MH 1810 Maths I Vectors Online Assessment

Date No.

1. $\|\underset{\sim}{u}\| = \sqrt{1^2+1^2+(-1)^2} = \sqrt{3} = 1.73$

$\underset{\sim}{u}\cdot\underset{\sim}{v} = \begin{pmatrix}1\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix} = 1-2-1 = -2$

$\cos\theta = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\|} = \dfrac{-2}{\sqrt{3}\,\sqrt{1^2+(-2)^2+1^2}} = \dfrac{-2}{\sqrt{3}\,\sqrt{6}}$

$\theta = \cos^{-1}\left(\dfrac{-2}{\sqrt{3}\,\sqrt{6}}\right) = 2.06 \text{ rad}$

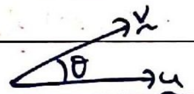
2. $\|7\underset{\sim}{a}\| = 7\|\underset{\sim}{a}\| = 7$

$3\underset{\sim}{a}\cdot 7\underset{\sim}{b} = 21\,\underset{\sim}{a}\cdot\underset{\sim}{b} = 21\left(\tfrac{1}{3}\right) = 7$

$\underset{\sim}{a}\cdot(\underset{\sim}{b}-\underset{\sim}{c}) = \underset{\sim}{a}\cdot\underset{\sim}{b} - \underset{\sim}{a}\cdot\underset{\sim}{c} = \tfrac{1}{3} - \tfrac{1}{8} = \tfrac{5}{24}$

$(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = \underset{\sim}{a}\cdot\underset{\sim}{a} - \cancel{\underset{\sim}{a}\cdot\underset{\sim}{b}} + \cancel{\underset{\sim}{a}\cdot\underset{\sim}{b}} - \underset{\sim}{b}\cdot\underset{\sim}{b} + \underset{\sim}{a}\cdot\underset{\sim}{c} - \underset{\sim}{b}\cdot\underset{\sim}{c}$

$= 1 - 1 + \tfrac{1}{8} - \tfrac{1}{7} = -\tfrac{1}{56}$

3. 

$\cos\theta = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\|} = \dfrac{3}{2(3)} = \dfrac{1}{2}$

$\|\underset{\sim}{u}+\underset{\sim}{v}\| = \sqrt{\left(2+3\left(\tfrac{1}{2}\right)\right)^2 + \left(3\sin\left(\cos^{-1}\left(\tfrac{1}{2}\right)\right)\right)^2} = 4.36$

4. $\|\underset{\sim}{u}+\underset{\sim}{v}\|^2 = (\underset{\sim}{u}+\underset{\sim}{v})\cdot(\underset{\sim}{u}+\underset{\sim}{v}) = \underset{\sim}{u}\cdot\underset{\sim}{u} + 2\underset{\sim}{u}\cdot\underset{\sim}{v} + \underset{\sim}{v}\cdot\underset{\sim}{v} = 4$ — (1)

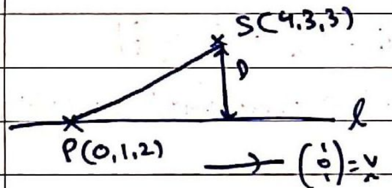
$\|\underset{\sim}{u}-\underset{\sim}{v}\|^2 = (\underset{\sim}{u}-\underset{\sim}{v})\cdot(\underset{\sim}{u}-\underset{\sim}{v}) = \underset{\sim}{u}\cdot\underset{\sim}{u} - 2\underset{\sim}{u}\cdot\underset{\sim}{v} + \underset{\sim}{v}\cdot\underset{\sim}{v} = 81$ — (2)

(1) − (2) $\Rightarrow 4\underset{\sim}{u}\cdot\underset{\sim}{v} = 4-81 = -77$

$\underset{\sim}{u}\cdot\underset{\sim}{v} = -\dfrac{77}{4}$

5. $\underset{\sim}{a}\times\underset{\sim}{b} = \begin{pmatrix}-3\\-1\\0\end{pmatrix}\times\begin{pmatrix}3\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\3\\0\end{pmatrix} = -1\begin{pmatrix}1\\-3\\0\end{pmatrix}$, since $x>0$

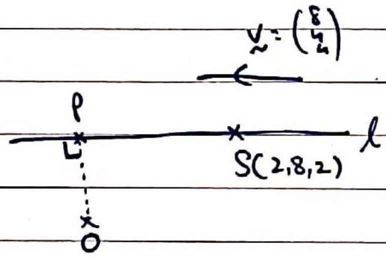
$\underset{\sim}{c} = \dfrac{1}{\sqrt{1^2+(-3)^2+0^2}}\begin{pmatrix}1\\-3\\0\end{pmatrix} = \dfrac{1}{\sqrt{10}}\begin{pmatrix}1\\-3\\0\end{pmatrix} = \begin{pmatrix}1/\sqrt{10}\\-3/\sqrt{10}\\0\end{pmatrix}$

6. 

$\overrightarrow{PS} = \begin{pmatrix}4\\3\\3\end{pmatrix} - \begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}4\\2\\1\end{pmatrix}$

$D = \dfrac{\|\overrightarrow{PS}\times\underset{\sim}{v}\|}{\|\underset{\sim}{v}\|} = \dfrac{\left\|\begin{pmatrix}4\\2\\1\end{pmatrix}\times\begin{pmatrix}1\\0\\1\end{pmatrix}\right\|}{\sqrt{2}} = \dfrac{\left\|\begin{pmatrix}2\\-3\\-2\end{pmatrix}\right\|}{\sqrt{2}} = \dfrac{\sqrt{2^2+(-3)^2+(-2)^2}}{\sqrt{2}}$

$= \dfrac{\sqrt{17}}{\sqrt{2}} = 2.92$

7. 

