NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005 Artificial Intelligence

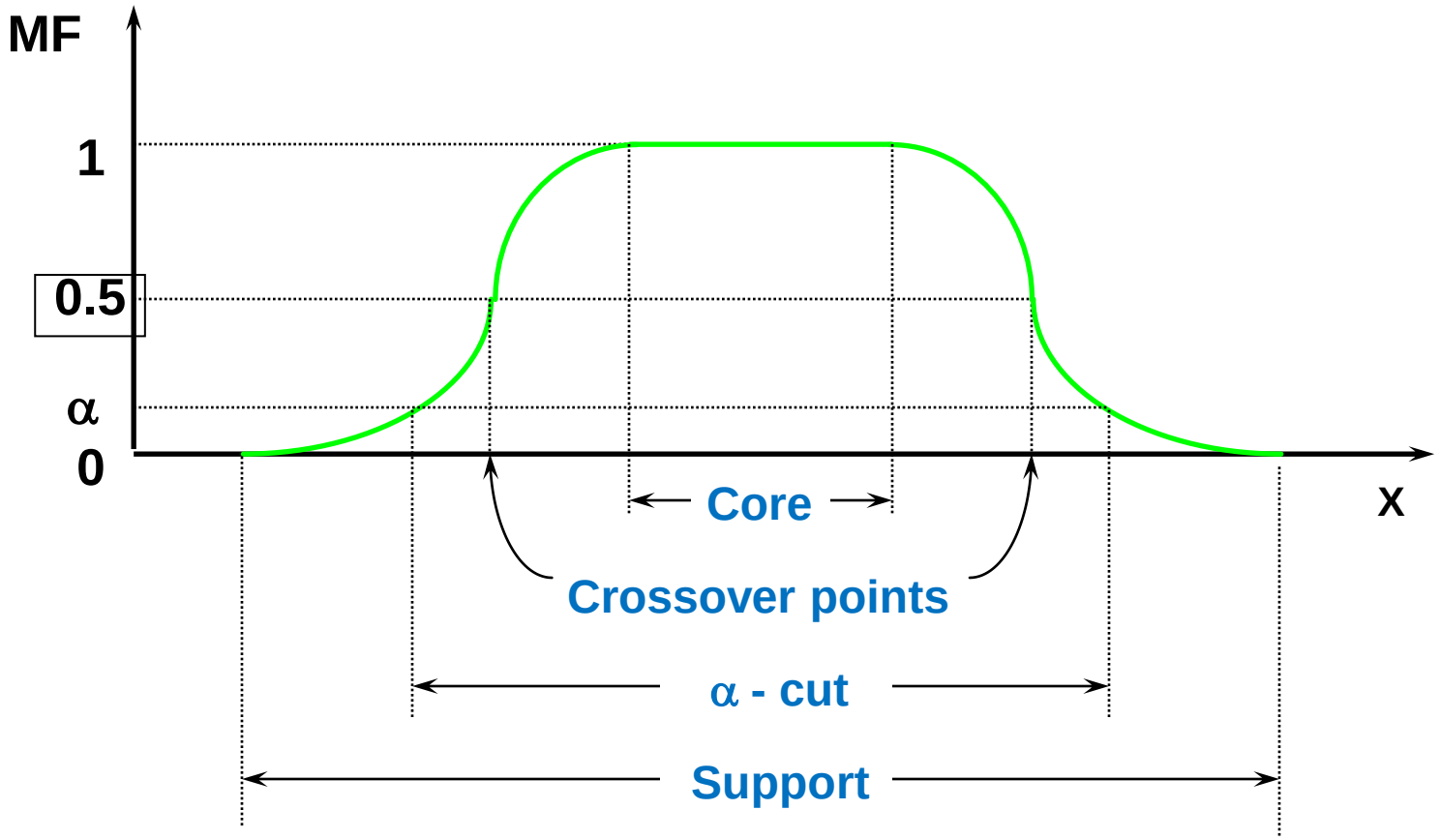
Week 12b – Fuzzy Logic

Yu Han

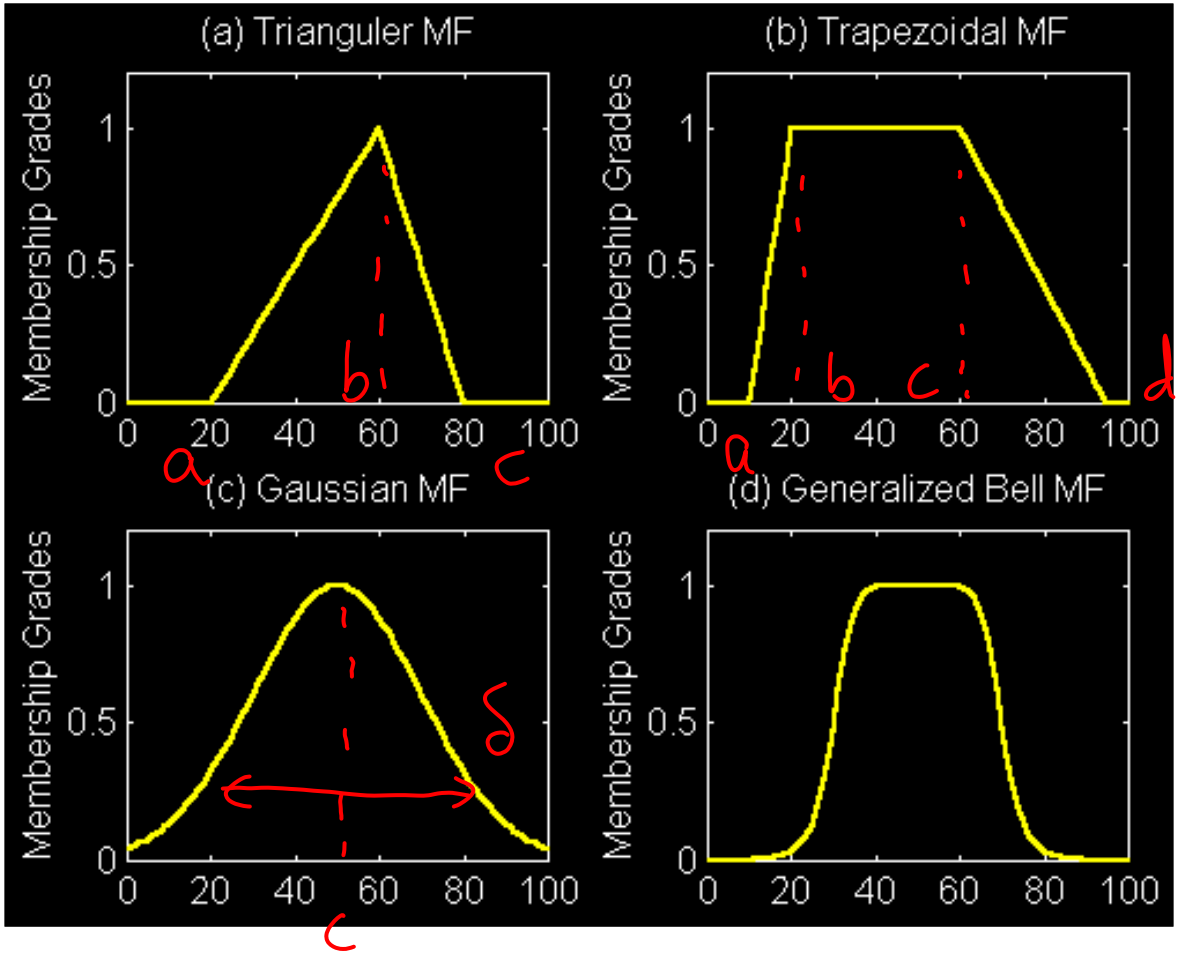
han.yu@ntu.edu.sg

Nanyang Assistant Professor
School of Computer Science and Engineering
Nanyang Technological University

Recap – MF Terminology

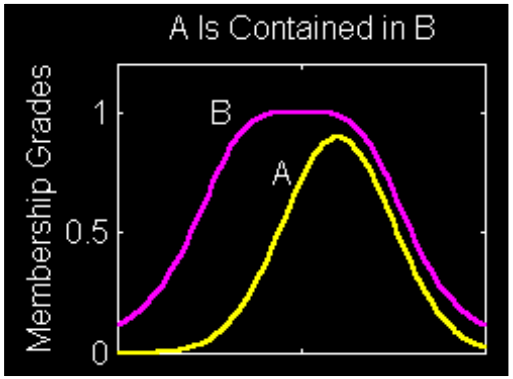


Recap – MF Formulation



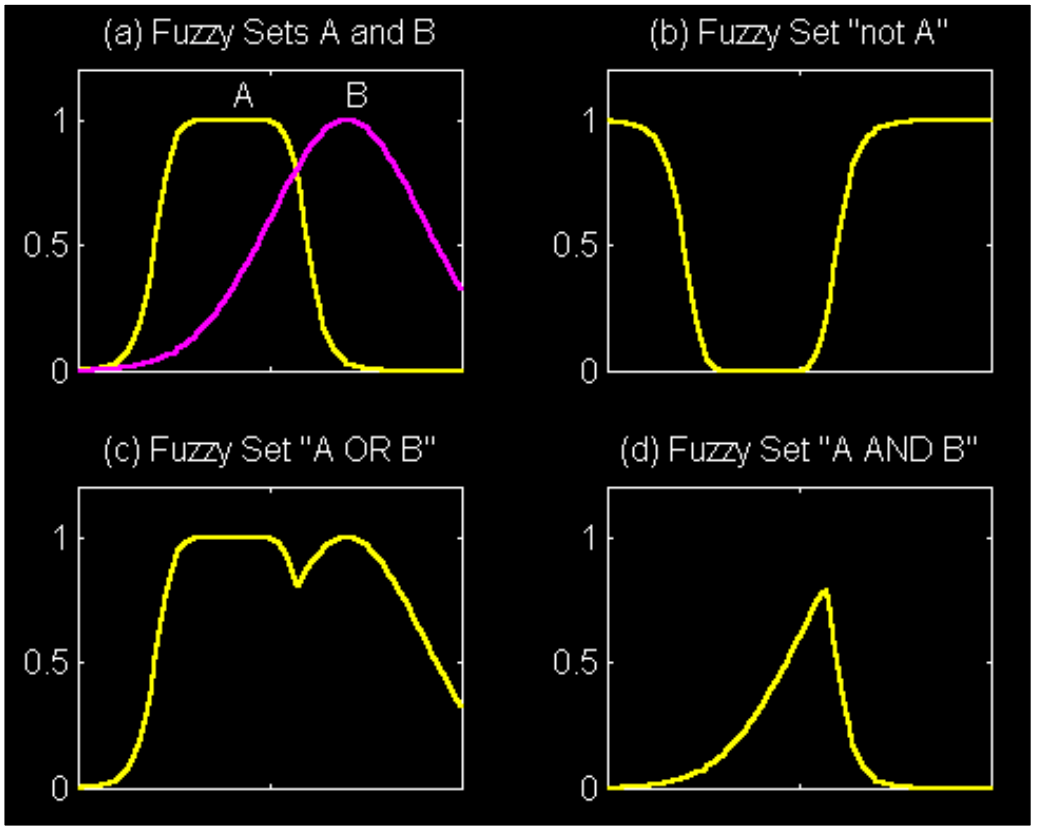
Recap – Set-Theoretic Operations

$1-\mu_A(x)$ **Complement**



$\mu_A \leq \mu_B$

Subset



$\max(\mu_A(x), \mu_B(x))$

Union

$\min(\mu_A(x), \mu_B(x))$

Intersection

Learning Goals

Understanding the:

- Linguistic modifier/hedges

- More on fuzzy union, intersection, and complement
  - Fuzzy complement
  - Fuzzy intersection and union
  - Parameterized T-norm and T-conorm

- Fuzzy Rule Based System
  - Fuzzy Rule, Fuzzy Inference, and defuzzification

Linguistic Hedge - Modifiers

- Linguistic Hedge/Modifiers are operations that modify the meaning of a term – fuzzy label (fuzzy set).
  - ‘‘*very Tall*’’, the word *very* modifies *“TALL”* which is a fuzzy set.

