ETH Quantum Hackathon 2024

Software: Process management problem

Hardware: Flux qubit quantum circuit

Group:	Banana State
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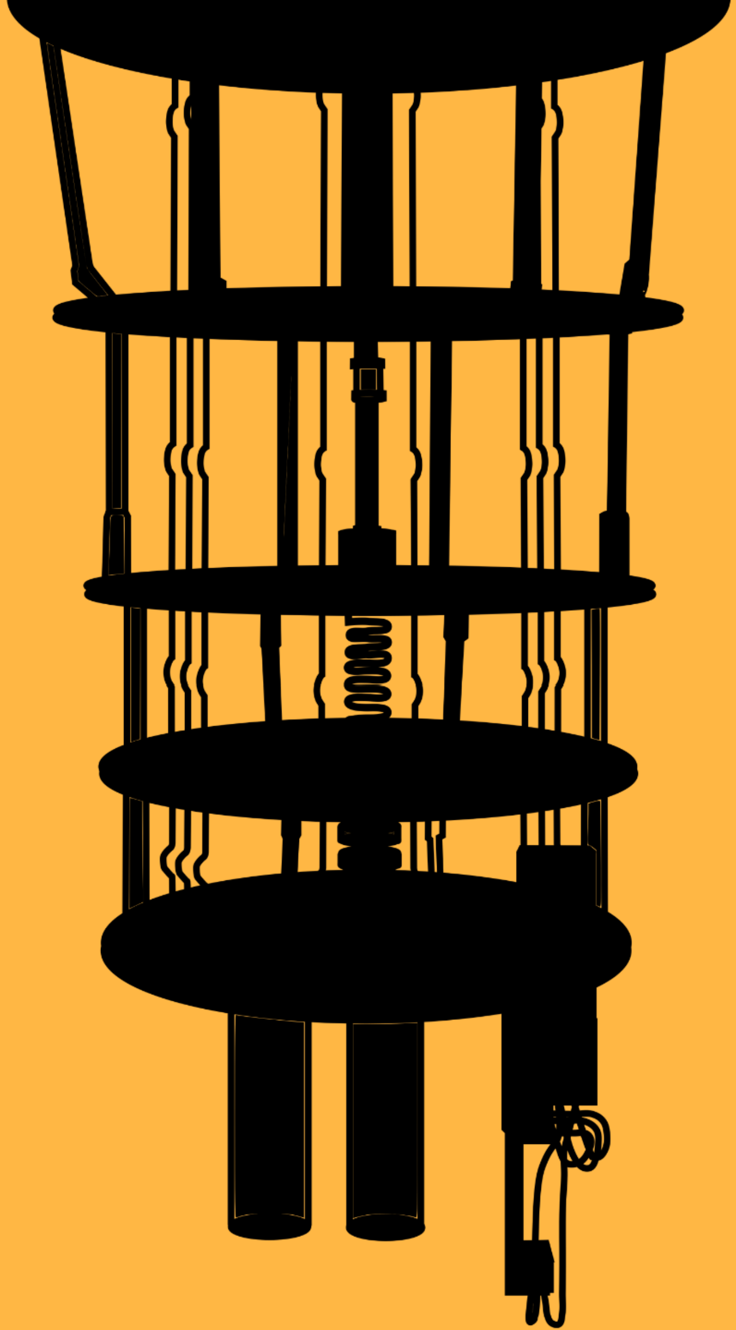
QILIMANJARO QUANTUM · TECH



Quantum Engineering Commission

ETH zürich

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Process management problem

Overview and mathematical formulation

- Set of processes p where each process p_i is associated with an importance v_i and a duration d_i
- Additionally, there exists a maximum allowed duration $d_{\max}$ within which the tasks have to be executed

Mathematical formulation:

Maximize: $$f(x) = \sum_i v_i \, x_i$$

Constraint: $$d_{\max} \geq \sum_i d_i \, x_i$$

$$x \in \{0,1\}^N$$

$$x_i = \begin{cases} 0 & problem\ i\ is\ not\ tackled \\ 1 & problem\ i\ is\ tackled \end{cases}$$

