General Solution Online Assignment Complex(Except Q1-4)

29 August 2021 13:36

(1) $z = 1+\sqrt{3}i \quad w = -\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}i$

$|z| = 2 \quad |w| = 1$

$\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{2}{1}$

$= 2$

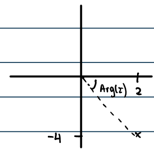
(2) $\frac{1}{1+3i} + \frac{1}{1+6i} = \frac{1-3i}{10} + \frac{1-6i}{37}$

$= \frac{37(1-3i) + 10(1-6i)}{370}$

$= \frac{37-111i+10-60i}{370}$

$= \frac{47}{370} - \frac{171}{370}i$

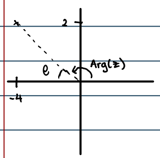
(3) $z = 2-4i$



$Arg(z) = \tan^{-1}\left(-\frac{4}{2}\right)$

$= -1.11$ rad (2dp)

(4) $z = -4+2i$



$\theta = \tan^{-1}\left(\frac{2}{4}\right)$

$= 0.4636$

$Arg(z) = \pi - 0.4636$

$= 2.68$ rad (2dp)

(5) $C+Di$ is a solution, $C-Di$ also solution.

$X(z-C-Di)(z-C+Di) = X(z^2 - Cz + Diz$

$-Cz + C^2 - CDi$

$-Diz + CDi + D^2)$

$= X(z^2 - 2Cz + C^2 + D^2)$

$A = -2XC, \quad B = X(C^2+D^2)$

(6) $\frac{A+ai}{B+bi} = C+Di$

$A+ai = (C+Di)(B+bi)$

$= CB + Cbi + DBi - Db$

$CB - Db - A + Cbi + DBi - ai = 0$

$CB - Db - A = 0 \qquad Cb + DB - a = 0$

$b = -\frac{A-CB}{D} \qquad a = Cb + DB$ (sub in b)

7) $z_1\bar{z}_2 = x+iy$

$|z_1 z_2| = |z_1||z_2| = |z_1||\bar{z}_2| = |z_1\bar{z}_2|$

$= \sqrt{x^2+y^2}$

(8) $Cz + \frac{D}{z}$ is a real number

Let $z = a+ib$

$c(a+ib) + \frac{D}{z} = c(a+ib) + \frac{D(a-ib)}{z\bar{z}}$

$= Ca + Cib + \frac{Da}{z\bar{z}} - \frac{Dib}{z\bar{z}}$

Real no., Imaginary part = 0,

$Cb - \frac{Db}{z\bar{z}} = 0$

$Cb = \frac{Db}{z\bar{z}}$

$z\bar{z} = \frac{D}{C}$

(9) $(C+Di)z = (C-Di)\bar{z}$

Let $z = a+ib$

$(C+Di)(a+ib) = (C-Di)(a-ib)$

$Ca + Cib + Dia - Db = Ca - Cib - Dia - Db$

$2Cib = -2Dia$

$\frac{b}{a} = -\frac{2Di}{2Ci}$

$= -\frac{D}{C}$

(10) $|z+\bar{z}| + |z-\bar{z}| = 2z + x$

$|2Re(z)| + |2Im(z)| = 2Re(z) + 2Im(z) + x$

RHS consists of magnitudes, so RHS is completely real.

$\therefore$ LHS also completely real $\Rightarrow Im(z) = 0$

$|2Re(z)| = 2Re(z) + x$

Open up modulus on RHS

$2Re(z) = 2Re(z) + x \qquad -2Re(z) = 2Re(z) + x$

$0 = x$ (rej $0 \neq x$) $\qquad -4Re(z) = x$

$Re(z) = -\frac{x}{4}$

(11) $\left(\frac{1+\sqrt{3}i}{2}\right)^x = (re^{i\theta})^x$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= 1$

$\theta = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Using De Moivre, $\left(\frac{1+\sqrt{3}i}{2}\right)^x = 1^x e^{i\frac{x\pi}{3}}$

