

3010 Submodule Applied Crypto

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NTU, SCSE, Week 10A

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Overview Week 10 Lecture-A

- Intro Crypto
- Caesar Cipher
- Simple Substitution
- Vignere Cipher
- One-Time Pad
- Randomness

What is Cryptography - Informal

- The Science and Math of **scrambling data** (using a specific method with a 'key') **into meaningless gibberish** to render data incomprehensible to eavesdropper.
- In use since ancient times (>2000 yrs)

Cryptography can be found **EVERYWHERE!**

- Cellphone calls
- Microsoft office password protection
- E-commerce with Amazon etc
- ATM card
- Smart Cards
- Whatsapp messages sent over the air
- Surfing **http**s websites
- Your digital signatures

At 11:55am a girl dropped a note to a boy

- She asked him to find out the meaning of the note by 12 noon & go to LT 13 to meet her if he manages to understand the message.
- So boy has 5 minutes to figure this note:

This is the Note –What does it mean?

L OLNH X

Steps to Decrypt

- L & X are single letters.
- Assuming text in English and all letters are just letters
- Only possibilities of L & X
 - A, I, O, U

Steps to Decrypt

L OLNH X

I I U

Steps to Decrypt - looks like shift 3 letters

L OLNH X

I I U

Steps to Decrypt - looks like shift 3 letters-YES

L OLNH X

I LIKE U

Steps to Decrypt - looks like shift 3 letters-YES

L OLNH X

I LIKE U

-Caesar Cipher

Caesar Cipher

- Used 2000 years ago!
- Extremely easy to break, just shift 3 letters to left to break it.
- How to make cipher harder?
- Use all possible shifts, 25 of them (26 alphabets)!
- Easy for computers – instant break!
- It's the simplest **mono-alphabetic** encryption
 - Means each letter means uniquely some other letter (1-1 match)

Caesar Cipher – Main Weakness

- Only 25 possible keys-- trivial for computers to try all keys
- We Say **Key space**

- **Too small!**

How Big Should Key Space be

- Depends who you are guarding against.
- A 3 GHz Pc can roughly crack a key space of 2^{34} in 1 day
- I am jumping the gun here, but I will state now:
- Present day Minimum acceptable security – 2^{128} !
- This is an extraordinary huge number

Ancient Famous Story

- A king wants to reward a wise man for his contribution to his kingdom
- King asked him what he wants as a reward
- Tricky question for the wise man
- Ask too much, his head might be gone
- Too little, he would have wasted an opportunity to get rich
- He told the king:

Ancient Famous Story

- Find a field and draw 8x8 squares (like a chessboard)
- Put 1 grain of rice in 1st square, 2 grains in 2nd, 4 grains in 3rd,...
- My reward: I want all the grains added up to the 64th square
- King thought it's a reasonable request. He agreed in front of court.
- Ordered the servants to do what wise man had asked.
- After a while the servants reported to the king.
- WE don't have enough grain in the whole kingdom for this wise man!
- What happened? (tutorial)

