NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

MH1810 – MATHEMATICS 1

April 2016 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. Reference materials permitted in this exam are limited in volume to a single two-sided sheet of A4 paper.

5. Candidates may use calculators. Nevertheless, they should write down systematically the steps in their workings.

QUESTION 1. **(15 marks)**

Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{(\tan\sqrt{x})^{2016}}{x} & \text{when } x > 0, \\ \sin(x^2) & \text{when } x \leq 0. \end{cases}$$

(a) Show that $\lim_{x \to 0} f(x) = 0$.

(b) Is f continuous at $x = 0$? Justify your answer.

QUESTION 2. **(20 marks)**

Consider an equilateral triangle T with side length a. A rectangle inscribed into T is a rectangle with two vertices lying on the base of T and two other vertices on the remaining sides of T.

(a) Use the Extreme Value Theorem to justify that there exists a rectangle inscribed into T which has the greatest area $A_{\max}$ among all such rectangles.

(b) Find the value of $A_{\max}$.

QUESTION 3. **(25 marks)**

(a) Evaluate the indefinite integral

$$\int \frac{x^3}{\sqrt{1-x^8}}\,dx.$$

(b) The planar region bounded by the curves $x = 1$, $x = 4$ and $y = \sqrt{x}\left(1 - \frac{x}{4}\right)$ is revolved about the x-axis. Calculate the exact volume of the resulting solid.

(c) Suppose f is a function given by

$$f(t) = \begin{cases} 0 & \text{when } t < 0, \\ t & \text{when } 0 \le t \le 1, \\ 2 - t & \text{when } t > 1. \end{cases}$$

(i) Sketch a graph of f.

(ii) Consider a new function F defined by $F(x) = \displaystyle\int_0^x f(t)\,dt$. Using the graph of f from part (i) or otherwise, find the value x for which $F(x)$ is maximal.

QUESTION 4. **(20 marks)**

Let L be the line passing through the points $A = (2, -1, -2)$ and $B = (-4, 2, 1)$.

(a) Find an equation of L.

(b) Find the point of intersection of L with the z-axis.

(c) Find an equation of the line which intersects both L and the z-axis at right angles.

QUESTION 5. **(10 marks)**

Find all values of the variables x, y, z for which the matrix

$$A = \begin{bmatrix} x & 0 \\ y & z \end{bmatrix}$$

satisfies $A \cdot A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$.

QUESTION 6. **(10 marks)**

For a complex number z, consider the expression

$$w = \frac{1+z}{1-z}.$$

Find all complex numbers z for which w is a well-defined **real** number.

END OF PAPER

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.