

DIGITALS

NOTES

GATE 2009

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Digitals

SUN.

17/08/08

Number Systems:

	<u>Base/Radix</u>	<u>Numbers</u>
1. Decimal	10	0, 1, ..., 9
2. Binary	2	0, 1
3. Octal	8	0, 1, ..., 7
4. Hexadecimal	16	0, 1, ..., 9, A, B, C, D, E, F.

Each Hexa digit \rightarrow 4 bits,

$$3F_{16} \rightarrow 0011\ 1111_2$$

Each octal digit \rightarrow 3 bits

$$316_8 \rightarrow 011\ 001\ 110_2$$

$$\text{Q. } 110010_2 = x_{16}$$

$$\text{Q. } 11011.01_2 = x_{16}$$

$$\frac{\overbrace{0011}^3 \overbrace{0010}^2}{3 \quad 2} = 32_{16} \quad \underbrace{0001}_{1} \underbrace{1011}_{B} \cdot \underbrace{0100}_{4} = 1B4_{16}$$

$$\text{Q. } 6728_{10} = x_2$$

$$6728_{10} \rightarrow 6728_{16} \rightarrow x_2$$

$$\Rightarrow 16 \left| \begin{array}{r} 6728 \\ 420 - 8 \\ \hline 26 - 4 \\ \hline 1 - 10(A) \end{array} \right. \qquad \begin{array}{l} 1A48_{16} \\ = \underline{0001}\ \underline{1010}\ \underline{0100}\ \underline{1000}_2 \end{array}$$

Q. Determine the possible bases of the following relations.

$$(1). \sqrt{41} = 5 \qquad \text{max. digit is } 5 \downarrow$$

so ^{min} value of base is 6, so base ≥ 6

Let base = b.

$$\sqrt{4 \times b^1 + 1 \times b^0}_{10} = 5 \times b^0_{10}$$

$$\Rightarrow \sqrt{4b+1} = 5$$

$$\Rightarrow 4b+1 = 25$$

$$\Rightarrow b = 6.$$

Q. $\frac{302}{20} = 12.1$

Let base = b .

Base ≥ 4 b'coz max

$$\Rightarrow \frac{3b^2+2}{2b} = b+2+\frac{1}{b}$$

$$\Rightarrow \frac{3b^2+2}{2b} = \frac{b^2+2b+1}{b}$$

$$\Rightarrow b = 4.$$

Q. $\frac{44}{4} = 11$

Let base = b . Observed base ≥ 5 , b'coz maximum value of digit = 4.

$$\frac{4b+4}{4} = b+1 \Rightarrow b+1 = b+1$$

The above relation is valid in all the no. system with base ≥ 5 .

Q. In a positional weight system x & y are two successive digits and $xy = 25_{10}$ & $yx = 31_{10}$
Determine the values of base x & y .

Here $b = ?$, $x = ?$ & $y = ?$

and $y = x+1$.

$$(x)(x+1) = 25_{10} \quad ((x+1)b+x) = 31_{10} \quad (1)$$

$$\Rightarrow [x \times b + (x+1)]_{10} = 25_{10} \Rightarrow x(b+1) + b = 31 \rightarrow (2)$$

$$\Rightarrow x(b+1) + 1 = 25 \rightarrow (1)$$

$$(1) - (2) \Rightarrow b = 7. \text{ Then from } (1) \Rightarrow x = 3, y = 4.$$

Complementary Number Representation :-

- Base = 2
- \Rightarrow (2-1)'s complement
- \Rightarrow 2's complement

Decimal system ($b=10$)

$$9\text{'s complement of } 168_{10} \Rightarrow \begin{array}{r} 999 \\ 168 \\ (-) \hline 831 \end{array}_{10}$$

10's complement of $168_{10} \Rightarrow 9\text{'s comp} + 1$

$$\Rightarrow \begin{array}{r} 999 \\ 168 \\ (-) \hline 831 + 1 \end{array}_{10} = 832_{10}$$

Q. 862_{10}

$$\begin{array}{r} 491_{10} \\ (-) \hline ? \end{array} = 862_{10} + (-491_{10})$$

$$(i) \quad \begin{array}{r} 862 \\ + (9\text{'s of } 491) \\ \hline 508 \end{array} \quad \begin{array}{r} 862 \\ (+) \hline 508 \end{array}$$

$$(ii) \quad \begin{array}{r} 862 \\ + (10\text{'s of } 491) \\ \hline 371 \end{array} \quad \begin{array}{r} 862 \\ (+) \hline 371 \end{array} \quad \xrightarrow{\text{EOC}}$$

$$+ (10\text{'s of } 491) = 509$$

EOC (1)
ignore

Q. 491_{10}

$$\begin{array}{r} 491_{10} \\ - 862_{10} \\ \hline ? \end{array} = \begin{array}{r} 491 \\ + (-862) \\ \hline 138 \end{array} \quad \leftarrow 10\text{'s}$$

NO EOC
↓
-ve sign

Digital System ($b=2$)

1's complement of $1011 \rightarrow 0100_2$

2's complement of $1011 \rightarrow 1\text{'s of } 1011 + 1$

$$\Rightarrow 0100 + 1 = 0101$$

$$Q. x = 1000111 \underbrace{000}_{\text{2's complement}}$$

$$\text{2's complement of } x = 0111001000$$

$$Q. x = 1011$$

$$\text{2's of } x = 0101$$

$$Q. \begin{array}{r} 11010_2 \\ - 01110_2 \end{array} = +(-01110)$$

$$(i). \begin{array}{r} 11010 \\ + (\text{1's of } 01110) \end{array} = \begin{array}{r} 11010 \\ + 10001 \end{array}$$

$$(ii). \begin{array}{r} 11010 \\ + (\text{2's of } 01110) \end{array} \quad \begin{array}{r} \xrightarrow{\text{EOC}} 01011 \\ \xrightarrow{\text{+1}} \\ \hline 01100 \end{array}$$

$$\begin{array}{r} 11010 \\ = \\ \boxed{\begin{array}{r} + 10010 \\ \hline \text{EOC ignore } 01100 \end{array}} \end{array}$$

$$Q. \begin{array}{r} 01110_2 \\ - 11010_2 \end{array} = 01110$$

$$= +(\text{2's of } 11010)$$

$$= \begin{array}{r} 01110 \\ + 00110 \end{array}$$

$$\begin{array}{c} \xrightarrow{\text{NO EOC}} \boxed{\begin{array}{r} 10100 \\ \downarrow \text{2's} \\ 01100 \end{array}} \\ \begin{array}{l} 2^4 = 16 \\ 16 - 2 = 14 \\ 16 - 1 = 15 \end{array} \end{array}$$

$$+0 = 0000$$

$$+0 = 0000$$

$$-0 = \text{1's comp of } +0$$

$$-0 = \text{2's comp of } +0$$

$$= \text{1's of } 0000$$

$$= 0000$$

$$= 1111 \leftarrow (\text{Disadv. of 1's complement})$$

* Range of numbers represented using 'n' bits

(to represent 1's comp. form) $\rightarrow + (2^{n-1} - 1)$ to $- (2^{n-1} - 1)$

(16) number Let $n=4 \Rightarrow +7$ to $-7 \rightarrow (14)$

2's comp. form $\Rightarrow + (2^{n-1} - 1)$ to -2^{n-1}

Let $n=4 \Rightarrow +7$ to $-8 \rightarrow (15)$

Q. How many bits are required to represent

-64_{10} in a). 1's comp. form b). 2's form

1's form $\Rightarrow + (2^{n-1} - 1)$ to $- (2^{n-1} - 1)$

Let $n=7 \Rightarrow +63$ to -63

$\checkmark n=8 \Rightarrow +127$ to -127 .

2's form $\Rightarrow + (2^{n-1} - 1)$ to -2^{n-1}

\checkmark Let $n=7 \Rightarrow +63$ to -64

Q. 10's comp for $(-731)_{11}$

Q. 9's comp of $(-731)_{10}$

$$\begin{array}{r} A A A \\ 7 3 1 \\ (-) \hline 3 7 9 \end{array}$$

$$\begin{array}{r} 999 \\ (-) 731 \\ \hline 268 \end{array}$$

Binary Numbers :

(a). Unsigned Numbers \rightarrow

$\boxed{n \text{ bits}}$

magnitude

(b). Signed Numbers

↓ represented by

$\boxed{\text{MSB}}$

↓ sign bit magnitude

(i). sign magnitude

(ii). 1's comp form

(iii). 2's comp form

$0 \rightarrow +ve$

$1 \rightarrow -ve$

These three representations are same for unsigned (+ve numbers).

$$(ii). \text{ sign magnitude} \Rightarrow +3 = \begin{array}{c} \downarrow \\ 011 \end{array}$$

$$-3 = \begin{array}{c} \downarrow \\ 111 \end{array}$$

$$(iii). 1's \text{ comp. form} \Rightarrow +3 = 011$$

$$-3 = \begin{array}{c} \text{1's comp of } +3 \\ = 100 \end{array}$$

$$(iv). 2's \text{ comp. form} \Rightarrow +3 = 011$$

$$-3 = \begin{array}{c} \text{2's comp of } +3 \\ = 101 \end{array}$$

Q. Decimal equivalent of 2's number $\underline{\underline{101}}$ is -?

$$\begin{array}{r} \downarrow \\ \begin{array}{r} \downarrow \\ \begin{array}{r} \downarrow \\ \begin{array}{r} \downarrow \\ \begin{array}{r} \downarrow \\ 011 \end{array} \end{array} \end{array} \end{array} \\ = -3_{10} \end{array}$$

Q. Decimal equivalent of sign mag. no. $\underline{\underline{111}}$ is -?

$$-3_{10}$$

Q. Represent $+53_{10}$ & -53_{10} in all the 3 forms of signed no. representation.

$$53_{10} \rightarrow \begin{array}{r} 53 \\ 2 \overline{)26} -1 \\ 2 \overline{)13} -0 \\ 2 \overline{)6} -1 \\ 2 \overline{)3} -0 \\ 1 -1 \uparrow \end{array} = 110101_2$$

$$+53 = 0110101$$

$+53_{10}$	Sign mag. form <u>0110101</u>	1's form 0110101	2's form 0110101
-53_{10}	\downarrow	$-53 = \text{1's of } +53$ $= 1001010$	$-53 = \text{2's of } +53$ $= 1001011$

Q. What are the decimal equivalents of the following signed no.s in all the 3 forms.

	Sign mag. form	1's form	2's form
01101	$+13_{10}$	$+13_{10}$	$+13_{10}$
101010	$\begin{array}{r} 101010 \\ -10_{10} \end{array}$	$\begin{array}{r} 101010 \\ \downarrow 1's \\ -010101 \end{array}$ $= -21_{10}$	$\begin{array}{r} 101010 \\ \downarrow 2's \\ -010110 \end{array}$ $= -22_{10}$
$\boxed{111111}$	$\begin{array}{r} 111111 \\ -11_{10} \end{array}$	$\begin{array}{r} 111111 \\ \downarrow 1's \\ -0 \end{array}$	$\begin{array}{r} 111111 \\ \downarrow 2's \\ -1 \end{array}$

Q. Decimal equivalent of 2's no. 1000 is - ?

$$\begin{array}{r} 1000 \\ \downarrow 2's \\ -1000 \end{array}$$

$$= -8_{10}$$

Q. Decimal equivalent of 2's no. 10000 is - ?

$$\begin{array}{r} 10000 \\ \downarrow 2's \\ -10000 \end{array}$$

$$= -16_{10}$$

Q. What is the equivalent 2's comp representation of a 2's comp. no. 1101 is - ?

- (a). 001101 (b). 011101 (c). 101101 (d). 111101

$$+6 = 0110$$

$$-6 = 2's \text{ of } +6 = 2's \text{ of } 0110$$

$$= 1010$$

$$= 2's \text{ of } 00110 = 11010$$

$$= 2's \text{ of } 000110 = 111010$$

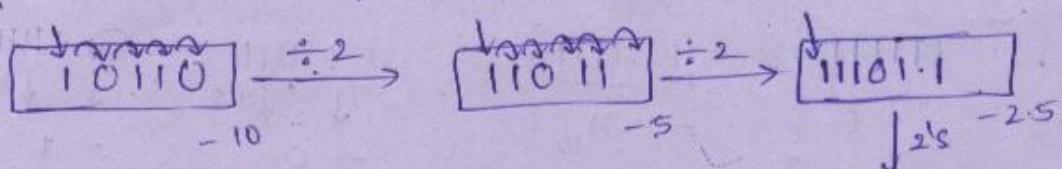
Q. A Register contains a 2's comp. no. 10110. What is the content of the register if it is divided by 2.

decimal equi. of 10110 = - 01010

$$= -\frac{10_{10}}{2} = -5$$

-5 = 2's of +5

(or) = 2's of 00101 = 11011

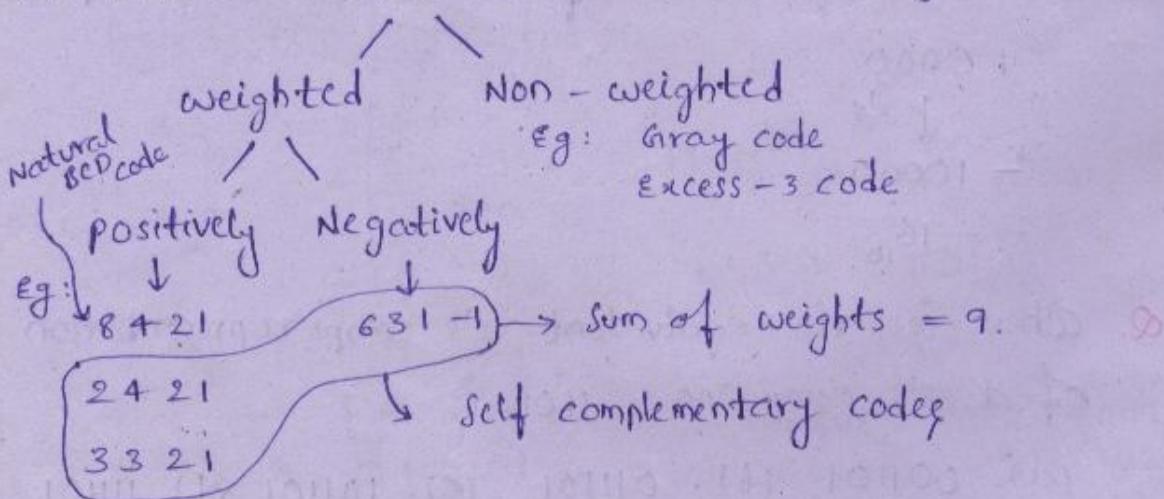


Binary Codes :-

a). Alpha numeric [ASCII code $000_7 = -2.5_{10}$
[7 bits, $2^7 = 128$ Alpha numericals]
EBCDIC

{ 8 bits, $2^8 = 256$ Alpha numericals]

b). Numeric $\xrightarrow{\text{BCD}}$ each decimal digit \rightarrow 4 bits



Excess-3 : self complementary code, sequential code.

8421 : sequential code.

Dec. digit	Natural BCD	Excess-3	2421	631-1	Gray
0	0000	0011	0000	0000	0000
1	0001	0100	0001	0010	0001
2	0010	0101	0010	0101	0011
3	0011	0110	0011	0100	0010
4	0100	0111	0100	0110	0100
5	0101	1000	1011	1001	0101
6	0110	1001	1100	1011	0101
7	0111	1010	1101	1010	0100
8	1000	1011	1110	1101	1100
9	1001	1100	1111	1111	1101

743_{10} in (1). BCD $\rightarrow 0111\ 0100\ 0011$ BCD.

(2). 3321 $\rightarrow 1101\ 0101\ 0011$ 3321

$\begin{array}{l} \text{3} \\ \downarrow \\ \{ \begin{array}{l} 1000 \\ 0100 \\ 0011 \\ 0010 \end{array} \} \end{array}$ Self complementary

$\begin{array}{l} \text{3} \\ \downarrow \\ (3321) \end{array}$ $\begin{array}{l} \text{2} = 0010 \\ (3321) \end{array}$ Now $7 = \text{comp. of } 0010 = 1101.$

(3). Binary, $\rightarrow 2^n \geq 743$, $n = 10.$

$\begin{array}{r} 16 | 743 \\ 16 | 46 - 7 \\ 16 | 2 \end{array} \quad \begin{array}{l} 2E7_{16} = 0010\ 1110\ 0111_2 \end{array}$

Gray code : (reflective code, unit distance code)

1-bit	2-bit	3-bit
$0 \oplus 0 = 0$ $0+1=1$ $1+0=1$ $1+1=0$ Modulo -2	$0 \oplus 0 = 0$ $0+1=1$ $1+0=1$ $1+1=0$ Modulo -2	$0 \oplus 0 = 0$ $0+1=1$ $1+0=1$ $1+1=0$ Modulo -2
	Addition (Exclusive OR)	differ by 1-bit

Binary : 10110 100

permits : 2^n

Gray : 11100110

2F1

458

Binary : 1011000000000000

0001

BCD Addition :-

$$\begin{array}{r}
 G_{10} = 0110_{BCD} \\
 + 2_{10} = 0010_{BCD} \\
 \hline
 1000_{BCD}
 \end{array}
 \quad
 \begin{array}{r}
 8_{10} = 1000_{BCD} \\
 + 6_{10} = 0110_{BCD} \\
 \hline
 1110 \rightarrow \text{not a valid BCD.}
 \end{array}$$

$$\begin{array}{r}
 + 9_{10} = 1001_{BCD} \\
 8_{10} = 1000_{BCD} \\
 \hline
 10001
 \end{array}
 \quad
 \begin{array}{r}
 + 0110 \\
 \hline
 10111_{BCD}
 \end{array}$$

Decimal Binary/Hexa

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	A
8	B
9	C
10	D
11	E
12	F
13	10
14	11
15	12

$8+2=10$

Q. In the following BCD additions how many BCD corrections are required.

$$\begin{array}{r}
 49_{10} = 0100\overset{R}{1}001 \\
 + 57_{10} = 0101\overset{R}{0}111 \\
 \hline
 1010\ 0000
 \end{array}$$

Ans: 2 times.

$$\begin{array}{r}
 + 0110\ 0110 \\
 \hline
 0000\ 0110
 \end{array}$$

$1\ 0\ 6_{16}$

$$\begin{array}{r}
 176_{10} = 0001\ 0111\ 0110 \\
 + 824_{10} = 1000\ 0010\ 0100 \\
 \hline
 1001\ 1001\ 1010
 \end{array}$$

Ans: 3 times

$$\begin{array}{r}
 176 \\
 824 \\
 \hline
 1000
 \end{array}$$

$$\begin{array}{r}
 0110 \\
 + 0110 \\
 \hline
 0000\ 0000\ 0000
 \end{array}$$

* SUNDAY. 31. Aug. 2008 *

Boolean Algebra:

<u>AND Law</u>	<u>OR Law</u>
$A \cdot 0 = 0$	$A + 0 = A$
$A \cdot 1 = A$	$A + 1 = 1$
Identity element $A \cdot A = A$	$A + A = A$
$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$

(1). Commutative Law:

$$A + B = B + A \quad * \text{ AND, OR operations}$$

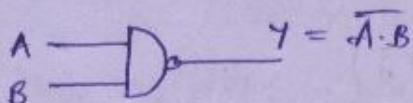
$$A \cdot B = B \cdot A \quad \text{are commutative &}$$

(2). Associative Law:

$$(A + B) + C = A + (B + C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

C find the commutative & associative operations of NAND.



$$(a). \overline{A \cdot B} = \overline{B \cdot A}$$

$$(b). (\overline{A \cdot B}) \text{NAND } C = \overline{\overline{A \cdot B} \cdot C}$$

$$A \text{ NAND } (B \text{ NAND } C) = A \text{ NAND } (\overline{B \cdot C})$$

$$\Rightarrow \overline{\overline{A \cdot B} \cdot C} \neq \overline{A \cdot \overline{B \cdot C}} = \overline{A \cdot \overline{B \cdot C}}$$

* NAND operation is commutative but not associative.

(3). Distribution law:

$$A \cdot (B + C) = AB + AC$$

$$A + (BC) = (A+B)(A+C)$$

$$\begin{aligned} (i). \quad A + \overline{A}B &= (A + \overline{A})(A + B) \\ &= (A + B). \end{aligned}$$

$$(ii). \quad \bar{A} + AB = (\bar{A} + A)(\bar{A} + B) \\ = (\bar{A} + B)$$

(4). Consensus Law:

$$AB + \bar{A}C + BC = AB + \bar{A}C.$$

Eg: $yx + \bar{y}\bar{z} + pwxz = yx + \bar{y}\bar{z}$

Proof: $AB + \bar{A}C + BC (A + \bar{A})$
 $= AB + \bar{A}C + ABC + \bar{A}BC$
 $= AB(1+C) + \bar{A}C(1+B)$
 $= AB + \bar{A}C.$

$$(A+B) \cdot (\bar{A}+C) \cdot (B+C) = (A+B) \cdot (\bar{A}+C)$$

(5). Transposition Law:

$$\boxed{AB + \bar{A}C = (A+C)(\bar{A}+B)}$$

Eg: $xy + \bar{y}\bar{z} = (x+\bar{y})(y+\bar{z})$

RHS: $(x+\bar{y})(y+\bar{z}) = xy + \bar{y}\bar{z} + x\bar{z}$
 $= xy + \bar{y}\bar{z}.$

$$\boxed{(A+B)(\bar{A}+C) = AC + \bar{A}B}$$

(6). De Morgan's Law:

$$\overline{A+B+C+\dots} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \dots$$

$$\overline{\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \dots} = \bar{A} + \bar{B} + \bar{C} + \dots$$

Additional Laws:

(1). $x \cdot f(x, \bar{x}, w, y, \dots, j)$
 $= x \cdot f(1, 0, w, y, \dots, j)$

$$x + f(x, \bar{x}, \omega, y, \dots) \\ = x + f(0, 1, \omega, y, \dots)$$

(7). Duality :

All the Boolean Expressions resulting from interchanging of operator and identity elements are valid.

Eg: $A \cdot 1 = A$

$$\Rightarrow A + 0 = A$$

Adv: To findout complement of a function f.

Step 1: find dual of f ie f_D .

Step 2: Compliment of all vars $\rightarrow \bar{F}$.

Eg: $A + B + C D \quad \bar{F} = \overline{A + B + C D}$

$$f_D = A \cdot B \cdot (C + D) \quad = \bar{A} \cdot \bar{B} \cdot (\bar{C} + \bar{D})$$

$$\bar{F} = \bar{A} \cdot \bar{B} (\bar{C} + \bar{D}) \quad = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot \bar{D}$$

$$= \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot \bar{D}.$$

Q. Simplify following Boolean functions.

(1). $f = AB + \bar{A}C + \bar{C}D + \bar{B}\bar{C}$

$$= AB + C(\bar{A} + \bar{B}) + \bar{C}D$$

$$= \underbrace{AB}_{\bar{x}} + \underbrace{\bar{A}\bar{B}C}_{\bar{x}} + \bar{C}D$$

$$= AB + (C + \bar{C}D)$$

$$= AB + C + D.$$

(2). $f = ABC\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

$$= ABC\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC$$

$$= AB(\bar{C} + C) + BC(\bar{A} + A) + AC(B + B)$$

$$= AB + BC + AC.$$

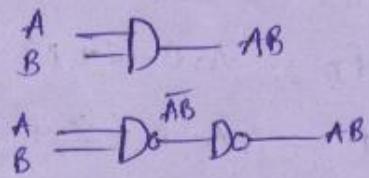
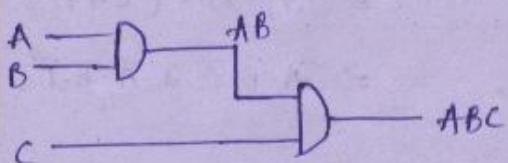
$$\begin{aligned}
 (3). \quad f &= \bar{x}\bar{y}j + \bar{x}y\bar{j} + \underline{\bar{x}yj} + xyj \\
 &= \bar{x}\bar{j} + \bar{x}y\bar{j} + \bar{x}yj + \bar{x}yj + \bar{x}yj + xyj \\
 &= \bar{x}z(\bar{y}+y) + \bar{x}y(\bar{x}+j) + yj(\bar{x}+x) \\
 &= \bar{x}j + \bar{x}y + yj.
 \end{aligned}$$

Q. How many two input NAND's are required to implement the following

$$\begin{aligned}
 (i). \quad f(A, B, C) &= A + AB + ABC \\
 &= A + AB(1+C) \\
 &= A + AB = A.
 \end{aligned}$$

Ans: zero NAND gates.

$$(ii). \quad f = ABC.$$

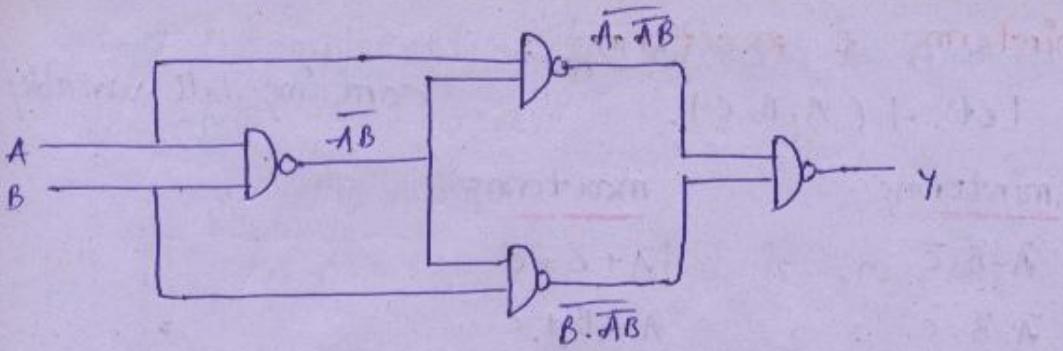


Each AND is replaced by two NAND's. so the total no. of NAND gates = 4.

Q. Complement EX-OR using min. no. of NAND gates.

$$\begin{aligned}
 \begin{array}{c} A \\ B \end{array} \overline{\oplus} &\quad \overline{\overline{AB} + A\bar{B}} \\
 &= \overline{AB} + \overline{A\bar{B}} + \overline{A\bar{B}} + \overline{B\bar{B}} \\
 &= (\overline{A} + \overline{B})A + (\overline{A} + \overline{B})B \\
 &= A\overline{AB} + B\cdot\overline{AB}
 \end{aligned}$$

$$\begin{aligned}
 Y = \overline{Y} &= \frac{\overline{(A \cdot \overline{AB} + B \cdot \overline{AB})}}{\overline{(A \cdot \overline{AB})} \cdot \overline{(B \cdot \overline{AB})}} \\
 &\Rightarrow 5 \text{ NAND's.}
 \end{aligned}$$



Here NAND's are replaced by NOR's
then we get Ex- NOR gate:

$$\begin{aligned}
 & \overline{\overline{A + \overline{A+B}} + \overline{B + \overline{A+B}}} \\
 = & (\overline{A + \overline{A+B}}) (\overline{B + \overline{A+B}}) \\
 = & (\overline{A + \overline{A} \cdot \overline{B}}) (\overline{B + \overline{A} \cdot \overline{B}}) \\
 = & (\overline{A + \overline{B}}) (\overline{B + \overline{A}}) \\
 = & \overline{AB + \overline{A}\overline{B}} = \overline{A \oplus B} =) \text{Do}
 \end{aligned}$$

Operator precedence:

- (1). parenthesis ()
- (2). NOT \rightarrow
- (3). AND .
- (4). OR +

Literal = variable (or) complement of a var.

Implement x-NOR using min. no. of NOR's.

minterms & maxterms:

Let $f(A, B, C)$.

containing all variables

minterms

$$\bar{A} \cdot \bar{B} \cdot \bar{C}$$

$$\bar{A} \cdot \bar{B} \cdot C$$

$$8 \quad \bar{A} \cdot B \cdot \bar{C}$$

\vdots

$$A \cdot B \cdot \bar{C}$$

$$A \cdot B \cdot C$$

maxterms

$$\bar{A} + \bar{B} + \bar{C}$$

$$\bar{A} + \bar{B} + C$$

$$8 \quad \bar{A} + B + \bar{C}$$

\vdots

$$A + B + \bar{C}$$

$$A + B + C$$

* for 'n' var. function $\rightarrow 2^n$ minterms
 Σ maxterms

* sum of all minterms = 1. $\sum_{i=0}^{2^n-1} m_i = 1$

* product of all maxterms = 0. $\prod_{i=0}^{2^n-1} M_i = 0$

* product of any two minterms = 0.

$$m_i \cdot m_j = 0, \text{ if } i \neq j$$

$$= m_i, \text{ if } i=j$$

* sum of any two maxterms = 1.

$$M_i + M_j = 1, \text{ if } i \neq j$$

$$= M_i, \text{ if } i=j$$

Let $f(x, y)$ $\begin{matrix} 1 = \text{var} \\ 0 = \text{var} \end{matrix}$ $\begin{matrix} 1 = \sqrt{\text{var}} \\ 0 = \text{var} \end{matrix}$

x y minterm

$$0 \quad 0 \quad \bar{x} \cdot \bar{y} \quad m_0$$

$$0 \quad 1 \quad \bar{x} \cdot y \quad m_1$$

$$1 \quad 0 \quad x \cdot \bar{y} \quad m_2$$

$$1 \quad 1 \quad x \cdot y \quad m_3$$

max term

$$x + y \quad M_0$$

$$x + \bar{y} \quad M_1$$

$$\bar{x} + y \quad M_2$$

$$\bar{x} + \bar{y} \quad M_3$$

\Rightarrow complement of minterm = maxterm
and vice-versa.

$$M_j = \overline{m_j}$$

Q. If $f(A, B, C, D, E)$. what is $m_{23} = ?$

$$m_{19} = ? \quad M_{28} = ? , \quad M_{23} = ?$$

$$23 \rightarrow 10\ 111$$

$$19 \rightarrow 1\ 00\ 11$$

$$m_3 = A \cdot \overline{B} \cdot C \cdot D \cdot E$$

$$m_{19} \rightarrow A \cdot \overline{B} \cdot \overline{C} \cdot D \cdot E$$

$$28 \rightarrow 111\ 00$$

$$23 \rightarrow 101\ 11$$

$$M_{28} \rightarrow \overline{A} + \overline{B} + \overline{C} + D + E \quad M_{23} = \overline{A} + B + \overline{C} + \overline{D} + \overline{E}$$

$$M_{23} = \overline{m_{23}} = \overline{A \cdot \overline{B} \cdot C \cdot D \cdot E}$$

$$= \overline{A} + B + \overline{C} + \overline{D} + \overline{E}$$

Q. $A \oplus A \oplus A \dots \oplus A = ?$

$A \oplus A \oplus A \oplus A$, if even no. of A's.

$$= 0 \oplus 0 = 0$$

$A \oplus A \oplus A$, if odd no. of A's.

$$= 0 \oplus A = A$$

* $A \oplus A \oplus A \dots \oplus A = 0$, if no. of terms = even
 $= A$, if " = odd

* $\overline{A} \oplus \overline{A} \oplus \overline{A} \oplus \dots \oplus \overline{A} = 0$, if no. of terms = Even
 $= \overline{A}$, " = odd

Q. How many Boolean fun's are possible, using 'n'-var's
Using n-var's $\rightarrow 2^n$ minterms

* minterms can be arranged in 2^n ways.

i.e. 2^2^n boolean functions are possible.

for 2 var.g $\rightarrow 2^{2^2} = 16$ functions.

