Models

2025-04-28

```
library(tidyverse)
library(janitor)
library(rstan)
library(rstanarm)
library(bayesplot)
library(MCMCpack)
library(lme4)
student_data <- read.csv("student-scores.csv");
clean_data <- read.csv("student-scores-clean.csv")
head(student_data)
```

```
##   id first_name last_name                              email gender
## 1  1       Paul     Casey        paul.casey.1@gslingacademy.com   male
## 2  2   Danielle  Sandoval danielle.sandoval.2@gslingacademy.com female
## 3  3       Tina   Andrews      tina.andrews.3@gslingacademy.com female
## 4  4       Tara     Clark        tara.clark.4@gslingacademy.com female
## 5  5    Anthony    Campos    anthony.campos.5@gslingacademy.com   male
## 6  6      Kelly      Wade        kelly.wade.6@gslingacademy.com female
##   part_time_job absence_days extracurricular_activities weekly_self_study_hours
## 1         False            3                      False                      27
## 2         False            2                      False                      47
## 3         False            9                       True                      13
## 4         False            5                      False                       3
## 5         False            5                      False                      10
## 6         False            2                      False                      26
##    career_aspiration math_score history_score physics_score chemistry_score
## 1             Lawyer         73            81            93              97
## 2             Doctor         90            86            96             100
## 3 Government Officer         81            97            95              96
## 4              Artist         71            74            88              80
## 5            Unknown         84            77            65              65
## 6            Unknown         93           100            67              78
##   biology_score english_score geography_score
## 1            63            80              87
## 2            90            88              90
## 3            65            77              94
## 4            89            63              86
## 5            80            74              76
## 6            72            80              84
```

```
head(clean_data)
```

```
##   id first_name last_name                              email gender
## 1  1       Paul     Casey        paul.casey.1@gslingacademy.com      1
## 2  2   Danielle  Sandoval danielle.sandoval.2@gslingacademy.com      0
## 3  3       Tina   Andrews      tina.andrews.3@gslingacademy.com      0
```

