SC4024/CZ4124
Data Visualization

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Chapter 11.2
Graph Visualization

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Outline

- What is a graph/network?
  - Real world networks
  - Common graph vocabulary
- Graph Visualization Methods
  - Node-link diagram
  - Adjacency matrix
  - Others
- Packages and tools for graph visualization

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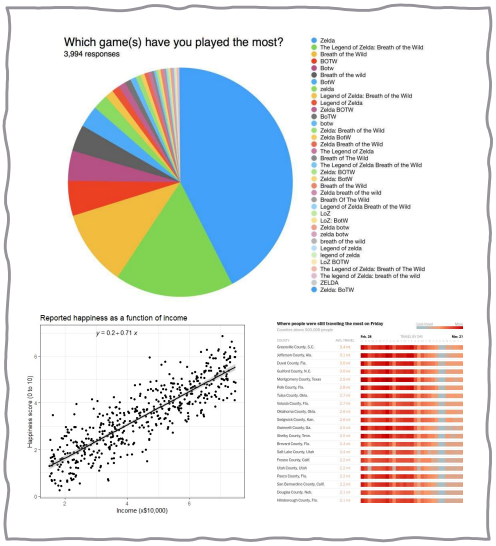
What is a Graph?



Graphs

Common charts that represent data are often referred to as "graphs", or "graph visualizations"

- Bar charts
- Line charts
- Pie charts
- Etc.

Clarification: for this lecture, when I refer to graphs, I do *not* mean the type of charts shown on the left.

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What is a Graph?

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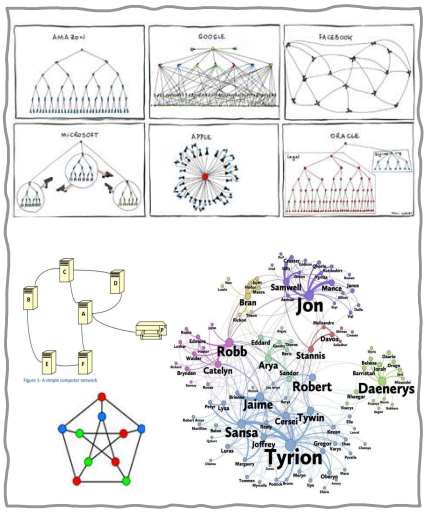
Graphs

Let's instead talk about graphs, networks, & trees in the mathematical sense: a model for representing items and the relationships between those items

- Social / friendship networks
- Computer networks
- Energy or transportation grids
- Organizational structures
- Etc.



Why do we care about visualizing graphs?

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Networks in Real World

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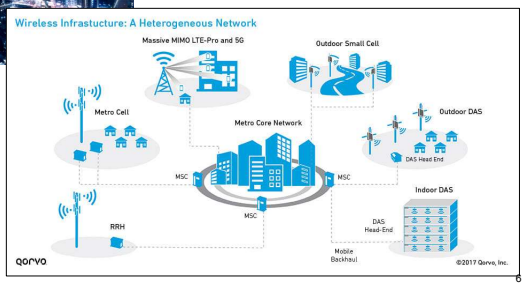
- Telecommunication network



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Networks in Real World

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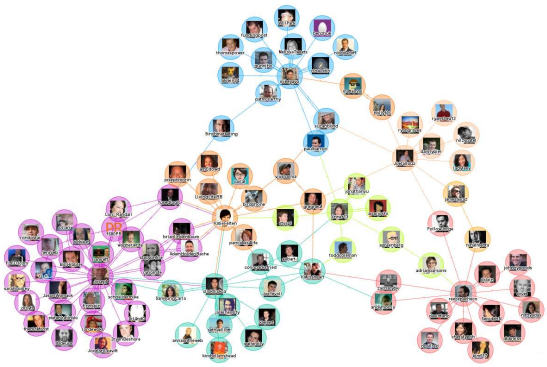
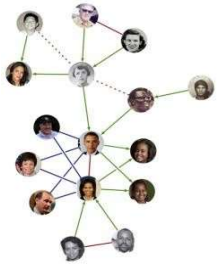
- Social network





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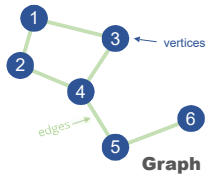
Common Graph Vocabulary

Graph G = {V, E}

V => collection of **vertices**

E=> set of **edges** consisting of vertex pairs



Graph
(represented as a node-link diagram)

