binomial $X \sim B(n, p)$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$\mu = np$

$\sigma^2 = npq$

Poisson $X \sim poiss(\mu)$

$$P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$$

Geometric $X \sim G(p)$

$$P(X = x) = (1-p)^{x-1} p$$

$\mu = \frac{1}{p}$

$\sigma^2 = \frac{1-p}{p^2}$

If $n \to \infty$, p not neat 0 or 1

then $X \to$ normal distribution $\to$ aprx binomial use .5 diff for Z

Normal distribution $X \sim N(\mu, \sigma^2)$

pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$P(Z \geq -z) = P(Z \leq z)$

$P(Z \leq -z) = 1 - P(Z \geq -z)$

$P(-z \leq Z \leq z) = 1 - 2P(Z \leq -z)$

$95\% \approx 2SE$ <u>Sampling distribution</u>

CLT

sum mean $\underline{n\mu}$ $\sigma = \underline{\sigma\sqrt{n}}$

mean mean $\underline{\mu}$ $\sigma = \underline{\sigma/\sqrt{n}}$

$Z = \frac{x-\mu}{\sigma/\sqrt{n}}$

margin of error $= 1.96\,SE = 1.96\left(\frac{s}{\sqrt{n}}\right) = 1.96\left(\sqrt{\frac{pq}{n}}\right)$

LCL
UCL $\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{2}}$ → standard error

use σ if hv σ else use s

Large-Sample Confidence

apx normal $SE = \sqrt{pq/n}$

$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{pq}{n}}$

3 ~ 20 years

$\mu = 8$ years

$\sigma = 4$

$n = 30$

$SE = 4/\sqrt{30} = 0.73$

$P(\bar{x} < 7)$

$P(\bar{x} > 7)$

$P(7 < \bar{x} < 9)$

$P\left(z < \frac{7-8}{0.73}\right)$

$P(z < -1.37)$

$P = 0.0853$

$P(-1.37 < z < 1.37)$

$P = 1 - 2(0.0853)$

$= 0.8294$

$P(\bar{x} > 7) = 1 - 0.0853$

$= 0.9147$

$n = 500$

$p = 0.6$

$q = 0.4$

$pn = 300 > 5$

$qn = 200 > 5$

$ND \sim SE = \sqrt{\frac{0.6 \cdot 0.4}{500}} = 0.022$

$\sim 2(0.022) = 0.044$

$\frac{0.6}{0.022}$

TUT 1 2000 part 2

1) $n = 10 \quad \sigma = 0.2 \quad \mu = 12.1$

$$P(\bar{x} < 12) = P\left(Z < \frac{12 - 12.1}{0.2/\sqrt{10}}\right)$$

$$= P(Z < -1.58) = 0.0571$$

2) $\hat{p} = 0.6$

$q = 0.4$

$n = 1700$

$\sigma = \sqrt{npq} = 20.2$

$\mu = np = 1020$

$$P(\bar{x} \geq 1061)$$

$$P\left(Z \geq \frac{x - \mu}{SE}\right)$$

$$P\left(Z \geq \frac{1060.5 - 1020}{20.2}\right)$$

$$P(Z \geq 2.025)$$

$$P(Z \leq -2.025)$$

$$0.0214$$

if $n \to \infty$, p not near 0 or 1

then $X \to$ normal distribution

3) $P(Rh+) = 0.85$

$q = 0.15$

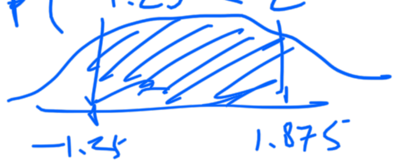
$n = 500$

$P(0.83 < \hat{p} < 0.88)$

$\sigma = \sqrt{\frac{pq}{n}} = 0.016$

$$P\left(\frac{0.83 - 0.85}{0.016} < Z < \frac{0.88 - 0.85}{0.016}\right)$$

$$P(-1.25 < Z < 1.875)$$



$$1 - P(Z \leq -1.25) - P(Z \leq -1.875)$$

$$1 - 0.1056 - 0.0301 = 0.8643$$

4) $\mu = 1.3M$

$\sigma = 300\,000$

$n = 60$

$P(x > 1.4M)$

$$P\left(Z > \frac{1.4M - 1.3M}{300\,000/\sqrt{60}}\right)$$

$$P(Z > 2.582) = 0.9951$$

$$1 - 0.9951 = 0.0049$$

240 . n

5) $\mu = 240$ $n = 10\,000$

$\sigma = 800$

$P(x > 2\,600\,000)$

$$P\left(Z > \frac{2\,600\,000 - 2\,400\,000}{800\,000/\sqrt{10\,000}}\right)$$

$$P\left(Z > \frac{\cancel{200\,000}}{\cancel{80\,000}}\right)$$

$$P(Z > 2.5) = 1 - P(Z < 2.5)$$
$$= 1 - 0.9938$$
$$= 0.0062$$

$= 10$

6) $n = 10$

$$P(x \geq 45) + P(x \leq 25) = 1 - P(25 \leq x \leq 45)$$

$\mu = 3.5$

$\sigma = 1.71$

$$= 1 - P\left(\frac{25.5 - 35}{1.71/\sqrt{10}} \leq Z \leq \frac{44.5 - 35}{1.71/\sqrt{10}}\right)$$

