

# Three-dimensional shape measurement with binary dithered patterns

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The previously proposed binary defocusing technique and its variations have proven successful for high-quality three-dimensional (3D) shape measurement when fringe stripes are relatively narrow, but they suffer if fringe stripes are wide. This paper proposes to utilize the binary dithering technique to conquer this challenge. Both simulation and experimental results show the phase error is always less than 0.6% even when the fringe stripes are wide and the projector is nearly focused. © 2012 Optical Society of America

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## 1. Introduction

The recently proposed binary defocusing technique [1] and its variations [2,3] have demonstrated their superiorities over conventional sinusoidal fringe projection techniques for three-dimensional (3D) shape measurement. Yet, it is very difficult for them to produce high-quality 3D shape measurement when the fringe stripes are wide and when the projector is nearly focused [4]. This is because for the binary defocusing techniques, binary patterns are fed to the projector and sinusoidal patterns are generated by properly defocusing the projector. However, due to the hardware limitation (projector lens can only be changed to a certain level), when the fringe stripes are wide, it is very difficult to generate high-quality sinusoidal patterns from these binary patterns through defocusing.

The aforementioned improved binary defocusing techniques [2,3] are, after all, modifying the patterns in one dimension; yet the desired sinusoidal fringe patterns are two-dimensional (2D) in nature. This indicates that if the binary patterns are modified in both dimensions, better sinusoidal patterns could be generated. Xian and Su demonstrated the

area-modulated patterns can indeed generate high-quality sinusoidal fringe patterns with a precise micromanufacturing technique [5]. However, this technique requires the cell to be about 1  $\mu\text{m}$ , which is still difficult to realize in a digital fringe projection system as the pixels are typically about 10 times larger. Lohry and Zhang proposed to approximate the triangular waveform by modifying  $2 \times 2$  pixels [6], which showed the promise of improving 3D shape measurement quality when fringe stripes are narrow, but suffers if the fringe stripes are wide.

Dithering [7], also called halftoning, is the process of representing an image with fewer colors or bits than they are in the real image. This technique has been extensively employed in digital signal (e.g., audio, video) processing. Figure 1 shows the binary dithered images and their original grayscale ones. It indicates that the dithered images could represent the original images with good quality; and the binary pattern appears sinusoidal even before any processing. However, to our knowledge, there is no prior attempt to apply such a technique to 3D shape measurement. This paper is thus the first to introduce the dithering technique to this community.

We propose to apply the dithering technique to optimize binary patterns so that high-quality fringe patterns can be generated even when the stripes

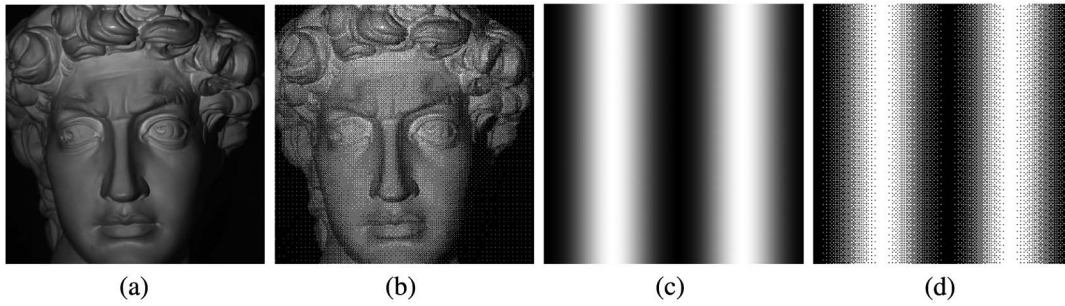


Fig. 1. Binary dithering technique for 8 bit grayscale images. (a) Original 8 bit image of David head. (b) Binary dithered image of (a). (c) Original 8 bit sinusoidal structured pattern. (d) Binary dithered pattern of (c).

are very wide. Specifically, the desired sinusoidal fringe patterns are binarized using the ordered-dithering algorithm proposed by Bayer [8]. These binary dithered structured patterns are then sent to a nearly focused or slightly defocused projector to produce ideal sinusoidal patterns, and then a fringe analysis technique can be used for high-quality 3D shape measurement. It is important to note that this paper only focuses on generating high-quality sinusoidal patterns with wide stripes for 3D shape measurement to overcome the problem of the existing binary defocusing technique, though the narrow ones can also be realized.

