Models

2025-04-28

```r
library(tidyverse)
library(janitor)
library(rstan)
library(rstanarm)
library(bayesplot)
library(MCMCpack)
library(lme4)
student_data <- read.csv("student-scores.csv");
clean_data <- read.csv("student-scores-clean.csv")
head(student_data)
```

```
##   id first_name last_name                                 email gender
## 1  1       Paul     Casey        paul.casey.1@gslingacademy.com   male
## 2  2   Danielle  Sandoval danielle.sandoval.2@gslingacademy.com female
## 3  3       Tina   Andrews      tina.andrews.3@gslingacademy.com female
## 4  4       Tara     Clark        tara.clark.4@gslingacademy.com female
## 5  5    Anthony    Campos    anthony.campos.5@gslingacademy.com   male
## 6  6      Kelly      Wade        kelly.wade.6@gslingacademy.com female
##   part_time_job absence_days extracurricular_activities weekly_self_study_hours
## 1         False            3                      False                      27
## 2         False            2                      False                      47
## 3         False            9                       True                      13
## 4         False            5                      False                       3
## 5         False            5                      False                      10
## 6         False            2                      False                      26
##    career_aspiration math_score history_score physics_score chemistry_score
## 1             Lawyer         73            81            93              97
## 2             Doctor         90            86            96             100
## 3 Government Officer         81            97            95              96
## 4             Artist         71            74            88              80
## 5            Unknown         84            77            65              65
## 6            Unknown         93           100            67              78
##   biology_score english_score geography_score
## 1            63            80              87
## 2            90            88              90
## 3            65            77              94
## 4            89            63              86
## 5            80            74              76
## 6            72            80              84
```

```r
head(clean_data)
```

```