Variables:

v_i	importance of task i
d_i	duration of task i

Process management problem

Cost function with ancilla qubits

Rewriting the problem

$$\sum_{i \in S} v_i$$
$$\sum_{i \in S} d_i \leq d_{max}$$

$$\longrightarrow$$

$$\sum_{i \in S} v_i$$
$$\sum_{i \in S} d_i + x = d_{max}$$
$$0 \leq x \leq d_{max}$$

$$x = \sum_{i=0}^{N} 2^i \times b_i$$

$$b_i \in \{0,1\}$$

Cost Hamiltonian

$$\mathrm{H} = -\sum_i v_i \frac{1-\hat{Z}_i}{2} + \left(\sum_i d_i \frac{1-\hat{Z}_i}{2} + \sum_j 2^j \frac{1-\hat{Z}_j}{2} - d_{max} \right)^2$$

In practice

- 6 qubits + 5 ancilla qubits
- Ground state $|011101\rangle$ reached

Process management problem

Discussion of a subspace solution

- Use the normal value Hamiltonian without constraint $f(x) = \sum_i v_i \, x_i$
- The constraint is naturall satisfied if we restrict the states to the subspace S

Analytical approach

- Start initially in the ground state
- Use a mixer Hamiltonian that conserves the duration or stays in the subspace

$$\left[\hat{H}_m, \sum_i d_i \frac{1 - \hat{Z}_i}{2}\right] = 0 \qquad \hat{H}_m\colon\ S \to S$$

Numerical approach

- Construct a mixer Hamiltonian that is more likely to map the states to ones with lower duration

$$\hat{H}_m\colon\ |\psi > \to |\phi >$$

$$\left\langle \psi \left| \sum_i d_i \tfrac{1-\hat{Z}_i}{2} \right| \psi \right\rangle > \left\langle \phi \left| \sum_i d_i \tfrac{1-\hat{Z}_i}{2} \right| \phi \right\rangle$$

Process management problem

Solution for a restricted qubit number

Cost Hamiltonian:

Penalty factor

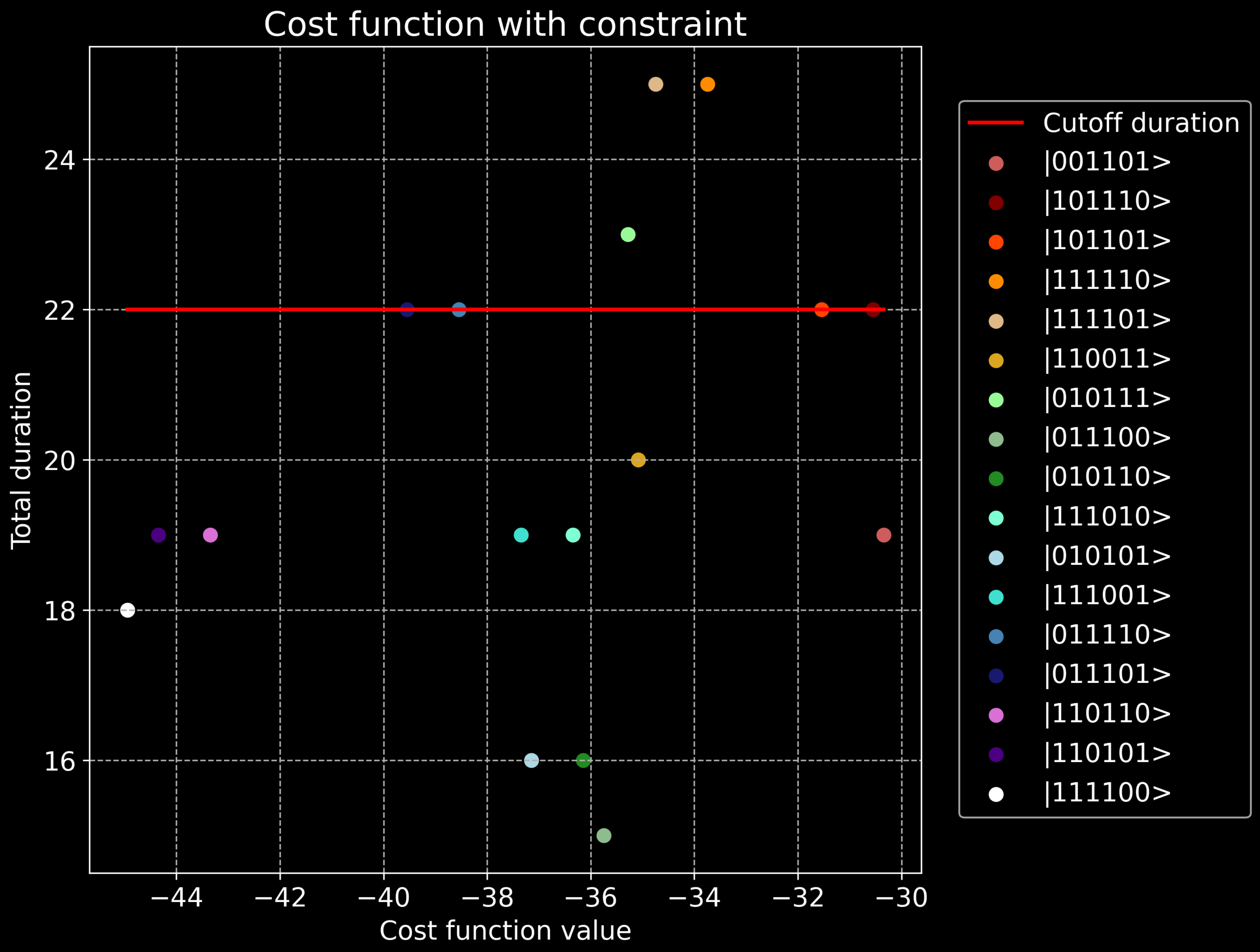
Center of optimization

$$H(x) = \underbrace{-\sum_i v_i \frac{1-\hat{Z}_i}{2}}_{\text{Optimization term}} + \underbrace{\frac{1}{3}\left(\frac{4}{5}\, d_{\max} - \sum_i d_i \frac{1-\hat{Z}_i}{2}\right)^2}_{\text{Constraint / Penalty term}}$$

Variables:

v_i	Importance of task i
d_i	Duration of task i
$\hat{Z}_i$	Pauli-z acting on qubit i

- A realistic solution is more oriented towards $d_{\max}$
- Calculate the total duration after ground state determination as controlling instance



Cost function with constraint
Total duration
Cost function value
24
22
20
18
16
−44
−42
−40
−38
−36
−34
−32
−30
Cutoff duration
|001101>
|101110>
|101101>
|111110>
|111101>
|110011>
|010111>
|011100>
|010110>
|111010>
|010101>
|111001>
|011110>
|011101>
|110110>
|110101>
|111100>

Process management problem

Comparison with the analytic solution

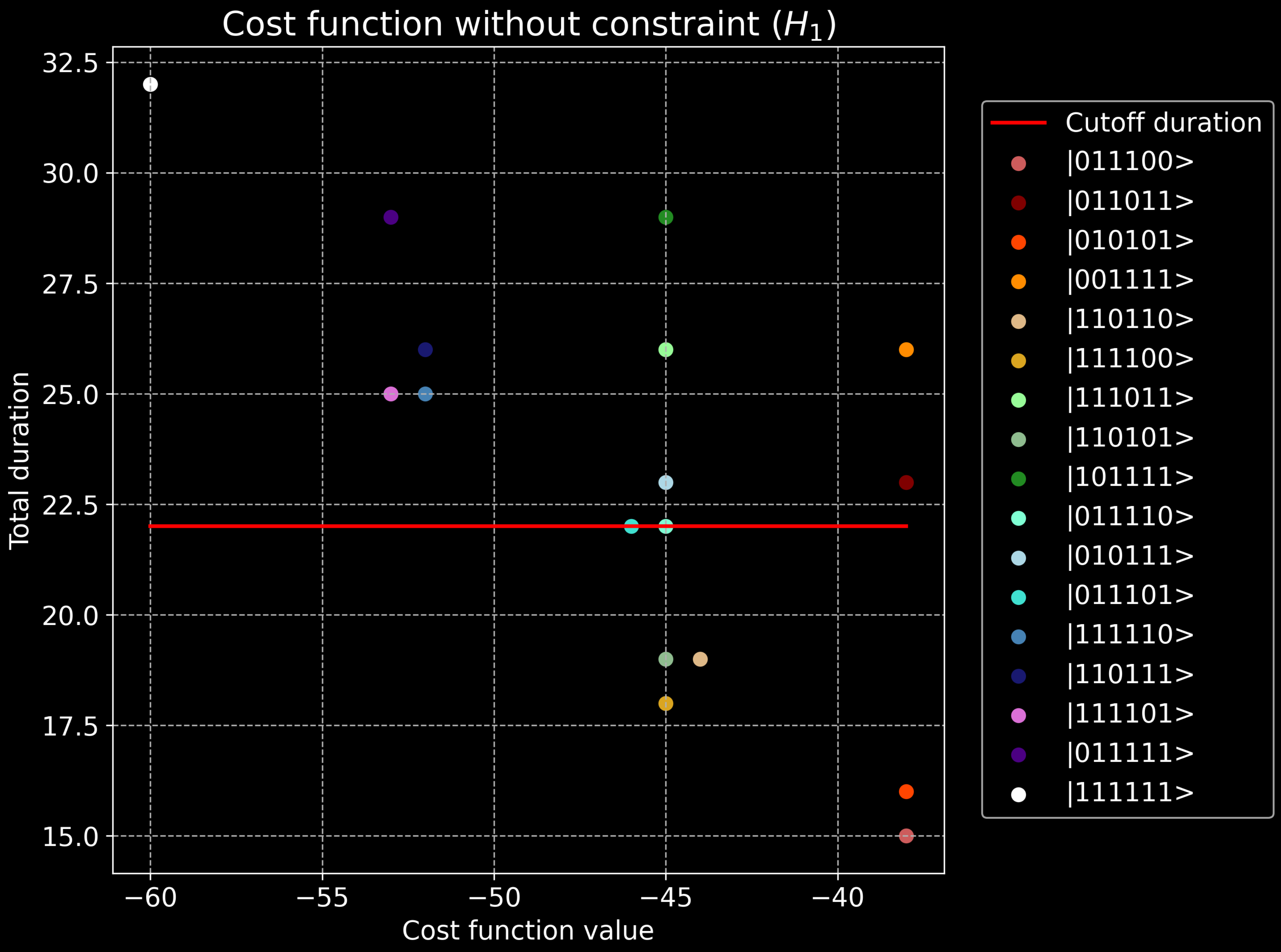
Optimization Hamiltonian (without constraints):