$f(x, y)$.

x	y	f_1	f_2	f_3	\dots	f_{16}
m_0	0 0	0	0	0		1
m_1	0 1	0	0	0		1
m_2	1 0	0	0	1		1
m_3	1 1	0	1	0		1
<hr/>						
\emptyset AND (Inhibition) $\bar{x}\bar{y} = \text{self}$						1

Algebraic forms of Boolean functions:

①. standard form stand. sop form
stand. pos form

②. canonical form cano. sop form (or) sum of minterms
cano. pos form (or) product of maxterms

$$f_1(A, B, C) = (A + B + \bar{C})(\bar{A} + \bar{B} + \bar{C}) \rightarrow \text{cano. pos}$$

$$f_2(A, B, C) = A\bar{B}\bar{C} + A\bar{B}C + A\bar{C} \rightarrow \text{stand. sop form}$$

* SAT. 11/10/08 *

Q. convert the following boolean eq. into canonical sop form.

$$1). f(A, B, C) = \bar{A} + A\bar{B}C + B\bar{C} \rightarrow \text{std. sop}$$

$$\begin{aligned}
 & \rightarrow \bar{A}(B + \bar{B})(C + \bar{C}) + A\bar{B}C + B\bar{C}(A + \bar{A}) \\
 & = \underline{\bar{A}BC} + \underline{\bar{A}\bar{B}\bar{C}} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC \\
 & \quad + \underline{ABC} \\
 & = m_3 + m_2 + m_1 + m_0 + m_5 + m_6 \\
 & = \sum m(0, 1, 2, 3, 5, 6).
 \end{aligned}$$

(OR)	\overline{A}	$A \quad B \quad C$	\overline{ABC}	$\overline{\overline{ABC}} \rightarrow m_5$
	\downarrow		$\overline{\emptyset} + \overline{BC}$	
102		$0 \quad 0 \quad 0 \rightarrow m_0$	$0 \quad 1 \quad 0 \rightarrow m_2$	
102		$0 \quad 0 \quad 1 \rightarrow m_1$	$1 \quad 1 \quad 0 \rightarrow m_6$	
102		$0 \quad 1 \quad 0 \rightarrow m_4$		
102		$0 \quad 1 \quad 1 \rightarrow m_3$		
		$f = \sum m(0, 1, 2, 3, 5, 6) \rightarrow \text{cano. SOP}$		

$$f = \pi M(4, 7) \rightarrow \text{cano. POS.}$$

Q. Convert the following boolean eq. into cano. POS form.

$$f(A, B, C) = \overline{A} \cdot (\overline{B} + \overline{C}) \cdot (\overline{A} + \overline{B} + \overline{C}) \rightarrow \begin{matrix} M_1 \\ \text{std. pos form} \end{matrix}$$

$$\begin{aligned} f &= (\overline{A} + B\overline{B} + C\overline{C})(\overline{B} + \overline{C} + A\overline{A})(A + B + \overline{C}) \\ &= (\overline{A} + B\overline{B} + C)(\overline{A} + B\overline{B} + \overline{C})(\overline{B} + \overline{C} + A)(\overline{B} + \overline{C} + \overline{A}) \\ &\quad (A + B + \overline{C}) \\ &= (\overline{A} + B + C)(\overline{A} + \overline{B} + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C}) \\ &\quad (\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})(A + B + \overline{C}) \\ &= M_4 \cdot M_6 \cdot M_5 \cdot M_7 \cdot M_3 \cdot M_7 \cdot M_1 \\ &= \pi M(1, 3, 4, 5, 6, 7) \rightarrow \text{cano. POS.} \\ &= \sum m(0, 2) \rightarrow \text{cano. SOP.} \end{aligned}$$

[OR]

A	B	C	M	01	11	11
1	—	—	—	—	—	0
1	0	0	$\rightarrow M_4$	0	1	1
1	0	1	$\rightarrow M_5$	0	1	1
1	1	0	$\rightarrow M_6$	1	1	1
1	1	1	$\rightarrow M_7$	1	1	1

Q. Convert the following into cano. pos form.

$$f(x, y, z) = \bar{x}\bar{y} + \bar{x}z \rightarrow \text{std. SOP}$$

$$\Rightarrow f = (\bar{x} + z)(\bar{x} + y)$$

$\bar{x} \quad y \quad z$	$\bar{x} \quad y \quad z$	\downarrow	\downarrow
$0 - 0$	$1 - 0$	\downarrow	\downarrow
		std. pos	cano. pos

$$\begin{array}{lll} m_0 & 000 & 100 \quad M_4 \\ m_2 & 010 & 101 \quad M_5 \end{array}$$

$$f = \prod M(0, 2, 4, 5) \rightarrow \text{cano. POS}$$

K-maps :-

2-variable k-map

A \ B	0	1
0	0	1
1	2	3

neighbours

$$m_0 \rightarrow m_1, m_2$$

$$m_2 \rightarrow m_0, m_3$$

A \ BC	3-var. k-map			
	00	01	11	10
0	0	1	3	2
1	4	5	7	6

neighbours

$$m_0 \rightarrow m_1, m_4, m_2, m_3$$

$$m_6 \rightarrow m_2, m_7, m_4$$

4 var. k-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

neighbours:

$$m_0 \rightarrow m_1, m_4, m_2, m_3$$

$$m_9 \rightarrow m_8, m_{11}, m_{13}, m_1$$

group of 8 \rightarrow octet

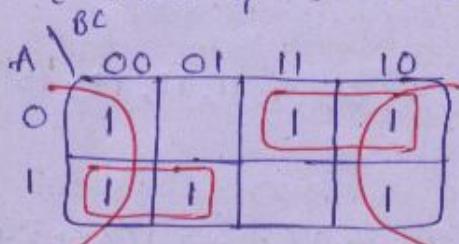
group of 4 \rightarrow quad

group of 2 \rightarrow pair

\rightarrow single minterm

8n 3-var. k-map: Quads: 0145, 1357,
3276, 0246, 0132, 4576 : Total = 6.

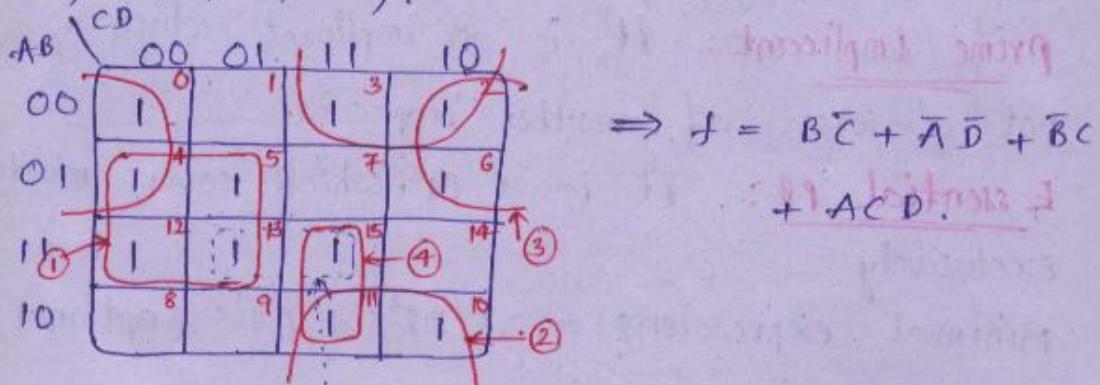
- Q. Simplify $f(A, B, C) = \sum m(0, 2, 3, 4, 5, 6)$
[That is from cano sop into std sop].



$$= \overline{A}B + A\overline{B} + \overline{C}$$

8n 4-var. k-map:
Total Octet = 8 ; columns 12, 23, 34, 41
Rows 12, 23, 34, 41

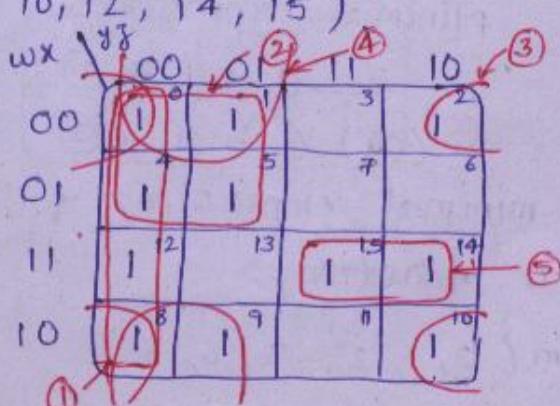
- Q. Simplify $f(A, B, C, D) = \sum m(0, 2, 3, 5, 6, 10,$
 $4, 11, 12, 13, 15)$.



$$\Rightarrow f = B\overline{C} + \overline{A}\overline{D} + \overline{B}C + ACD$$

Simplified k-map eq. is a minimal eq.
but not unique.

- Q. Simplify $f(w, x, y, z) = \sum m(0, 1, 2, 4, 5, 8, 9,$
 $10, 12, 14, 15)$



$$f = \overline{y}\overline{z} + \overline{w}\overline{y} + \overline{x}\overline{y} + \overline{x}\overline{y} + wxy$$

Q. $f(A, B, C) = \prod M(0, 1, 2, 4, 5, 6) \rightarrow$ Cano. POS

		BC		A		m	
		00	01	11	10		
A	0	0	0		0		
	1	0	0		0		

{ convert it into }
std pos form

$$f = B \cdot (A + C)$$

Q. $f(w, x, y, z) = \prod M(0, 1, 2, 4, 5, 9, 11, 13, 14, 15)$.

		wx\yz		wz		wy		xy	
		00	01	11	10				
wz	00	0	0		0				
	01	0	0		0				

$$f = (w+y)(\bar{w}+\bar{z})(\bar{w}+\bar{x}+\bar{y})$$

$$(x+w+z)$$

Implicant: It indicates the set of all adjacent minterms.

Prime Implicant: It is an implicant which is not a subset of another implicant.

Essential PI: It is a PI which covers minterms exclusively.

Minimal expressions = EPI's + PIs (optional)

Eg: $f(A, B, C) = \sum m(1, 2, 5, 6, 7)$

		BC		A		m	
		00	01	11	10		
A	0	0	1	1	1		
	1	1	0	1	0	1	1

All are PIs.

EPI's = ①, ④

Minimal expressions

= ① + ④ + ②

(or) ① + ④ + ③

Q. find EPI's and minimal expressions for the following boolean functions.

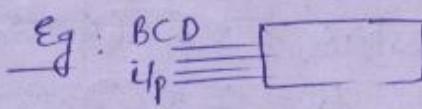
$$f(A, B, C) = \sum m(0, 1, 2, 5, 6, 7)$$

	BC	00	01	11	10	
A	0	1	1	1	1	0
O	1	1	1	1	1	1
I		2	3	7	6	5

$EPIF = 0 \text{ (Nil)}$
Minimal expression
 $= ① + ③ + ⑤$
(or) $② + ④ + ⑥$

Dont care conditions :-

for non-occurring IP^f the o/p can be assumed as 0 or 1. and this is called as Dont care condition.

Eg :  Non-occurring IP^f . o/p

valid BCD IP^f

0 - 0000

1 - 0001

:

9 - 1001

10 - 1010 $\rightarrow \times$

11 - 1011 $\rightarrow \times$

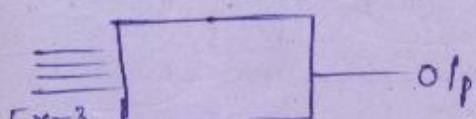
12 - 1100 $\rightarrow \times$

13 - 1101 $\rightarrow \times$

14 - 1110 $\rightarrow \times$

15 - 1111 $\rightarrow \times$

} dont care's



Dont care's : $0000 \rightarrow \times$
 $0001 \rightarrow \times$
 $0010 \rightarrow \times$

Q. $f(A, B, C, D) = \sum m(0, 1, 3, 6, 10, 13, 15) + d(2, 5, 8, 11)$

AB \ CD	00	01	11	10	
00	1	1	1	x	①
01	x			1	④
11		1	1		
10	x	x		1	②
					③

$f = \bar{A}\bar{B} + \bar{B}C + ABD$

$+ \bar{A}C\bar{D}$

: input fault output

Q. $f_1 = \sum m(0, 2, 4, 7); f_2 = \sum m(1, 2, 4, 6)$
 $f = f_1 \cdot f_2 \Rightarrow f = ?$

$$f = \sum m(2, 4)$$

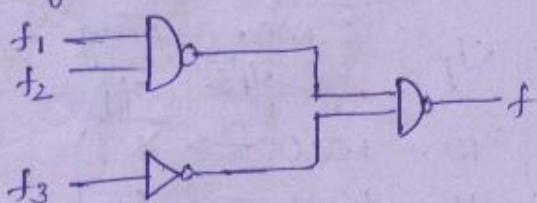
Similarly $f_3 = f_1 - f_2 \Rightarrow f_3 = ?$

$$f_3 = \sum m(0, 7).$$

$$f_4 = f_2 - f_1 \Rightarrow f_4 = ?$$

$$f_4 = \sum m(1, 6).$$

Q. Determine the function f_5 in the following logic ckt.



$$\text{where } f = \sum m(0, 1, 3, 5)$$

$$f_1 = \sum m(2, 3, 6, 7)$$

$$f_2 = \sum m(0, 1, 5).$$

$$f = \overline{\overline{f_1} \cdot \overline{f_2} \cdot \overline{f_3}}$$

$$= f_1 \cdot f_2 + f_3$$

$$\Rightarrow f_3 = f - f_1 \cdot f_2$$

$$\text{But } f_1 \cdot f_2 = \emptyset$$

$$\Rightarrow f_3 = f = \sum m(0, 1, 3, 5).$$

~~Q.~~ $f = f_1 \cdot f_2$ where $f_1 = \sum m(0, 1, 5) + d(2, 3, 7)$

$$f_2 = \sum m(1, 2, 4, 5) + d(0, 7).$$

$$f = f_1 \cdot f_2 = \sum m(1, 5)$$

$$+ d(0, 2, 7).$$

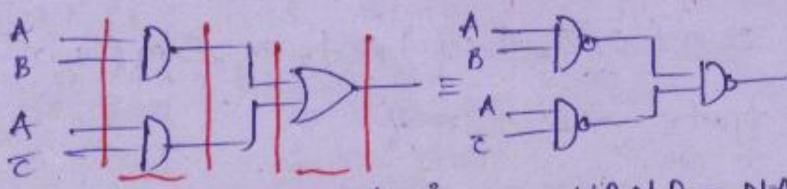
minterm
 in one fun. ↓ dont care
 ↓ in another fun.
 1. d = d
 0. d = 0

$$1+d = 1$$

$$0+d = d$$

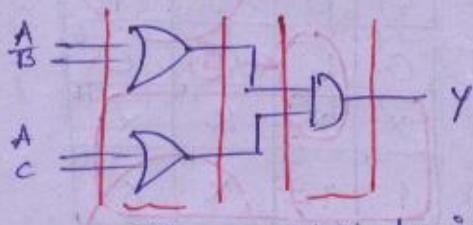
Two level logic :-

$$\text{SOP form} \rightarrow Y = AB + A\bar{C}$$



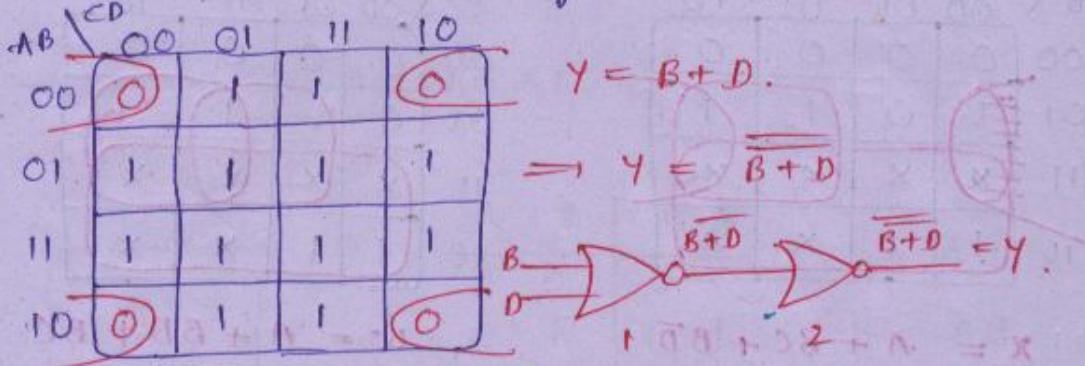
AND - OR logic \equiv NAND - NAND

$$\text{POS form} : \rightarrow y = (A + \bar{B})(A + C)$$

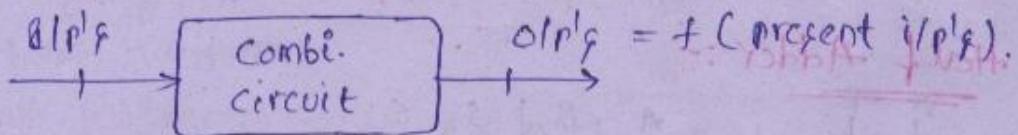


OR - AND logic \equiv NOR - NOR.

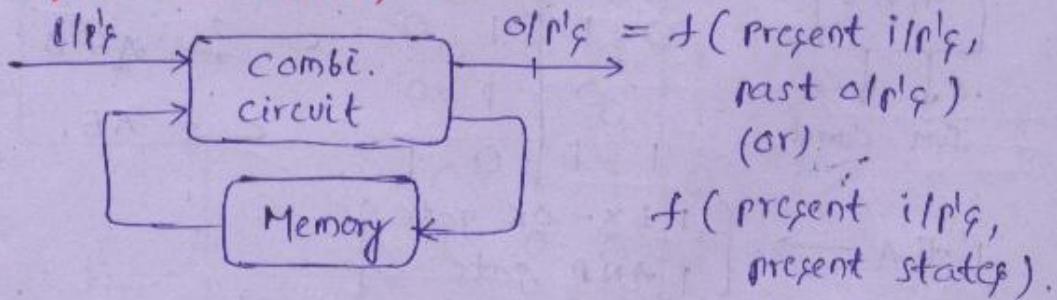
- Q. How many two i/p NOR gates are required to implement the following k-map.



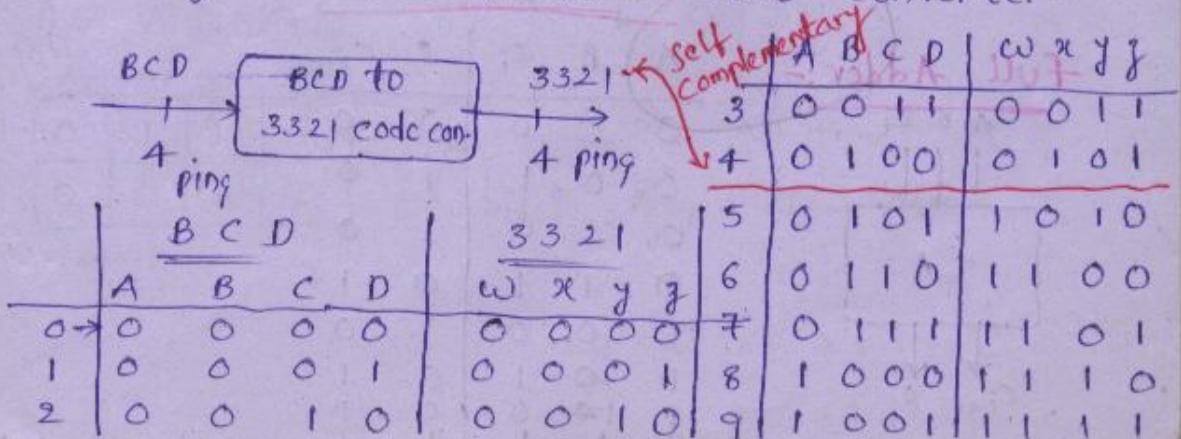
combinational circuits :-



Sequential circuits :-



- Q. Design a BCD to 3321 code converter.



AB	CD	Z	BD
00	00	01	11
00	01	1	0
01	1	0	5
11	x	x	x
10	0	8	9
		1	10
		x	x
		1	11
		x	x

AB	CD	Z	BD
00	00	01	11
00	01	1	0
01	1	0	5
11	x	x	x
10	1	1	10
		x	x
		1	11
		x	x

$$Z = \overline{BD} + CD + B\overline{C}D$$

AB	CD	X	10
00	00	00	00
01	10	1	1
11	x	x	x
10	1	1	x

$$X = A + BC + B\overline{D}$$

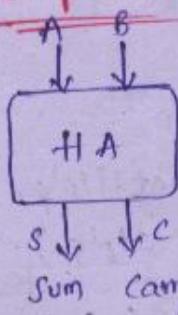
$$J = A + \overline{BC} + B\overline{C}D$$

AB	CD	J	10
00	00	00	00
01	01	11	1
11	x	x	x
10	1	1	x

$$W = A + BD + BC$$

Arithmetic combi. circuit :-

Half Adder :-



A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = A'B + AB'$$

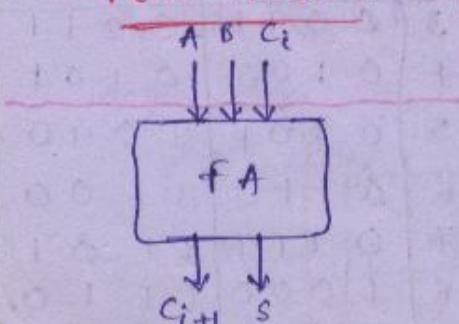
$$\Rightarrow A \oplus B$$

$$C = AB.$$

1 HA \rightarrow { 1 EX-OR gate
1 AND gate }

* SUNDAY, 12, 10/10/08 *

Full Adder :-



A	B	C_i	S	C_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

	BC _i	for. S		
	00	01	11	10
0	0	1	0	00
1	1	0	1	0

diagonal Adjacency

$$S = \overline{B} (\underbrace{A \oplus c_i}_x) + B (\underbrace{A \otimes \overline{c_i}}_{\bar{x}})$$

$$= B \oplus x = B \oplus A \oplus c_i$$

$$\Rightarrow S = \underline{A \oplus B \oplus c_i}$$

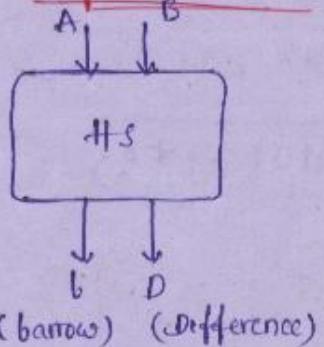
$$C_{i+1} = \overline{ABC}_i$$

$$+ \overline{AB} \underline{c_i} + \overline{A} \underline{B} \overline{c_i} + \underline{A} \underline{B} \overline{c_i}$$

$$= \underline{\underline{AB}} + BC_i + c_i A$$

$$(or) \quad \underline{c_i (A \oplus B)} + AB.$$

Half subtractor :-



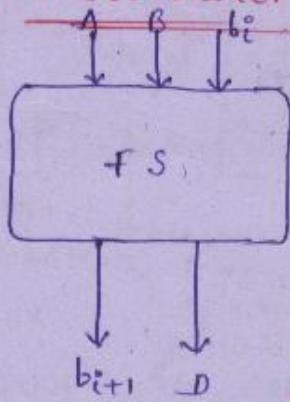
(borrow) (Difference)

A	B	D	b
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = A \oplus B$$

$$b = \overline{AB}.$$

full subtractor:-



A	B	b _i	D	b _{i+1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0

$$D = A \oplus B \oplus b_i$$

	00	01	11	10
0	0	1	0	1
1	0	0	1	0

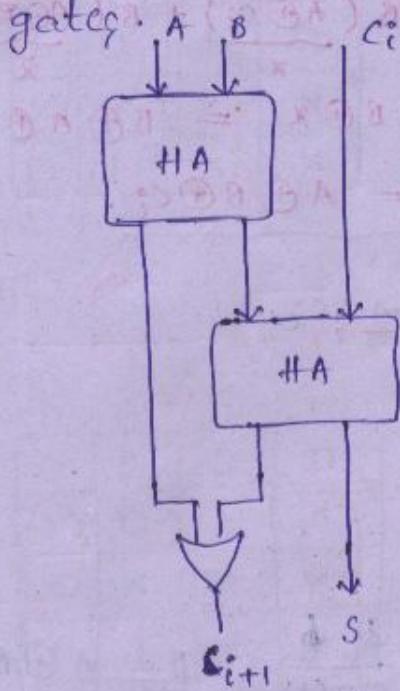
$$\Rightarrow b_{i+1} = \overline{A} b_i + B b_i + \overline{AB}$$

for b_{i+1},

$$> (\overline{A} + B) + \overline{AB} = 0 \quad : 102$$

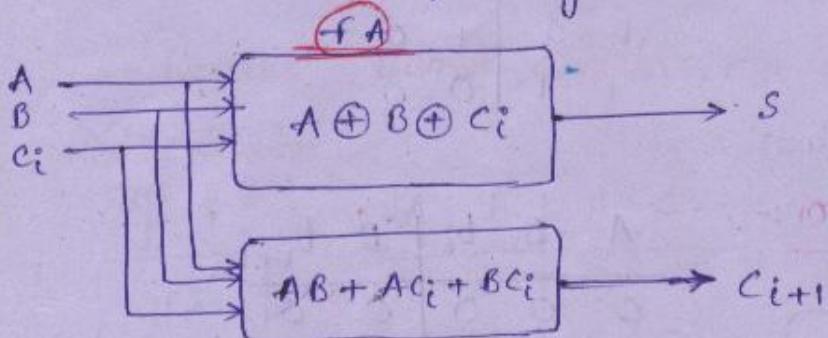
$$102 \otimes A = 2 \cdot \overline{B} A + \overline{B} A =$$

Q. Implement a FA by using HA's and logic gates.



FA requires, one OR gate and two HA's.

Q. Convert the following FA into a FS.



Q. Implement the following boolean exp's using only HA's.

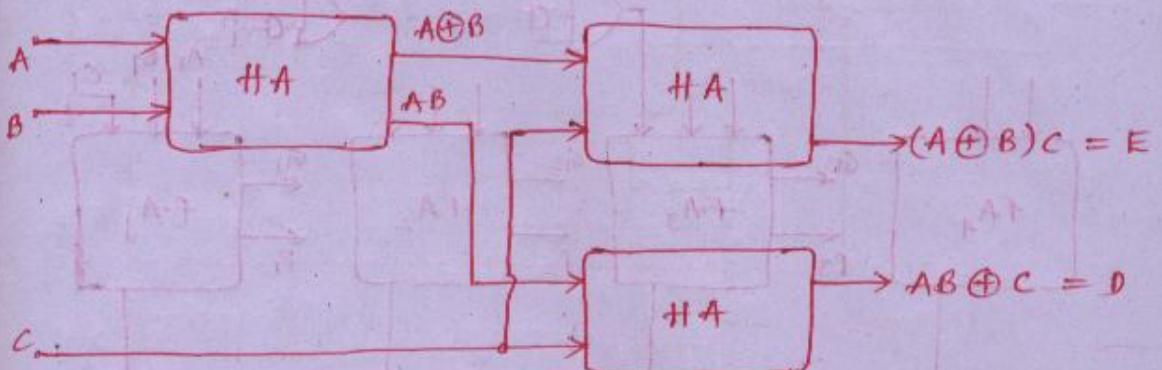
$$D = ABC + \bar{A}C + \bar{B}C$$

$$E = \bar{A}BC + A\bar{B}C$$

$$\text{Sol: } D = AB\bar{C} + (\bar{A} + \bar{B})C$$

$$= AB\bar{C} + \bar{A}\bar{B} \cdot C = AB \oplus C$$

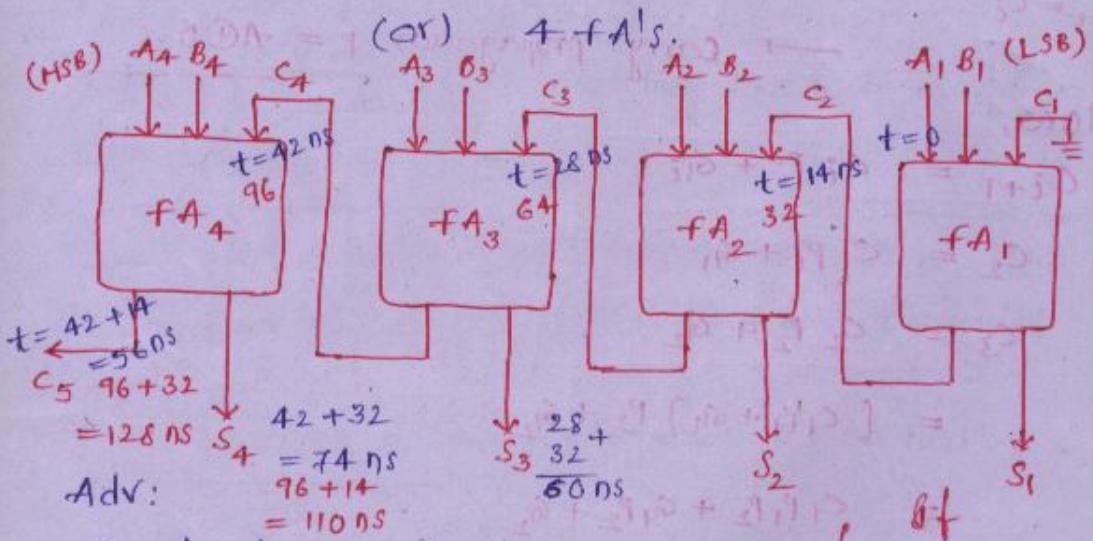
$$E = (\bar{A}B + A\bar{B})C \quad \text{[from De Morgan's law]} \\ = (A \oplus B)C$$



(1) 4-bit parallel binary Adder :-

$$\begin{array}{r} A \rightarrow A_4 \ A_3 \ A_2 \ A_1 \\ B \rightarrow B_4 \ B_3 \ B_2 \ B_1 \\ \hline \end{array} \quad \text{(combi. circuit)}$$

Required : 3 full adders + 1 HA



Simple to construct

Drawback:

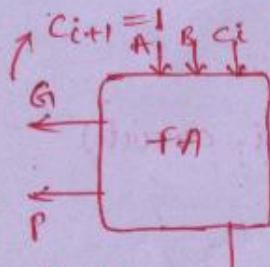
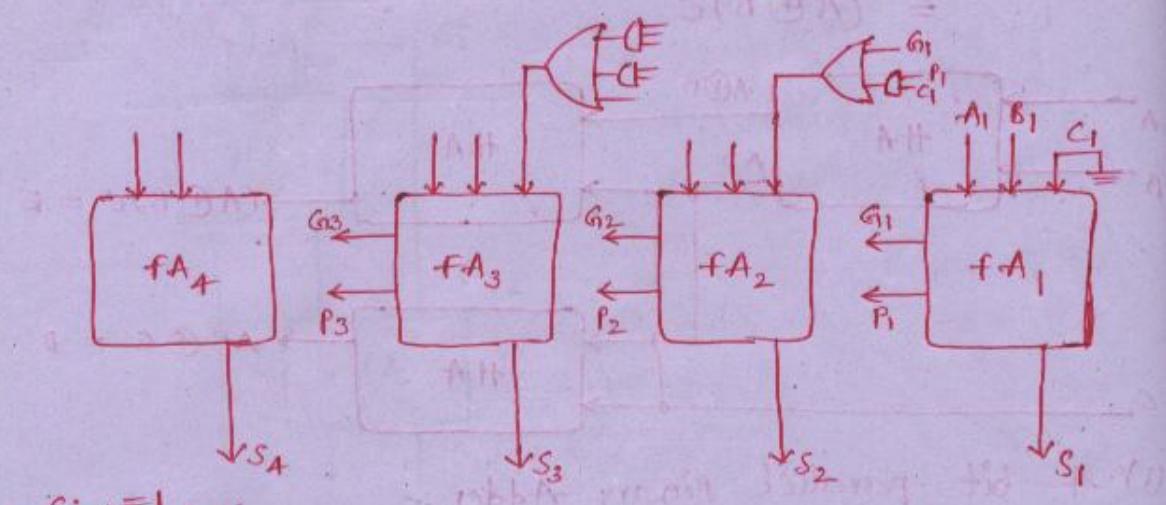
Speed of operation is less if the size of the adder increases.

