Nanyang Technological University
SPMS/Division of Mathematical Sciences

2021/22 Semester 1 MH1810 Math 1 Take Home Test

Version J

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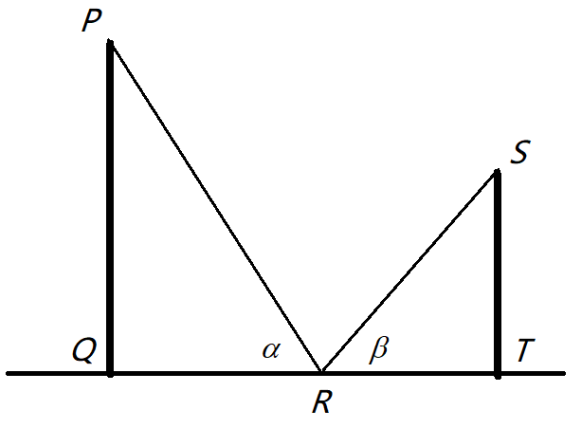
All questions carry the same marks. Answer ALL questions.

1. Two vertical poles PQ and ST are secured by a rope PRS going from the top of the first pole to a point R on the ground between the two poles and then to the top of the second pole as shown in the figure. Show that the shortest length of such a rope occurs when $\alpha = \beta$, where $\alpha = \angle PRQ$ and $\beta = \angle SRT$.



2. Let $f(x) = \sqrt{1 + \dfrac{1}{x}}$. Use the *definition of deivatives* to show that

$$f'(x) = -\frac{1}{2x^2\sqrt{\frac{1}{x} + 1}}.$$

3. Express the following as a definite integral $\int_0^1 f(x)\,dx$ and find its exact value.
$$\lim_{n\to\infty}\left(\sqrt[3]{\frac{1}{n^4}}+\sqrt[3]{\frac{2}{n^4}}+\sqrt[3]{\frac{3}{n^4}}+\cdots+\sqrt[3]{\frac{n}{n^4}}\right).$$

4. Show that

   (a) $\displaystyle\int_0^{\pi/2} e^{-x}\cos 2x dx = a\left(e^b+1\right)$, where the numbers a, b are to be determined.

   (b) $\displaystyle\int_0^1 \frac{3^x}{3^x+4^x}dx = \frac{\ln A}{\ln B}$, where the numbers A, B are to be determined.

5. Let R be the region bounded by the curve $y = \dfrac{x}{1+3x^2+x^3}$, $x = 1$, $x = 0$ and $y = 0$. Find the volume when R is rotated 2π radians about the the line $x = -2$. Express your answer in terms of π.