##   id first_name last_name                                 email gender
## 1  1       Paul     Casey        paul.casey.1@gslingacademy.com      1
## 2  2   Danielle  Sandoval danielle.sandoval.2@gslingacademy.com      0
## 3  3       Tina   Andrews      tina.andrews.3@gslingacademy.com      0
```

```
## 4  4       Tara     Clark        tara.clark.4@gslingacademy.com      0
## 5  5    Anthony    Campos    anthony.campos.5@gslingacademy.com      1
## 6  6      Kelly      Wade        kelly.wade.6@gslingacademy.com      0
##   part_time_job absence_days extracurricular_activities weekly_self_study_hours
## 1             0            3                          0                      27
## 2             0            2                          0                      47
## 3             0            9                          1                      13
## 4             0            5                          0                       3
## 5             0            5                          0                      10
## 6             0            2                          0                      26
##    career_aspiration math_score history_score physics_score chemistry_score
## 1             Lawyer         73            81            93              97
## 2             Doctor         90            86            96             100
## 3 Government Officer         81            97            95              96
## 4             Artist         71            74            88              80
## 5            Unknown         84            77            65              65
## 6            Unknown         93           100            67              78
##   biology_score english_score geography_score average_score
## 1            63            80              87         82.00
## 2            90            88              90         91.43
## 3            65            77              94         86.43
## 4            89            63              86         78.71
## 5            80            74              76         74.43
## 6            72            80              84         82.00
```

$Y_i|B_0, B_1, \sigma^2 \sim N(\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+\beta_3 x_{3i}+\beta_4 x_{4i}, \sigma^2)$ where: $x_{1i}, x_{2i}, x_{3i}, x_{4i}$are the predictors for observation i

$$\beta_j \sim N(\mu, \tau^2) \text{ where j=}0, 1, 2, 3, 4$$

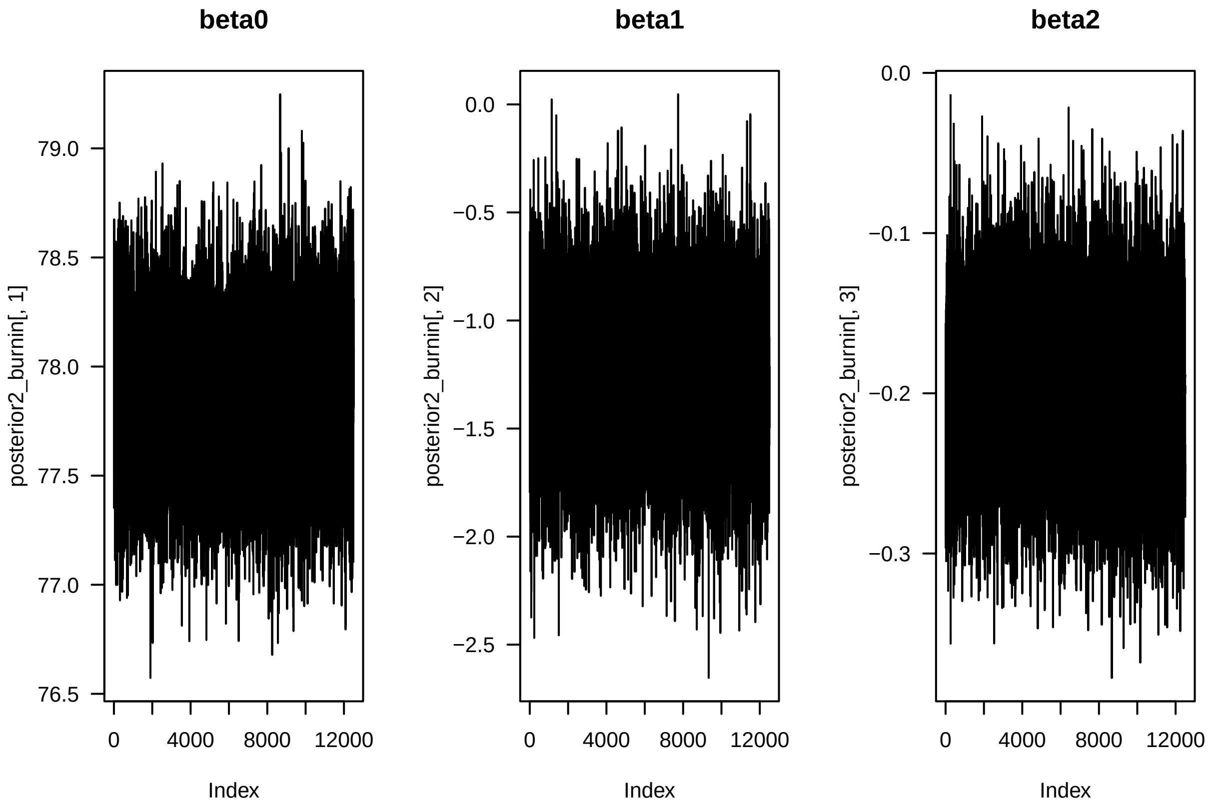
$$\sigma^2 \sim InvGamma(\alpha_1, \alpha_2)$$