$$= 1 - P(-1.76 \leq Z \leq 1.76)$$

0.0392 0.9608

$$= 0.0392 + (1 - 0.9608)$$

$$= 0.0784$$

7) $\sigma = 4$

$2SE = 0.5$

$2\frac{4}{\sqrt{n}} = 0.5$

$\frac{80}{5} = \sqrt{n}$

$n = 256$

8) $\sigma = \sqrt{2}$
$\mu = 2$

$P\left(\sum_{i=1}^{30} x_i > 50\right)$

$P\left(z > \frac{50-60}{\sqrt{60}}\right) =$

9) SKIP

TUT 2 part 2 5/2000

1) $n = 200$
$\mu = 0.72$ / 12 month
$\sigma = 0.56$

$$SE = 1.96\frac{\sigma}{\sqrt{n}} = 1.96\frac{0.56}{\sqrt{200}}$$

$$= 0.078$$

$$0.72 \pm 0.078$$

2) $p = 0.49$
$n = 1034$
$q = 0.51$

$$SE = 1.96\sqrt{\frac{pq}{n}} = 1.96\sqrt{\frac{0.51 \cdot 0.49}{1034}}$$

$$0.49 \pm = 1.96(0.0155)$$
$$0.49 \pm 0.0305$$

3) $n = 30$
$\mu = 0.145$
$\sigma = 0.0037$

$$1-\alpha = 0.9$$
$$\alpha = 0.1$$
$$\alpha/2 = 0.05$$

$$\bar{\mu} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$\bar{\mu} \pm 1.645\frac{0.0051}{\sqrt{30}}$$

$$0.145 \pm 0.00153$$

$$0.1435 < \mu < 0.14\cancel{7}5$$

4) $n = 500$
$p = \frac{68}{500} = 0.136$
$q = 0.864$

$$1-\alpha = 0.95$$
$$\alpha = 0.05$$
$$\alpha/2 = 0.025$$

$$0.136 \pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$$

$$0.136 \pm 0.03$$

$$0.106 < p < 0.166$$

5) $n = 356$

$p = \frac{201}{356} = 0.565$

$q = 0.435$

$1.96\sqrt{\frac{pq}{n}}$

0.0515

6) $n = 100$

$\mu = 40$

$\sigma = 10$

$40 \pm 1.96 \frac{\cancel{10}}{\cancel{\sqrt{100}}} \cancel{10}$

$(38.04, 41.96)$

7) a) $n = 50$

$\mu = 654.16$

$\sigma = 164.43$

$654.16 \pm 1.96 \frac{164.43}{\sqrt{50}}$

654.16 ± 45.58

$(608.58, 699.74)$

b) $2 \cdot 1.96 \frac{175}{\sqrt{n}} = 50$

$n = \left(\frac{2 \cdot 175}{25.51}\right)^2 = 188.24 \approx 189$

8) skip

$p_0 = 0.2 \quad q = 0.8$

$n = 100$

$\hat{p} = \frac{15}{100} = 0.15$

$H_0 : p = 0.2$

$H_1 : p < 0.2$

$$\frac{0.15 - 0.2}{\sqrt{\frac{0.2 \cdot 0.8}{100}}} = \frac{-0.05}{0.04} = -1.25$$

One tailed $\quad 1.282$

$-1.25 < 1.282$ ✓

$p = P(z < 1.282)$

$0.8997 \quad > 0.05$

unable to reject

$P(z < -1.25)$

$\cancel{p} = 0.1056 \quad > 0.1$

tut 3

part 2

1) $\frac{1}{4} = 0.25$ $H_0 = p = 0.25$

$p = \frac{290}{1000} = 0.29$ $H_a = p > 0.25$

One sided

$Z = \frac{0.29 - 0.25}{\sqrt{0.25 \cdot 0.75 / 1000}} = 2.92$

$\alpha = 0.05$
1.645

$2.92 > 1.645$
true reject

6) $\mu = 5.5$ H_0 $\mu = 5.5$
$n = 16$ H_a $\mu -$
$\sigma = 0.3$ $Z = \frac{5.25 - 5.5}{0.3 / \sqrt{16}}$
$\bar{x} = 5.25$
$Z = -3.33$
two sided
p-value $= 0.0008 \approx 0.08\%$
true reject H_0