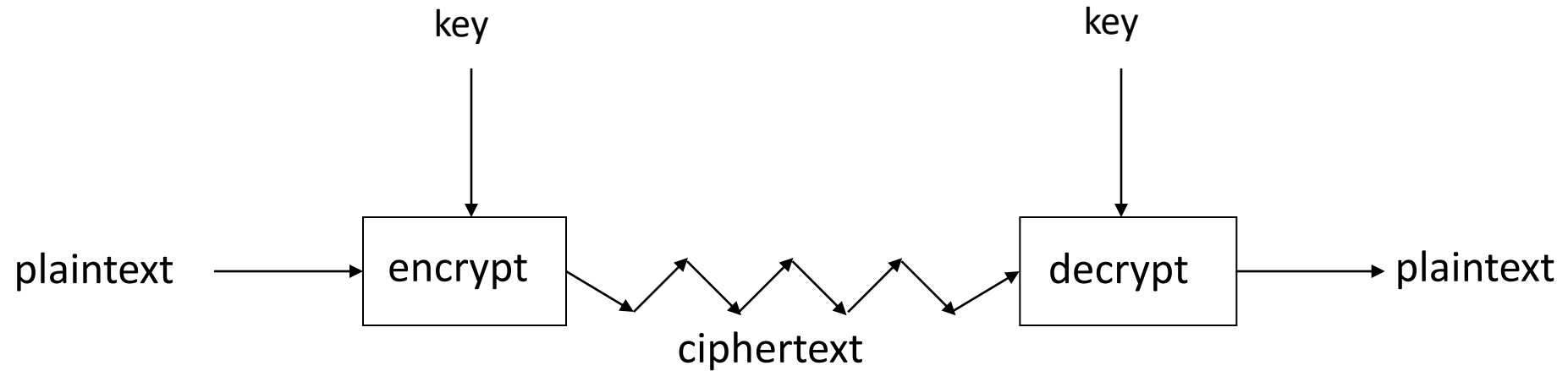
Crypto Terminology

- **Cryptology** — The art and science of making and breaking “secret codes”
- **Cryptography** — making “secret codes”
- **Cryptanalysis** — breaking “secret codes”
- **Crypto** — all of the above (and more)

How to Speak Crypto

- A *cipher* or *cryptosystem* is used to *encrypt* the *plaintext*
- The result of encryption is *ciphertext*
- We *decrypt* ciphertext to recover plaintext
- A *key* is used to configure a cryptosystem
- A *symmetric key* cryptosystem uses the same key to encrypt as to decrypt
- A *public key* cryptosystem uses a *public key* to encrypt and a *private key* to decrypt - 2nd half course

Crypto as Black Box



A generic view of symmetric key crypto

Caesar Cipher-Simple Substitution

- Plaintext: **fourscoreandsevenyearsago**
- Key:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- ❑ Ciphertext:

IRXUVFRUHDQGVHYHQBHDUVDJR

- ❑ Shift by 3 is “Caesar’s cipher”

Caesar Cipher Decryption

- Suppose we know a Caesar's cipher is being used:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Given ciphertext:

VSRQJHEREVTXDUHSDQWV

- Plaintext: spongebobsquarepants

Easy to harden Caesar Cipher

- Instead of shifting all letters by 3 letters, we can scramble the 26 letters A-Z into other random permutations of A-Z.
- How many possible permutations of A-Z?
- $26!$ ($26 \times 25 \times 24 \times \dots \times 3 \times 2 \times 1$)
- Then encrypt accordingly based on your scrambled key.
- Decrypt using same scrambled key!

Simple Substitution: General Case

- In general, simple substitution key can be any **permutation** of letters
 - Not necessarily a shift of the alphabet
- For example

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	J	I	C	A	X	S	E	Y	V	D	K	W	B	Q	T	Z	R	H	F	M	P	N	U	L	G	O