```r
#Block Gibbs Sampler
set.seed(4889)
clean_data <- read.csv("student-scores-clean.csv")


set.seed(8451)
y <- clean_data$average_score
x1 <- clean_data$part_time_job
x2 <- clean_data$absence_days
x3 <- clean_data$extracurricular_activities
x4 <- clean_data$weekly_self_study_hours


# Design matrix
X <- cbind(1, x1, x2, x3, x4)
n <- length(y)
p <- ncol(X)

# Hyperparameters
tau2 <- 10000^2
```

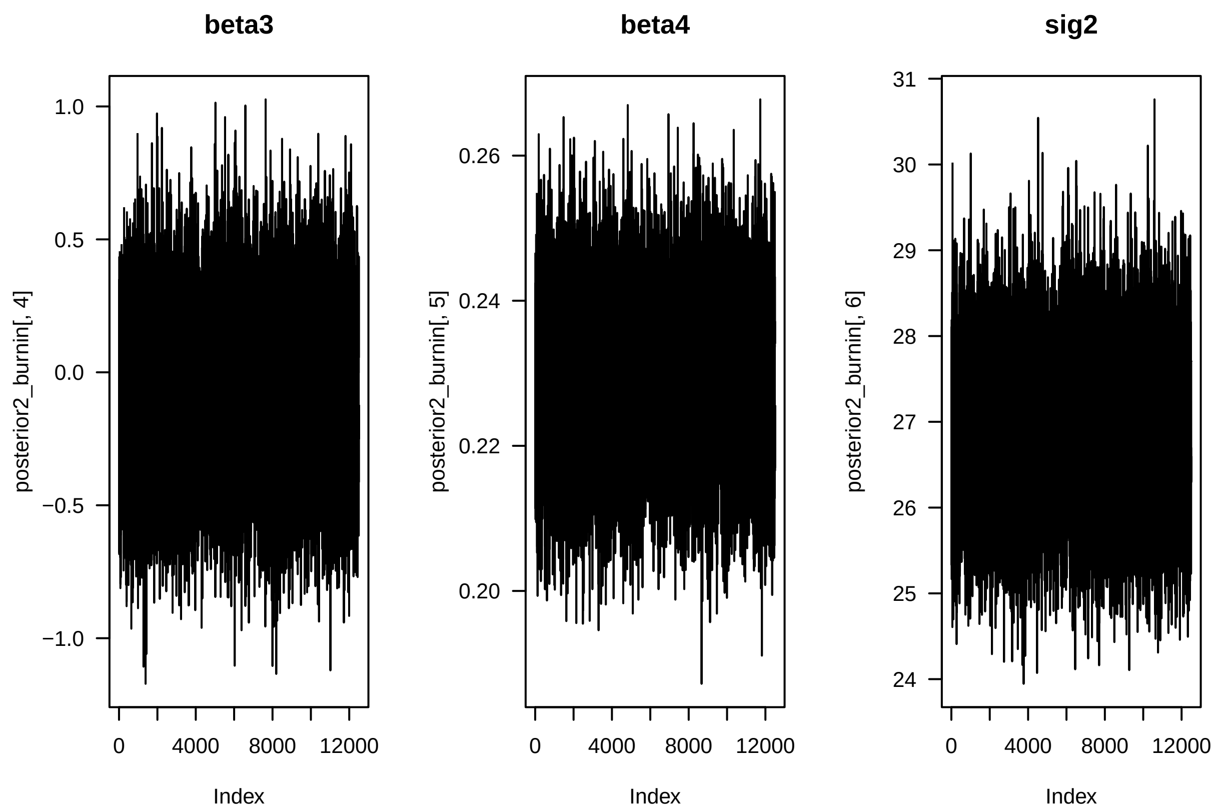
```r
a <- b <- 1
mu0 <- rep(0, p)

S <- 2.5e4

#place to store data
posterior_beta <- matrix(NA, S, p)
posterior_sig2 <- rep(NA, S)

beta <- rep(0, p)
sig2 <- 1

XX <- t(X) %*% X
Xy <- t(X) %*% y

# block Gibbs sampler
for (s in 1:S) {

  # Update beta0
  v <- solve(XX / sig2 + diag(rep(1/tau2, p)))
  m <- v %*% (Xy / sig2 + mu0 / tau2)
  beta <- m + t(chol(v)) %*% rnorm(p)

  # Update sig2 (variance)
  sig2 <- rinvgamma(1, a + n/2,
                    b + t(y - X %*% beta)%*%(y-X%*%beta) / 2)

  # Store results
  posterior_beta[s, ] <- beta
  posterior_sig2[s] <- sig2
}

posterior2 <- cbind(posterior_beta, posterior_sig2)
colnames(posterior2) <- c("beta0", "beta1", "beta2",
                          "beta3", "beta4", "sigma")

#remove burn-in
posterior2_burnin <- posterior2[1:round(s/2),]

head(posterior2_burnin)
```

```
##         beta0     beta1      beta2      beta3     beta4    sigma
## [1,] 77.87654 -1.281166 -0.2003770 -0.1229279 0.2255845 27.69401
## [2,] 77.90033 -0.589276 -0.2092001  0.1524083 0.2279868 25.71107
## [3,] 78.04359 -1.461563 -0.2041925 -0.2506536 0.2197806 26.16819
## [4,] 77.76850 -1.559494 -0.2369275 -0.0571564 0.2424850 27.25814
## [5,] 78.24938 -1.271030 -0.2965047 -0.1193520 0.2139665 27.47846
## [6,] 78.64778 -1.082733 -0.2756962 -0.6835690 0.2114292 27.02233
```

```r
# Block Gibbs Sampler Trace plots
par(mfrow=c(1,3))
plot(posterior2_burnin[,1], type="l", las=1, main="beta0")
plot(posterior2_burnin[,2], type="l", las=1, main="beta1")
plot(posterior2_burnin[,3], type="l", las=1, main="beta2")
```




```r
plot(posterior2_burnin[,4], type="l", las=1, main="beta3")
plot(posterior2_burnin[,5], type="l", las=1, main="beta4")
plot(posterior2_burnin[,6], type="l", las=1, main="sig2")
```




```r
# fit model in rstanarm
grades_lmer <- stan_lmer(average_score ~ part_time_job +
                           absence_days + extracurricular_activities +