$FA \rightarrow \left\{ \begin{array}{l} 32 \text{ ns} \rightarrow \text{sum} \\ 14 \text{ ns} \rightarrow \text{carry} \end{array} \right\}$ Then the total time required for the operations

Ans: $42 + 32$ for S_4 ; for $C_5 \Rightarrow 42 + 14 = 56$
 $= 74 \text{ ns.}$

\therefore To complete addition
 Total time = 74 ns.

(2) Carry Look Ahead Adder :- (combi. circuit)



$$C_{i+1} = C_i (A \oplus B) + AB.$$

(1). $C_{i+1} = 1$ if $AB = 1$

→ Carry Generation $G = AB$.

$$P = 1 \\ C_{i+1} = C_i$$

(2). When $C_{i+1} = C_i$ then $A \oplus B = 1$

→ Carry propagation $P = A \oplus B$.

Now,

$$C_{i+1} = C_i P_i + G_i$$

$$\Rightarrow C_2 = C_1 P_1 + G_1$$

$$C_3 = C_2 P_2 + G_2$$

$$= [C_1 P_1 + G_1] P_2 + G_2$$

$$= C_1 P_1 P_2 + G_1 P_2 + G_2$$

$$\left. \begin{array}{l} C_4 = \\ C_3 P_3 + G_3 \end{array} \right\}$$

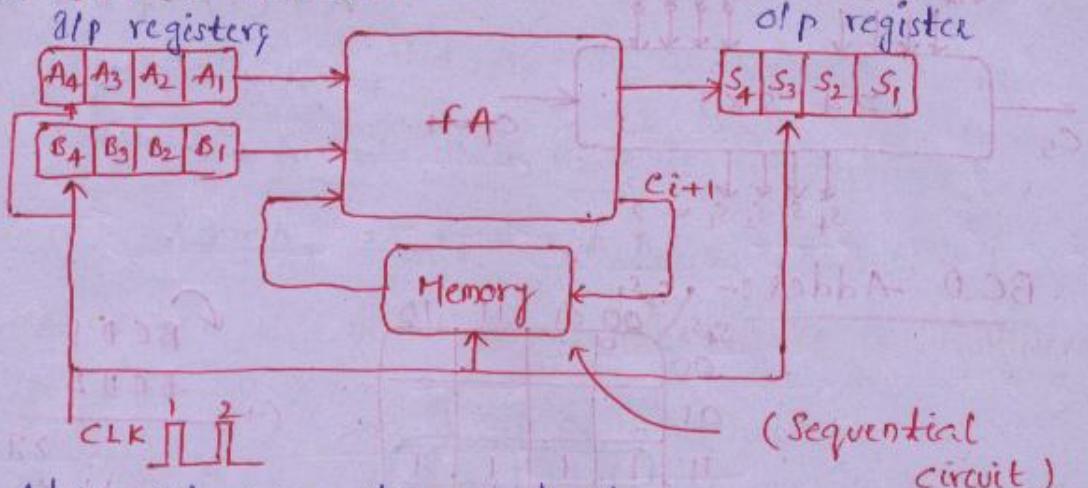
$$= C_1 P_1 P_2 P_3 + G_1 P_2 P_3 + G_2 P_3 + G_3$$

Adv: speed is more

Dis Adv: More hardware complexity

hardware cost not less

(3). Serial Adder :-



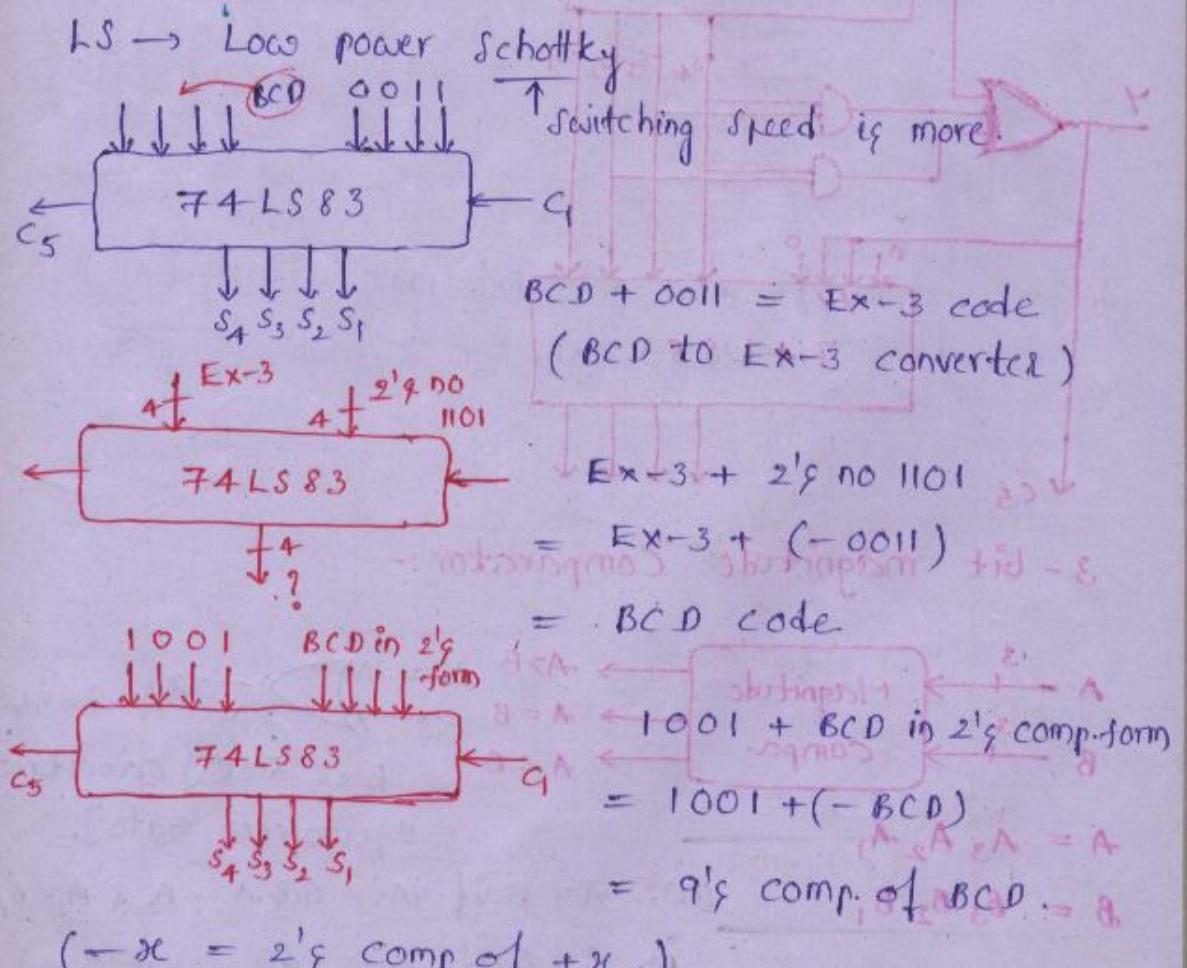
Adv: (1) easy to construct

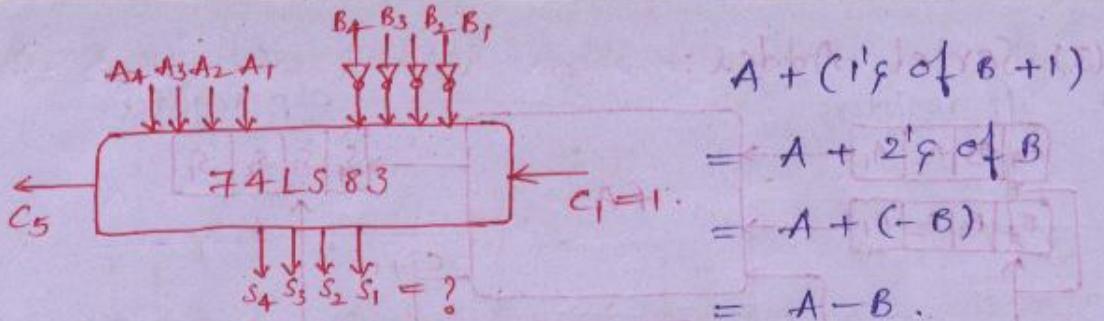
(2). only one FA is used.

Dis-Adv:

speed of operation is less.

4 Bit parallel Binary Adder (74 LS 83)





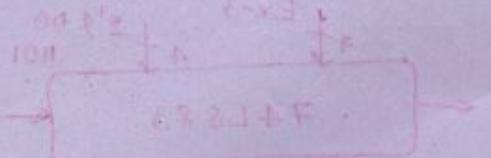
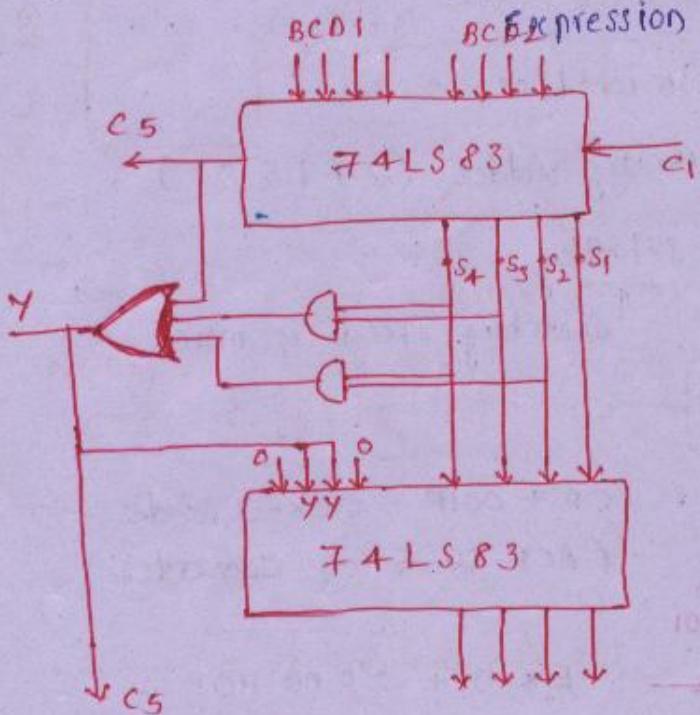
BCD Adder :-

	$S_4 S_3$	00	01	11	10
	00	0			
	01				
	11	1	1	1	1
	10		1	1	1

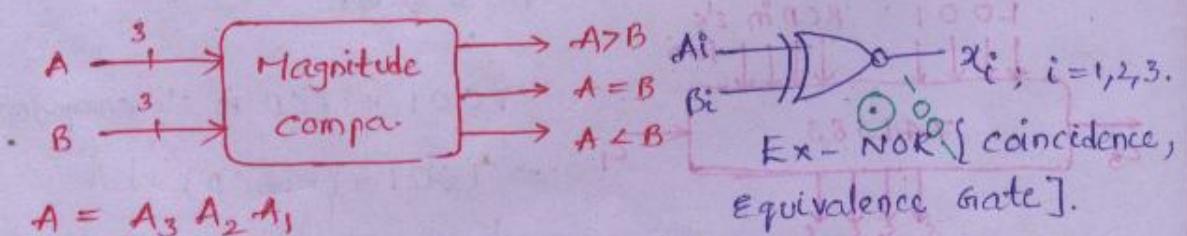
$\begin{array}{r} \text{BCD 1} \\ \text{BCD 2} \\ (+) \end{array}$
 $\rightarrow \text{add 6}$

Invalid BCD = BCD Expression

$$Y = S_4 S_3 + S_4 S_2 + C_5$$



3-bit magnitude comparator :-



$A_i \rightarrow D$, x_i , $i = 1, 2, 3$.
 Ex- NOR { coincidence, equivalence gate].

$$A = A_3 A_2 A_1$$

$$B = B_3 B_2 B_1$$

- (a). $A = B$ if $A_3 = B_3 \& A_2 = B_2 \& A_1 = B_1$
 ie $A = B$ if $x_3 = 1 \& x_2 = 1 \& x_1 = 1$
 ie $A = B$ if $x_3 x_2 x_1 = 1$

(b). $A > B$ if $A_3 > B_3$ (or) $A_3 = B_3$ and $A_2 > B_2$

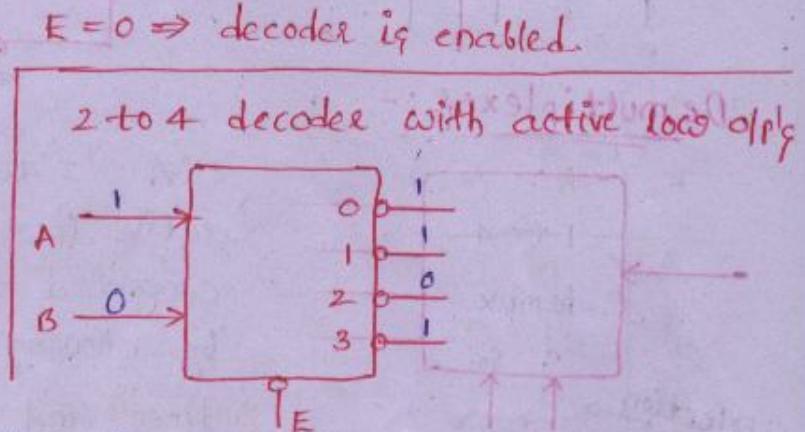
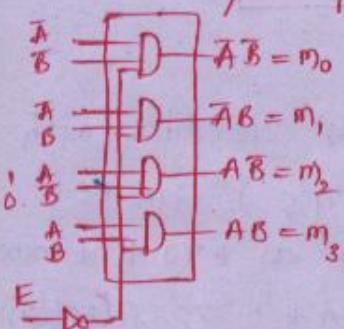
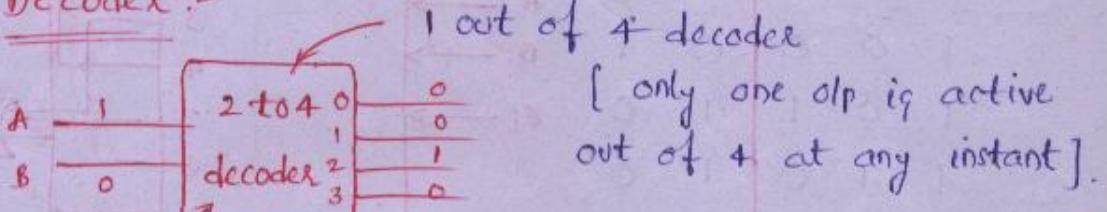
(or) $A_3 = B_3$ and $A_2 = B_2$ and $A_1 > B_1$

$A > B$ if $A_3 \bar{B}_3 + x_3 A_2 \bar{B}_2 + x_3 x_2 A_1 \bar{B}_1 = 1$.

(c). $A < B$ if $\bar{A}_3 B_3 + x_3 \bar{A}_2 B_2 + x_3 x_2 \bar{A}_1 B_1 = 1$.

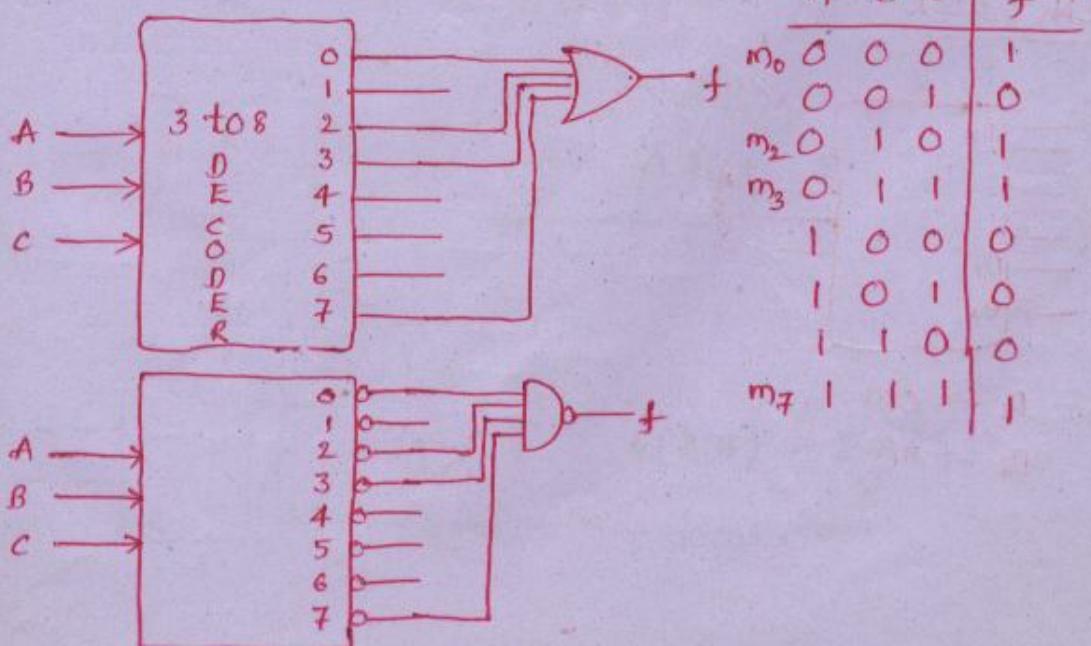
(1). decoder (2). demultiplexer (3). Encoder (4). Multiplexer

Decoder :-



c. Implement the following sum of minterm eq by using a decoder and logic gates.

$$f(A, B, C) = \sum m(0, 2, 3, 7)$$

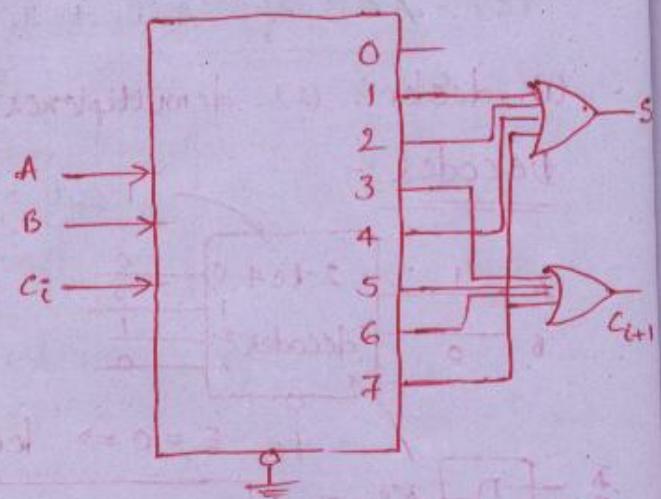


Q. Implement a FA by using decoder and logic gates.

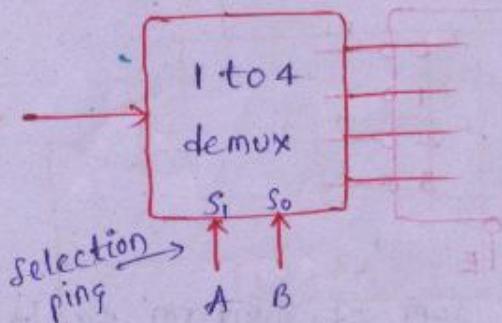
A	B	C_i	C_{i+1}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = \sum m(1, 2, 4, 7)$$

$$C_{i+1} = \sum m(3, 5, 6, 7)$$



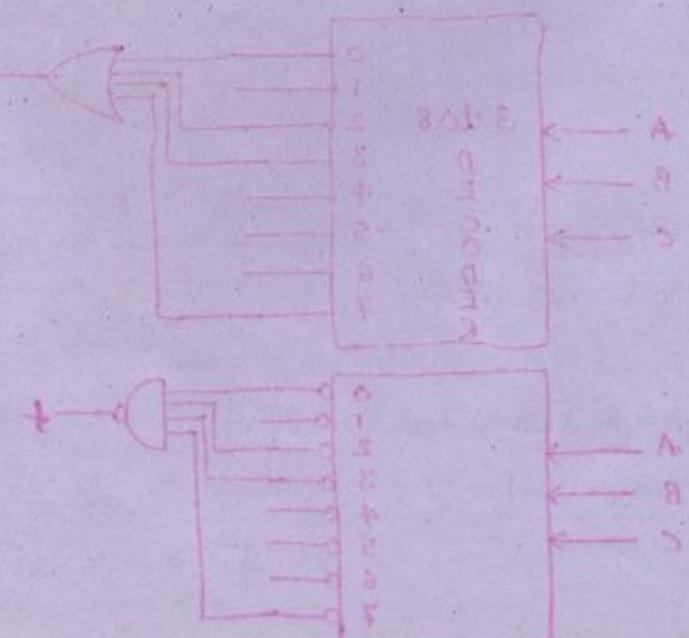
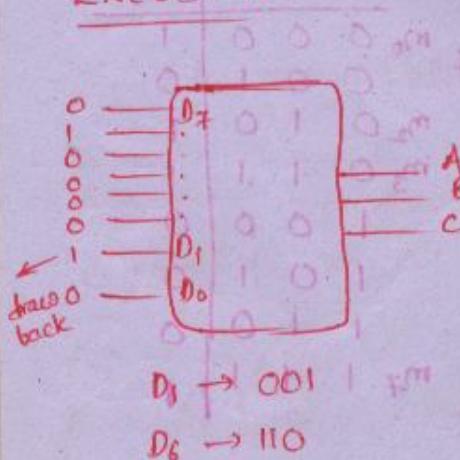
Demultiplexed :-



A 2 to 4 decoder [with active low output] can be converted to a 1 to 4 demux by choosing A & B as selection lines and the enable pin as the serial input.

* 29/11/08 *

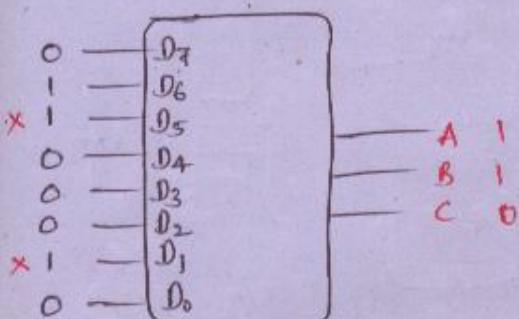
ENCODER :-



PRIORITY ENCODER: (74 LS 148)

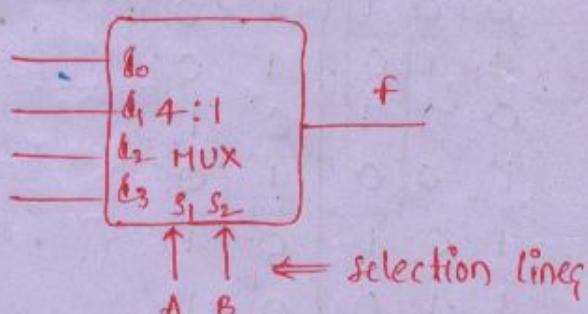
D₇ - highest priority

D₀ - lowest priority



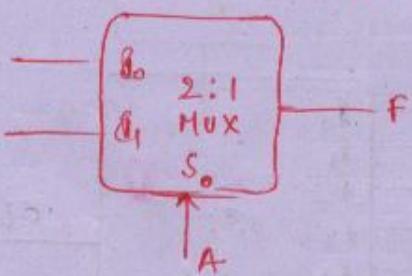
x - ignored

MULTIPLEXER:



for 4:1 MUX,

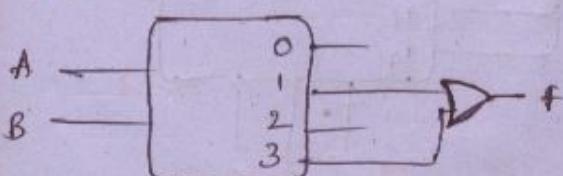
$$\begin{aligned} f &= \bar{A}\bar{B}d_0 + \bar{A}Bd_1 + A\bar{B}d_2 + ABd_3 \\ &= m_0d_0 + m_1d_1 + m_2d_2 + m_3d_3. \end{aligned}$$



for 2:1 MUX,

$$f = \bar{A}d_0 + Ad_1$$

Q.

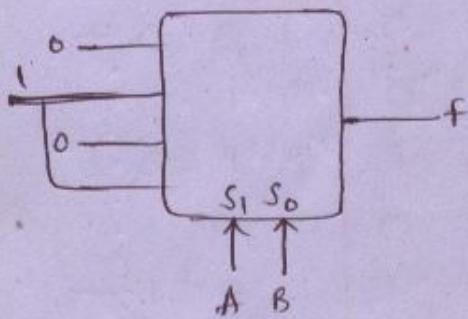


$$f(A, B) = \sum m(1, 3)$$

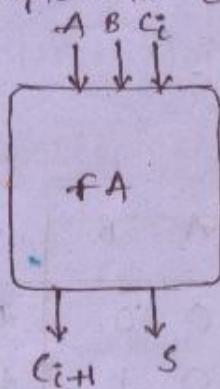
decoder

Q. Implement the following sum of minterms exp. by using multiplexer.
 (sum of minterms)

$$f(A, B) = \sum m(1, 3).$$



Q. Implement a FA by using multiplexers:

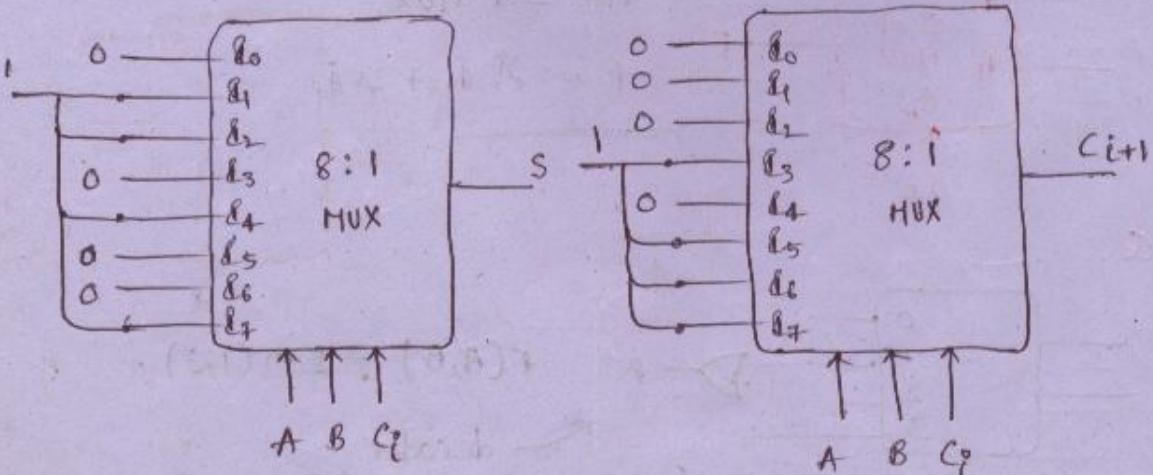


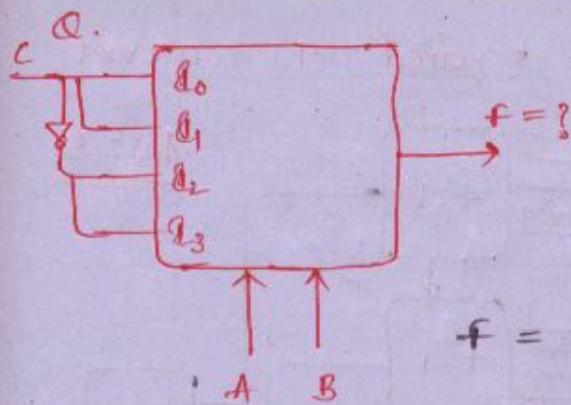
FA \rightarrow MUX
 (sum of minterms)

A	B	C _i	S	C _{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \sum m(1, 2, 4, 7)$$

$$C_{i+1} = \sum m(3, 5, 6, 7).$$





Given that

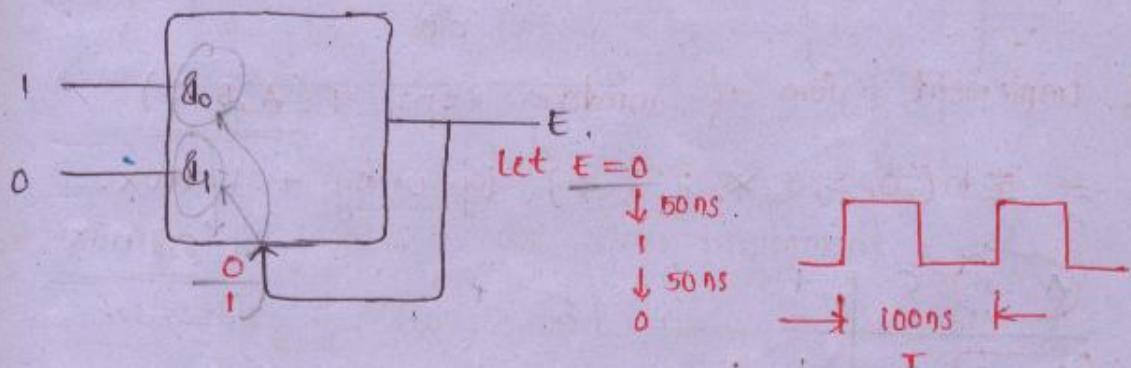
$$d_0 = d_1 = c$$

$$d_2 = d_3 = \bar{c}$$

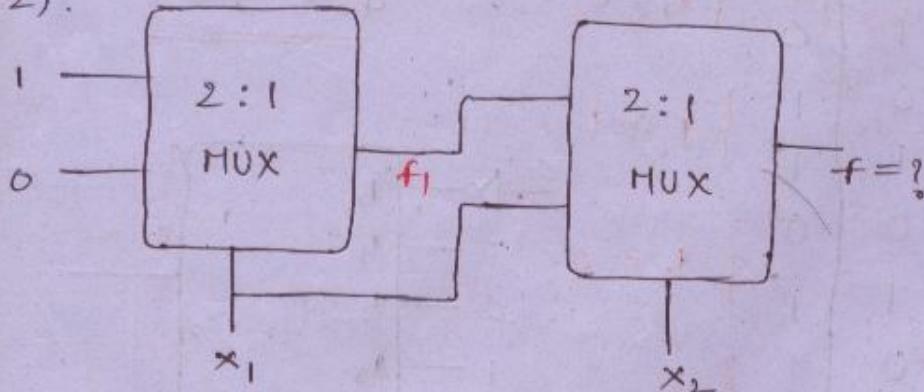
$$\begin{aligned} f &= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + ABC \\ &= \bar{A}c(\bar{B}+B) + A\bar{c}(\bar{B}+B) \\ &= A \oplus c. \end{aligned}$$

Q. Determine the op's of the following Mux's?

1). Switching speed is 50 ns.