□ Then $26! > 2^{88}$ possible keys (tutorial)-**beyond BF**

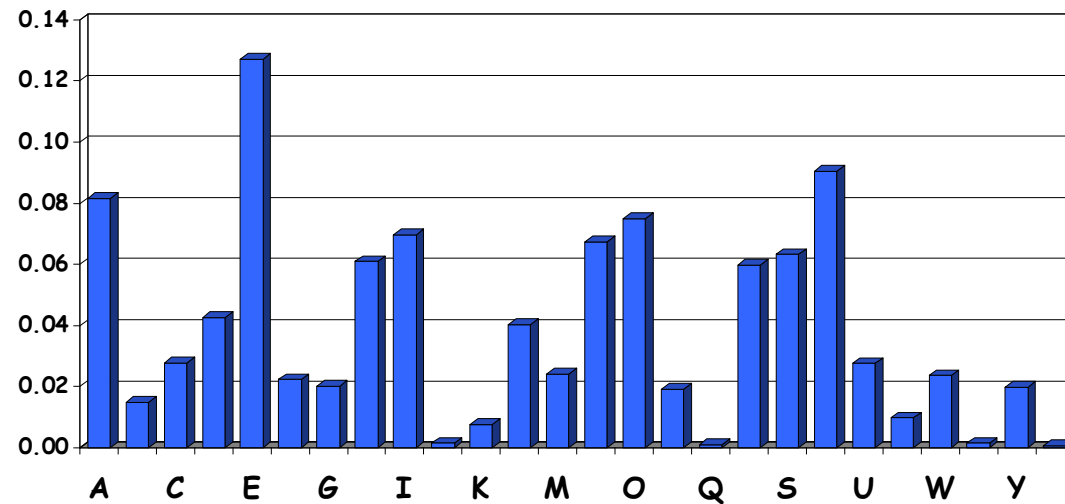
Cryptanalysis II: Be Clever

- We know that a simple substitution is used
- But not necessarily a shift by n
- Find the key given the ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQWAXBVCXQW
AXFQJVWLEQNTQZQGGQLFXQWAKVWLXQWAEBIPBFXFQVXGTVJVWLBTPQWAEBFP
BFHCVLXBQUFEVWLXGDPEQVPQGVPBPBFTIXPFHXZHVFAGFOTHFEFBQUFTDHzBQPO
THXTYFTODXQHFTDPTOGHFQPBQWAQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFW
PFHPBFIPBQWKFABVYYDZBOTHBPBPQJTQOTOGHFQAPBFEQJHDXXQVAVXEBQPEFZ
BVFOJIWFFACFCFHQWAUVWFLQHGFVAFXQHUFHILTAVWAFFAWTEVOITDHFHF
QAITIXPFHXAFQHEFZQWGFLVWPTOFFA

Cryptanalysis II

- Cannot try all 2^{88} simple substitution keys
- Can we be more clever?
- English letter frequency counts...



Letter Frequencies of English Letter

Letter	Frequency	Letter	Frequency
A	0.0817	N	0.0675
B	0.0150	O	0.0751
C	0.0278	P	0.0193
D	0.0425	Q	0.0010
E	0.1270	R	0.0599
F	0.0223	S	0.0633
G	0.0202	T	0.0906
H	0.0609	U	0.0276
I	0.0697	V	0.0098
J	0.0015	W	0.0236
K	0.0077	X	0.0015
L	0.0403	Y	0.0197
M	0.0241	Z	0.0007

Cryptanalysis: Terminology

- Cryptosystem is **secure** if best known attack is to try all keys (brute force -Exhaustive key search)
- Cryptosystem is termed '**broken**' if **any** shortcut attack is known without trying all keys)
- Although substitution = $26!$ Keys (beyond brute-force range), it succumbs to frequency analysis.
- Main weakness – letters used in English are very unevenly distributed! (eg compare Q with E)
- ***So Don't ever use Substitution ciphers!!!***

Lessons Learned So Far

- Key space must be large
- All letters must be equally likely to happen for ciphers to be maximally secure!
- Average freq for equal appearance is $1/26$, which is roughly 4%
- “e” occurs 12% in English text, 3 times higher than average
- “z” appears 0.07%
- So e appears abt 170 times more frequent than z!

Vigenere Cipher

- Took 1500 years to see a meaningful improvement of the Caesar cipher - Vigenère cipher, created in the 16th century
- Used during the American Civil War by Confederate forces and during WWI by the Swiss Army, among others.

