CZ3005 Artificial Intelligence

Week 11a – Default Logic

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Learning Goals

Understanding the:

- Basics of Reiter's Default Logic (RDL)

- Limitations of RDL

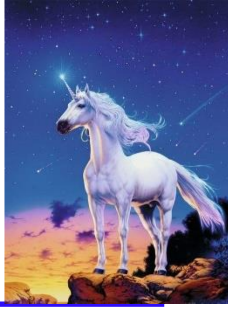
Default Logic

- Standard logic can only express that something is true or that something is false.

- Default logic is proposed to formalize reasoning with default assumptions.

- It can express facts like
  – “by default, something is true”.

Unicorns



- **Given:**
  - If the unicorn is mythical, then it is immortal.
  - If it is not mythical, then it is a mortal mammal.
  - If it is either immortal or a mammal, then it is horned.
  - The unicorn is magical if it is horned.
- **Question:**
  - Based on Modus Ponens, can we show that the unicorn is mythical, magical and horned?

Unicorns

- **KB:**

  Mythical $\Rightarrow$ ¬ Mortal
  ¬ Mythical $\Rightarrow$ Mortal
  ¬ Mythical $\Rightarrow$ Mammal
  ¬ Mortal $\Rightarrow$ Horned
  Mammal $\Rightarrow$ Horned
  Horned $\Rightarrow$ Magical

- **Assumptions**

  I can believe $\frac{:\text{Mythical}}{\text{Mythical}}$ or $\frac{:\neg\text{Mythical}}{\neg\text{Mythical}}$

Mythical	¬ Mythical
¬ Mortal	Mammal
Horned	Horned
Magical	Magical

Default Logic

$$\frac{\text{Prerequisite} : \text{Justification}_1, \ldots, \text{Justification}_n}{\text{Conclusion}}$$

According to this default:

- If we believe that Prerequisite is true;
- AND each of Justifications is consistent with our current beliefs;
- THEN, we are led to believe that the Conclusion is true.

Default Logic Syntax

prerequisite

justification

$$\frac{\alpha(\boldsymbol{x}) \; : \; \beta_1(\boldsymbol{x}), \ldots, \beta_n(\boldsymbol{x})}{\gamma(\boldsymbol{x})}$$

conclusion

where $\boldsymbol{x} = x_1, \ldots, x_m$, and $\alpha(\boldsymbol{x}), \beta_1(\boldsymbol{x}), \ldots, \beta_n(\boldsymbol{x}), \gamma(\boldsymbol{x})$ are formulae whose free variables are among $x_1, \ldots, x_m$.

The default is **applied** by substituting **c** (the ground instance) into α and β to infer γ:

- Trigger: $\alpha(\boldsymbol{c})$ belongs to our set of beliefs.
- Justification: the set of our beliefs is <u>consistent</u> with each $\beta(\boldsymbol{c})$.

Our KB does not entail $\neg\beta(\boldsymbol{c})$

Default Theory

Delta: Set of Defaults

$$\langle \Delta, \Phi \rangle$$

Phi: Set of "Facts"

Example – the default rule that "birds typically fly":

- $\Delta = \left\{ \frac{bird(x) \ : \ flies(x)}{flies(x)} \right\}$
  - This rule means that, "if x is a bird, and it can be assumed that x flies, then we can conclude that x flies".
- $\Phi = \{bird(Tweety), cat(Sylvester)\}$

Default Theory

A common default assumption:

- What is not known to be true is believed to be false.
- This is known as the Closed-World Assumption.
- It is formalized in default logic using a default as follows for every fact F.

$$\frac{:\neg F}{\neg F}$$

Types of Defaults

- Normal Defaults: $\frac{\alpha(x)\ :\ \gamma(x)}{\gamma(x)}$

- Semi-Normal Defaults: $\frac{\alpha(x)\ :\ \beta(x)}{\gamma(x)}$, where $\beta(x) \vdash \gamma(x)$

E.g., $\frac{bird(x)\ :\ flies(x) \wedge \neg swim(x)}{flies(x)}$,

where $flies(x) \wedge \neg swim(x) \vdash flies(x)$

Types of Defaults

- Open Defaults (Default Schemas) have unbounded variables, e.g., *x*

$$\frac{\alpha(x) \quad : \quad \beta_1(x), \ldots, \beta_n(x)}{\gamma(x)}$$

- Closed (Grounded) Defaults use ground terms, e.g., *x=c*

$$\frac{\alpha(c) \quad : \quad \beta_1(c), \ldots, \beta_n(c)}{\gamma(c)}$$

Reiter Inference with Default Theory

- Guess the extension Ξ (pronounced as “Xi”)
- Initialise beliefs $\Xi^* = \Phi$
- (loop over) c-ground instance of an (unused) default $\frac{\alpha(x)\ :\ \beta(x)}{\gamma(x)}$:
  - Check two conditions
    - Triggered?: $\Xi^* \vdash \alpha(c)$
    - Justified?: $\beta(c)$ is consistent with Ξ
  - If yes: update beliefs $\Xi^* \leftarrow \Xi^* \cup \{\gamma(c)\}$
- (end loop)
- If $\Xi = \Xi^*$ then extension found/confirmed

