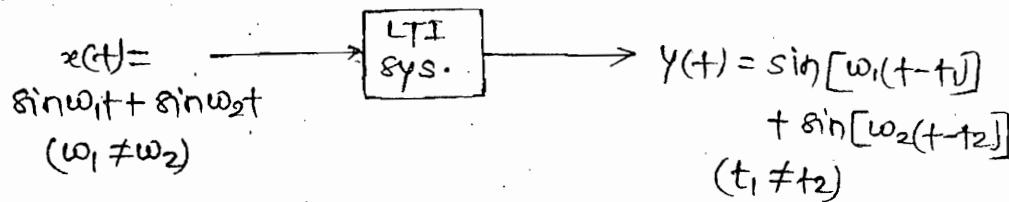
$$\begin{aligned} x(t) &= \sin \omega_0 t & y(t) &= \sin \omega_0 t + \sin 2\omega_0 t \\ \downarrow \omega_0 & & \downarrow 2\omega_0 & \end{aligned}$$

\* For production of harmonics, nature of sys. should be either NL (or) TV.

(1) Magnitude/Amplitude distortion → If sys. provides unequal amount of amplification (or) attenuation two to diff. freq. components present in i/p sys., then sys. is having magnitude distortion.

$$\begin{aligned} x(t) &\xrightarrow{\text{LTI sys.}} y(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t \\ &= \sin \omega_1 t + \sin \omega_2 t \\ &\quad \omega_1 \neq \omega_2 \end{aligned}$$

(2) Delay (or) phase distortion → If sys. provides unequal amount of time delays to diff. freq. components present in i/p signal then sys. is having delay (or) phase distortion.



\* Cond'n for LTI sys. to be distortionless  $\rightarrow$

$$x(t) = \sin\omega_1 t + \sin\omega_2 t$$

$$y(t) = kx(t-t_0)$$

$$= k \sin[\omega_1(t-t_0)] + k \sin[\omega_2(t-t_0)]$$

So Laplace transform of above eqn

$$Y(s) = k \times X(s) e^{-s t_0}$$

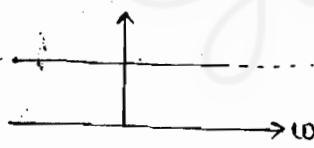
$$H(s) = \frac{Y(s)}{X(s)}$$

$$= k e^{-s t_0}$$

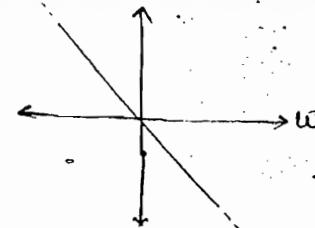
$$(s = j\omega)$$

$$H(j\omega) = k e^{-j\omega t_0}$$

$$|H(\omega)| = k$$



$$\angle H(\omega) = -\omega t_0$$



\* For distortionless LTI sys., magnitude of TF should be independent of freq. & phase of TF should be linear.

\* Differential eqn for LTI sys.  $\rightarrow$

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t)$$

$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \dots + b_0 x(t)$$

for linearity  $\rightarrow$

All initial cond'n should be zero.

Time-invariance  $\rightarrow$

Coefficients  $a_n, a_{n-1}, \dots, a_0, b_m, b_{m-1}, \dots, b_0$  should be independent of time.

Que.  $\rightarrow$  Check time invariance & linearity of sys. (initial cond'n are zero).

$$(1) \frac{2d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = x(t)$$

$$(2) \frac{2d^2y(t)}{dt^2} + 3t \cdot \frac{dy(t)}{dt} + y(t) = x(t)$$

$$(3) 2\left[\frac{dy(t)}{dt}\right]^2 + \frac{3dy(t)}{dt} + y(t) = x(t)$$

Ans.  $\rightarrow$  (1) L, TIU

(2) L, TV

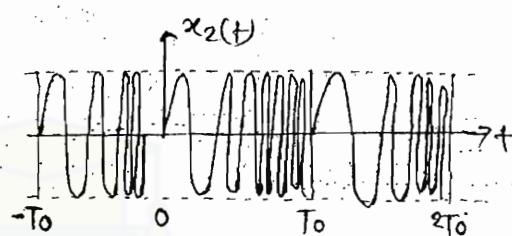
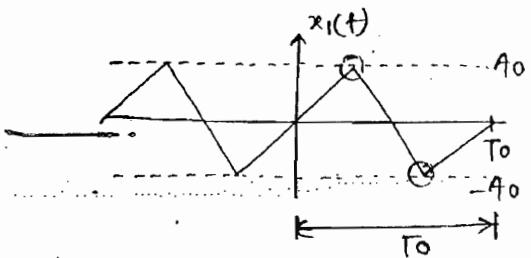
(3) NL, TIU

Chapter-04  
Fourier Series.

- \* FS expansion is used only for periodic signal.
- \* In FS sig. is expanded in terms of its harmonics which are sinusoidal & orthogonal to one another.

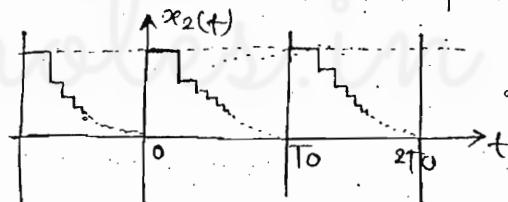
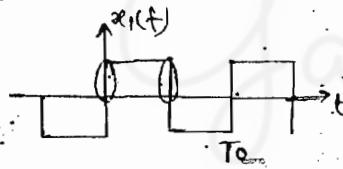
Cond'n for existence of FS expansion  $\rightarrow$  (Dirichlet cond'n)

- (1) Signal should have finite no. of maxima & minima over its time-period.



$x_1(t) \rightarrow$  FS expansion is possible       $x_2(t) \rightarrow$  FS expansion is not possible.

- (2) Signal should have finite no. of discontinuities over its time-period.



$x_1(t) \rightarrow$  FS expansion is possible  
No. of discontinuities = 2

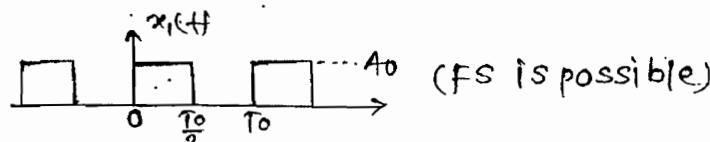
$x_2(t) \rightarrow$  FS expansion is not possible

- (3) Signal should be absolutely integrable over its time-periods.

i.e.

$$\boxed{\int_{T_0} |x(t)| dt < \infty}$$

Eg:- (1)



(2.)  $x_2(t) = \tan(t)$

(FS is not possible)

### \* Types of FS expansion →

#### (1.) Trigonometrical FS exp. →

$$x(t) = a_0 + \sum_{n=-\infty}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where,  $a_0 = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) dt$  = Represents avg. (or) dc value of signal.

$$a_n = \frac{2}{T_0} \int_{0}^{T_0} x(t) \cos n\omega_0 t dt = a_{(n)} \quad [\text{even f^n wrt 'n'}]$$

$$b_n = \frac{2}{T_0} \int_{0}^{T_0} x(t) \sin n\omega_0 t dt = -b_{(n)} \quad [\text{odd f^n wrt 'n'}]$$

#### (2.) Exponential FS exp. →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where;  $C_n$  = Complex exponential FS coefficient

$$= \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) e^{-jn\omega_0 t} dt \quad \text{(i)}$$

$$C_n = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) e^{-jn\omega_0 t} dt$$

↓ \* (conjugate)

$$C_n^* = \frac{1}{T_0} \int_{-T_0}^{T_0} x^*(t) e^{-jn\omega_0 t} dt \quad \text{(ii)}$$

for Conjugate Symmetry (CS)  $C_n$  :-

$$C_n = C_n^*$$

from eqn (i) & (ii)

$x(t) = x^*(t)$	IF $x(t)$ is real.
-----------------	--------------------

\* If time domain signal is real then its exponential FS coefficient will be conjugate symmetric.

$$C_n = |C_n| e^{j\angle C_n} \quad \text{(3.)}$$

where;  $|C_n|$  = magnitude of  $n$ th harmonic ( $n\omega_0$ )

from eqn (3)  $n = -n$

$$C_{(-n)} = |C_n| e^{j(\angle C_n)} \\ \overset{*}{\downarrow} \\ C_{(-n)}^* = |C_n| e^{-j(\angle C_n)} \quad \text{--- (4)}$$

for cs  $C_n$ :

$$C_n = C_{-n}^*$$

from eqn (3) & (4)

$$|C_n| = |C_{-n}| \rightarrow \text{Even}$$

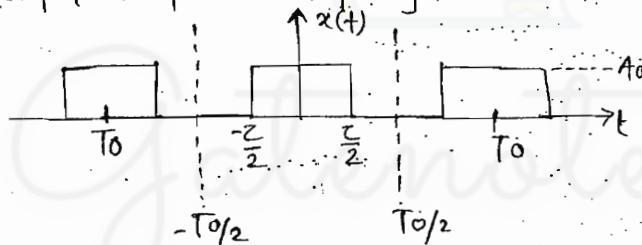
$$\angle C_n = \cancel{\angle C_n} - \angle C_{-n} \rightarrow \text{Odd.}$$

Note:-

For Real signal :- (R.M.F.P.)

- (i) Real part of  $C_n$  will be even & imaginary part of  $C_n$  will be odd.
- (ii) magnitude of  $C_n$  will be even & phase of  $C_n$  will be odd.

Ques → Find exp. FS expansion of signal.



Soln →

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn\omega_0 t} dt \\ (x(t) = A_0) = \frac{A_0}{T_0} \int_{-T_0/2}^{T_0/2} e^{-jn\omega_0 t} dt \\ = \frac{A_0}{T_0} \left( \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right) \Big|_{-T_0/2}^{T_0/2} \\ = \frac{A_0}{T_0(j\omega_0)} \left[ -e^{-jn\omega_0 \frac{T_0}{2}} + e^{jn\omega_0 \frac{T_0}{2}} \right] \\ = \frac{A_0}{T_0(j\omega_0)} \times 2j \sin\left(\frac{n\omega_0 T_0}{2}\right)$$

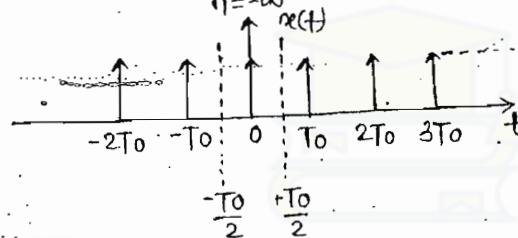
$$e_n = \frac{A_0}{T_0(n\omega_0)} (2j) \times \left[ \frac{\sin\left(\frac{n\omega_0 z}{2}\right)}{\left(\frac{n\omega_0 z}{2}\right)} \right] \times \left(\frac{n\omega_0 z}{2}\right)$$

$$c_n = \frac{A_0 z}{T_0} \operatorname{Sa}\left(\frac{n\omega_0 z}{2}\right)$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \left[ \frac{A_0 z}{T_0} \operatorname{Sa}\left(\frac{n\omega_0 z}{2}\right) \right] \cdot e^{jn\omega_0 t}$$

Que → Find  $c_n$  for sig.  $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$



Sol ↗

$$\begin{aligned} c_n &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cdot e^{-jn\omega_0 t} dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cdot e^{-jn\omega_0 t} dt \end{aligned}$$

$$\because f(t) \cdot \delta(t) = f(0) \cdot \delta(t)$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cdot e^0 dt$$

$$c_n = \frac{1}{T_0}$$

Que → The sig.  $x(t)$  has 'T\_0=2' & following coefficients

$$c_k = \begin{cases} (1/2)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

The value of  $x(0)$  will be.

Soln

$$c_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \frac{1}{2} e^{-jk\omega_0 t} dt$$

 $\frac{1}{2}$ 

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\downarrow t=0$$

$$x(0) = \sum_{k=-\infty}^{\infty} c_k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$x(0) = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots$$

$$= \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

Que. → The sig.  $x(t)$  has FTP  $T_0 = 1$  & the following Fourier coefficients

$$c_k = \begin{cases} \left(\frac{-1}{3}\right)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

'Find  $x(t) = ?$

- (a.)  $\frac{1}{1 - \frac{1}{3}e^{j2\pi t}}$  (b.)  $\frac{1}{1 + \frac{1}{3}e^{j2\pi t}}$  (c.)  $\frac{1}{1 - \frac{1}{3}e^{j2\pi t}}$  (d.)  $\frac{1}{1 + \frac{1}{3}e^{j2\pi t}}$

Soln

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3} e^{j\omega_0 t}\right)^k$$

$$= 1 + \left(\frac{-1}{3} e^{j\omega_0 t}\right) + \left(\frac{-1}{3} e^{j\omega_0 t}\right)^2 + \dots$$

$$= \frac{1}{1 - \left(\frac{-1}{3} e^{j\omega_0 t}\right)}$$

$$= \frac{1}{1 + \frac{1}{3} e^{j\omega_0 t}}$$

$$= \frac{1}{1 + \frac{1}{3} e^{j2\pi t}}$$

Note:-

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn\omega_0 t} dt$$

$$C_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$C_0 = a_0 = \text{dc value of } x(t)$ .

$$\textcircled{*} x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = \dots + C_{-1} e^{-j\omega_0 t} + C_0 + C_1 e^{j\omega_0 t} + \dots$$

Ques. Consider the periodic sig.

$$x(t) = 1 + \sin \omega_0 t + 2 \cos \omega_0 t + \cos(2\omega_0 t + \frac{\pi}{4})$$

Determine  $C_n$ .

$$\text{Soln} \rightarrow x(t) = 1 + \left( \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right) + \left( e^{j\omega_0 t} + e^{-j\omega_0 t} \right) + \frac{1}{2} \left[ e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})} \right]$$

$$x(t) = 1 + \left( \frac{1+j}{2j} \right) e^{j\omega_0 t} + \left( \frac{1-j}{2j} \right) e^{-j\omega_0 t} + \left( \frac{1}{2} \right) e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})(1/2)}$$

$$= 1 + \left( \frac{1+j}{2j} \right) e^{j\omega_0 t} + \left( -\frac{1}{2j} \right) + \left( \frac{e^{j\pi/4}}{2} \right) \times e^{j2\omega_0 t} + \left( \frac{e^{-j\pi/4}}{2} \right) \times e^{-j2\omega_0 t}$$

$$= C_0 + C_1 e^{j\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_{-2} e^{-j2\omega_0 t}$$

$$C_0 = 1$$

$$C_2 = \frac{e^{j\pi/4}}{2} = \frac{1+j}{2\sqrt{2}}$$

$$C_1 = \left( \frac{1+j}{2j} \right)$$

$$C_{-1} = \frac{e^{-j\pi/4}}{2} = \frac{1-j}{2\sqrt{2}}$$

$$C_{-2} = \left( \frac{1-j}{2j} \right)$$

Que. Consider a periodic sig.  $x(t)$  with ' $T_0=8$ ' & FS coefficients.

$$c_1 = c_{-1} = 2$$

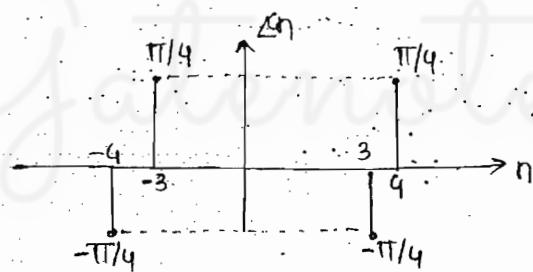
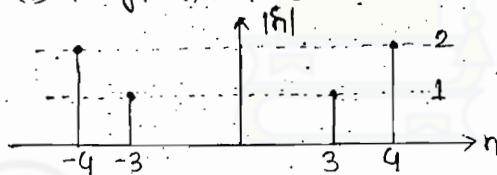
$$c_3 = 4j$$

$$c_{-3} = -4j$$

Find  $x(t)$

$$\begin{aligned}\text{Soln} \rightarrow x(t) &= c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t} + c_3 e^{j3\omega_0 t} + c_{-3} e^{-j3\omega_0 t} \\ &= 2[e^{j\omega_0 t} + e^{-j\omega_0 t}] + 4j[e^{j3\omega_0 t} - e^{-j3\omega_0 t}] \\ &= 2 \times 2 \cos \omega_0 t + 4j \times 2j \sin 3\omega_0 t \\ &= 4 \cos \omega_0 t - 8 \sin 3\omega_0 t \quad (\omega_0 = \frac{\pi}{4})\end{aligned}$$

Que.  $c_n$  for sig.  $x(t)$  is given below.  
( $\omega_0 = \pi$ )



Find  $x(t)$

$$\text{Soln} \rightarrow c_3 = |c_3| e^{j\angle c_3} = 1 \times e^{-j\pi/4}$$

$$c_{-3} = e^{j\pi/4}$$

$$c_4 = 2e^{j\pi/4} \quad c_{-4} = 2e^{-j\pi/4}$$

$$x(t) = c_3 e^{j3\omega_0 t} + c_{-3} e^{-j3\omega_0 t} + c_4 e^{j4\omega_0 t} + c_{-4} e^{-j4\omega_0 t}$$

$$= e^{-j\pi/4} e^{j3\omega_0 t} + e^{j\pi/4} e^{-j3\omega_0 t} + 2e^{j\pi/4} e^{j4\omega_0 t} + 2e^{-j\pi/4} e^{-j4\omega_0 t}$$

$$= e^{j3\pi/4} t / e^{j(3\pi/4)} + e^{j5\pi/4} t / e^{-j(5\pi/4)}$$

$$= 2 \cos(\frac{3\pi}{4}t) \sin(\frac{5\pi}{4}t) + 4 \cos(\frac{4\pi}{4}t)$$

$$(d) 4 \cos(4\pi t + \frac{\pi}{4}) + 2 \cos(3\pi t - \frac{\pi}{4})$$

$x(t) - c_n$  pairs  $\rightarrow$

	$x(t)$	$c_n$
(1)	Real	CS
(2)	CS	Real
(3)	Img.	CAS
(4)	CAS	Img.
(5)	Real even	Real + Even
(6)	Img. + Even	Img. + Even
(7)	Real + Odd	Img. + Odd
(8)	Img. + Odd	Real + Odd

Que.  $\rightarrow f(t) = \sum_{n=-\infty}^{\infty} \cos n\pi t e^{j n \pi t}$   $f(t)$  will be?

Sol.  $\rightarrow c_n = \cos n\pi t$

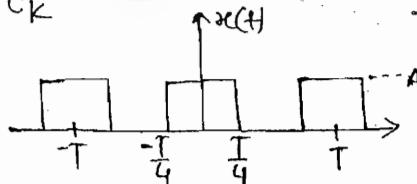
$$= R + E$$

Que.  $\rightarrow \sum_{n=-\infty}^{\infty} j \sin \frac{n\pi}{2} t e^{j n \pi t}$   $f(t)$  will be?

Sol.  $\rightarrow c_n = I + O = j \sin \frac{n\pi}{2}$

$$f(t) = R + O.$$

Que.  $\rightarrow$  find  $c_k$



- (a.)  $\frac{A}{j\pi k} \sin \frac{\pi}{2} k$  (b.)  $\frac{A}{j\pi k} \cos \left( \frac{\pi}{2} k \right)$  (c.)  $\frac{A}{\pi k} \sin \left( \frac{\pi}{2} k \right)$  (d.)  $\frac{A}{\pi k} \cos \left( \frac{\pi}{2} k \right)$

Sol.  $\rightarrow x(t) = R + E$

$$c_k = R + E \quad (A)$$

$$\frac{\sin \left( \frac{\pi}{2} k \right)}{(\pi k)} \frac{0}{0} = E$$

$$\frac{\cos \left( \frac{\pi}{2} k \right)}{(\pi k)} = \frac{E}{0} = 0$$

$$f(k) = \pi k$$

$$f(-k) = -\pi k$$

Ans. (d)  $\rightarrow$

Que.  $\rightarrow x(t) = c_n = \begin{cases} 2 & , n=0 \\ j(\frac{1}{2})^{|n|} & , \text{otherwise} \end{cases}$

which of the following is true?

- (a)  $x(t)$  is a real sig.      (c)  $\frac{dx(t)}{dt}$  is an even sig.  
 (b)  $x(t)$  is an even sig.      (d) both (b) & (c)

Sol.

$$c_n \rightarrow n = -n$$

so  $c_n = c_{-n}$  (Even signal)

$\therefore c_n = \text{imag.}$

$$c_n = E + I$$

Ans. (b)

$\frac{dx(t)}{dt}$  is always odd. (derivative of even is odd)

Relation between  $a_n$ ,  $b_n$  &  $c_n$

- \*  $a_0 = c_0$
  - \*  $c_n = \frac{1}{2}[a_n - j b_n]$
  - \*  $a_n = 2 \text{Real}[c_n]$
  - \*  $b_n = 2 \text{Img.}[c_n]$
- } valid for any type  
of signal  $x(t)$
- } valid only for real signal  $x(t)$

Que.  $\rightarrow$  FS expansion of real sig  $f(t)$  is

$$f(s) = \sum_{n=-\infty}^{\infty} \frac{3}{4 + (3n\pi)^2} e^{j n \pi t}$$

Determine:-

- To
- A term in that expansion is  $A_0 \cos \omega_0 t$ , calculate the value of  $A_0$ .
- Repeat (ii) for  $A_0 \sin \omega_0 t$ .

Sol.

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j n \omega_0 t}$$

$$c_n = \frac{3}{4 + (3n\pi)^2} = R + E$$

$$\omega_0 = \pi$$

$$(i) T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$$

$$(ii) A_0 \cos 6\pi t = a_n \cos n\omega_0 t = a_n \cos n\pi t$$

$n=6$

$$A_0 = 96$$

$$a_n = 2 \operatorname{Re}[c_n] = 2c_n = \frac{6}{4 + (3n\pi)^2}$$

$$a_6 = a_6 = \frac{6}{4 + (3\pi)^2}$$

$$a_6 = \frac{6}{4 + (18\pi)^2}$$

$$(iii) A_0 \sin 6\pi t = b_n \sin n\omega_0 t = b_n \sin n\pi t$$

$$A_0 = b_6 = 0 \quad (n=6)$$

$$b_n = 2 \operatorname{Im}[c_n] = 0$$

\* Symmetry is FS  $\rightarrow$

(1.) Even symm.  $\rightarrow$

$$x(t) = A_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

Even      Even      Odd

\* FS expansion of an even signal does not contain sine terms.

(2.) Odd symm.  $\rightarrow$

\* Odd symm. signal contains only odd-sine terms in the FS expansion.

(3.) Half wave symm.  $\rightarrow$

$$x(t) = -x(t + \frac{T_0}{2})$$

Cn       $-c_{(n)} = -c_n$

$$\underline{c_n = c_m} \quad c_n = -c_m e^{-j\pi \omega_0 T_0 / 2}$$

$$j = -e^{j\pi} \quad \left( \because \frac{\omega_0 T_0}{2} = \pi \right)$$

<b>time shifting</b> $x(t) = c_n$ $x(t - t_0) = c_n e^{-j\pi \omega_0 t_0}$
---

$$1 + e^{j\pi} = 0$$

$$1 + (-1)^n = 0$$

$$e^{j\pi} = (e^{j\pi})^n$$

$$= (\cos \pi + j \sin \pi)^n$$

$$= (-1)^n$$

The above relation will be satisfied only when

$n \rightarrow$  odd-integer

$\downarrow$   $n \in \{ \text{odd-integer} \}$

∴ FS expansion of any HWS signal contains only odd harmonics.

#### (4) Even + HWS $\rightarrow$

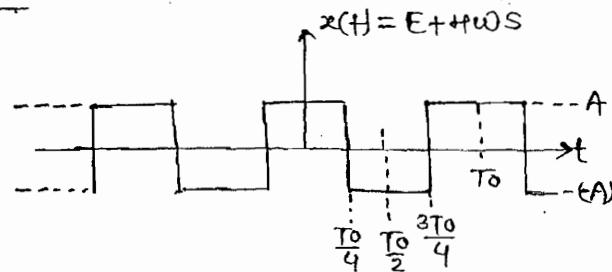
- \* Contains only odd harmonics. } Because of HWS
  - \* Avg/dc value is zero ( $a_0 = 0$ )
  - \* Does not contain sine terms.  $\rightarrow$  Because of even.
- Note  $\rightarrow$  FS expansion of an even HWS signal contains odd harmonics of cos.

#### (5) Odd + HWS $\rightarrow$

- \* Contains only sine terms.
  - \* Contains only odd harmonics.
- Note  $\rightarrow$  FS expansion of an odd HWS signal contains sine terms without harmonics.