G = {V, E}

V = {1, 2, 3, 4, 5, 6}
E = {(1,2), (1,3), (2,4), (3,4), (4,5), (5,6)}

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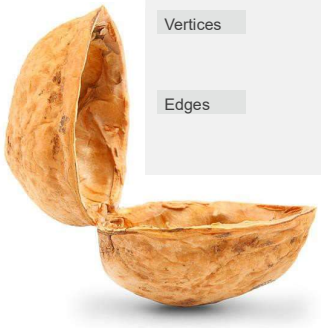
Common Graph Vocabulary



Graphs

Vertices

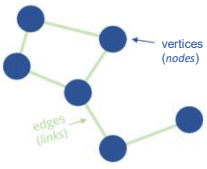
Edges

aka

Networks

Nodes

Links



Graph (*network*)

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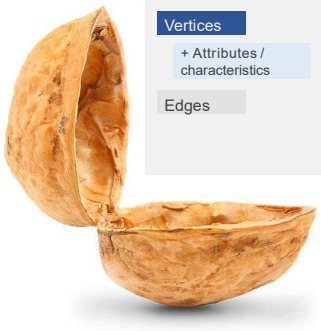
Common Graph Vocabulary

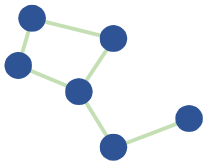


Graphs

Vertices

+ Attributes / characteristics

Edges



Vertex id	Name	Favorite color	Popularity
1	Sam	Blue	6
2	Sebastian	Green	7
3	Abigail	Purple	8
4	Haley	Pink	2
5	Shane	Orange	4
6	Leah	Purple	7

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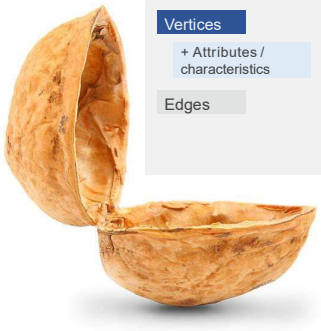
Common Graph Vocabulary



Graphs

Vertices

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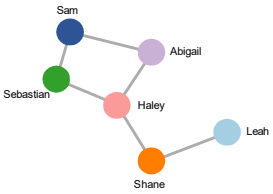
Edges



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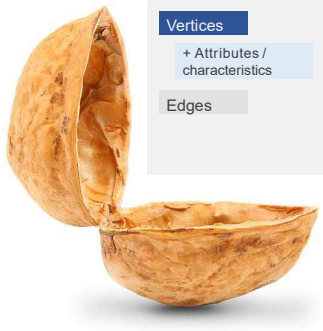
Common Graph Vocabulary

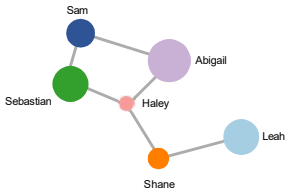


Graphs

Vertices

+ Attributes / characteristics

Edges



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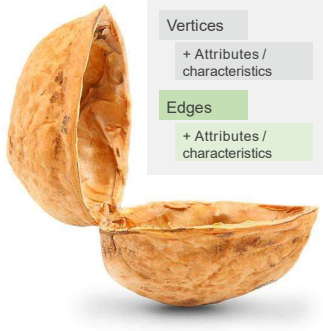
Common Graph Vocabulary



Graphs

Vertices

+ Attributes / characteristics

Edges

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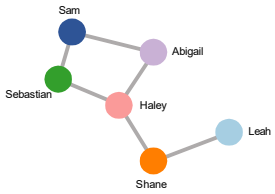

Edge id	Source	Target	Friend value
1	Sam	Sebastian	10
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6	Shane	Leah	2

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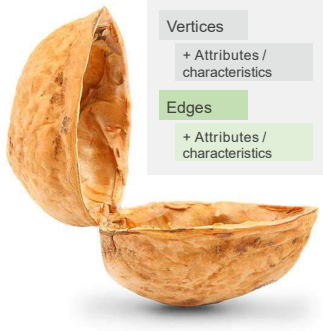
Common Graph Vocabulary