- Other modifiers are:
  - “more or less” (morl), “possibly”, and “definitely”

Linguistic Hedge – Modifiers

- very **a** = $\mathbf{a}^2$
- morl **a** = $\mathbf{a}^{0.5}$
- extremely **a** = $\mathbf{a}^3$
- slightly **a** = $\mathbf{a}^{0.333}$
- somewhat **a** = morl **a** and not slightly **a**

E.g., young = [1/0, 0.6/20, 0.1/40, 0.0/60, 0.0/80]

very young = young^2

= [1/0, 0.36/20, 0.01/40, 0.0/60, 0.0/80]

Summary of Membership Functions

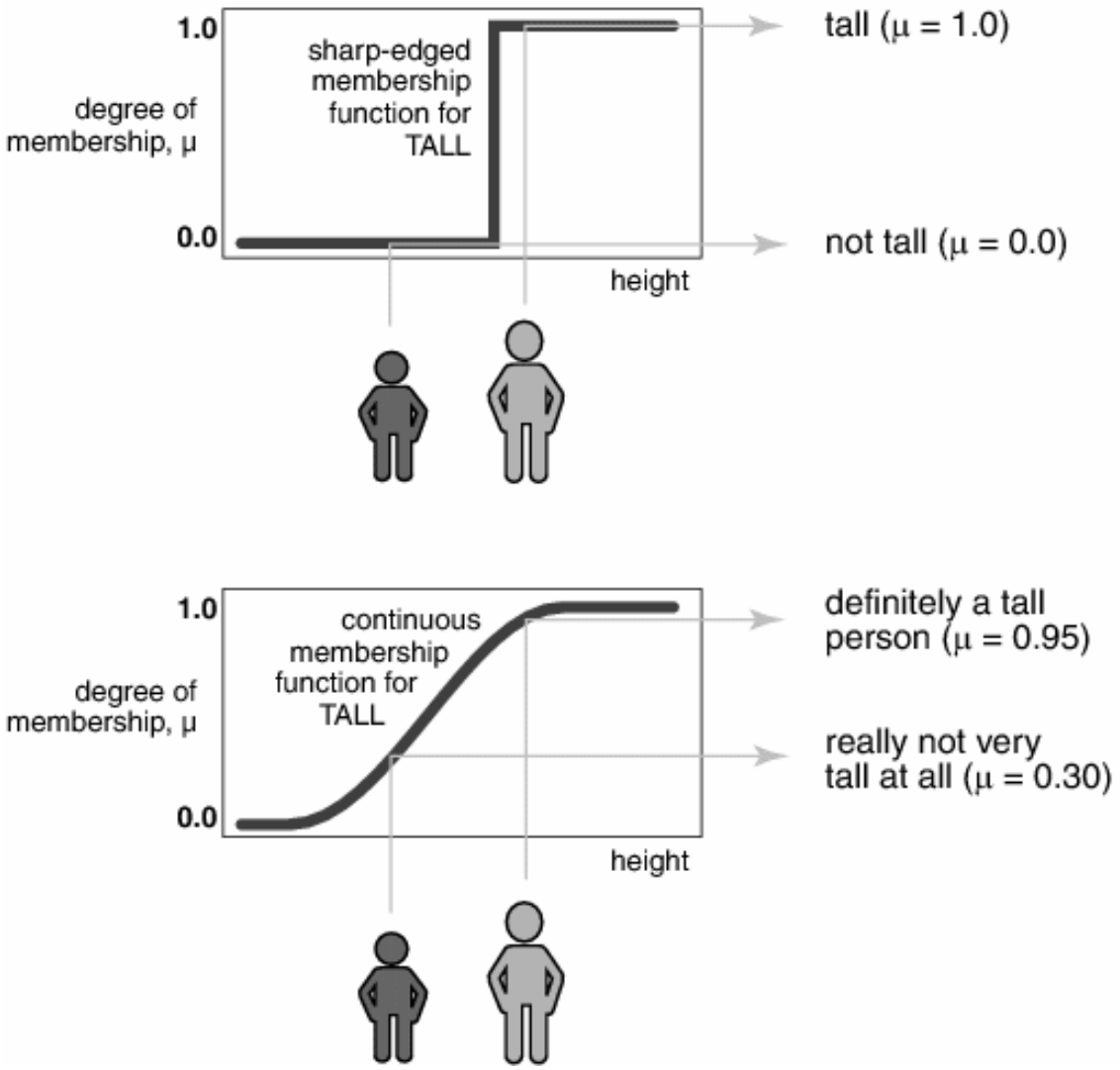
- Fuzzy sets allow the description of vague concepts (e.g., *SLOW, MEDIUM* and *FAST*) for a fuzzy variable (e.g., *SPEED*)

- This provides the semantics (concepts) to linguistic rules involving fuzzy variables.

  e.g., The *SPEED* is *FAST*.

- The fuzzy set admits the possibility of partial memberships in it. (e.g., Friday is sort of a weekend day, the weather is rather hot).