$\|\overrightarrow{SP}\| = \dfrac{\overrightarrow{SO}\cdot\underset{\sim}{v}}{\|\underset{\sim}{v}\|} = \dfrac{\begin{pmatrix}-2\\-8\\-2\end{pmatrix}\cdot\begin{pmatrix}8\\4\\4\end{pmatrix}}{\sqrt{8^2+4^2+4^2}} = \dfrac{-56}{\sqrt{96}}$

$P = S + \left(\dfrac{-56}{\sqrt{96}}\right)\left(\dfrac{1}{\sqrt{96}}\right)\begin{pmatrix}8\\4\\4\end{pmatrix} = \begin{pmatrix}2\\8\\2\end{pmatrix} - \dfrac{56}{96}\begin{pmatrix}8\\4\\4\end{pmatrix} = \begin{pmatrix}-8/3\\17/3\\-1/3\end{pmatrix}$

8. From equation $(0,0,\frac{1}{2})$ lies on plane.

$$\text{dist} = \frac{\begin{pmatrix}0\\0\\0.5\end{pmatrix}\cdot\begin{pmatrix}3\\2\\2\end{pmatrix}}{\sqrt{3^2+2^2+2^2}} = \frac{1}{\sqrt{17}} = 0.24$$

9. $$\alpha = \frac{\begin{pmatrix}2\\2\\2\end{pmatrix}\cdot\begin{pmatrix}5\\5\\-4\end{pmatrix}}{\sqrt{2^2\times 3}\,\sqrt{5^2+5^2+(-4)^2}} = \frac{10+10-8}{\sqrt{12}\sqrt{66}} = \frac{12}{\sqrt{12}\sqrt{66}} > 0$$

$$\therefore,\ \theta = \alpha = \cos^{-1}\left(\frac{12}{\sqrt{12}\sqrt{66}}\right) = 1.13 \text{ rad}$$

10. A vector // plane $= \begin{pmatrix}4\\8\\7\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}3\\6\\4\end{pmatrix}$

normal to plane $= \begin{pmatrix}3\\6\\4\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\-3\end{pmatrix} = 2\begin{pmatrix}1\\0.5\\-1.5\end{pmatrix}$ since coeff of $x=1 \Rightarrow a=\frac{1}{2},\ b=-\frac{3}{2}$

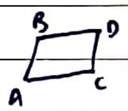
$c = \begin{pmatrix}1\\0.5\\-1.5\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix} = 1+1-4.5 = -\frac{5}{2}$

11. normal to new plane $= \begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}5\\8\\10\end{pmatrix} = \begin{pmatrix}2\\-5\\3\end{pmatrix} = 2\begin{pmatrix}1\\-2.5\\1.5\end{pmatrix}$ since coeff of $x=1 \Rightarrow a=-2.5,\ b=1.5$

$c = \begin{pmatrix}5\\8\\9\end{pmatrix}\cdot\begin{pmatrix}1\\-2.5\\1.5\end{pmatrix} = -4$

12. 1st plane contains $(0,0,0)$ & 2nd plane contains $(0,0,\frac{d}{3})$

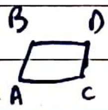
$$\frac{\begin{pmatrix}0\\0\\d/3\end{pmatrix}\cdot\begin{pmatrix}7\\8\\3\end{pmatrix}}{\sqrt{7^2+8^2+3^2}} = \frac{d}{\sqrt{122}} = 14 \qquad d = 14\sqrt{122} = 154.64$$

13. 

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}0\\4\\0\end{pmatrix} - \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}-2\\4\\0\end{pmatrix}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} \Rightarrow \overrightarrow{OD} = \overrightarrow{CD} + \overrightarrow{OC} = \overrightarrow{AB} + \overrightarrow{OC} = \begin{pmatrix}-2\\4\\0\end{pmatrix} + \begin{pmatrix}0\\0\\3\end{pmatrix} = \begin{pmatrix}-2\\4\\3\end{pmatrix}$

D is $(-2,4,3)$

14. 

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}0\\9\\0\end{pmatrix} - \begin{pmatrix}6\\0\\0\end{pmatrix} = \begin{pmatrix}-6\\9\\0\end{pmatrix}$ $\quad \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix}0\\0\\4\end{pmatrix} - \begin{pmatrix}6\\0\\0\end{pmatrix} = \begin{pmatrix}-6\\0\\4\end{pmatrix}$

$\text{Area} = \|\overrightarrow{AB}\times\overrightarrow{AC}\| = \left\|\begin{pmatrix}-6\\9\\0\end{pmatrix}\times\begin{pmatrix}-6\\0\\4\end{pmatrix}\right\| = \left\|\begin{pmatrix}36\\24\\54\end{pmatrix}\right\| = \sqrt{36^2+24^2+54^2} = 69.20$

15. Let $\underset{\sim}{v} = \overrightarrow{AB}\times\overrightarrow{AC} = \begin{pmatrix}2\\6\\0\end{pmatrix}\times\begin{pmatrix}0\\-4\\3\end{pmatrix} = \begin{pmatrix}30\\-10\\-8\end{pmatrix}$

$\text{Volume} = |\overrightarrow{AD}\cdot\underset{\sim}{v}| = \left|\begin{pmatrix}30\\-10\\-8\end{pmatrix}\cdot\begin{pmatrix}3\\-5\\7\end{pmatrix}\right| = |90+50-56| = 84$