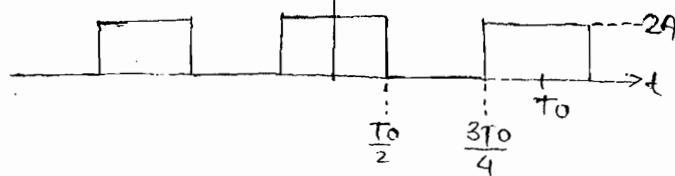
\*

#### (6) Hidden Symm. $\rightarrow$

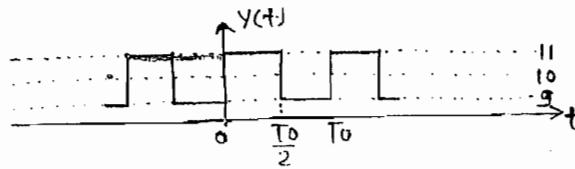
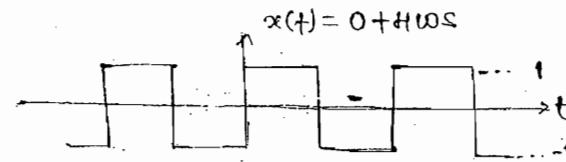


$$x(t) = a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t + a_5 \cos 5\omega_0 t + \dots$$

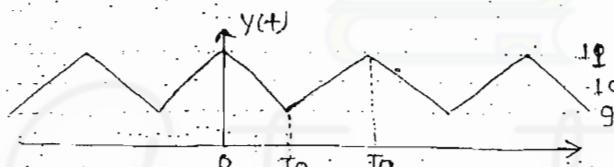
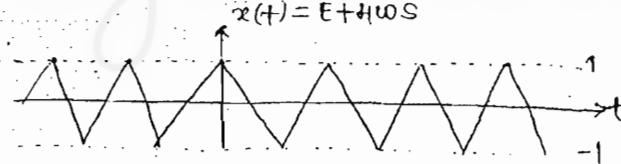
$$Y(t) = A + x(t)$$



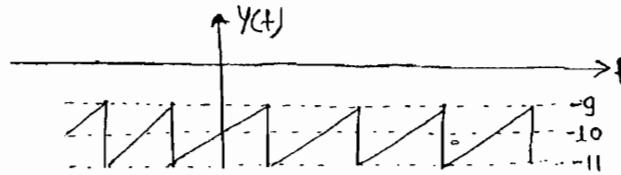
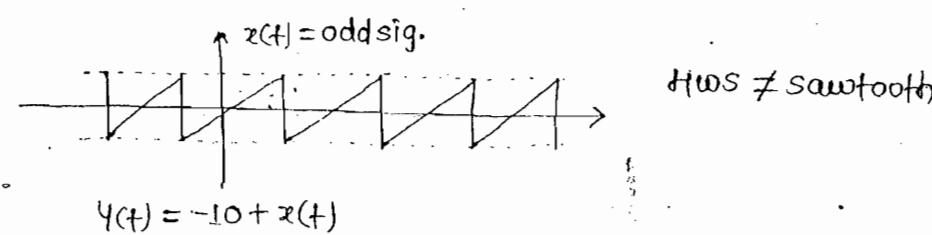
$$\begin{aligned}
 y(t) &= A + x(t) \\
 &= A + a_1 \cos \omega_0 t + 3 \cos 3\omega_0 t + \dots \\
 &= \text{dc} + \text{odd harmonics of cos.}
 \end{aligned}$$

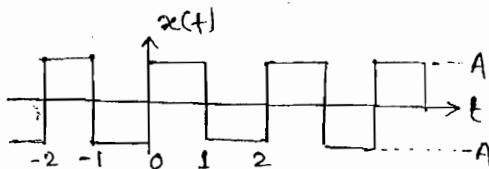
Que. →Soln →

$$\begin{aligned}
 y(t) &= 10 + x(t) \\
 &= 10 + b_1 \sin \omega_0 t + b_3 \sin 3\omega_0 t + \dots \\
 &= \text{dc} + \text{odd harmonics of sine.}
 \end{aligned}$$

Que. →Soln →

$$\begin{aligned}
 y(t) &= 10 + x(t) \\
 &= \text{dc} + \text{odd harmonics of cos}
 \end{aligned}$$

Que. →Soln →

Que →

- (a.)  $\frac{A}{n\pi} [1 - (-1)^n]$       (c.)  $\frac{A}{n\pi} [1 - (-1)^n]$   
 (b.)  $\frac{A}{n\pi} [1 + (-1)^n]$       (d.)  $\frac{A}{n\pi} [1 + (-1)^n]$   
 $c_n = ?$

Soln →

$$\rightarrow x(t) = R + O$$

$$\text{So; } c_n = I + O$$

$$x(t) = HWS$$



Only odd harmonics are present.

$$c_n = \begin{cases} \neq 0, n = \text{odd} \\ = 0, n = \text{even} \end{cases}$$

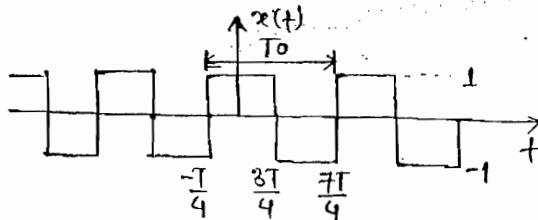
From option (c.)  $\frac{A}{n\pi} [1 - (-1)^n] \rightarrow 0, n = \text{even}$   
 $\neq 0, n = \text{odd}$

Que → A sig.  $x(t)$  is given by

$$x(t) = \begin{cases} 1 & -T/4 < t \leq 3T/4 \\ -1 & 3T/4 < t \leq 7T/4 \\ *x(t+T) & \end{cases}$$

which of the following gives the fundamental Fourier term of  $x(t)$ ?

- (a.)  $\frac{\pi}{4} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$  (b.)  $\frac{\pi}{4} \cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$  (c.)  $\frac{4}{\pi} \sin\left(\frac{\pi t}{4} - \frac{\pi}{4}\right)$  (d.)  $\frac{4}{\pi} \sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

Soln →

$$T_0 = \frac{T}{4} + \frac{7T}{4} = 2T$$

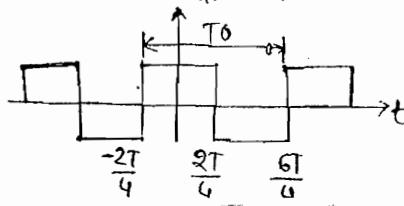
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

for HWS →

$$x(t) = -x(t + \frac{T_0}{2}) = *x(t+T)$$

\* The above signal is NENO (Neither even nor odd)

$$y(t) = E + HWS.$$



$$y(t) = a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t + \dots$$

$$x(t) = y(t - \frac{T}{4})$$

$$= a_1 \cos \omega_0 \left(t - \frac{T}{4}\right) + a_3 \cos 3\omega_0 \left(t - \frac{T}{4}\right) + \dots$$

$$= a_1 \cos \left[\frac{\pi}{4} \left(t - \frac{T}{4}\right)\right]$$

$$\boxed{x(t) = a_1 \cos \left[\frac{\pi t}{T} - \frac{\pi}{4}\right]} \quad \text{Ans.}$$

Fundamental Fourier term

$$= a_1 \cos \omega_0 t + b_1 \sin \omega_0 t$$

$$x = c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t}$$

Note → Polarity of periodic signal at any time instant is decided by polarity of its fundamental Fourier term which is dominant as compare to all other terms in the expansion of periodic signal (This rule is applicable for those periodic signals in which  $\infty$  no. of harmonics are present).

From the above options:-

~~(A)  $\frac{\pi}{4} \cos \left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0} +ve$~~

~~(B)  $\frac{\pi}{4} \sin \left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0} -ve$~~

\* Properties of FS →

(1.) Linearity →

$$a_1 x_1(t) + a_2 x_2(t) \iff a_1 c_{1n} + a_2 c_{2n}$$

where;  $x_1(t) = c_{1n}$

$$x_2(t) = c_{2n}$$

(2.) Time-reverse →

$$x(t) \iff c_{en}$$

(3.) Conjugation →

$$x^*(t) \iff c_{-n}^*$$

(4.) Time-shifting  $\rightarrow$

$$x(t-T_0) \iff c_n e^{-jn\omega_0 t}$$

(5.) freq. shifting  $\rightarrow$

$$e^{+jm\omega_0 t} x(t) \iff c_{n-m}$$

(6.) Convolution in time  $\rightarrow$

$$x_1(t) * x_2(t) \iff T_0 [c_{1n} * c_{2n}]$$

(7.) Multiplication in time  $\rightarrow$

$$x_1(t) \cdot x_2(t) \iff [c_{1n} * c_{2n}]$$

(8.) Differentiation  $\rightarrow$

$$\frac{d^m x(t)}{dt^m} \iff (j n \omega_0)^m c_n$$

(9.) Integration in time  $\rightarrow$

$$\int_{-\infty}^{t_0} x(t) dt \iff \frac{c_n}{jn\omega_0}$$

(10.) Parseval's power theorem  $\rightarrow$

$$P = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Ques.  $\rightarrow$  find  $c'_n$  in term of  $c_n$

where;

$$y(t) \iff c'_n$$

$$x(t) \iff c_n$$

$$(i) y(t) = e^{-j2\omega_0 t} \cdot x(t)$$

$$(ii) y(t) = x(t-T_0) + x(t+T_0)$$

$$(iii) y(t) = \frac{d^2 x(t)}{dt^2}$$

$$(iv) y(t) = \text{even}[x(t)]$$

$$(v) y(t) = \text{Real}[x(t)]$$

$$\text{Soln} \rightarrow (i) \quad c'_n = c_{n+2}$$

$$(ii) \quad c'_n = c_n e^{-jn\omega_0 t_0} + c_n e^{jn\omega_0 t_0}$$

$$= c_n [e^{jn\omega_0 t_0} + e^{-jn\omega_0 t_0}]$$

$$c'_n = 2c_n \cos(n\omega_0 t_0)$$

$$(iii) c_n' = (j n \omega_0)^2 c_n = -n^2 \omega_0^2 c_n$$

$$(iv) y(t) = \frac{x(t) + x^*(t)}{2}$$

↓ (time reversal)

$$c_n' = \frac{c_n + c_{-n}}{2}$$

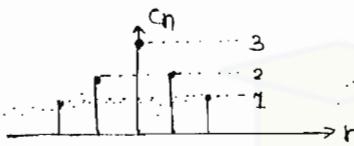
$$(v) y(t) = \text{Real}[x(t)]$$

$$= \frac{x(t) + x^*(t)}{2}$$

↓ (conjugation)

$$c_n' = \frac{c_n + c_n^*}{2}$$

Que. → Calculate power of signal  $x(t)$



$$\begin{aligned} \text{Sol'n} \rightarrow P &= \sum_{n=-\infty}^{\infty} |c_n|^2 \\ &= 1^2 + 2^2 + 3^2 + 2^2 + 1^2 \\ &= 19 \end{aligned}$$

Que. → Let  $x(t)$  be the periodic signal with  $T_0$  s

$$\begin{aligned} y(t) &= x(t-t_0) + x(t+t_0) \\ y(t) &= b_k \\ x(t) &= c_k \end{aligned} \quad \left. \begin{array}{l} \text{exp. FS coefficients.} \\ \end{array} \right\}$$

If  $b_k = 0$  for odd integer 'k' then 't<sub>0</sub>' can be equal to

- (a)  $\frac{T}{8}$  (b)  $\frac{T}{4}$  (c)  $\frac{T}{2}$  (d)  $2T$

$$\text{Sol'n} \rightarrow y(t) = x(t-t_0) + x(t+t_0)$$

$$b_k = 2c_k \cos k \omega_0 t_0 \quad \dots \dots \dots (i)$$

Given that;

$$b_k = 0; k = \text{an odd-integer}$$

$$= 2m+1$$

$$m = \text{int}$$

$$b_{2m+1} = 0$$

$$\Rightarrow 2c_{2m+1} \cos[(2m+1)\omega_0 t_0] = 0 \quad \text{from } \textcircled{1}$$

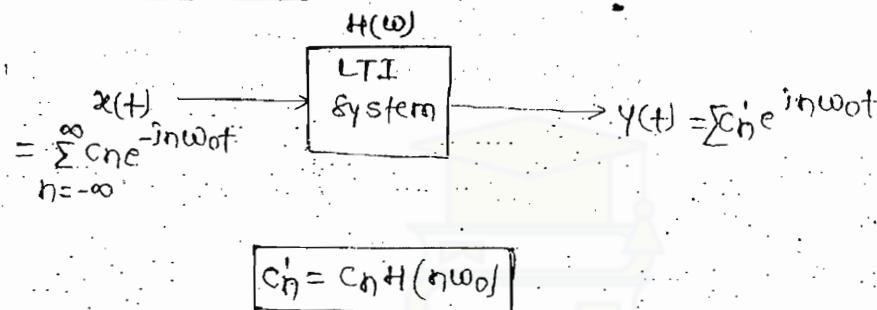
$$\cos[(2m+1)\omega_0 t_0] = 0$$

$$\cos(2m+1)\omega_0 t_0 = \cos(2m+1)\frac{\pi}{2}$$

$$\frac{2\pi}{T}t_0 = \frac{\pi}{2}$$

$$t_0 = \frac{\pi}{4}$$

\* FS for LTI system  $\rightarrow$



Que.  $\rightarrow$  Consider a continuous time LTI sys. whose i/p  $x(t)$  & o/p  $y(t)$  are related by the following DE:

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

find  $c_n$  for o/p  $y(t)$  if i/p  $x(t) = \cos \omega_0 t$ ,  $\omega_0 = 2\pi$

Sol<sup>n</sup>

$$sY(s) + 4Y(s) = X(s)$$

$$Y(s)[s+4] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(s+4)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+4}$$

$$\therefore H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{j\omega + 4}$$

$$\therefore x(t) = \cos \omega_0 t$$

$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$$c_1 = 1/2 \quad \& \quad c_{-1} = 1/2$$

$$c_n^i = H(n\omega_0) c_n$$

$$n=1, \quad c_1^i = H(\omega_0) c_1$$

$$= \frac{1}{4+j\omega_0} \times \left(\frac{1}{2}\right)$$

$$= \frac{1}{4+j2\pi} \times \left(\frac{1}{2}\right)$$

$$c_1^i = \frac{1}{4+j2\pi} \left(\frac{1}{2}\right) \quad c_{-1}^i = \frac{1}{4-j2\pi} \left(\frac{1}{2}\right)$$

Que. → Suppose we have given following information about a sig.  $x(t)$

(1.)  $x(t)$  is real & odd.

(2.)  $x(t)$  is periodic with  $T_0 = 2$

(3.) Fourier Coefficients

$$c_n = 0, \quad |n| > 1$$

$$(4.) \frac{1}{2} \int_0^2 |x(t)|^2 dt = 1$$

The sig. that satisfies this cond. is...

(a.)  $\sqrt{2} \sin \pi t$  & unique

(c.)  $2 \sin \pi t$  & unique

(b.)  $\sqrt{2} \sin n\pi t$  & but not unique

(d.)  $2 \sin n\pi t$  & but not unique.

Soln.

$$T_0 = 2, \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$$

The avg. of odd signal  
is 0.

$$c_n = 0, \quad |n| > 1$$

$$c_1 \neq 0$$

$$c_0 = 0 \quad \leftarrow x(t) \text{ is odd.}$$

$$x(t) = A_0 \bar{\sin} \omega_0 t = A_0 \sin \pi t$$

$$\frac{A_0^2}{2} = 1, \quad A_0 = \pm \sqrt{2} \quad (\text{Because of power signal})$$

Ans. (B).

Chapter-05  
Fourier Transform

- \* FT is a mathematical tool for freq. analysis of sig. whereas LT is a convenient mathematical tool for ckt analysis.
- \* FT exists for energy & power signals whereas LT also exists for NENP signals. (upto certain extent only)
- \* In the category of NENP signal unit impulse is the only fn for which FT also exists.

$$u(t) \xrightarrow{LT} \frac{1}{s}$$

$\downarrow s = j\omega (\text{FT})$

$$\frac{1}{j\omega} + \pi s(\omega)$$

$$e^{2t} u(t) \xrightarrow{LT} \frac{1}{s-2} \quad (\text{FT does not exist})$$

- \* The replacement ( $s = j\omega$ ) is used for Laplace to Fourier conversion only for absolutely integrable signal.
- \* Impulse fn & energy signals are absolutely integrable signals.

Fourier Xform →

$$x(t) \xleftrightarrow{} X(\omega) \xrightarrow{\text{rad/sec}}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Conditions for existence of FT:- (Dirichlet's cond'n)

- (1) sig. should have finite no. of maxima & minima over finite interval.
- (2) sig. should have finite no. of discontinuities over finite interval.
- (3) sig. should be absolutely integrable.

i.e.  $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

→ Impulse sig.  
→ Energy sig.

\* Dirichlet's cond<sup>n</sup> are sufficient but not necessary.

Que. → Cal. FT for sig.  $x(t) = e^{-at} u(t)$ ,  $a > 0$

Sol<sup>n</sup> →

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[ \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_{-\infty}^{\infty}$$

$$A = \frac{e^{-(a+j\omega)\infty} - e^{-(a+j\omega)(-\infty)}}{-(a+j\omega)}$$

$$e^{(a+j\omega)\infty} = e^{-a\infty - j\omega\infty}$$

$$= e^{-a\infty} e^{-j\omega\infty} \quad (\text{undefined})$$

$$e^{-a\infty} = 0, a > 0$$

$\left. \begin{array}{l} e^{-j\infty} = \cos\infty - j\sin\infty \\ \text{These cos & sin are not} \\ \text{defined in the given} \\ \text{Range.} \end{array} \right\}$

$$A = \frac{0 - 1}{-(a+j\omega)}$$

$$\boxed{X(\omega) = \frac{1}{a+j\omega}}$$

\* At  $t = \pm\infty$ , complex exponentials & sinusoidal f<sup>n</sup> are undefined.

### Properties of FT →

\* (1.) Linearity →  $a_1 x_1(t) + a_2 x_2(t) \iff a_1 X_1(\omega) + a_2 X_2(\omega)$

\* (2.) Time reversal →  $x(-t) \iff X(-\omega)$

\* (3.) Conjugation →  $x^*(t) \iff X^*(-\omega)$

\* (4.) Time shifting →  $x(t-t_0) \iff X(\omega)e^{-j\omega t_0}$

\* (5.) Time scaling →  $x(at) \iff \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

\* (6.) Freq. shifting →  $e^{-j\omega_0 t} x(t) \iff X(\omega + \omega_0)$

\* (7.) Diff. in time →  $\frac{d^n x(t)}{dt^n} \iff (j\omega)^n X(\omega)$

\* (8.) Integration in time →  $\int_{-\infty}^{+\infty} x(t) dt \iff \frac{X(\omega)}{j\omega} + \pi X(0) \cdot \delta(\omega)$

where;  $X(0) = X(\omega) \Big|_{\omega=0}$

\* (9.) Convolution in time →  $x_1(t) * x_2(t) \iff [X_1(\omega) \cdot X_2(\omega)]$

\* (10.) Multiplication in time →  $x_1(t) \cdot x_2(t) \iff \frac{1}{2\pi} [X_1(\omega) * X_2(\omega)]$

$$x_1(t) \cdot x_2(t) \iff X_1(f) * X_2(f)$$

\* (11.) Diff. in freq. →  $t^n x(t) \iff (j\omega)^n \frac{d^n X(\omega)}{d\omega^n}$

\* (12.) Parseval's energy theorem →  $E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$

\* (13.) Modulation →  $x_1(t) \cdot \cos\omega_0 t \iff \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$

$$x(t) \cdot \sin\omega_0 t \iff \frac{j}{2} [X(\omega + \omega_0) - X(\omega - \omega_0)]$$

\* (14.) Area of time-domain →  $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$\downarrow \omega = 0$

$X(0) = \int_{-\infty}^{\infty} x(t) dt$

$$\text{Eq:- } x(t) = e^{-at} u(t), a > 0 \Rightarrow X(\omega) = \frac{1}{a+j\omega}$$

$$\text{Area of } x(t) = X(\omega) \Big|_{\omega=0} = \frac{1}{a}$$

\* (15.) Area under freq. domain  $\rightarrow$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega$$

$$\int_{-\infty}^{\infty} X(\omega) d\omega = 2\pi x(0)$$

$$\boxed{\text{Area under } X(\omega) = 2\pi x(t) \Big|_{t=0}}$$

[DATE-20/10/14]

$$\text{Que.} \rightarrow x(t) = e^{-at} u(-t) \Leftrightarrow X(\omega) = ? \quad a > 0$$

$$\text{Soln} \rightarrow e^{-at} u(-t) \Leftrightarrow X(\omega)$$

$\downarrow (t = -t) \quad \downarrow (\omega = -\omega) \rightarrow \text{time reversal.}$

$$e^{-at} u(-t) \Leftrightarrow \frac{1}{a-j\omega}$$

$$\text{Que.} \rightarrow y(t) = e^{-at} H, a > 0 \Leftrightarrow Y(\omega) = ?$$

$$\text{Soln} \rightarrow y(t) = e^{-at} H$$

$$= \begin{cases} e^{-at}, t < 0 \\ e^{-at}, t > 0 \end{cases}$$

$$= e^{-at} u(-t) + e^{-at} u(t)$$

$$Y(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$Y(\omega) = \frac{2a}{a^2 + \omega^2}$$

$$e^{-at} H, a > 0 \Leftrightarrow \frac{2a}{a^2 + \omega^2}$$

\* Property of duality →

(1.)

$$\begin{aligned} x(t) &\rightleftharpoons; X(\omega) \\ (\omega = +) & \quad \quad \quad t = -\omega \\ x(t) &\rightleftharpoons; 2\pi x(-\omega) \\ x(t) &\rightleftharpoons; X(f) \\ (\mathcal{F} = t) & \quad \quad \quad t = -f \\ x(t) &\rightleftharpoons; x(-f) \end{aligned}$$

Q. →  $x(t) = \frac{1}{q+it} \rightleftharpoons; X(\omega) = ?$

Soln

$$\begin{aligned} x(t) &= \frac{1}{q+it} \\ (t = \omega) & \downarrow \\ e^{-qt} u(t), q > 0 &\rightleftharpoons; \frac{1}{q+i\omega} \\ (\omega = +) & \quad \quad \quad (t = -\omega) \end{aligned}$$

$$\boxed{\frac{1}{q+it} = 2\pi e^{q\omega} u(-\omega), q > 0}$$

Q. →  $x(t) = \frac{2q}{q^2+t^2} \rightleftharpoons; X(\omega) = ?$

Soln

$$\begin{aligned} x(t) &= \frac{2q}{q^2+t^2} \\ (t = \omega) & \downarrow \\ e^{-q|\omega|}, q > 0 &\rightleftharpoons; \frac{2q}{q^2+\omega^2} \\ (\omega = +) & \quad \quad \quad (t = -\omega) \end{aligned}$$

$$\boxed{\frac{2q}{q^2+t^2} \rightleftharpoons; 2\pi e^{-q|\omega|}, q > 0}$$

\* \* \*

$$\boxed{A_0 s(t) \rightleftharpoons; A_0}$$

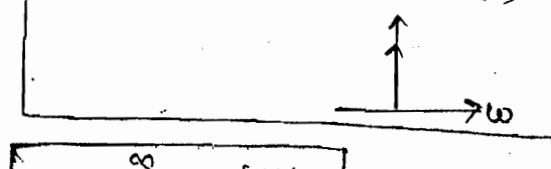
Q. →  $x(t) = A_0 \rightleftharpoons; X(\omega) = ?$

Soln

$$A_0 s(t) \rightleftharpoons; A_0$$

$$\begin{aligned} \cancel{\omega = +} &\quad \cancel{t = -\omega} \\ A_0 &\rightleftharpoons; 2\pi A_0 [\delta(\omega)] \cdot 2\pi A_0 \delta(-\omega) \end{aligned}$$

$$\boxed{A_0 = \text{dc signal} \rightleftharpoons; 2\pi A_0 \delta(\omega)}$$



Q → Find  $y(\omega)$  in terms of  $X(\omega)$

SOPK  $\rightarrow x(t) = X(\omega)$

$$y(t) = Y(\omega)$$

i)  $y(t) = e^{j2t} x(t)$

SOL  $\rightarrow y(\omega) = X(\omega - 2)$  } freq. shifting property

ii)  $y(t) = x(-2t)$

SOL  $\rightarrow y(\omega) = \frac{1}{2} X\left(\frac{-\omega}{2}\right)$  } time scaling

iii)  $y(t) = x(2t - 3)$

SOL  $\rightarrow y(t) = x(2t - 3) = x\left[2\left(t - \frac{3}{2}\right)\right]$

$$x(t) \longrightarrow x(2t) \longrightarrow y(t) = x\left[2(t - 1.5)\right] = F(t - 1.5)$$

$$X(\omega) \cdot F(\omega) = \frac{1}{2} X\left(\frac{\omega}{2}\right) \quad Y(\omega) = F(\omega) e^{-j1.5\omega}$$

$$= \frac{1}{2} X\left(\frac{\omega}{2}\right) e^{-j1.5\omega}$$

(OR)

$$y(t) = x(2t - 3) = x\left[2(t - 1.5)\right]$$

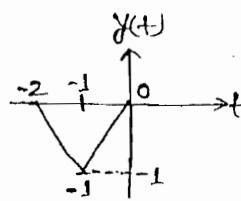
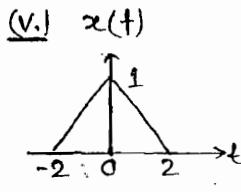
$$= \frac{1}{2} X\left(\frac{\omega}{2}\right) e^{-j\omega 1.5}$$

iv)  $y(t) = x(-2t - 4)$

SOL  $\rightarrow y(t) = x\left[-2(t + 2)\right]$

$$\downarrow t_0 = 2$$

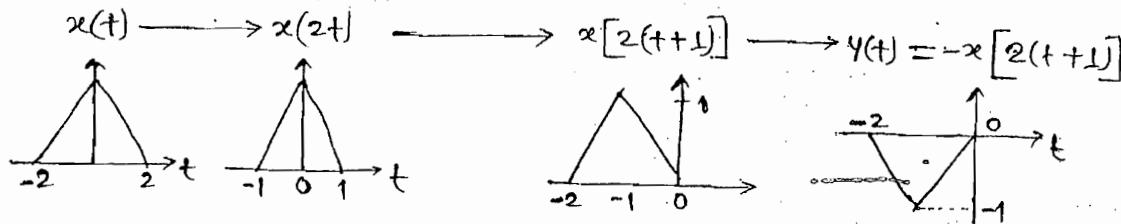
$$Y(\omega) = \frac{1}{2} X\left(\frac{-\omega}{2}\right) e^{j\omega 2}$$



$$y(\omega) = ?$$

SOLN

$$T_{01} = 4 \quad T_{02} = 2$$



$$y(t) = -x[2(t+1)]$$

$$Y(\omega) = -\frac{1}{2} \times \left(\frac{\omega}{2}\right) e^{j\omega} \quad (t_0 = 1)$$

Q.  $\Rightarrow y(t) = x(t) * h(t) \dots \text{(i)}$

$$g(t) = x(3t) * h(3t) \dots \text{(ii)}$$

If  $g(t) = A y(Bt)$  then calculate values of A & B.