```
## 4  4       Tara     Clark        tara.clark.4@gslingacademy.com      0
## 5  5    Anthony    Campos    anthony.campos.5@gslingacademy.com      1
## 6  6      Kelly      Wade        kelly.wade.6@gslingacademy.com      0
##   part_time_job absence_days extracurricular_activities weekly_self_study_hours
## 1             0            3                          0                      27
## 2             0            2                          0                      47
## 3             0            9                          1                      13
## 4             0            5                          0                       3
## 5             0            5                          0                      10
## 6             0            2                          0                      26
##    career_aspiration math_score history_score physics_score chemistry_score
## 1             Lawyer         73            81            93              97
## 2             Doctor         90            86            96             100
## 3 Government Officer         81            97            95              96
## 4             Artist         71            74            88              80
## 5            Unknown         84            77            65              65
## 6            Unknown         93           100            67              78
##   biology_score english_score geography_score average_score
## 1            63            80              87         82.00
## 2            90            88              90         91.43
## 3            65            77              94         86.43
## 4            89            63              86         78.71
## 5            80            74              76         74.43
## 6            72            80              84         82.00
```

$Y_i|B_0, B_1, \sigma^2 \sim N(\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+\beta_3 x_{3i}+\beta_4 x_{4i}, \sigma^2)$ where: $x_{1i}, x_{2i}, x_{3i}, x_{4i}$are the predictors for observation i

$$\beta_j \sim N(0, 10000^2) \text{ where j=}0, 1, 2, 3, 4$$

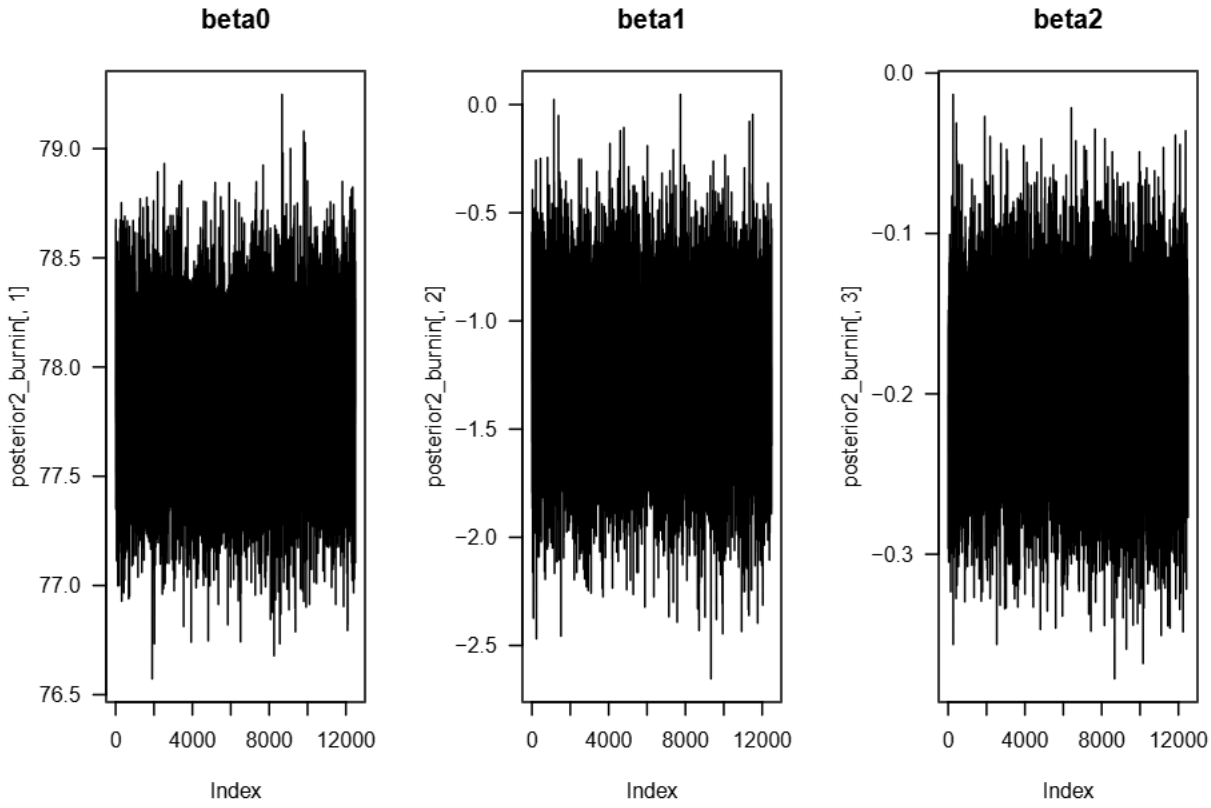
$$\sigma^2 \sim InvGamma(1, 1)$$

```r
#Block Gibbs Sampler
set.seed(4889)
clean_data <- read.csv("student-scores-clean.csv")


set.seed(8451)
y <- clean_data$average_score
x1 <- clean_data$part_time_job
