Binary = base 2

$10110_2 = 2 + 4 + 16 = 22$

$2^4 \; 2^3 \; 2^2 \; 2^1 \; 2^0$

A	10	$= 1010_2$
B	11	$= 1011_2$
C	12	$= 1100_2$
D	13	
E	14	
F	15	

Hexadecimal = base 16

$AC7_{16} = 7 + 12 \times 16 + 10 \times 16^2 =$

$\underbrace{1010}\underbrace{1100}\underbrace{0111}$

Binary addition.

$$\begin{array}{r} 0101 \\ +\ 0111 \\ \hline 1100 \end{array} \quad \begin{array}{r} = 5 \\ = 7 \\ \hline 12 \end{array}$$

overflow.

$$\begin{array}{r} 1101 \\ 0011 \\ \hline 0000 \end{array}$$

Subtraction? Negative number?

1st idea: use 1st bit as a 'sign bit'

sign | magnitude

e.g. 0|101 = +5

1|101 = −5

But this doesn't work well!

- What is 1000? −0? Two different 0's.
- Need a lot of if's to handle different situations.

2's complement arithmetic

0 0 1 1	3
0 0 1 0	2
0 0 0 1	1
0 0 0 0	0
1 1 1 1	−1 ?
1 1 1 0	−2 ?
1 1 0 1	−3 ?

Sign

−8 .... +7

1000 0111

```
  0110    6
+ 0011    3
------  ---
  1001
```

```
② 1111    −1
 + 0001    1
 ------  ---
   0000    0 ✓

   0 1111    −1
 + 1111     −1
 ------   ---
   1110     −2 ✓

① 0010     2
 + 1111    −1
 ------   ---
   0001     1 ✓
```

How to negate? $-x = \overline{x} + 1$

e.g. $0010 \xrightarrow[\text{inv}]{} 1101 \xrightarrow[+1]{} 1110$

$1110 \xrightarrow[\text{inv}]{} 0001 \xrightarrow[+1]{} 0010.$

-1000? $\xrightarrow[\text{inv}]{} 0111 \xrightarrow[+1]{} 1000$

Addition

– Half adder : add two 1-bit numbers.

a	b	carry	sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

```
  0111
+ 1011
------
     0
```

– Full adder : add three 1-bit numbers.

a	b	c	carry	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

A bunch of full adders in a chain can add n-bit #'s.

"Ripple carry adder"

ALU = arithmetic logic unit

inputs

x (16)
y (16)
zx, zy (1) — should we replace x (y) with 0?
nx, ny (1) — should we invert x (y)?
f — 0: AND 1: ADD
no — should we invert the output?

outputs

out (16)
zr (1) — is the result zero?
ng (1) — is the result negative?

	zx	zy	nx	ny	f	no
x AND y	0	0	0	0	0	0
x OR y	0	0	1	1	0	1
x + y	0	0	0	0	1	0
$\bar{x}$	0	1	1	0	1	0
$\bar{x}$	0	1	1	1	0	0