Semantic Meaning & Partial Membership

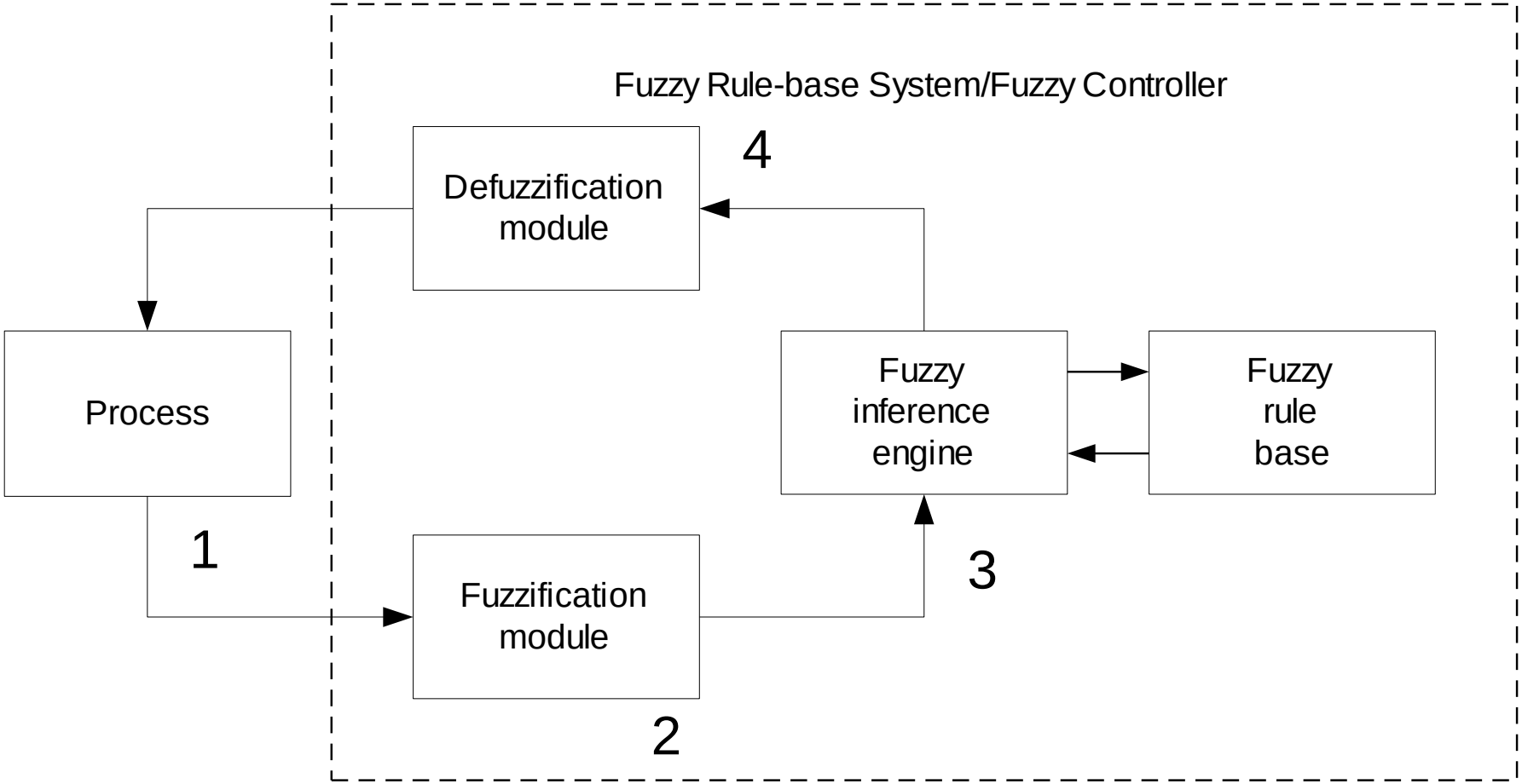


A person who is 180 cm is tall

A person who is 179 cm is **NOT** tall!

A person who is 180 cm is tall **to a higher degree than** another person who is 179 cm

Fuzzy Rule-Based (FRB) Systems



Operation of FRB Systems

1. Firstly, measurements are taken of all variables from the process.

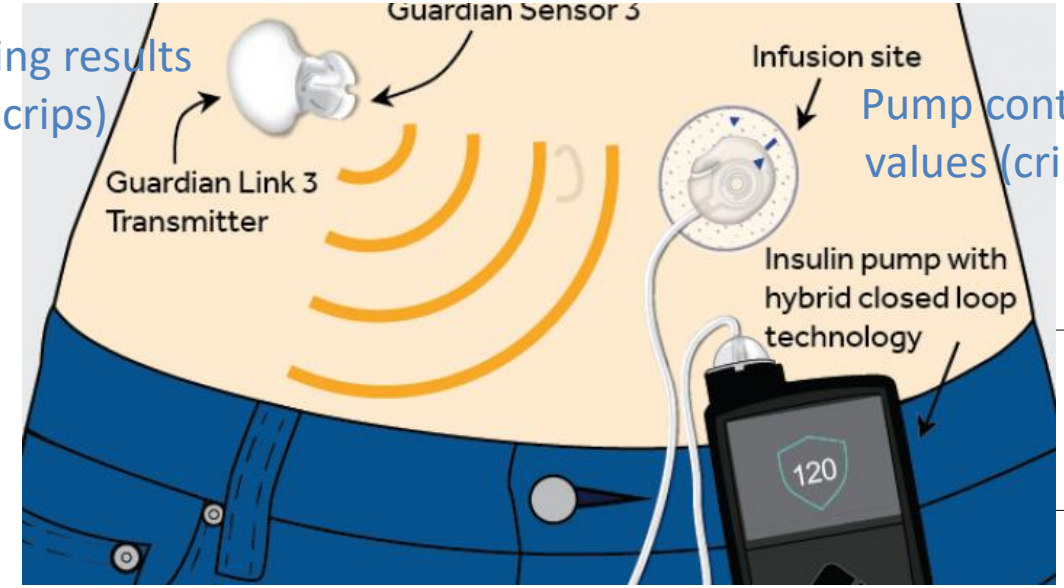
2. Next, these measurements are converted into appropriate fuzzy sets to express measurement uncertainties - fuzzification.