x2 <- clean_data$absence_days
x3 <- clean_data$extracurricular_activities
x4 <- clean_data$weekly_self_study_hours


# Design matrix
X <- cbind(1, x1, x2, x3, x4)
n <- length(y)
p <- ncol(X)

# Hyperparameters
tau2 <- 10000^2
```

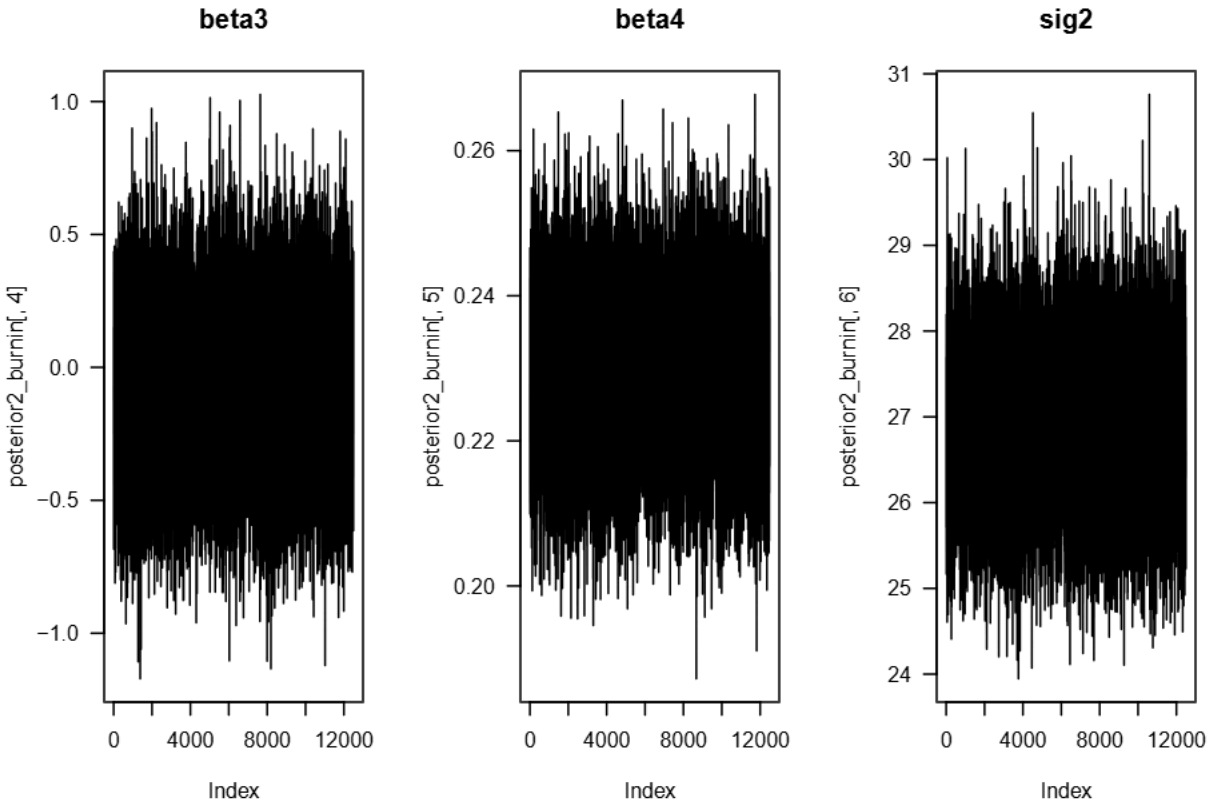
```r
a <- b <- 1
mu0 <- rep(0, p)

S <- 2.5e4

#place to store data
posterior_beta <- matrix(NA, S, p)
posterior_sig2 <- rep(NA, S)

beta <- rep(0, p)
sig2 <- 1

XX <- t(X) %*% X
Xy <- t(X) %*% y

# block Gibbs sampler
for (s in 1:S) {

  # Update beta0
  v <- solve(XX / sig2 + diag(rep(1/tau2, p)))
  m <- v %*% (Xy / sig2 + mu0 / tau2)
  beta <- m + t(chol(v)) %*% rnorm(p)

  # Update sig2 (variance)
  sig2 <- rinvgamma(1, a + n/2,
                    b + t(y - X %*% beta)%*%(y-X%*%beta) / 2)

  # Store results
  posterior_beta[s, ] <- beta
  posterior_sig2[s] <- sig2
}

posterior2 <- cbind(posterior_beta, posterior_sig2)
colnames(posterior2) <- c("beta0", "beta1", "beta2",
                          "beta3", "beta4", "sigma")

#remove burn-in
posterior2_burnin <- posterior2[1:round(s/2),]

head(posterior2_burnin)
```

```
##         beta0     beta1      beta2      beta3     beta4    sigma
## [1,] 77.87654 -1.281166 -0.2003770 -0.1229279 0.2255845 27.69401
## [2,] 77.90033 -0.589276 -0.2092001  0.1524083 0.2279868 25.71107
## [3,] 78.04359 -1.461563 -0.2041925 -0.2506536 0.2197806 26.16819
## [4,] 77.76850 -1.559494 -0.2369275 -0.0571564 0.2424850 27.25814
## [5,] 78.24938 -1.271030 -0.2965047 -0.1193520 0.2139665 27.47846
## [6,] 78.64778 -1.082733 -0.2756962 -0.6835690 0.2114292 27.02233
```

```r
# Block Gibbs Sampler Trace plots
par(mfrow=c(1,3))
plot(posterior2_burnin[,1], type="l", las=1, main="beta0")
plot(posterior2_burnin[,2], type="l", las=1, main="beta1")
plot(posterior2_burnin[,3], type="l", las=1, main="beta2")
```




```
plot(posterior2_burnin[,4], type="l", las=1, main="beta3")
plot(posterior2_burnin[,5], type="l", las=1, main="beta4")
plot(posterior2_burnin[,6], type="l", las=1, main="sig2")
```




```
lm(y~x1+x2+x3+x4, data=clean_data)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + x4, data = clean_data)
##
## Coefficients:
## (Intercept)           x1           x2           x3           x4
##    77.84436     -1.27204     -0.19562     -0.08946      0.22935
```

```
# fit model in rstanarm
grades_lmer <- stan_lmer(average_score ~ part_time_job +
                         absence_days + extracurricular_activities +