SOLN  $\Rightarrow$  from eqn (i)

$$Y(\omega) = X(\omega) H(\omega) \dots \text{(iii)}$$

From eqn (ii)

$$G(\omega) = \left[ \frac{1}{3} X\left(\frac{\omega}{3}\right) \right] \left[ \frac{1}{3} H\left(\frac{\omega}{3}\right) \right]$$

$$G(\omega) = \frac{1}{9} \left[ X\left(\frac{\omega}{3}\right) H\left(\frac{\omega}{3}\right) \right]$$

$$= \frac{1}{9} [Y\left(\frac{\omega}{3}\right)] \quad \text{from eqn (iii)}$$

$$= \frac{1}{3} \left[ \frac{1}{3} Y\left(\frac{\omega}{3}\right) \right]$$

$$g(t) = \frac{1}{3} Y(3t)$$

$$g(t) = A Y(Bt) = \frac{1}{3} Y(3t)$$

$$A = \frac{1}{3}, B = 3$$

2nd method  $\Rightarrow$

$$x(t) * h(t) = y(t)$$

$$x(at) * h(at) = \frac{1}{|a|} y(a(t))$$

$|a|=3$

$$x(3t) * h(3t) = \frac{1}{3} y(3t)$$

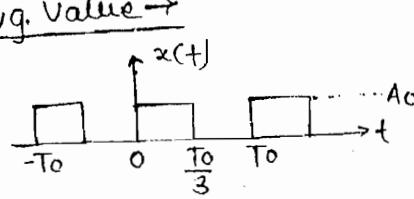
$$g(t) = A Y(Bt)$$

By comparison;

$$A = \frac{1}{3} \text{ (or)} B = 3$$

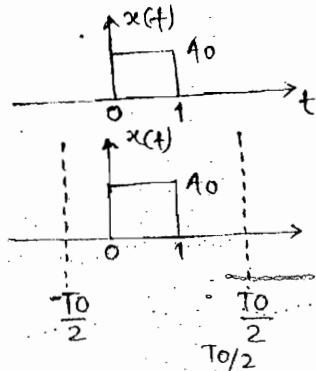
Ques  
Avg. Value  $\rightarrow$

(1)



$$\text{avg.} = \frac{A_0}{3}$$

(2)



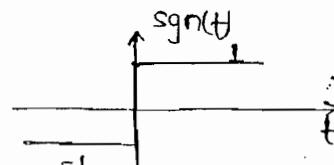
$$\text{Avg.} = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$= \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_0^{T_0} A_0 dt = \frac{A_0}{T_0} \cdot T_0 = A_0$$

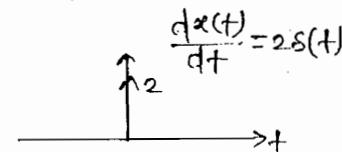
$$\text{Avg.} = 0$$

for any finite duration pulse Avg. value  
will be = 0

Q.  $\rightarrow x(t) = \text{sgn}(t) \rightleftharpoons X(\omega) = ?$

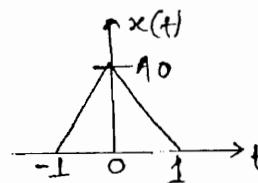


Soln  $\rightarrow$



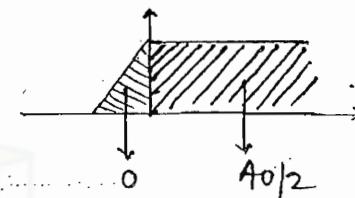
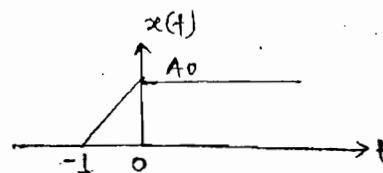
$$\frac{d x(t)}{dt} = 2\delta(t)$$

FT.



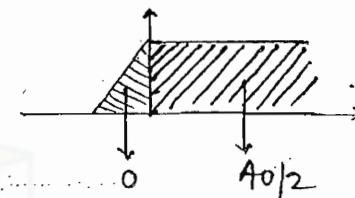
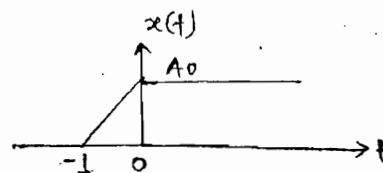
$$\text{avg.} = A_0$$

(3.)



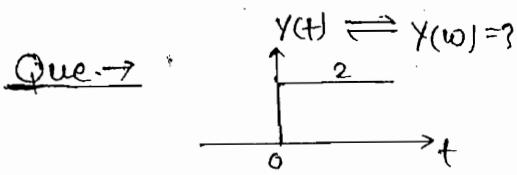
$$\text{avg.} = \frac{A_0}{2}$$

(4.)



$$\text{avg.} = \frac{A_0}{2}$$

$$x(t) = \text{sgn}(t) \rightleftharpoons X(\omega) = \frac{2}{j\omega}$$

Soln  $\rightarrow$ 

$$\frac{dy(t)}{dt} \xrightarrow{t=0} \text{avg} = \frac{2}{2} = 1$$

$$\frac{dy(t)}{dt} = 2\delta(t)$$

$$[\text{J}(\omega)] y(\omega) = 2$$

$$y(\omega) = \frac{2}{j\omega} X$$

2nd method  $\rightarrow$ 

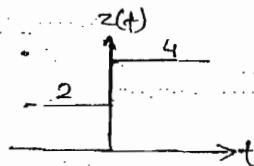
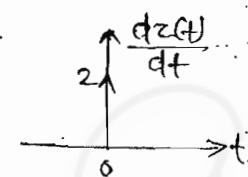
$$Y(t) = 1 + x(t)$$

$$\downarrow \text{FT}$$

$$Y(\omega) = 2\pi\delta(\omega) + X(\omega)$$

$$= \frac{2}{j\omega} + 2\pi\delta(\omega)$$

$$Y(\omega) = \boxed{\frac{2}{j\omega} + 2\pi\delta(\omega)}$$

Que.  $\rightarrow$ Soln  $\rightarrow$ 

$$j\omega z(\omega) = 2$$

$$z(\omega) = \frac{2}{j\omega} X$$

2nd method  $\rightarrow$ 

$$Z(t) = 3 + z(t)$$

 $\downarrow \text{FT}$ 

$$Z(\omega) = 6\pi\delta(\omega) + \frac{2}{j\omega} X(\omega)$$

$$= X(\omega) + 6\pi\delta(\omega)$$

$$Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)$$

$$Z(\omega) = \boxed{\frac{2}{j\omega} + 6\pi\delta(\omega)}$$

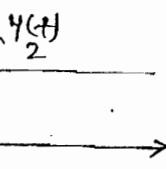
$$\text{Avg.} = \frac{4+2}{2} = 3$$

$$\downarrow$$

$$3 \times 2\pi\delta(\omega)$$

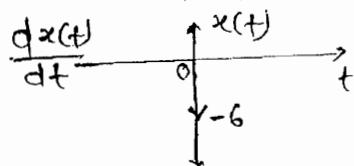
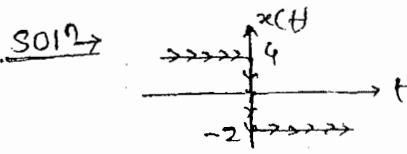
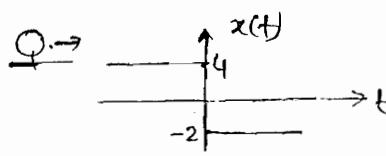
$$6\pi\delta(\omega)$$

$$z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)$$



$$2u(t) = \left[ \frac{2}{j\omega} + 6\pi\delta(\omega) \right]$$

$$\frac{2u(t)}{2} = \frac{1}{2} \left[ \frac{2}{j\omega} + 6\pi\delta(\omega) \right]$$



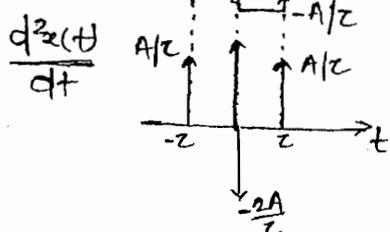
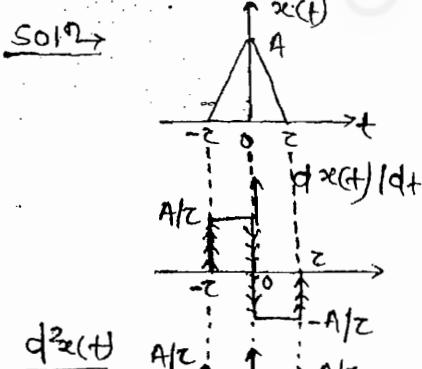
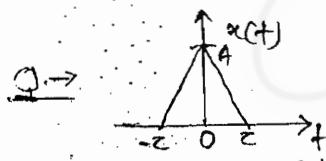
$$\frac{d^2x(t)}{dt^2} = -6\delta(t)$$

$$j\omega x(\omega) = -6$$

$$x(\omega) = \frac{-6}{j\omega}$$

$$\text{Avg.} = \frac{4+2}{2} = 1$$

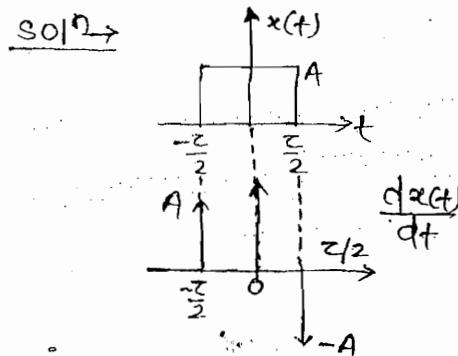
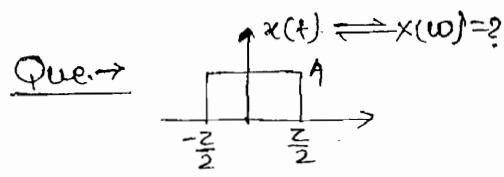
$$X(\omega) = \frac{-6}{j\omega} + 2\pi S(\omega)$$



$$\frac{d^2x(t)}{dt^2} = \frac{A}{2}\delta(t+2) + \frac{A}{2}\delta(t-2) - \frac{2A}{2}\delta(t)$$

FT

$$(j\omega)^2 X(\omega) = A(1 - \frac{\omega^2}{2\omega_0^2})$$



$$\frac{d^2x(t)}{dt^2} = A\delta(t+\frac{\pi}{2}) - A\delta(t-\frac{\pi}{2})$$

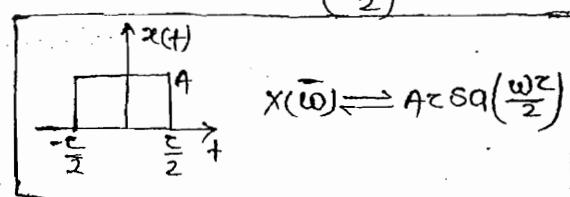
$$j\omega x(\omega) = Ae^{-j\omega/2\omega_0} - Ae^{j\omega/2\omega_0}$$

$$X(\omega) = \frac{A}{j\omega} \times \left[ e^{-j\omega/2\omega_0} - e^{j\omega/2\omega_0} \right]$$

$$= \frac{A}{j\omega} \times 2j \times \sin\left(\frac{\omega}{2}\right)$$

$$= \frac{2A}{\omega} \left[ \frac{\sin(\frac{\omega}{2})}{j\omega} \right] \times \left[ \frac{j\omega}{\sin(\frac{\omega}{2})} \right]$$

$$= 4\pi \sin\left(\frac{\omega}{2}\right)$$



$$X(\omega) = \frac{A}{\tau(-\omega^2)} \left[ (e^{j\omega z} + e^{-j\omega z}) - 2 \right]$$

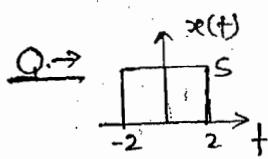
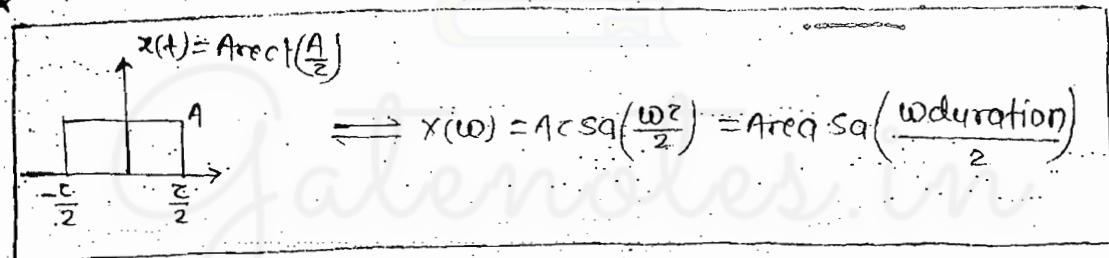
$$X(\omega) = \frac{A}{-\tau\omega^2} \left[ 2\cos\omega z - 2 \right]$$

$$X(\omega) = \frac{2A}{\tau\omega^2} (1 - \cos\omega z)$$

$$= \frac{2A}{\tau\omega^2} \left[ \sin^2\left(\frac{\omega z}{2}\right) \right]$$

$$= \frac{2A}{\tau\omega^2} \times 2 \frac{\sin^2\left(\frac{\omega z}{2}\right)}{\left(\frac{\omega z}{2}\right)^2} \times \left(\frac{\omega z}{2}\right)^2$$

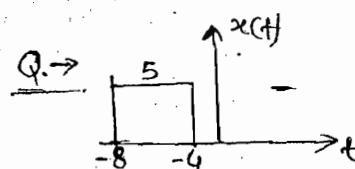
$$X(\omega) = A z \operatorname{sq}^2\left(\frac{\omega z}{2}\right)$$



$$\underline{\text{Soln}} \rightarrow X(\omega) = 20 \operatorname{sq}\left(\frac{\omega 4}{2}\right)$$

$$= 20 \operatorname{sq}(2\omega)$$

$$X(\omega) = 20 \operatorname{sq}(2\omega)$$

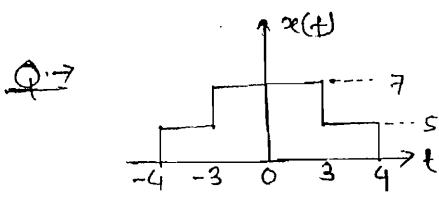
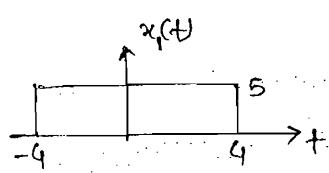


$$\underline{\text{Soln}} \rightarrow Y(t) = x(t+6)$$

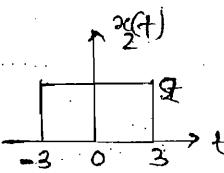
$$Y(\omega) = X(\omega) e^{j\omega 6}$$

$$= 20 \operatorname{sq}(2\omega) e^{j\omega 6}$$

$$Y(\omega) = 20 \operatorname{sq}(2\omega) e^{j\omega 6}$$

Soln.

$$x_1(\omega) = 40 \operatorname{Sa}(4\omega)$$



$$x_2(\omega) = 12 \operatorname{Sa}(3\omega)$$

$$x(t) = x_1(t) + x_2(t)$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$X(\omega) = 40 \operatorname{Sa}(4\omega) + 12 \operatorname{Sa}(3\omega)$$

Q.  $x(t) = \operatorname{rect}(t - \frac{1}{2})$

$$y(t) = x(t) + x(-t) \Rightarrow y(\omega) \quad \text{Find } y(\omega) = ?$$

(a.)  $\operatorname{sinc}(\frac{\omega}{2\pi})$       (c.)  $2\operatorname{sinc}(\frac{\omega}{\pi}) \cos(\frac{\omega}{2})$

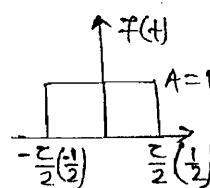
(b.)  $2\operatorname{sinc}(\frac{\omega}{2\pi})$

Soln.  $x(t) = \operatorname{rect}(t - \frac{1}{2})$

$$x(-t) = \operatorname{rect}(-t - \frac{1}{2})$$

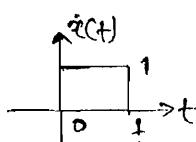
$$y(t) = x(t) + x(-t) = \operatorname{rect}(t - \frac{1}{2}) + \operatorname{rect}(-t - \frac{1}{2})$$

$$f(t) = \operatorname{rect}(t) = A \operatorname{rect}(\frac{t}{2})$$

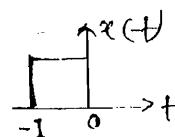
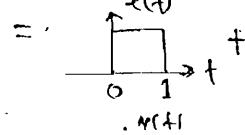


$$\therefore \operatorname{Sa}(k) = \operatorname{sinc}(\frac{k}{\pi})$$

$$x(t) = f(t - \frac{1}{2})$$



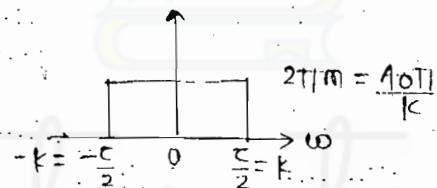
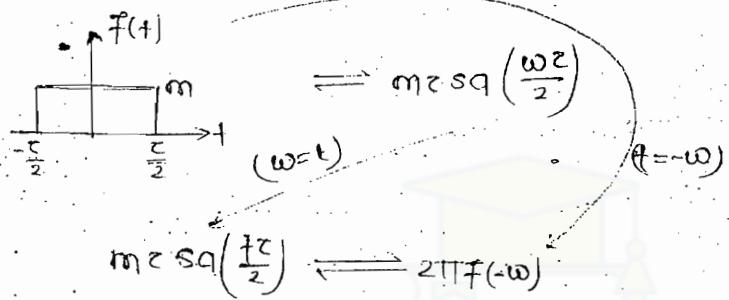
$$y(t) = x(t) + x(-t)$$



$$\begin{aligned}
 Y(\omega) &= 2\text{sinc}(\omega) = 2\text{sinc}\left(\frac{\omega}{\pi}\right) \\
 &= \frac{2\sin\omega}{\omega} = 2 \times \left(\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\right) \times \cos\frac{\omega}{2} = 2\text{sinc}\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right) \\
 &= 2 \times \cancel{\sin\frac{\omega}{2}} \cdot \cancel{\cos\frac{\omega}{2}} \quad \underline{\omega} \quad = 2\text{sinc}\left(\frac{\omega}{2\pi}\right) \cdot \cos\left(\frac{\omega}{2}\right) \\
 &\boxed{Y(\omega) = 2\text{sinc}\left(\frac{\omega}{2\pi}\right) \cdot \cos\left(\frac{\omega}{2}\right)}
 \end{aligned}$$

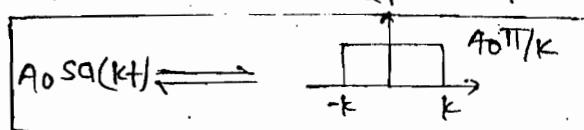
Q.  $\rightarrow x(t) = A_0 \text{sinc}(t) \iff \text{Draw FT } X(\omega)$

Soln  $\rightarrow$



$$m \cdot 2 = A_0, \quad K = \frac{\pi}{2}$$

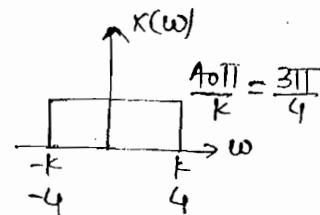
$$2\pi/m = 2\pi \cdot \frac{A_0}{2} = \frac{2\pi \cdot A_0}{2K} = \frac{A_0\pi}{K}$$



Q.  $\rightarrow x(t) = 3 \text{sinc}(4t) \iff X(\omega)$

Soln  $\rightarrow$

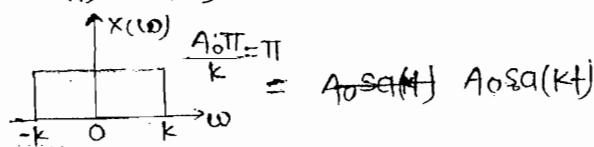
$$\begin{aligned}
 A_0 &= 3 & A_0 \text{sinc}(Kt) \\
 K &= 4
 \end{aligned}$$



Q. → Calculate area & energy of  $x(t) = \text{sq}(t)$

Soln →

$$x(t) = \text{sq}(t)$$



$$x(t) = A_0 \text{sq}(kt) = \text{sq}(t)$$

$$A_0 = 1, k = 1$$

area under time-domain →

$$\text{area of } x(t) = x(\omega)|_{\omega=0}$$

$$A = \pi$$

Parseval's energy theorem →

$$\begin{aligned} E &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 d\omega \\ &= \frac{1}{2\pi} \int_{-1}^{1} \pi^2 d\omega \end{aligned}$$

$$E = \pi$$

Q. →  $h(t) \Leftrightarrow H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

Find  $h(0) = ?$

- (a) 1/4 (b) 1/2 (c) 1 (d) 2

Soln →

$$H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$$

$$= \frac{\sin(3\omega) + \sin\omega}{\omega}$$

$$= \frac{\sin(3\omega)}{\omega} + \frac{\sin\omega}{\omega}$$

$$= \frac{3\sin(3\omega)}{3\omega} + \frac{\sin\omega}{\omega}$$

$$h(\omega) = 3\text{sq}(3\omega) + \text{sq}(\omega)$$

$$= H_1(\omega) + H_2(\omega)$$

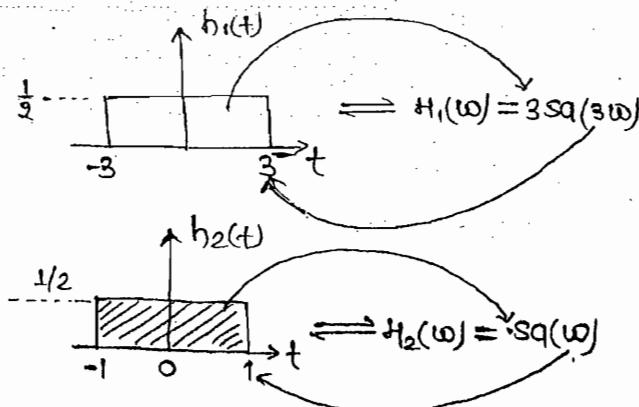
$$h(t) = h_1(t) + h_2(t)$$

$$\downarrow t=0$$

$$h(0) = h_1(0) + h_2(0)$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$



Area

$$x(t) \rightarrow A$$

$$x(kt) \rightarrow \frac{A}{k}$$

$$kx(t) \rightarrow kA$$

$$Sa(\omega) \rightarrow \pi$$

$$Sa(3\omega) \rightarrow \pi/3$$

$$3 \cdot Sa(3\omega) \rightarrow 3 \times \frac{\pi}{3} = \pi$$

2nd method  $\rightarrow$ 

Area under freq. domain

$$2\pi h(0) = \text{Area of } H(\omega)$$

$$h(0) = \frac{\text{Area of } H(\omega)}{2\pi}$$

$$h(0) = \frac{2\pi + \pi}{2\pi} = 1$$

$$\boxed{h(0) = 1}$$

Q.  $\rightarrow y(t) = x(t) \cos t \iff y(\omega) = \begin{cases} 2, & |\omega| \leq 2 \\ 0, & \text{otherwise.} \end{cases}$

Find  $x(t)$ .

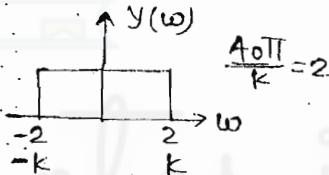
- (a)  $\frac{4}{\pi} \frac{\sin t}{t}$    (b)  $\frac{2\sin t}{t}$    (c)  $\frac{4\sin t}{t}$    (d)  $2\pi \frac{\sin t}{t}$

Sol'n  $\rightarrow$  1st method  $\rightarrow$

$$y(t) = A_0 Sa(kt) \iff$$

$$= \frac{4}{\pi} \sin(2t)$$

$$K=2, A_0 = 4/\pi$$



$$= \frac{4}{\pi} \cdot \frac{\sin 2t}{2t} = \left[ \frac{4}{\pi} \cdot \frac{2\sin t}{2t} \right] \cdot \cos t$$

$$= \left[ \frac{4}{\pi} \frac{\sin t}{t} \right] \cdot \cos t$$

$$= x(t) \cdot \cos t$$

2nd method  $\rightarrow$

Area under freq. domain

$$2\pi y(0) = \text{Area of } y(\omega)$$

$$2\pi y(0) = 8$$

$$y(0) = \frac{8}{2\pi} = \frac{4}{\pi} = x(0)$$

$y(0) = x(0) \text{ ok.}$

Q. → Find FT of  $\cos \omega_0 t$

$$x(t) = \cos \omega_0 t \iff X(\omega) = ?$$

Soln

$$x(t) = \overset{\text{~~~~~}}{\cos \omega_0 t}$$

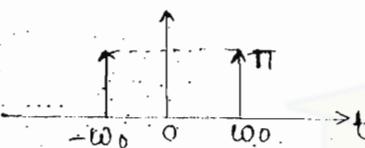
$$x(t) = \frac{1}{2} [e^{j\omega_0 t} + e^{-j\omega_0 t}]$$

$$X(\omega) = \frac{1}{2} [2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0)]$$

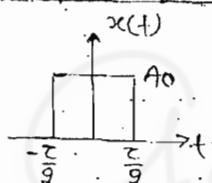
$$X(\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$

$$\cos \omega_0 t = \pi \delta(\omega - \omega_0) + \delta(\omega + \omega_0)$$

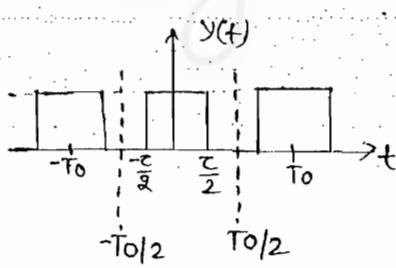
$$\begin{cases} A_0 = 2\pi A_0 \delta(\omega) \\ \downarrow A_0 = 1 \\ 1 = 2\pi \delta(\omega) \\ 1 \cdot e^{j\omega_0 t} = 2\pi \delta(\omega - \omega_0) \\ e^{-j\omega_0 t} = 2\pi \delta(\omega + \omega_0) \end{cases}$$



\* Calculation of  $c_n$  by using FT →



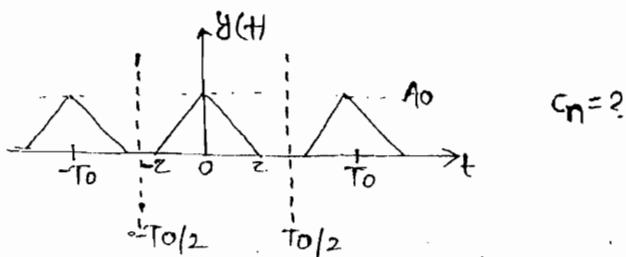
$$X(\omega) = A_0 \frac{\sin(\omega T_0)}{\omega T_0}$$



$$c_n = \frac{X(n\omega_0)}{T_0}$$

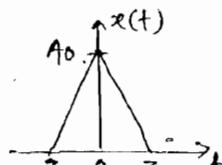
$$X(\omega) = \frac{A_0 \omega}{T_0} \sin\left(\frac{n\omega_0 T_0}{2}\right)$$

Q. →



$$c_n = ?$$

Soln



$$X(\omega) = A_0 \frac{\sin^2(\omega T_0/2)}{\omega T_0/2}$$

$$c_n = Y(n\omega_0) - A_0 \frac{\sin^2(n\omega_0 T_0/2)}{n\omega_0 T_0/2}$$

\* FT For periodic Signal  $\rightarrow$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad X(\omega) = ?$$