3. The fuzzified measurements are then used by the inference engine to evaluate the control rules stored in the **fuzzy rule base**. - (fuzzy rules defined with fuzzy (linguistic) variables using fuzzy labels – fuzzy sets)

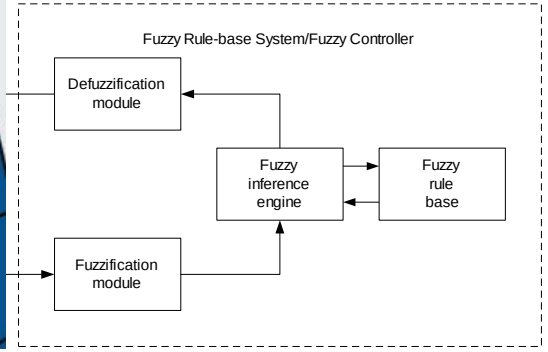
4. The result of this evaluation is one or several fuzzy sets defined on the universe of possible control actions. This fuzzy set is then converted, in the final step of the cycle, into a single crisp value or a vector of values which best represents the resulting fuzzy set or sets – defuzzification.

Example FRB Continuous Insulin Pump

Sensing results (crips)



Pump control values (crips)



Components of a Fuzzy Rule

- A single fuzzy if – then rule:
  - If *x* is *A* then *y* is *B*.

A and *B* are linguistic values defined by fuzzy sets on the range of discourse for fuzzy variables *x* and *y*.

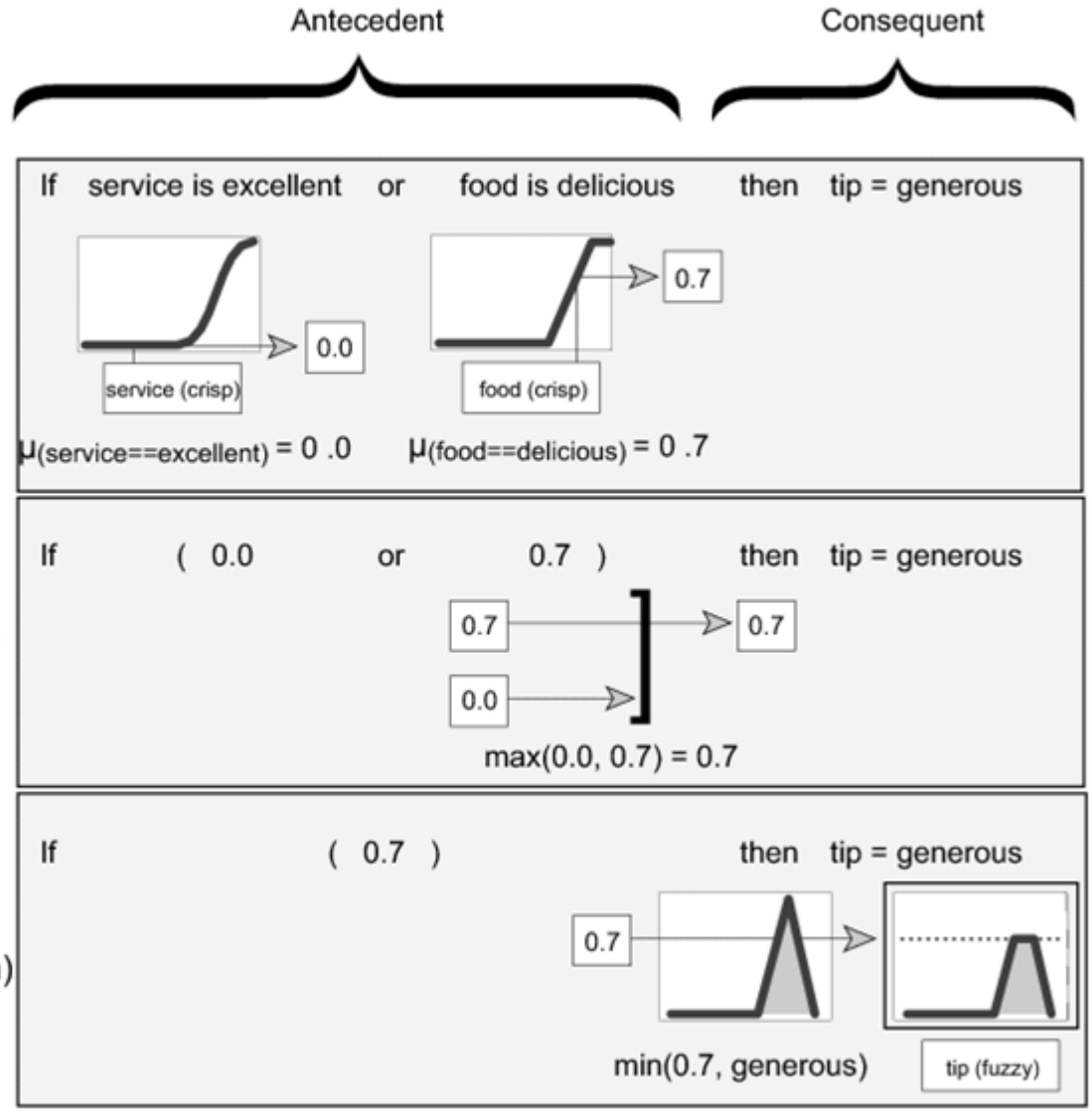
E.g., If *service* is *good* then *tips* is *average*

x *A* *y* *B*

Interpreting fuzzy if-then rule:

- Evaluating antecedent *(fuzzifying input and necessary fuzzy operators)*
- Applying the result to the conclusion/consequent *(known as implication)*

Fuzzy Inference – Example



Steps of Fuzzy Inference

1. **Fuzzify inputs**: Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1

2. **Apply Fuzzy Operators**: If there are multiple parts to the antecedent, apply fuzzy logic operators (AND, OR, etc.) and resolve the antecedent to a single number between 0 and 1.

3. **Apply Implication Method**: Use the degree of support for the entire rule to shape the output fuzzy set. The consequent of a fuzzy rule assigns an entire fuzzy set to the output. This fuzzy set is represented by a membership function that is chosen to indicate the qualities of the consequent.

4. **Aggregation** of the consequents across all rules (if there are multiple rules).

