Proof of the Collatz Conjecture

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Abstract

Take any positive integer N. If it is odd, multiply it by three and add one. If it is even, divide it by two. Repeatedly do the same operations to the results, forming a sequence. It is found that, whatever the initial number we choose, the sequence will eventually descend and reach number 1, where it enters a closed loop of 1- 4 - 2 - 1. This is known as the Collatz conjecture which states that the sequence always converges to 1. So far no proof has ever been found that this holds for every positive integer. In this paper we prove the Collatz conjecture by studying what happens as the sequence becomes infinitely long, thereby avoiding the chaos of finite length sequences. We have noted that the ratio between the number of odd operations and even operations continues to decrease as the sequence length increases, approaching zero for infinite sequence length. This leads to the only possibility that the sequence must be eventually decoupled from its staring value and enter a cycle, with the only possible cycle being the 1-4-2-1 cycle. We have obtained an equation for the final sequence of infinite length, which is the 1-4-2-1 closed loop:

$$C_n = \frac{\frac{3}{2^{Kn-2}} + 1}{2^{Kn-1}}$$

Introduction

The Collatz conjecture, originally proposed by Lothar Collatz in 1937[1], is a famous unsolved problem in number theory. It concerns the behavior of the iteration defined on positive integers by:

$$\mathrm{C(N)} = \begin{cases} 3\mathrm{N} + 1\,, & \text{if N is odd} \\ \dfrac{\mathrm{N}}{2}, & \text{if N is even} \end{cases}$$

The conjecture asserts that for every positive integer N, repeated iterations of C eventually reaches the number 1. Despite extensive computational evidence and numerous partial results, a full proof has remained illusive[2,3].

Several surveys and studies have explored the problem's rich structure and its generalizations (see, e.g. Lagarias[2]). This paper presents a novel approach that studies the properties of an infinite Collatz sequence, particularly the ratio between the number of even operations and odd operations.

Collatz Sequence of Odd Terms

Definition

Odd Collatz Sequence is an alternative representation containing only the odd terms of the sequence.

A term in a Collatz sequence can be even or odd. We start from the fact that even terms always lead to odd terms because of successive divisions by 2. Therefore, for any initial odd number N, the Collatz sequence can be re-defined as:

$$N \longrightarrow \frac{3N+1}{2^{K1}} \longrightarrow \frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}} \longrightarrow \frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}$$

$$\longrightarrow \frac{3\left(\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}\right)+1}{2^{K4}} \longrightarrow \frac{3\left(\frac{3\left(\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}\right)+1}{2^{K4}}\right)+1}{2^{K5}}$$

$$\longrightarrow \frac{3\left(\frac{3\left(\frac{3\left(\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}\right)+1}{2^{K4}}\right)+1}{2^{K5}}\right)+1}{2^{K6}} \longrightarrow \cdots$$

All terms of the above sequence are odd numbers because every time an even term occurs in the sequence it is successively divided by 2 until an odd term occurs. That is, when an even term occurs, we don't include it into the sequence but divide it by an integral power of 2 until we get an odd term, which is included into the sequence. Thus, with this we have created an alternative Collatz sequence as shown above with all terms odd.

By expanding each term we get the following:

$$\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}} = \frac{3^2N+3+2^{K1}}{2^{K1+K2}}$$

$$\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}} = \frac{3\left(\frac{3^2N+3+2^{K1}}{2^{K1+K2}}\right)+1}{2^{K3}} = \frac{3^3N+3^2+3*2^{K1}+2^{K1+K2}}{2^{K1+K2+K3}}$$

$$\frac{3\left(\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}\right)+1}{2^{K4}} = \frac{3\left(\frac{3^3N+3^2+3*2^{K1}+2^{K1+K2}}{2^{K1+K2+K3}}\right)+1}{2^{K4}}$$