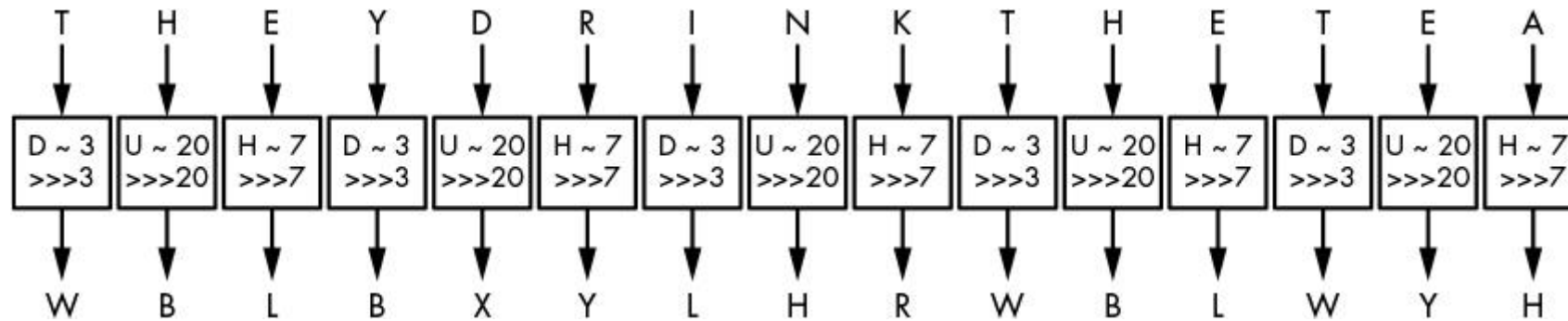
Vigenere Cipher

- Vigenère cipher is similar to the Caesar cipher, except that letters aren't shifted by three places but rather by values defined by a *key*, a collection of letters that represent numbers based on their position in the alphabet.
- For example, if the key is **DUH**, letters in the plaintext are shifted using the values **3, 20, 7**
 - because *D* is 3 letters after *A*,
 - *U* is 20 letters after *A*, and
 - *H* is 7 letters after *A*.
- The 3, 20, 7 pattern repeats until you've encrypted the entire plaintext.

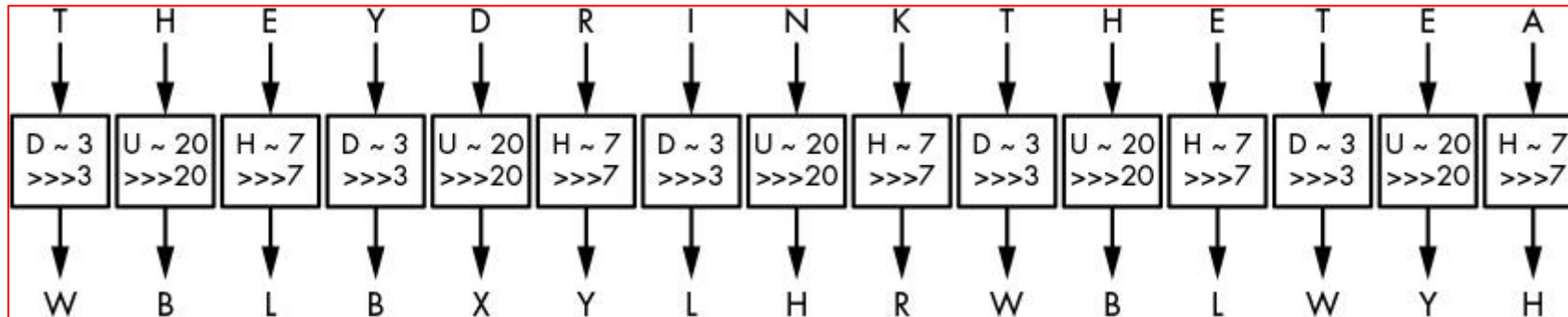
Vigenere Cipher

- For example, the word CRYPTO would encrypt to FLFSNV using DUH as the key:
 - *C* is shifted 3 positions to *F*,
 - *R* is shifted 20 positions to *L*, and so on.
- [Figure 1-3](#) illustrates this principle when encrypting the sentence THEY DRINK THE TEA.

Vigenere Cipher



Vigenere Cipher : E maps to both L & Y!