Section 2 presents the phase-shifting algorithm we are going to use. Section 4 shows some simulation results. Section 5 shows some experimental results, and Section 6 summarizes this paper.

## 2. Phase-Shifting Algorithm

We used a five-step phase-shifting algorithm to verify the performance of the the proposed technique. For a five-step phase-shifting algorithm with a phase shift of  $2\pi/5$ , the fringe patterns can be described as

$$I_n(x, y) = I'(x, y) + I''(x, y) \cos(\phi + 2\pi n/5), \quad (1)$$

where  $n = 1, 2, \dots, 5$ ,  $I'(x, y)$  is the average intensity,  $I''(x, y)$  is the intensity modulation, and  $\phi(x, y)$  is the phase to be solved for using the following equation:

$$\phi(x, y) = \tan^{-1} \left[ \frac{\sum_{n=1}^5 I_n(x, y) \sin(2\pi n/5)}{\sum_{n=1}^5 I_n(x, y) \cos(2\pi n/5)} \right]. \quad (2)$$

The arctangent function will result a value ranging  $[-\pi, +\pi)$  with  $2\pi$  discontinuities. A spatial or temporal phase unwrapping algorithm can be used to remove the  $2\pi$  discontinuities, and then 3D shape can be reconstructed from the unwrapped phase if the system is calibrated.

## 3. Binary Dithering Technique

Conventionally, sinusoidal patterns are generated by the computer and projected by the projector in a digital fringe projection system. Recently, our research showed that the binary patterns can also be used for high-quality 3D shape measurement when the projector is properly defocused. This technique is

called the binary defocusing technique. The binary defocusing technique has many potential advantages: (1) no requirement for gamma calibration since only two grayscale values are used [9], (2) no strict requirement for camera exposure time even when the digital-light-processing (DLP) projector is utilized [10], and (3) possibilities of achieving unprecedentedly high-speed 3D shape measurement [11]. However, the binary defocusing technique is not trouble-free, especially when the fringe stripes are wide where the high-order harmonics cannot be effectively suppressed by defocusing due to the limitation of the projector lens [4].

Dithering, which is one technique used to achieve satisfactory image rendering and color reduction, has the potential to tackle the aforementioned problem of the squared binary technique. A structured sinusoidal fringe pattern is typically stored as an 8 bit image with 256 grayscale values. In contrast, a binary pattern is a 1 bit image that only requires two grayscale values to represent. To approximate a sinusoidal pattern with a binary pattern, various dithering techniques can be used, such as the simple thresholding [12], the random dithering [12], the Bayer-ordered dithering [8], and the error-diffusion dithering [13] techniques. For the simple thresholding method [12], each pixel grayscale value is compared against a fixed threshold value: if it is larger than the threshold, the pixel is set to be 1, otherwise to be 0. This technique is very simple, yet it results in immense loss of details and contouring. The random dithering technique [12] could alleviate the problem of thresholding to some extent by comparing each pixel value against a random threshold. However, this technique still cannot preserve the details of the original image. The error-diffusion dithering technique [13] involves area operation since the quantization error residuals distribute to the neighboring pixels that have not been processed. Though working well for representing the grayscale images, the algorithm is not efficient since it is difficult to operate in a parallel manner.

Due to its good performance and easy operation, the Bayer ordered dithering technique [8] was adopted in this paper. The ordered dithering consists of comparing blocks of the original image to a 2D grid of thresholds called dither patterns. Each element of

the original block is quantized according to the corresponding threshold value in the dither pattern. The values in the dither matrix are fixed, but are typically different from each other. Because the threshold changes between adjacent pixels, some decorrelation from the quantization error is achieved. Bayer has shown that if the sizes of the matrices are  $2^N$  ( $N$  is an integer), there is an optimal dither pattern that results in the pattern noise being as high-frequency as possible. It is well known that the high-frequency noise can be effectively reduced by applying a low-pass filter. The Bayer patterns can be obtained as follows:

$$M_1 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}, \quad (3)$$

which is the smallest  $2 \times 2$  base dither pattern. Larger dither patterns can be obtained using

$$M_{n+1} = \begin{bmatrix} 4M_n & 4M_n + 2U_n \\ 4M_n + 3U_n & 4M_n + U_n \end{bmatrix}, \quad (4)$$

where  $U_n$  is  $n$ -dimensional unit matrix. Using these standard dither patterns, a grayscale image can be efficiently quantized depending upon the chosen size of dither pattern.