5. **Defuzzification**

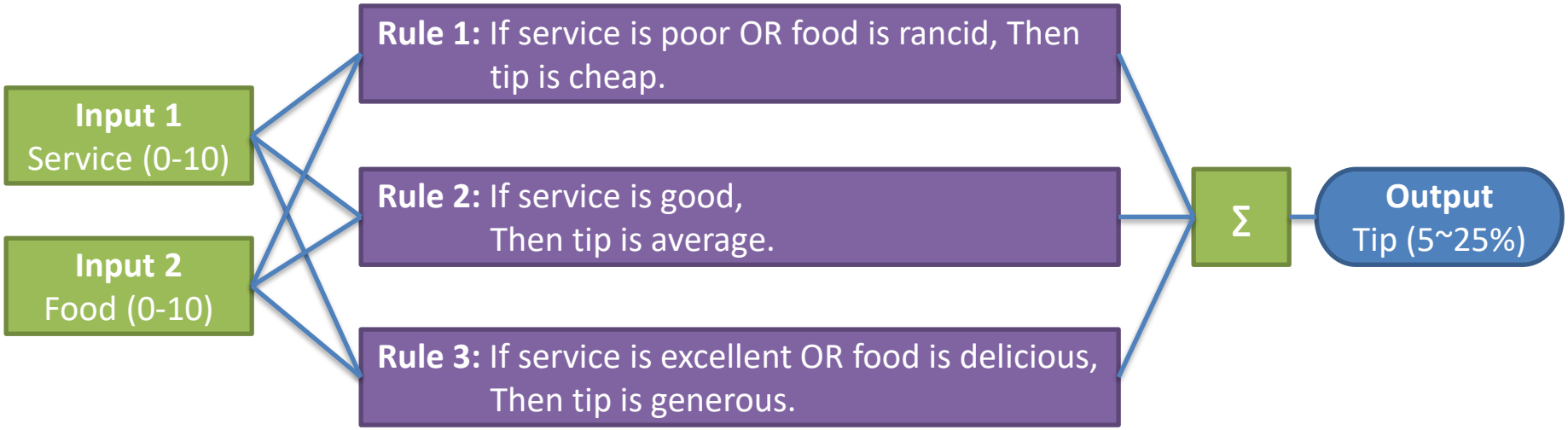
Fuzzy Inference – Cont'd

- Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic.

- The process of fuzzy inference involves all of the pieces that are described in the previous sections:
  - Membership Functions
  - Logical Operations
  - If-Then Rules

Example – Multiple Rules

Consider the example of the service for a dinner for two



The inputs are crips (non-fuzzy) numbers limited to a specific range.

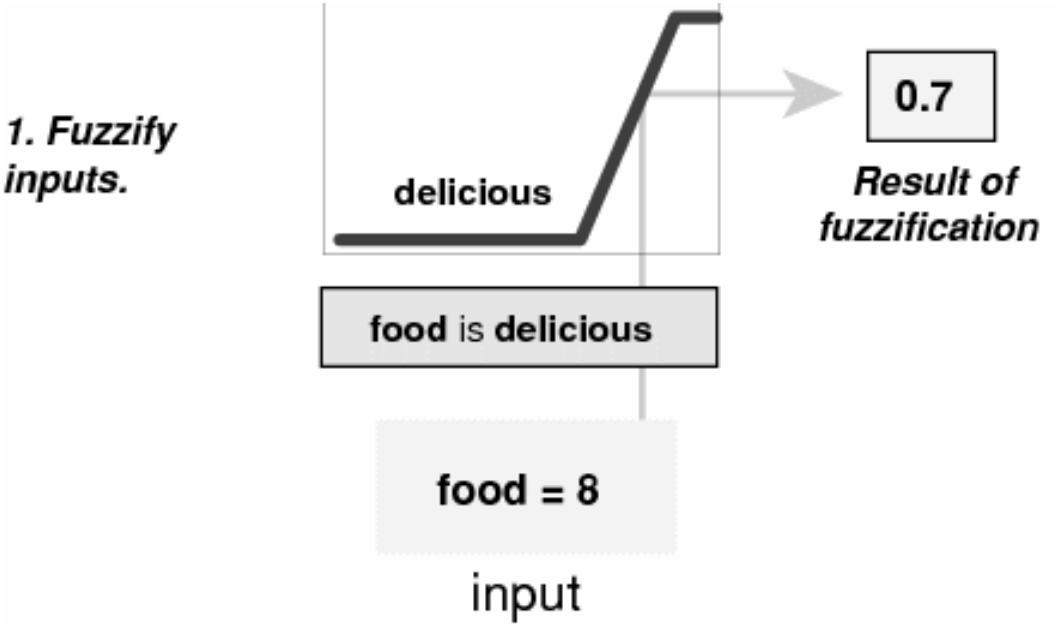
All rules are evaluated in parallel using fuzzy reasoning.

The results of the rules are combined and distilled (defuzzified).

The result is a crips (non-fuzzy) number.