$= e^{i\frac{x\pi}{3}}$

$= \underbrace{\cos\frac{x\pi}{3}}_{a} + i\underbrace{\sin\frac{x\pi}{3}}_{b}$

(12) $z = re^{ik\pi} \qquad z^n = r^x e^{i\frac{\pi}{y}}$

$z^n = (re^{ik\pi})^n$

$= r^n e^{ikn\pi}$

$n = x, \quad kn = \frac{1}{y}$

$k = \frac{1}{xy}$

(13) $z^2 + z\bar{w} = x$ —① $\qquad 2\bar{z} = \bar{w}(1-i)$ —②

$\bar{w} = \frac{2\bar{z}}{1-i}$ —③

sub ③ into ①

$z^2 + z\left(\frac{2\bar{z}}{1-i}\right) = x$

Let $z = a+ib$

$(a+ib)^2 + \frac{2z\bar{z}}{1-i} = x$

$a^2 - b^2 + 2abi + \frac{2(a^2+b^2)}{1-i} = x$

multiply this term by $\frac{1+i}{1+i}$

$a^2 - b^2 + 2abi + (a^2+b^2)(1+i) = x$

$a^2 - b^2 + 2abi + a^2 + b^2 + a^2i + b^2i = x$

$2a^2 + 2abi + a^2i + b^2i = x$

$2a^2 = x \qquad 2ab + a^2 + b^2 = 0 \quad [(a+b)^2 = a^2+b^2+2ab]$

$a = \sqrt{\frac{x}{2}} \qquad (b+a)^2 = 0$

$b = -a = -\sqrt{\frac{x}{2}}$

Sub a and b in ③

$\bar{w} = \frac{2\left(\sqrt{\frac{x}{2}} + \sqrt{\frac{x}{2}}i\right)}{1-i}$

$= \left(\sqrt{\frac{x}{2}} + \sqrt{\frac{x}{2}}i\right)(1+i)$

$= \sqrt{\frac{x}{2}} + \sqrt{\frac{x}{2}}i + \sqrt{\frac{x}{2}}i - \sqrt{\frac{x}{2}}$

$= 2\sqrt{\frac{x}{2}}i$

$w = -2\sqrt{\frac{x}{2}}i$

(14) Let $z = x+iy$

$|x+iy-i| \geqslant |x+iy-1|$ and $|x+iy+i| \geqslant |x+iy-1|$

$|x+(y-1)i| \geqslant |(x-1)^2+iy|$ ← same method

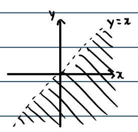
$x^2 + (y-1)^2 \geqslant (x-1)^2 + y^2$

$x^2 + y^2 + 1 - 2y \geqslant x^2 - 2x + 1 + y^2 \qquad x^2+y^2+2y+1 \geqslant x^2 - 2x + 1 + y^2$

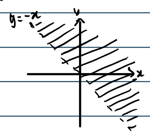
$-2y \geqslant -2x \qquad 2y \geqslant -2x$

$y \leq x$ and $y \geqslant -x$

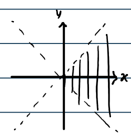
Use graph to visualize



and



combine ⇒



(15) $z - \frac{1}{z} = C + Di$

Note $|\alpha| = \sqrt{\alpha^2}$

$\left|z + \frac{1}{z}\right| = \sqrt{\left(z+\frac{1}{z}\right)^2}$

$= \sqrt{\left(z^2 + \frac{1}{z^2} + 2\right)}$

$\left(z - \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} - 2$

$(C+Di)^2 = C^2 + 2CDi - D^2$

$z^2 + \frac{1}{z^2} - 2 = C^2 - D^2 + 2CDi$

$z^2 + \frac{1}{z^2} + 2 = z^2 + \frac{1}{z^2} - 2 + 4$

$= C^2 - D^2 + 4 + 2CDi$

$= \left[(C^2-D^2+4)^2 + (2CD)^2\right]^{\frac{1}{2}} e^{ik\theta}$ —①

$\left|z + \frac{1}{z}\right| = $ sqrt(①)

$= \left\{\left[(C^2-D^2+4)^2 + (2CD)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$

$= \left[(C^2-D^2+4)^2 + (2CD)^2\right]^{\frac{1}{4}}$