Figure 2 shows the differences of different dithering techniques. Figure 2(b) shows the result after applying the simple thresholding technique to binarize the image. It can be seen that almost all details are lost. The random dithering technique improves the simple thresholding technique, yet a lot of details are still missing, as shown in Fig. 2(c). The ordered Bayer dithering technique can well preserve most features of the original image. Figure 2(d) shows the result after applying the ordered Bayer dithering technique. The binarized image of Fig. 2(e) using the error-diffusion dithering technique can also well represent the original image.

#### 4. Simulation Results

We firstly verified the performance of the proposed technique through simulations. Fringe patterns with wide fringe stripes are generated by the binary dithering algorithm [8]. Figure 3 shows two examples:

one fringe pattern has a fringe pitch ( $T$ ), the number of pixels per fringe period, of 600, and the other has a fringe pitch of 150. Figures 3(a) and 3(d) show the dithered patterns and their associated Gaussian smoothed patterns. It is interesting to see that even before applying a Gaussian smoothing filter, the patterns visually appear to have good sinusoidality. Even though the Gaussian filter is quite small (size of  $9 \times 9$  pixels with a standard deviation of 3.0 pixels), the resultant patterns depict clear sinusoidal structures. Figures 3(b) and 3(e) show the horizontal cross sections of the blurred patterns. To better visualize the sinusoidality of these blurred patterns, the frequency-amplitude spectra of cross sections were plotted in Figs. 3(c) and 3(f). These figures clearly show that high-frequency harmonics almost completely disappear, which means that they have good-quality sinusoidality.

We further simulated the influence of different amounts of defocusing by varying the Gaussian filter size ( $F_s$ ) with the standard deviation being always  $F_s/3$ . The phase was calculated by using the aforementioned five-step phase-shifting algorithm, and the resultant phase errors are calculated by comparing with the phase obtained from the ideal sinusoidal fringe patterns. Figure 4(c) shows the root-mean-square (rms) error percentages. The error percentages were calculated as the ratios based of rms error and the total phase for the same size image. In our case,  $T = 150$  has a total phase of  $8\pi$  and  $T = 600$  has  $2\pi$  since the image has a resolution of  $600 \times 600$ . In these simulations, we used filter sizes of  $F_s = 5, 7, 11, 15, 19, 23, 27, 39$  pixels, for levels 1 through 8. These simulations show that (1) the phase errors are all very small (approximately 0.4% for  $T = 600$  pixels) even when fringe patterns are almost in focus [top images in Figs. 4(a) and 4(b)], and (2) the phase error reduces when increasing the amount of defocusing. This means that the proposed technique can be used for large depth-range 3D shape measurement, because the measurement can start from almost in focus to significantly defocused.

#### 5. Experimental Results

We also developed a 3D shape measurement system to verify the proposed method. In this system, we

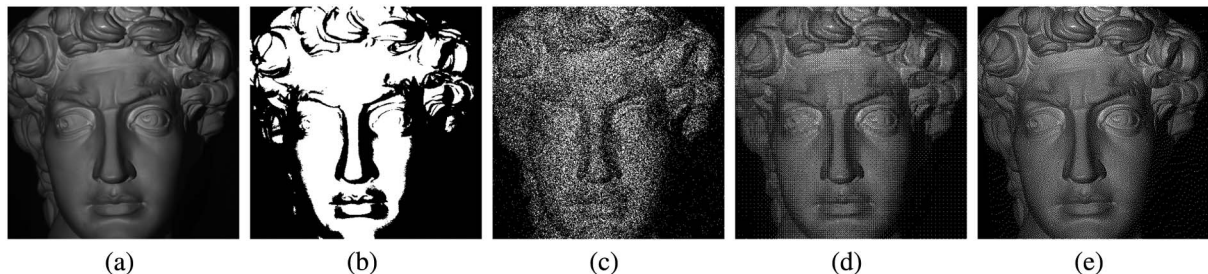


Fig. 2. Binary dithering techniques for a 8 bit grayscale image. (a) Original 8 bit image of the David head. (b) Binary dithered image of (a) by applying the thresholding technique. (c) Binary dithered image of (a) by applying random dithering technique. (d) Binary dithered image of (a) by applying the Bayer ordered dithering technique. (e) Binary dithered image of (a) by applying the error-diffusion dithering technique.