2) $p = 0.14 = \mu$ $H_0 = p = 0.14$
$H_a = p > 0.14$
$p = \frac{104}{590} = 0.18$
$\alpha = 0.01$ $Z = \frac{0.18 - 0.14}{\sqrt{0.14 \cdot 0.86 / 590}} = 2.52$

$2.52 > 2.326$

7) a) $H_a : \theta > 0.5$ $\alpha = 0.06$

$p \frac{140}{250} = 0.56$ 0.44

$Z = \frac{0.56 - 0.5}{\sqrt{0.5 \cdot 0.5 / 250}} = \frac{0.06}{0.031} = 1.897$

one side $Z > 0.0287 \sim \alpha = 1 - 0.9713$
$= 0.0287$
2 side Z Z, 0.0287 $= 0.0574$
two sided $\sim 6\%$

3) $\mu = 85$ One Sided
$n = 25$ $H_0 = \mu = 85$
$\bar{x} = 80.44$ $H_a = \mu < 85$
$\sigma = 11.6$
0.05
1.645 $Z = \frac{80.44 - 85}{11.6 / \sqrt{25}} = -1.75$

$P = P(Z < Z_\alpha) = 0.0401 < 0.05$
$= P(Z < -1.75)$

b) $\alpha = 10\%$ $6 < 10$ H_0 reject
$\alpha = 5\%$ $6 \not< 5$ cannot reject

$0.01 \sim 2$ sided $\sim 0.005 \sim 2.576$

4) C1 174 $H_0 = (\mu_1 - \mu_2) = 0$
C2 355 $H_a = (\mu_1 - \mu_2) > 0$
$\mu_1 > \mu_2$

$Z = \frac{(3.51 - 3.24)}{\sqrt{\frac{0.51^2}{174} + \frac{0.52^2}{355}}} = 5.684$
One sided

$1\% = 0.01$ $5.684 > 2.326$
true

c) $Z < 2.576$

1 side) $\frac{p - 0.5}{\sqrt{0.5 \cdot 0.5 / 250}} < -2.576$

$p < 104.635$

2 side $\frac{p - 0.5}{\sqrt{0.5 \cdot 0.5 / 250}} < +2.576$

$p < 145.365$

5) $Z = \frac{94.32 - 95}{1.20 / \sqrt{16}} = \frac{-0.68}{0.3}$

two sided $1 - P(Z < 2.27)$ $1 - P(Z < 2.27)$
$P = P(Z < -2.27) + P(Z > 2.27)$
$= (1 - 0.9884) + (1 - 0.9884)$ reject
$= 0.0232 > 0.01$ true H_0

TUT 2 PT2

1) $\mu = 7.2\%$ $\quad 1.96 \frac{\sigma}{\sqrt{200}} = 0.776$

$\sigma = 5.6\%$

$n = 200$

2) $p = 0.49$ $\quad q = 0.51$ $\quad 1.96\sqrt{\frac{0.49 \cdot 0.51}{1034}} = 0.0305$

$n = 1034$

3) $n = 30$ $\quad 1.645 \cdot \frac{0.0051}{\sqrt{30}} = 0.00153$

$\bar{x} = 0.145$

$S = 0.0051$

0.1 $\quad 0.145 \pm 0.00153$

4) $P = \frac{68}{500} = 0.136$ $\quad q = 0.864$ $\quad n = 500$

$\alpha = 0.95 \quad Z = 1.96$

margin error $\quad 1.96\sqrt{\frac{0.136 \cdot 0.864}{500}} = 0.03$

$0.136 \pm 0.03 \sim$ 0.106, 0.166

5) $n = 356$ $\quad 1.96\sqrt{\frac{0.565 \cdot 0.435}{356}} = 0.0515$

$p = \frac{201}{356} = 0.565$

$q = 0.435$ $\quad 0.565 \pm 0.0515$ $\quad 0.5135$, 0.6165

6) $n = 100$ $\quad 1.96 \frac{10}{\sqrt{100}}$

$\mu = 40$

$\sigma = 10$ $\quad 40 \pm 1.96$ $\quad 38.04$, 41.96

7) $n = 50$

$\bar{x} = 654.16$

$S = 164.43$

a) $1.96 \frac{164.43}{\sqrt{50}}$ $\quad 654.16 \pm 45.58$

b) $S = 175$ $\quad \frac{175}{\sqrt{n}} 1.96 = 50/2$

$n = 188.24 \sim 189$

TUT 1 PT2

1) $n = 10$

$\mu = 12.1$

$\sigma = 0.2$

$P(\bar{x} < 12)$

$$z = \frac{12 - 12.1}{0.2/\sqrt{10}} = -1.58$$

$P(z < 1.58)$

2) $p = 0.6$

$q = 0.4$

$n = 1700$

$P < 1060$

$(P \geq 1061)$

$$SE = \sqrt{\frac{0.6 \cdot 0.4}{1700}}$$

$= 0.01188$

$P \quad \underline{1060.5}$