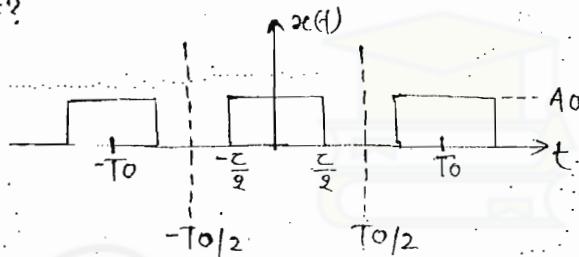
$$j = 2\pi \delta(\omega)$$

$$c_n = 2\pi c_n \delta(\omega)$$

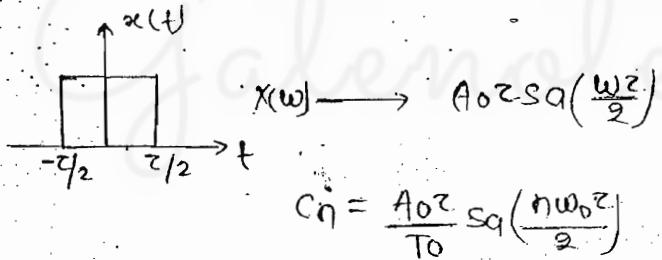
$$c_n e^{jn\omega_0 t} = 2\pi c_n \delta(\omega - n\omega_0) \quad (\text{freq: shifting})$$

$$\sum c_n e^{jn\omega_0 t} = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

Que.  $\rightarrow X(\omega) = ?$



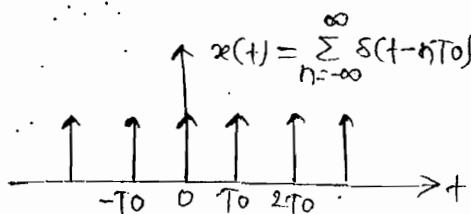
Soln  $\rightarrow$



$$X(\omega) = 2\pi \sum c_n \delta(\omega - n\omega_0)$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \left[ A_0 \cdot 2 \cdot \frac{\sin(\frac{n\omega_0}{2})}{\frac{n\omega_0}{2}} \right] \delta(\omega - n\omega_0)$$

Que.  $\rightarrow X(\omega) = ?$



Soln  $\rightarrow$

$$X(\omega) = 2\pi \sum c_n \delta(\omega - n\omega_0)$$

$$= 2\pi \sum \frac{1}{T_0} \delta(\omega - n\omega_0)$$

\*Important Signal  $\rightarrow$

$x(t)$

$X(\omega)$

(1.)  $\delta(t)$

1

(2.)  $u(t)$

$$\frac{1}{j\omega} + \pi \delta(\omega)$$

(3.)  $\text{sgn}(t)$

$$\frac{2}{j\omega}$$

(4.)  $A_0$

$$2\pi A_0 \delta(\omega)$$

(5.)  $e^{-at} u(t), a > 0$

$$\frac{1}{a+j\omega}$$

(6.)  $e^{-at+1} u(t), a > 0$

$$\frac{2a}{a^2 + \omega^2}$$

(7.)  $\cos \omega_0 t$

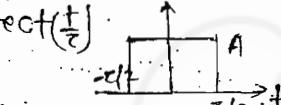
$$\pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

(8.)  $\sin \omega_0 t$

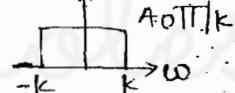
$$\pi j [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

(9.)  $A \text{rect}\left(\frac{t}{2}\right)$

$$A \text{rect}\left(\frac{\omega}{2}\right)$$



(10.)  $A_0 \sin(\omega_0 t)$

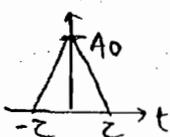


(11.) Periodic sig.

$$2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$$

(12.)  $\sum \delta(t - nT_0)$

$$w_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$



(13.)  $A_0 e^{j\omega_0 t}$

$$4\pi \text{rect}^2\left(\frac{\omega_0}{2}\right)$$

(14.)  $e^{-j\omega_0 t}$

$$2\pi \delta(\omega - \omega_0)$$

$$2\pi \delta(\omega + \omega_0)$$

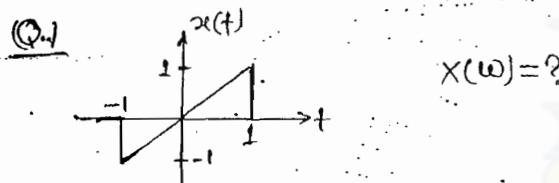
[DATE-21/10/14]

\*  $x(t)-X(\omega)$  pairs  $\rightarrow$

$x(t)$	$X(\omega)$
Real	CS
CS	Real
Img.	CAS
CAS	Img.
R + I	R + I

R + I	I + 0
I + 0	R + I

$x(t)$	$X(\omega)$
Continuous	Non-periodic
Non-periodic	continuous
Discrete	periodic
periodic	discrete
C+P	D+NP
C+NP	C+NP
D+P	D+P
D+NP	C+P



(a)  $4\pi j \left[ \frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega} \right]$

(c)  $2j \left[ \frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega^2} \right]$

(b)  $4\pi j \left[ \frac{\cos \omega}{\omega^2} - \frac{\sin \omega}{\omega} \right]$

(d)  $2j \left[ \frac{\cos \omega}{\omega^2} + \frac{\sin \omega}{\omega} \right]$

SOLN → For soln go through the option.

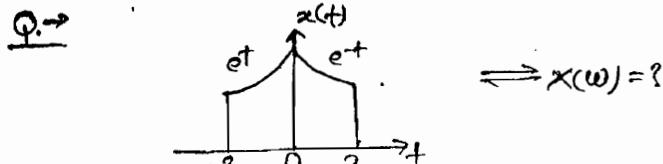
ans. (c)

$x(t) \rightarrow R+O$

I+O

$\frac{\sin \omega}{\omega} = E$

$X(\omega) = 2j \left[ \frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega^2} \right]$



(a)  $2 - 2e^{-2} \sin 2\omega + 2\omega e^{-2} \sin 2\omega$

(b)  $2 + 2e^{-2} \cos 2\omega - 2\omega e^{-2} \cos 2\omega$

(c)  $2 + 2e^{-2} \cos 2\omega - 2\omega e^{-2} \sin 2\omega$

(d)  $2 - 2e^{-2} \cos 2\omega + 2\omega e^{-2} \sin 2\omega$

SOL<sup>n</sup>  $\Rightarrow x(t) = R+E ; x(\omega) = R+E$

so; option (a) & (b)  $\neq R+E$

Area under time domain;

$$\begin{aligned} x(0) &= \int_{-\infty}^{\infty} x(t) dt \\ &= \int_{-2}^{2} x(t) dt = 2 \int_0^2 x(t) dt \\ &= 2 \int_0^2 e^t dt \\ &= 2 \left[ e^t \right]_0^2 = 2(1 - e^2) \\ &= 2 - 2e^2 \end{aligned}$$

Now; put  $x(0)$  in the option (C) & (D)

**Ans. (d)**

Q.  $\rightarrow f(t) \iff F(\omega)$

$$g(t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

what is the relationship bet<sup>n</sup>  $f(t)$  &  $g(t)$ ?

- (a.)  $g(t)$  would always be proportional to  $f(t)$ .
- (b.)  $g(t)$  would always be proportional to  $f(t)$  if  $f(t)$  is an even sig.
- (c.)  $g(t)$  would proportional to  $f(t)$  only if  $f(t)$  is sinusoidal f?
- (d.)  $g(t)$  would never be proportional to  $f(t)$ .

SOL<sup>n</sup> IFT (inverse FT)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(u) e^{jut} du$$

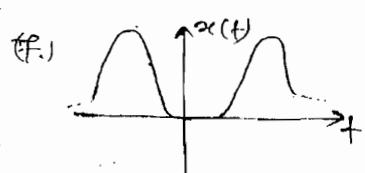
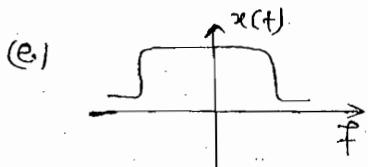
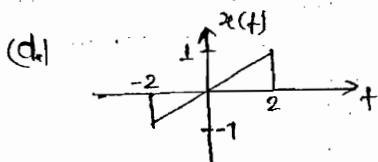
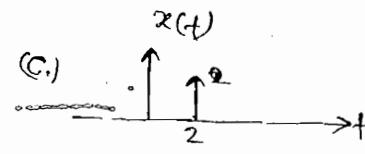
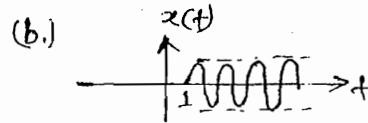
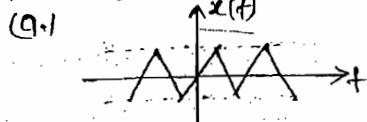
$$\downarrow (t = -u)$$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(u) e^{jut} du$$

$$\boxed{g(t) = 2\pi f(t)}$$

If  $f(t)$  is even  
 $f(-t) = f(t)$

Q.  $\rightarrow$  sig.  $x(t)$  is a real sig.



(iv)  $\operatorname{Re}[x(\omega)] = 0$

- (a) a,d (b) e,f (c) b,c (d) b,d.

Sol 2

$$x(t) \rightarrow \operatorname{Real}$$

$$\begin{aligned} x(\omega) &\rightarrow \text{CS} \quad (\text{given}) \\ &= \operatorname{Real}[x(\omega)] + i \operatorname{Imag}[x(\omega)] \\ &\quad \downarrow \quad \downarrow \\ &\text{even} \quad \text{odd} \end{aligned}$$

$\cdots x(\omega) \rightarrow \text{CS} \& \text{ nature is Imag. odd.}$

$$\text{so } x(t) \rightarrow R + 0i$$

[ans(s) (a)]

(ii)  $\int_{-\infty}^{\infty} x(\omega) d\omega = 0$

- (a) e (b) a,b,c,d,f (c) b,c (d) a,d,e,f.

Sol 3  $\rightarrow$  Area under freq. domain

$$2\pi x(0) = \int_{-\infty}^{\infty} x(\omega) d\omega$$

$$2\pi x(0) = 0$$

[x(0) = 0]

[ans.(b)]

iii)  $\int_{-\infty}^{\infty} i\omega x(\omega) d\omega = 0$

- (a) a, b, c, d, f    (b) b, c, e, f    (c) e    (d) b, c

Soln  $x(t) \iff x(\omega)$

$$\frac{dx(t)}{dt} = (j\omega)x(\omega)$$

$$\frac{1}{j} \frac{dx(t)}{dt} = \omega x(\omega)$$

$$y(t) = y(\omega)$$

area under freq. domain

$$\begin{aligned} 2\pi y(0) &= \int_{-\infty}^{\infty} y(\omega) d\omega \\ &= \int_{-\infty}^{\infty} \omega x(\omega) d\omega \end{aligned}$$

$$2\pi y(0) = 0$$

$$y(0) = 0$$

$$y(t) \Big|_{t=0} = 0$$

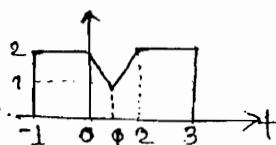
$$\frac{1}{j} \frac{dx(t)}{dt} \Big|_{t=0} = 0$$

$$\boxed{\frac{dx(t)}{dt} = 0}$$

(slope is zero at the origin)

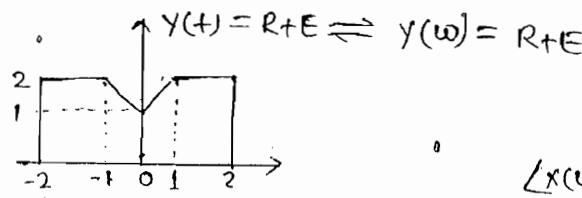
ans. (b)

Q.  $\rightarrow x(t) \iff x(\omega)$



Find  $\angle x(\omega) = ?$

Soln



$$\angle x(\omega) = \angle y(\omega) + (-\omega)$$

$$x(t) = y(t-1)$$

$$\therefore y(\omega) = R + E \text{ & } \angle y(\omega) = 0$$

Where;  $f(0^-) = \lim_{t \rightarrow 0^-} f(t)$

$$f'(0^-) = \left. \frac{df(t)}{dt} \right|_{t=0^-} \quad f''(0^-) = \left. \frac{d^2f(t)}{dt^2} \right|_{t=0^-}$$

### (8.) Integration in time →

$$\int_{-\infty}^{+\infty} f(t) dt = \begin{cases} \frac{F(s)}{s}, & \text{Bilateral FT.} \\ \frac{F(s)}{s} + \frac{\int_0^{-} f(t) dt}{s}, & \text{unilateral LT.} \end{cases}$$

### (9.) Convolution in time →

$$f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$$

### (10.) Multiplication in time →

$$f_1(t) \cdot f_2(t) = \frac{1}{2\pi j} [F_1(s) * F_2(s)]$$

### (11.) Differentiation in freq. →

$$t^n f(t) = (-j)^n \frac{d^n F(s)}{ds^n}$$

### (12.) Integration in freq. →

$$\frac{f(t)}{t} = \int_s^{\infty} F(s) ds$$

### (13.) Initial value theorem →

$$x(0) = \lim_{s \rightarrow \infty} [sx(s)]$$

Condition:- It is applicable only for causal type signals.

$$\text{i.e. } x(t) = 0; t < 0$$

### (14.) Final value theorem →

$$x(\infty) = \lim_{s \rightarrow 0} [sx(s)]$$

Condition →

(i) It is applicable only for causal type signals.

$$\text{i.e. } x(t) = 0, t < 0$$

\* (ii) "sx(s)" should have only RHS poles in s-plane.

Que →  $F(s) = \frac{1}{s^2 + 1} \Rightarrow f(t)$ . Calculate  $f(\infty) = ?$

- (a) -1 (b) 0 (c) 1 (d)  $-1 \leq f(\infty) \leq 1$ .

Soln →  $sF(s) = \frac{s}{s^2 + 1}$

Poles:  $s = \pm i \neq \text{LHS poles}$

\* FVT is not applicable because  $s = \pm i$  poles.

$$f(t) = \text{Im} \{ f(t) \}$$

$$f(\infty) = \text{Im} \{ \infty u(\infty) \}$$

$$(-1)^{\frac{1}{2}}$$

$$f(\infty) = -1 \leq f(\infty) \leq 1$$

Que →  $F(t) = F(s) = \frac{1}{s(s-1)}$ ,  $f(\infty) = ?$

- (a) 0, (b)  $\infty$  (c) -1 (d) 1

Soln → (1) signal is causal because depend on past ( $s-1$ )

$$(2) sF(s) = \frac{s}{s(s-1)} = \frac{s}{s-1}$$

Pole:  $s=1 \neq \text{LHS plane}$

FVT is not applicable.

$$F(s) = \frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{s-1}$$

$$f(t) \in e^{At}x(t) + c \quad f(t) = e^t u(t) - u(t)$$

$$f(\infty) = \infty - 1 + \infty$$

$$\boxed{f(\infty) \approx \infty}$$

$$Q \rightarrow f(t) = e^{-at} u(t) \Rightarrow F(s) = ?, \text{ ROC} = ?$$

Soln

$$f(t) = e^{-at} u(-t)$$

$$e^{-at} u(t) \Leftrightarrow \frac{1}{s+a}; (\sigma > -a)$$

$$-e^{-at} u(t) \Leftrightarrow \frac{-1}{s+a}; (\sigma > -a)$$

$\downarrow (t = -t)$        $\downarrow (s = -s)$        $\xrightarrow{\text{By time Reversal}}$   
 $(\sigma = -\sigma)$

$$-e^{at} u(-t) \Leftrightarrow \frac{-1}{s-a}; (-\sigma > -a) \quad \left\{ \begin{array}{l} s = \sigma + j\omega \\ -s = -\sigma - j\omega \end{array} \right.$$

$\downarrow (a = -a)$        $\downarrow (a = -a)$

$$-e^{at} u(-t) \Leftrightarrow \frac{-1}{s-a}; (-\sigma > a)$$

$$-e^{-at} u(-t) \Leftrightarrow \frac{1}{s+a}; (\sigma < -a)$$

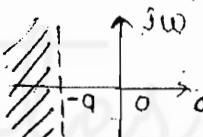
$$-e^{-at} u(t) \Leftrightarrow \frac{1}{s+a}; (\sigma < -a)$$

$\therefore$  also;

$$e^{-at} u(t) \Leftrightarrow \frac{1}{s+a}; (\sigma > -a)$$

$$\Rightarrow u(t) \rightarrow \frac{1}{s}; \sigma < 0$$

$$\Rightarrow u(t) \rightarrow \frac{1}{s}; \sigma > 0$$



$e^{-at} u(t) \rightarrow \frac{1}{s+a}; (\sigma > -a)$
$-e^{-at} u(-t) \rightarrow \frac{1}{s+a}; (\sigma < -a)$

$u(-t) \rightarrow \frac{1}{s}; \sigma < 0$
$u(t) \rightarrow \frac{1}{s}; \sigma > 0$

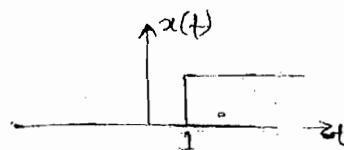
..

Region of Convergence (ROC) → It is defined as the range of complex variable as in s-planes for which LT of signal is convergent (or) finite.

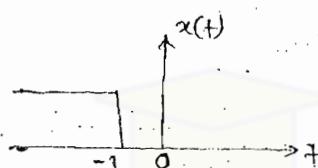
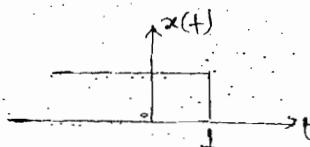
Properties →

(i) ROC doesn't include any pole.

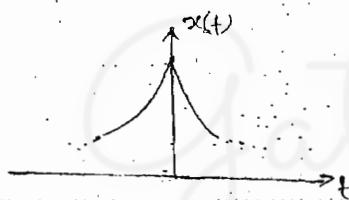
Right sided signal →



Left sided signal →



Both sided signal →



(ii) For right side signal, ROC will be right side to the right most pole.  
for left sided signal, ROC will be left side to the left most pole.

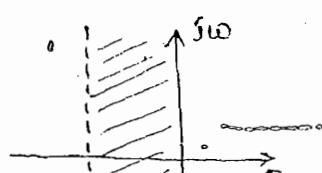
(iii) For stability ROC includes imaginary axis in s-plane.

(iv) For both sided sig, ROC is a strip in s-plane.

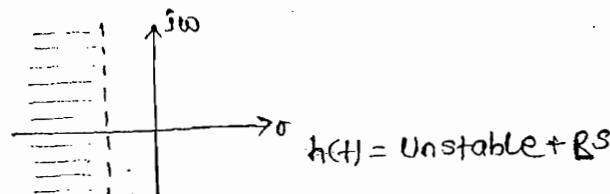
(v) For  $\infty$  duration signal ROC is entire s-plane excluding possibly  $s=0$  or  $\pm\infty$ .

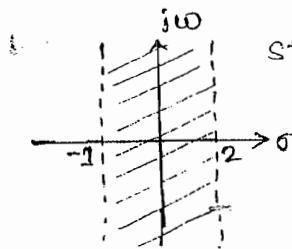
Q. → Check stability of LTI sys. of & comment about extension of  $h(t)$ .

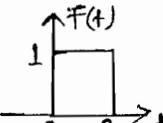
(i) ROC:  $\sigma > -1$



(ii) ROC:  $\sigma < -2$



(3) ROC:  $-1 < \sigma < 2$ strip type  $h(t) = \text{stable + Both side.}$ 

Que.  $\rightarrow$    $\Leftrightarrow F(s) = ? ; \text{ROC} = ?$

Soln  $\rightarrow F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$

$$= \int_0^3 e^{-st} dt = \left[ \frac{e^{-st}}{-s} \right]_0^3 = \frac{1 - e^{-3s}}{s}$$

$\text{At } (s=0)$   $F(0) = \frac{1 - e^{0}}{s} = \frac{0}{0}$  then use L-Hospital Rule

$$\text{diff wrt } s = 3e^{-3s} \Big|_{s=0} = 3$$

$\text{At } (s=\infty)$   $F(\infty) = 0$

$\text{At } (s=-\infty)$   $F(-\infty) = 3e^{-3s} \Big|_{(s=-\infty)} = 3e^{\infty} = \infty$

ROC: Entire s-plane excluding  $s = -\infty$ 

$$e^{2t} u(-t+3)$$

ROC:  $\sigma < 2$

$$e^{3t} u(-t-4)$$

ROC:  $\sigma < 2$

Que.  $\rightarrow F(t) = e^{2t} u(-t) + e^{3t} u(t)$

Soln  $\rightarrow F(t) = \underbrace{e^{2t} u(-t)}_{\text{ROC: } \sigma < -2} + \underbrace{e^{3t} u(t)}_{\sigma > 3}$

LT of  $f(t)$  will not exist  
because of no common ROC

Que.  $\rightarrow f(t) = -e^{-2t} u(-t) + e^{-3t} u(t)$

Soln  $\rightarrow F(t) = \underbrace{-e^{-2t} u(-t)}_{\text{ROC: } \sigma < -2} + \underbrace{e^{-3t} u(t)}_{\sigma > 3}$

common ROC:  $\underline{-3 < \sigma < -2}$   
strip

Q.  $\rightarrow f(t) = e^{-3t+1}$ ; ROC = ?

Soln.  $\rightarrow f(t) = \begin{cases} e^{3t} & ; t < 0 \\ e^{-3t} & ; t > 0 \end{cases}$

$$\text{ROC: } -3 < \sigma < 3 \quad f(t) = \frac{e^{3t}u(t) + e^{-3t}u(t)}{\downarrow \quad \downarrow} \quad \begin{matrix} \sigma < 3 \\ \sigma > -3 \end{matrix}$$

Ques.  $\rightarrow f(t) = e^{\sigma t}u(t)$ ; ROC = ?

Soln.  $\rightarrow f(t) = \begin{cases} 1 & ; t < 0 \\ e^{\sigma t} & ; t > 0 \end{cases}$  (No common ROC)

$$= \frac{u(-t) + e^{\sigma t}u(t)}{\downarrow \quad \downarrow} \quad \begin{matrix} \sigma < 0 \\ \sigma > 1 \end{matrix}$$

Ques.  $\rightarrow f(t) = e^{\sigma t}u(t)$

Soln. where  $a = b+ic$

$$f(t) = e^{(b+ic)t}u(t)$$

for existence of LT

$$= \int_{-\infty}^{\infty} |f(t)| e^{-\sigma t} dt < \infty = \int_{-\infty}^{\infty} |e^{(b+ic)t} e^{-\sigma t}| dt < \infty = \int_{-\infty}^{\infty} e^{-(\sigma - b)t} |e^{ict}| dt < \infty$$

$$= \int_0^{\infty} e^{-(\sigma - b)t} dt < \infty \quad (\sigma - b) > 0 \quad \Rightarrow \sigma > b \quad \Rightarrow \sigma > -\operatorname{Re}(a)$$

Ques.  $\rightarrow f(t) = \cos \omega_0 t u(t) \iff F(s) = ?$ , ROC = ?

Soln.  $\cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$f(t) = \frac{1}{2} [e^{j\omega_0 t}u(t) + e^{-j\omega_0 t}u(t)]$$

$$e^{-at}u(t) \iff \frac{1}{s+a}; \quad \sigma > -\operatorname{Re}(a)$$

$$a = \sigma + i\omega_0$$

$$e^{-j\omega_0 t}u(t) \iff \frac{1}{s+j\omega_0}; \quad \sigma > 0$$

$$a = \sigma - j\omega_0$$

$$e^{j\omega_0 t}u(t) \iff \frac{1}{s-j\omega_0}; \quad \sigma > 0$$

## Important signals →

$F(t)$	$F(s)$	ROC
$s(t)$	1	entire s-plane
$u(t)$	$1/s$	$\sigma > 0$
$e^{at}u(t)$	$\frac{1}{s-a}$	$\sigma > a$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma < -a$
$t u(t)$	$\frac{1}{s^2}$	
$e^{at}t u(t)$	$\frac{1}{(s-a)^2}$	$\sigma > a$
$\cos \omega_0 t u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\sigma > 0$
$-t e^{at} u(t)$	$\frac{1}{(s-a)^2}$	$\sigma < -a$
$\sin \omega_0 t u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\sigma > 0$
$t^n u(t), n \geq 0$	$\frac{n!}{s^{n+1}}$	$\sigma > 0$

Que.  $\rightarrow r(t) = t u(t) \Leftrightarrow R(s) = 1/s^2$

Que.  $\rightarrow f(t) = t u(t-1) \Leftrightarrow [t-1] \cdot u(t-1) = (t-1) u(t-1) + u(t-1)$   
 $= \frac{1}{s^2} e^{-s} + \frac{e^{-s}}{s}$

$$y(t) = r(t-1) \Leftrightarrow Y(s) = R(s) e^{-s} = \frac{1}{s^2} e^{-s}$$

$$= (t-1) u(t-1)$$

Que.  $\rightarrow f(t) = (t^2 + 5t - 2) u(t-1)$

Sol.  $\rightarrow [t-1]^2 + 7(t-1) u(t-1)$

$$= [t-1]^2 + 7(t-1) u(t-1)$$

$$= (t-1)^2 u(t-1) + 7(t-1) u(t-1) + 4 u(t-1)$$

$$F(s) = \frac{2}{s^3} e^{-s} + \frac{7}{s^2} e^{-s} + \frac{4}{s} e^{-s}$$

Que.  $\rightarrow f(t) = (t^3 + 5t^2 + 3t + 1) u(t-1)$

Sol.  $\rightarrow f(t+1) = [(t+1)^3 + 5(t+1)^2 + 3(t+1) + 1] u(t)$

$$f(t+1) = [t^3 + 8t^2 + 16t + 10] u(t)$$

↓ LT

$$F(s) = \underline{16} + \underline{16} + \underline{16} + \underline{10} e^{-s}$$

DATE-28/10/14

$$\text{Q.} \rightarrow f(t) = F(s) = \log\left(\frac{s+5}{s+6}\right)$$

$$\text{Find } f(t) : \text{(a)} \frac{1}{t}(e^{-6t} - e^{-5t})u(t) \quad \text{(c)} + (e^{-6t} - e^{-5t})u(t)$$

$$\text{(b)} \frac{1}{t}(e^{-5t} - e^{-6t})u(t) \quad \text{(d)} + (e^{-5t} - e^{-6t})u(t)$$

Sol<sup>n</sup> Diff. in freq. domain;

$$F(t) \iff F(s)$$

$$tf(t) \iff -\frac{dF(s)}{ds} = -\left[\frac{1}{s+5} - \frac{1}{s+6}\right]$$

$$tf(t) = \frac{1}{s+6} - \frac{1}{s+5}$$

$$tf(t) = e^{-6t}u(t) - e^{-5t}u(t)$$

$$f(t) = \frac{1}{t}(e^{-6t} - e^{-5t})u(t)$$

$$\text{Q.} \rightarrow f(t) = \frac{(1-e^t)u(t)}{t}; \quad F(s) = ?$$

$$\text{(a)} \log\left(\frac{s}{s-1}\right)$$

$$\text{(c)} \log\left(\frac{s-1}{s+1}\right)$$

$$\text{(b)} \log\left(\frac{s-1}{s}\right)$$

$$\text{(d)} \log\left(\frac{s+1}{s-1}\right)$$

Sol<sup>n</sup> Integration in freq. domain:-

$$y(t) = (1-e^t)u(t) \iff Y(s)$$

$$f(t) = \frac{y(t)}{t} \iff F(s) = \int_s^\infty y(s) \cdot ds$$

$$F(s) = \int_s^\infty y(s) \cdot ds = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$F(s) = \left[ \log(s) - \log(s-1) \right]_s^\infty = \left[ \log\left(\frac{s}{s-1}\right) \right]_s^\infty$$

$$= \log \left[ \lim_{s \rightarrow \infty} \sqrt[s-1]{\frac{s}{s-1}} \right] - \log\left(\frac{s}{s-1}\right)$$

$$= \log(1) - \log\left(\frac{s}{s-1}\right)$$

$$= 0 - \log\left(\frac{s}{s-1}\right)$$

Ques. → Find inverse LT of  $F(s) = \frac{s^2 + 2s + 5}{(s+3)(s+5)^2}$

For (i)  $\sigma > -3$  (ii)  $\sigma < -5$  (iii)  $-5 < \sigma < -3$

Soln →

$$F(s) = \frac{A}{(s+3)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$A = 2, B = -1, C = -10$$

$$F(s) = \frac{2}{s+3} - \frac{1}{s+5} - \frac{10}{(s+5)^2}$$

Poles :-  $s = -3, -5$

(i)  $\sigma > -3$ ;  $f(t)$  will be Right sided.

$$f(t) = 2e^{3t}u(t) - e^{-5t}u(t) - 10te^{-5t}u(t)$$

(ii)  $\sigma < -5$ ;  $f(t)$  will be left sided.

$$f(t) = -2e^{3t}u(-t) - [-e^{-5t}u(-t)] - 10[-te^{-5t}u(-t)]$$

(iii)  $-5 < \sigma < -3$ ;  $f(t)$  will be both sided.