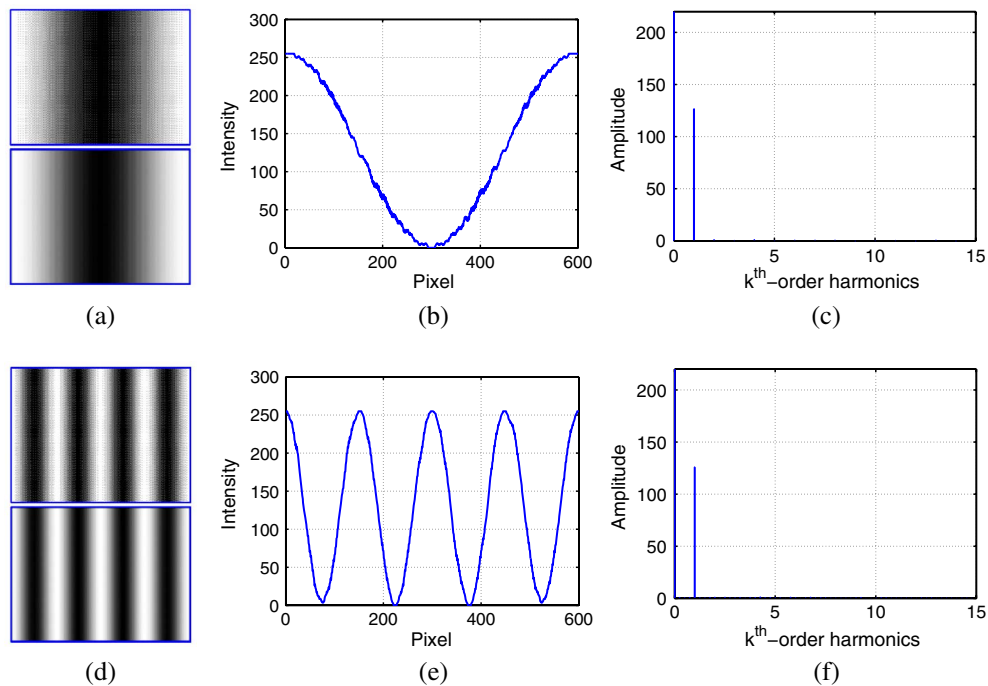


Fig. 3. (Color online) Simulation results of the proposed technique. (a) Binary dithered pattern and its smoothed pattern ( $T = 600$ ); top image shows the dithered pattern and the bottom image shows the smoothed pattern. (b) Cross section of the blurred pattern. (c) Fourier spectrum of the cross section shown in (b). (d)–(f) Corresponding results to above image when  $T = 150$ .

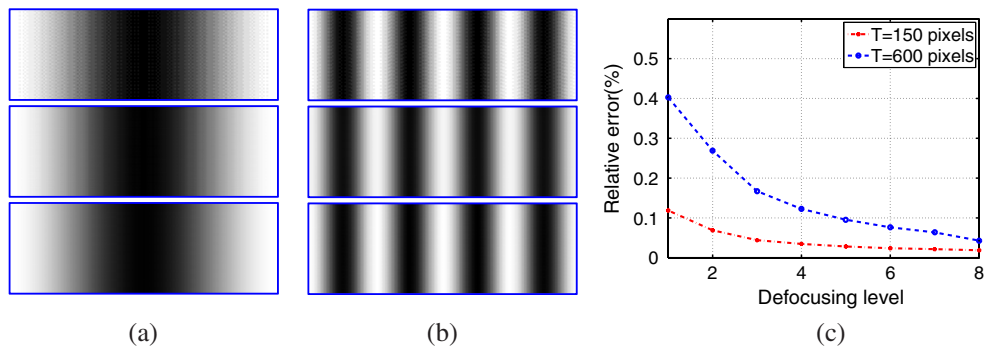


Fig. 4. (Color online) Phase errors with varying amount of defocusing by simulations. (a) Fringe patterns ( $T = 600$ ). (b) Fringe patterns ( $T = 150$ ). (c) Phase error percentage. (a) and (b) from top to bottom show defocusing levels of 1, 4, and 8, respectively.