$$H(x) = -\sum_i v_i \frac{1-\hat{Z}_i}{2} + \cancel{\frac{1}{3}\left(\frac{4}{5}d_{\max} - \sum_i d_i \frac{1-\hat{Z}_i}{2}\right)}$$

Variables:

v_i	Value of task i
d_i	Duration of task i
$\hat{Z}_i$	Pauli-z acting on qubit i

- The analytical solution is np hard to find (here only used for comparison)
- The most valuable solutions will exceed the maximal duration
- The approximated Hamiltonian yields a ground state which is one of the 4 lowest states of the analytical solution (overall $2^6 = 64$ states)



Cost function without constraint (H_1)
Total duration
Cost function value
32.5
30.0
27.5
25.0
22.5
20.0
17.5
15.0
−60
−55
−50
−45
−40
Cutoff duration
|011100>
|011011>
|010101>
|001111>
|110110>
|111100>
|111011>
|110101>
|101111>
|011110>
|010111>
|011101>
|111110>
|110111>
|111101>
|011111>
|111111>

Digital and analog quantum algorithms

Quantum Approximate Optimization Algorithm (QAOA)
Quantum Adiabatic Evolution (QAE)
Comparison between QAOA and QAE

Digital and analog quantum algorithms

Quantum Approximate Optimization Algorithm (QAOA)

- Application of two unitary operators sequentially on initial state
- Encoding of the problem in the phase separation operator
- The other operator is called "mixer" operator

Mixing operator & unitary:

$$H_m = \sum_i X_i \longrightarrow U_m = \exp\left(-i\,\gamma \sum_i X_i\right)$$

Cost unitary:

$$U_m = \exp(-i\,\beta H_c)$$

for H_c as cost Hamiltonian

- Here β and γ are trainable parameters

Digital and analog quantum algorithms

Quantum Adiabatic Evolution (QAE)

- Definition of a simple Hamiltonian with known ground state
- Adiabatic transfer to the cost Hamiltonian in a time interval $t \in [0,1]$

Adiabatic transfer:

H_c as cost Hamiltonian $\qquad H_0 = \sum_i X_i$ with known ground state

$H(s) = s(t)\, H_c + \left(1 - s(t)\right) H_0$ for $s(t=0) = 0$ and $s(t=1) = 1$