$$= \frac{3^4N+3^3+3^2*2^{K1}+3*2^{K1+K2}+2^{K1+K2+K3}}{2^{K1+K2+K3+K4}}$$

$$\frac{3\left(\frac{3\left(\frac{3\left(\frac{3N+1}{2^{K1}}\right)+1}{2^{K2}}\right)+1}{2^{K3}}\right)+1}{2^{K4}}\right)+1}{2^{K5}} = \frac{3\left(\frac{3^4N+3^3+3^2*2^{K1}+3*2^{K1+K2}+2^{K1+K2+K3}}{2^{K1+K2+K3+K4}}\right)+1}{2^{K5}}$$

$$= \frac{3^5N + 3^4 + 3^3 * 2^{K1} + 3^2 * 2^{K1+K2} + 3 * 2^{K1+K2+K3} + 2^{K1+K2+K3+K4}}{2^{K1+K2+K3+K4+K5}}$$

and so on.

From the above, we can see that, if we take N to be the first term, then the n^{th} odd Collatz term (C_n) will be:

$$C_n = \frac{3^{n-1}N + 3^{n-2} + 3^{n-3} * 2^{K1} + \ldots + 3^2 * 2^{K1+K2+\ldots Kn-4} + 3 * 2^{K1+K2+\ldots Kn-3} + 2^{K1+K2+K3+\ldots+Kn-2}}{2^{K1+K2+K3+\ldots+Kn-1}} \quad \ldots (1)$$

C_n can be written as:

$$C_n = \frac{3^{n-1}N + 3^{n-2}}{2^{K1+K2+K3+\ldots+Kn-1}} + \frac{3^{n-3} * 2^{K1} + \ldots + 3^2 * 2^{K1+K2+\ldots Kn-4} + 3 * 2^{K1+K2+\ldots Kn-3} + 2^{K1+K2+K3+\ldots+Kn-2}}{2^{K1+K2+K3+\ldots+Kn-1}}$$

The number of even Collatz operations compared to the number of odd operations

Intuitively, the number of even operations is significantly greater than the number of odd operations, and increasingly so at very large or infinitely large numbers. This follows from the fact that Collatz operation on an odd number always results in an even number, whereas an even operation may result in several successive even operations before an odd number occurs again, and this effect is even more pronounced at very large numbers. After every odd operation, the next term is always an even term and the probability that this even term has large powers of two as a factor (e.g. 2^{1000}) indefinitely increases as the length of the sequence approaches infinity. The rigor of such proof should not be questioned (claiming that it is based on probability) because one can arbitrarily increase this probability, making it approach 1 in the limit, that is by assuming infinite length of the sequence.

Assumption 1

The ratio of the number of odd operations to the number of even operations continuously decreases with increase in length of the sequence and approaches zero for infinite length of the sequence. That is:

$$\frac{n}{K1 + K2 + K3 + \ldots} \quad \longrightarrow \quad 0 \quad \text{at infinity}$$

Where n is the total number of Collatz odd operations and K1+K2+K3+ . . . is the total number of Collatz even operations in a sequence

Assumption 2

This assumption concerns the terms in equation (1):

$$\frac{3^n}{2^{K1+K2+K3+\,.\,.\,.\,+Kn-1}} \qquad \longrightarrow \quad 0 \;\; as \quad n \;\; and \;\; K1 + K2 + K3 + \cdots \;\; \longrightarrow \infty$$

We postpone the proof of these assumptions. Later on we will show that Assumption 2 follows from Assumption 1. We can see that the first term is the coefficient of N. The convergence of these terms to zero at infinity (infinite length of the sequence) would reflect the fact that the number of even Collatz operations is significantly greater than the number of odd operations in a sequence, which intuitively means that the sequence always follows a general descent, eventually reaching the number 1.

Proof of the Collatz Conjecture

Theorem 1

All Collatz sequences converge to the trivial loop 1-4-2-1.