                         weekly_self_study_hours + career_aspiration + (1|gender),
                       data = clean_data)
```

```
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.000277 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 2.77 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
```

```
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 26.075 seconds (Warm-up)
## Chain 1:                15.158 seconds (Sampling)
## Chain 1:                41.233 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.000204 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 2.04 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 23.003 seconds (Warm-up)
## Chain 2:                7.963 seconds (Sampling)
## Chain 2:                30.966 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.000198 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 1.98 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
```

```
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 16.719 seconds (Warm-up)
## Chain 3:                24.955 seconds (Sampling)
## Chain 3:                41.674 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'continuous' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.00021 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 2.1 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 23.607 seconds (Warm-up)
## Chain 4:                16.178 seconds (Sampling)
## Chain 4:                39.785 seconds (Total)
## Chain 4:
```

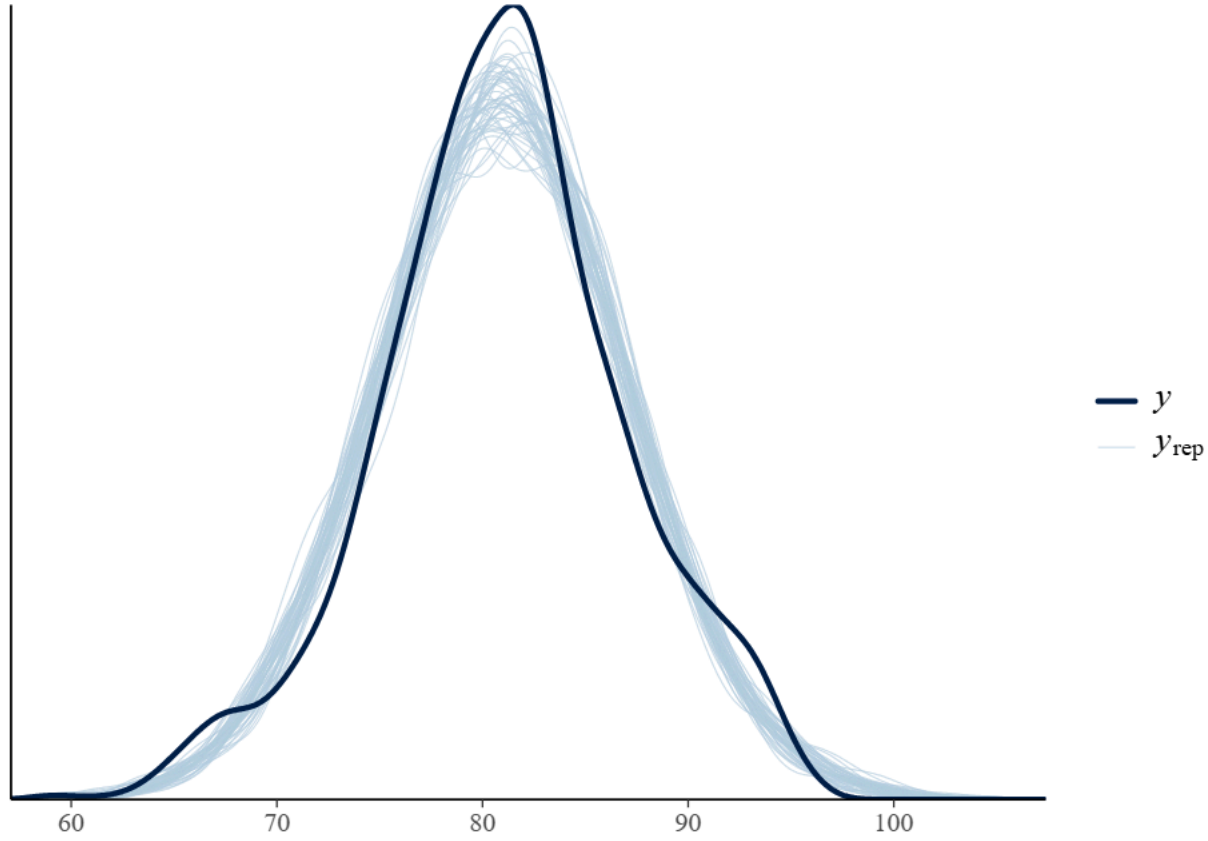
```r
# show results
summary(grades_lmer, digits = 3)
```

```
##
## Model Info:
##  function:     stan_lmer
##  family:       gaussian [identity]
##  formula:      average_score ~ part_time_job + absence_days + extracurricular_activities +
##     weekly_self_study_hours + career_aspiration + (1 | gender)
##  algorithm:    sampling
##  sample:       4000 (posterior sample size)
##  priors:       see help('prior_summary')
##  observations: 2000
##  groups:       gender (2)
##
## Estimates:
##                                  mean   sd     10%    50%    90%
## (Intercept)                     76.155  1.354 74.839 76.126 77.534
## part_time_job                   -0.138  0.322 -0.553 -0.142  0.275
```

```