- If the transfer is sufficiently slow, the system remains in the ground state

Digital and analog quantum algorithms

Comparison between QAOA and QAE

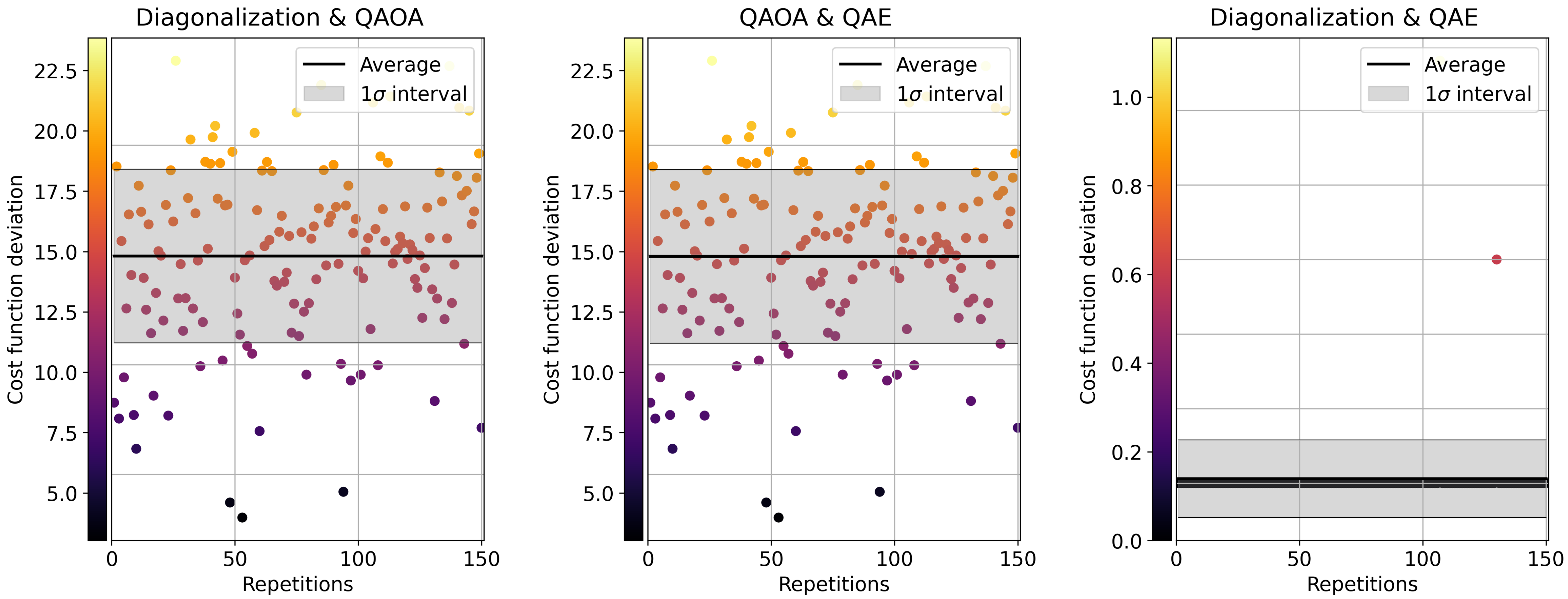
- Randomize initial values and record 150 runs
- Compare the ground states and quantify energy deviations for each run

Ground state comparison:

Method 1	Method 2	Matching probability
Diagonalization	QAOA	22.67%
QAOA	QAE	22.67%
QAE	Diagonalization	96.0%