Step 1: Fuzzification

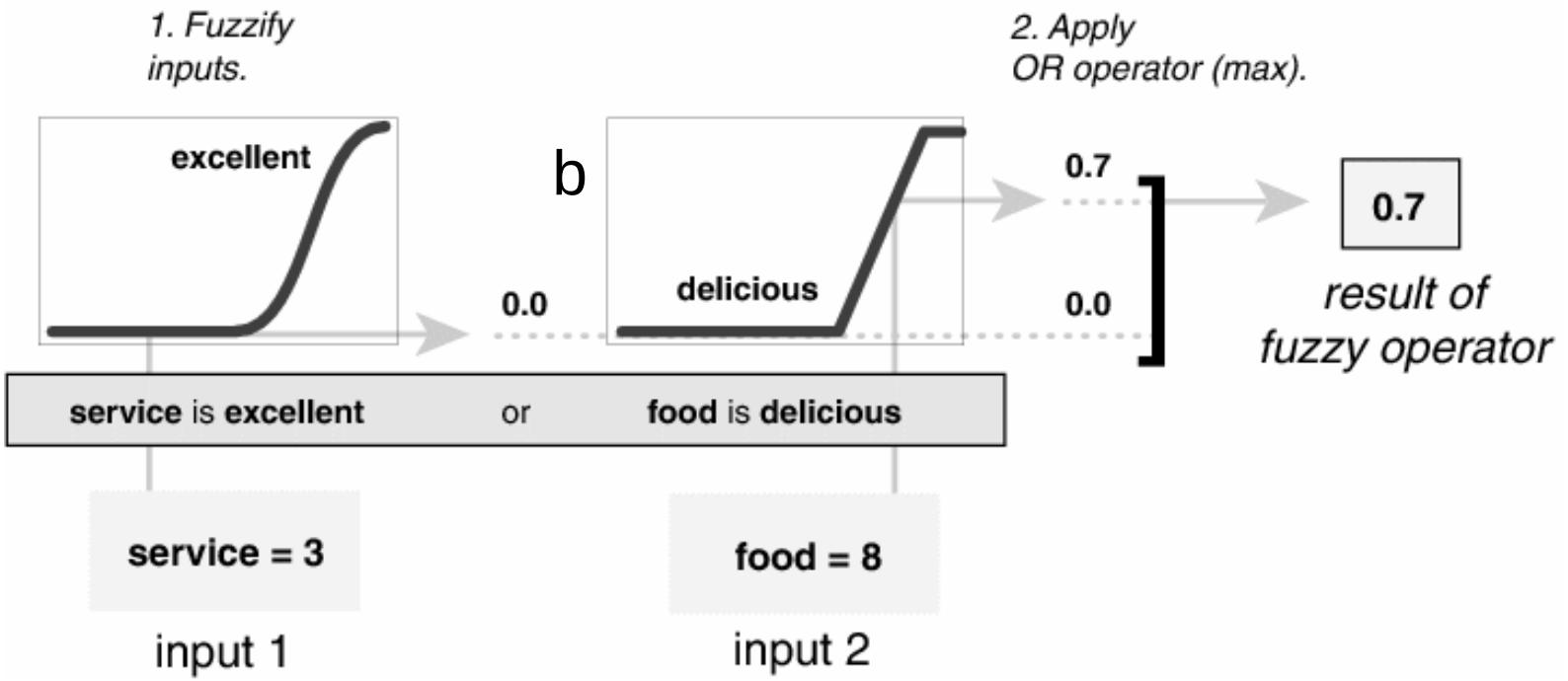
- Fuzzification of the input amounts to either a table lookup or a function evaluation.



- In this manner, each input is fuzzified over all the qualifying membership functions required by the rules.

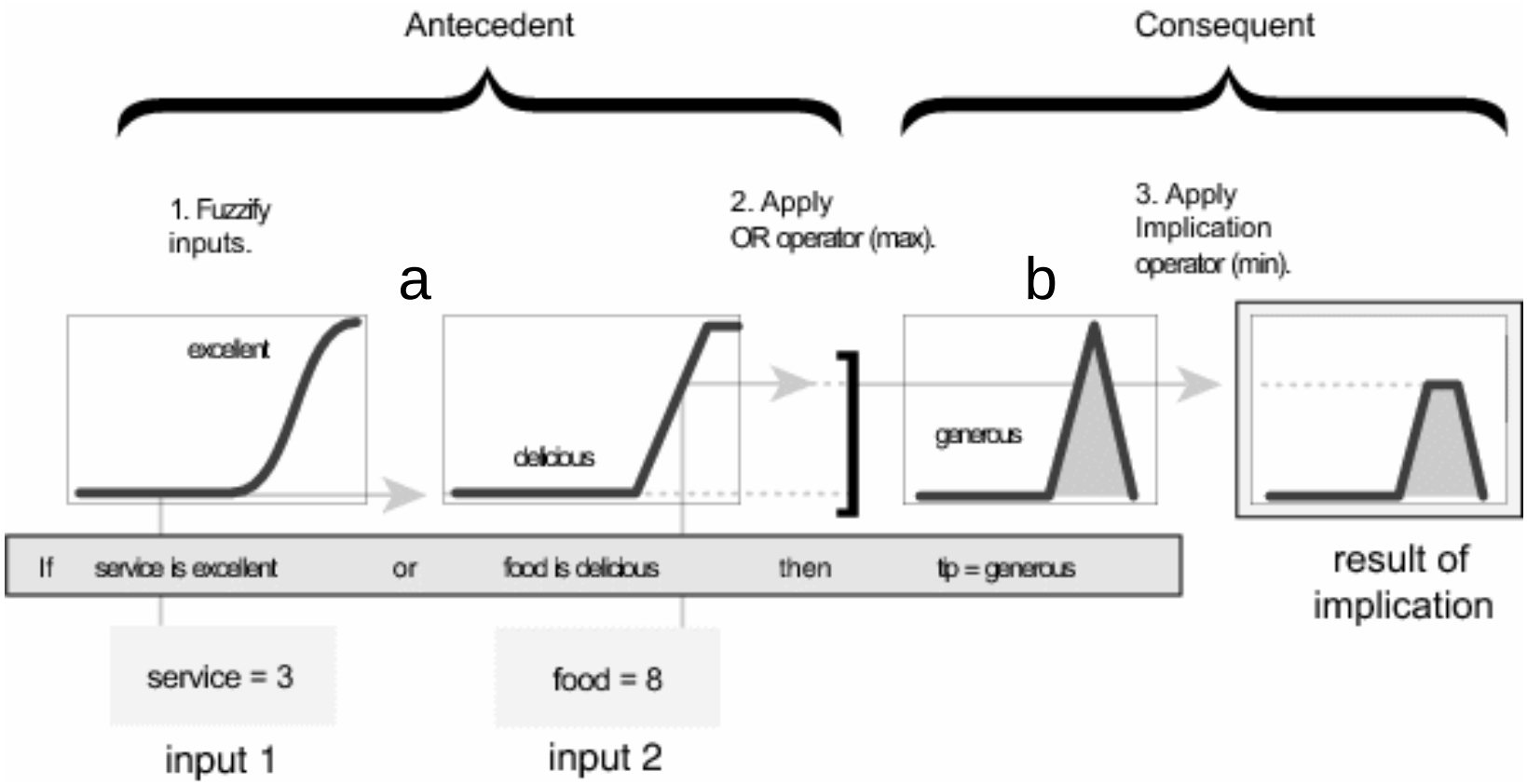
Step 2: Applying Fuzzy Operators

- OR - max(a, b)