Proof

Using Assumption 2, we can see that the first terms of equation (1) diminish to zero for a sequence of infinite length.

$$C_n \;=\; \cancelto{0}{\frac{3^{n-1}N + 3^{n-2}}{2^{K1+K2+K3+\,...\,+Kn-1}}} \;+\; \frac{3^{n-3} * 2^{K1} + \,...\, +3^2 * 2^{K1+K2+\,...\,Kn-4} +3 * 2^{K1+K2+\,...\,Kn-3} + 2^{K1+K2+K3+\,...\,+Kn-2}}{2^{K1+K2+K3+\,...\,+Kn-1}}$$

$$C_n \;\rightarrow\; \frac{3^{n-3} * 2^{K1} + 3^{n-4} * 2^{K1+K2} + \,...\, +3^2 * 2^{K1+K2+\,...\,Kn-4} +3 * 2^{K1+K2+\,...\,Kn-3} + 2^{K1+K2+K3+\,...\,+Kn-2}}{2^{K1+K2+K3+\,...\,+Kn-1}}$$

We now factor out the term 2^{K1} ,

$$C_n \;\rightarrow\; \frac{2^{K1}\,(3^{n-3} + 3^{n-4} * 2^{K2} + 3^{n-5} * 2^{K2+K3} +\,...\, +3^2 * 2^{K2+\,...\,Kn-4} +3 * 2^{K2+\,...\,Kn-3} + 2^{K2+K3+\,...\,+Kn-2})}{2^{K1+K2+K3+\,...\,+Kn-1}}$$

Cancelling the 2^{K1} term from both the numerator and the denominator,

$$C_n \;\rightarrow\; \frac{(3^{n-3} + 3^{n-4} * 2^{K2} + 3^{n-5} * 2^{K2+K3} +\,...\, +3^2 * 2^{K2+\,...\,Kn-4} +3 * 2^{K2+\,...\,Kn-3} + 2^{K2+K3+\,...\,+Kn-2})}{2^{K2+K3+\,...\,+Kn-1}}$$

This can be re-written as:

$$C_n \;\longrightarrow\; \frac{(3^{n-3})}{2^{K2+K3+\,...\,+Kn-1}} + \frac{(3^{n-4} * 2^{K2} + 3^{n-5} * 2^{K2+K3} + ... + 3^2 * 2^{K2+\,...\,Kn-4} + 3 * 2^{K2+\,...\,Kn-3} + 2^{K2+K3+\,...\,+Kn-2})}{2^{K2+K3+\,...\,+Kn-1}}$$

Again, as n and K2+K3+ … +Kn-1 approach infinity, the first term diminishes to zero:

$$\frac{(3^{n-3})}{2^{K2+K3+\,...\,+Kn-1}} \;\longrightarrow\; 0 \qquad \left(\textit{Note that this follows from } \frac{3^n}{2^{K1+K2+K3+\,...\,+Kn-1}} \;\longrightarrow\; 0 \right)$$

Therefore:

$$C_n \;\longrightarrow\; \cancelto{0}{\frac{(3^{n-3})}{2^{K2+K3+\,...\,+Kn-1}}} + \frac{(3^{n-4} * 2^{K2} + 3^{n-5} * 2^{K2+K3} + ... + 3^2 * 2^{K2+\,...\,Kn-4} + 3 * 2^{K2+\,...\,Kn-3} + 2^{K2+K3+\,...\,+Kn-2})}{2^{K2+K3+\,...\,+Kn-1}}$$

$$C_n \;\longrightarrow\; \frac{(3^{n-4} * 2^{K2} + 3^{n-5} * 2^{K2+K3} + ... + 3^2 * 2^{K2+\,...\,Kn-4} + 3 * 2^{K2+\,...\,Kn-3} + 2^{K2+K3+\,...\,+Kn-2})}{2^{K2+K3+\,...\,+Kn-1}}$$

