Nanyang Technological University

SPMS/Division of Mathematical Sciences

2019/20 Semester 1 **MH1810 Mathematics 1** **Take-Home Test (10%)**

Name: | **Tutorial Group:**

Matriculation Number: | $a =$ | $b =$

Instructions

Print the question paper and fill in your Name, Tutorial Group, Matriculation Number, and the values of a and b. (See below for how a and b are determined.)

Attach the question paper to the your (handwritten) answer script.

Submit to your tutor by **15 November 2019 1 pm**

Throughout this assignment, let z = last numerical digit of your matriculation number.

Set

$$a = \begin{cases} z+1 & \text{if } z < 9 \\ 2 & z = 9 \end{cases}, \text{ and}$$

$$b = 9 - a.$$

E.g., if matric number = U1912345Z, then $a = 6$ and $b = 3$. Use these values of a and b in the questions.

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1. Show, by performing integration, that

   (a)
   $$\int \frac{x+2a}{x^2+2ax+2a^2} dx = A \ln |f(x)| + B \tan^{-1} g(x) + C,$$
   where constants A, B and functions $f(x)$, $g(x)$ are to be determined.

   (b)
   $$\int e^{ax} \sin(bx)\, dx = e^{ax} (A \sin(bx) + B \cos(bx)) + C,$$
   where constants A and B are to be determined.

2. (a) Use trigonometric substitution $x = a\sin\theta$, or otherwise, to show that

$$\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C.$$

(b) Hence evaluate $\int_{-a}^{a} \left[\sqrt{a^2 - x^2} + \sin\left(x^3\right)\right] dx$.

3. Find the following integrals:

(a) $\int_0^{1/2} \frac{1}{x^2 - (a+b)\,x + ab} dx$

(b) $\int_0^{\frac{\pi}{2}} \frac{a\sin x}{a\sin x + b\cos x} dx$

4. Find the following integrals:

(a) $\int x\,(1-x)^{a+2b}\,dx$

(b) $\int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right) dx.$

5. (a) Evaluate

$$\lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+an}\right).$$

(b) Use integrals to estimate

$$\sum_{i=1}^{1000000} \sqrt{i}.$$

Express the answer in standard notation $m \times 10^n$, where $1 \leq m < 10$ in three decimal places, and $n \in \mathbf{Z}^+$.