CZ3005 Artificial Intelligence

Week 9a – Propositional Logic

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Logic is a Formal Language

- **Propositions:**
  - Anil is Intelligent
  - Anil is hardworking
  - If Anil is Intelligent and Anil is Hardworking, then Anil scores a high mark

Propositional Logic

- **Syntax** of the representation language specifies all the sentences that are well-formed.

- **Semantics** of the language defines the truth of each sentence with respect to each possible world.

Elements of Propositional Logic

- **Symbols**
  - Logical constants: TRUE, FALSE
  - Propositional symbols: P, Q, etc. (uppercase)
  - Logical connectives: $\wedge$, $\vee$, $\Leftrightarrow$, $\Rightarrow$, $\neg$
  - Parentheses: ( )
- **Sentences**
  - Atomic sentences: constants, propositional symbols
  - Combined with connectives, e.g. $P \wedge Q \vee R$
    also wrapped in parentheses, e.g. $(P \wedge Q) \vee R$

Elements of Propositional Logic

- Anil is intelligent = Intelligent(Anil)
- Anil is hardworking = Hardworking(Anil)

Propositions

- Objects and relations or Functions

Anil hardworking, intelligent

- A proposition (statement) can be true or false

Logical Connectives

- **Conjunction** Λ
  - Binary op., e.g. P Λ Q, “P and Q”, where P, Q are the *conjuncts*
- **Disjunction** $\vee$
  - Binary op., e.g. P $\vee$ Q, “P or Q”, where P, Q are the *disjuncts*
- **Implication** $\Rightarrow$
  - Binary op., e.g. P $\Rightarrow$ Q, “P implies Q”, where P is the *premise* (antecedent) and Q the *conclusion* (consequent)
  - Conditionals, “if-then” statements, or ***rules***
- **Equivalence** $\Leftrightarrow$
  - Binary op., e.g. P $\Leftrightarrow$ Q, “P equivalent to Q”
    - Biconditionals
- **Negation** $\neg$
  - Unary op., e.g. $\neg$ P, “not P”

Syntax of Propositional Logic
(Backus-Naur Form)

Sentence	$\rightarrow$	AtomicSentence | ComplexSentence
AtomicSentence	$\rightarrow$	LogicalConstant | PropositionalSymbol
ComplexSentence	$\rightarrow$	(Sentence) | Sentence LogicalConnective Sentence | ¬Sentence
LogicalConstant	$\rightarrow$	TRUE | FALSE
PropositionalSymbol	$\rightarrow$	P | Q | R | …
LogicalConnective	$\rightarrow$	Λ | $\vee$ | $\Leftrightarrow$ | $\Rightarrow$ | $\neg$

Precedence (from highest to lowest): $\neg, \Lambda, \vee, \Rightarrow, \Leftrightarrow$

e.g.: $\neg P \Lambda Q \vee R \Rightarrow S$ (not ambiguous), equal to: $(((\neg P) \Lambda Q) \vee R) \Rightarrow S$

Semantics of Propositional Logic

- **Validity**
  - A sentence is valid if it is true in all models.
  - Valid sentences are known as **tautologies**.
  - Every valid sentence is logically equivalent to *True*.

Semantics of Propositional Logic

- **Satisfiability**
  - A sentence is satisfiable if it is true in some models.
  - Satisfiability can be checked by enumerating the possible models until one is found that satisfies the sentence.
  - Most problems in computer sciences are satisfiability problems.
    - E.g., Constraint satisfaction problem, Search problems.

Semantics of Propositional Logic

- **Interpretation of symbols**
  - Logical constants have fixed meaning
    - True: always means the fact is the case; valid
    - False: always means the fact is not the case; unsatisfiable
  - Propositional symbols mean "whatever they mean"
    - e.g.: **P** "we are in a pit", etc.
    - Satisfiable, but not valid (true only when the fact is the case)
- **Interpretation of sentences**
  - Meaning derived from the meaning of its parts
    - Sentence as a combination of sentences using connectives
  - Logical connectives as (boolean) functions:

TruthValue ***f*** (TruthValue, TruthValue)

Example 1

- Let P stands for Intelligent(Anil)
- Let Q stands for Hardworking(Anil)

- What does $P \Lambda Q$ mean?
- What does $P \vee Q$ mean?

- $P \Lambda Q$, $P \vee Q$ are compound propositions

Example 2

- Use parenthesis to ensure that the syntax is completely unambiguous:
  - A: John likes Kate.
  - B: John likes Chocolate.
  - C: John buys Chocolate

- $(A \wedge B) \Rightarrow C$
  - If John likes Kate and John likes Chocolate, John buys Chocolate
- $A \wedge (B \Rightarrow C)$
  - John likes Kate, and
  - If John likes Chocolate, then John buys Chocolate

Semantics of Propositional Logic

- **Interpretation of connectives**
  - Truth-table
    - Define a mapping from input to output

P	Q	$\neg P$	$P \land Q$	$P \lor Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	False	True	False	False	True	True
False	True	True	False	True	True	False
True	False	False	False	True	False	False
True	True	False	True	True	True	True

  - Interpretation of sentences by decomposition
    - e.g.: $\neg P \land Q \lor R \Rightarrow S$, with $P \leftarrow F$, $Q \leftarrow T$, $R \leftarrow F$, $S \leftarrow T$:

$\neg P \leftarrow T$ $\quad$ $((\neg P) \land Q) \lor R) \leftarrow T$

$(\neg P) \land Q \leftarrow T$ $\quad$ $(((\neg P) \land Q) \lor R) \Rightarrow S \leftarrow T$

Validity and Inference

- **Testing for validity**
  - Using truth-tables, checking all possible configurations
    - e.g.: $((P \vee Q) \wedge \neg Q) \Rightarrow P$

P	Q	$P \vee Q$	$\neg Q$	$(P \vee Q) \wedge \neg Q$	$((P \vee Q) \wedge \neg Q) \Rightarrow P$
False	False	False	True	False	True
False	True	True	False	False	True
True	False	True	True	True	True
True	True	True	False	False	True

- The proposition says:
  - If $((P \vee Q) \wedge \neg Q)$ is True, then P is True.
  - If $((P \vee Q) \wedge \neg Q)$ is False, then ? (didn't specify, so P can be either True or False) -> overall, this proposition is *valid*

Summary

- **Valid** sentence – TRUE under all interpretations
- **Satisfiable** sentence – TRUE under at least 1 interpretation
- **Unsatisfiable** sentence – FALSE under all interpretations

Exercise

A	B	C	A ∧ B	$B \Rightarrow C$	$(A \wedge B) \Rightarrow C$	$A \wedge (B \Rightarrow C)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Thank you!