Again, factoring out 2^{K2} ,

$$C_n \;\longrightarrow\; \frac{2^{K2}(3^{n-4} + 3^{n-5} * 2^{K3} + 3^{n-6} * 2^{K3+K4} + ... + 3^2 * 2^{K3+\,...\,Kn-4} + 3 * 2^{K3+\,...\,Kn-3} + 2^{K3+K4+\,...\,+Kn-2})}{2^{K2+K3+\,...\,+Kn-1}}$$

and cancelling the 2^{K2} term from both the numerator and the denominator,

$$C_n \;\longrightarrow\; \frac{(3^{n-4} + 3^{n-5} * 2^{K3} + 3^{n-6} * 2^{K3+K4} + ... + 3^2 * 2^{K3+\,...\,Kn-4} + 3 * 2^{K3+\,...\,Kn-3} + 2^{K3+K4+\,...\,+Kn-2})}{2^{K3+\,...\,+Kn-1}}$$

Again, this can be re-written as:

$$C_n \longrightarrow \frac{3^{n-4}}{2^{K3+\ ...\ +Kn-1}} + \frac{(3^{n-5} * 2^{K3} + 3^{n-6} * 2^{K3+K4}\ +...\ +3^2 * 2^{K3+\ ...\ Kn-4}\ +3 * 2^{K3+\ ...\ Kn-3} + 2^{K3+K4+\ ...\ +Kn-2})}{2^{K3+\ ...\ +Kn-1}}$$

Again applying our assumption about convergence to zero at infinity, that is as *n* and K2+K3+ … +Kn-1 approach infinity, the first term diminishes to zero.

$$C_n \longrightarrow \cancelto{0}{\frac{3^{n-4}}{2^{K3+\ ...\ +Kn-1}}} + \frac{(3^{n-5} * 2^{K3} + 3^{n-6} * 2^{K3+K4}\ +...\ +3^2 * 2^{K3+\ ...\ Kn-4}\ +3 * 2^{K3+\ ...\ Kn-3} + 2^{K3+K4+\ ...\ +Kn-2})}{2^{K3+\ ...\ +Kn-1}}$$

$$C_n \longrightarrow \frac{(3^{n-5} * 2^{K3} + 3^{n-6} * 2^{K3+K4}\ +...\ +3^2 * 2^{K3+\ ...\ Kn-4}\ +3 * 2^{K3+\ ...\ Kn-3} + 2^{K3+K4+\ ...\ +Kn-2})}{2^{K3+\ ...\ +Kn-1}}$$

Next we factor out 2^{K3} and repeat the above procedure, then factor out 2^{K4} , and so on.

It can be shown that eventually we get:

$$C_n \longrightarrow \frac{3}{2^{Kn-2+Kn-1}} + \frac{2^{Kn-2}}{2^{Kn-2+Kn-1}}$$

$$C_n \longrightarrow \frac{3}{2^{Kn-2+Kn-1}} + \frac{1}{2^{Kn-1}}$$

$$C_n \longrightarrow \frac{1}{2^{Kn-1}}\left(\frac{3}{2^{Kn-2}} + 1\right) = \frac{\frac{3}{2^{Kn-2}} + 1}{2^{Kn-1}}$$

Now, since C_n can only be a whole number, then the term:

$$\left(\frac{3}{2^{Kn-2}} + 1\right)$$

must be some factor of 2. The only possible case is if 2^{Kn-2} is equal to 1. That is:

$$2^{Kn-2} = 1$$

Therefore,

$$\left(\frac{3}{2^{Kn-2}} + 1 \right) = \frac{3}{1} + 1 = 4$$

Therefore:

$$C_n \longrightarrow \frac{\frac{3}{2^{Kn-2}} + 1}{2^{Kn-1}} = \frac{4}{2^{Kn-1}}$$

Since C_n can only be a whole number, only three possible values are possible for C_n .