2).



$$f_1 = \bar{x}_1 \cdot 1 + x_1 \cdot 0 = \bar{x}_1$$

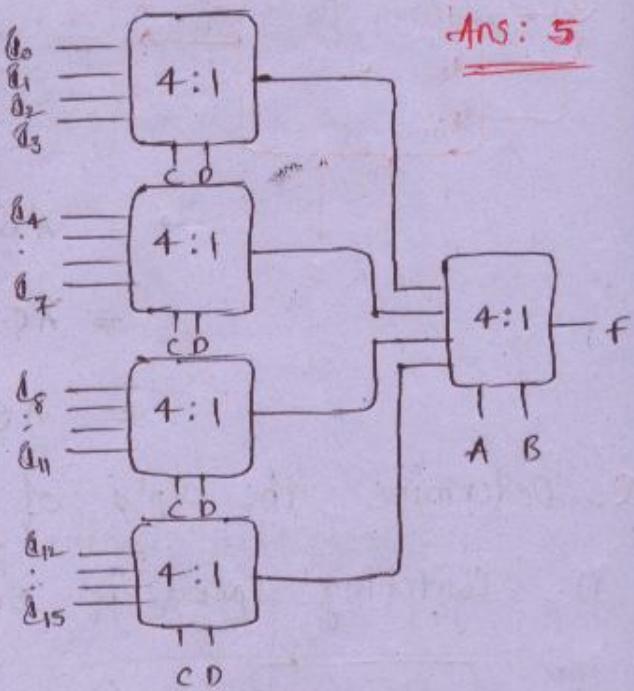
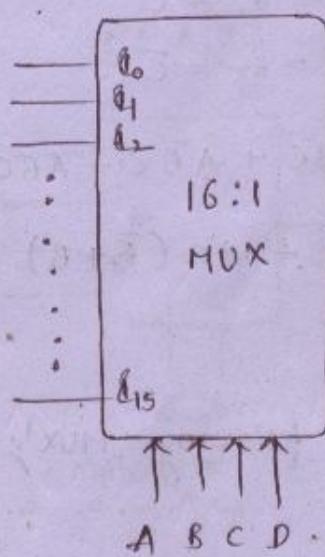
$$f_1 = \bar{x}_1 \cdot 1 + x_1 \cdot 0 = \bar{x}_1$$

$$f = \bar{A}d_0 + Ad_1$$

$$= \bar{x}_2 \cdot \bar{x}_1 + x_2 \cdot x_1$$

$$= x_2 \oplus x_1$$

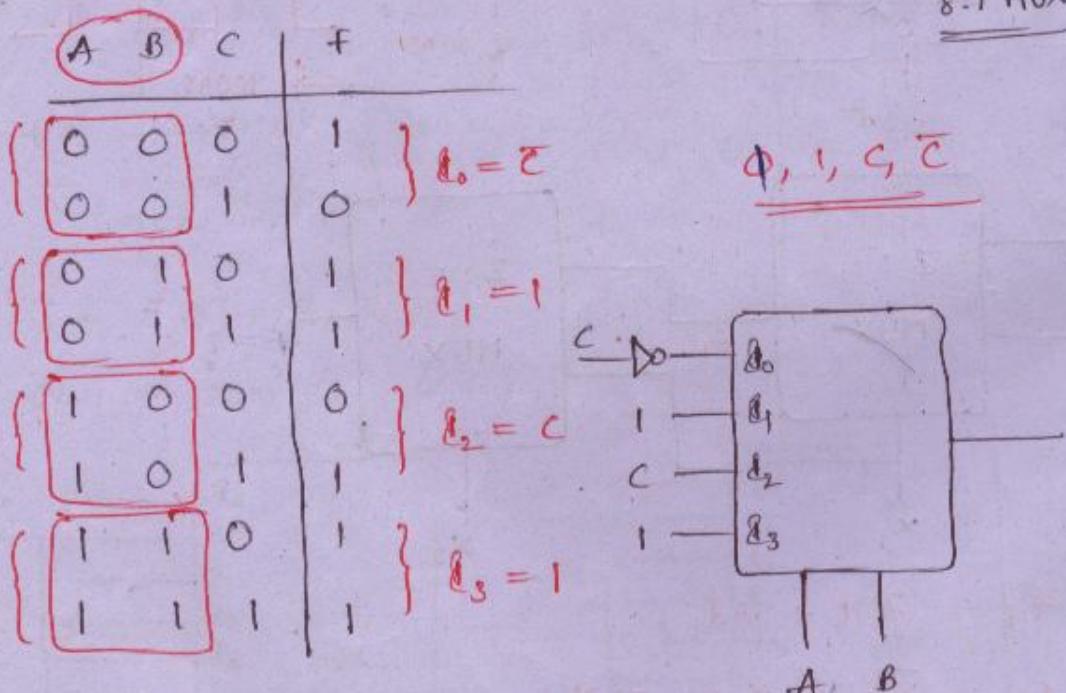
Q. How many 4:1 mux's are required to construct a 16:1 MUX.



Ans: 5

Q. Implement sum of minterm exp. $f(A, B, C)$

$= \sum m(0, 2, 3, *, 5, 6, 7)$. by using 4:1 MUX.



[OR]		\oplus			
		00	01	10	11
		d_0	d_1	d_2	d_3
0	\bar{C}	0	0	0	0
1	C	1	1	1	1

Below the table, the binary values are converted to decimal:

\bar{C}	000	010	100	110	6
0	0	2	4	6	6
1	1	3	5	7	7

<u>AB</u>	d_0	d_1	d_2	d_3
\bar{C}	0	2	4	6
C	1	3	5	7

$\bar{C} \quad 1 \quad C \quad 1$

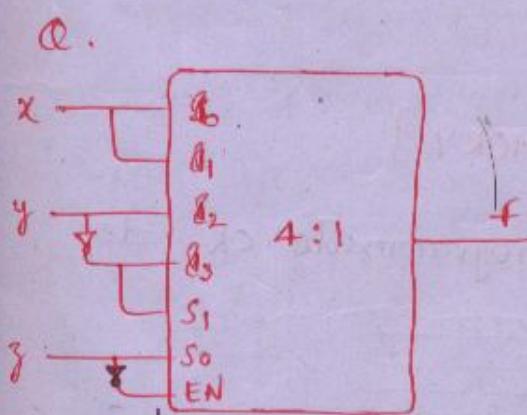
Implement above problem by choosing B & C as selection lines.

<u>BC</u>	d_0	d_1	d_2	d_3
0 A	0	1	4	6
1 A	4	5	6	7

$\bar{A} \quad A \quad 1 \quad 1$

* Using 4:1 MUX, we can implement all 2 variable functions and some 3 variable functions.

\downarrow (P.S.C) \downarrow
 Require some logic gates like NOT GATE.



If $Z=0$, MUX is enabled and with $Z=1$, MUX is disabled.

$$S_1 = \bar{Y}$$

$$S_0 = Z$$

x	y	z	s_1	s_0	f
0	0	0	1	0	$\bar{s}_2 = y = 0$
0	1	0	0	0	$\bar{s}_3 = x = 0$
1	0	0	1	0	$\bar{s}_2 = z = 0$
1	1	0	0	0	$\bar{s}_3 = x = 1$
$1 \rightarrow \text{disabled}$			$f = xy\bar{z}$		

ANOTHER WAY :

$$f = \bar{A}\bar{B}s_0 + \bar{A}Bs_1 + A\bar{B}s_2 + ABs_3$$

$$\text{where } s_1 \ A = \bar{y} \quad s_0 = s_1 = x$$

$$s_0 \ B = \bar{z} \quad s_2 = y; \ s_3 = \bar{y}$$

$$\Rightarrow f = \bar{y}\bar{z}x + \bar{y}z\bar{x} + \bar{y}\bar{z}y + \bar{y}zy$$

$$= xy\bar{z} + \cancel{xyz} + 0 + \cancel{yz}$$

$$x=1 \quad x=1 \quad y=0$$

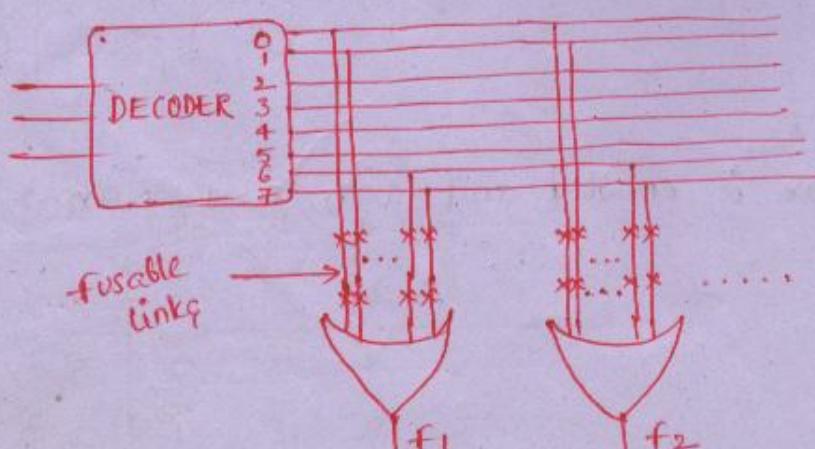
$$y=1 \quad y=1 \quad \underline{z=1}$$

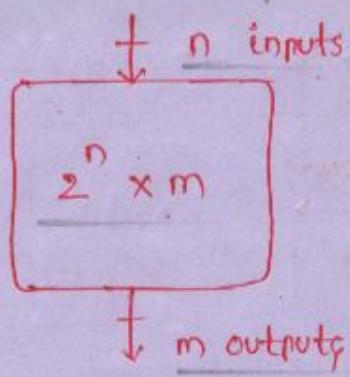
$$z=0 \quad \underline{z=1}$$

$$\Rightarrow f = xy\bar{z}$$

ROM [READ ONLY] MEMORY]

ROM \Rightarrow DECODER + programmable OR gates





Size of the ROM indicates the no. of fuses at the beginning.

PLA : programmable AND gates & programmable OR gates.

PAL : Programmable AND gates & fixed OR gates.

Decoder }
MUX }
ROM } ← Sum of minterms ie $\sum m(\dots)$.
 (canonical SOP form).

PLA ← std. SOP form. is sufficient.

Determine the size of the ROM for the following

$$(i). f_1(x, y, z) = \sum m(0, 1, 3).$$

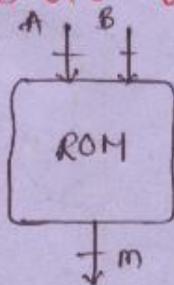
$$f_2(x, y, z) = x\bar{y} + \bar{x}\bar{y}\bar{z} + \bar{y}z$$

$$f_3(x, y, z) = \bar{x}yz.$$

$$n = 3 \text{ & } m = 3.$$

$$\text{ROM size} = 2^3 \times 3 = 24.$$

(ii). 3 bit binary Multiplier

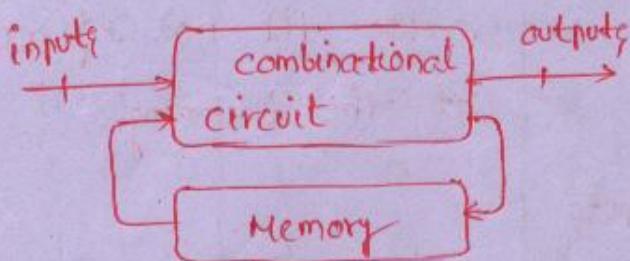


$$\begin{array}{r}
 111 \times 111 \\
 \hline
 7_{10} \times 7_{10} = 49_{10} \\
 \Rightarrow 2^m \geq 49 \\
 \Rightarrow m = 6
 \end{array}$$

$$\therefore \text{Size} = 2^6 \times 6$$

=

SEQUENTIAL CIRCUITS:

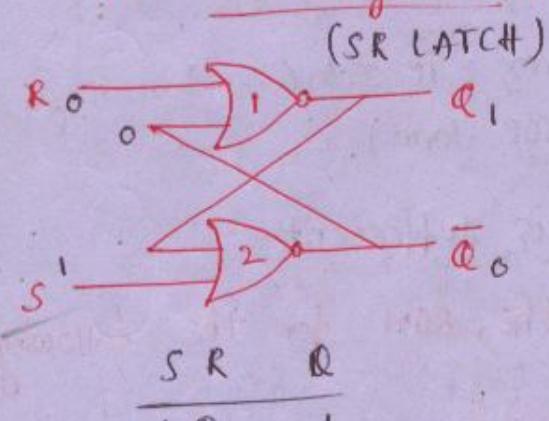


Output = $f(\text{present i/p's, past o/p's})$

or

+ (" , present state)

1 Bit Memory Element:



$\begin{array}{c} \swarrow \text{SET} \\ S \\ \searrow \text{RESET} \\ R \end{array}$

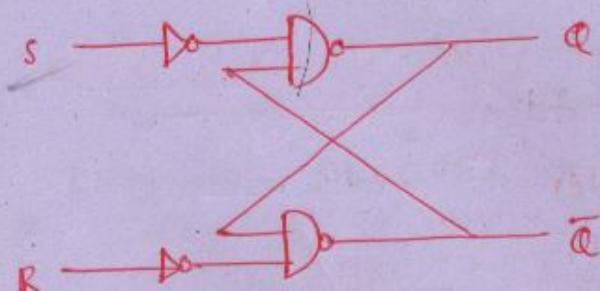
$\begin{array}{ccc} 0 & 0 & \text{No change in o/p} \end{array}$

$\begin{array}{ccc} 0 & 1 & 0 \end{array}$

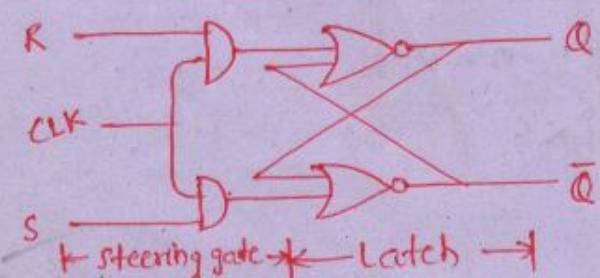
$\begin{array}{ccc} 1 & 0 & 1 \end{array}$

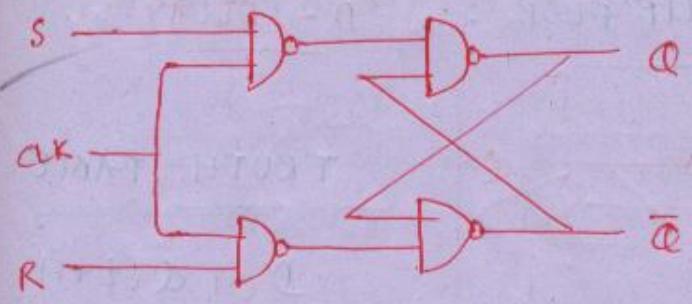
$\begin{array}{ccc} 1 & 1 & \text{Im practical state} \end{array}$

$\begin{array}{ccc} S & R & Q \\ \hline 1 & 0 & 1 \\ 0 & 0 & 1 \end{array}$ → Even if i/p's are removed the o/p will be 1 ie it stored the o/p. → ^{16bit} memory unit



CLOCKED S-R FLIP FLOP:

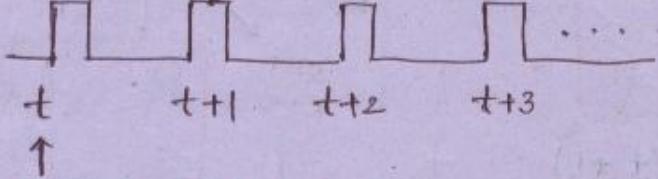




\leftarrow Steering \rightarrow Catch

gate $Q(t) = \text{present o/p}$

$Q(t+1) = \text{Next o/p}$



CLK waveform.

TRUTH TABLE :

S	R	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	(Ambiguous state)

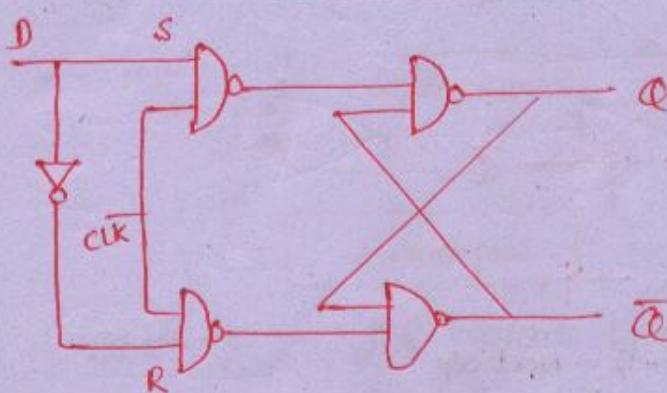
CHAR. TABLE :

S	R	$Q(t)$	$Q(t+1)$
0	0	0	0
	0	1	1
0	1	0	0
	1	1	0
1	0	0	1
	0	1	1
1	1	0	x
	1	1	x

		S	R	Q	
		00	01	11	10
S	R	0	0	1	0
0	0	0	1	0	0
0	1	0	1	1	0
1	0	1	1	x	x
1	1	x	x	x	x

$$Q(t+1) = S + \bar{R}Q$$

CLOCKED D - FLIP FLOP :



TRUTH TABLE

D	Q(t+1)
0	0
1	1
S=1 R=0	

CHAR. TABLE :

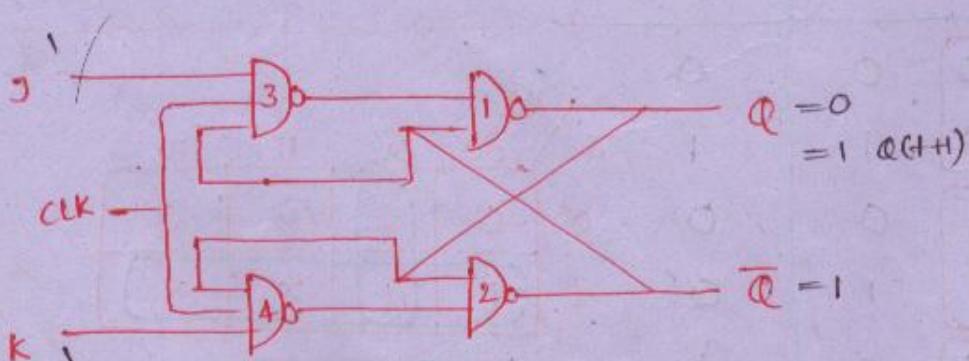
D	Q(t)	Q(t+1)
0	0	0
0	1	0
1	0	1
1	1	1

$$\begin{aligned} Q(t+1) &= \bar{D}\bar{Q} + DQ \\ &= D. \end{aligned}$$

CLOCKED JK - FLIP FLOP :

$$S = J\bar{Q}$$

$$R = KQ$$



TRUTH TABLE :

J	K	Q(t)	Q(t+1)
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

CHAR. TABLE :

J	K	$Q(t)$	$Q(t+1)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

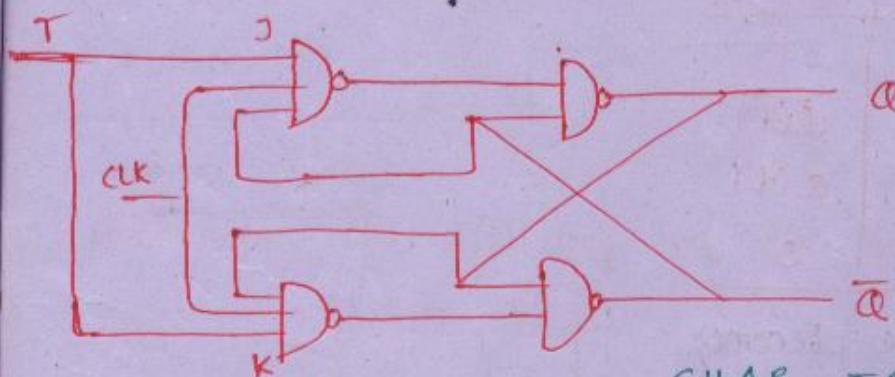
$\begin{matrix} & J \\ & \swarrow \\ \begin{matrix} 00 \\ 01 \\ 11 \\ 10 \end{matrix} & \end{matrix}$

 $\begin{matrix} K \\ \searrow \\ \begin{matrix} 00 \\ 01 \\ 11 \\ 10 \end{matrix} & \end{matrix}$

$$Q(t+1) = J\bar{Q} + \bar{K}Q$$

CLOCKED T- FLIP FLOP:

T - TOGGLE



TRUTH TABLE :-

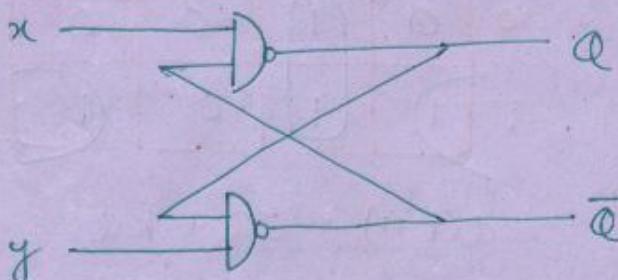
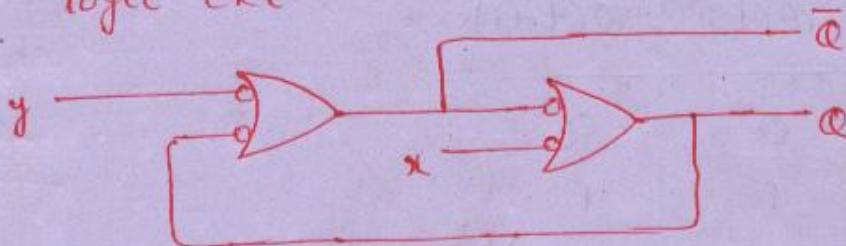
T	$Q(t+1)$
$J=K=0 \leftarrow 0$	$Q(t)$
$J=K=1 \leftarrow 1$	$\bar{Q}(t)$

CHAR. TABLE :-

T	$Q(t)$	$Q(t+1)$
0	0	0
0	1	1
1	0	1
1	1	0

$$Q(t+1) = T \oplus Q$$

a. Determine the fun. table of the following logic ckt.



x	y	Q
0	0	$Q=1, \bar{Q}=1$
0	1	1
1	0	0
1	1	No change

a. obtain char. eq. of x-y flip flop whose truth table as shown below:-

x	y	$Q(t+1)$
0	0	1
0	1	$\bar{Q}(t)$
1	0	$\bar{Q}(t)$
1	1	0

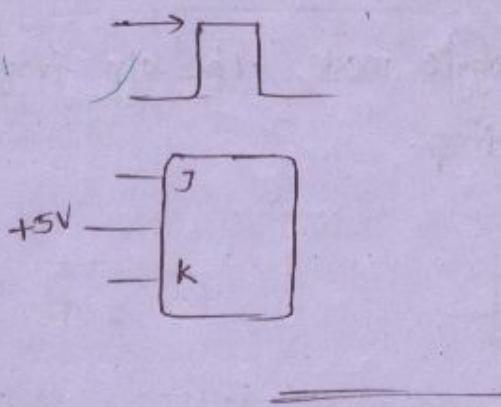
char. table becomes :-

x	y	$Q(t)$	$Q(t+1)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

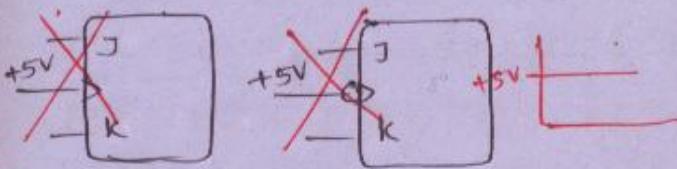
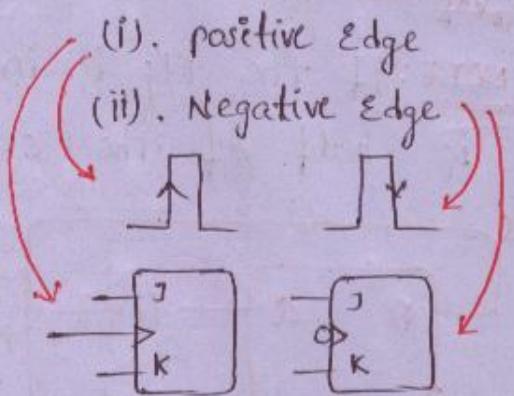
$Q(t+1) = \bar{x}\bar{Q} + \bar{y}Q$

TYPES OF TRIGGERING:

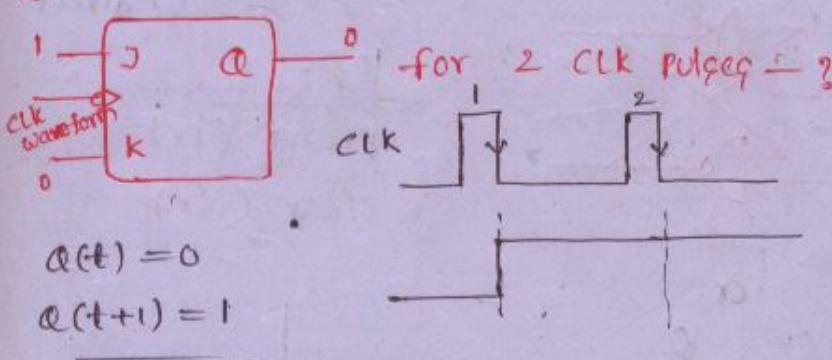
(1). LEVEL TRIGGER



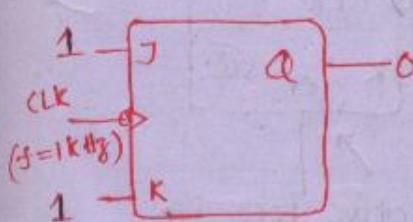
(2). EDGE TRIGGERED



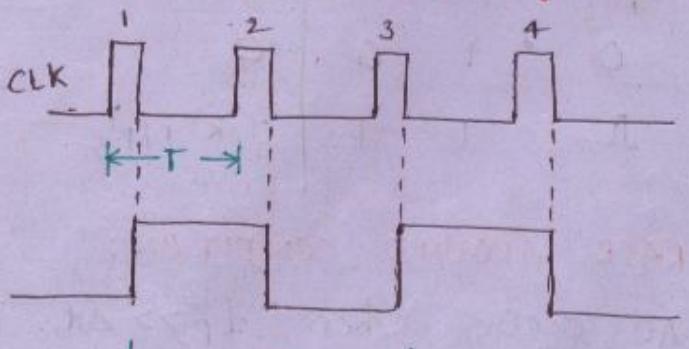
Q.



a. Determine the off freq. of the following f/f - :



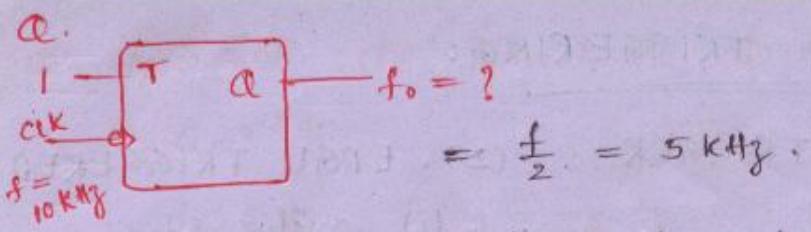
$$\begin{matrix} J & K \\ 1 & 1 \end{matrix} \rightarrow Q(t+1) = \overline{Q(t)}$$



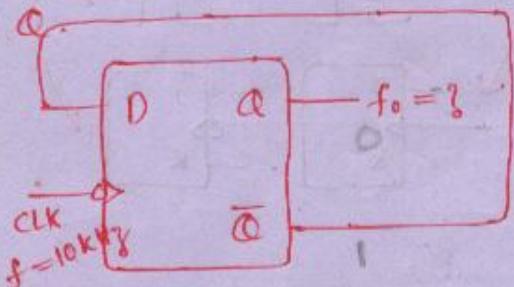
$$T_0 = 2T$$

$$f_0 = \frac{1}{T_0}$$

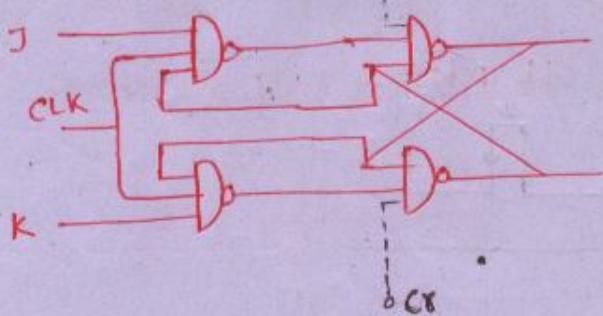
$$f_0 = \frac{f}{2} = \frac{1 \text{ kHz}}{2} = 500 \text{ Hz}$$



NOTE: If the flf is in toggle mode, the o/p freq. is half of the clk freq.



* SUM. OF (12/08) *



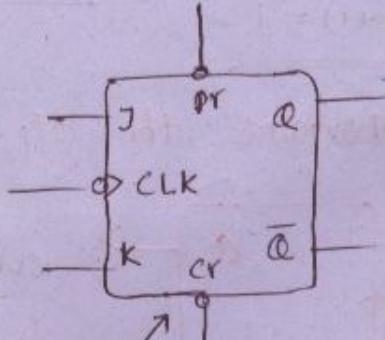
CLK	PR	CR	Q
0	0	1	1
0	1	0	0
1	1	1	J, K if 1's

Asynchronous / direct

Blng:

Preset (Pr)

clear (Cr)



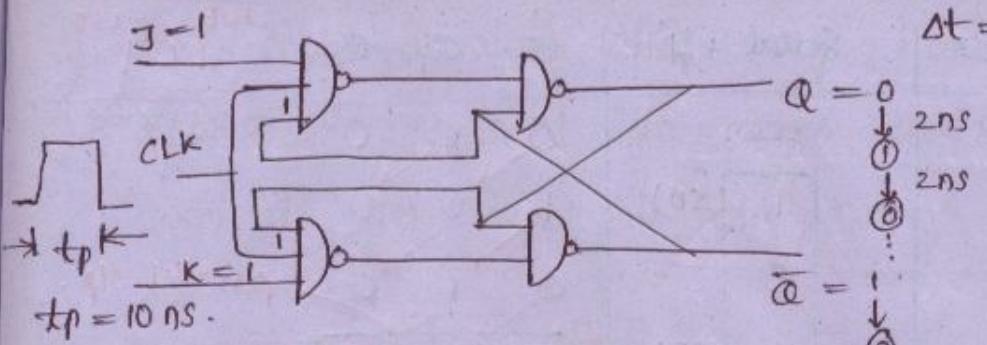
Active low i/p

RACE AROUND CONDITION:

RAC occurs when $t_p \gg \Delta t$ and $J = K = 1$.

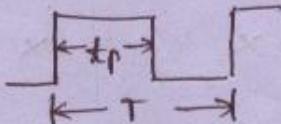
$t_p \rightarrow$ applied CLK pulse width

$\Delta t \rightarrow$ propagation delay of flf.



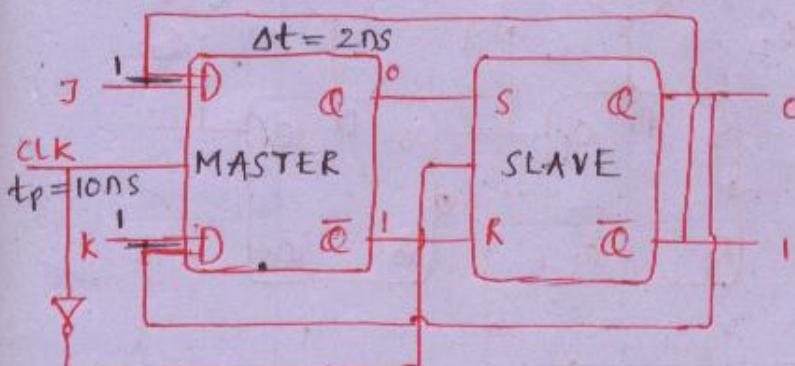
TO AVOID RAC:

$$t_p \leq \Delta t < T$$

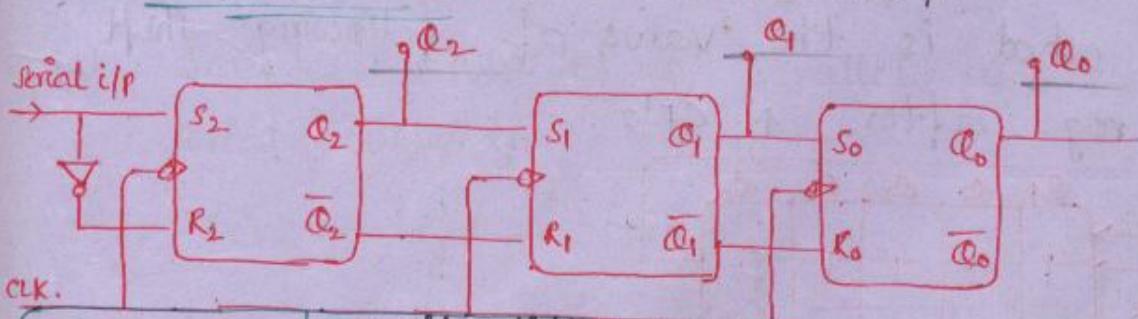


RAE occurs only in level triggered f/f but not in edge triggered f/f.