Digital and analog quantum algorithms

Comparison between QAOA and QAE



Hardware implementation via Flux Qubits

Walkthrough
Energy spectrum
Research

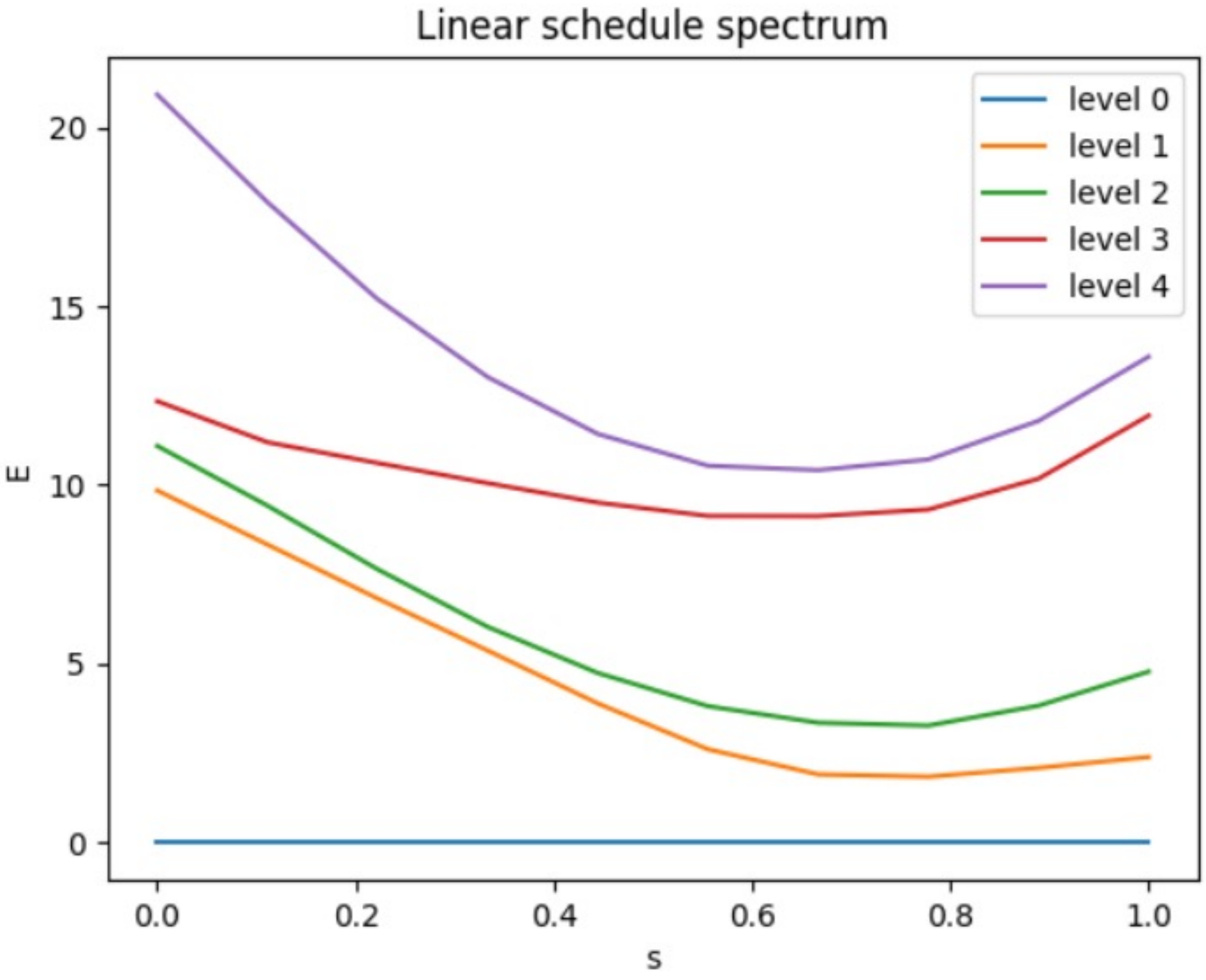
Hardware implementation via Flux Qubits

Walkthrough

- 3 qubits + 3 couplers
- Only lowest eigenvalues and eigenenergies
- Hamiltonian coefficients
- Ising schedule
- Flux schedule
- Energy spectrum