$$2^{Kn-1} = 1 \implies C_n = 4$$

$$2^{Kn-1} = 2 \implies C_n = 2$$

$$2^{Kn-1} = 4 \implies C_n = 1$$

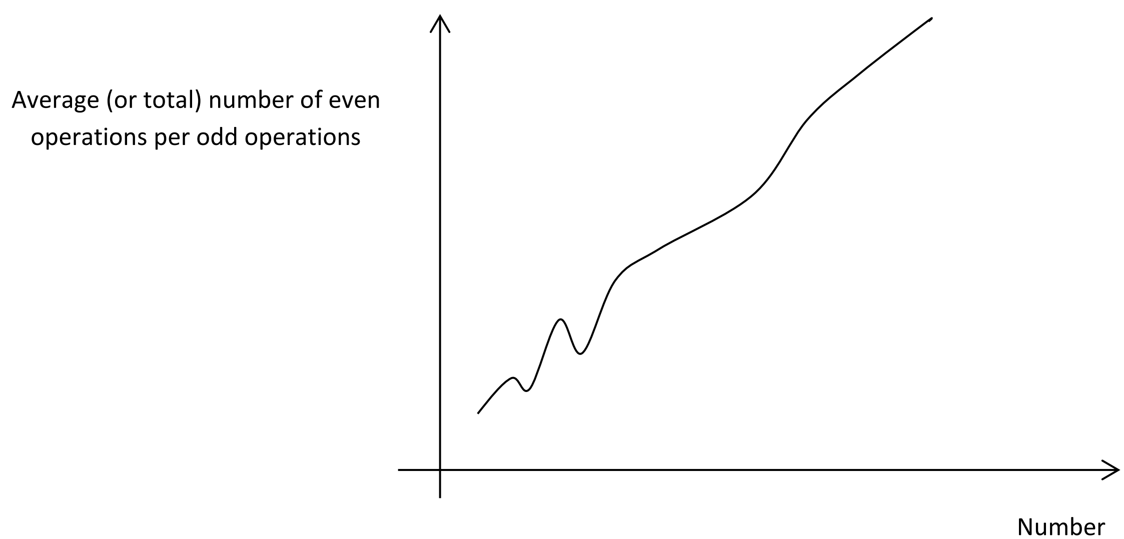
Thus we have proved that all Collatz sequences eventually converge to the 1-4-2-1 loop.

Proof of convergence to zero at infinity

So far, we have reduced the problem of the Collatz conjecture to a problem of proving that those terms assuming as converging to zero at infinity indeed converge to zero. Next, I present the proof of convergence to zero of the terms we have assumed as converging to zero at infinity, and perhaps the rigor of the proof is the only possible one for the Collatz conjecture.

We start from the fact that the total number of even Collatz operations in a given Collatz sequence is significantly greater than the number of total odd Collatz operations. And this effect indefinitely increases for infinitely large numbers and infinitely long sequences. That is, if one starts with a very, very large number, the total number of even operations will be very large compared to the total number of odd operations, resulting in steep descent of the sequence. In fact, the steepness of the general descent indefinitely increases as the starting value approaches infinity. The steepness of the general descent will gradually decrease as the sequence descents towards finitely large numbers and the descent of the sequence becomes less and less pronounced at relatively small numbers, but eventually reaching 1 in all cases.

Why does the ratio of the number (average or total) of even operations to the number of odd operations increase vastly if the sequence diverged to infinity? The reason for this is that at infinitely large numbers, unlike at smaller numbers, even integers can have very large powers of two as a factor, say 2^{10} , 2^{100} , 2^{1000} etc. Thus, since every Collatz operation on an odd number results in an even number, once the sequence lands on an even number it may go through tens or hundreds or thousands of successive even operations before ending in an odd number, that is before another odd operation occurs again, with successive divide by 2 operations, vastly dropping the sequence. There can never be successive odd operations, but there can always be successive even operations. Therefore, although every odd operation results in a term greater than the term before it, that operation always results in an even operation, which may vastly drop the sequence, much more than it was lifted by the single odd operation. In other words, at relatively small numbers, an even number can have 2, 2^2 , 2^3 , 2^4 , etc. as a factor and at very large numbers this could be all the way from 2, 2^2 , . . . to 2^{100} and still at immensely large numbers 2, 2^2 , . . . to 2^{10000} and so on , following the same trend indefinitely. Therefore, as the starting odd number increases, the average and total ratio of even to odd operations also increases accordingly. Thus, after just one odd operation, there could be thousands or billions of successive even operations before the next odd operation occurs, always resulting in a steep average descent of the sequence. As the length of the sequence increases towards infinity, the ratio between the total number of odd operations and total number of even operations continuously tends towards zero. This results from the fact that as the starting value increases more and more even operations occur *per every odd operation*.