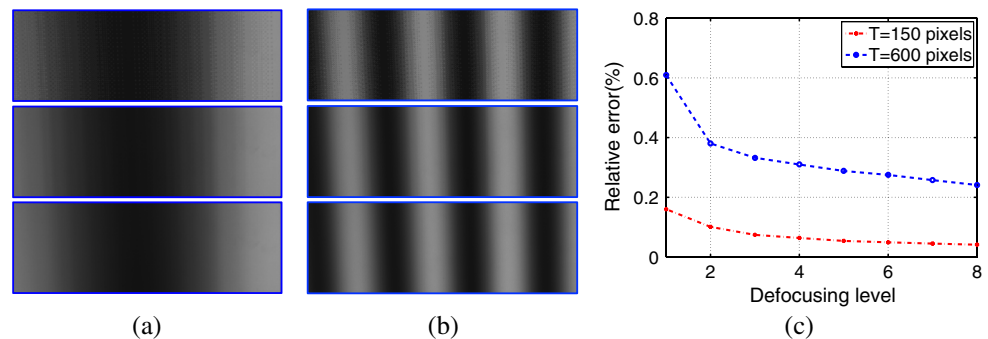


Fig. 5. (Color online) Phase errors with varying amount of defocusing by experiments. (a) Fringe patterns ( $T = 600$ ). (b) Fringe patterns ( $T = 150$ ). (c) Phase error percentage. (a) and (b) from top to bottom show defocusing levels of 1, 4, and 8, respectively.



utilized a DLP projector (Samsung SP-P310MEMX) and a digital CCD camera (Jai Pulnix TM-6740CL). The camera is attached with a 16 mm focal length megapixel lens (Computar M1614-MP) with an image resolution is  $640 \times 480$ . The projector has  $800 \times 600$  resolution and has 0.49–2.80 m projection distance.

The phase error caused by different amounts of defocusing was experimented first. In this experiment,

a white flat board was measured, and the amount of defocusing was realized by manually adjusting the focal length of the projector. The projector was adjusted from nearly focused to significantly defocused. We used the dithered patterns generated in Fig. 3 and two fringe pitches ( $T = 600$  and  $T = 150$ ) to determine the phase error percentage for each case. The phase error is calculated, again, by comparing the phase obtained from the dithered