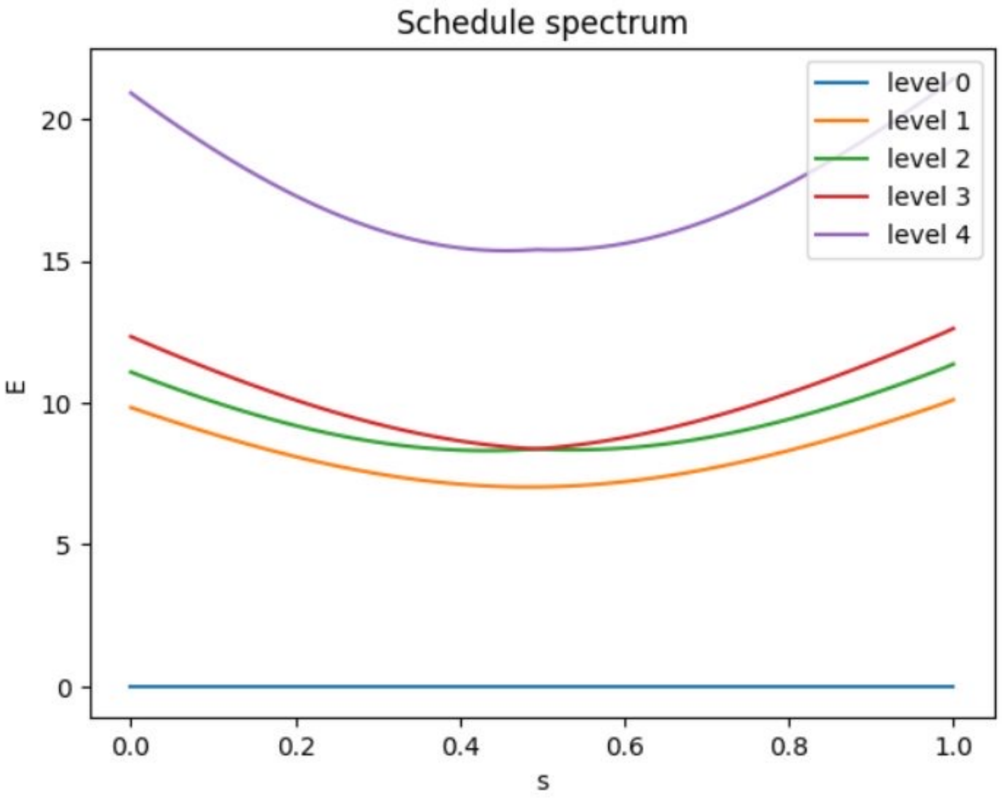
Hardware implementation via Flux Qubits

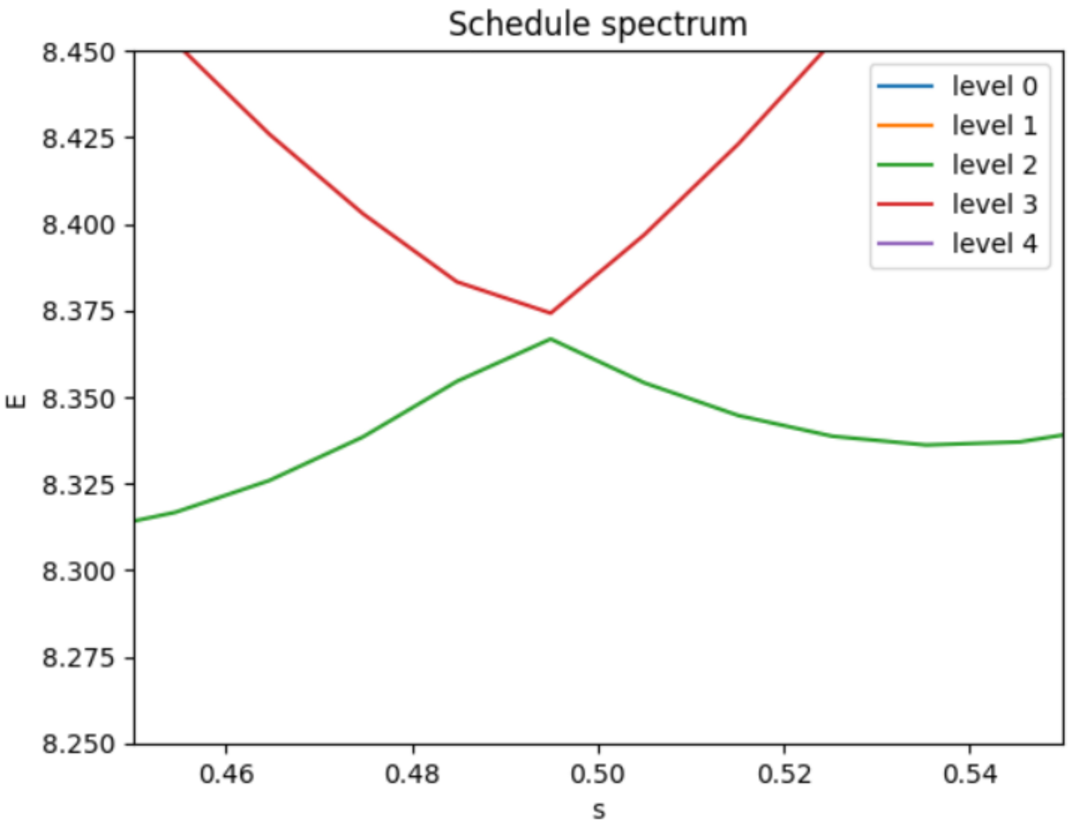
Energy spectrum – Cost Hamiltonian



Hardware implementation via Flux Qubits

Energy spectrum – modified Hamiltonian





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