                           weekly_self_study_hours + career_aspiration + (1|gender),
                       data = clean_data)
```

```
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.000251 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 2.51 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
```

```
## Chain 1:
## Chain 1:  Elapsed Time: 26.162 seconds (Warm-up)
## Chain 1:                15.196 seconds (Sampling)
## Chain 1:                41.358 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.000135 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 1.35 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 23.168 seconds (Warm-up)
## Chain 2:                7.954 seconds (Sampling)
## Chain 2:                31.122 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.00014 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 1.4 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 16.776 seconds (Warm-up)
## Chain 3:                24.946 seconds (Sampling)
## Chain 3:                41.722 seconds (Total)
```

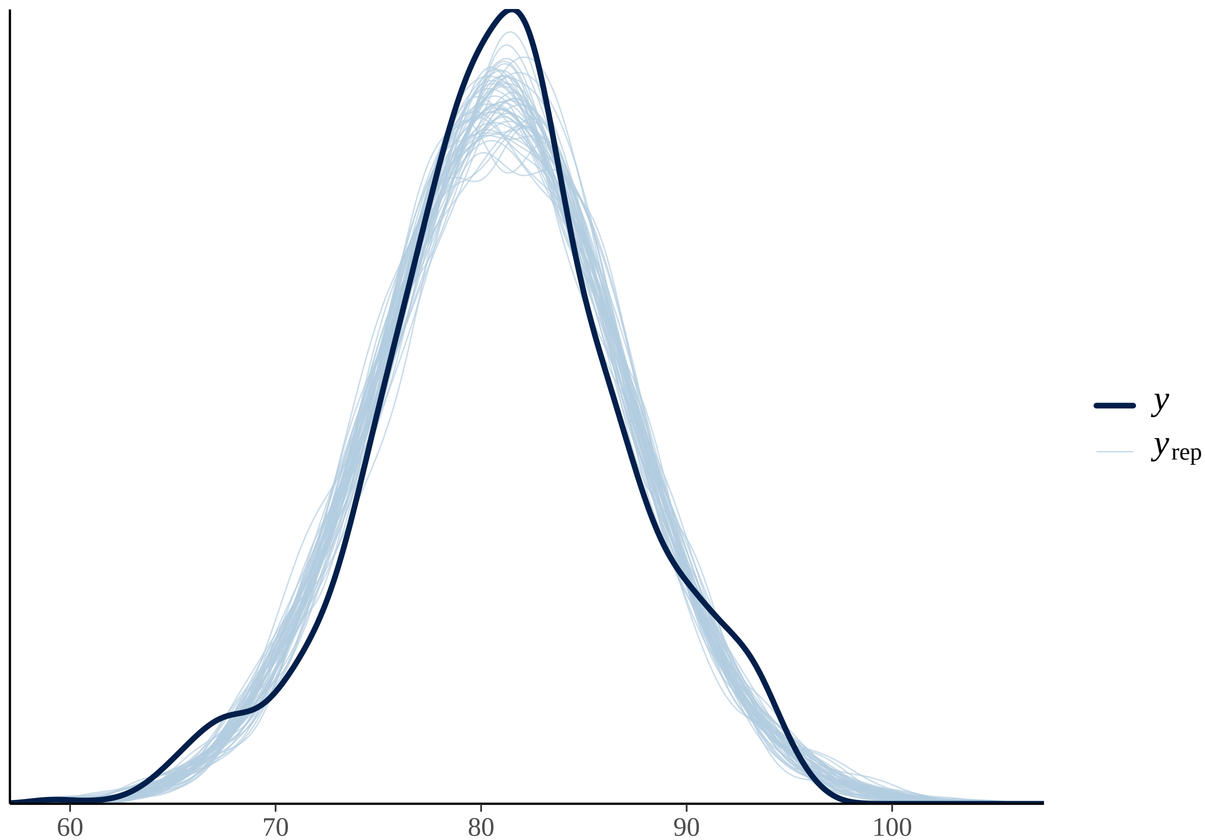
```
## Chain 3:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.000146 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 1.46 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 23.785 seconds (Warm-up)
## Chain 4:                18.383 seconds (Sampling)
## Chain 4:                42.168 seconds (Total)
## Chain 4:
```

```
# show results
summary(grades_lmer, digits = 3)
```

```
##
## Model Info:
##  function:     stan_lmer
##  family:       gaussian [identity]
##  formula:      average_score ~ part_time_job + absence_days + extracurricular_activities +
##     weekly_self_study_hours + career_aspiration + (1 | gender)
##  algorithm:    sampling
##  sample:       4000 (posterior sample size)
##  priors:       see help('prior_summary')
##  observations: 2000
##  groups:       gender (2)
##
## Estimates:
##                                         mean   sd     10%    50%    90%
## (Intercept)                           76.155  1.354 74.839 76.126 77.534
## part_time_job                         -0.138  0.322 -0.553 -0.142  0.275
## absence_days                           0.052  0.048 -0.009  0.051  0.114
## extracurricular_activities            -0.092  0.262 -0.426 -0.086  0.245
## weekly_self_study_hours                0.134  0.016  0.113  0.133  0.155
## career_aspirationArtist                