NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER I EXAMINATION 2016–2017
MH1810 – Mathematics 1

NOVEMBER 2016 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **SEVENTEEN (17)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the attachments.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only

Questions	Marks
1 (12)	
2 (13)	
3 (10)	
4 (10)	

Questions	Marks
5 (15)	
6 (15)	
7 (25)	

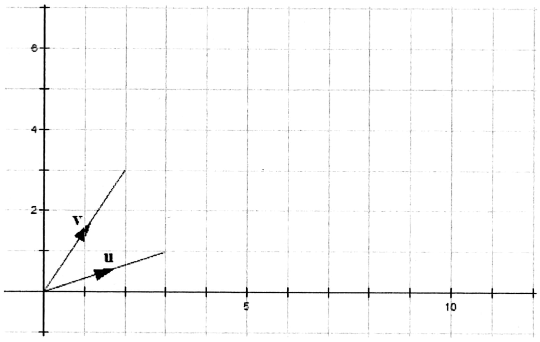
Total (100)	

QUESTION 1. **(12 Marks)**

(a) The diagram below shows two vectors **u** and **v**. Draw, on the diagram,

(i) the vector $\mathbf{u}+2\mathbf{v}$

(ii) the line $\ell : \mathbf{r} = \mathbf{v} + \lambda\mathbf{u},\ \lambda \in \mathbb{R}$.



(b) Find the distance between the planes

$$\Pi_1 : \mathbf{r}\cdot(\mathbf{i}-2\mathbf{j}-2\mathbf{k}) = 3 \text{ and } \Pi_2 : 2x - 4y - 4z = 0.$$

Question 1 continues on Page 3.

(c) Let r be a constant. Consider the following system of equations for variables x, y and z.

$$\begin{array}{rcrcl} x^3 & + & z & = & 1 \\ x^3 & + & y^5 & = & 1 \\ y^5 & + & rz & = & 1 \end{array}$$

(i) Find the values of r for which Cramer's rule is applicable.

(ii) For $r = 1$, use Cramer's Rule to find the unknown z.

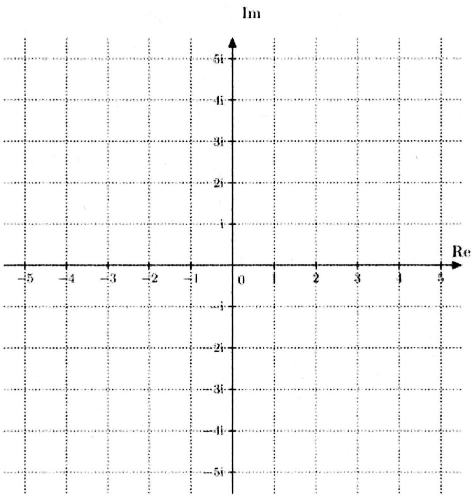
QUESTION 2. **(13 Marks)**

(a) Solve the equation $z^4 = -1024$. Express the roots in polar form $re^{i\theta}$, $r > 0$, $-\pi < \theta \leq \pi$.

(b) Express the roots found in part (a) in the form $x + iy$.

Question 2 continues on Page 5.

(c) Plot the four roots in the diagram below.



(d) Use your answers from part (b) to find the roots of the equation

$$\left(\frac{w}{2} + 4i\right)^4 = -1024.$$

Express your answers in the form $x + iy$.