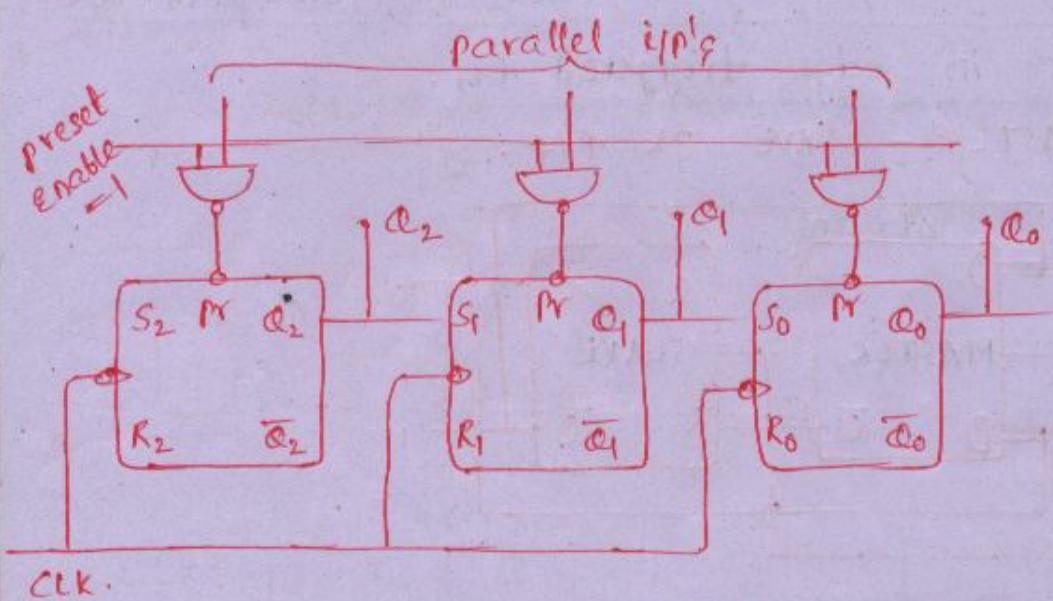
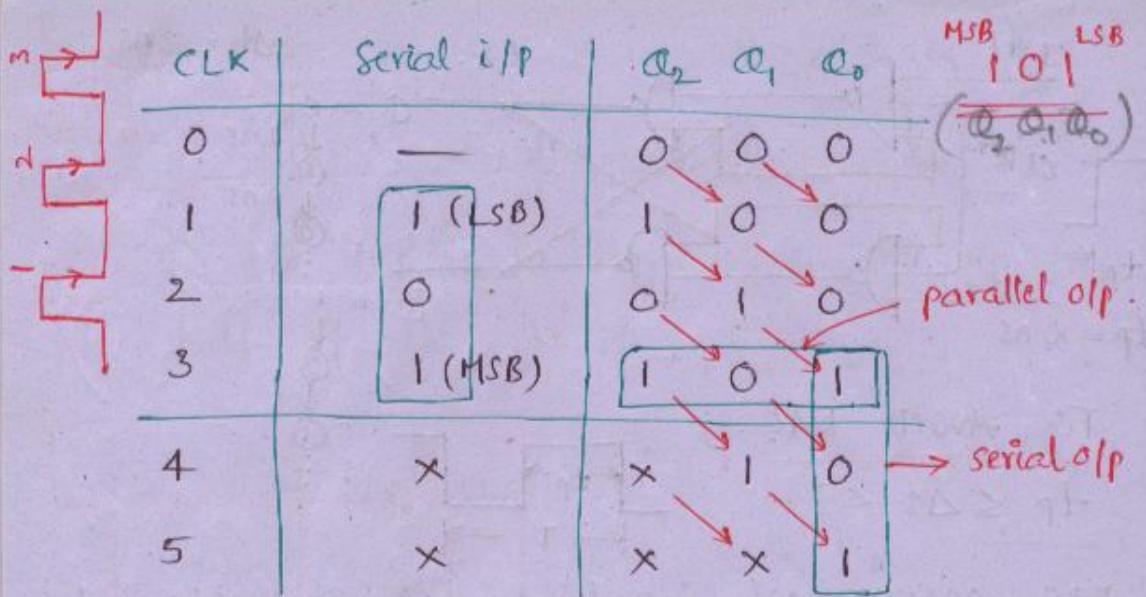
MASTER - SLAVE JK f/f:



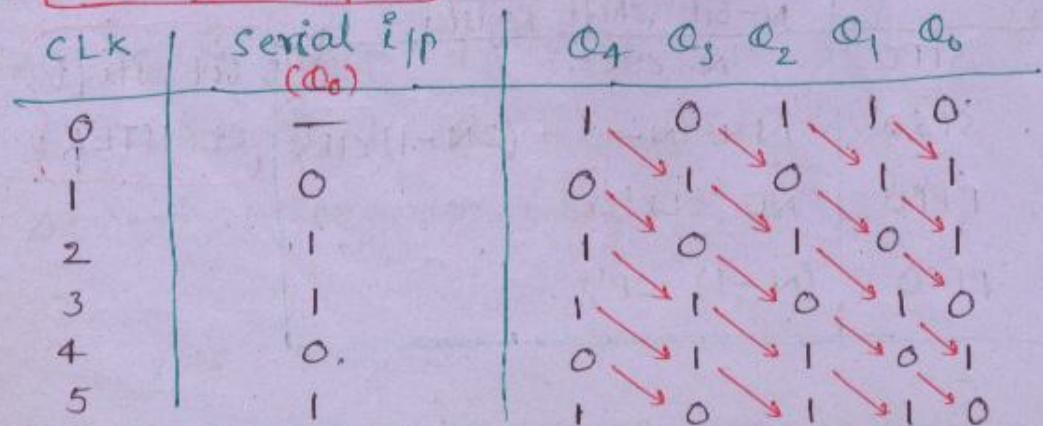
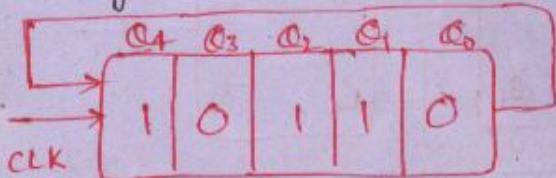
SHIFT REGISTER \rightarrow D - f/f's.



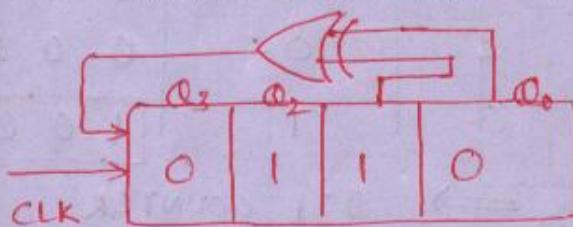
N-bit shift Register	
(1). SIPO	'N' CLK's.
(2). SISO	$N + (N-1) = (2N-1)$ CLK's.
(3). PIPO	NO CLK's.
(4). PISO	$(N-1)$ CP's.



Q. what is the value of following shift reg. after 4 CP's.



Q. In the following shift reg. how many CP's are required to make shift reg. content to have all one's.



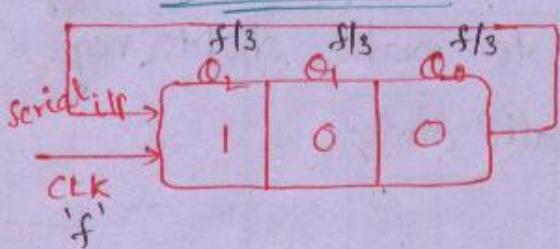
CLK	Serial i/p ($Q_4 \oplus Q_0$)	$Q_3 \quad Q_2 \quad Q_1 \quad Q_0$
0	-	0 1 0
1	1	0 1
2	0	0 1 0 1
3	1	0 1 0
4	1	1 0 1
5	1	1 1 0
6	1	1 1 1

APPLICATIONS OF SHIFT REG'S:

- (1). Serial to parallel & parallel to serial conversion
- (2). Time delays - SISO
- (3). Sequence Generator
 -
- (4). Counter
 - RING
 - JOHNSON.

..... 011011011011

RING COUNTER:

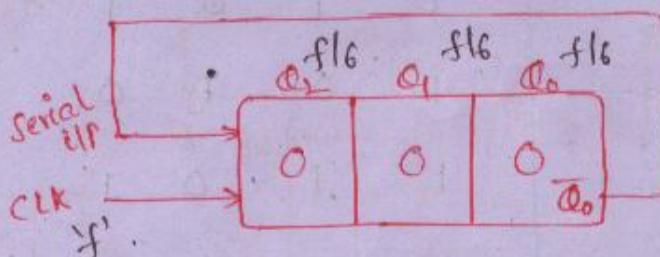


CLK	Serial i/p (Q0)	Q2 Q1 Q0
0	-	1 0 0
1	0	0 1 0
2	0	0 0 1
3	1	1 0 0

N-bit Ring Counter:

- Counting capacity = $N:1$
- Output frequency = f/N .

JOHNSON COUNTER: [TWISTED RING COUNTER]



CLK	Serial i/p (Q0)	Q2 Q1 Q0
0	-	0 0 0
1	1	1 0 0
2	1	1 1 0
3	1	1 1 1
4	0	0 1 1
5	0	0 0 1
6	0	0 0 0

→ 6:1 COUNTER.

N-Bit Johnson Counter:

- Counting capacity = $2N:1$
- Output frequency = $f/2N$.

Q. what is the o/p freq. of a 3bit Johnson counter if its clk freq is 18 kHz.
The initial content of the reg. is 101.

clk	Serial i/p (\bar{Q}_0)	Q_2	Q_1	Q_0
0	-	1	0	1
1	0	0	1	0
2	1	1	0	1

→ 2:1 COUNTER.

$$2N = 6$$

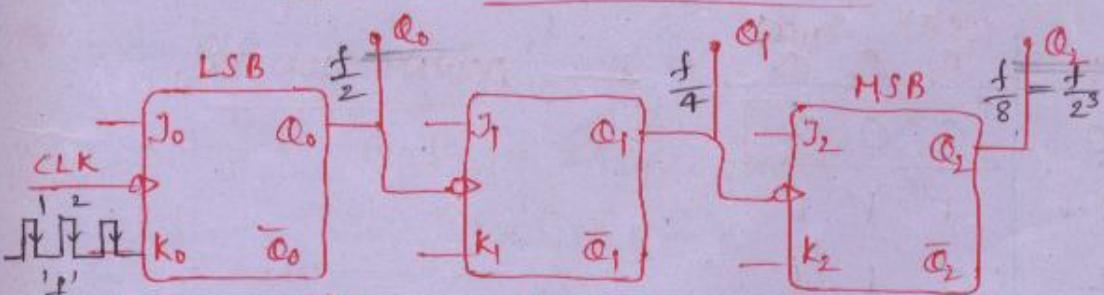
$$\therefore f_o = \frac{f}{2} = 9 \text{ kHz. } \quad \frac{f}{2^N} = \frac{f}{6}$$

COUNTERS:

(1). Asynchronous / Ripple. → $T \cdot f_{IF}$.

(2). Synchronous / parallel.

3-bit Asynchronous / Ripple counter:



CLK	Q_0 (LSB)	Q_1	Q_2 (MSB)
0	0 0 0		
1	1 0 0		
2	0 1 0		
3	1 1 0		
4	0 0 1		
5	1 0 1		
6	0 1 1		
7	1 1 1		
8	0 0 0		

← UP COUNTER

{ CLK PULSE is given to LSB f/f }

8:1 COUNTER

→ N -bit Asynchronous counter:

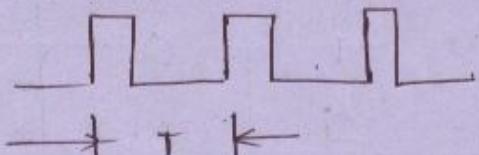
→ $2^N : 1$ counter

→ final output freq = $f/2^N$.

Let $t_{pd/ff} = 10 \text{ ns}$.

Then Max. conversion time = 30 ns .

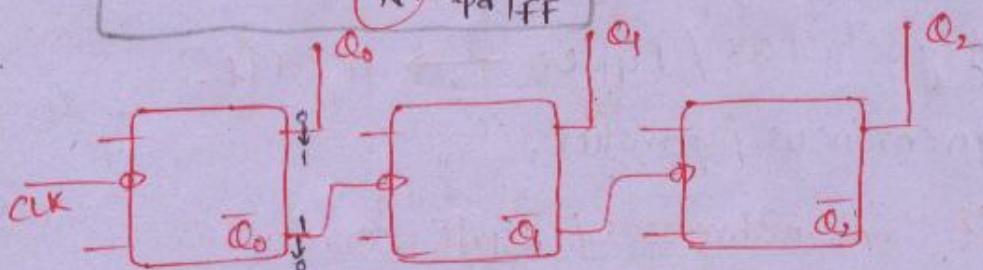
⇒ $T \geq 30 \text{ ns}$.



$$f = \frac{1}{T} \leq \frac{1}{30 \text{ ns}}$$

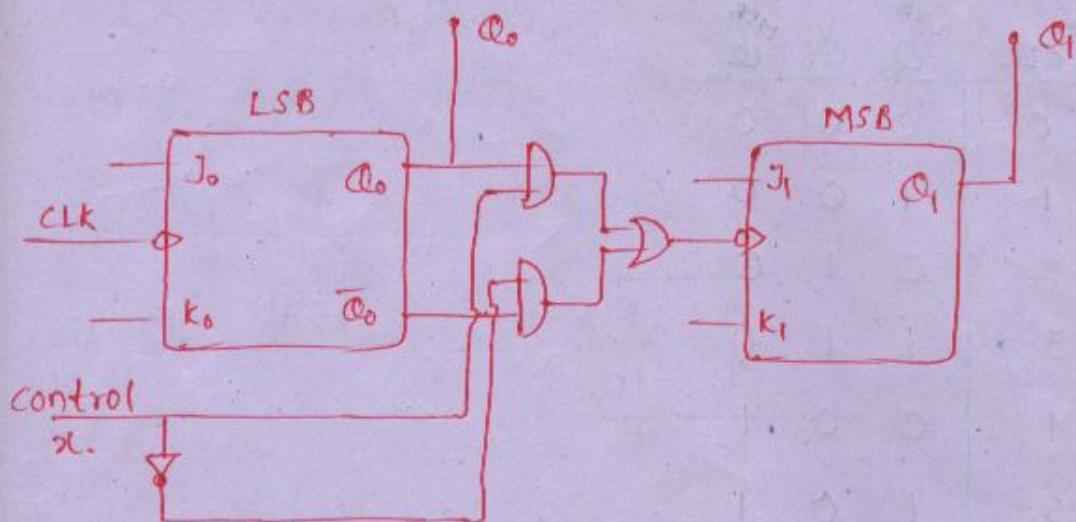
→ $f_{\max} = \frac{1}{30 \text{ ns}}$.

$$f_{\max} = \frac{1}{N \cdot t_{pd/ff}} \quad \text{no. of flip-flops}$$



CLK	(LSB)			(MSB)		
	Q_0	Q_1	Q_2	Q_0	Q_1	Q_2
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	0	1	1	0	1	1
3	1	0	1	1	0	1
4	0	0	1	0	0	1
5	1	1	0	1	1	0
6	0	1	0	0	1	0
7	1	0	0	1	0	0
8	0	0	0	0	0	0

2-Bit Asynchronous up/down counter:



$x=1 \rightarrow Q_0 \rightarrow \text{clk} \rightarrow \text{up counter. (00, 01, 10, 11, 00...)}$

$x=0 \rightarrow \bar{Q}_0 \rightarrow \text{clk} \rightarrow \text{down counter. (00, 11, 10, 01, 00...)}$

MODULUS OF A COUNTER:

→ It is the no. of cpl's required to bring the counter to the initial state.

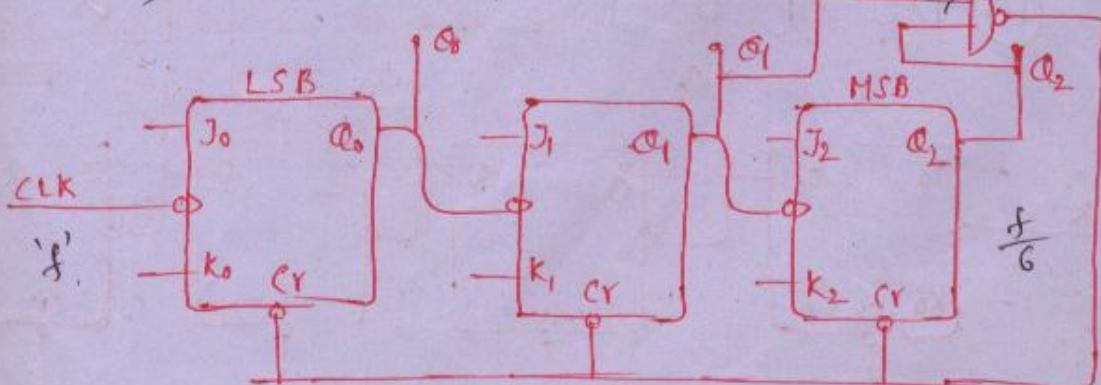
→ A Mod-N counter counts from 0 to (N-1).

$$\text{and o/p freq.} = \frac{f}{N}$$

Q. Construct Mod-6 Asy. counter.

Mod-6 Asy. Counter:

$$2^N \geq \text{mod} \Rightarrow 2^N \geq 6 \Rightarrow N = 3$$



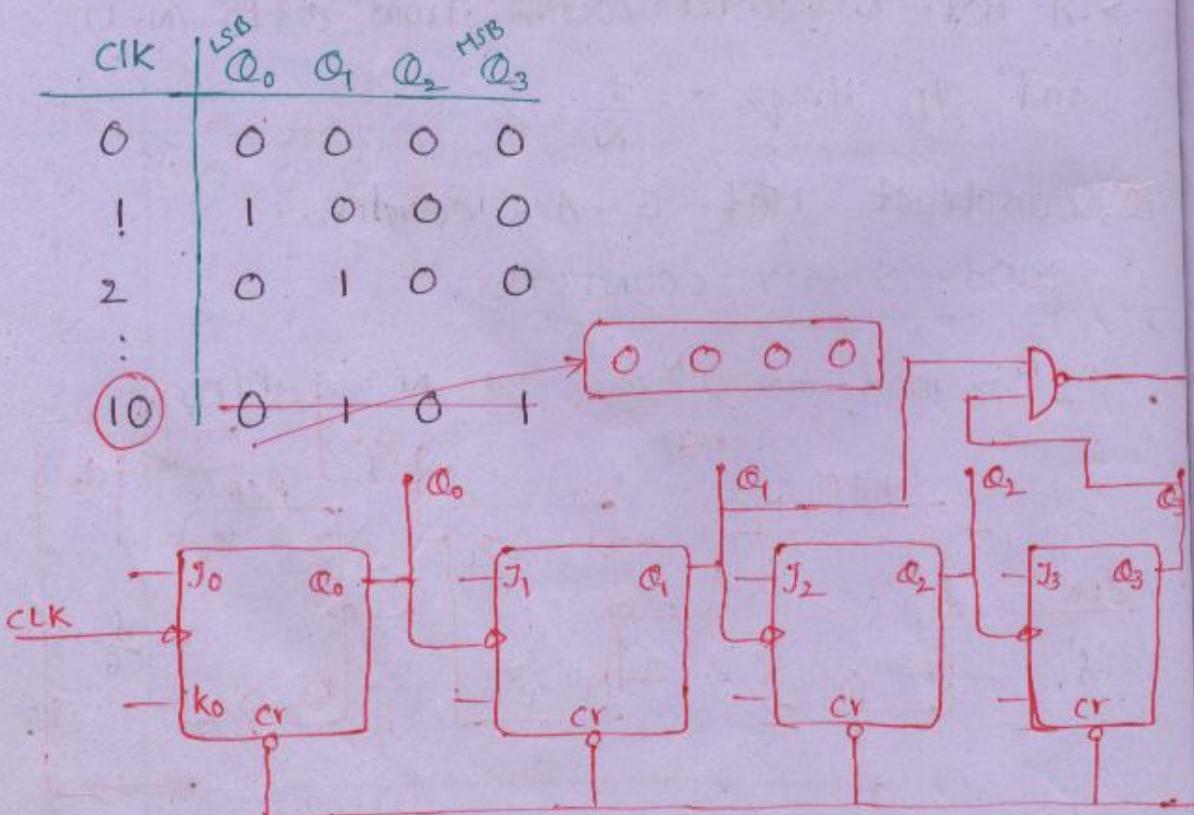
UP COUNTER

CLK	LSB	Q_0	Q_1	MSB	Q_2
0	0	0	0		
1		1	0	0	
2		0	1	0	
3		1	1	0	
4		0	0	1	
5		1	0	1	
6		0	1	1	→ 0 0 0
7					

a. Construct a Asy. decade counter?

Mod - 10

$$2^N \geq 10 \rightarrow N = 4 + \text{fif's.}$$



Excitation Table

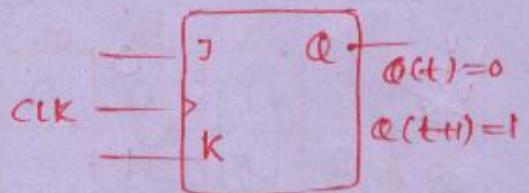
J K Q(t+1)

0 0 Q(t)

0 1 0

1 0 1

1 1 \overline{Q(t)}



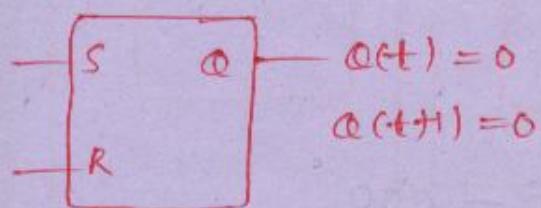
J K

1 0

1 1

1 x

S R f/f :



S R

0 1

0 0

0 x

<u>Q(t) Q(t+1)</u>	<u>J K</u>	<u>S R</u>	<u>T</u>	<u>D</u>
$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$	0 x	0 x	0	0
$\begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{matrix}$	1 x	1 0	1	1
$\begin{matrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{matrix}$	x 1	0 1	1	0
$\begin{matrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix}$	x 0	x 0	0	1

Q. Obtain excitation table of x-y f/f:-

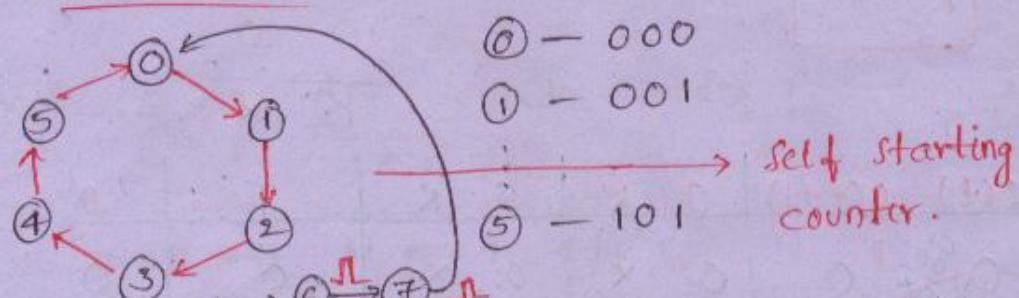
<u>x y</u>	<u>Q(t+1)</u>
0 0	1
0 1	$\overline{Q(t)}$
1 0	$Q(t)$
1 1	0

$Q(t)$	$Q(t+1)$	x	y
10	0	1	x
00	1	0	x
11	0	x	1
01	1	x	0
10			

Q. Design a mod-6 syn. counter using JK flip-flops.

$$\text{mod-6} \Rightarrow 0 \text{ to } 5.$$

state diagram



present state $Q_2\ Q_1\ Q_0$	Next state $Q_2\ Q_1\ Q_0$	ff inputs		
		$J_2\ K_2$	$J_1\ K_1$	$J_0\ K_0$
0 0 0	0 0 1	0x	0x	1x
0 0 1	0 1 0	0x	1x	x1
0 1 0	0 1 1	0x	x0	1x
0 1 1	1 0 0	1x	x1	x1
1 0 0	1 0 1	x0	0x	1x
1 0 1	0 0 0	x1	0x	x1

$$J_0 = k_0 = 1.$$

\bar{Q}_2	00	01	J_1	101	10
0	0	1	X	X	
1	0	0	X	X	

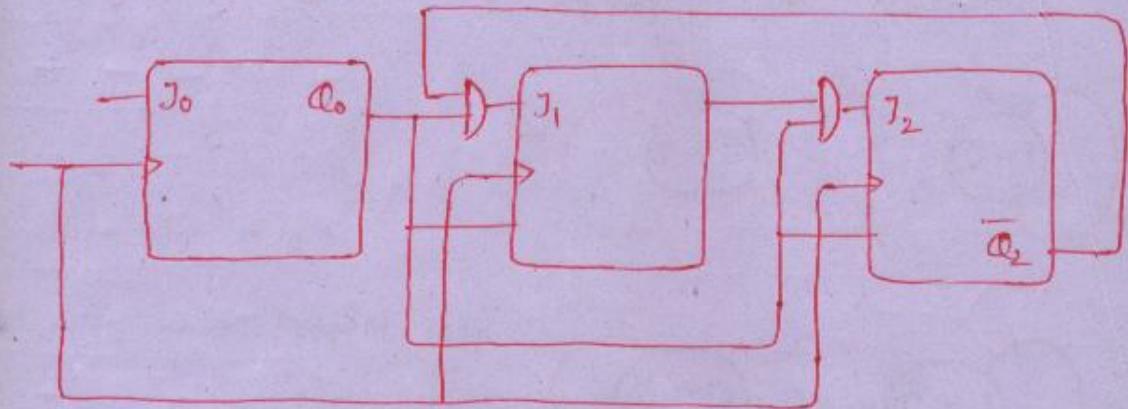
$$J_1 = \bar{Q}_2 Q_0$$

$$J_2 = Q_1 Q_0$$

\bar{Q}_2	00	01	J_2	101	10
0	0	0	1	0	
1	X	X	X	X	X

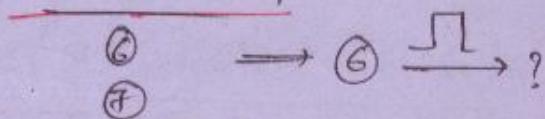
$$J_2 = Q_1 Q_0$$

and $k_1 = Q_0$, $k_2 = Q_0$.



$$f_{\max} = \frac{1}{t_{pd/ff}}$$

unfixed states



present state			flip flops			Next state		
\bar{Q}_2	Q_1	Q_0	J_2	k_2	J_1	k_1	J_0	k_0
1	1	0	00	00	11		1	1
1	1	1	11	01	11		0	0

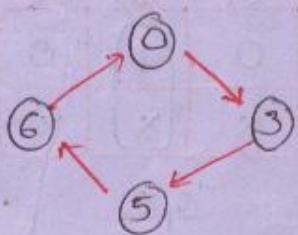
Q. Design a syn. counter using T flip-flops.

which counts +10 0, 3, 5, 6, 0, ...

Is it a self starting counter?

state diagram

→ Not a self starting counter.



(1).

P.S.

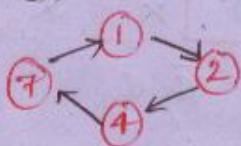
$Q_2 \ Q_1 \ Q_0$

N.S.

$Q_2 \ Q_1 \ Q_0$

Unused states:

①, ②, ④, ⑦



F/F i/p's

$T_2 \ T_1 \ T_0$

(2).

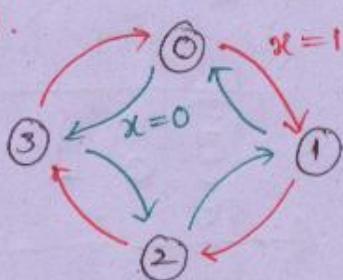
P.S

F/F i/p's

N.S.

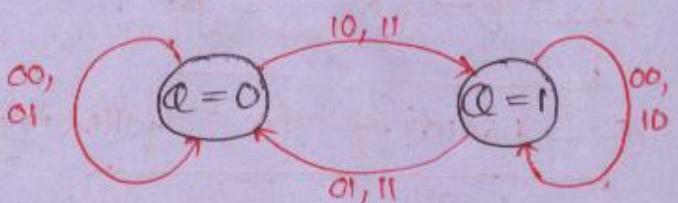
a. Draw the state diagram of following digital circuit (ix) 2 bit syn. up/down counter).

(i).



(ii). JK - f/f

present
input $\{J, K\}$ → branches of each state.
p.s. $\{\emptyset\} \rightarrow$ states

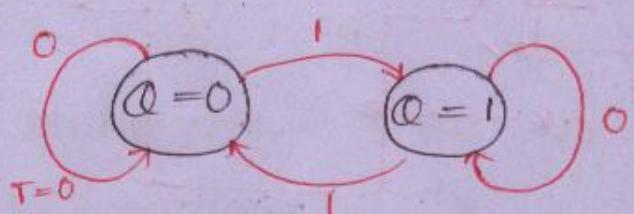


(iii). T - f/f

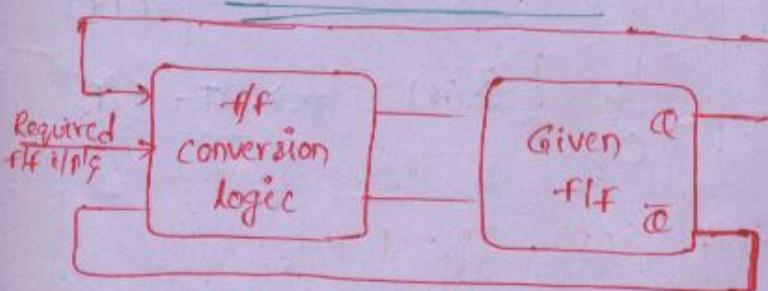
$T \rightarrow$ present i/p

$Q \rightarrow$ p.s.

T	$Q(t+1)$
0	$Q(t)$
1	$\overline{Q(t)}$



CONVERSION OF f/f's:



b. Convert SR-f/f into T-f/f.

$$\begin{array}{c} \text{SR - f/f} \longrightarrow \text{T - f/f} \\ \text{exc. table} \qquad \qquad \qquad \text{char. table} \end{array}$$

T	$Q(t)$	$Q(t+1)$		
			S	R
0	0	0	0	x
0	1	1	x	0
1	0	1	1	0
1	1	0	0	1

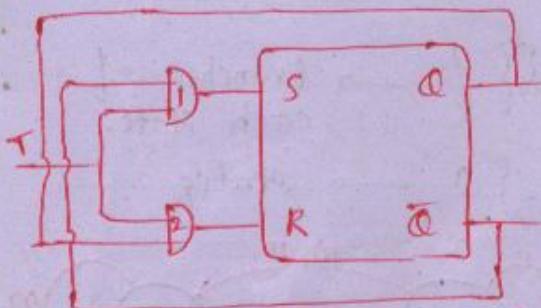
T	Q	Q'
0	0 X	
1	1 0	

$$S = T\bar{Q}$$

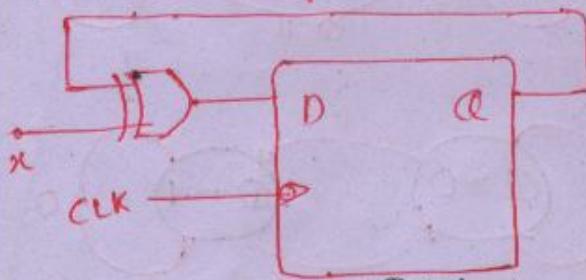
T	Q	Q'
0	X 0	
1	0 1	

$$R = \bar{T}Q$$

$$\overline{T - R - f/f}$$



c) Identify the following f/f.