↳ strip

ROC;  $\sigma > -5$



Right sided

ROC;  $\sigma < -3$



left sided

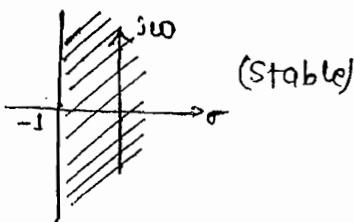
$$f(t) = -2e^{3t}u(-t) - e^{-5t}u(t) - 10te^{-5t}u(t).$$

### \*Causal system →

(i)  $h(t) = 0; t < 0$

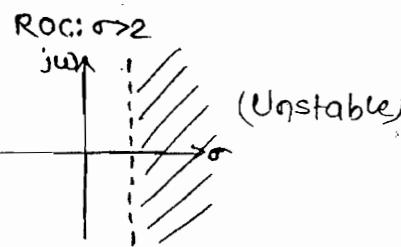
For causal sys. ROC will be right side to the right most pole.

Eg:- ROC:  $\sigma > -1$



$$H(s) = \frac{1}{s+1}, \sigma > -1$$

$$h(t) = e^{-t}u(t) \text{ (energy)}$$

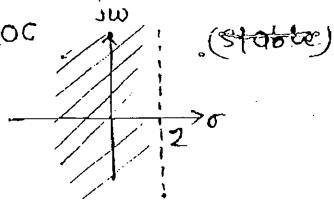


$$H(s) = \frac{1}{s-2}; \sigma > 2$$

$$h(t) = e^{2t}u(t) \text{ (N.E.N.P.)}$$

Note → For stability of causal sys., poles of TF should lie in the LHS of s-plane.

Eg:- ROC

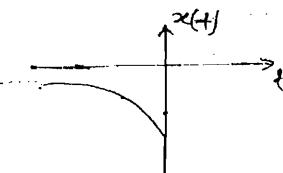


$$H(s) = \frac{1}{(s-2)} ; \sigma < 2$$

$$h(t) = -e^{2t} u(-t)$$

(Energy)

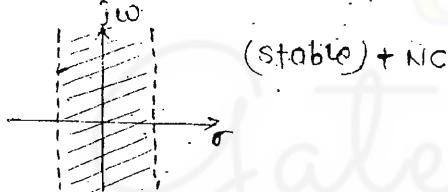
stable.



\* For anticausal sys. ROC will be left side to the left most pole.

\* For stability of anticausal sys., poles of TF should lie in the RHS of s-plane.

Eg:- (4) ROC  $-1 < \sigma < 3$



Ques → Consider a continuous time LTI sys. whose i/p  $x(t)$  & o/p  $y(t)$  are related by the differential eqn:-

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

determine  $h(t)$  of sys.

(a) When the sys. is causal.

(b) When the sys. is stable.

(c) Neither stable nor causal.

Soln

$$s^2Y(s) - sy(s) - 2y(s) = X(s)$$

$$Y(s)[s^2 - s - 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

= -1

$$H(s) = \frac{\left(\frac{1}{3}\right)}{(s-2)} + \frac{\left(\frac{1}{3}\right)}{(s+1)}$$

Poles:- -1, 2

(i) When the sys is causal  $h(t)$  will be Right sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

(ii) When sys is stable.  $h(t)$  will be both sided.

ROC:-  $-1 < \sigma < 2$  (because Imag axis is included)



$s+1 \rightarrow$  Right sided.

$s-2 \rightarrow$  Left sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$$

(iii) ROC:  $\sigma < -1$ ;  $h(t)$  will be left sided.

$$h(t) = \left(\frac{1}{3}\right)e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$

Que  $\rightarrow$  For diff. eqn

$$\frac{d^2y(t)}{dt^2} + \frac{6dy(t)}{dt} + 8y(t) = 0$$

With initial condn  $y(0^-) = 1$ ,  $y'(0^-) = 0$ ; the soln of  $y(t)$  is:

- (a)  $e^{2t} - e^{4t}$  (b)  $2e^{6t} - e^{2t}$  (c)  $e^{6t} + 2e^{4t}$  (d)  $e^{2t} + e^{4t}$

Ans  $\rightarrow$

$$s^2y(s) + sy(s) + 8y(s) = 0$$

$$y(s)(s^2 + 6s + 8) = 0$$

By applying LT on given diff. eqn

$$[s^2y(s) - sy(0^-) - y'(0^-)] + s[y(s) - y(0^-)] + y(s) = 0$$

$$s^2y(s) - s + 6[sy(s)]_{-1} + y(s) = 0$$

$$y(s)[s^2 + 6s + 1] = s + 6$$

$$y(s) = \frac{s+6}{s^2 + 6s + 1} = \frac{s+6}{(s+2)(s+4)}$$

$$Y(s) = \frac{2}{(s+2)} - \frac{1}{(s+4)}$$

$$y(t) = 2e^{-2t} - e^{-4t}$$

Ques. → For DE  $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = 8(t)$

with initial condn;  $y(0) = 2, y'(0) = 0$

the value of  $\left. \frac{dy(t)}{dt} \right|_{t=0^+}$  is

- (a) -1 (b) 1 (c) 0 (d) 2

Soln

$$s^2 Y(s) - s y(0) - y'(0) + 2[s Y(s) - y(0)] - y(s) + y(s) = 1$$

$$Y(s)[s^2 + 2s + 1] - (-2s + 2 \times 2) = 1$$

$$Y(s) = \frac{-2s - 3}{s^2 + 2s + 1}$$

$$Y(s) = \frac{-2s - 3}{(s+1)^2} = \frac{-2(s+1) - 1}{(s+1)^2}$$

$$Y(s) = \frac{-2}{(s+1)} - \frac{1}{(s+1)^2}$$

$$y(t) = -2e^{-t} - t e^{-t}$$

$$\frac{dy(t)}{dt} = -2e^{-t} - (e^{-t} - te^{-t})$$

$$\left. \frac{dy(t)}{dt} \right|_{t=0^+} = 2(2) - (1 - 0)$$

$$\boxed{\text{Ans. } = 1}$$

Ques. →  $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$  Find  $y(s)$  in terms of  $F(s)$

Soln  $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$

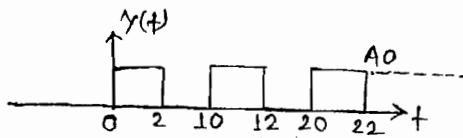
$$y(t) = f(t) + f(t-T_0) + f(t-2T_0) + \dots$$

| LT

$$y(s) = F(s) [1 + e^{-sT_0} + (e^{-sT_0})^2 + \dots]$$

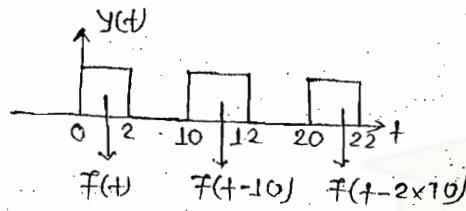
$$y(s) = \frac{F(s)}{1 - e^{-sT_0}}$$

Que. →



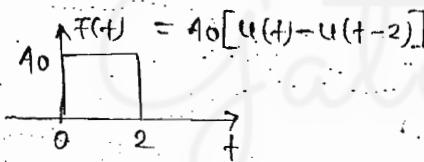
Find  $y(s)$

Soln →



$$y(t) = f(t) + f(t-10) + f(t-20) + \dots$$

$$= \sum_{n=0}^{\infty} f(t-nT_0); T_0 = 10$$

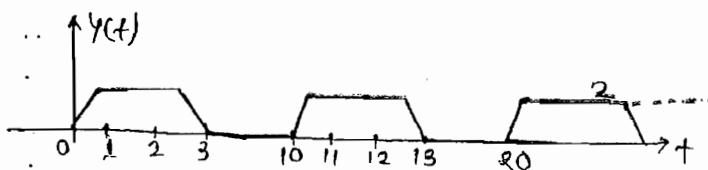


$$F(s) = \frac{A_0}{s} (1 - e^{-2s})$$

$$Y(s) = \frac{F(s)}{(1 - e^{-sT_0})}$$

$$Y(s) = \frac{A_0(1 - e^{-2s})}{s(1 - e^{-10s})}$$

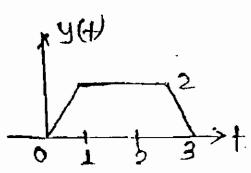
Que. →



Find LT of the signal.

Soln →

$$Y(s) = \sum_{n=0}^{\infty} f(t-nT_0)$$



$$f(t) = 2\tau(t) - 2\tau(t-1) - 2\tau(t-2) + 2\tau(t-3)$$

$$F(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$$

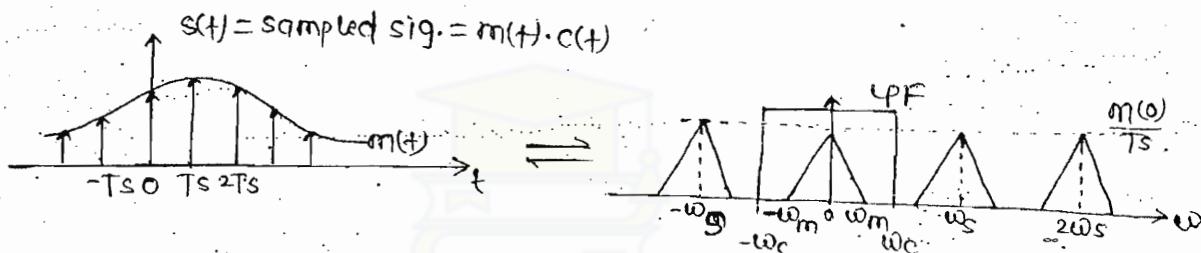
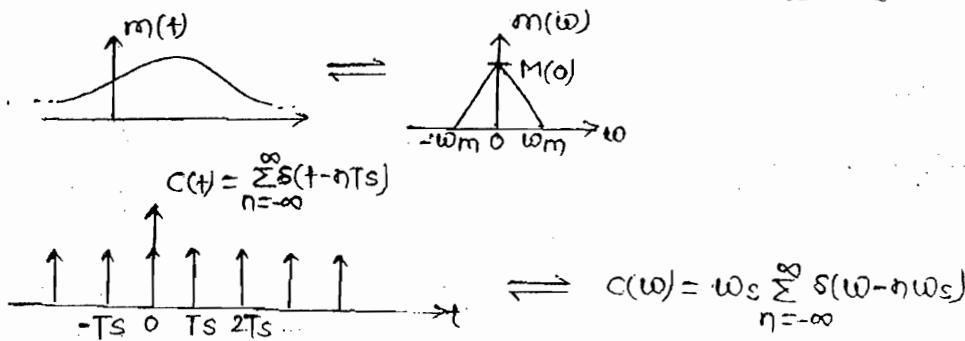
$$F(s) = \frac{2}{s^2} [1 - e^{-s} + e^{-2s} + e^{-3s}]$$

$$\gamma(s) = \frac{F(s)}{1 - e^{-st_0}}$$

$$\gamma(s) = \frac{\frac{2}{s^2}(1 - e^{-s} + e^{-2s} + e^{-3s})}{1 - e^{-t_0 s}}$$

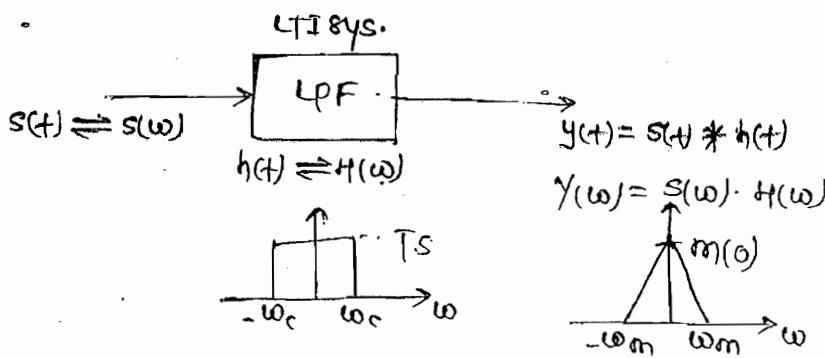
Chapter-07  
Sampling Theorem

Sampling theorem →



$T_s = \text{sampling interval}$ $= \frac{2\pi}{\omega_s}$
---

$$\begin{aligned}
 s(\omega) &= \frac{1}{2\pi} [m(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)] \\
 &= \frac{1}{T_s} \left[ \sum_{n=-\infty}^{\infty} m(\omega - n\omega_s) \right] \\
 &= \frac{1}{T_s} \left[ \dots + m(\omega + \omega_s) + m(\omega) + m(\omega - \omega_s) + m(\omega - 2\omega_s) + \dots \right]
 \end{aligned}$$



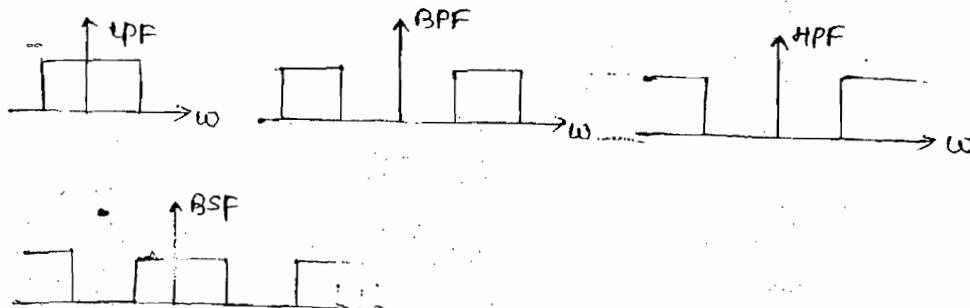
$$\boxed{\omega_m \leq \omega_c \leq \omega_s - \omega_m}$$

\* To avoid overlapping in sampled sig. spectrum:-

$$\omega_m \leq \omega_s - \omega_m$$

$$\boxed{\omega_s \geq 2\omega_m}$$

Note:-



Statement → A sig. can be represented by its samples (or) recovered back from its samples if sampling freq. is greater than <sup>(or)</sup> equal to twice of max<sup>m</sup> freq. component present in signal.

Nquist Rate →

$$f_{Ny} = 2f_m$$

Nquist-interval →

$$T_{Ny} = \frac{1}{f_{Ny}} = \frac{1}{2f_m}$$

Over Sampling →

\*  $f_s > 2f_m$

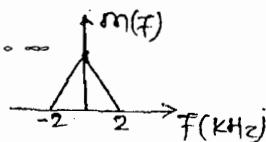
\* Allowable case

Under-sampling →

\*  $f_s < 2f_m$

\* not allowable.

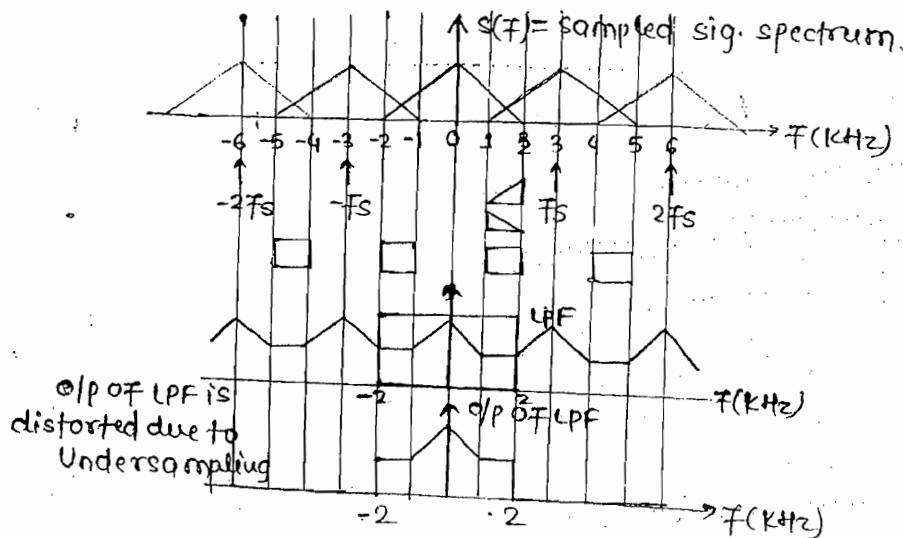
Eg:-  $m(t) \rightleftharpoons$



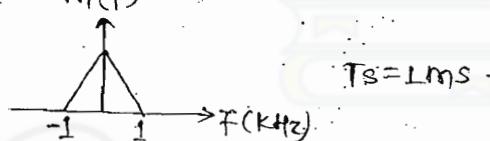
\*  $f_m = 2\text{kHz}$

\*  $f_s = 3\text{kHz} < 2f_m$

\* Undersampling



Ques  $\rightarrow m(t) \Leftrightarrow M(f)$



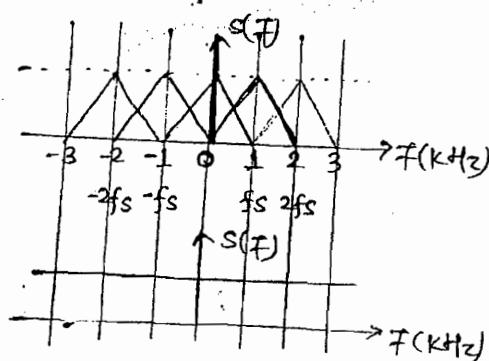
$T_s = Lms$

draw sampled sig. spectrum.

Soln

$f_m = 1 \text{ kHz}$

$f_s = \frac{1}{T_s} = 1 \text{ kHz} < 2f_m \text{ (Undersampling)}$



Ques Calculate Nyquist Rate in (rad/sec)

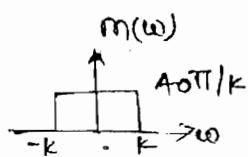
(i)  $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$ .

Soln  $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$

$$= \sin 6\pi t + \sin 2\pi t$$

$$\text{(ii)} \quad m(t) = \sin(4\pi t)$$

Solt  $m(t) = \sin(4\pi t)$   
 $= A_0 \sin(\omega_m t)$



$$\omega_m = k = 4\pi$$

$$\omega_{ny} = 2\omega_m = 8\pi$$

$$\text{(iii)} \quad m(t) = \sin^3(5\pi t)$$

Solt  $\omega_m = 3 \times 5\pi$   
 $\omega_{ny} = 2\omega_m$   
 $= 30\pi$

$$y(t) = [\alpha(t)]^n$$

$$\downarrow$$

$$\omega_m$$

$$\omega_m' = n\omega_m$$

$$\text{(iv)} \quad m(t) = \sin^2(4\pi t) \cdot \sin^2(3\pi t)$$

Solt  $\omega_{m1} = 2 \times 4\pi$   
 $\omega_{m2} = 4 \times 3\pi$   
 $\omega_m = \omega_{m1} * \omega_{m2} = 20\pi$   
 $\omega_{ny} = 2\omega_m = 40\pi$

$$m(t) = m_1(t) \cdot m_2(t)$$

$$\downarrow \quad \downarrow$$

$$\omega_{m1} \quad \omega_{m2}$$

$$\omega_m = \omega_{m1} + \omega_{m2}$$

$$\text{(v)} \quad m(t) = m_1(t) * m_2(t)$$

$$\downarrow \quad \downarrow$$

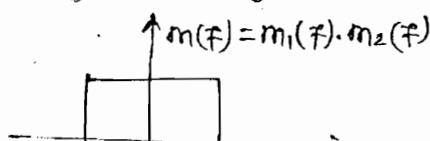
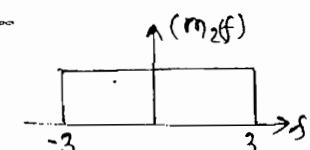
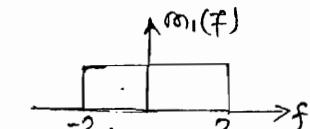
$$f_{m1} = 2 \text{ kHz} \quad f_{m2} = 3 \text{ kHz}$$

$$f_{ny} = ?$$

- (a) 4 kHz (b) 6 kHz (c) 10 kHz (d) 12 kHz

Solt

$$m(f) = m_1(f) \cdot m_2(f)$$



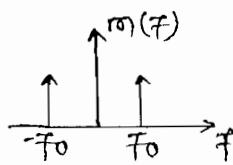
$$f_m = 2 \text{ kHz}$$

$$f_{ny} = 2f_m = 4 \text{ kHz}$$

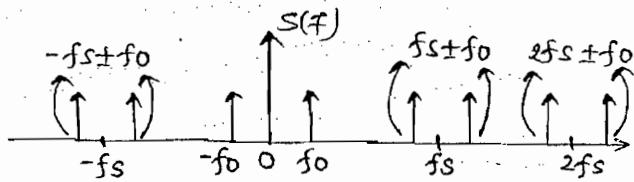
QUESTION

Important points →

$$(1) m(t) = \cos 2\pi f_0 t \implies$$



$$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Freq. components present in  $s(f)$

$$\therefore \pm f_0, f_s \pm f_0, 2f_s \pm f_0, \dots$$

$$\therefore n f_s \pm f_0$$

where;  $n$  = an integer

$$= \pm 1, \pm 2, \pm 3, \dots$$

$$(2) m(t) = \cos 2\pi f_1 t + \cos 2\pi f_2 t$$

Freq. component present in  $s(f)$

$$\therefore n f_s \pm f_1, n f_s \pm f_2$$

Ques. → A sig.  $m(t) = 100 \cos(2\pi \times 10^3 t)$  is ideally sampled at  $T_s = 50 \mu s$  & passed through an LPF with  $f_c = 15 \text{ kHz}$ . Which of the following freq. is/are present at the o/p of the LPF?