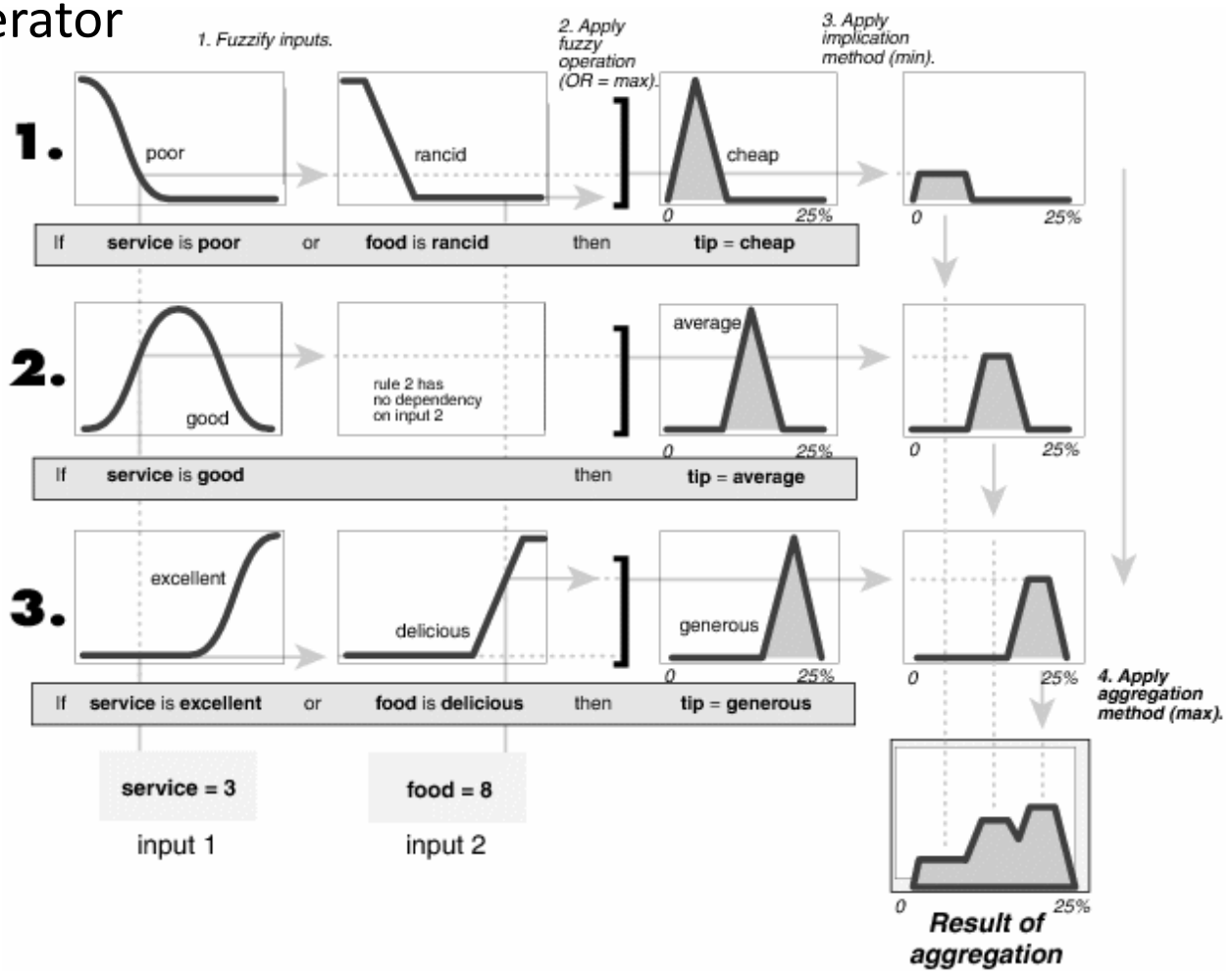
Step 3: Applying Implication Method

- min(a, b), Rule 3:



Step 4: Aggregating All Outputs (3 Rules)

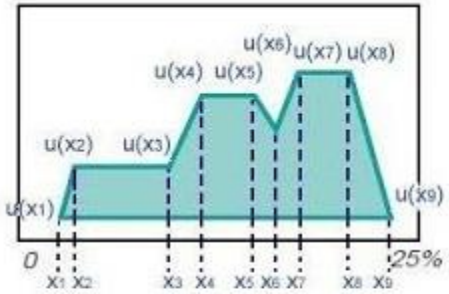
- Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set.
- E.g., max operator



Step 5: Defuzzification

- The input for the defuzzification process is a fuzzy set (the aggregate output fuzzy set)
- The output is a single number.

5. Defuzzify the aggregate output (centroid)



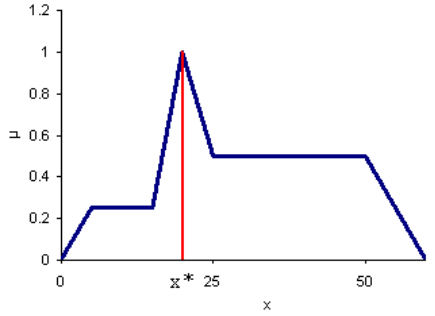
$$g = \frac{\sum_{i=1}^{9} x_i \cdot u(x_i)}{\sum_{i=1}^{9} u(x_i)} = 16.7$$

Result of defuzzification

- The final tip to pay for a rating of service=3 and food=8 is 16.7%.

Types of Defuzzifications

- Max-membership $\sup_x(\mu(X))$ defuzzification method



- Centroid defuzzification

$$x^* = \frac{\int \mu_i(x)\, x\, dx}{\int \mu_i(x)\, dx}$$

- Weighted Average Defuzzification Technique

$$x^* = \frac{\sum_{i=1}^{n} m^i w_i}{\sum_{i=1}^{n} m^i}$$

Thank you!