Reiter Extension: Compact Description

Given a default theory $\langle \Delta, \Phi \rangle$, Ξ is an extension if and only if

$$\Xi = \mathrm{Cn}\left(\bigcup_{i=1}^{\infty} \Xi_i\right)$$

Cn means applying any known inference rules to expand the KB

where:

- $\Xi_0 = \Phi$
- $\Xi_{\mathrm{i+1}} = \Xi_{\mathrm{i}} \cup \left\{\gamma(c) \middle| \frac{\alpha(x):\ \beta_1(x),\ldots,\beta_n(x)}{\gamma(x)} \in \Delta, \Xi_i \vdash \alpha(c), \neg\beta_1(c), \ldots, \neg\beta_n(c) \notin \Xi\right\}$

Example

Given Theory: $\mathrm{T} = \left\langle \Delta = \left\{ \frac{bird(x)\ :\ flies(x)}{flies(x)} \right\}, \Phi = \{bird(Tweety), cat(Sylvester)\} \right\rangle,$

- Guess the extension $= \mathrm{Cn}(\{flies(Tweety)\} \cup \Phi)$
- Our initial knowledge is $F = \Phi$
- Sylvester-instance of default not applicable:
  - not hold $\Phi \vdash bird(Sylvester)$
- $\Phi \vdash bird(Tweety)$ and $flies(Tweety)$ is consistent with *F*
- $F = \Phi \cup \{flies(Tweety)\}$
- No more default rules to apply
- An extension is reached

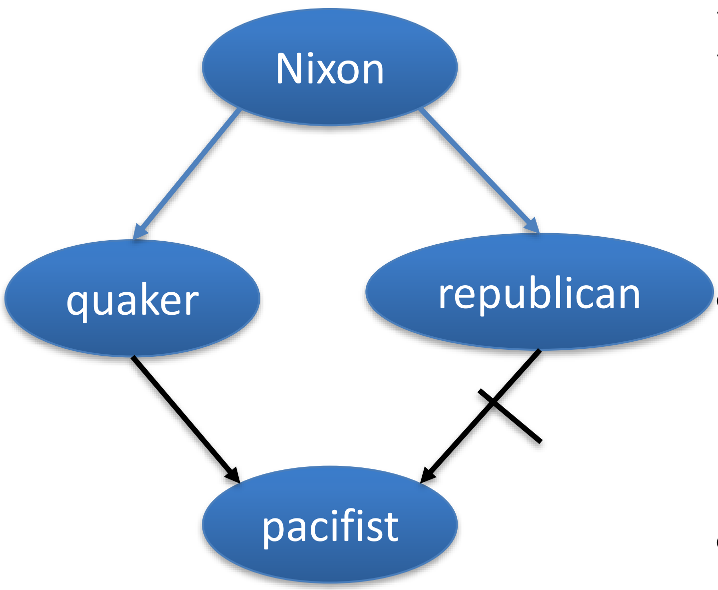
Implications of Default Theory

- A default rule can be applied to a theory
  - if its precondition is entailed by the theory; and
  - its justifications are all consistent with the theory.

- The application of a default rule leads to the addition of its consequence to the theory.
- Other default rules may then be applied to the resulting theory.
- When the theory is such that no other default can be applied, the theory is called an <u>extension</u> of the default theory.

Nixon Diamond

The default rules may be applied in <u>different orders</u>, and this may lead to <u>different extensions</u>. E.g.:



Let Theory $\mathrm{T} = \langle \Delta, \Phi \rangle$

- $\Delta = \left\{ \dfrac{quaker(x)\ :\ pacifist\ (x)}{pacifist(x)}, \dfrac{republican(x)\ :\neg pacifist\ (x)}{\neg pacifist(x)} \right\}$
- $\Phi = \{quaker(Nixon), republican(Nixon)\}$

Nixon Diamond

Given $\langle \Delta, \Phi \rangle$

- $\Delta = \left\{ \frac{quaker(x)\ :\ pacifist(x)}{pacifist(x)}, \frac{republican(x)\ :\neg pacifist(x)}{\neg pacifist(x)} \right\}$
- $\Phi = \{quaker(Nixon), republican(Nixon)\}$

There are **two** extensions:

1. One that contains: pacifist(Nixon)
2. .. and the one that contains: ¬ pacifist(Nixon)

Entailment

- A default theory can have 0, 1 or more extensions.
- Entailment of a formula from a default theory can be defined in one of two ways:
  - Skeptical:
    - a formula is entailed by a default theory if it is entailed by all its extensions.
  - Credulous:
    - a formula is entailed by a default theory if it is entailed by at least one of its extensions.

Example

- The Nixon diamond example theory has two extensions:
  - one in which Nixon is a pacifist; and
  - one in which Nixon is not a pacifist.

- Thus, we have:
  - Neither Pacifist(Nixon) nor ¬Pacifist(Nixon) are skeptically entailed.
  - Both Pacifist(Nixon) and ¬Pacifist(Nixon) are credulously entailed.

- The credulous extensions of a default theory can be inconsistent with each other.

Thank you!