X	Q(t)	D	Q(t+1)
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

X	Q(t)	$X \oplus Q$	D	Q(t+1)
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	0	0

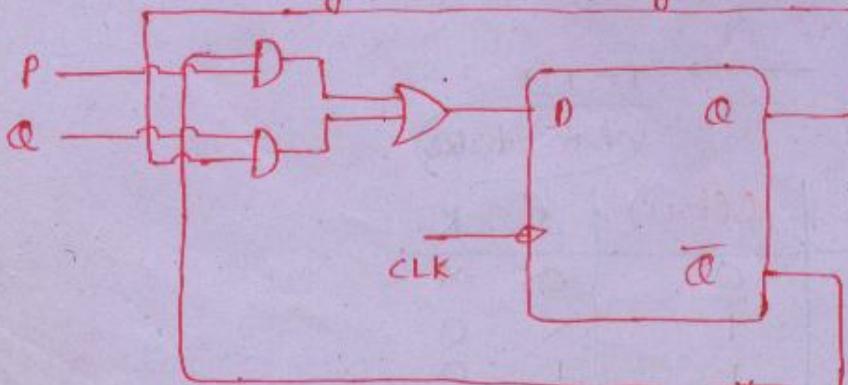
$$X \quad Q(t+1)$$

$$\begin{array}{|c|c|} \hline 0 & Q(t) \\ \hline 1 & \bar{Q}(t) \\ \hline \end{array}$$

$$\Rightarrow T-f/f$$

c) Convert D-f/f into JK-f/f.

c2) Identify the following f/f.



Q. In which of the following counters lockout doesn't occur.

(1). Mod - 13 counter (2). Mod - 30 counter

- (3). Mod - 32 " (4). Mod - 36 "
- $2^4 - 13 = 3$ unused states
- $2^5 - 32 = 0$ unused states
- $2^5 - 30 = 2$ unused states
- $2^6 - 36 = 28$ unused states

MULTI VIBRATORS USING LOGIC GATES:

1. ASTABLE MV:

→ 2 quasi stable states

→ Square wave Generator.

2. BISTABLE MV:

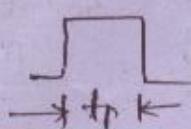
→ 2 stable states

→ 1-bit memory element

3. MONOSTABLE MV: [One shot]

→ 1 quasi & 1 stable

→ pulse generator

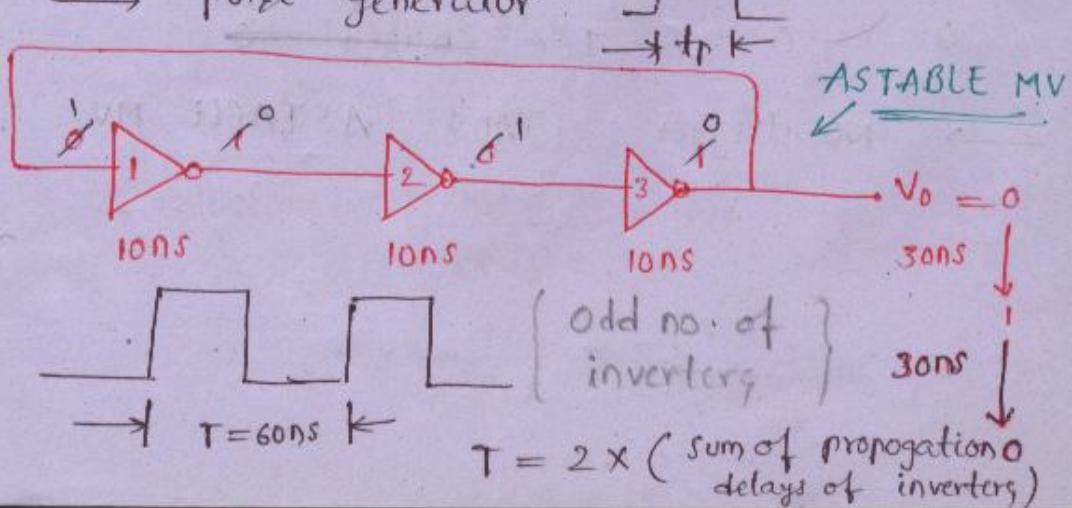


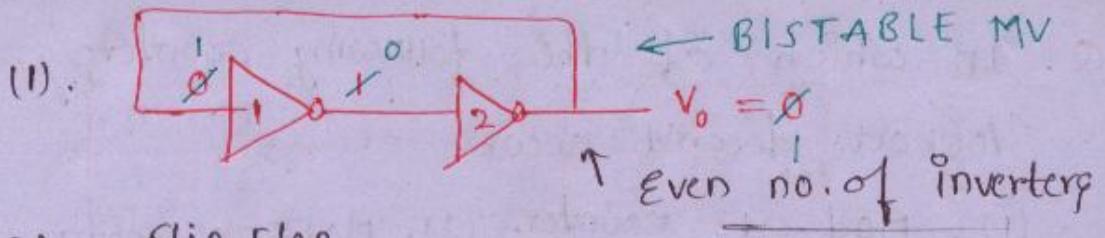
ASTABLE MV

$$V_0 = 0$$

30ns

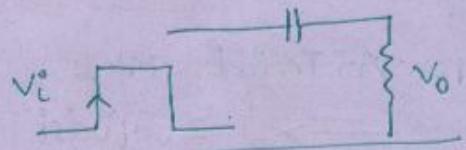
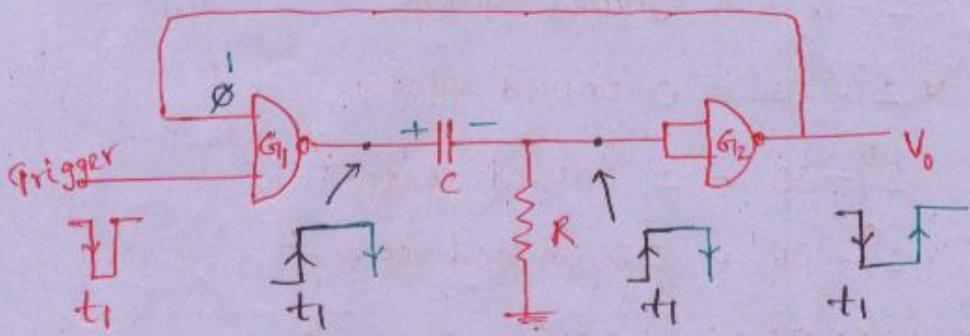
30ns



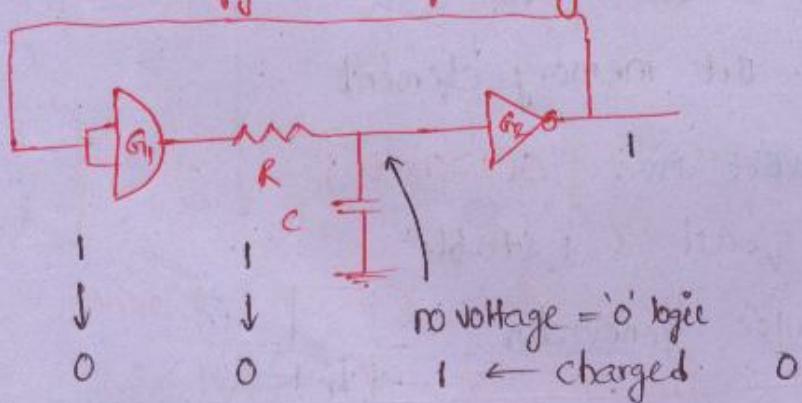


(2). flip flop

MONO STABLE :



Q. Identify the following MV's.



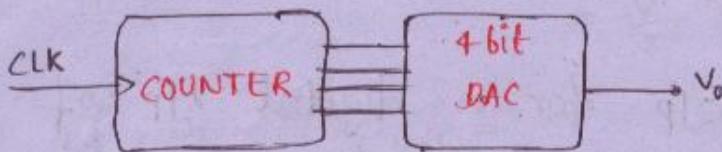
no voltage = '0' logic

I O 1 ← charged 0

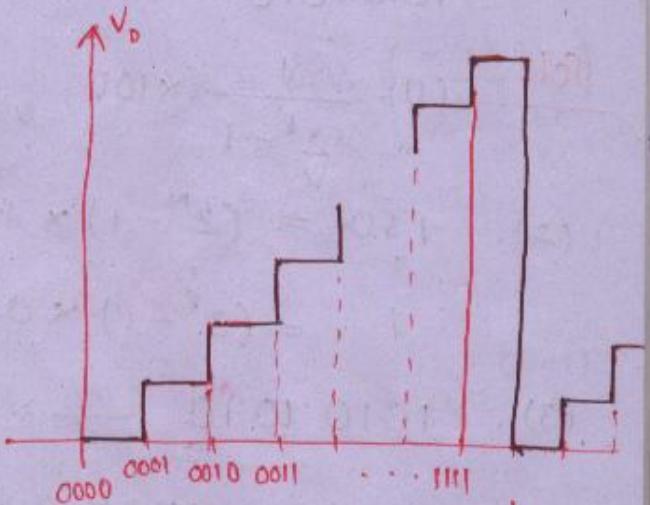
→ NO trigger ; Ans: ASTABLE MV.

DATA CONVERTERS :

- (1). DAC [Digital to Analog]
- (2). ADC [Analog to Digital].



CLK	count	V _o
0	0000	0V
1	0001	1V
2	0010	2V
:	:	:
15	1111	15V
16	0000	0V



no. of steps = 15

fs0 (full scale o/p) = 15V

Resolution = Step size (V)

It is the smallest possible change at the o/p of DAC for any change in i/p.

$$'N' \text{ bit DAC} \rightarrow (2^N - 1)$$

$$= \text{no. of steps} \times \text{step size} \quad \xrightarrow{\text{fs0}}$$

$$= (2^N - 1) \times \text{step size}$$

$$\rightarrow \% \text{ Resolution} = \frac{\text{Step size}}{\text{fs0}} \times 100$$

$$= \frac{1}{2^N - 1} \times 100$$

Q The o/p of a 8-bit DAC is 0.15V when the i/p is 00000001.

Determine (1). % resolution

(2). fso

(3). DAC o/p for a digital i/p of 10101010.

Sol: (1) $\frac{1}{2^8 - 1} \times 100$

(2) $\therefore f_{SO} = (2^N - 1) \times \text{step size}$
 $= (2^8 - 1) \times 0.15V$

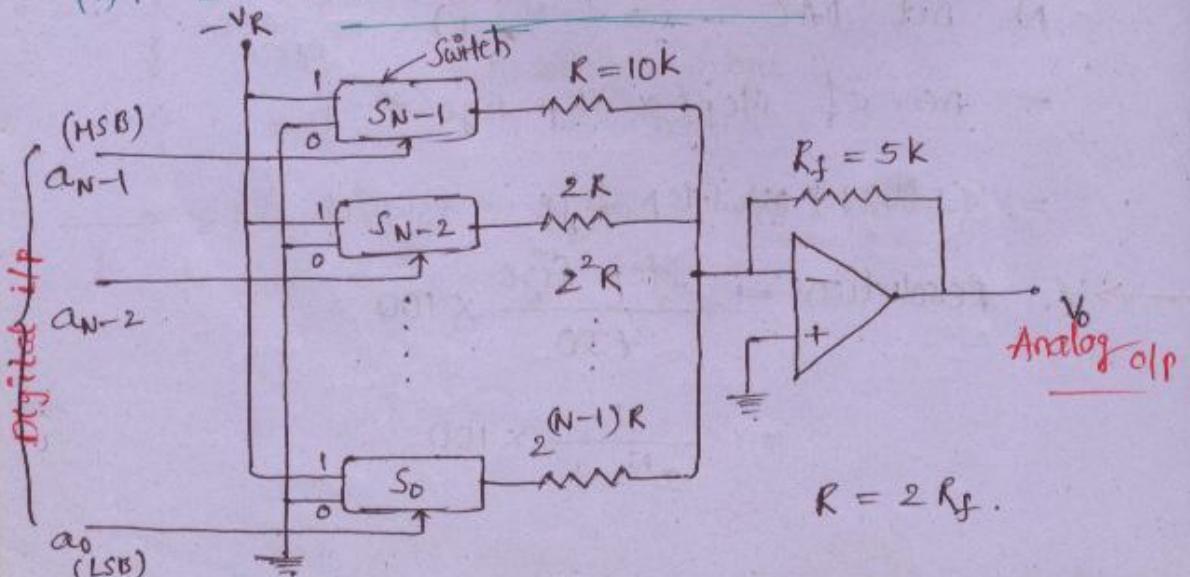
(3) $10101010_2 \rightarrow 170_{10}$

$\therefore o/p = 170 \times 0.15V$.

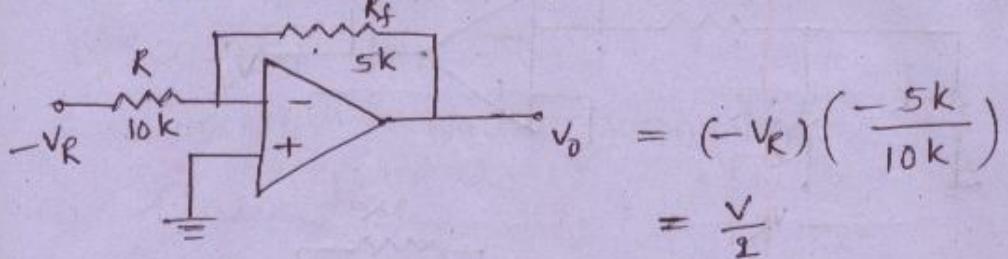
Resolution ← voltage (should be less)

8 bit DAC $0.1V$
 16 " $0.5V$
 32 " $1V$

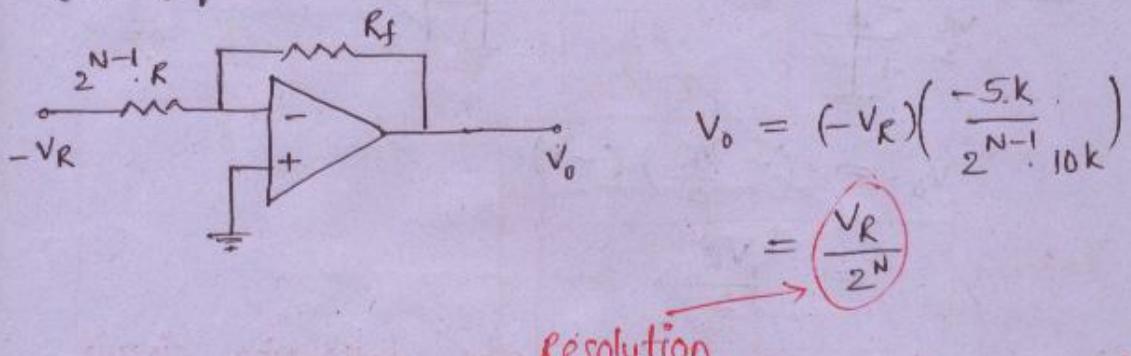
(1). BINARY WEIGHTED DAC:



(1). If $a_{N-1} = 1; a_{N-2} = \dots = a_1 = a_0 = 0$



(2). If $a_0 = 1, a_{N-1} = \dots = a_1 = 0$.



Resolution

$$V_o = (a_{N-1} \cdot 2^{-1} + a_{N-2} \cdot 2^{-2} + \dots + a_1 \cdot 2^{-(N-1)} + a_0 \cdot 2^{-N}) V_R$$

Eg: for a 3 bit DAC.

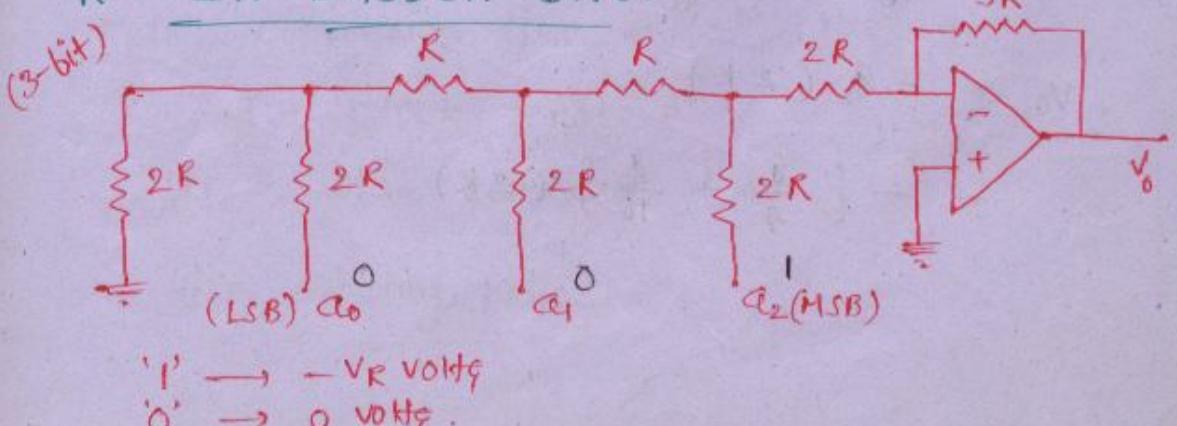
$$V_o = (a_2 \cdot 2^{-1} + a_1 \cdot 2^{-2} + a_0 \cdot 2^{-3}) V_R$$

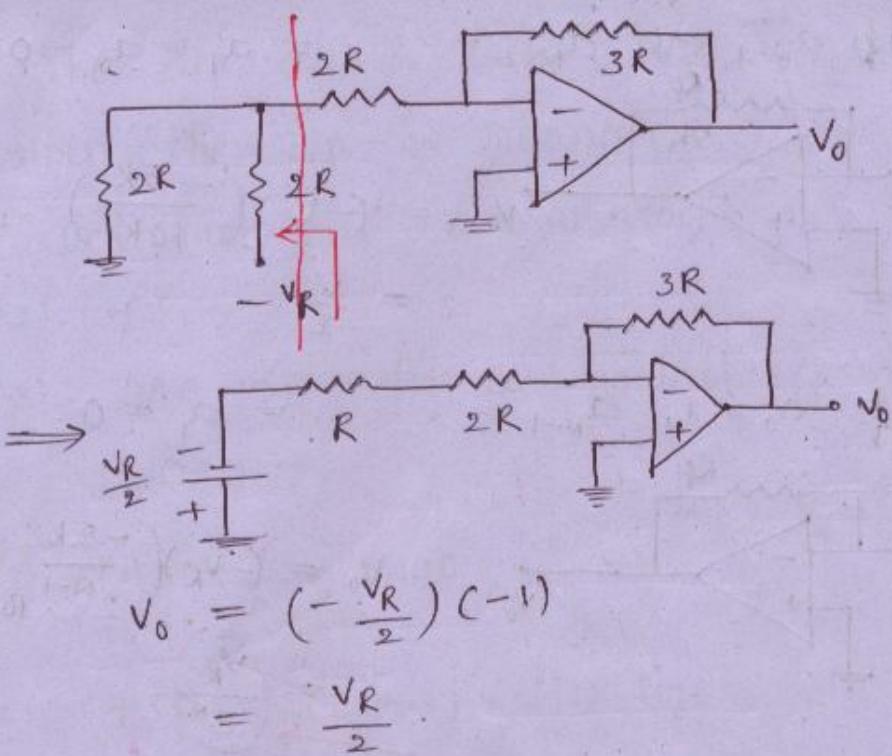
$$\text{Resolution} = \frac{V_R}{2^3}$$

Draw back \rightarrow for 32 bit DAC $\rightarrow 2^{31} \times R$

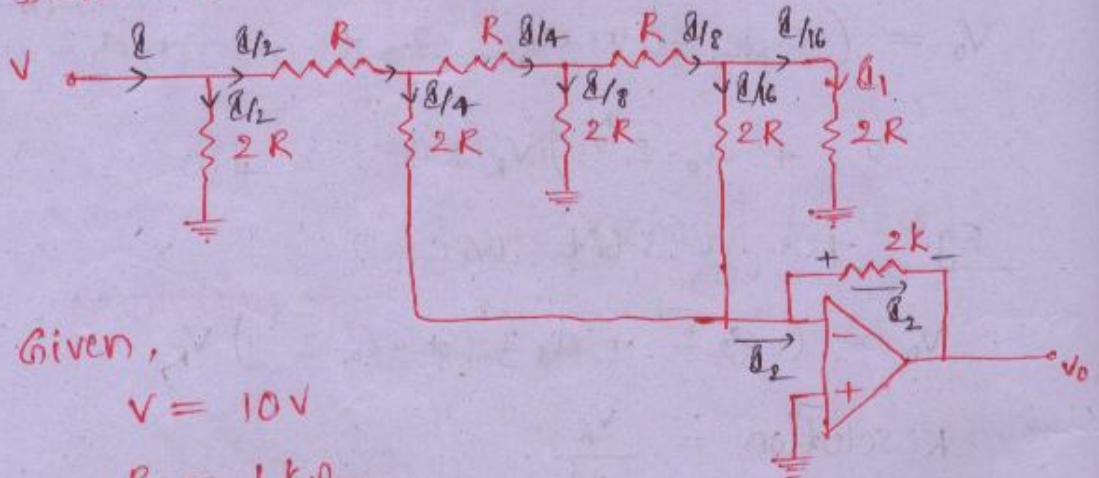
if required . . . and so.

R - 2R LADDER DAC:





Q: Determine i_1 & v_o in the following circuit.



Given,

$$V = 10V$$

$$R = 1 \text{ k}\Omega.$$

$$I = \frac{V}{R} = \frac{10}{1k\cdot} = 10mA$$

$$I_1 = \frac{8}{16} = \frac{10 \text{ mA}}{16}$$

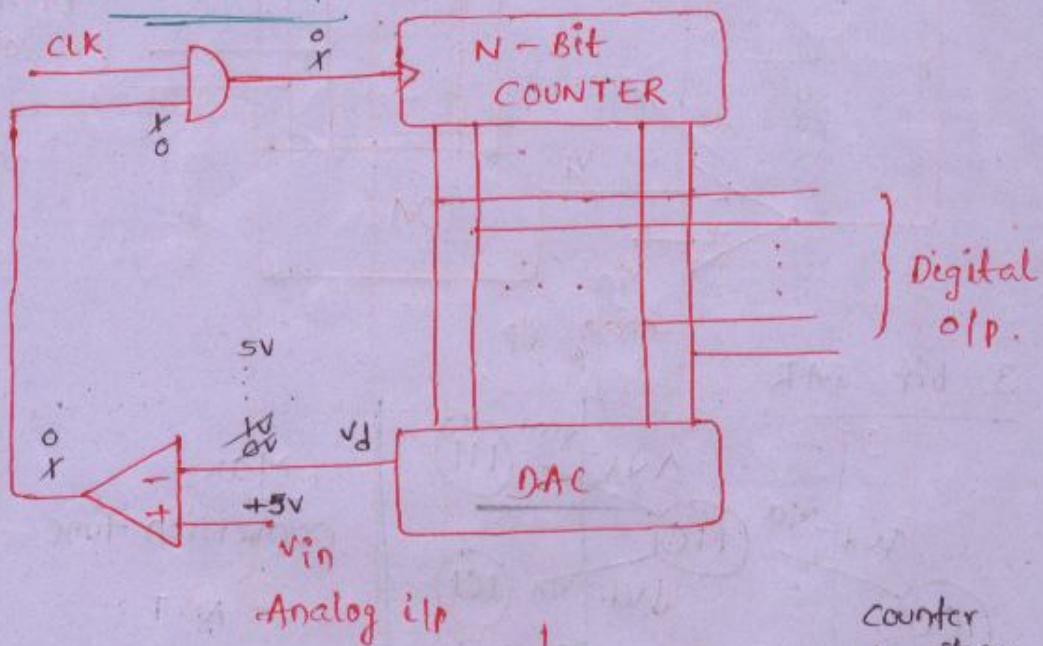
$$V_0 = -\Phi_2(2k).$$

$$= - \left[\frac{8}{4} + \frac{8}{16} \right] (2k)$$

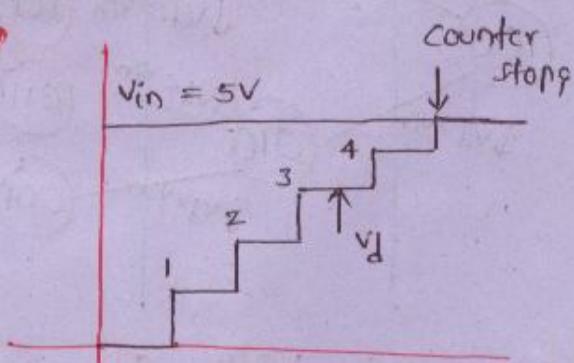
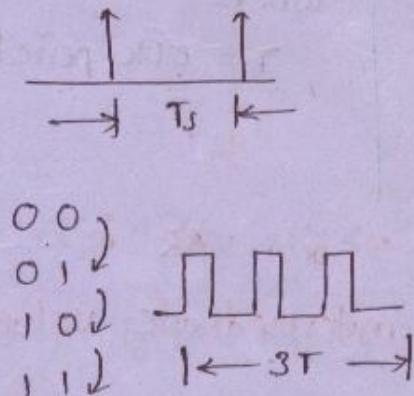
ADC's :-

1. counter type
2. successive approximation type.
3. flash type
4. dual slope.

COUNTER TYPE:-



Analog i/p



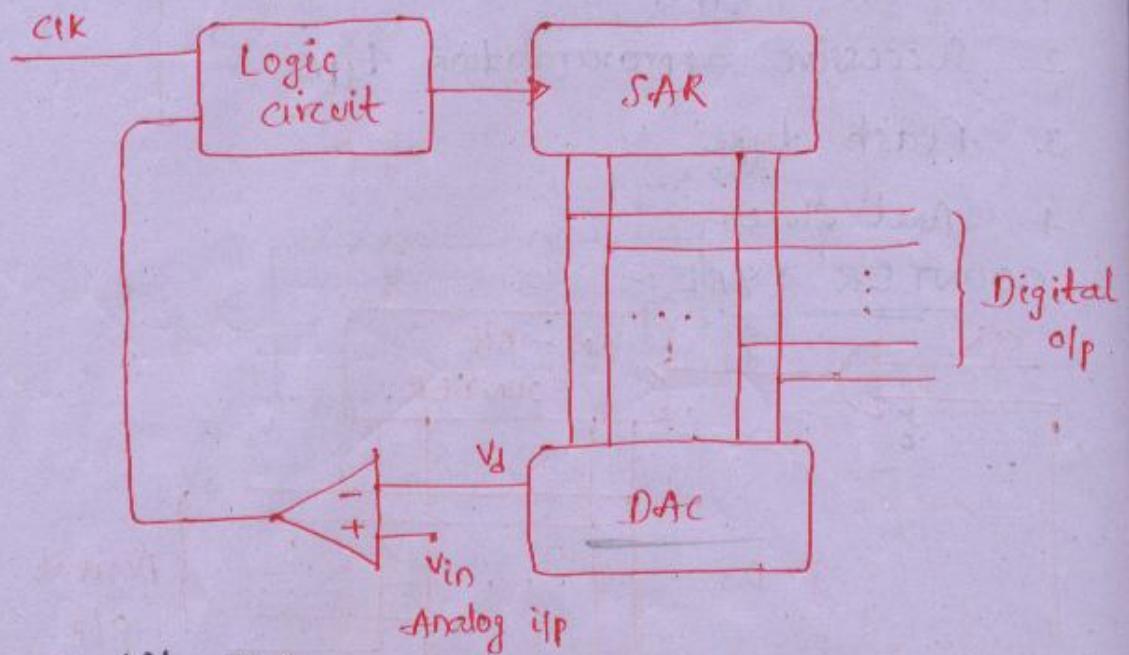
$$\text{Max. conversion time} = (2^N - 1) \cdot T ;$$

T - clock period

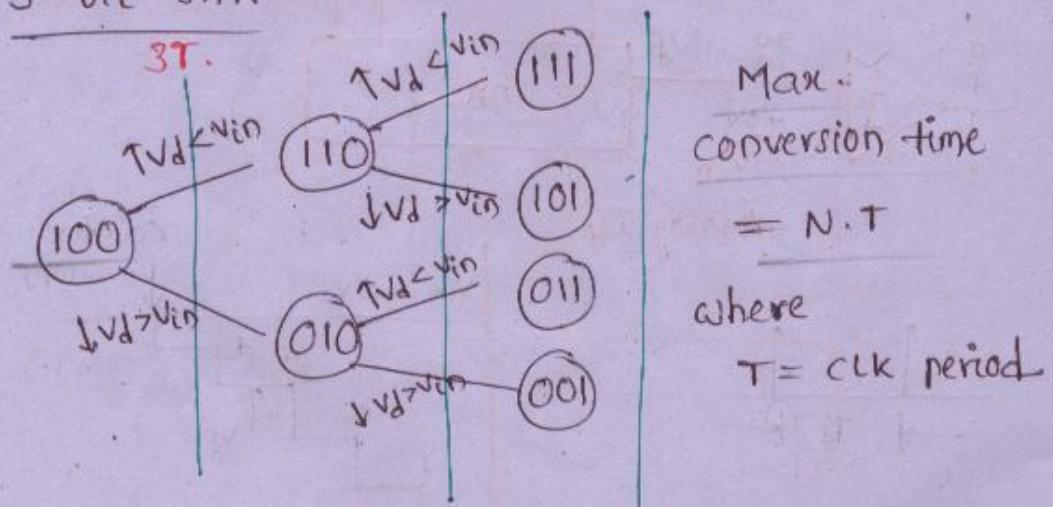
$T_s \geq \text{Max. conversion time}$

T_s = Sampling period

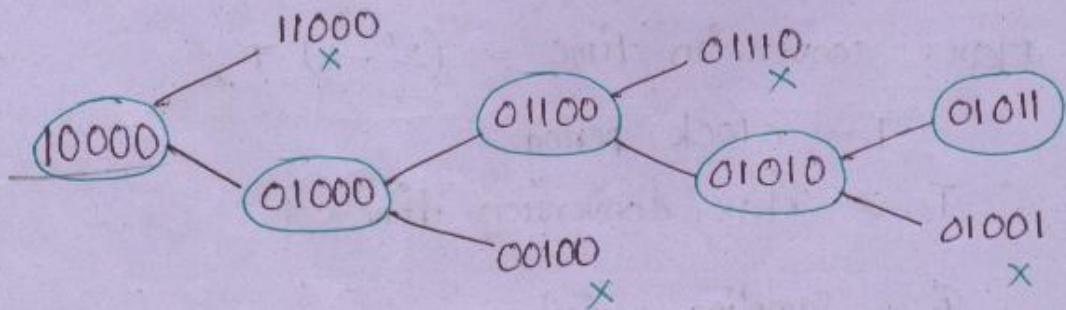
SUCCESSIVE APPROXIMATION ADC



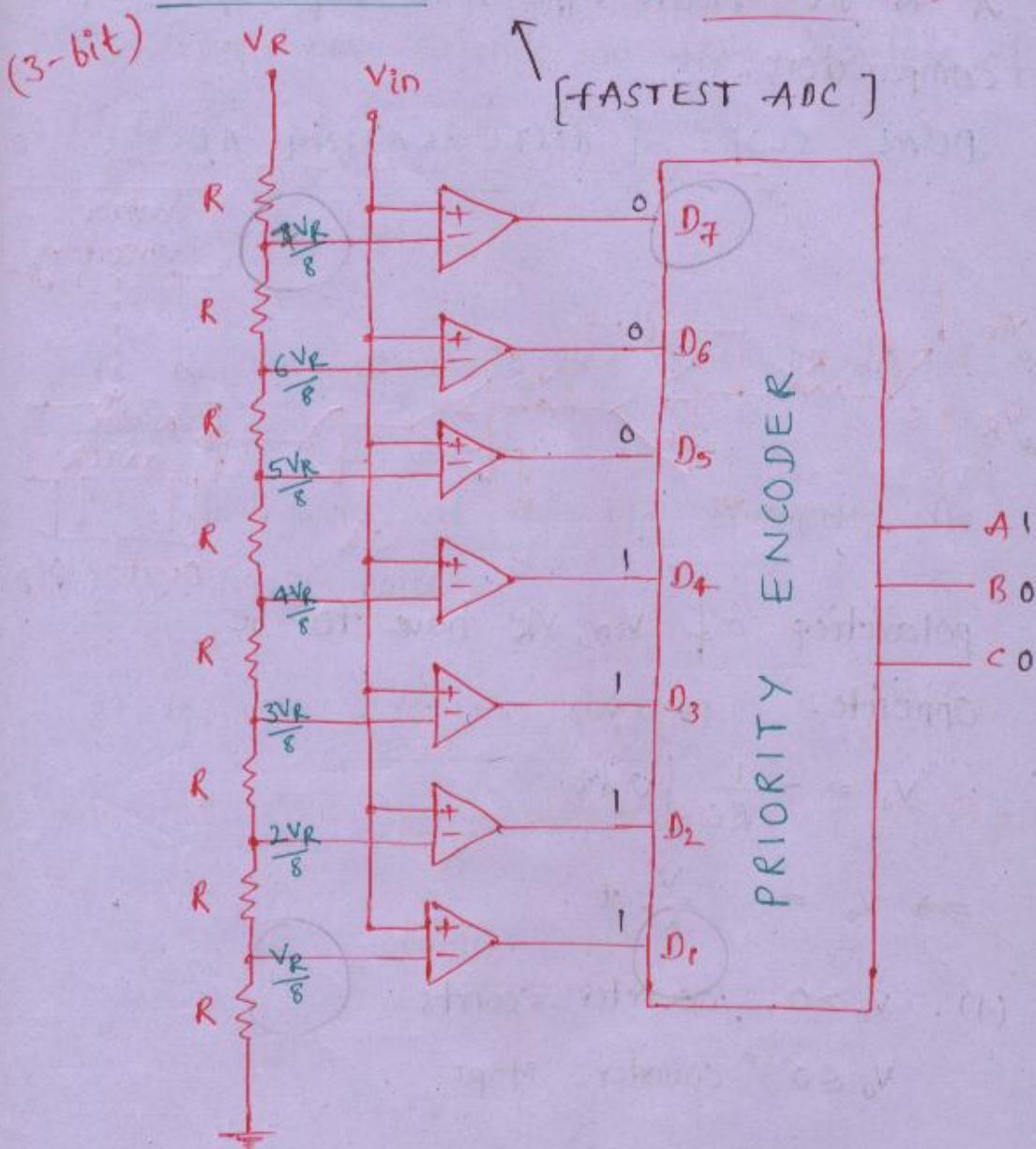
3-bit SAR



Q. The final value of a 5-bit SAR is 01011. What are its intermediate values?



FLASH TYPE ADC : (PARALLEL ADC)



$$\text{let } \frac{4V_R}{8} < V_{in} < \frac{5V_R}{8}$$

\Rightarrow Digital output = 100

$$\text{let } V_R = 8$$

$$4V < V_{in} < 5V$$

$\Rightarrow 100$

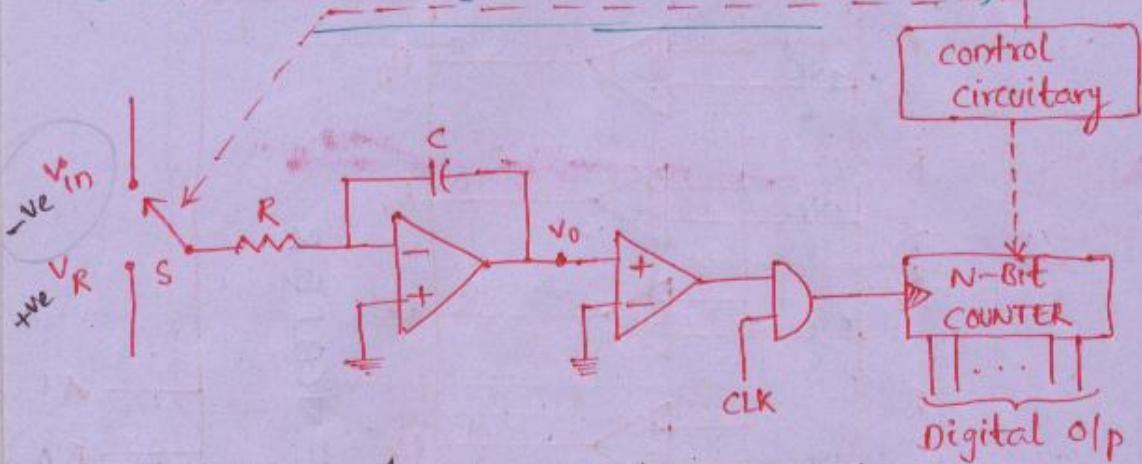
$$\text{if } \frac{1.V_R}{8} < V_{in} < \frac{2V_R}{8}$$

$\Rightarrow 001.$

Draw back :-

A N-bit flash type ADC requires $2^N - 1$ comparators.