- (a) 8 kHz      (c) 8.8 kHz  
 (b) 12 kHz      (d) 8.8 12 kHz

Sol'n →

$$f_0 = 12 \text{ kHz}$$

$$f_s = \frac{1}{T_s} = 20 \text{ kHz}$$

$$\text{LPF: } f_c = 15 \text{ kHz}$$

Freq. compo. present in  $s(f)$

$$\therefore f_0, f_s \pm f_0, 2f_s \pm f_0, \dots$$

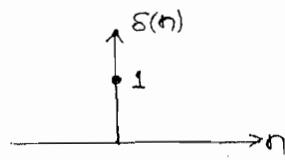
$$\therefore \pm 2, 20 \pm 12, 40 \pm 12, \dots$$

$$\therefore 12, 8, 32, 28, 52, \dots$$

Chapter - 08  
Discrete time signal

1) Unit-impulse signal  $\delta(n) \rightarrow$

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$



properties →

- (1.)  $\delta(n)$  is an even signal.
- (2.)  $\delta(n)$  is an energy signal.
- (3.)  $\delta(0) = \delta(n)$

$$\delta(at) = \frac{1}{|a|} \delta(t)$$

$$(4.) x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$$

$$(5.) x(n) * \delta(n-n_0) = x(n-n_0)$$

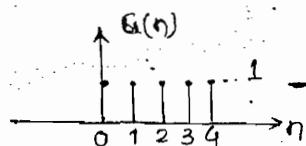
$$(6.) \int_{-\infty}^{+\infty} \delta(z) dz = u(t)$$

$$\int = \sum, z = n, z = k$$

$$\sum_{k=-\infty}^n \delta(k) = u(n)$$

(2) Unit-step sig. →

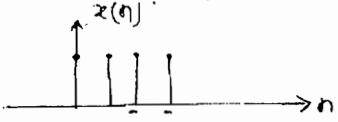
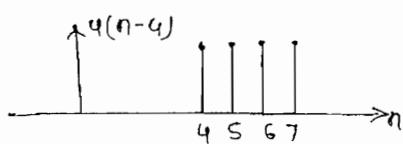
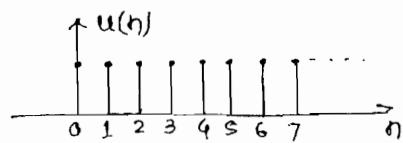
$$\bar{u}(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Q. → Draw sig.  $x(n)$

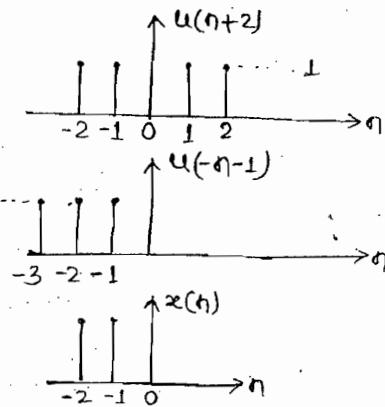
$$x(n) = u(n) - u(n-4)$$

Soln →



Ques. → Draw the signal  $x(n) = u(n+2) \cdot u(-n-1)$

Soln →



$$\begin{aligned} -n-1 &= 0 \\ \uparrow n &= -1 \rightarrow \text{starting point} \\ \text{because LS.} \end{aligned}$$

Operations on signal →

(1) Time-shifting →

$$x(n) = \left\{ \begin{array}{l} 5, 3, 7, 4, 8, 9 \\ \uparrow \end{array} \right. \quad \begin{array}{l} n=2 \\ n=3 \end{array}$$

axis not shifts only signal shifts

$$x(n-1) = \left\{ \begin{array}{l} 5, 3, 7, 4, 8, 9 \\ \uparrow \end{array} \right. \quad \begin{array}{l} (RS) \end{array}$$

$$x(n+2) = \left\{ \begin{array}{l} 5, 3, 7, 4, 8, 9 \\ \uparrow \end{array} \right. \quad \begin{array}{l} (LS) \end{array}$$

(2) Time Compression → (Decimation) →

$$x(n) = \left\{ \begin{array}{l} 5, 3, 7, 8, -2, 4, 9 \\ \uparrow \end{array} \right. \quad \begin{array}{l} n=3 \\ n=2 \end{array}$$

$$f(-2) = x(-4) = 0$$

$$f(-1) = x(-2) = 3$$

$$f(0) = x(0) = 8$$

$$f(1) = x(2) = 4$$

$$f(2) = x(4) = 0$$

(3) Time expansion → (Interpolation)

$$x(n) = \left\{ \begin{array}{l} 4, 3, 5 \\ \uparrow \end{array} \right. \quad \begin{array}{l} n=-1 \\ n=1 \end{array}$$

$$f(-3) = x(-3/2) = 0$$

$$f(n) = x(\frac{n}{2}) = \left\{ \begin{array}{l} 4, 0, 3, 0, 5 \\ \uparrow \end{array} \right. \quad \begin{array}{l} n=2-1 \end{array}$$

$$f(-2) = x(-1) = 4$$

$$f(-1) = x(-1/2) = 0$$

$$f(0) = x(0) = 3$$

$$f(1) = x(1/2) = 0$$

$$f(2) = x(1) = 5$$

$$x(\frac{n}{3}) = \left\{ \begin{array}{l} 4, 0, 0, 3, 0, 0, 5 \\ \uparrow \end{array} \right. \quad \begin{array}{l} n=3-1 \end{array}$$

$$f(3) = x(3/2) = 0$$

Que.  $\rightarrow x(n) = \{1, 2, 3, 4, 5\}$  Find  $y(n)$

$$(i) y(n) = x\left(\frac{2n}{3}\right)$$

$$\text{Sol} \rightarrow x(n) \xrightarrow{\text{Dec.}} x(2n) \xrightarrow{\text{Int.}} x\left(\frac{2n}{3}\right)$$

$$\begin{array}{c} \{1, 2, 3, 4, 5\} \\ \uparrow \\ \{1, 3, 5\} \end{array} \quad \begin{array}{c} \{1, 2, 3, 4, 5\} \\ \uparrow \\ \{1.0, 0, 3, 0, 0.5\} \end{array}$$

$$(ii) y(n) = x(-2n)$$

$$\text{Sol} \rightarrow x(n) \longrightarrow x(2n) \longrightarrow x(-2n) \quad (\text{-ve-folding about origin})$$

$$\begin{array}{c} \{1, 2, 3, 4, 5\} \\ \uparrow \\ \{1, 3, 5\} \end{array} \quad \begin{array}{c} \{1, 2, 3, 4, 5\} \\ \uparrow \\ (5, 3, 1) \end{array}$$

$$(iii) y(n) = x(-n-1)$$

$$\text{Sol} \rightarrow x(n) \longrightarrow x(n-1) \longrightarrow x(-n-1)$$

$$\begin{array}{c} (1, 2, 3, 4, 5) \\ \uparrow \\ (1, 2, 3, 4, 5) \end{array} \quad \begin{array}{c} (1, 2, 3, 4, 5) \\ \uparrow \\ (5, 4, 3, 2, 1) \end{array}$$

$$(iv) y(n) = x(2n-1)$$

$$\text{Sol} \rightarrow x(n) \longrightarrow x(n-1) \longrightarrow x(2n-1)$$

$$\begin{array}{c} (1, 2, 3, 4, 5) \\ \uparrow \\ (1, 2, 3, 4, 5) \end{array} \quad \begin{array}{c} (1, 2, 3, 4, 5) \\ \uparrow \\ (2, 4) \end{array}$$

#### (A) Convolution $\rightarrow$

$$y(n) = x_1(n) * x_2(n)$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k)$$

signal	extension	Length
$x_1(n)$	$n_1 \leq n \leq n_2$	$L_1$
$x_2(n)$	$n_3 \leq n \leq n_4$	$L_2$
$y(n)$	$n_1 + n_3 \leq n \leq n_2 + n_4$	$* L_1 + L_2 - 1$

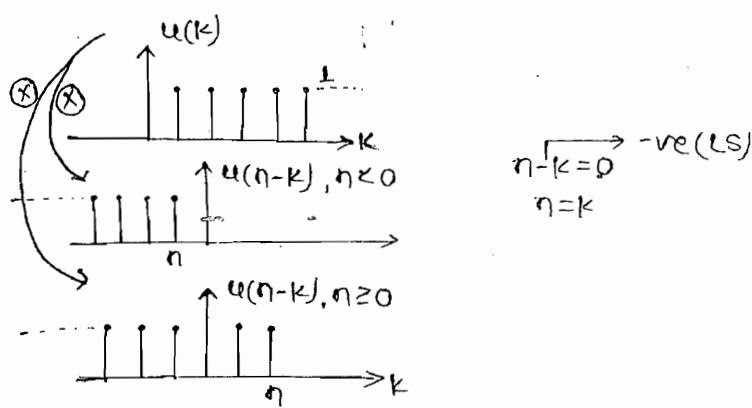
length = no. of samples

Que.  $\rightarrow y(n) = u(n) * u(n)$

Sol  $\rightarrow$

$$y(n) = u(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k) \cdot u(n-k)$$



$$y(n) = u(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k) \cdot u(n-k)$$

$$= \begin{cases} 0, & n < 0 \\ \sum_{k=0}^n 1, & n \geq 0 \end{cases}$$

$$= \begin{cases} 0, & n < 0 \\ n+1, & n \geq 0 \end{cases}$$

$$\boxed{y(n) = (n+1)u(n)}$$

Que.  $\rightarrow x_1(n) = (1, 2, -2)$  ,  $x_2(n) = (2, 0, 1)$

$$y(n) = x_1(n) * x_2(n) = ?$$

Sol. Tabular method  $\rightarrow$

$x_2(n)$	1	2	-2
2	2	4	-4
0	0	0	0
$\rightarrow 1$	1	2	-2

$$x_1(n) = (1, 2, -2) \leftarrow 2^{\text{nd}} \text{ element}$$

$$x_2(n) = (2, 0, 1) \leftarrow 3^{\text{rd}} \text{ element}$$

$$y(n) = (2, 4, -3, 2, -2)$$

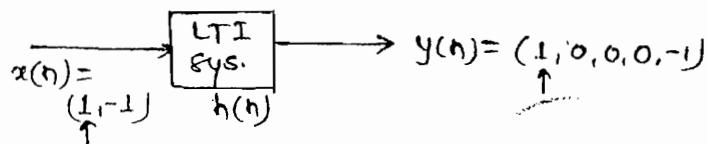
$$\begin{aligned} 2+3 &= 5 \\ 5-1 &= 4 \\ (\text{arrow point}) \end{aligned}$$

Que.  $\rightarrow x_1(n) = (-1, 2, 0, 1)$  ,  $x_2(n) = (3, 1, 0, -1)$

$$y(n) = x_1(n) * x_2(n)$$

$x_1(n)$	-1	2	0	1
$x_2(n)$	3	1	0	-1
3	-3	6	0	3
1	-1	2	0	1
$\rightarrow 0$	0	0	0	0
-1	1	-2	0	-1

$$y(n) = (-3, 5, 2, 4, -1, 0, -1)$$

Quefind  $h(n) = ?$ 

- (a)  $\begin{pmatrix} 1, 0, 0, 1 \end{pmatrix}$       (b)  $\begin{pmatrix} 1, 0, 1 \end{pmatrix}$   
 (c)  $\begin{pmatrix} 1, 1, 1, 1 \end{pmatrix}$       (d)  $\begin{pmatrix} 1, 1, 1 \end{pmatrix}$

Sol

$$y(n) = x(n) * h(n)$$

$$x(n) = 2$$

$$y(n) = 5$$

$$5 = 2 + l_2 - 1$$

$$l_2 = 4 \quad \{ \text{ans (c) or (d)} \}$$

Tabular method

$x(n)$	1	-1
$h(n)$	1	1
1	1	1
1	1	1
1	1	1

$$\text{Que} \rightarrow y(n) = h(n) * g(n), \quad h(n) = \left(\frac{1}{2}\right)^n u(n)$$

 $g(n)$  is causal sequence.If  $y(0) = 1, y(1) = 1/2$  then  $g(1)$  is equal to

- (a) 0 (b) 1/2 (c) 1 (d) 3/2

Sol

$h(n)$	1	1/2	1/4	1/8	-----
$g(n)$	1	1/2	1/4	1/8	-----
0	0	0	0	0	-----

Ans-(a)

\* Energy & power signal →

\* Energy signal →

\*  $E = \text{finite}$ ;  $P=0$

\* These are absolutely summable signals i.e.

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Ques → Calculate energy of signal:-

(i)  $x(n) = \delta(n)$

Sol →  $x(n) = \delta(n)$

$$= (\dots, 0, \underset{\uparrow}{1}, 0, \dots)$$

$$E = \sum |x(n)|^2 = 1$$

(ii)  $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol →  $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= 1 + \frac{1}{9} + \left(\frac{1}{9}\right)^2 + \dots$$

$$= \frac{1}{1 - 1/9} = \frac{9}{8}$$

(iii)  $x(n) = (1+j, 1-j, -2, 2)$

Sol →  $E = \sum |x(n)|^2$

$$= (\sqrt{2})^2 + (\sqrt{2})^2 + (-2)^2 + (2)^2$$

$$= 12$$

Ques → calculate energy of  $y(n)$

$$x(n) = (1, 2, 3, 4, 5)$$

(i)  $y(n) = x(-n)$     (ii)  $y(n) = x(n-1)$     (iii)  $y(n) = x\left(\frac{n}{2}\right)$     (iv)  $y(n) = -x(n)$     (v)  $y(n) = x(3n)$

Sol → (i)  $y(n) = x(-n)$     (ii)  $y(n) = x(n-1)$

$$x(n) = E[x(n)] = 55$$

$$x(-n) = 55$$

$$Ex(n-1) = 55$$

(iii)  $y(n) = x\left(\frac{n}{2}\right)$

$$= (1, 0, 2, 0, 3, 0, 4, 0, 5)$$

$$Ex(n/2) = 55$$

(iv)  $y(n) = -x(n)$

$$Ey(n) = 55$$

(v)  $y(n) = x(3n)$

$$= (1, 4)$$

$$Ey(n) = 1^2 + 4^2 = 17$$

Note:- Energy calculation is independent of time shifting, time reversal, amp. reversal & interpolation.

\* Power signal  $\rightarrow$

(1)  $P = \text{finite}$ ,  $E = \infty$

(2)  $P = \left\{ \begin{array}{l} \frac{1}{N} \sum_{n=N}^N |x(n)|^2 ; \text{ for periodic signal.} \\ \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2 ; \text{ for Non-periodic signal.} \end{array} \right.$

$$\lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2 ; \text{ for Non-periodic signal.}$$

Que.  $\rightarrow$  Calculate power of signal.

$$(i) x(n) = A_0 u(n)$$

$$\text{Soln} \rightarrow x(n) = A_0 u(n)$$

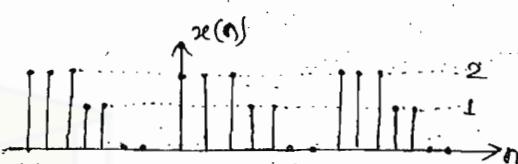
$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N A_0^2 (N+1)$$

$$= \lim_{N \rightarrow \infty} \frac{A_0^2 (N+1)}{2N+1}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Soln

$$P = \frac{1}{N} \sum |x(n)|^2$$

$$= \frac{2^2 + 2^2 + 2^2 + 1^2 + 1^2 + 0^2 + 0^2}{7}$$

$$= \frac{14}{7}$$

$$\boxed{P = 2}$$

Que.  $\rightarrow$  Find even, odd, CS & CAS for sig.  $x(n)$

$$x(n) = (-4-5j, 1+2j, 4)$$

Soln  $\rightarrow$  \* Even part  $\rightarrow$

$$\frac{x(n) + x(-n)}{2}$$

$$x(-n) = (4, 1+2j, -4-5j)$$

$$= \left( \frac{-5j}{2}, 1+2j, \frac{-5j}{2} \right)$$

\* Odd part  $\rightarrow$

$$\frac{x(n) - x(-n)}{2}$$

$$= \left( -4 - \frac{5j}{2}, 0, 4 + \frac{5j}{2} \right)$$

\* CS part  $\rightarrow$

$$\frac{x(n) - x^*(-n)}{2}$$

$$x^*(-n) = (4, 1-2j, -4+5j)$$

\* Periodic Signal  $\rightarrow$

$$x(n) = x(n \pm KN)$$

where;  $K = \text{an integer}$

$N = \text{FTP} = \text{integer}$

$$x(n) = x_1(n) + x_2(n)$$

$$\begin{array}{c} \downarrow \\ N_1 \end{array} \quad \begin{array}{c} \downarrow \\ N_2 \end{array}$$

$$\rightarrow \frac{N_1}{N_2} = \frac{\text{integer}}{\text{integer}} = \text{Rational no. (always)}$$

Note:- The sum of 2 (or) more 2 periodic signals in case of discrete time system will be always periodic.

Complex-exponential  $\rightarrow$  Complex exponential & sinusoidal signals are always periodic in case of continuous time signal.

e.g:-  $x(n) = A_0 e^{j\omega_0 n}$

let  $N$  be the FTP of  $x(n)$  i.e.

$$x(n) = x(n+N)$$

$$A_0 e^{j\omega_0 n} = A_0 e^{j\omega_0(n+N)}$$

$$e^{j\omega_0 N} = 1 = e^{j2\pi k} \quad (k = \text{an integer})$$

$$\omega_0 N = 2\pi k$$

$$\frac{2\pi}{\omega_0} = \frac{N}{k} = \text{Rational no.}$$

In case of discrete time sys; complex exponentials & sinusoidal sig. will be periodic only if ratio  $2\pi/\omega_0$  is rational no.

$$N = \frac{2\pi}{\omega_0} k$$

$k$  is a least int. for which  $N$  is an integer.

Ques. Calculate FTP of sig. if it is periodic

(i)  $x(n) = e^{j2n}$ .

(ii)  $x(n) = \cos \frac{3\pi}{4} n$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi = \text{irrational no.}$$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{8}{3} \quad (\text{R. no.})$$

$$(iii) x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$$

Sol?  $x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$

$\downarrow \quad \downarrow$

$N_1 = 8 \quad N_2$

$$N_2 = \frac{2\pi}{\omega_2} k_2 = \frac{2\pi}{5\pi/7} k_2 = \frac{14}{5} k_2 = 14$$

$$\begin{aligned} N &= \text{LCM}(N_1, N_2) \\ &= \text{LCM}(8, 14) \\ &= 56. \end{aligned}$$

$e^{j2t} \rightarrow \text{periodic}$

$e^{j2n} \rightarrow \text{NP}$

$\sin 4t \rightarrow P$

$\sin 4n \rightarrow \text{NP}$

$\sin 2t + \cos 4t \xrightarrow{(P)} P$

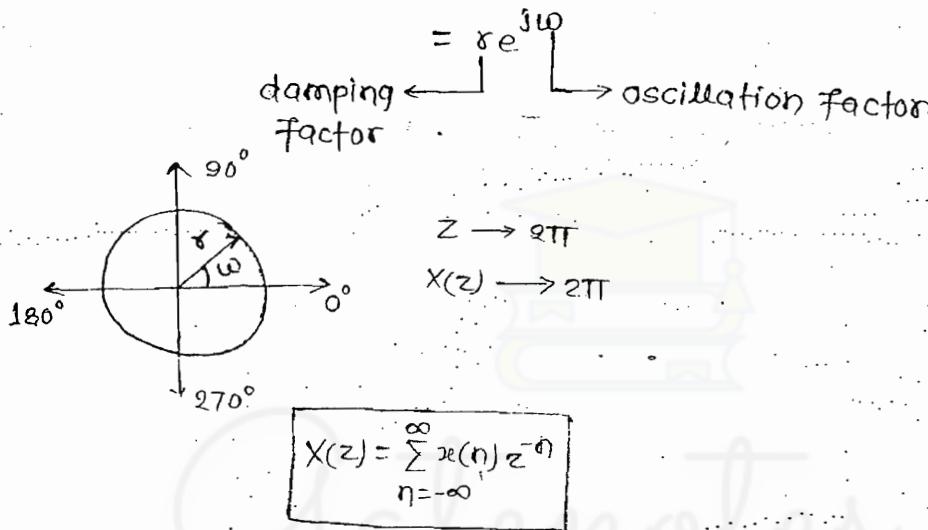
$\sin 3n + \cos 4n \xrightarrow{(NP)} \text{NP}$

Chapter-09  
Z-transform

- \* Discrete time fourier transform (DTFT) exists for E & P signals where all Z-TF also exist for NENP sig. (upto certain only).
- \* The replacement  $z = e^{j\omega}$  is used for z-transform to DTFT conversion only for absolutely summable signals.

$$x(n) \rightleftharpoons X(z)$$

where;  $z = \text{complex variable}$



$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

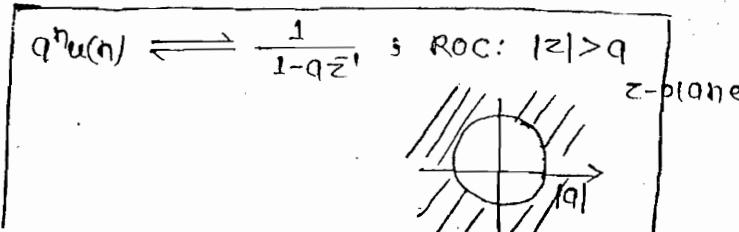
Que.  $\rightarrow x(n) = q^n u(n) \rightleftharpoons X(z) = ?$

Soln

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} q^n z^{-n} \\ &= \sum_{n=0}^{\infty} q^n z^{-n} \\ &= \sum_{n=0}^{\infty} (qz^{-1})^n \end{aligned}$$

$$= (qz^{-1})^0 + (qz^{-1})^1 + (qz^{-1})^2 + \dots$$

$$X(z) = \frac{1}{1 - qz^{-1}} ; |qz^{-1}| < 1$$



Que.  $\rightarrow x(n) = -q^n u(-n-1)$  cal. ZTF & ROC.

$$\underline{\text{Soln}} \rightarrow X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} -q^n u(-n-1) z^{-n}$$

$$= \sum_{-\infty}^{-1} (-q^n) z^{-n}$$

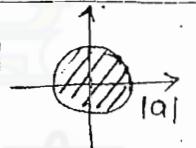
$$= -\sum_{-\infty}^{-1} (qz')^n$$

$$X(z) = -[(qz')^1 + (qz')^2 + \dots]$$

$$= -[q'z + (q'z)^2 + \dots]$$

$$= \frac{-q'z}{1-q'z} ; |q'z| < 1$$

$$X(z) = \frac{1}{1-qz} ; |z| < |q|$$

$$-q^n u(-n-1) \Leftrightarrow \frac{1}{1-qz} ; |z| < |q|$$


$$-q^n u(-n-1) \Leftrightarrow \frac{1}{1-qz} ; |z| < |q|$$

$$(q=1) \quad q^n u(n) \Leftrightarrow \frac{1}{1-z} ; |z| > 1$$

$$(q=1) \quad u(n) \Leftrightarrow \frac{1}{1-z} ; |z| > 1$$

$$-u(n-1) \Leftrightarrow \frac{1}{1-z^{-1}} ; |z| < 1$$

## Properties of z-transform →

1.) linearity :-  $a_1x_1(n) + a_2x_2(n) \iff a_1X_1(z) + a_2X_2(z)$

2.) Time-reversal :-  $x(n) \iff X(z^{-1})$

3.) Conjugation :-  $x^*(n) \iff X^*(z^*)$

4.) Time-shifting :-  $x(n-n_0) \iff X(z) \cdot z^{-n_0}$

$$\downarrow (n_0=1)$$

$$x(n-1) \iff z^{-1}X(z)$$

5.) Scaling of z :-  $a^n x(n) \iff X(a'z)$

6.) Convolution in time :-

$$x_1(n) * x_2(n) \iff X_1(z) \cdot X_2(z)$$

7.) Multiplication in time :-

$$x_1(n) \cdot x_2(n) \iff \frac{1}{2\pi j} [X_1(z) * X_2(z)]$$

8.) Successive diff./difference in time →

$$\begin{aligned} \frac{d x(n)}{d n} &= \frac{x(n) - x(n-1)}{n - (n-1)} \\ &= x(n) - x(n-1) \iff X(z) - z^{-1}X(z) \end{aligned}$$

$$x(n) - x(n-1) \iff (1 - z^{-1})X(z)$$

9.) Accumulation/Integration in time →

$$\sum_{k=-\infty}^n x(k) \iff \frac{X(z)}{1 - z^{-1}}$$

(10.) Differentiation in freq. →

$$n \cdot x(n) \iff -z \frac{d X(z)}{dz}$$

(11.) Initial value theorem →

$$x(t) \Big|_{t=0} = \lim_{s \rightarrow \infty} s X(s)$$

Cond'n:- Applicable only for causal type signal i.e.

$$\text{if } n \geq 0, x(n) \neq 0$$

### (12) Final value theorem $\rightarrow$

$$x(t) \Big|_{t=\infty} = \lim_{s \rightarrow 0} [s X(s)]$$

$$x(n) \Big|_{n=\infty} = \lim_{z \rightarrow 1} [(1-z^{-1}) X(z)]$$

cond<sup>n</sup>: - (i) Applicable only for causal signals. i.e.

$$x(n) = 0, n < 0$$

(ii) poles of term  $[(1-z^{-1}) X(z)]$  should lie inside unit circle in z-plane.

[DATE-30/10/14]

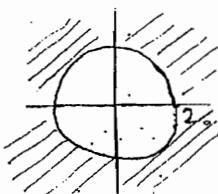
Region of Convergence (ROC)  $\rightarrow$  It is defined as the range of complex variable z in z-plane for which z-transform of signal is convergent (or) finite.

### Properties of ROC $\rightarrow$

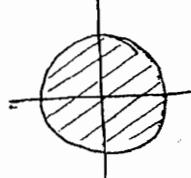
- (1) ROC does not include any pole.
- (2) For right sided signal ROC will be outside circle in z-plane. } RS is ROC is outside any side.
- (3) For left sided signal ROC will be inside circle in z-plane.
- (4) For both sided signal ROC is a ring in z-plane.
- (5) For stability, ROC includes unit circle in z-plane.
- (6) For finite duration sig. ROC is entire z-plane excluding possibly  $z=0$  & or  $\pm\infty$ .

Que.  $\rightarrow$  Check stability of sys. & comment about extension of  $h(n)$ .

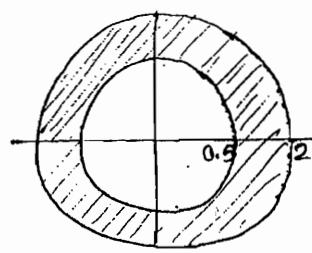
(i) ROC:  $|z| > 2$



(ii) ROC:  $|z| < 2$



(iii) ROC:  $0.5 < |z| < 2$



Sol<sup>n</sup>  $\rightarrow h(n) = RS + US$

$h(n) = LS + S$

$h(n) = BS + S$

Que.  $x(n) = [2, 5, 3, 7, 8]$   $X(z) = ?$ , ROC = ?

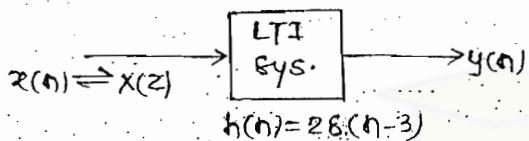
Soln.

$$\begin{aligned} X(z) &= \sum_{-\infty}^{\infty} x(n) z^{-n} \\ &= \sum_{n=1}^{3} x(n) z^{-n} \\ &= x(-1)z^1 + x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} \\ &= 2z + 5 + 3z^{-1} + 7z^{-2} + 8z^{-3} \end{aligned}$$

ROC → Entire z-plane excluding

$z = 0, +\infty, -\infty$  (excluded) [Because they give  $\infty$  soln in  $X(z)$ ]

Que. →



$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}. \text{ Find } y(4)$$

- (a) -6 (b) 0 (c) 2 (d) -4

Soln. →

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

$$x(n) = [1, 0, 1, -2, 2, 0, 0, 0, +\infty]$$

$$y(n) = x(n) * h(n)$$

$$\begin{array}{l} \delta(0) \rightleftharpoons 1 \\ 2\delta(n) \rightleftharpoons 2 \\ 2\delta(n-3) \rightleftharpoons 2z^{-3} \end{array}$$

$$Y(z) = X(z) \cdot H(z)$$

$$= 2z^3(z^4 + z^2 - 2z + 2 - 3z^{-4})$$

$$Y(z) = 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$$

$$y(n) = (2, 0, 2, -4, 4, +0, 0, 0, -6)$$

$$y(9) = 0$$

$$\boxed{y(4)=0}$$

Que. →  $X(z) = 1 - 3z^{-1} \Rightarrow x(n) \neq i/p$

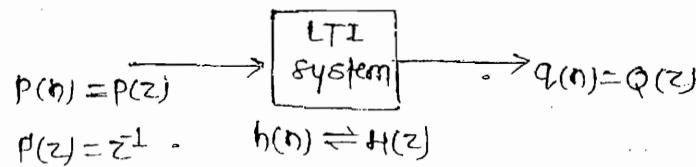
$$Y(z) = 1 + 2z^{-2} \Rightarrow y(n) \neq o/p$$

An LTI sys. has impulse response  $h(n)$  defined as  $h(n) = x(n-1) * y(n)$ .