Note that the above graph is qualitative, meant only to explain the concept.

Proof of Assumption 2

From Assumption 1 :

$$\frac{n}{K1 + K2 + K3 + \dots} \quad \longrightarrow \quad 0 \quad \text{at infinity}$$

Let

$$\frac{n}{K1 + K2 + K3 + \dots} = r$$

$$\Longrightarrow \; n = r \; (K1 + K2 + K3 + \cdots)$$

Therefore,

$$\frac{3^{n}}{2^{K1+K2+K3+\;.\;.\;.\;+Kn-1}} = \frac{3^{r\;(K1+K2+K3+\cdots)}}{2^{K1+K2+K3+\;.\;.\;.\;+Kn-1}} = (\frac{3^{r}}{2})^{(K1+K2+K3+\cdots)}$$

Now since *r* approaches 0 at infinity (Assumption 1), *r* = 0 at infinity:

$$(\frac{3^{r}}{2})^{(K1+K2+K3+\cdots)} = (\frac{3^{0}}{2})^{(K1+K2+K3+\cdots)} = (\frac{1}{2})^{(K1+K2+K3+\cdots)} = 0 \qquad at\ infinity\ , that\ is\ for\ K1 + K2 + K3 + \;.\;.\;. \rightarrow \infty$$

Proof of Assumption 1

Lemma 1

Let n_1 and n_2 be any two positive integers, with $n_2 > n_1$. Let $n_1 = m_1 \times 2^{k1}$ and $n_2 = m_2 \times 2^{k2}$, where m_1 and m_2 are odd numbers. The probability that $k_2 > k_1$ is always than the probability that $k_1 > k_2$. In other words, the probability that a larger even number has a given power (k) of two is greater than the probability that a smaller even number has the same power of two.

Lemma 2 follows from the fact that the probability that an even number $n = m * 2^k$ having greater *k* increases as *n* increases. This follows from the fact that larger numbers can have higher factors of 2 than smaller numbers. For example, even numbers up to 100 can have 2^1 , 2^2 , 2^3 , 2^4 , 2^5 , 2^6 as factors, whereas even numbers up to 1000 can have 2^1 , 2^2 , 2^3 , 2^4 , 2^5 , 2^6 , 2^7 , 2^8 , 2^9 , as factors, and even numbers up to 1,000,000 can have 2^1 , 2^2 , 2^3 , 2^4 , 2^5 , 2^6 , 2^7 , 2^8 , 2^9 , 2^{10} , 2^{11} , 2^{12} , 2^{13} , 2^{14} , 2^{15} , 2^{16} , 2^{17} , 2^{18} , 2^{19} and even numbers up to 1,000,000,000 can have 2^1 , 2^2 , 2^3 , 2^4 , 2^5 , 2^6 , 2^7 , 2^8 , 2^9 , 2^{10} , 2^{11} , 2^{12} , 2^{13} , 2^{14} , 2^{15} , 2^{16} , 2^{17} , 2^{18} , 2^{19} , 2^{20} , 2^{21} , 2^{22} , 2^{23} , 2^{24}, 2^{25} , 2^{26} , 2^{27} , 2^{28} , 2^{29} and so on.