DUAL SLOPE [INTEGRATING ADC]



polarities of V_{in} , V_R have to be opposite.

$$V_o = -\frac{1}{RC} \int v dt$$

$$\Rightarrow V_o = -\frac{V}{RC} t$$

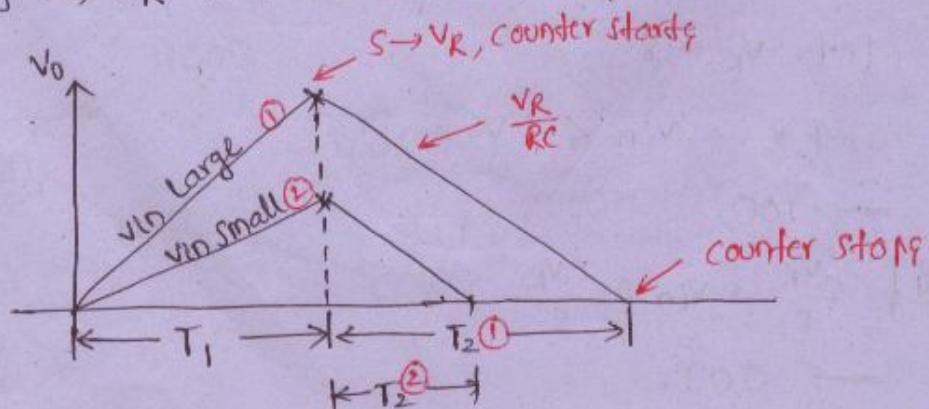
(1). $V_o > 0$, counter counts

$V_o \leq 0$, counter stops.

(2). Control circuitry :

$S \rightarrow V_{in}$ for fixed time T_1

$S \rightarrow V_R$ and counter starts.



→ Max. conversion time = $(2^N - 1) T$.

Conversion time depends on the magnitude of i/p.

$$T_2 = \frac{|V_{in}| T_1}{|V_R|}$$

Advantages :

1. It is very accurate and used in digital voltmeters.
2. The "integrator" at the i/p eliminates the power supply noise.

Draw back :

It is very slow in conversion.

Q. 8-bit ADC, i/p voltage range is -10 to $+10$

Resolution = ?

$$\text{Resolution} = \frac{+10 - (-10)}{2^8}$$

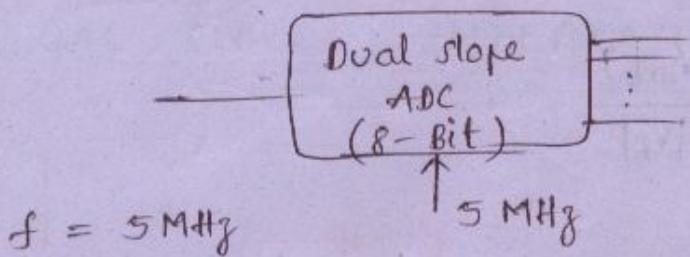
$$= \frac{20}{256}$$

Q. To convert $V_{in} = 5V$ into digital, a SAR ADC takes 10s and dual slope takes 10ns.

Then for $V_{in} = 2.5V$, what is time required. ?

$$V_{in} = 2.5V \quad \begin{cases} \text{SAR ADC} \rightarrow 10s \\ \text{Dual ADC} \rightarrow 5s. \end{cases}$$

c. what is sampling rate of 8 bit dual slope if its CLK freq. is 5 MHz.



$$T = \frac{1}{f} = 0.2 \mu\text{sec}$$

$T_s \geq \text{max conversion time}$

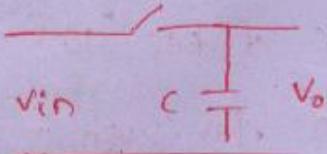
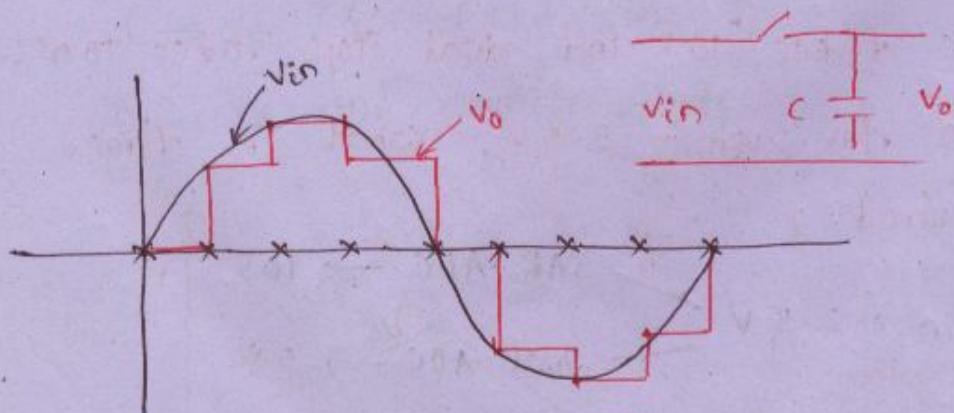
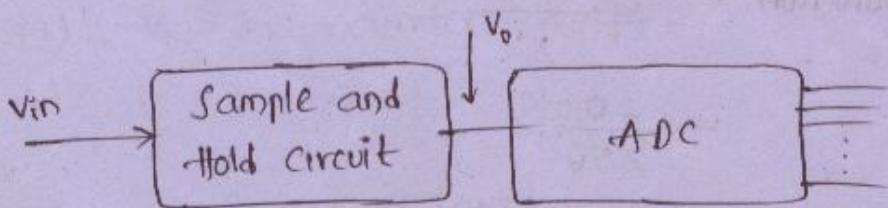
$$\text{ie } T_s \geq (2^8 - 1) T$$

$$T_s \geq (255 * 0.2 \mu\text{sec})$$

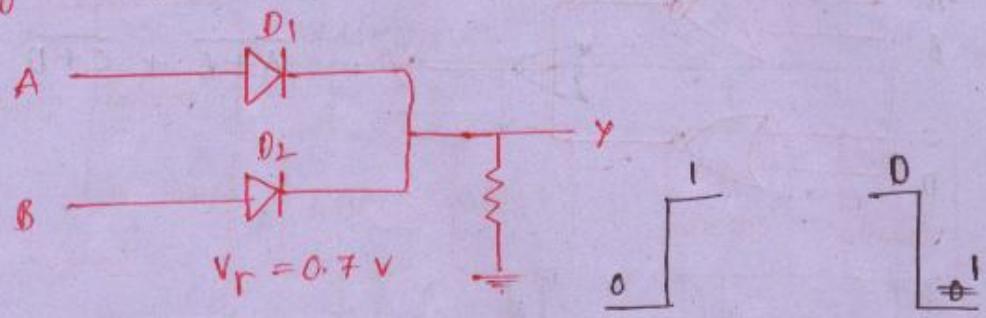
$$T_s = 51 \mu\text{sec}$$

$$\text{Sampling rate } f_s = \frac{1}{T_s}$$

$$= \frac{1}{51 \mu} \text{ samples/sec.}$$



Q. Identify the following logic gate in +ve logic - ?



A	B	Y
0	0	0
0	+5	$4.3V \leq 5V$
+5	0	$4.3V \leq 5V$
+5	+5	$4.3V \leq 5V$

A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0

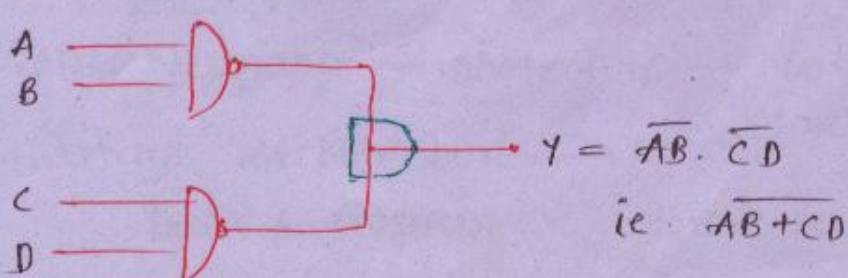
+ve logic -ve logic

AND gate

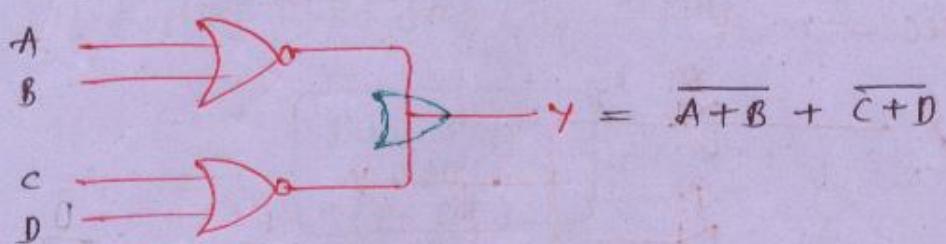
* The OR gate in +ve logic is equal to AND gate in -ve logic.

+ve logic	-ve logic
NAND	NOR
NOR	NAND
Ex-OR	Ex-NOR
Ex-NOR	Ex-OR

WIRED-AND LOGIC :-

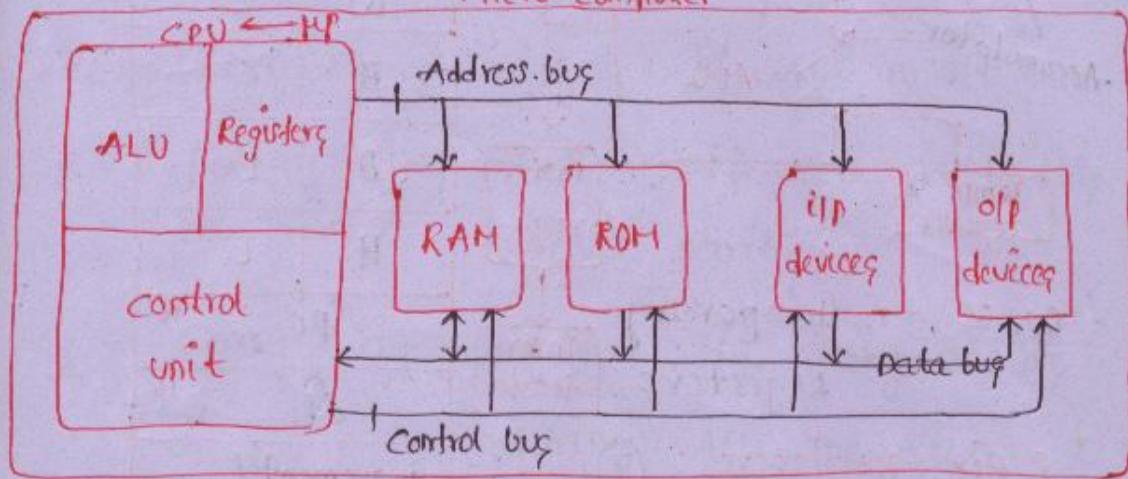


WIRED-OR LOGIC:



MICRO PROCESSORS

Micro computer.



8085 MP :-

(1). 16 Adr. lines $\rightarrow A_0$ to A_{15}

$$\begin{aligned}
 \text{Memory capacity} &= 2^{16} \\
 &= 2^6 \cdot 2^{10} \\
 &= 64 \cdot 1 \text{ KB} \\
 &= 64 \text{ KB}.
 \end{aligned}$$

$A_8 - A_{15}$

$A_0 - A_7$

(2). 8 Data lines $\rightarrow D_0$ to D_7 .

(3). freq of MP = ~~3.688~~ 3.072 MHz.
(f).

(4). Clock freq 'f',

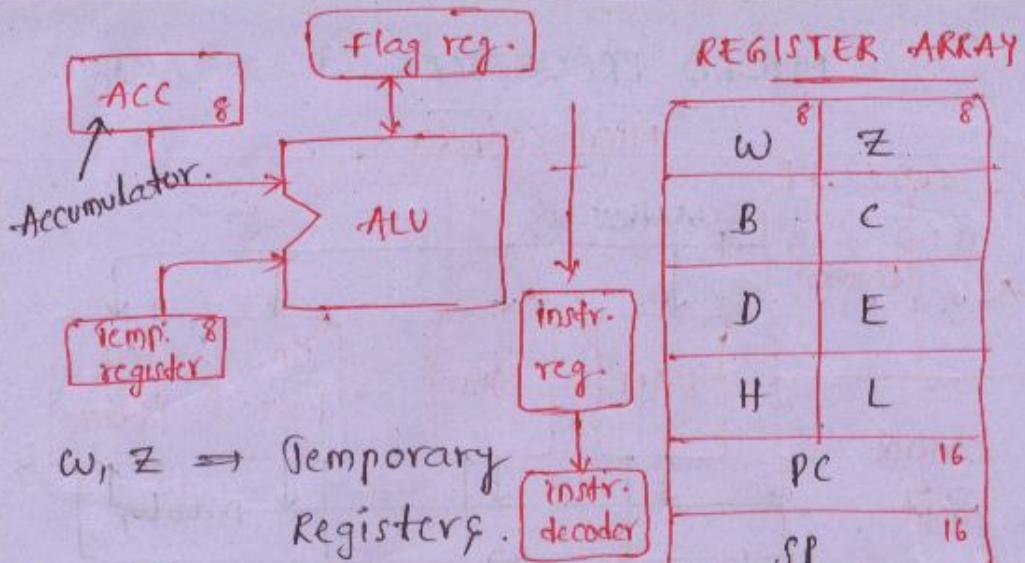
$$\text{clock period } T = \frac{1}{f} = 320 \text{ ns.}$$

'NMOS' Tech :

Von Neumann's Architecture \rightarrow Data & program stored in the same

Harvard's Architecture \rightarrow

Data & program are stored separately



W	Z
B	C
D	E
H	L
PC	16
SP	16
Increment / decrement latch	

B
C
D
E
H
L
ACC

16 Bit Registers : ③

$$\text{PSW}_{16} = \text{program status word}$$

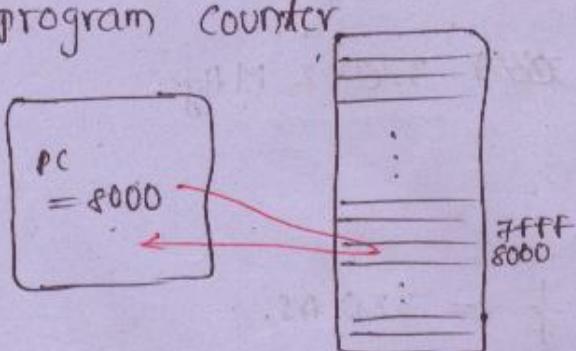
$$= \text{ACC}_8 + \text{flag reg}_8$$

BC
DE

HL \rightarrow Memory pointer

PC:

program counter



It indicates memory location from where μP has to fetch its next instr.

FLAG REGISTER:

S	Z	X	AC	X	P	X	CY
---	---	---	----	---	---	---	----

S - Sign flag \Rightarrow $s=1$, if MSB of ALU result = 1.

Z - Zero flag \Rightarrow $z=1$, if ALU result = 0.

P - Parity flag \Rightarrow $p=1$, if ALU result has even parity.

Cy - Carry flag \Rightarrow $cy=1$, if carry occurs during ALU operations.

AC - Auxiliary carry flag \Rightarrow $ac=1$, if carry occurs from D_3 to D_4 bit.

→ Can't accessed by the programmer.

→ Used in BCD arithmetic operations.

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \\ | \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \\ \hline 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \end{array}$$

$$S = 1, P = 0, Z = 0, Cy = 1, AC = 1.$$

Over flow flag \rightarrow Signed Addition

$$+ 011 (+3) \text{ (or)} \quad 110 (-2) \quad \begin{array}{r} 101 \\ \downarrow 2^7 \\ -011 -3 \end{array}$$

$$+ 010 (+2) \quad 101 (-3) \quad \begin{array}{r} 011 \\ \hline -011 -3 \end{array}$$

$$\begin{array}{r} 101 \\ \uparrow -ve \ number \\ 011 \end{array} \quad \leftarrow +ve \ number$$

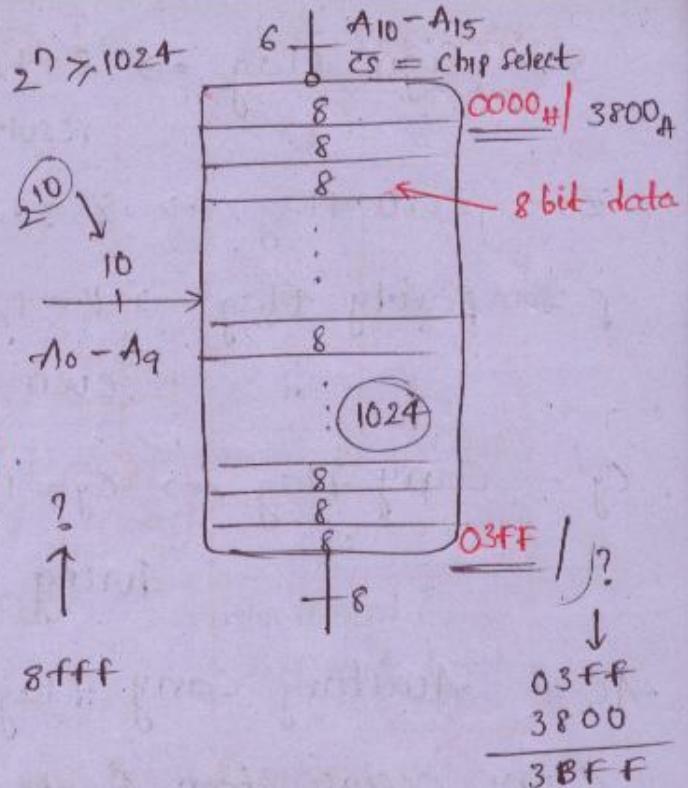
\rightarrow So over flow flag will set in this case.

MEMORY 8C's:

$$\begin{aligned} & \text{1 KB Memory} \\ & = 1024 \times 8 \\ & = 0000 \text{ } 0011 \text{ } 1111 \text{ } 1111 \end{aligned}$$

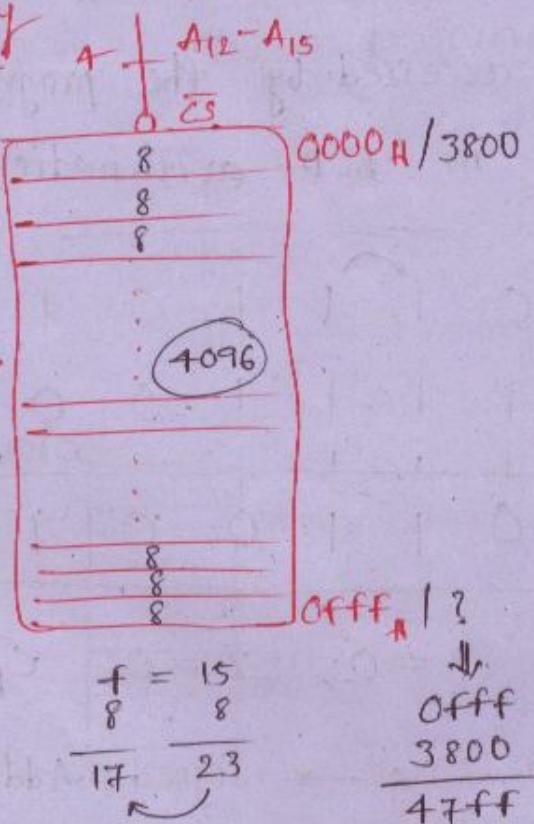
16 bit Address

$$\begin{array}{r} 8\text{ffff} \\ - 03\text{ff} \\ \hline 8\text{c00} \end{array}$$



4 KB Memory

$$\begin{aligned} & 4 \text{ KB} \\ & = 2^2 \cdot 2^{10} \\ & = 2^{12} \\ & = 4 \times 1024 \times 8 \\ & = 4096 \times 8 \\ & = 4096 \times 8 \end{aligned}$$

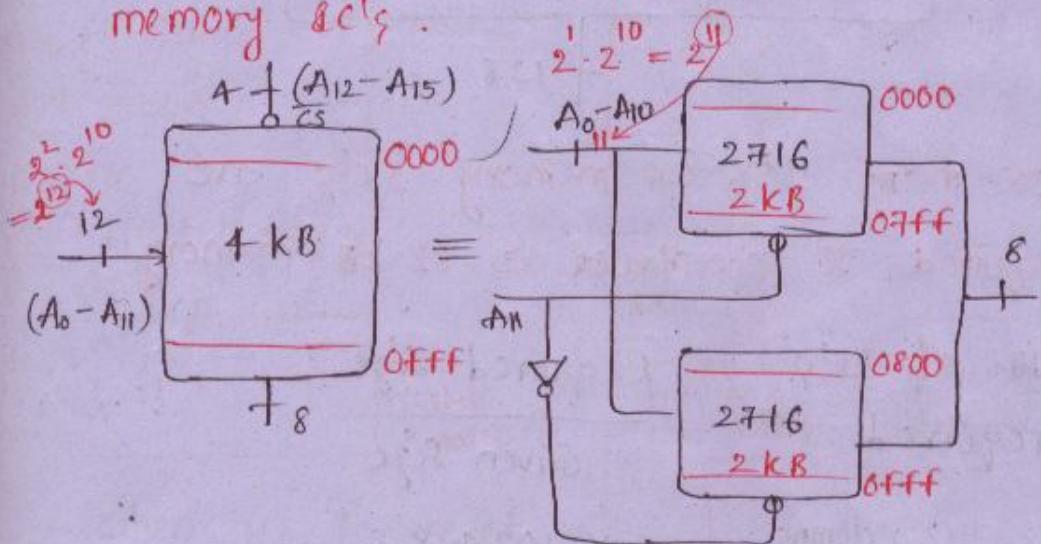


Q. for a 32 KB memory the ending location address is "AFFF". what is its starting address. ?

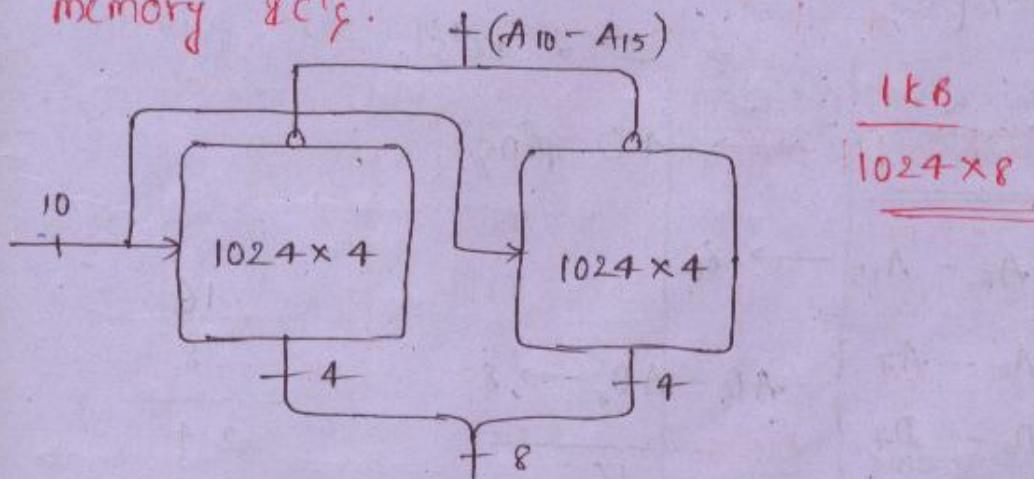
Ans: 3000H

EPROM	RAM
$27\frac{16}{8}$	$\leftarrow 2 \text{ KB} \quad \boxed{G116}$
$27\frac{32}{8}$	$\leftarrow 4 \text{ KB} \quad \boxed{G132}$
$27\frac{64}{8}$	$\leftarrow 8 \text{ KB} \quad \boxed{G164}$
$27\frac{128}{8}$	$\leftarrow 16 \text{ KB} \quad \boxed{G1128}$

e. Construct a 4 kB memory using 2716 memory IC's. , 10 (1)



c. Construct a 1KB memory using 1024×4 memory 8C's. $(A_{10} - A_{15})$

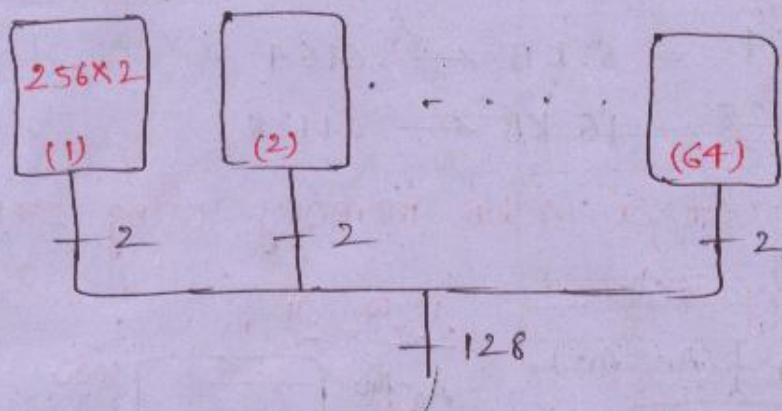


c) The add. lines of 64 memory &c having capacity of 256×2 are connected together. what is the size of resulting memory.