4.077  0.775  3.077  4.075  5.050
## career_aspirationBanker                2.689  0.557  1.980  2.691  3.399
## career_aspirationBusiness Owner       -2.229  0.625 -3.033 -2.235 -1.410
## career_aspirationConstruction Engineer  4.424  0.703  3.525  4.423  5.297
## career_aspirationDesigner              4.029  0.752  3.069  4.025  4.965
## career_aspirationDoctor                8.694  0.641  7.851  8.702  9.530
```

```
## career_aspirationGame Developer         5.147  0.807  4.118  5.141  6.200
## career_aspirationGovernment Officer     3.465  0.753  2.499  3.476  4.409
## career_aspirationLawyer                 3.948  0.600  3.177  3.961  4.723
## career_aspirationReal Estate Developer  2.479  0.729  1.539  2.476  3.397
## career_aspirationScientist              5.900  0.886  4.773  5.920  7.039
## career_aspirationSoftware Engineer      2.732  0.495  2.081  2.739  3.352
## career_aspirationStock Investor         2.530  0.717  1.628  2.542  3.441
## career_aspirationTeacher                2.543  0.763  1.567  2.555  3.537
## career_aspirationUnknown                1.167  0.526  0.481  1.174  1.844
## career_aspirationWriter                 4.587  0.945  3.357  4.579  5.812
## b[(Intercept) gender:0]                 0.081  1.227 -0.981  0.082  1.138
## b[(Intercept) gender:1]                -0.205  1.233 -1.320 -0.109  0.809
## sigma                                   4.739  0.074  4.644  4.737  4.834
## Sigma[gender:(Intercept),(Intercept)]   4.357 11.847  0.018  0.570 11.062
##
## Fit Diagnostics:
##            mean   sd     10%    50%    90%
## mean_PPD 80.979  0.145 80.793 80.981 81.163
##
## The mean_ppd is the sample average posterior predictive distribution of the outcome variable (for de
##
## MCMC diagnostics
##                                        mcse  Rhat  n_eff
## (Intercept)                            0.069 1.014  386
## part_time_job                          0.005 1.001 4064
## absence_days                           0.001 1.000 3728
## extracurricular_activities             0.004 1.001 3857
## weekly_self_study_hours                0.000 1.002 1716
## career_aspirationArtist                0.021 1.002 1412
## career_aspirationBanker                0.018 1.003  968
## career_aspirationBusiness Owner        0.021 1.004  902
## career_aspirationConstruction Engineer 0.019 1.002 1436
## career_aspirationDesigner              