MH1810 OA3: Matrices

Date No.

1. $AB = \begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 5 & -1 \\ 2 & 7 & -3 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$

2. $\det(A) = 8-9 = -1$

$A^{-1} = \frac{1}{-1}\begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

3. Not invertible $\Rightarrow \det(A) = k-14 = 0 \quad k = 14$

4. $A^2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$A^3 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 6 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

5. $\det\begin{bmatrix} 2 & 2 & 2 \\ 0 & 5 & 5 \\ 0 & 0 & 5 \end{bmatrix}\begin{matrix} 2 & 2 \\ 0 & 5 \\ 0 & 0 \end{matrix} = 50+0+0-0-0-0 = 50$

6. $\det\begin{bmatrix} 1 & 5 & 1 & 1 \\ 0 & 0 & 13 & 1 \\ 0 & 0 & 0 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix} = 0-0+0-6\begin{vmatrix} 1 & 5 & 1 \\ 0 & 0 & 13 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} 1 & 5 \\ 0 & 0 \\ 1 & 1 \end{matrix} = -6(0+65+0-0-13-0) = -312$

7. $\begin{pmatrix} 3 & 2 \\ 6 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ is the Ax=b matrix equation.

For no solutions, $\det(A) = 3k-12 = 0 \Rightarrow k = 4$

8. $\begin{pmatrix} 3 & 3 & 0 \\ 2 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ is the Ax=b matrix equation. $\det(A) = \begin{vmatrix} 3 & 3 & 0 \\ 2 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} 3 & 3 \\ 2 & 0 \\ 1 & 1 \end{matrix} = 0-6+0-0-(-6)-6 = -6$

$x = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 0 & 3 & 0 \\ 2 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} 0 & 3 \\ 2 & 0 \\ 1 & 1 \end{matrix}}{-6} = \frac{0-6+0-0-0-6}{-6} = \frac{-12}{-6} = 2$

9. $\det(A) = \begin{vmatrix} 4 & 6 & -6 \\ 0 & 7 & -2 \\ 0 & 0 & 9 \end{vmatrix}\begin{matrix} 4 & 6 \\ 0 & 7 \\ 0 & 0 \end{matrix} = 252+0+0-0-0-0 = 252$

$C_{ij}(A) = \begin{pmatrix} 63 & -0 & 0 \\ -(54) & 36 & -0 \\ 30 & -(-8) & 28 \end{pmatrix} = \begin{pmatrix} 63 & 0 & 0 \\ -54 & 36 & 0 \\ 30 & 8 & 28 \end{pmatrix} \quad adj(A) = \begin{pmatrix} 63 & -54 & 30 \\ 0 & 36 & 8 \\ 0 & 0 & 28 \end{pmatrix} \quad A(adj(A)) = \det(A)I$

$A^{-1} = \frac{1}{\det(A)}adj(A) = \frac{1}{252}\begin{pmatrix} 63 & -54 & 30 \\ 0 & 36 & 8 \\ 0 & 0 & 28 \end{pmatrix}$

10. As $A_{1j} = -\frac{1}{2}A_{3j}$, row 3 is linear combination of row 1 $\Rightarrow \det(A) = 0 \Rightarrow A^{-1}$ does not exist.

11. $\underset{2\times3}{\begin{pmatrix}1 & a & b\\ a & 1 & b\end{pmatrix}}\underset{3\times2}{\begin{pmatrix}1 & 4\\ 4 & 4\\ 2 & 2\end{pmatrix}} = \underset{2\times2}{\begin{pmatrix}1+4a+2b & 4+4a+2b\\ a+4+2b & 4a+4+2b\end{pmatrix}} = \begin{pmatrix}1 & c\\ 16 & d\end{pmatrix}$

$1+4a+2b=1 \quad 4a+2b=0$
$a+4+2b=16 \quad a+2b=12$
$\Big\} \; a=-4,\ b=8$

$c = 4+4a+2b = 4+4(-4)+2(8) = 4$

$d = 4a+4+2b = 4(-4)+4+2(8) = 4$

12. $A: 4\times6$
$B: 7\times4$
$A^T: 6\times4$
$B^T: 4\times7$

(a) $A^T(B^TB)^{-1}A \quad 6\times4 \cdot 4\times4 \cdot 4\times6 = 6\times6$

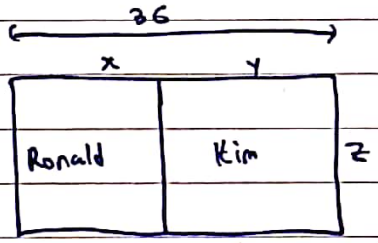
(b) $B^T(BA)(BA) = 4\times7 \cdot 7\times6 \cdot 7\times6 =$ undefined

13. $B + J = 876$
$B + \frac{1}{2}J = 584$
$\Rightarrow$ (1) − (2) $\Rightarrow \frac{1}{2}J = 876-584$

$J = 2(876-584) = 584$

$B = 876 - J = 292$

14.



$x+y+0z=36$
$2x+0y+2z=54$
$0x+2y+2z=72$
$\Big\} \; z = \frac{27}{2}$ (inner wall, from GC)

15. As $A_{1j} = \frac{1}{2}A_{3j}$, row 3 is linear combination of row 1 $\Rightarrow$ det.(A) = 0