64 Memory IC's

$$256 \times 2$$

$$256 \times (64 \times 2) \\ = \underline{\underline{256 \times 128}}$$



Q. How many 256×4 memory IC's are required to construct a 32 kB memory.

$$\text{No. of IC's required} = \frac{\text{Required size}}{\text{Given size}}$$

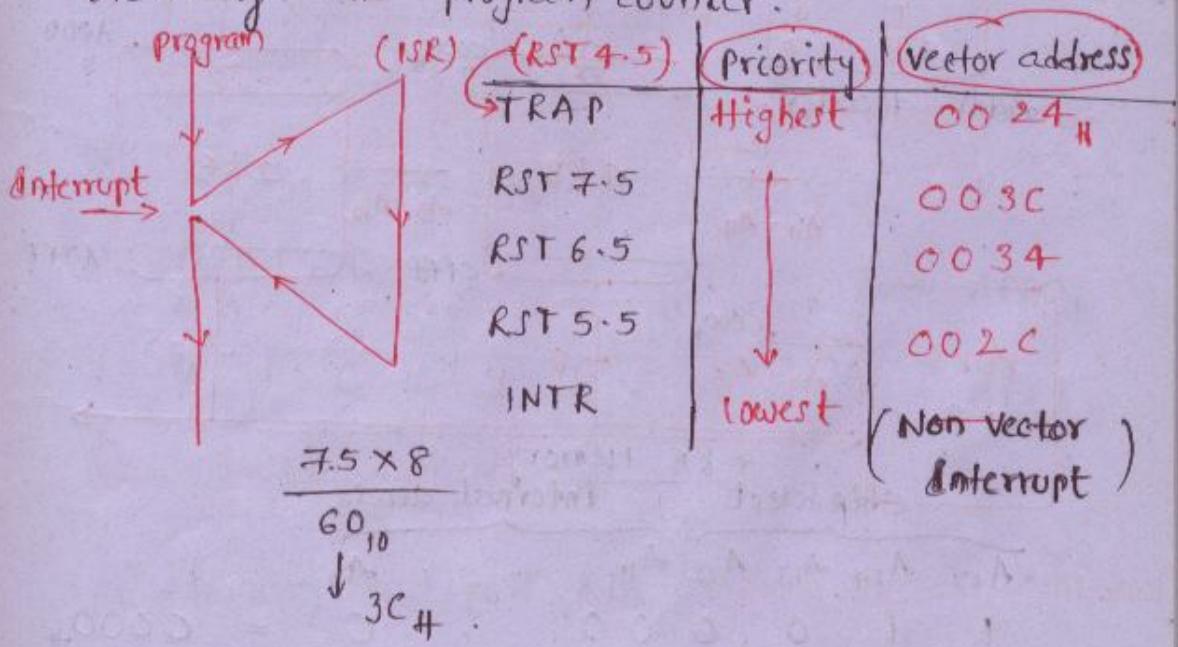
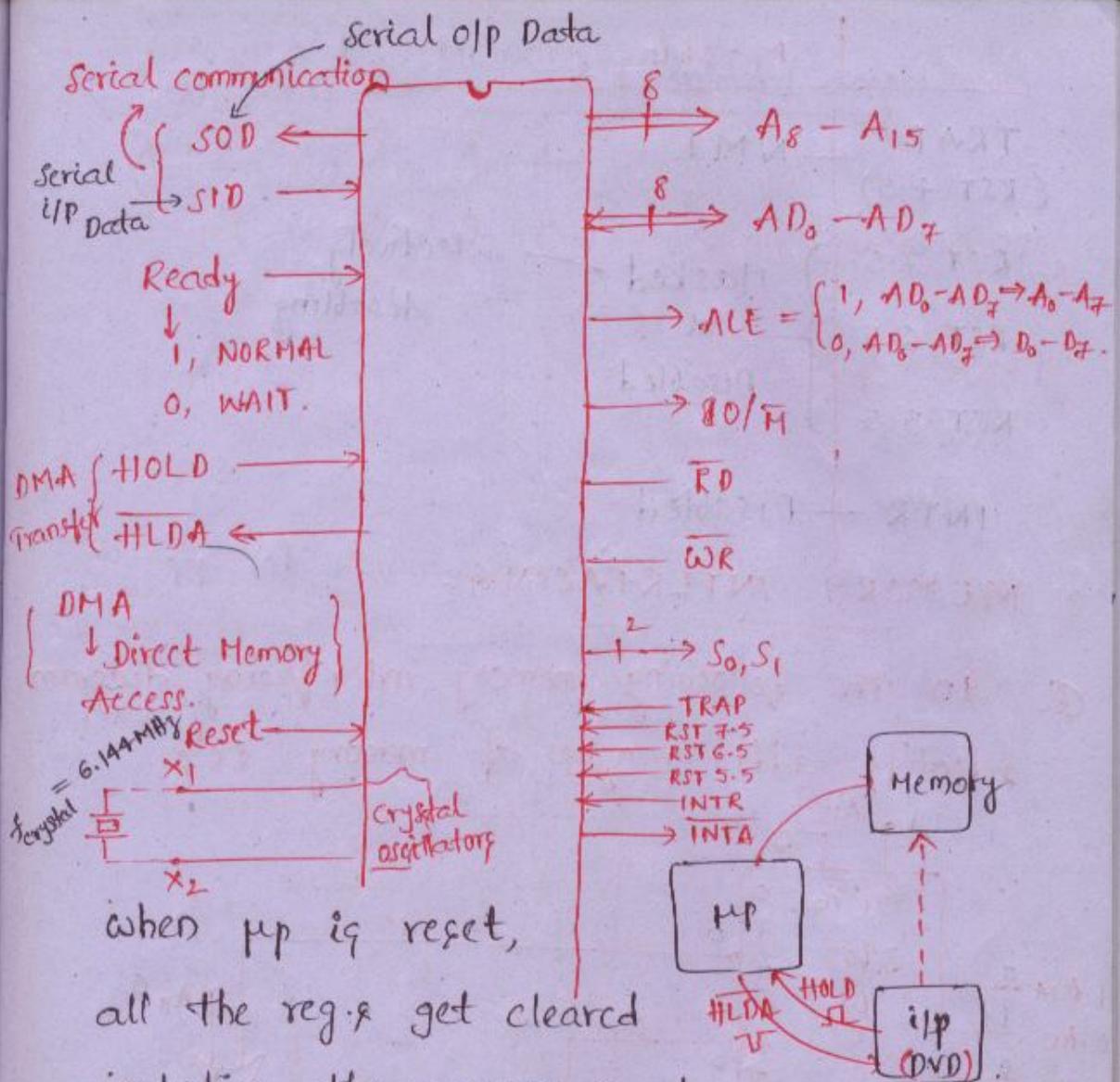
$$128 \left\{ \begin{array}{c} \text{2 columns} \\ \boxed{\text{ }} \end{array} \right. = \frac{32 \times 1024 \times 8}{256 \times 4} \\ = 256 \text{ IC's}$$

8085 HP \rightarrow 10 pins

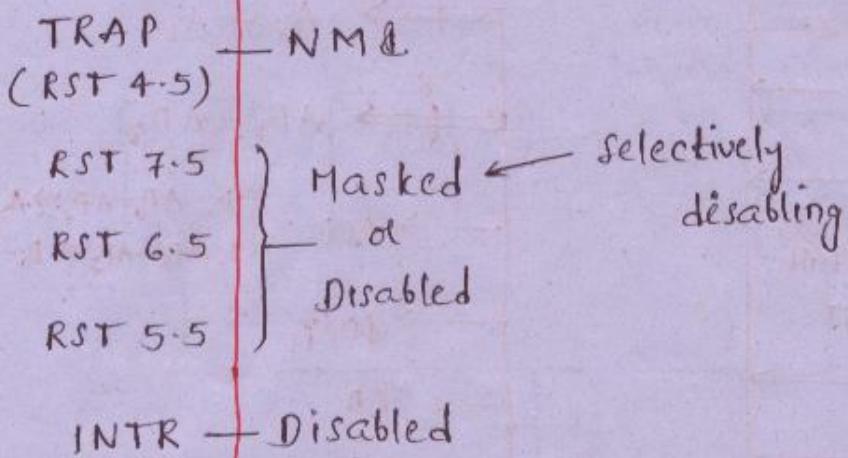
$$A_8 - A_{15} \rightarrow 8$$

$$\left. \begin{array}{l} A_0 - A_7 \\ D_0 - D_7 \end{array} \right\} AD_8 - AD_7 \rightarrow 8 \quad \begin{array}{r} 16 \\ 8 \\ \hline 24 \end{array}$$

* Ready pin used to interface up with slow speed peripherals.

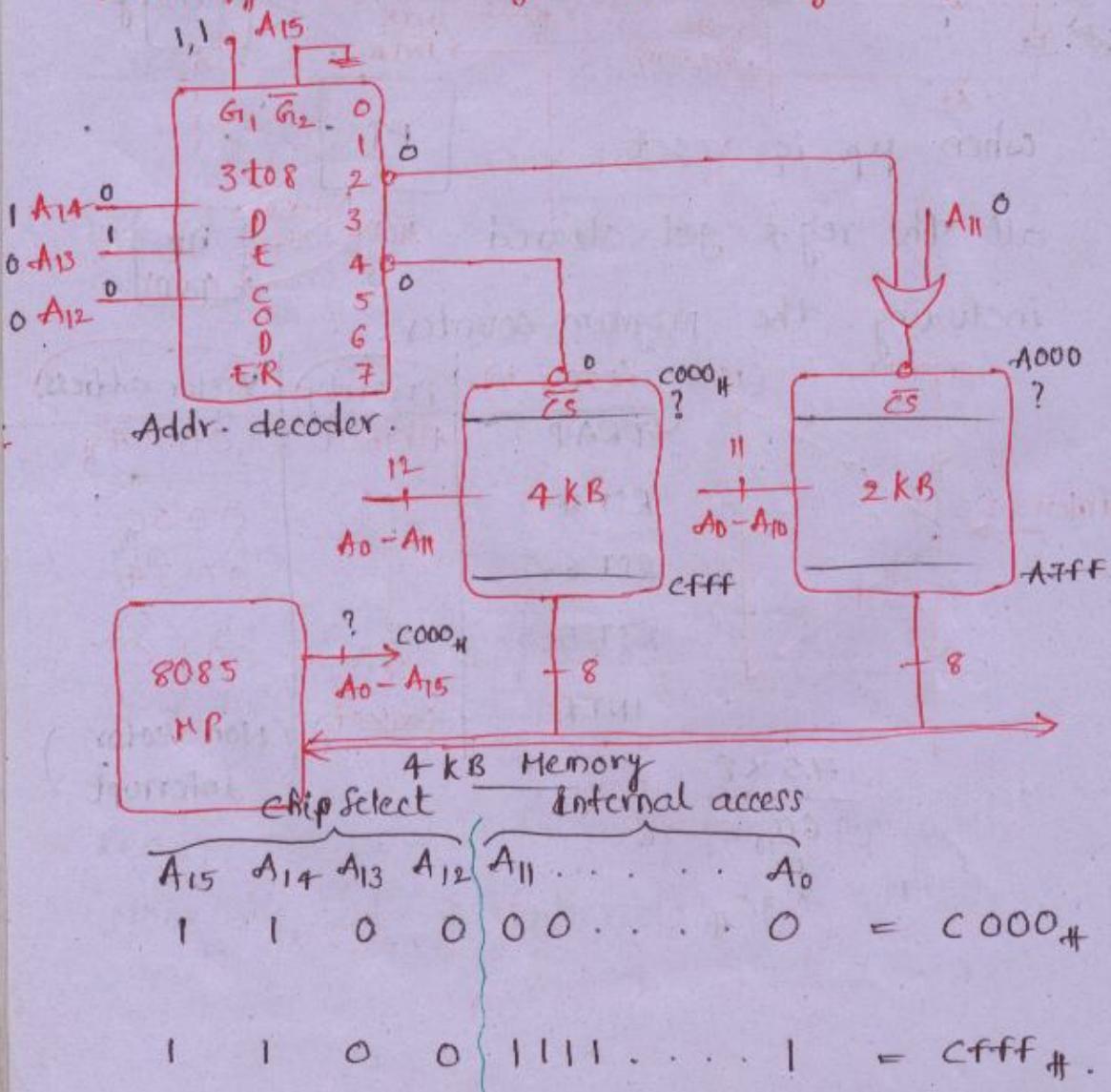


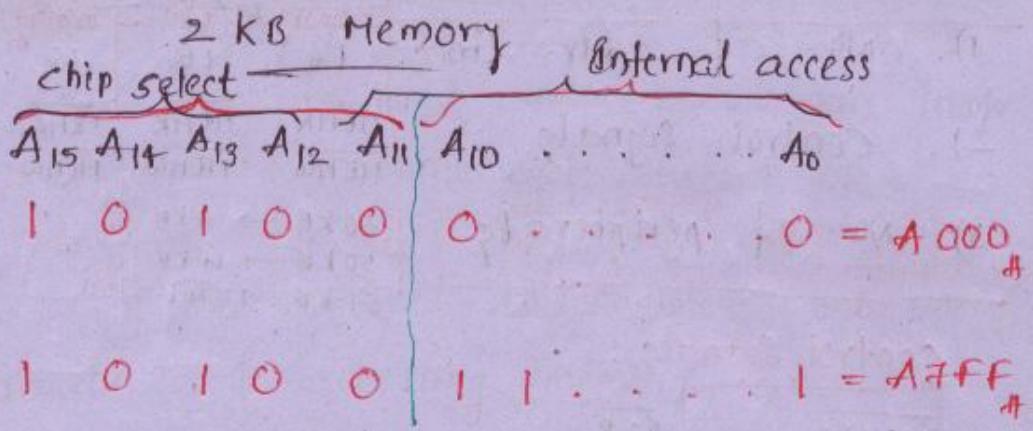
Masking



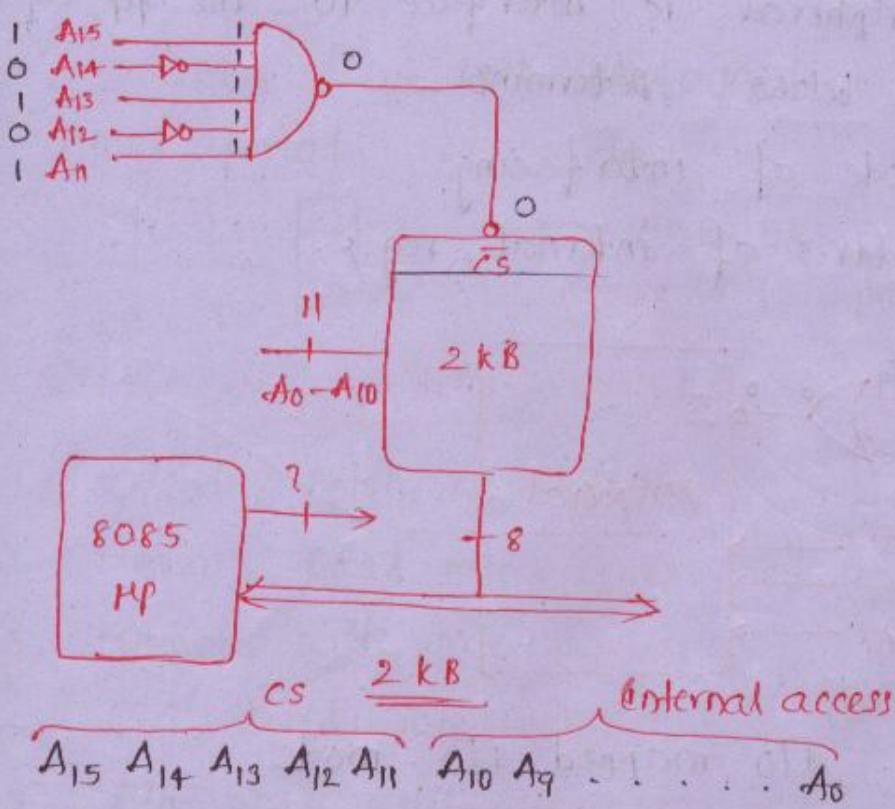
MEMORY INTERFACING:

- Q In the following memory interfacing diagram identify addr. ranges of memory &c'tg.





Q.



1 0 1 0 1 0 0 . . . 0 = A 800₁₆

1 0 1 0 1 1 1 . . . 1 = A FFF₁₆

I/O INTERFACING:

1. Memory mapped I/O → I/O devices are considered as memory Ic.
2. I/O mapped I/O. → I/O & Memory are considered separately

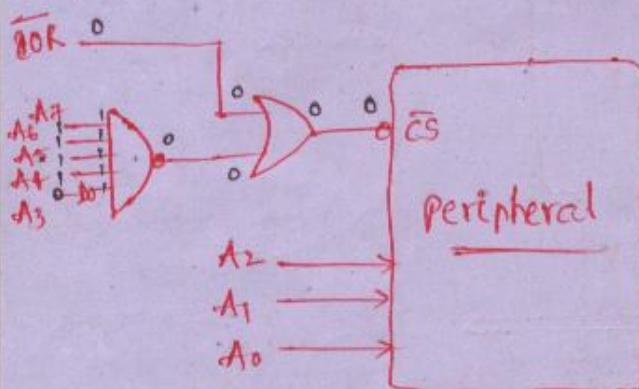
	Memory mapped Memory		I/O mapped memory	
1). NO. of addr. lines	16	16	16	8(16)
2). Control signals	MEHR MEHW	MEHR MEHW	MEHR MEHW	IOR IOW
3). NO. of peripherals	60 KB → 4 KB 50 KB → 14 KB 64 KB → nil			$2^8 = 256$ 8 I/O devices

control signals:

MEHR IOR
MEHW IOW

Q. A peripheral is interface to the CPU as shown below. Determine —

- (1). Mode of interfacing
- (2). Addr.s of internal reg.s



Ans: (1). I/O mapped I/O mode.

(2). peripheral has 8 reg.s, peripheral is I/O device b'coz

	A ₇	A ₆	A ₅	A ₄	A ₃	A ₂	A ₁	A ₀	=	IOR
R ₁	1	1	1	1	0	0	0	0	=	f0
R ₂										
R ₃										
R ₄										
R ₅										
R ₆										
R ₇										
R ₈	1	1	1	1	0	1	1	1	=	f7

INSTRUCTION CYCLE:

Time required to execute an instr.

Range : 1 machine cycle ~~to~~ ^{Memory} 5 m/c.



np **ACK** Time required to complete one operation of accessing memory, accessing I/O devices & sending an acknowledgement.

Range : 3 T states to 6 T.

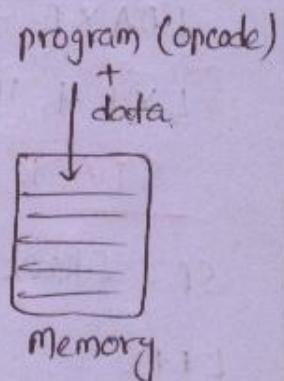
T - STATE :

It is sub task performed in one clock period



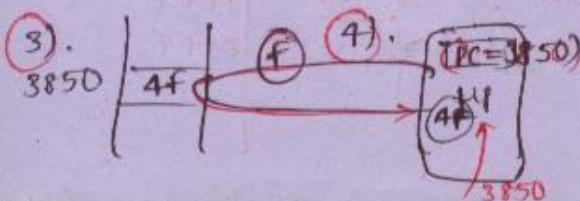
TYPES OF MACHINE CYCLES:

1. Opcode fetch m/c - (f) → 4T
 2. Memory Read m/c - (R) → 3T
 3. Memory write m/c - (W) → 3T
 4. S10 Read m/c - (I) → 3T
 5. S10 write m/c - (O) → 3T
 6. Hold ACK m/c
 7. Interrupt ACK m/c

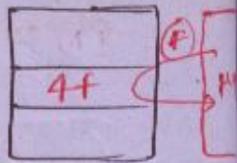


opcode fetch m/c \rightarrow $4T = 3T + T$

- ①. MOV C, A → ②. opcode
=01001111₂ = 4FH ↑ fetching

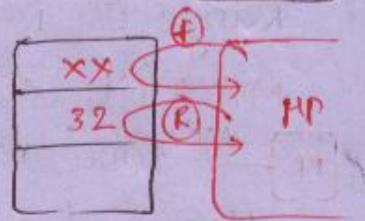


1 - Byte Instruction: $\rightarrow \text{MOV C, A}$



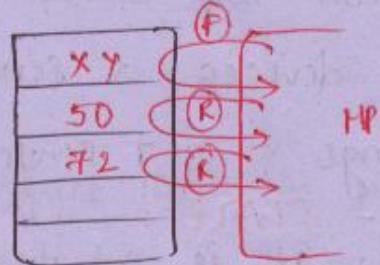
2 - Byte Instruction:

$\rightarrow \text{MOVW MV8 C, 32}$
let xx



3 - Byte Instruction:

$\rightarrow \text{LDA F250}$
xy



XTHL \rightarrow 1B

ANI f2 \rightarrow 2B

LDA X B \rightarrow 1B

LXI H, 1122 \rightarrow 3B

STACK:

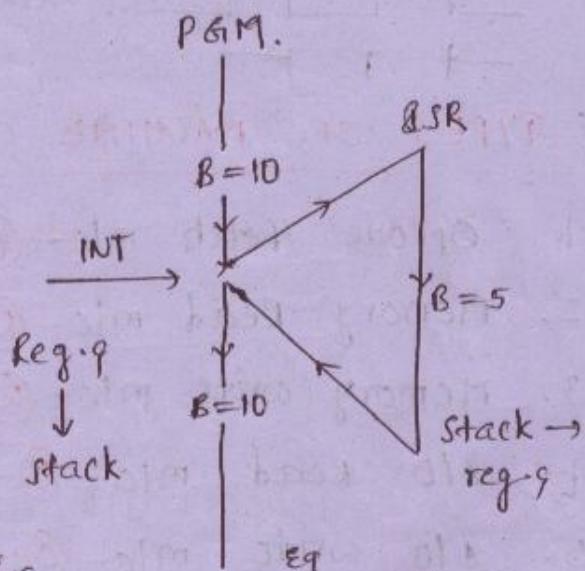
SP: stack pointer

LIFO:

Last in first out

(1). PUSH R

↑ reg. pair



PUSH B

PUSH D

PUSH H

decrement sp + push higher reg.

(2)

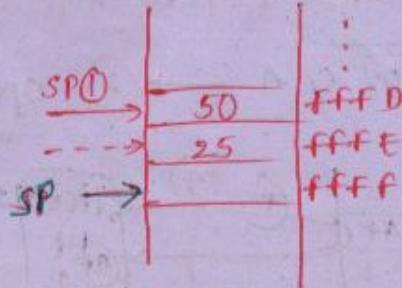
decrement sp + push lower reg.

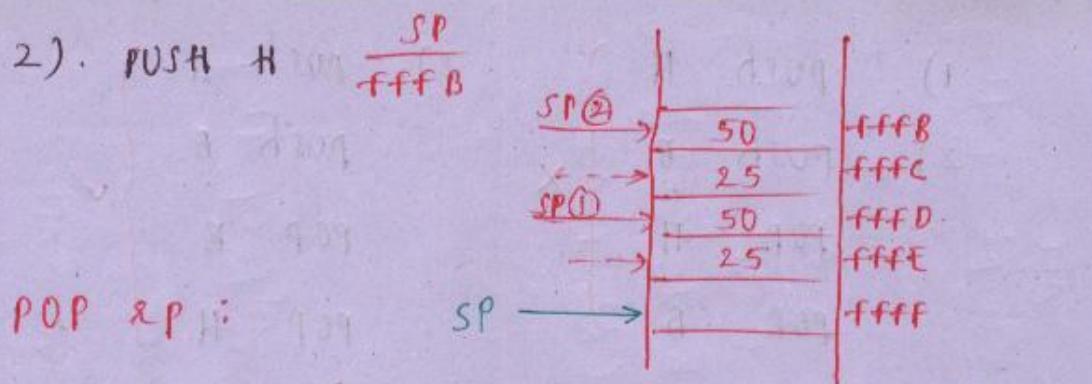
Eg: Let SP = ffff

HL = 2550

(3). PUSH H

$\frac{SP}{ffff}$

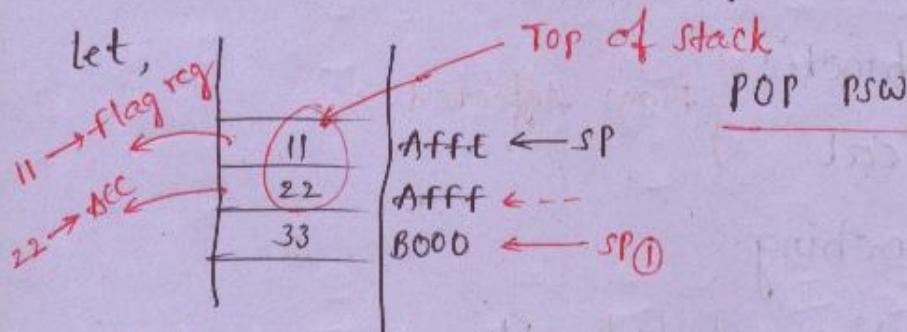




POP & P :

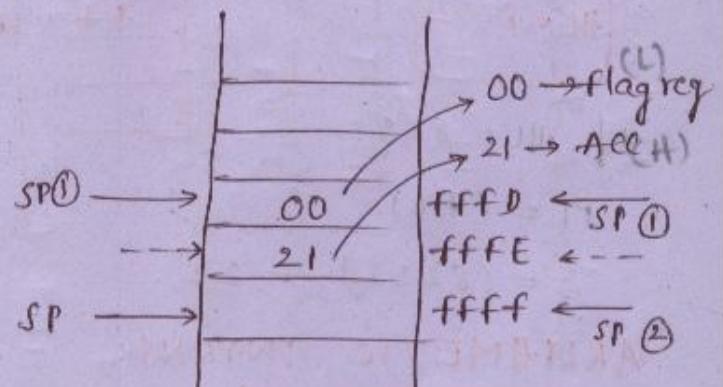
Get 1 Byte into lower reg + Increment sp

Get 1 byte into higher reg + Increment sp



Q what are the contents of Acc & flag reg. after executing following instructions.

- (i). SP = fffff (ii). Push H
 $\#L = 2100$ (iii). POP PSW



The above program is used to clear the flag register.

1). push H	X	2). push H
2) push B		push B
POP H		POP B
POP B		POP H

INSTRUCTIONS:

1. Data Transfer
2. Arithmetic } Flags Affected.
3. Logical }
4. Branching
5. Machine related, & I/O

6. Additional

RP
 BC = 8250
 HL = 8252

$$\text{if } HL = 8252$$

$$M = (HL)$$

$$= (8252) = 22$$

Memory

data	Addr.
11	8250 $(BC) = (8250)$
22	8251 = 11
33	8252 $(HL) = (8252)$
44	8253 = 33.
	M = (HL) = 33.

$$(BC) = (8250)$$

$$= 11$$

$$(HL) = (8252)$$

$$= 33.$$

$$M = (HL) = 33.$$

ARITHMETIC INTRNS:

MLC → get the instr + operation

X	1	+	0	←
ADD FF	1	+	1	
ADD FF	2	+	0	←

Instruction Operation Bytes/HLC/R Types of HLC Flags affected

1). ADD R \rightarrow B, C, D,
E, H, L, A.

ALL

F

1/1/4

A

ADD R

H

ALL

F

1/2/7

A

ADD 8bit data

A

ALL

F, R

2/2/7

SUB R

H

ALL

F, R

2/2/7

SUB 8bit data

A

ALL

F, R

2/2/7

ADC R

H

ALL

F, R

1/1/4

A

ADC R

H

ALL

F, R

1/2/7

A

ADC 8bit data

A

ALL

F, R

2/2/7

SBB R

H

ALL

F, R

1/1/4

A

SBB R

H

ALL

F, R

1/2/7

SBB R

A

ALL

F, R

1/2/7

A

SBB 8bit data

A

ALL

F, R

1/2/7

A

SBB 8bit data

A

ALL

F, R

1/2/7

A

SBB 8bit data

A

ALL

F, R

1/2/7

All
} except 'c' flag

1/14

$$R+1 \rightarrow R$$

5). INR DCR

$1/2/10$	f, R, ω	All
$1/2/10$	f, R, ω	except 'c' flag
$1/1/6$	$S = \text{opcode fetch mtc } (6+)$	
$1/1/6$	$B = \text{Bus idle mtc } (3N) \text{ flags}$	

$$(\#L)^{+1} \rightarrow (\#L)$$

INR M
DCR M

$$x^p + 1 \rightarrow x^p$$

NP

$$\frac{BC}{ANR} = \frac{8250}{6000}$$

$$BC = 8251$$

二〇

f, B, B only 'cy' flag.

1 / 3 / 10

十一

6) ADD UP

LOGICAL INSTRUCTIONS:

$$A \vee R \rightarrow A$$

DRAK

$$A \vee (\neg A) \rightarrow A$$

H
OKA

A + 8 bit data \rightarrow A

ORI 8bit data

Downloaded From www.gatenotes.in

2). ANA R

ANA H

~~ANL~~ \rightarrow A

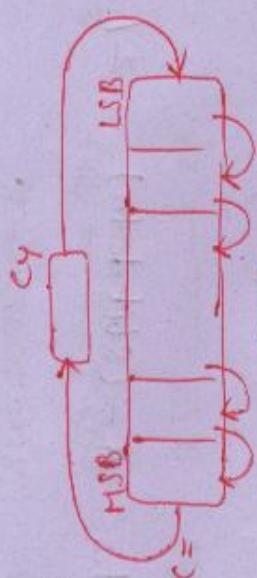
~~ANL~~ = ~~ANL~~
~~ANL~~ = ~~ANL~~
+ +

3). XRA R

XRA H

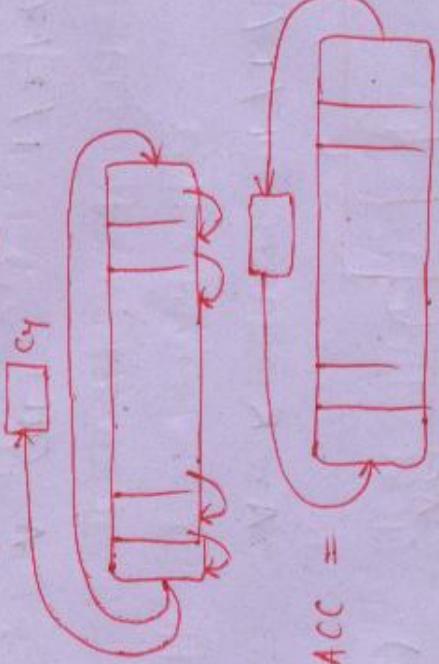
$A \oplus R \rightarrow A$

XRL validate



4). RAL
(with c_7)

RLC
(without c_7)

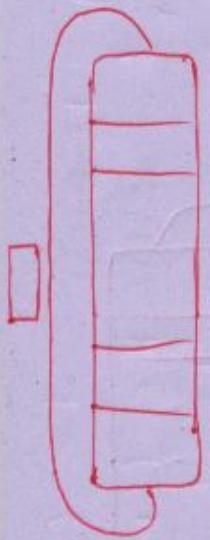


5). RAR
(with c_7)



Affected only
 c_7

RRC
(without c_y)



$A_{CC} =$

6).

CHP R

$$A - R \quad 1 / 1 / 4 \quad + \begin{cases} A > R \\ A < R \\ A = R \end{cases}$$

$c_y = 0, \quad Z = 0$

$c_y = 1, \quad Z = 0$

$c_y = 0, \quad Z = 1$

S, P, AC affected

CHP H

$A - (HL)$ $1 / 2 / \bar{4}$

$$+ \begin{cases} A > R \\ A < R \\ A = R \end{cases}$$

$c_y = 0, \quad Z = 0$

$c_y = 1, \quad Z = 1$

$$\text{cpd 8 bit data} \quad A - (8 \text{ bit data}) \quad 2 / 2 / \bar{4}$$

$c_y = 0, \quad Z = 0$

$$+ \begin{cases} A > R \\ A < R \\ A = R \end{cases}$$

$c_y = 0, \quad Z = 0$

$$A = \boxed{\text{AND OF}} \quad \begin{matrix} \text{Masking} \\ \text{A} \end{matrix}$$

$$A \rightarrow \begin{array}{r} 11110000 \\ 00001111 \\ \hline 00000010 \end{array}$$

$c_y = ?$

No flags

CMA

$\overline{A} \rightarrow A$

$c_y \rightarrow c_y$

only 'y'

STC

$c_y = 1$

only 'y'

DATA TRANSFER INSTR.S:

- MOV $R_d, R_s \rightarrow R_d$
MOV $R, M \quad (HL) \rightarrow R$
 - MOV $M, R \quad R \rightarrow (HL)$
MOV $M, 8\text{bit data} \quad 8\text{bit data} \rightarrow R$
 $MUL M, 8\text{bit data} \quad 8\text{bit data} \rightarrow (HL)$
 $MUL M, 16\text{bit data} \quad 16\text{bit data} \rightarrow RP$

$fR + \omega$
 \uparrow
 $2/3/10$
 $fR + \omega$
 \uparrow
 $3/3/0$
 $3/3/10$
 - LXI $RP, 16\text{bit data}$
(Load Immediate)
LDA $16\text{bit address} \quad (16\text{bit addr.}) \rightarrow A$
(Load Accumulator)
STA $16\text{bit address} \quad A \rightarrow (16\text{bit addr.})$
(Store Accumulator)
 - LDX $RP \quad (RP) \rightarrow A$
STAX $RP \quad A \rightarrow (RP)$

- Q). LHD 16 bit addr. (16 bit addr.) \rightarrow L
 (16 bit addr + 1) \rightarrow H
 \downarrow
 3 | 5 | 16
 $\frac{3+2}{\text{f} \text{R} \text{R}}$
- LHD 8252 ie (8252) \rightarrow L
 (8253) \rightarrow H
 \downarrow
 3 | 5 | 16
 $\frac{\text{f} \text{R} \text{R}}{\text{H} \text{W}}$
- SHLD 16 bit addr. L \rightarrow (16 bit addr.)
 H \rightarrow (16 bit addr. + 1)
 \downarrow
 3 | 5 | 16
 $\frac{\text{f} \text{R} \text{R}}{\text{H} \text{W}}$
- SHLD 8254,
 \downarrow
 (H) = 8090
- | | |
|----|------|
| 44 | 8253 |
| 90 | 8254 |
| 60 | 8255 |

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