The Unofficial MH1810 Weekly Challenge
Week 7 Midterm Preparation

1 Find the conditions such that the lines l: $\mathbf{r} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda_1 \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$, m: $\mathbf{r} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + \lambda_1 \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}$ are not parallel and do not intersect. Under these conditions, derive an expression for the shortest distance between l and m.

A plane π, parallel to both l and m, passes through the origin. Find the conditions such that the shortest distance between π and either l or m is the same as that between l and m, and that this distance is positive.

2 A given tetrahedron has surface area 18 cm^2 and volume 48 cm^3. Given that the height of the tetrahedron is 4 cm, calculate the smallest angle between either of its slanted faces and its base.

3 Let A be a real $n \times n$ matrix such that $A^3 = 0$. Find a real matrix X that satisfies the equation

$$X + AX + XA^2 = A.$$

4 If $\det(A - I) = 1$, $\det(A + I) = 2$ and $\det(A + 2I) = 4$, what can you say about $\det A$, where A is a real 3×3 matrix? If A is invertible, find an example of A^{-1}.

5 Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} \frac{a^{x^2 \cos(\frac{1}{x})}}{b-a} & \text{if } x \neq 0 \\ b & \text{if } x = 0 \end{cases}.$$

Find a relation connecting a, b such that f is continuous on $\mathbb{R}$. On what interval(s) is/are f differentiable?

6 Use the definition of the derivative to differentiate $\tan\frac{1}{x}$ with respect to x.
Given that $f(x) = \frac{\tan\frac{1}{x}}{2^{\frac{1}{x}}}$, evaluate $f'(3)$.