The o/p of sys. for i/p  $x(n-1)$  has  $-zT$ .

Soln

$$i/p \rightarrow s(n-1) = z^{-1}$$



$$h(n) = x(n-1) * y(n)$$

$$H(z) = z^{-1} X(z) * Y(z)$$

$$q(n) = p(n) * h(n)$$

$$Q(z) = P(z) \cdot H(z)$$

$$= (z^{-1}) [z^1 X(z) * Y(z)]$$

$$Q(z) = z^2 X(z) * Y(z)$$

$$= z^2 (1 - 3z^{-1})(1 + 2z^{-2})$$

$$= z^2 (1 - 3z^{-1} + 2z^{-2} - 6z^{-3})$$

$$Q(z) = z^2 - 3z^3 + 2z^4 - 6z^5$$

$$q(n) = s(n-2) - 3s(n-3) + 2s(n-4) - 6s(n-5)$$

Ans.(b).

2nd method →

$$q(n) = p(n) * h(n)$$

$$= s(n-1) * h(n)$$

$$= h(n-1)$$

$$h(n) = x(n-1) * y(n)$$

$$X(z) = 1 - 3z^{-1} \iff x(n) = (1, -3)$$

$$\xrightarrow{\quad\quad\quad} x(n-1) = (0, 1, -3)$$

$$\therefore Y(z) = 1 + 2z^{-2} \iff y(n) = (1, 0, 2)$$

$$\xrightarrow{\quad\quad\quad} h(n) = (0, 1, 2) * (1, 0, 2)$$

$$\text{Que.} \rightarrow x(n) = \left(-\frac{1}{2}\right)^n u(-n+1) + 3^n u(n)$$

$$X(z) = ?$$

Sol<sup>n</sup> →

$$|z| < \left|\frac{-1}{2}\right| \quad |z| > |3|$$

$$|z| < \left(\frac{1}{2}\right) \quad |z| > 3$$

$$\left(\frac{1}{2}\right) < |z| < 3$$

$$(-2)^n u(-n-3)$$

$$|z| < | -2 |$$

$$|z| < 2$$

$$x(n) = \left(-\frac{1}{2}\right)^n u(-n+1) + 3^n u(n)$$

$$= (-2)^n u(-n+1) + 3^n u(n)$$

$$\begin{array}{c} \downarrow \\ |z| < 2 \end{array} \quad \begin{array}{c} \downarrow \\ |z| > 3 \end{array}$$

ZT of the  $x(n)$  will not exist because no common ROC

$$\text{Que.} \rightarrow x(n) = \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n u(n) \quad \text{ROC} = ?$$

Sol<sup>n</sup> →

$$x(n) = \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n u(n)$$

$$\left(\frac{1}{3}\right)^n = \begin{cases} \left(\frac{1}{3}\right)^{-n} ; n < 0 \\ \left(\frac{1}{3}\right)^n ; n \geq 0 \end{cases}$$

$$= \begin{cases} 3^n ; n < 0 \\ \left(\frac{1}{3}\right)^n ; n \geq 0 \end{cases}$$

$$= 3^n u(-n-1) + \left(\frac{1}{3}\right)^n u(n)$$

$$= |z| < 3 \quad |z| > \frac{1}{3}$$

$$x(n) = \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n u(n)$$

$$= 3^n u(-n-1) + \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n)$$

$$|z| < 3 \quad |z| > \frac{1}{3} \quad |z| > \frac{1}{2}$$

$$|z| \geq 3$$

$$|z| > \frac{1}{3}$$

$$\frac{1}{3} < |z| < 3$$

$$\boxed{\text{ROC} = \frac{1}{3} < |z| < 3}$$

Que.  $\rightarrow x(n) = 2^{x(n)}$  ZT=?

$$\underline{\text{Soln}} \rightarrow x(n) = 2^{x(n)}$$

$$X(z) = \sum_{-\infty}^{\infty} 2^{x(n)} z^{-n}$$

$$x(n) = 2^{x(n)}$$

$$= 2^{n u(n)}$$

$$= \begin{cases} 1 & ; n < 0 \\ 2^n & ; n \geq 0 \end{cases}$$

$$= u(-n-1) + 2^n u(n)$$

$$|z| < 1 \quad |z| > 2$$

ZT will not exist because no common ROC.

Que.  $\rightarrow x(n) = \cos \omega_0 n \cdot u(n)$  X(z)=?, ROC=?

Soln  $\rightarrow$

$$x(n) = \cos \omega_0 n u(n)$$

$$\cos \omega_0 n = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$x(n) = \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n} u(n)$$

$$X(z) = \frac{1}{2} \left[ \frac{z}{z - e^{j\omega_0}} + \frac{z}{z - e^{-j\omega_0}} \right], \quad |z| > 1$$

$$= \frac{1}{2} \left[ \frac{z(z - e^{-j\omega_0}) + z(z - e^{j\omega_0})}{z^2 - z(e^{j\omega_0} + e^{-j\omega_0}) + 1} \right], \quad |z| > 1$$

$$= \frac{1}{2} \left[ \frac{2z^2 - z(e^{j\omega_0} + e^{-j\omega_0})}{z^2 - 2z \cos \omega_0 + 1} \right], \quad |z| > 1$$

$$= \frac{1}{2} \left[ \frac{2z^2 - 2z \cos \omega_0}{z^2 - 2z \cos \omega_0 + 1} \right], \quad |z| > 1$$

$$= \frac{z^2 - z \cos \omega_0}{z^2 - 2z \cos \omega_0 + 1}, \quad |z| > 1$$

$$a^n u(n) \Leftrightarrow \frac{z}{z-a}, \quad |z| > |a|$$

$$a = e^{j\omega_0} \quad e^{j\omega_0 n} u(n) \rightarrow \frac{z}{z - e^{j\omega_0}}, \quad |z| > 1$$

$$a = e^{-j\omega_0} \quad e^{-j\omega_0 n} u(n) \rightarrow \frac{z}{z - e^{-j\omega_0}}, \quad |z| > 1$$

### Important Signals →

$x(n)$	$X(z)$	ROC
$\delta(n)$	1	entire z-plane
$a^n u(n)$	$\frac{1}{1-a\bar{z}}$	$ z  >  a $
$-a^n u(-n-1)$	$\frac{1}{1-a\bar{z}}$	$ z  <  a $
$u(n)$	$\frac{1}{1-z^{-1}}$	$ z  > 1$
$n \cdot a^n u(n)$	$\frac{az^1}{(1-a\bar{z})^2}$	$ z  >  a $
$-n a^n u(-n-1)$	$\frac{az^{-1}}{(1-a\bar{z})^2}$	$ z  <  a $
$\cos \omega_0 n$	$\frac{z^2 z \cos \omega_0}{z^2 - 2z \cos \omega_0 + 1}$	$ z  > 1$
$\sin \omega_0 n$	$\frac{z \sin \omega_0}{z^2 - 2z \cos \omega_0 + 1}$	$ z  > 1$

Que.  $\rightarrow x(n) \Leftrightarrow X(z) = \frac{0.5}{1-2z^{-1}}$

It is given that ROC of  $X(z)$  includes unit circle. i.e

The value of  $x(0)$  is :-

- (a) 0.5 (b) 0 (c) 0.25 (d) 0.5

Soln

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Pole:-  $1-2\bar{z}^1 = 0$

$z=2$

ROC:-  $|z| < 2$  (Given)

so inverse will be left sided,  $x(n)$

$x(n) = -0.5 (2)^n u(-n-1)$

$u(-n-1) \rightarrow (-\infty \leftarrow 0-1)$

Ques. → Find inverse ZT of

$$X(z) = \frac{z}{(z-1)(z-2)^2} \quad \text{if}$$

(i)  $|z| > 2$     (ii)  $|z| < 1$     (iii)  $1 < |z| < 2$

Soln →

$$X(z) = \frac{z}{(z-1)(z-2)^2}$$

$$X(z) = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$\frac{X(z)}{z} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$X(z) = \frac{Az}{(z-1)} + \frac{Bz}{(z-2)} + \frac{Cz}{(z-2)} \times \frac{z^{-2}}{z^{-2}}$$

$$X(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-2z^{-1}} + \frac{C}{2} \left[ \frac{z^{-1}}{(1-2z^{-1})^2} \right]$$

$$A=1, B=-1, C=1$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{z}{1-2z^{-1}} + \frac{1}{2} \left[ \frac{z^{-1}}{(1-2z^{-1})^2} \right]$$

Poles →  $z=1, 2$

(i) ROC:  $|z| > 2$

\*  $x(n)$  will be RS.

$$* x(n) = u(n) - 2^n u(n) + \frac{1}{2} n 2^n u(n)$$

(ii) ROC:  $|z| < 1$

\*  $x(n)$  will be LS.

$$* x(n) = -u(-n-1) - [-2^n u(-n-1)] + \frac{1}{2} [-n 2^n u(-n-1)]$$

(iii) ROC:  $1 < |z| < 2$

\*  $x(n)$  will be both sided.

$$* x(n) = u(n) - [-2^n u(-n-1)] + \frac{1}{2} [-n 2^n u(-n-1)]$$

$$f(n) = a^n u(n) \rightarrow F(z) = \frac{z}{z-a}$$

$$f(n-1) = a^{n-1} u(n-1)$$

$$\bar{z}^1 F(z) = \frac{1}{(z-a)}$$

$$\text{Que.} \rightarrow x(n) = 4^n u(n) \iff x(z)$$

$$y(n) \iff y(z) = x^2(z) \quad \text{Find } y(n)$$

- (a)  $4^n u(n)$  (b)  $(n+1)4^n u(n+1)$  (c)  $(n+1)4^n u(n)$  (d)  $n \cdot 4^n u(n+1)$

Soln.

$$x(z) = \frac{1}{1-4z^{-1}}$$

$$x^2(z) = \left( \frac{1}{1-4z^{-1}} \right)^2 = \frac{1}{(1-4z^{-1})^2} = \frac{1}{1+16z^{-2}-8z^{-1}}$$

$$Y(z) = X^2(z) = \frac{1}{(1-4z^{-1})^2}$$

$$\begin{aligned} Y(z) &= \frac{z^2}{(z-4)^2} = \frac{z(z-4)+4z}{(z-4)^2} \\ &= \frac{z}{(z-4)} + \frac{4z}{(z-4)^2} \times \frac{z^2}{z^2} \end{aligned}$$

$$Y(z) = \frac{1}{1-4z^{-1}} + \frac{4z^{-1}}{(1-4z^{-1})^2}$$

$$y(n) = 4^n u(n) + n 4^n u(n)$$

$$\boxed{y(n) = (n+1)4^n u(n)}$$

Ans: (b) & (c)

$$(n+1)4^n u(n+1) = (0, 1, 8, \dots)$$

$$(n+1)4^n u(n) = (1, 8, \dots)$$

$$\text{Que.} \rightarrow X(z) = \log(1+az^{-1}) ; |z| > |a| \quad \text{Find } x(n) = ?$$

Soln.  $\rightarrow$  diff in freq:

$$x(n) \iff x(z)$$

$$nx(n) \iff -z \frac{dx(z)}{dz} = -z \left[ \frac{1}{1+az^{-1}} (-az^{-2}) \right]$$

$$n \cdot x(n) = \frac{az^{-1}}{1+az^{-1}} ; |z| > |a|$$

$$nx(n) = 1 - \frac{1}{1+az^{-1}} ; |z| > |a|$$

$$nx(n) = \delta(n) - (-a)^n u(n)$$

$$\text{Que} \rightarrow X(z) = \frac{1+z^{-1}}{1+\frac{1}{3}z^{-1}} \iff x(n)$$

(a) Assuming ROC to be  $|z| < 1/3$ .

determine  $x(0), x(-1), x(-2)$

(b.) Assuming ROC to be  $|z| > 1/3$

determine  $x(0), x(1) \& x(2)$

Soln

$$X(z) = \frac{1+z^{-1}}{1+\left(\frac{1}{3}\right)z^{-1}}$$

(a)  $|z| < \frac{1}{3}$ ;  $x(n)$  will be LS sig.

Arrange numerator & denominator polynomials in ascending powers of  $z$ .

$$X(z) = \frac{z^1 + z^0}{\left(\frac{1}{3}z^1 + z^0\right)}$$

LFD (LA)

$$\begin{array}{r} \frac{1}{3}z^1 + 1 \\ \underline{+ z^1 + 3} \\ -2 \\ \underline{-2 - 6z} \\ 6z \\ \underline{6z + 18z^2} \\ -18z^2 \end{array}$$

$$X(z) = 3 - 6z + 18z^2 + \dots$$

$$= x(0) + x(-1)z + x(-2)z^2 + \dots$$

$$x(0) = 3, x(-1) = -6, x(-2) = 18$$

(b.)  $|z| > \frac{1}{3}$ ;  $x(n)$  will be RS sig.

Arrange nume. & deno. poly. in decensing powers of  $z$ .

$$X(z) = \frac{1+z^{-1}}{1+\left(\frac{1}{3}\right)z^{-1}} = \frac{\left(1+\frac{1}{3}z^1\right)z^1 + z^0}{1+\frac{1}{3}z^1} = \frac{\left(1+\frac{2}{3}z^1 - \frac{2}{3}z^2 + \dots\right)z^1 + z^0}{1+\frac{1}{3}z^1}$$

$$\begin{array}{r} \frac{2}{3}z^1 \\ \underline{- \frac{2}{3}z^1} \\ - \frac{2}{3}z^2 \end{array}$$

$$X(z) = 1 + \frac{2}{3}z^{-1} - \frac{2}{9}z^2 + \dots$$

$$= x(0) + x(1)z^{-1} + x(2)z^2 + \dots$$

$$x(0) = 1, x(1) = \frac{2}{3}, x(2) = -\frac{2}{9}$$

Que.  $\rightarrow x(n) \Leftrightarrow X(z) = \frac{z}{9z^2 - 3z + 1}, |z| < \frac{1}{2}$ . Find  $x(-2)$

- (a) 0 (b) 1 (c) 2 (d) 3

Sol<sup>n</sup> →

$$X(z) = \frac{z}{1 - 3z + 2z^2}$$

$$\begin{array}{r} 1 - 3z + 2z^2 \end{array} \overline{) z} \quad (z + 3z^2 + 7z^3 + \dots$$

$$\underline{-z - 3z^2 - 2z^3}$$

$$\begin{array}{r} 3z^2 - 2z^3 \\ 3z^2 - 9z^3 \\ \hline - \end{array}$$

$$\begin{array}{r} 7z^3 \\ 7z^3 - 21z^4 + 14z^5 \\ \hline \end{array}$$

$$\boxed{x(-2) = 3}$$

Que.  $\rightarrow x(n) \Leftrightarrow X(z) = \frac{z + z^{-3}}{z + z^{-1}}$ ;  $x(n)$  series has

- (a) Alternate -is (c) alternate 1s  
 (b) Alternate 0's (d) alternate 2s.

Sol<sup>n</sup> →

$$X(z) = \frac{z + z^{-3}}{z + z^{-1}}$$

Here ROC is not given, so devide in the given form.

$$\begin{array}{r} z + z^{-1} \end{array} \overline{) z + z^{-3}} \quad (1 - z^2 + 2z^4 - 2z^6 + 2z^8 - 2z^{10}$$

$$\underline{-z - z^{-1}}$$

$$\begin{array}{r} -z + z^3 \\ -z + z^3 \\ \hline - \end{array}$$

$$\begin{array}{r} 9z^3 \\ 9z^3 + 2z^5 \\ \hline -2z^5 \\ -2z^5 - 2z^7 \\ \hline \end{array}$$

$$X(z) = 1 - z^2 + 2z^4 + 2z^6 + 2z^8 + \dots$$

$$x(n) = (1, 0, -1, 0, 2, 0, -2, 0, 2, \dots)$$

Ans. (b).

### \* Causal system $\rightarrow$

$$(1) h(n) = 0, n < 0$$

$$(2) H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \dots = \frac{N(z)}{D(z)}$$

Note  $\rightarrow$  For causal sys., expansion of TF does not include tve. powers of  $z$ .

$$(3) \lim_{z \rightarrow \infty} H(z) = h(0) = 0 \text{ (or) finite.}$$

Note  $\rightarrow$  For causal sys., order of numerator can't exceed order of denominator.

\* For causal sys. ROC will be outside circle in  $z$ -plane.

\* For stability of dff discrete-time causal sys., poles of TF should lie inside unit circle in  $z$ -plane.

### \* Anticausal system $\rightarrow$

\* For this sys. ROC will be inside circle in  $z$ -plane.

\* For stability of this sys., poles of TF should lie outside unit circle in  $z$ -plane.

Que  $\rightarrow$  A causal LTI sys. is described by the difference eqn

$$2y(n) = ay(n-2) - 2x(n) + Bx(n-1)$$

The sys. is stable only if

- (a)  $|a|=2$ ,  $|B| < 2$
- (b)  $|a| > 2$ ,  $|B| > 2$
- (c)  $|a| < 2$ , for any value of ' $B$ '
- (d)  $|B| < 2$ , for any value of ' $a$ '

Soln  $2y(z) = \alpha y(z) \cdot z^2 - 2x(z) + \beta x(z) z^{-1}$

$$H(z) = \frac{y(z)}{x(z)} = \frac{-2 + \beta z^{-1}}{2 - \alpha z^2}$$

Poles:-  $2 - \alpha z^2 = 0$

$$z = \sqrt{\frac{\alpha}{2}}$$

For stability of causal sys.

$$|Pole| < 1$$

$$\sqrt{\frac{\alpha}{2}} < 1$$

$$|\alpha| < 2$$

Ques  $\rightarrow x(n) \Leftrightarrow X(z) = \frac{z^4(1-z^4)}{4(1-z^4)^2}$

Find  $x(\infty) = ?$  (a) 1/4 (b) 0 (c) 1 (d)  $\infty$

Soln

$$x(n) \Leftrightarrow X(z) = \frac{z^4(1-z^4)}{4(1-z^4)^2}$$

(i)  $\lim_{z \rightarrow \infty} X(z) = 0$

Cond'n for causality is satisfied.

$$\begin{aligned} \text{(ii)} \quad (1-z^4)X(z) &= \frac{z^4(1-z^4)}{4(1-z^4)^2} \\ &= \frac{z^4(1+z^2)(1-z^2)}{4(1-z^4)} \\ &= \frac{z^4(1+z^2)(1+z^1)(1-z^1)}{4(1-z^1)} \\ &= \frac{z^4(1+z^2)(1+z^1)}{4} \end{aligned}$$

Pole:-  $z = 0 < 1$

Both the cond'n are satisfied. so we can use final value theorem.

Final value theorem  $\rightarrow$

$$\begin{aligned} x(\infty) &= \lim_{z \rightarrow 1} [(1-z^{-1})x(z)] \\ &= \lim_{z \rightarrow 1} \left[ \frac{z^1(1+z^2)(1+z^{-1})}{4} \right] \\ &= \frac{2 \times 2}{4} \\ &= 1 \end{aligned}$$

Que  $\rightarrow$  A stable & causal sys. is described by the diff-eq

$$y(n) + \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = -2x(n) + \frac{5}{4}x(n-1)$$

$h(n)$  of the sys. is

- (a.)  $(\frac{1}{4})^n u(n) + 3(\frac{-1}{2})^n u(n)$       (b.)  $(\frac{1}{4})^n u(n) - 3(\frac{-1}{2})^n u(n)$   
 (c.)  $(\frac{1}{4})^n u(n) - 3u(n)$       (d.)  $(\frac{1}{4})^{n-1} u(n-1) - 3u(n-1)$

SOL

$$Y(z) + \frac{1}{4}Y(z)z^{-1} + \frac{1}{8}Y(z)z^{-2} = -2X(z) + \frac{5}{4}X(z)z^{-1}$$

$$Y(z) \left[ 1 + \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2} \right] = X(z) \left[ -2 + \frac{5}{4}z^{-1} \right]$$

$$H(z) = \frac{-2 + \frac{5}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

From option

- (a.)  $|z| > \frac{1}{4}$        $|z| > \frac{1}{2}$       common ROC  $|z| > \frac{1}{2}$  (stable)  
 (b.) common ROC  $|z| > \frac{1}{2}$  (stable)  
 (c.) common ROC  $|z| > 1$  (US)  
 (d.) common ROC  $|z| > 1$  (US)

Here sys. is causal so follow the initial value theorem

$$h(0) = \lim_{z \rightarrow \infty} H(z) = -2$$

- (a.)  $h(0) = 1+3=4$       (b.)  $h(0)=1-3=-2$

### \* Jury Test →

- (1) To check stability of continuous time causal LTI sys. Routh-Hurwitz criteria is used.
- (2) Jury test is used to check stability of discrete time causal LTI sys.

$$H(z) = \frac{k_m z^m + k_{m-1} z^{m-1} + \dots + k_0}{q_n z^n + q_{n-1} z^{n-1} + \dots + q_0} = \frac{N(z)}{D(z)}$$

For causality :-  $n \geq m$

c/s eqn :-  $D(z) = 0$

Necessary cond'n for stability →

- (1)  $D(1) > 0$
- (2)  $D(-1) \begin{cases} > 0 & ; n = \text{even} \\ < 0 & ; n = \text{odd} \end{cases}$

Jury table → no. of rows =  $2n-3$

	$z^0$	$z^1$	$z^2$	$\dots$	$z^n$
1.	$q_0$	$q_1$	$q_2$	$\dots$	$q_n$
2.	$q_n$	$q_{n-1}$	$q_{n-2}$	$\dots$	$q_0$
3.	$b_0$	$b_1$	$b_2$	$\dots$	$b_{n-1}$
4.	$b_{n-1}$	$b_{n-2}$	$b_{n-3}$	$\dots$	$b_0$
5.	$c_0$	$c_1$	$c_2$	$\dots$	$c_{n-2}$

$$b_i = \begin{vmatrix} q_0 & q_{n-i} \\ q_n & q_1 \end{vmatrix} = q_0 q_1 - q_n q_{n-i}$$

$$b_0 = q_0 q_0 - q_n q_{n-0} = q_0^2 - q_n^2$$

$$b_1 = q_0 q_1 - q_n q_{n-1}$$

$$c_0 = b_0^2 - b_{n-1}^2$$

$$c_1 = b_0 b_1 - b_{n-1} b_{n-2}$$

sufficient cond<sup>n</sup> →

- (1)  $|a_n| > |a_0|$
- (2)  $|b_{n-1}| < |b_0|$
- (3)  $|c_{n-2}| < |c_0|$
- (4)  $|d_{n-3}| < |d_0|$

Que. →  $H(z) = \frac{2z^3 + 2z^2 + 3z + 1}{2z^4 + 3z^3 + z^2 - 1}$  check stability of sys.

Sol<sup>n</sup> →  $D(z) = 2z^4 + 3z^3 + z^2 - 1$

order →  $n=4$ , (even)

necessity cond<sup>n</sup>:

- (i)  $D(1) > 0$  ✓
- (ii)  $D(-1) > 0, n=\text{even} \times$

Que. → check stability of sys. having

$$D(z) = 5z^3 + 2z^2 + 4z + 1$$

Sol<sup>n</sup> →  $D(z) = 5z^3 + 2z^2 + 4z + 1$

order - odd ( $n=3$ )

cond<sup>n</sup> → (i)  $D(1) > 0$

- (ii)  $D(-1) < 0, n=\text{odd}$

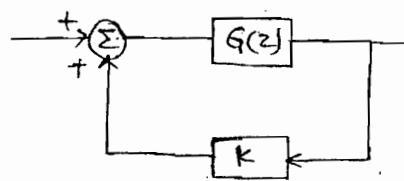
↓  
-6

Jury table → No. of rows  $2n-3 = 2 \times 3 - 3 = 3$

	$z^0$	$z^1$	$z^2$	$z^3$	
$a_0$	1	1	4	2	5 $\curvearrowright a_n$
	2.	5	2	4	1
$b_0$	3.	-24	-6	-18	0 $\curvearrowright b_{n-1}$
	4	8	-18	-16	-24

Sufficient cond<sup>n</sup>:

- (1)  $|a_n| > |a_0|$  ✓
- (2)  $|b_{n-1}| < |b_0|$  ✓

Que.

$$g(n) \Leftrightarrow g(z)$$

$$g(n) = (0, 1, 1)$$

The sys. is stable for range of value

value of 'K'

- (a)  $(-1, \frac{1}{2})$  (b)  $(-1, 1)$  (c)  $(-\frac{1}{2}, 1)$  (d)  $(-\frac{1}{2}, 2)$

Soln.

$$G(z) = z^1 + z^2$$

$$H(z) = \frac{G(z)}{1 - KG(z)}$$

$$= \frac{z^1 + z^2}{1 - K(z^1 + z^2)} \times \frac{z^2}{z^2}$$

$$= \frac{z+1}{z^2 - K(z+1)}$$

$$D(z) = z^2 - Kz + 1 \xrightarrow{\text{even}}$$

\*  $D(1) > 0, 1 - 2K > 0 \Rightarrow K < \frac{1}{2} \text{ ---- (i)}$

\*  $D(-1) > 0 \Rightarrow 1 > 0$

\*  $|q_n| > |q_0|$

$$1 > |K|$$

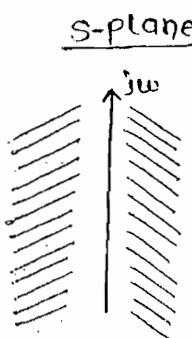
$$(-1 < K < 1) \text{ ---- (ii)}$$

from (i) & (ii)

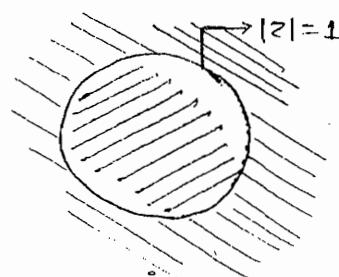
$$\boxed{-1 < K < \frac{1}{2}}$$

DATE-31/10/14

mapping between S-plane & z-plane →



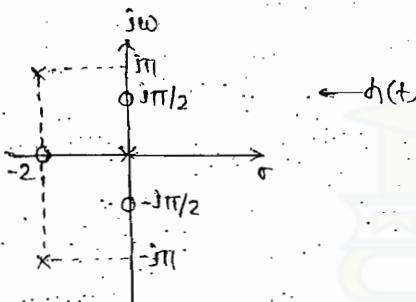
z-plane



Pole mapping -  $[z = e^{st}]$

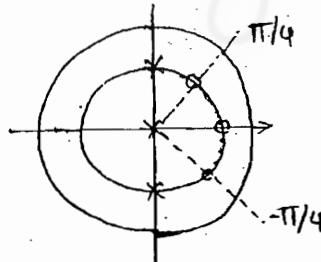
T-sampling time interval

Que. →

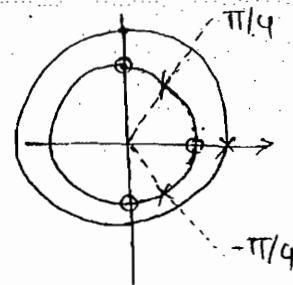


The impulse response  $h(t)$  is sampled at 2kHz to get  $h(n)$ . Which one of the following represents equivalent pole-zero plot of  $H(z)$  in z-plane?  
(The concentric circles are  $|z|=1$ ;  $|z|=\frac{1}{2}$ )

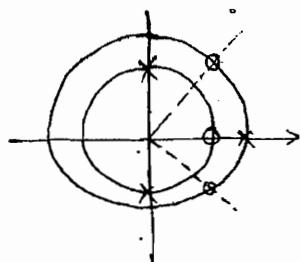
(a.)