Vigenere Cipher

- In this cipher, same letter can be mapped to different letters
- Also different letters can be mapped into a single letter
- We call this poly-alphabetic substitution
- Much more secure than simple mono-alphabetic sub!

Vigenere Cipher

- Vigenère cipher > Caesar cipher, yet it's still fairly easy to break.
- The **first step** to breaking it is to figure out the **key's length**.
- **THEYDRINKTHETEA** encrypts to
- **WBLBXYLHRWBLWYH** , with the key **DUH**.
- Notice in ciphertext **WBLBXYLHRWBLWYH**, the group of three letters WBL appears twice in the ciphertext at **nine-letter intervals**.

Vigenere Cipher

- Suggest same 3-letter word was encrypted using the same shift values, producing WBL each time.
- A cryptanalyst can then deduce that the key's length is either **9** or a value that divides nine (that is, **3**).
- Furthermore, they may guess that this repeated 3-letter word is THE and **therefore determine DUH** as a possible encryption key.

Vigenere Cipher -Status

- Long keywords implies stronger Vigenere (Tutorial)
- Short message implies stronger Vigenere (Tutorial)
- Not good enough for modern use!

Classical Ciphers

- The previous class of ciphers are all known as classical ciphers
- There are many more such ciphers, but all insecure due to computers
- Used before WW2, way before invention of computers
- Question:
- Any unbreakable ciphers, even with computers in attacker's hand?
- **YES- ONE-TIME PAD!**

Perfect Encryption: The One-Time Pad

- Essentially, a classical cipher can't be secure unless it comes with a huge key, but encrypting with a huge key is impractical.
- However, the one-time pad is such a cipher, and it is the most secure cipher.
- In fact, it guarantees *perfect secrecy*:
 - even if an attacker has unlimited computing power, it's impossible to learn anything about the plaintext except for its length.

Perfect Encryption: The One-Time Pad

- The one-time pad takes a plaintext, P , and a “random key”, K , that’s the same length as P and produces a ciphertext C , defined as

$$C = P \oplus K,$$

where C , P , and K are bit strings of the same length and where \oplus is the bitwise exclusive OR operation (XOR), defined as

$$0 \oplus 0 = 0,$$

$$0 \oplus 1 = 1,$$

$$1 \oplus 0 = 1,$$

$$1 \oplus 1 = 0.$$

Easy Encryption: The One-Time Pad

- Since $C = P \oplus K$, we have
- $C \oplus K = P \oplus K \oplus K$, yielding
- $C \oplus K = P \oplus 0 = P$, the plaintext!
- So for both encrypt & decrypt, just XOR!
- XOR is superfast – *instantaneous* encryption & decryption!
- So course should be over.
- Everyone just use one-time pad, end of story!
- Will answer this question shortly.

Easy Encryption: The One-Time Pad -Letters

- Can be used for encrypting just letters too
- We need long random pads, as long as the message
- Suppose msg is YES & random pad generated is CAB
- For encryption
 - C – shift 3 to right
 - A – shift 1 to right
 - B- shift 2 to right
- **YES + CAB** (Y+C = B , becos Y shift 3 right –sequence Y,Z,A,**B**)
- **BFU (E shift 1 right, S shift 2 right)**

Easy Decryption: The One-Time Pad -Letters

- If user receive cipher BFU, and we know one time pad is CAB, we apply “inverse of add – that is minus
- So since encrypt means shift right, **Decrypt means shift left**
- For decryption
 - C – shift 3 to left
 - A – shift 1 to left
 - B- shift 2 to left
- **BFU–CAB** ($B-C=Y$, becos B shift 3 left –sequence B,A,Z,**Y**)
- **YES (F shift 1 left, U shift 2 right)**
- Useful to list ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ...