0.020 1.003 1361
## career_aspirationDoctor                0.018 1.003 1302
## career_aspirationGame Developer        0.022 1.002 1308
## career_aspirationGovernment Officer    0.021 1.002 1293
## career_aspirationLawyer                0.018 1.004 1081
## career_aspirationReal Estate Developer 0.021 1.003 1218
## career_aspirationScientist             0.020 1.001 2007
## career_aspirationSoftware Engineer     0.017 1.005  812
## career_aspirationStock Investor        0.020 1.002 1254
## career_aspirationTeacher               0.019 1.003 1598
## career_aspirationUnknown               0.017 1.003  914
## career_aspirationWriter                0.021 1.001 2027
## b[(Intercept) gender:0]                0.064 1.012  372
## b[(Intercept) gender:1]                0.064 1.012  371
## sigma                                  0.001 1.000 3147
## Sigma[gender:(Intercept),(Intercept)]  0.316 1.006 1404
## mean_PPD                               0.002 1.000 3881
## log-posterior                          0.111 1.001 1125
##
## For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective sample
```

```r
pp_check(grades_lmer)
```




```r
set.seed(12344)
library(MCMCpack)
# Gibbs
data <- read.csv("student-scores-clean.csv")
  y <- clean_data$average_score
  x1 <- clean_data$part_time_job
  x2 <- clean_data$absence_days
  x3 <- clean_data$extracurricular_activities
  x4 <- clean_data$weekly_self_study_hours
#design matrix
  z <- model.matrix(~as.factor(career_aspiration)-1, data=data)

gibbs <- function(y,x1,x2,x3,x4,z,a,b,a_kappa,b_kappa,mu0,tau2,S) {

  n <- length(y)

  X <-cbind(1,x1,x2,x3,x4,z)

  p <-ncol(X)



  # hyperparameters
```

```r
  #fixed tau for the first 3 something
  tau2 <- 100^2
  beta <- rep(0,p)
  kappa2 <- 1

  a <- b <- 1 # complete
  a_kappa <- b_kappa <- 1
  mu0 <- 0

  #draws
  S <- 1000

  #place to store data
  posterior_beta <- matrix(NA,S,p)
  posterior_sig2 <- rep(NA,S)
  posterior_kappa2 <- rep(NA, S)


  #starting values
  beta <- rep(0,p)
  sig2 <- 1

  XX <- t(X)%*%X
  Xy <- t(X)%*%y

  for(s in 1:S){
    # update beta0
    prior_cov <- diag(c(rep(1/tau2,3),rep(1/kappa2,p-3)))
    v <- solve(XX/sig2 + prior_cov)
    m <- v %*% (Xy/sig2 + mu0*diag(prior_cov))
    beta <- m + t(chol(v)) %*% rnorm(p)
    #update sig2
    sig2 <- rinvgamma(1, a + n/2,
                     b + t(y-X%*%beta)%*%(y-X%*%beta)/2)