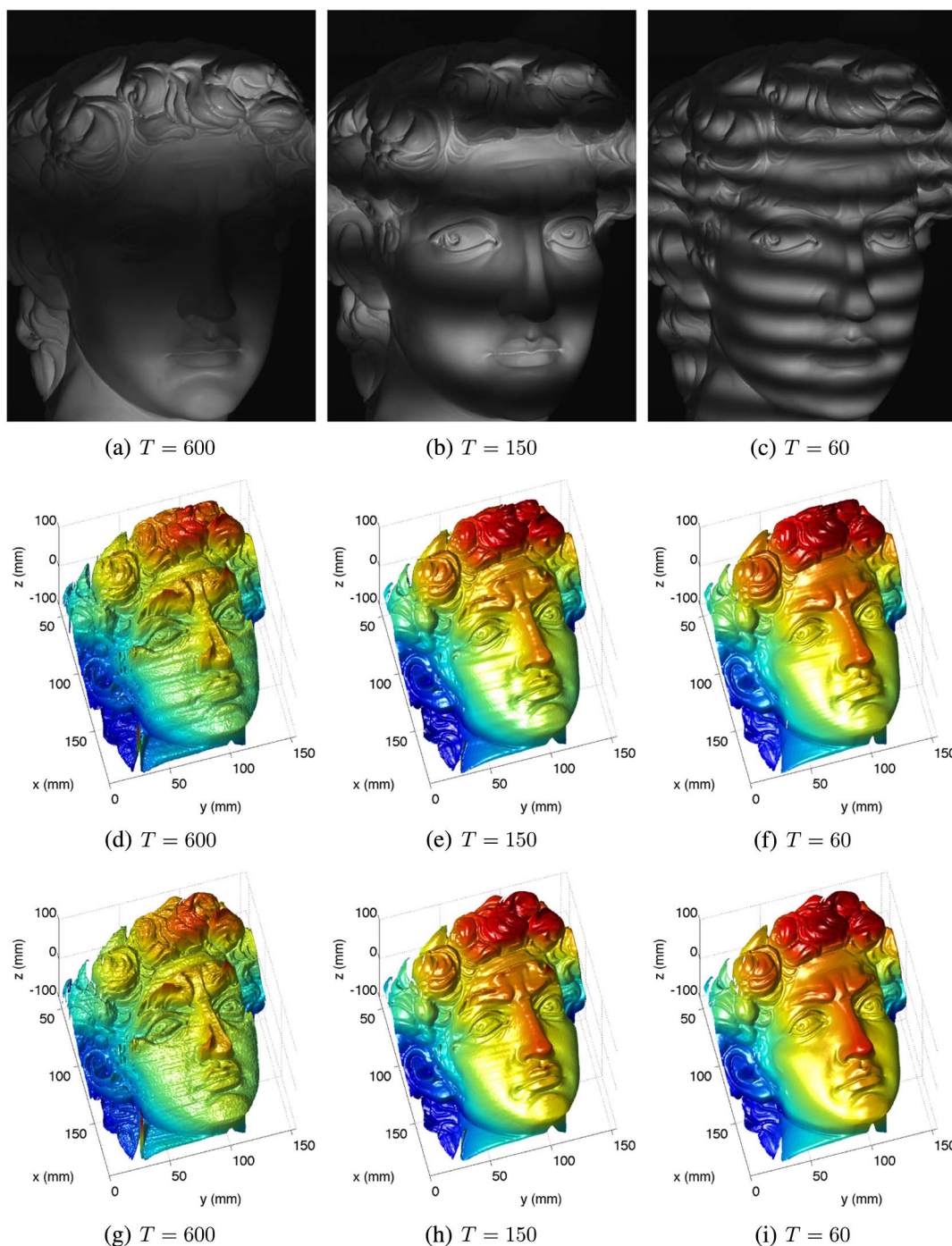


Fig. 6. (Color online) Experimental results of measuring a complex 3D object. (a)–(c) Representative captured dithered patterns. (d)–(f) 3D results using the dithered patterns. (g)–(i) 3D results using ideal sinusoidal patterns.

patterns with the phase obtained the ideal sinusoidal fringe patterns. To reduce the noise influence of the ideal sinusoidal fringe patterns, the fringe pitch used for the ideal sinusoidal fringe patterns is much smaller ( $T = 30$ ). Figure 5 shows the results. It can be seen that the projector starts with nearly focused to significantly defocused, but the phase errors all remain small (less than 0.6% even for the wide fringe,  $T = 600$ ). Clearly, these experimental results are similar to our simulation results, and thus confirm one merit of the proposed technique: that it is suitable for large depth-range 3D shape measurement.

We also measured a complex 3D object—David head shown in Fig. 1(a). Figure 6 shows the measuring results with different fringe breadths ( $T = 600$ , 150, and 60). During all the measurements, the projector and the camera remain untouched. To test our proposed technique, we firstly used the dithered patterns with projector being slightly defocused. Figures 6(a)–6(c) show the representative captured fringe patterns with different fringe breadths. Five phase-shifted dithered patterns are captured and used to recover the 3D shape. Figures 6(d)–6(f) show the 3D result for each case. It is not necessary to unwrap the phase from the widest fringe patterns, because the single fringe covers the whole measurement range. The narrower phase maps are unwrapped with the assistance of the phase from the largest fringe period using a temporal phase-unwrapping algorithm. To convert the unwrapped phase to depth, we used a simple phase-to-height conversion algorithm described in [14]. It is well known that when fringe stripes are wide, the random noise is very large, and with the fringe period decreasing, the quality of the 3D results becomes better. The experimental results shows that the 3D shape measurements are of very high quality when the fringe period is  $T = 60$ .

To evaluate the measurement quality, we also did an experiment using sinusoidal fringe patterns. Figures 6(g)–6(i) show the corresponding results with the same fringe size and the same phase-shifting algorithm. The nonlinearity of the projector was corrected using the approach described in [15]. This experiment clearly shows that the measurement quality using the proposed technique is comparable to that using the conventional techniques after projector's nonlinearity corrections, but the proposed technique does not need any nonlinearity correction because it naturally utilizes the binary patterns.

## 6. Conclusion

We have presented a novel 3D shape measurement technique using the binary dithered patterns and the phase-shifting technique. This method overcomes one of the challenges of the existing binary defocusing techniques: that it is difficult to generate high-quality sinusoidal fringe pattern when fringe stripes are very wide. Both simulation and experimental results have demonstrated the success of the proposed method.

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