QUESTION 3. **(10 Marks)**

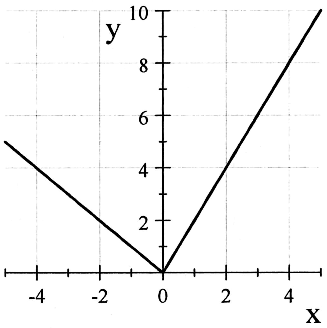
Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{e^{2x}+1}{e^{3x}+1} \cos(x+1)$

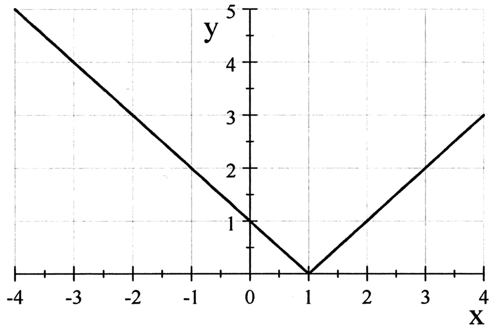
(b) $\displaystyle\lim_{x\to 1^-} \frac{\sqrt[3]{x}-1}{x-1}$

QUESTION 4. (10 Marks)

The graphs of piecewise linear functions f and g are given below.



Graph of $y = f\,(x)$



Graph of $y = g\,(x)$

(a) Find $\lim\limits_{x \to -1} f\,(g\,(x))$.

(b) Find $(f \circ g)'\,(3)$.

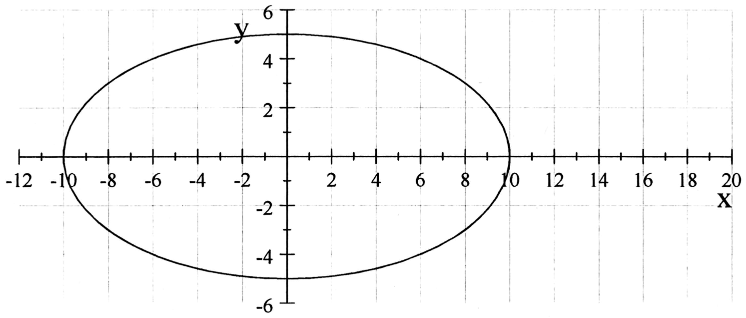
QUESTION 5 **(15 Marks)**

(a) (i) State the definition of the **derivative** of a function $f(x)$ at a point $x = a$.

(ii) Let $f(x) = x^3 \sin|x|$. Use the definition of derivative you have given in part (a)(i) to find the derivative of f at 0.

Question 5 continues on Page 9.

(b) (i) The graph of an ellipse with equation $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is given below, where a and b are positive integers. Find the values of a and b. Draw, on the graph below, the tangent line at the point $(6,4)$.



(ii) Show that the equation of the tangent line to the ellipse at the point (x_0, y_0) can be written as $\dfrac{x_0 x}{a^2}+\dfrac{y_0 y}{b^2}=1$.

QUESTION 6. **(15 Marks)**

(a) After a particular drug is taken, the concentration of the drug in the bloodstream is modeled by the function

$$c(t) = 27\left(e^{-0.4t} - e^{-0.6t}\right),$$

where the time t is measured in hours and the concentration c is measured in $\mu g/m\ell$. Find the maximum concentration of the drug during the first 6 hours after it is taken.

Question 6 continues on Page 11.

(b) Find the volume of the largest cylinder that can be inscribed in a sphere of radius r.

QUESTION 7. **(25 Marks)**

(a) Evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{x^4 - 1}\, dx.$

(ii) $\displaystyle\int \frac{x}{\sqrt{6x - 8 - x^2}}\, dx.$

Question 7 continues on Page 13.

(b) (i) Use integration by parts to prove the reduction formula

$$\int \sin^n x \; dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \; dx, \;\; n \geq 1.$$

(ii) Evaluate $\displaystyle\int_0^{\pi/2} \sin^5 x \; dx.$

Question 7 continues on Page 14.

(c) Let R be the region bounded by the curve of $y = e^{x^2}$, x-axis, y-axis and $x = 1$. Find the volume of the solid obtained by rotating R about the y-axis.

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln |x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln |\sec x| + C \qquad \int \cot x \, dx = \ln |\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1} \left(\frac{x}{a}\right) + C, \; |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C \qquad \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1} \left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.