    #update kappa
    kappa2 <- rinvgamma(1, a_kappa + (p-3)/2,
                       b_kappa +0.5* sum((beta[-c(1:3)])^2))

    #store results
    posterior_beta[s,] <- beta
    posterior_sig2[s] <- sig2
    posterior_kappa2[s] <- kappa2

  }
  return(cbind(posterior_beta, posterior_sig2, posterior_kappa2))
}
post_samples <- gibbs(y,x1,x2,x3,x4,z,a,b,a_kappa,b_kappa,mu0,tau2,S);
```

```r
set.seed(222)
num_beta <- ncol(post_samples) - 2
post_samples_burnin <- post_samples[-c(1:500), ]

results <- data.frame(
  mean = colMeans(post_samples_burnin),
```

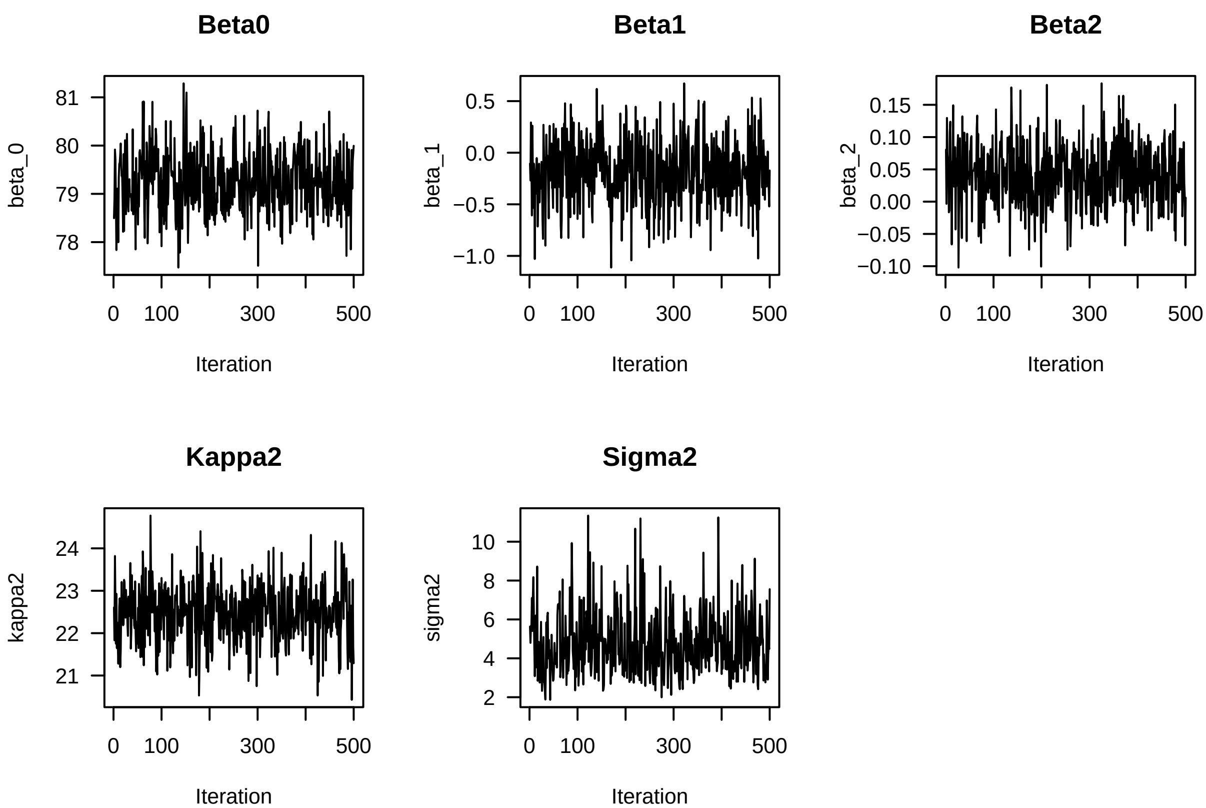
```r
  sd = apply(post_samples_burnin, 2, sd),
  lower = apply(post_samples_burnin, 2, quantile, 0.025),
  upper = apply(post_samples_burnin, 2, quantile, 0.975)
)
row.names(results) <- c(paste0("beta_", 0:(num_beta - 1)), "kappa2", "sigma")

head(results)
```

```
##                mean         sd      lower      upper
## beta_0 79.25575749 0.63076484 78.0541024 80.5077657
## beta_1 -0.18646772 0.31430105 -0.8258711  0.4512410
## beta_2  0.04093165 0.04806883 -0.0584664  0.1327248
## beta_3 -0.08944629 0.26686664 -0.5760893  0.4404568
## beta_4  0.14347027 0.01562466  0.1151566  0.1742610
## beta_5 -3.25428319 0.66480210 -4.4870964 -1.9247567
```

```r
tail(results)
```

```
##                mean        sd      lower      upper
## beta_18 -0.7099384 0.7307113 -2.2098239  0.6776092
## beta_19 -0.5515927 0.7699941 -2.0917382  0.9335377
## beta_20 -2.0857356 0.6145579 -3.2910615 -0.8390151
## beta_21  1.0633372 0.9114244 -0.7085883  2.8821142
## kappa2  22.4517279 0.6819069 21.0734367 23.8529279
## sigma    4.7665742 1.6235547  2.4283475  8.7411536
```

```r
par(mfrow = c(2, 3))

for (i in 1:3) {
  plot(post_samples_burnin[, i], type = "l", las = 1,
       main = paste0("Beta", i - 1),
       xlab = "Iteration", ylab = paste0("beta_", i - 1))
}
plot(post_samples_burnin[, num_beta + 1], type = "l", las = 1,
     main = "Kappa2", xlab = "Iteration", ylab = "kappa2")

plot(post_samples_burnin[, num_beta + 2], type = "l", las = 1,
     main = "Sigma2", xlab = "Iteration", ylab = "sigma2")
```


Beta0
beta_0
81
80
79
78
0
100
300
500
Iteration
Beta1
beta_1
0.5
0.0
−0.5
−1.0
0
100
300
500
Iteration
Beta2
beta_2
0.15
0.10
0.05
0.00
−0.05
−0.10
0
100
300
500
Iteration
Kappa2
kappa2
24
23
22
21
0
100
300
500
Iteration
Sigma2
sigma2
10
8
6
4
2
0
100
300
500
Iteration