## absence_days                              0.052  0.048 -0.009  0.051  0.114
## extracurricular_activities               -0.092  0.262 -0.426 -0.086  0.245
## weekly_self_study_hours                   0.134  0.016  0.113  0.133  0.155
## career_aspirationArtist                   4.077  0.775  3.077  4.075  5.050
## career_aspirationBanker                   2.689  0.557  1.980  2.691  3.399
## career_aspirationBusiness Owner          -2.229  0.625 -3.033 -2.235 -1.410
## career_aspirationConstruction Engineer    4.424  0.703  3.525  4.423  5.297
## career_aspirationDesigner                 4.029  0.752  3.069  4.025  4.965
## career_aspirationDoctor                   8.694  0.641  7.851  8.702  9.530
## career_aspirationGame Developer           5.147  0.807  4.118  5.141  6.200
## career_aspirationGovernment Officer       3.465  0.753  2.499  3.476  4.409
## career_aspirationLawyer                   3.948  0.600  3.177  3.961  4.723
## career_aspirationReal Estate Developer    2.479  0.729  1.539  2.476  3.397
## career_aspirationScientist                5.900  0.886  4.773  5.920  7.039
## career_aspirationSoftware Engineer        2.732  0.495  2.081  2.739  3.352
## career_aspirationStock Investor           2.530  0.717  1.628  2.542  3.441
## career_aspirationTeacher                  2.543  0.763  1.567  2.555  3.537
## career_aspirationUnknown                  1.167  0.526  0.481  1.174  1.844
## career_aspirationWriter                   4.587  0.945  3.357  4.579  5.812
## b[(Intercept) gender:0]                   0.081  1.227 -0.981  0.082  1.138
## b[(Intercept) gender:1]                  -0.205  1.233 -1.320 -0.109  0.809
## sigma                                     4.739  0.074  4.644  4.737  4.834
## Sigma[gender:(Intercept),(Intercept)]     4.357 11.847  0.018  0.570 11.062
##
## Fit Diagnostics:
##            mean   sd     10%    50%    90%
## mean_PPD 80.979  0.145 80.793 80.981 81.163
##
## The mean_ppd is the sample average posterior predictive distribution of the outcome variable (for de
##
## MCMC diagnostics
##                                        mcse  Rhat  n_eff
## (Intercept)                            0.069 1.014  386
## part_time_job                          0.005 1.001 4064
## absence_days                           0.001 1.000 3728
## extracurricular_activities             0.004 1.001 3857
## weekly_self_study_hours                0.000 1.002 1716
## career_aspirationArtist                0.021 1.002 1412
## career_aspirationBanker                0.018 1.003  968
## career_aspirationBusiness Owner        0.021 1.004  902
## career_aspirationConstruction Engineer 0.019 