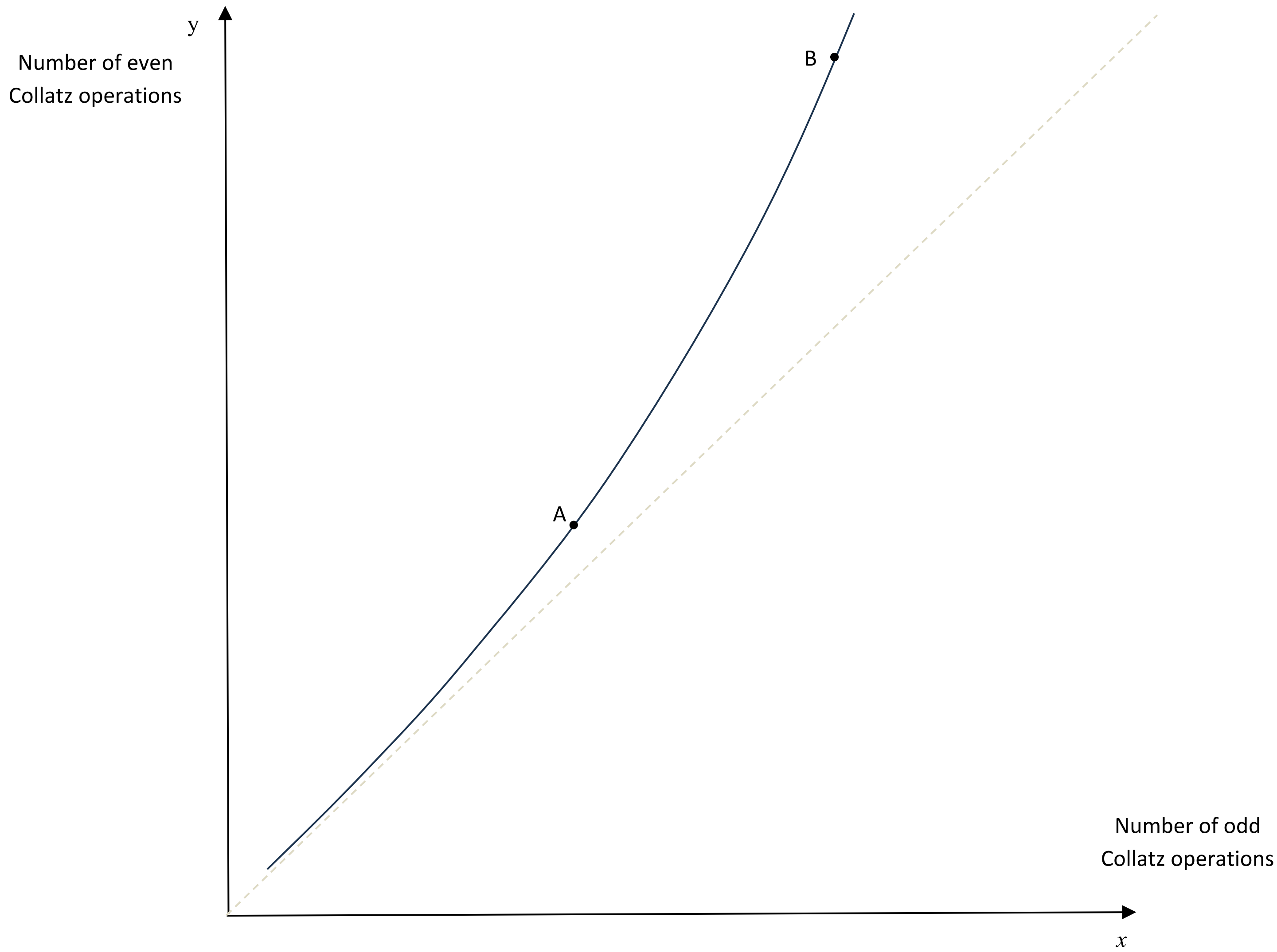
This clearly proves that the probability that a bigger number has a given powers of 2 is greater than the probability that a smaller number has the same power of 2. It follows that the number of even operations after every odd operation increases with increase of the starting value, and increases towards infinity as the starting number goes to infinity.