OTP In Computation (letters)-encryption

- Earlier by shifting argument, we have $Y + C = B$
- If letters to add are large, not easy to see shifted letter by listing
- Use this trick: A=1, B=2,...,Y=25,Z=26
- $Y + C$ (numerical) (mod 26) , $\alpha(\text{mod}26)$ means remainder of α when divided by 26.
- $= 25 + 3 \pmod{26}$
- $= 28 \pmod{26}$
- $= 2$
- $= B$, same answer as before

OTP In Computation (letters)-decryption

- Earlier by shifting argument, we have $B - C = Y$
- If letters to subtract are large, not easy to see shifted letter by listing
- Use this trick: $A=1, B=2, \dots, Y=25, Z=26$
- $B - C$ (numerical) (mod 26)
- $= 2 - 3$ (mod 26)
- $= -1$ (mod 26)
- $= -1 + 26$ (mod 26) $= 25$
- $= Y$, same answer as before.

One-Time Pad Summary

- **Provably** secure
 - Ciphertext gives **no** useful info about plaintext
 - All plaintexts are ***equally likely, so for a 3 letter cipher, we will not be able to tell if plaintext is YES or NOT!***
- BUT, only when it is used correctly
 - Pad must be random – (why? – Tutorial)
 - Pad used only once (why? -Tutorial)
 - Keep track of bits used (why? –Tutorial)
 - Pad is known only to sender and receiver
- Note: pad (key) is same size as message
- Got to generate a long random pad to your buddy so that you do not need to frequently meet up.

OTP instantaneous & unbreakable –End of Story?

- We have noted OTP operations are instantaneous using computers
- And its unbreakable.
- So should be end of story for crypto.
- Why not?

Challenges of Using OTP

- How to generate truly random LONG one-time pad (OTP)?
- How to store OTP securely?
- How to encrypt and decrypt securely
- Both parties have to keep in synchronization portions of pad that has already been used, so that both can keep on talking
- How to agree on new OTP if old OTP is used up or compromised?

Informal Notions of Random used in crypto

- Suppose X_i is bit i of OTP.
- Random OTP (bits) informally means
 1. $P(X_i = 0) = P(X_i = 1) = 0.5$, both equally likely
 2. Successive bits are indep of each other i.e. $P(X_{i+1}/X_i) = P(X_{i+1})$
- One way out: Think of unbiased coin and repeatedly toss it & record heads or tails
- In practice: how to manufacture unbiased coin?
- Truly random source – from eg radioactive decay... slow to generate

Randomness

- Randomness is found everywhere in cryptography:
 - in the generation of secret keys,
 - in encryption schemes, and
 - even in the attacks on cryptosystems.
- Without randomness, cryptography would be impossible because all operations would become predictable, and therefore insecure.

Randomness

- This section introduces you to the concept of randomness in the context of cryptography and its applications.
- We discuss pseudorandom number generators and how operating systems can produce reliable randomness, and we conclude with real examples showing how flawed randomness can impact security.

Randomness

- You've probably already heard the phrase "random bits," but strictly speaking there is no such thing as a series of random bits.
- What is random is actually the algorithm or process that produces a series of random bits; therefore, when we say "random bits," we actually mean randomly generated bits.

Randomness –how they look like

Is the 8-bit string
11010110 more random
than **00000000**?
(tutorial)

Randomness

- This example illustrates two types of errors people often make when identifying randomness:
- **Mistaking non-randomness for randomness** Thinking that an object was randomly generated simply because it *looks* random.
- **Mistaking randomness for non-randomness** Thinking that patterns appearing by chance are there for a reason other than chance.
- The distinction between random-looking and actually random is crucial. Indeed, in crypto, **non-randomness is often synonymous with insecurity.**

Informal Notions of Random used in crypto

- Please note **crypto notion of randomness needed is much more stringent than randomness needed in simulations** (monte-carlo) used in video games and computing probabilities of complicated events
- It is also more stringent than random numbers generated from pseudo-random number generators
- This type of generation won't produce truly robust random numbers needed for crypto.
- Later in course I will suggest some famously good CSPRNG, crypto-secure pseudo random number generators