1.002 1436
## career_aspirationDesigner              0.020 1.003 1361
## career_aspirationDoctor                0.018 1.003 1302
## career_aspirationGame Developer        0.022 1.002 1308
## career_aspirationGovernment Officer    0.021 1.002 1293
## career_aspirationLawyer                0.018 1.004 1081
## career_aspirationReal Estate Developer 0.021 1.003 1218
## career_aspirationScientist             0.020 1.001 2007
## career_aspirationSoftware Engineer     0.017 1.005  812
## career_aspirationStock Investor        0.020 1.002 1254
## career_aspirationTeacher               0.019 1.003 1598
## career_aspirationUnknown               0.017 1.003  914
## career_aspirationWriter                0.021 1.001 2027
## b[(Intercept) gender:0]                0.064 1.012  372
```

```
## b[(Intercept) gender:1]                0.064 1.012  371
## sigma                                  0.001 1.000 3147
## Sigma[gender:(Intercept),(Intercept)]  0.316 1.006 1404
## mean_PPD                               0.002 1.000 3881
## log-posterior                          0.111 1.001 1125
##
## For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective sample
```

```
pp_check(grades_lmer)
```




```
set.seed(222)
num_beta <- ncol(posterior2_burnin) - 2
post_samples_burnin <- posterior2_burnin[-c(1:500), ]

results <- data.frame(
  mean = colMeans(post_samples_burnin),
  sd = apply(post_samples_burnin, 2, sd),
  lower = apply(post_samples_burnin, 2, quantile, 0.025),
  upper = apply(post_samples_burnin, 2, quantile, 0.975)
)
row.names(results) <- c(paste0("beta_", 0:(num_beta - 1)), "kappa2", "sigma")

head(results)
```

```
##               mean         sd      lower      upper
## beta_0 77.84057028 0.31414949 77.2338982 78.4530387
## beta_1 -1.27436664 0.32701048 -1.9089270 -0.6320920
```

```
## beta_2 -0.19519928 0.04666249 -0.2882050 -0.1041172
## beta_3 -0.08447862 0.28730845 -0.6483447  0.4843899
## kappa2  0.22939711 0.01016277  0.2094566  0.2494970
## sigma  26.86494957 0.85450731 25.2349431 28.5756726
```

```
tail(results)
```

```
##               mean         sd      lower      upper
## beta_0 77.84057028 0.31414949 77.2338982 78.4530387
## beta_1 -1.27436664 0.32701048 -1.9089270 -0.6320920
## beta_2 -0.19519928 0.04666249 -0.2882050 -0.1041172
## beta_3 -0.08447862 0.28730845 -0.6483447  0.4843899
## kappa2  0.22939711 0.01016277  0.2094566  0.2494970
## sigma  26.86494957 0.85450731 25.2349431 28.5756726
```