The next graph illustrates this effect. We can see that the slope of the graph continues to increase as the number of Collatz operations increases. We can see that not only the local slope *dy/dx*, but also *y/x* , increases with the number of operations.

We can see that the ratio of the total number of even operations to the total number of odd operations is greater at point B than at point A on the curve, indicating a continuous increase of this ratio with increase in the number of Collatz operations (or sequence length), the ratio approaching infinity for infinite sequence length and, conversely, the ratio of odd to even operations approaching zero.

$$\frac{Number\ of\ even\ Collatz\ operations}{Number\ of\ odd\ Collatz\ operations} \simeq \infty \quad (for\ infinite\ length\ of\ sequence)$$

$$\Rightarrow \frac{Number\ of\ odd\ Collatz\ operations}{Number\ of\ even\ Collatz\ operations} \simeq 0 \quad (for\ infinite\ length\ of\ sequence)$$


y
Number of even
Collatz operations
B
A
Number of odd
Collatz operations
x

Now, there are two possibilities for the Collatz conjecture to be disproved.

1. The sequence indefinitely diverges to infinity.
2. The sequence may enter some closed loop (there might be more than one loop.

Since we have already proved that the Collatz sequence cannot diverge to infinity, next we consider the possibility of another closed loop other than the trivial loop.

Proof that no closed loops exist other than the 1-4-2-1 loop

Corollary 1

There are no closed loops of the Collatz sequence other than the trivial loop of 1-4-2-1.

We propose a property of a closed loop as follows: *any closed loop is decoupled from the sequences and the numbers in the sequences leading to it*. Since multiple Collatz sequences lead to the closed loop, the closed loop must be independent from any Collatz sequence (and its terms) leading to that closed loop. This necessitates the vanishing of the coefficient of N at infinity, in the following equation:

$$C_n = \frac{3^{n-1}N + 3^{n-2}}{2^{K1+K2+K3+\,...\,+Kn-1}} + \frac{3^{n-3} * 2^{K1} + ... +3^2 * 2^{K1+K2+\,...\,Kn-4} +3 * 2^{K1+K2+\,...\,Kn-3} + 2^{K1+K2+K3+\,...\,+Kn-2}}{2^{K1+K2+K3+\,...\,+Kn-1}}$$

Note that the coefficient of N here is:

$$\frac{3^{n-1}}{2^{K1+K2+K3+\,...\,+Kn-1}}$$

which in turn brings us back to our previous assumption about the convergence of terms to zero at infinity, which in turn leads to the 1-4-2-1 loop! So we began with an assumption that another closed loop could exist somewhere in the darkness of infinity, which demanded the vanishing of the coefficient of N, which led back to the only known closed loop of 1-4-2-1. This should be a complete mathematical proof (by induction) that no other closed loop exists.

In other words, assume that a closed loop exists at some immensely large number. The idea presented here is that any closed loop requires that the coefficient of N in the above equation should diminish to zero in the limit. Assume that the sequence is not decoupled from the starting number even at infinity, that is for infinite length of the sequence. By contradiction, there cannot be such a closed loop because C_n cannot be shown to converge to constant numbers (possible terms in the supposed closed loop), in the same way that we have proved that every Collatz sequence converges to the number 4, 2 or 1. In summary, any closed loop requires the coefficient of N to diminish to zero at infinity, but then this leads to the 1-4-2-1 loop. This proves that no other closed loop can exist.

Conclusion

In this paper, we have been able to prove the Collatz sequence, by studying the property of the sequence of infinite length, thereby avoiding the chaos of the finite length sequences. We noted that at infinite lengths, the ratio of odd operations to even operations diminishes, approaching zero. This leads to the eventual decoupling of the sequence from the starting value, ending in the only possible path of all sequences: the 1-4-2-1 loop.

Glory Be To Almighty God Jesus Christ and His Mother, Our Lady Saint Virgin Mary

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