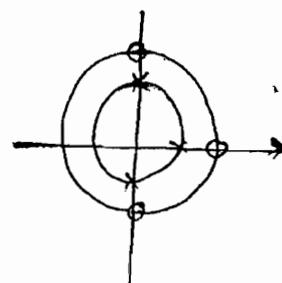
(b.)



(c.)



(d.)



Sol<sup>n</sup> → S-plane

$$s = 0, -2 + j\pi, -2 - j\pi$$

z-plane

$$z = e^{sT} = e^{s/T} = e^{s/2}$$

$$= 1, e^{-j\frac{3\pi}{2}}, e^{-j\frac{\pi}{2}}$$

$$= 1, \frac{1}{e} e^{j\frac{\pi}{2}}, \frac{1}{e} e^{-j\frac{\pi}{2}}$$

\* DTFT →

$$x(n) = X(e^{j\omega})$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\downarrow z = e^{j\omega}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

$$\downarrow (\omega = 0)$$

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n)$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{jn\omega} d\omega.$$

$$\downarrow (n=0)$$

$$x(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

$$2\pi x(0) = \int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

$$\begin{aligned}
 \textcircled{16} \quad y(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} y(n) \\
 &= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \\
 &= 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \dots \\
 &= \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{4}{3}
 \end{aligned}$$

$$\textcircled{20} \quad h(n) = \frac{1}{2} [\delta(n) + \delta(n-2)] \quad |H(e^{j\omega})| = ?, \quad \omega = \Omega$$

$$\begin{aligned}
 H(z) &= \frac{1}{2}(1+z^{-2}) \\
 H(e^{j\omega}) &= \frac{1}{2}(1+e^{-j2\omega}) \\
 H(e^{j\omega}) &= e^{-j\omega} \left[ \frac{e^{j\omega} + e^{-j\omega}}{2} \right]
 \end{aligned}$$

$$|H(e^{j\omega})| = |e^{-j\omega}| |\cos\omega|$$

$$|H(e^{j\omega})| = |\cos\omega|$$

$$\begin{aligned}
 \textcircled{31} \quad x(n) &\xrightarrow[\sin(\omega_0 n + \phi)]{} \boxed{\text{LTI Sys.}} \xrightarrow[H(e^{j\omega})]{} y(n) = A x(n-n_0) \\
 &\quad \downarrow z = e^{j\omega}
 \end{aligned}$$

$$H(e^{j\omega_0}) = ?$$

$$y(n) = A x(n-n_0)$$

$$y(z) = A x(z) z^{-n_0}$$

$$H(z) = A z^{-n_0}$$

$$\downarrow z = e^{j\omega}$$

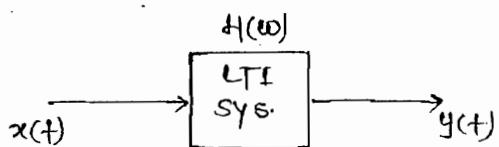
$$H(e^{j\omega}) = A e^{-j\omega n_0} \quad \downarrow \omega = \omega_0$$

$$H(e^{j\omega_0}) = A e^{-j\omega_0 n_0} e^{j2\pi k} \quad \rightarrow k = \text{an int.}$$

$$= A e^{j(2\pi k - \omega_0 n_0)}$$

$$H(e^{j\omega_0}) = 2\pi k - \omega_0 n_0$$

\*\*



$$x(t) = A_0 \sin(\omega_0 t + \phi)$$

$$y(t) = A_0 \times |H(\omega_0)| \times \sin[(\omega_0 t + \phi) + \angle H(\omega_0)]$$

(21)  
19

$$h(t) = e^{-2t} u(t)$$

$$x(t) = 2 \cos(2t); \omega_0 = 2$$

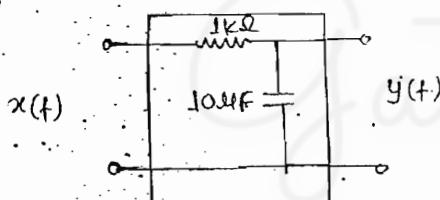
$$y(t) = ?$$

$$H(\omega) = \frac{1}{j\omega + 2}$$

$$H(\omega_0) = \frac{1}{2+2j} \quad |H(\omega_0)| = \frac{1}{2\sqrt{2}}, \angle H(\omega_0) = -\pi/4$$

$$y(t) = 2 \times \frac{1}{2\sqrt{2}} \times \cos(2t - \pi/4)$$

$$= e^{-0.5} \cos(2t - 0.25\pi)$$

(11)  
22

$$x(t) = 3 + 4 \sin 100t$$

$$\omega_0 = 100$$

$$H(s) = \frac{1}{1+sCR}$$

$$(s=j\omega_0)$$

$$H(\omega) = \frac{1}{1+j\omega RC}$$

$$(\omega = \omega_0)$$

$$H(\omega_0) = \frac{1}{1+j\omega_0 RC}$$

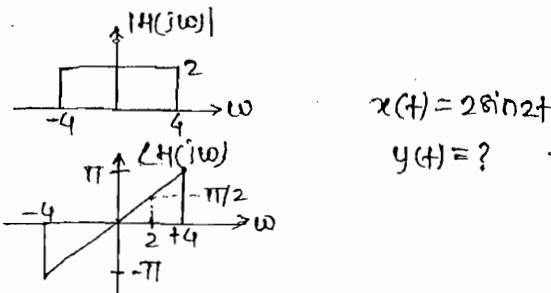
$$= \frac{1}{1+j}$$

$$\left\{ \begin{array}{l} \omega_0 RC = 100 \times 10^3 \times 10 \times 10^{-6} \\ = 1 \end{array} \right.$$

$$|H(\omega_0)| = \frac{1}{\sqrt{2}}, \angle H(\omega_0) = -\frac{\pi}{4}$$

$$y(t) = 3 + 4 \times \frac{1}{\sqrt{2}} \times \sin[100t + (-\pi/4)]$$

$$= 3 + \frac{4}{\sqrt{2}} \sin(100t - \pi/4)$$

Que.

$$x(t) = 2\sin 2t$$

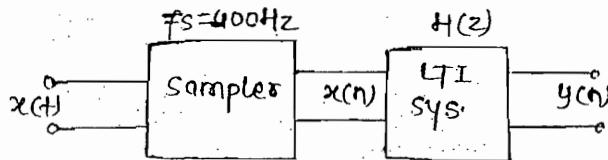
$$y(t) = ?$$

Sol.

$$\omega_0 = 2 \quad x(t) = 2\sin 2t$$

$$y(t) = 2 \times 2 \times \sin(2t + \pi/2)$$

$$y(t) = 4\cos 2t$$

Que.

$$x(t) = 2 + 5\sin(100\pi t)$$

$$H(z) = \frac{1}{N} \left[ \frac{1-z^N}{1-z} \right] \quad \text{where } N = \text{no. of samples per cycle}$$

The o/p.  $y(n)$  of sys. under steady state is

- (a) 0 (b) 1 (c) 2 (d) 5

Sol.  $y(\infty) = ?$ ;  $N = \text{no. of samples per cycle} = \text{Sampling freq.} = 400$

$$x(t) = 2 + 5\sin(100\pi t)$$

$$\downarrow t = nT_s = \frac{n}{f_s} = \frac{n}{400}$$

$$x(n) = 2 + 5\sin\left(100\pi \times \frac{n}{400}\right)$$

$$= 2 + 5\sin\left(\frac{n\pi}{4}\right) \quad ; \quad \omega_0 = \frac{\pi}{4}$$

$$(z = e^{j\omega}) \quad H(\omega) = \frac{1}{N} \left[ \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} \right] \quad \left| \begin{array}{l} \\ (w = \omega_0 = \frac{\pi}{4}) \end{array} \right.$$

$$H(\omega_0) = \frac{1}{N} \left[ \frac{1 - e^{-j\pi N}}{1 - e^{-j\pi/4}} \right]_{N=400/100} \quad (\because e^{-j100\pi} = 1)$$

$$H(\omega_0) = 0$$

$y(n) = 2 + \text{o/p due to sinusoidal part}$

$$= 2$$

$$\boxed{y(\infty) = 2}$$

$$\text{Due to } x(n) \iff x(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$$

$x(n)$  is orthogonal to the signal.

$$(a) y_1(n) = Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$$

$$(c) y_3(n) = Y_3(z) = \sum_{n=-\infty}^{\infty} z^{-|n|} z^{-n}$$

$$(b) y_2(n) = Y_2(z) = \sum_{n=0}^{\infty} (5^n n) z^{-(2n+1)}$$

$$(d) y_4(n) = Y_4(z) = 2z^4 + 3z^2 + 1$$

Q12

$$\sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n) = 0$$

(orthogonal)

$y_2(n)$  is available for odd instants of 'n'

(qns. b)

$$x_1(n) y_2(n) = 0 \quad \text{so} \quad \sum_{n=-\infty}^{\infty} x(n) y_2(n) = 0$$

so; they are orthogonal.

3  
4

$$3y^2(t) + 2y^2(t) + y(t) = x^2(t) + x(t)$$

(all values are present (static))

• NL (sq. term)

10  
4

anticipator - anticausal sys.

11  
4

$$h(n) = \delta(n+2) - \delta(n-2) \quad (\text{OR}) \quad H(z) = z^2 - \bar{z}^2$$

$$y(n) = x(n+2) - x(n-2) \quad (\text{a}) \quad H(z) =$$

$\int_{n=0}^{n-2}$

$$y(n-2) = x(n) - x(n-4)$$

14  
5

$$y(n) = x(n) * h(n)$$

$$= \sum x(k) \cdot h(n-k)$$

$$= \sum x(k) u(n-k) \cdot h(n-k) u(n-k)$$

(16)  
5

$$x(n) = \begin{cases} 0, & n < -2 \text{ or } n > 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$(n-2)$$

$$x(n-2) = 0 \quad n-2 < -2 \quad (-n-2) > 4$$

$$n > 0 \quad n < -6$$

(22)  
6

$$\text{even part of } u(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$$

$$\text{odd part of } u(t) = \frac{u(t) - u(-t)}{2}$$

$$u(t) + u(-t) = 1$$

$$u(-t) = 1 - u(t)$$

$$\begin{aligned} &= \frac{u(t) - 1 + u(t)}{2} \\ &= \frac{2u(t) - 1}{2} \\ &= \frac{\text{sgn}(t)}{2} \\ &= \frac{x(t)}{2} \end{aligned}$$

$$u(t) = \frac{1 + \text{sgn}(t)}{2}$$

$$\text{sgn}(t) = 2u(t) - 1$$

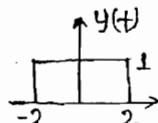
$$\text{Ans. } \left( \frac{1}{2}, \frac{x(t)}{2} \right)$$

(27)  
7

$$x(t) = \delta(t+2) - \delta(t-2)$$

$$y(t) = \int_{-\infty}^t x(z) dz$$

$$= u(t+2) - u(t-2)$$



$$E[y] = \int_{-\infty}^{\infty} |y(t)|^2 dt$$

$$= \int_{-2}^2 1^2 dt = 4$$

(28)  $\frac{7}{7}$  scaling property of convoln

$$(t^2+t) \quad (t)$$

$$\downarrow$$

$$x_1(t) * x_2(t) = y(t)$$

$$(t=t+3)$$

$$x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$$

$$\downarrow a=3$$

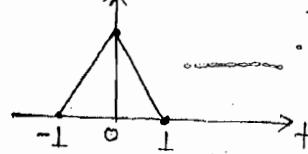
$$x_1(3t) * x_2(3t) = \frac{1}{3} y(3t)$$

$$\rightarrow (9t^2 + 3t) \quad (3t)$$

(32)  
8

$$f(t) = \begin{cases} 1-t & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$$

$$= \begin{cases} 1+t & -1 \leq t \leq 0 \\ 1-t & 0 \leq t \leq 1 \\ 0 & |t| > 1 \end{cases}$$



## Chapter-02

(1)  
9

$$y(n+2) = 5y(n+1) + 6y(n) = x(n)$$

$$H(z) = \frac{1}{z^2 - 5z + 6}$$

$$H(z) = \frac{1}{(z-3)(z-2)}$$

Poles:- 3, 2.

(9)

$$y(t) = \int_0^\infty y(z)x(t-z)dz = \delta(t) + x(t)$$

$$y(t) + y(t) * x(t) = \delta(t) + x(t)$$

$$\downarrow (4)$$

$$y(s) + y(s) \cdot x(s) = 1 + x(s)$$

$$y(s) [1 + x(s)] = 1 + x(s)$$

$$h(n) = (1, \frac{1}{2}, \frac{1}{4})$$

$$x(n) = (1, 0, 1)$$

$$y(n) = x(n) * h(n)$$

Tabular method

(15)  
10

$$x(t) = u(t) \rightarrow y(t) = 0.5(1-e^{-2t})u(t)$$

$$X(s) = \frac{1}{s} \quad Y(s) = 0.5 \left( \frac{1}{s} - \frac{1}{s+2} \right)$$

$$= \frac{1}{s(s+2)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$$h(t) = e^{-2t}u(t)$$

(17)  
10

$$(i) y(t) = t x(t) \rightarrow L$$

$$(ii) y(t) = t x^2(t) \rightarrow NL$$

$$(iii) y(t) = x(2t) \rightarrow L$$

(18)  
11

$$y(t) = \frac{1}{T} \int_{-T/2}^{+T/2} x(\tau) d\tau$$

$$\downarrow y(t) = h(t)$$

$$h(t) = \frac{1}{T} \int_{-T/2}^{+T/2} s(\tau) d\tau$$

$$h(t) = \frac{1}{T} \left[ u(\tau) \right]_{-T/2}^{+T/2}$$

$$= \frac{1}{T} [u(t+T/2) - u(t-T/2)]$$

$$\downarrow \downarrow \downarrow \downarrow h(t)$$

(19)  
11

$$b(t) = \delta(t) + \delta(t-1) \Rightarrow H(s) = 1 + e^{-s}$$

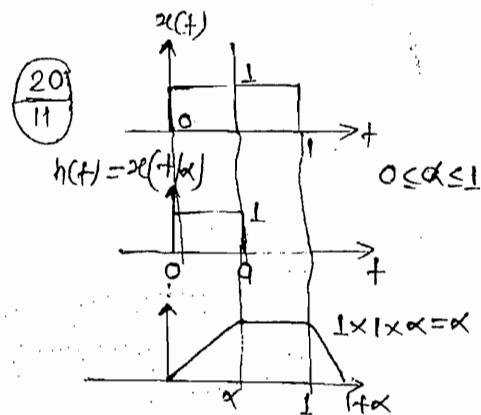
$$y(t) = u(t) + u(t-1) \Rightarrow Y(s) = \frac{1}{s} + \frac{1}{s} e^{-s}$$

$$= \frac{1}{s}(1 + e^{-s})$$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$X(s) = \frac{1}{s}$$

$$x(t) = u(t)$$

21  
12LPF;  $f_c = 100\text{Hz}$ 

$$v(t) = 30\sqrt{2}\sin(1256t)$$

$$f_i = \frac{1256}{2\pi} \approx 200\text{Hz}$$

### Chapter-03

9  
14

$$\begin{aligned} x_1(t) &\rightarrow e^{j20t} (P) \\ x_2(t) &\rightarrow e^{-2t+jt} \downarrow e^{-2t} e^{jt} \downarrow \\ &\downarrow \text{NP} \qquad \downarrow \text{N} \qquad \downarrow \text{P} \end{aligned}$$

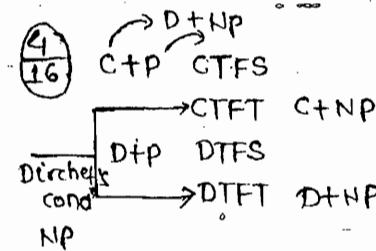
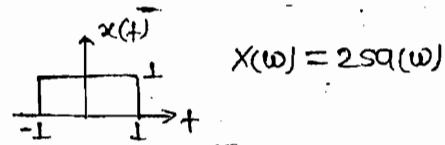
13  
14

$$e^{-1t} \downarrow \text{NP}$$

$$\begin{aligned} \text{15} & \quad v(t) = (10\sin 2\pi t) 100t \\ & \downarrow \text{NP} \qquad \downarrow \text{P} \qquad \downarrow \text{NP} \end{aligned}$$

### Chapter-04

16

20  
19

$$X(\omega) = 2s(\omega)$$

$$X(\omega) = 0, \omega = ?$$

$$2s(\omega) = 0$$

$$s(\omega) = 0$$

$$\frac{\sin \omega}{\omega} = 0$$

$$\sin \omega = 0$$

$$\omega = n\pi, n \neq 0$$

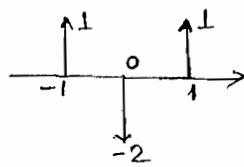
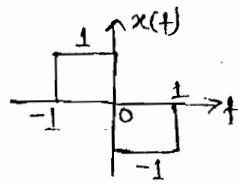
8  
17

$$y(n) = \frac{1}{2}y(n-1) = x(n) = k\delta(n)$$

$$y(z) - \frac{1}{2}y(z)z^{-1} = x(z) = k$$

$$y(z) = \frac{k}{1 - \frac{1}{2}z^{-1}}$$

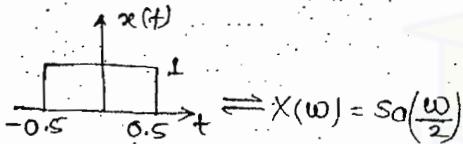
$$y(n) = k\left(\frac{1}{2}\right)^n u(n)$$

(24)  
19

$$\frac{dx(t)}{dt} = \delta(t+1) + \delta(t-1) - 2\delta(t)$$

$$j\omega X(\omega) = (e^{j\omega} + e^{-j\omega}) - 2$$

$$X(\omega) = \frac{2\cos\omega - 2}{j\omega}$$

25  
19

$$h(t) = e^{j\omega_0 t} \Leftrightarrow H(\omega) = 2\pi\delta(\omega - \omega_0)$$

$$Y(t) = 0, \omega_0 = ?$$

$$Y(\omega) = 0$$

$$X(\omega) \cdot H(\omega) = 0$$

$$S\delta\left(\frac{\omega}{2}\right) \cdot 2\pi\delta(\omega - \omega_0) = 0$$

$$S\delta\left(\frac{\omega_0}{2}\right) 2\pi\delta(\omega - \omega_0) = 0$$

$$S\delta\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\sin\left(\frac{\omega_0}{2}\right)}{\left(\frac{\omega_0}{2}\right)} = 0$$

$$\sin\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\omega_0}{2} = n\pi, n \neq 0$$

$$\omega_0 = 2n\pi, n \neq 0$$

### Chapter-05

(2)  
21

$$x(t) = u(t) \Leftrightarrow y(t) = t^2 e^{-2t} u(t)$$

$$t^2 u(t) \Leftrightarrow \frac{2}{s^3}$$

$$e^{-2t} t^2 u(t) \Leftrightarrow \frac{2}{(s+2)^3}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{(s+2)^3} = \frac{2s}{(s+2)^3}$$

(4)  
21

$$\frac{d^2y(t)}{dt^2} = x(t-2)u(t-2) + \frac{d^2x(t)}{dt^2}$$

$$H(s) \rightarrow LT$$

$$f(t) \Leftrightarrow F(s) = \frac{s+2}{s^2+1}$$

$$g(t) \Leftrightarrow G(s) = \frac{s^2+1}{(s+2)(s+2)}$$

$$h(t) = \int_0^t f(z) \cdot g(t-z) dz \Leftrightarrow H(s) = ?$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau) d\tau$$

$$h(t) = f(t) * g(t)$$

let assume  $\rightarrow f(t) \& g(t) \rightarrow \text{causal}$

$$H(s) = F(s) G(s) = \frac{1}{s+3}$$

$$x(t) = \sin\omega_0 t$$

$$Y(\omega) = 0, \omega = ?$$

$$Y(t) = 0, \omega = ?$$

$$Y(s) = 0$$

$$H(s) \cdot X(s) = 0$$

$$(s=j\omega) \quad H(s) = 0$$

$$H(\omega) = 0, \omega = ?$$

$$\frac{26}{24} f(t) \rightleftharpoons F(s)$$

$$f(t-z) \rightleftharpoons F_2(s) = F_1(s) e^{-sz}$$

$$g(t) \rightleftharpoons G(s) = \frac{F_2(s) \cdot F_1(s)}{|F(s)|^2}$$

$$G(s) = \frac{F_2(s) \cdot F_1(s)}{F_1(s) \cdot F_1(s)} = e^{-sz}$$

$$g(t) = \delta(t-z)$$

(29)  
24

$$y(t)=0, \omega=?$$

$$Y(s)=0$$

$$\frac{1}{s} - \frac{3}{s+1} + \frac{3}{s+2} = 0$$

$$(s+1)(s+2) - 3s(s+2) + 3s(s+1) = 0$$

$$s^2 + 3s + 2 - 3s^2 - 6s + 3s^2 + 3s = 0$$

$$s^2 + 2 = 0$$

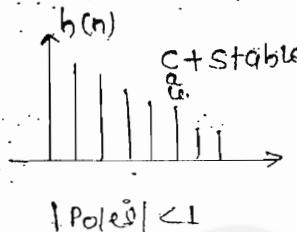
$$(j\omega)^2 + 2 = 0$$

$$-\omega^2 + 2 = 0$$

$$\omega = \sqrt{2} \text{ rad/s.}$$

### Chapter-06

(25)

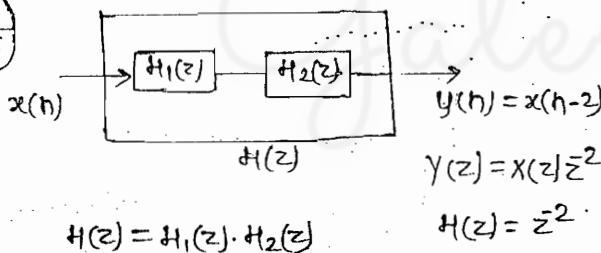


(26)

$$a^n u(n) \rightleftharpoons \frac{z}{z-a}, |z| > |a|$$

$$+a^n u(-n-1) \rightleftharpoons \frac{-z}{z-a} |z| < |a|$$

(25)



$$H(z) = H_1(z) \cdot H_2(z)$$

$$H_2(z) = \frac{H(z)}{H_1(z)}$$

$$H_2(z) = \frac{z^2}{z-0.8}$$

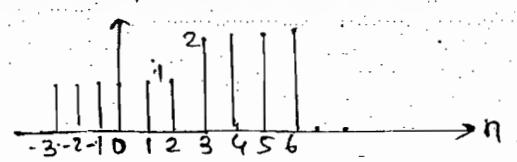
$$= \frac{z^2 - 0.8z^2}{z-0.5} \times \frac{z^2}{z^2}$$

$$= \frac{z^2(1-0.8z^{-2})}{1-0.5z^{-1}}$$

$$= \frac{z^2(0.8z^2 - 1)}{1-0.5z^{-1}}$$

(27)

$$h(n) = u(n+3) + u(n-3) - 2u(n-7)$$



(30)

$$x(n) \quad y(n) \quad \text{LT[sys.}$$

$$u(n) \rightarrow s(n) = \text{step response}$$

$$\downarrow d/dn$$

$$s(n) \rightarrow h(n) = \frac{ds(n)}{dn}$$

$$= s(n) - s(n-1)$$

$$h(n) = s(n) - s(n-1)$$

$$s(n) = (\underbrace{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots})$$

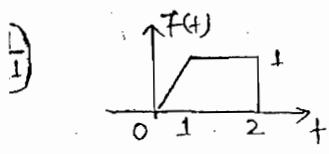
$$s(n-1) = (0, 1, \frac{1}{2}, \frac{1}{4}, \dots)$$

$$h(n) = (1, -1, -1, -1, \dots)$$

(32)

$$G(s) \rightarrow \text{Poles} \Rightarrow s = 0, 5$$

$$z = e^{sT} = 1, e^{sT}$$



$$f(t) = \tau(t) - \tau(t-1) - u(t-2)$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

$$= \frac{1}{s^2} (1 - e^{-s} - s e^{-2s})$$

$$\frac{d^2y}{dt^2} = x(t-2) + \frac{d^2x}{dt^2}$$

$$s^2 Y(s) = e^{-2s} X(s) + s^2 x(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s^2 + e^{-2s}}{s^2} = 1 + \frac{e^{-2s}}{s^2}$$

$$F(s) = \frac{27s+97}{s(s+33)} \Rightarrow f(t)$$

$$\frac{27s+97}{s(s+33)} = \frac{A}{s} + \frac{B}{s+33} \quad s=0, A=\frac{97}{33}$$

$$s=-33, B = \frac{-27 \times 33 + 97}{-33 \times 33} = \frac{794}{1089}$$

$$f(t) = \frac{97}{33} u(t) + \frac{794}{1089} e^{-33t} u(t)$$

$$f(0^+) = \frac{97}{33} u(0^+) + \frac{794}{1089} e^{-33(0^+)} = \frac{97}{33}$$

$$x_1(t) = e^{k_1 t} u(t) \quad x_2(t) = e^{k_2 t} u(t)$$