

# Multiplication (efficient)

$$\begin{array}{r}
 100100 \quad x \\
 \times 10110 \quad y \\
 \hline
 100100 \\
 100100 \\
 100100 \\
 +100100 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 527 \\
 \times 36 \\
 \hline
 \dots \leftarrow 6 \times 527 \\
 \dots \leftarrow 30 \times 527
 \end{array}$$

Pseudocode to mul(x, y):

res = 0

for i in 0..15:

if bit i of y is 1:

res = res + x

x = x + x

(in a real CPU, shift x left 1 bit)

2 ways:

① shift y right 1 bit every loop, test last bit  
~~X~~ can't do on Hack CPU.

② Test w/ a mask.

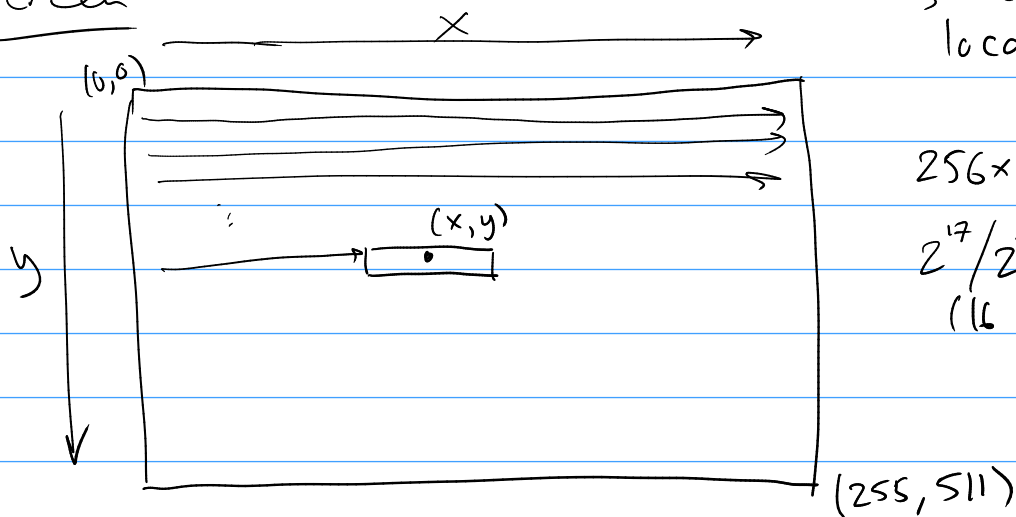
To test bit i:

$$\begin{array}{r}
 101010110 \\
 \& 000001000 \\
 \hline
 000001000
 \end{array}$$

↖ ?  
 ← bit mask, =  $2^b$  where b is bit to test.  
 ← See if this = 0.

keep track in a loop or precompute in array.

Screen



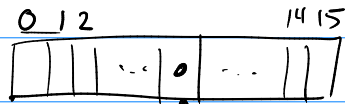
Screen starts at memory location  $2^{14} = 16384$ .

$$256 \times 512 = 2^8 \cdot 2^9 = 2^{17} \text{ pixels.}$$

$$2^{17} / 2^4 = 2^{13} \text{ memory locations (16 pixels per memory loc.)}$$

Pixel  $(x,y)$  is the  $(512y + x)^{\text{th}}$  pixel.

Thus it is stored in memory location  $(512y + x) / 16$   
 $= 32y + x/16$  (offset from  $2^{14}$ ).



At bit  $x \% 16$ . ie.  $x \& 1111$

To set pixel  $(x,y)$ :

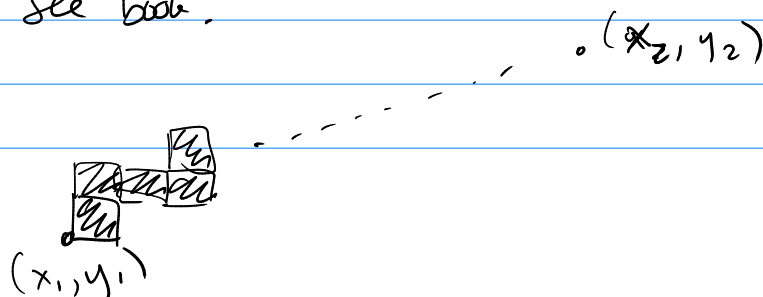
- Get value @  $SCREEN + 32y + x/16$
- Set bit  $x/16$
- Put updated value back.

To set bit  $i$ :

- Use mask  $000010000 = 2^i$
- To set bit  $i$  to 1,  $v = v | 2^i$
- To clear bit  $i$  (set to 0),  $v = v \& (\sim(2^i))$

↓  
111101111

drawLine: Bresenham's algorithm.  
 See book!



drawRectangle



Loop + draw horizontal lines.

↳ drawLine

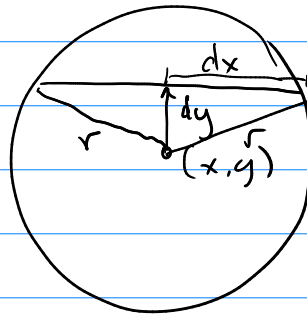
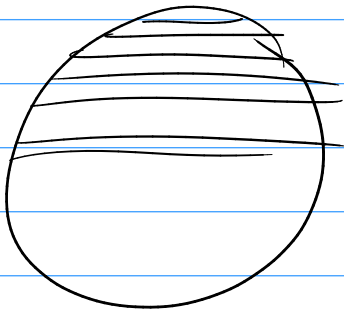
OR helps drawHorizLine

↳ pixel by pixel

OR 16 pixels @ once.

drawCircle

Fill in by  
horiz. rows



$$dx^2 + dy^2 = r^2$$

$$\rightarrow dx = \sqrt{r^2 - dy^2}$$