Take home test J
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1) Find basis of Span $\left\langle \begin{pmatrix}-2\\-4\\-1\end{pmatrix}, \begin{pmatrix}-1\\-7\\2\end{pmatrix}, \begin{pmatrix}1\\7\\2\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rangle$

Take $X = \begin{pmatrix}-2 & -1 & 1 & 1\\-4 & -7 & 7 & 1\\-1 & 2 & 2 & 1\end{pmatrix}$ $\begin{matrix}R_2 \leftarrow R_2 - 2R_1\\R_3 \leftarrow R_3 - \frac{1}{2}R_1\end{matrix} \rightarrow \begin{pmatrix}-2 & -1 & 1 & 1\\0 & -5 & 5 & -1\\0 & 5/2 & 3/2 & 1/2\end{pmatrix} \rightarrow R_3 \leftarrow R_3 + \frac{1}{2}R_2 \rightarrow \begin{pmatrix}-2 & -1 & 1 & 1\\0 & -5 & 5 & -1\\0 & 0 & 4 & 0\end{pmatrix}$

Basis is linear independent $= \left\{ \begin{pmatrix}-2\\-4\\-1\end{pmatrix}, \begin{pmatrix}-1\\-7\\2\end{pmatrix}, \begin{pmatrix}1\\7\\2\end{pmatrix} \right\}$

2) Find LU Factorization of $A = \begin{pmatrix}-4 & -1 & -1\\-2 & -5 & -2\\-2 & 2 & -3\end{pmatrix}$

$\begin{pmatrix}-4 & -1 & -1\\-2 & -5 & -2\\-2 & 2 & -3\end{pmatrix}$ $\begin{matrix}R_2 \leftarrow R_2 - \frac{1}{2}R_1\\R_3 \leftarrow R_3 - \frac{1}{2}R_1\end{matrix} \rightarrow \begin{pmatrix}-4 & -1 & -1\\0 & -9/2 & -3/2\\0 & 5/2 & -5/2\end{pmatrix}$ $R_3 \leftarrow R_3 + \frac{5}{9}R_2 \rightarrow$ $U = \begin{pmatrix}-4 & -1 & -1\\0 & -9/2 & -3/2\\0 & 0 & -10/3\end{pmatrix}$

$L = \begin{pmatrix}1 & 0 & 0\\1/2 & 1 & 0\\1/2 & -\frac{5}{9} & 1\end{pmatrix}$

$LU = \underset{L}{\begin{pmatrix}1 & 0 & 0\\1/2 & 1 & 0\\1/2 & -5/9 & 1\end{pmatrix}} \underset{U}{\begin{pmatrix}-4 & -1 & -1\\0 & -9/2 & -3/2\\0 & 0 & -10/3\end{pmatrix}}$

3) Find range and kernel of linear operator L, Null Space, Column Space and rank of matrix of operator L

$L(A) = \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} \cdot A + A \cdot \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix}$

take basis $b_1 = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$, $b_2 = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$, $b_3 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

$L(b_1) = \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} \cdot \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}-1 & 0\\2 & 0\end{pmatrix} + \begin{pmatrix}-1 & 2\\0 & 0\end{pmatrix} = \begin{pmatrix}-2 & 2\\2 & 0\end{pmatrix} \rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix}$

$L(b_2) = \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} \cdot \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}0 & 2\\0 & 2\end{pmatrix} + \begin{pmatrix}0 & 0\\2 & 2\end{pmatrix} = \begin{pmatrix}0 & 2\\2 & 4\end{pmatrix} \rightarrow \begin{pmatrix}0\\4\\2\end{pmatrix}$

$L(b_3) = \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} \cdot \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} + \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}2 & -1\\2 & 2\end{pmatrix} + \begin{pmatrix}2 & 2\\-1 & 2\end{pmatrix} = \begin{pmatrix}4 & 1\\1 & 4\end{pmatrix} \rightarrow \begin{pmatrix}4\\4\\1\end{pmatrix}$

Range (L) $= \text{Span} \left\langle \begin{pmatrix}-2 & 2\\2 & 0\end{pmatrix}, \begin{pmatrix}0 & 2\\2 & 4\end{pmatrix}, \begin{pmatrix}4 & 1\\1 & 4\end{pmatrix} \right\rangle$

Kernel (L) $= \left\{ \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix} \right\}$

$B = \begin{pmatrix}-2 & 0 & 4\\0 & 4 & 4\\2 & 2 & 1\end{pmatrix}$ $R_3 \leftarrow R_3 + R_1$

$\downarrow$

$\begin{pmatrix}-2 & 0 & 4\\0 & 4 & 4\\0 & 2 & 5\end{pmatrix}$ $R_3 \leftarrow R_3 - \frac{1}{2}R_2$

$\downarrow$

$\begin{pmatrix}-2 & 0 & 4\\0 & 4 & 4\\0 & 0 & 3\end{pmatrix}$

Null Space $N(B) = \{0\}$ because B has trivial Solution

Column Space $C(B) = \text{Span} \left\langle \begin{pmatrix}-2\\0\\2\end{pmatrix}, \begin{pmatrix}0\\4\\2\end{pmatrix}, \begin{pmatrix}4\\4\\1\end{pmatrix} \right\rangle$

Rank $(B) = 3$