SC4000 Machine Learning Tutorial
Support Vector Machines and Linear Regression

Question 1: Consider a 2-dimensional dataset for two-class classification by SVM, as shown in Figure 1, where the red "square" and blue "circle" denote the positive and negative classes respectively. Is this dataset separable by a linear SVM classifier? If no, why? If yes, what is the decision boundary of the linear SVM? And what are the pair of parallel hyperplanes associated with the decision boundary? (No need to provide proofs)

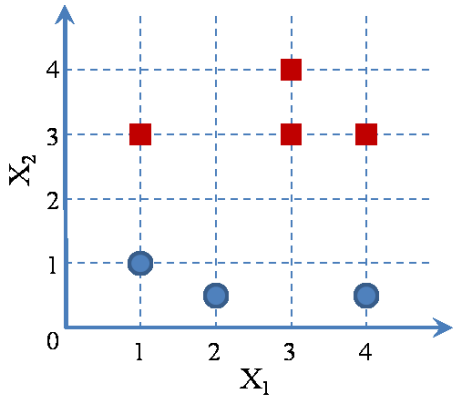

Figure 1: Dataset for Question 1.

Question 2: Please induce why the two parallel hyperplanes of a decision boundary,

$$\mathbf{w} \cdot \mathbf{x} + b = 0,$$

can be written as

$$\mathbf{w} \cdot \mathbf{x} + b = \bar{k}, \text{ and } \mathbf{w} \cdot \mathbf{x} + b = -\bar{k},$$

where $\bar{k} > 0$, respectively.

Question 3: In an SVM, we have two support vectors $(2, 3)$ from class 1 and $(-1, 4)$ from class 2. The separating hyperplane can be written as $3x_1 + x_2 + b = 0$. Compute the margin of separation.

Question 4: In the slide deck of Lecture 8b, we presented the closed-form solution for the regularized regression model,

$$\boldsymbol{w} = (\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I})^{-1}\boldsymbol{X}^T\boldsymbol{y}. \tag{1}$$

Please derive this solution and show the steps.