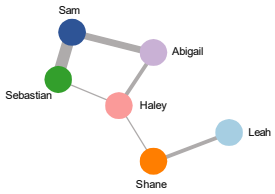
Graphs

Vertices

+ Attributes / characteristics

Edges

+ Attributes / characteristics



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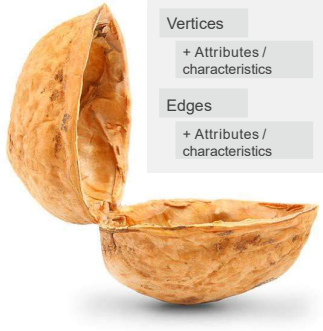
Common Graph Vocabulary



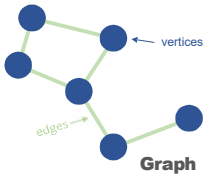
Graphs

Vertices

+ Attributes / characteristics

Edges

+ Attributes / characteristics



Questions?

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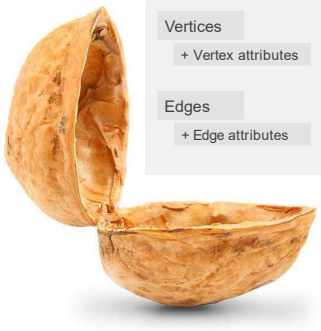
Common Graph Vocabulary



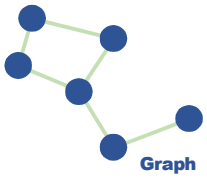
Graphs & Trees

Vertices

+ Vertex attributes

Edges

+ Edge attributes



Graph

Tree

How do trees differ from graphs?

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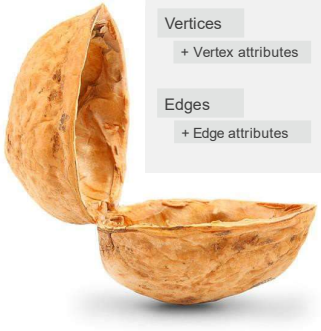
Common Graph Vocabulary



Trees

Vertices

+ Vertex attributes

Edges

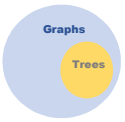
+ Edge attributes

Tree

Properties of trees

- Connected
- Hierarchical structure
- One path between any pair of vertices
- No cycles in the graph
- Removing an edge would create a disconnected graph

Every tree is a graph but not every graph is a tree!



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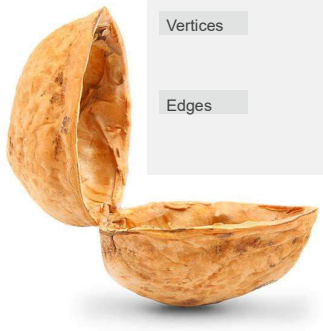
Common Graph Vocabulary



Graphs

Vertices

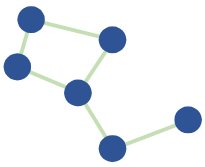
Edges

$\stackrel{?}{=}$

Trees

Vertices

Edges



Is this graph a tree?

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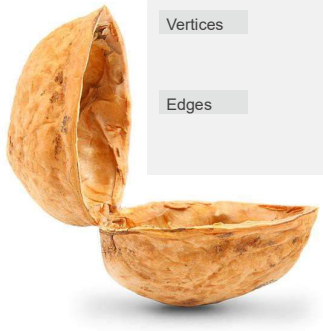
Common Graph Vocabulary



Graphs

Vertices

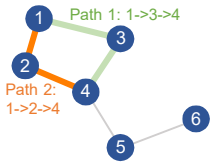
Edges

$\supseteq$

Trees

Vertices

Edges



Is this graph a tree?
No

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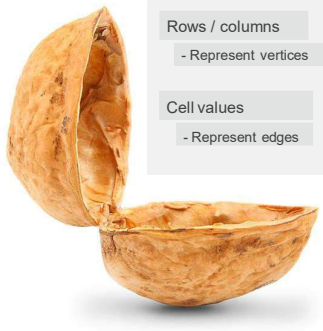
Common Graph Vocabulary



Adjacency Matrix

Rows / columns
- Represent vertices

Cell values
- Represent edges

	1	2	3	4
1	0	1	1	1
2	1	0	1	0
3	1	1	0	1
4	1	0	1	0

Properties of adjacency matrices

- Another graph representation
- Symmetrical along the diagonal
  - Can read from top or bottom half
  - Typically, all 0's on the diagonal (unless self-loops)
- Non-zero cell value means an edge exists between that pair
  - Zero cell value means no edge exists
  - Cell values can also be edge 'weights' (so not just 0/1)

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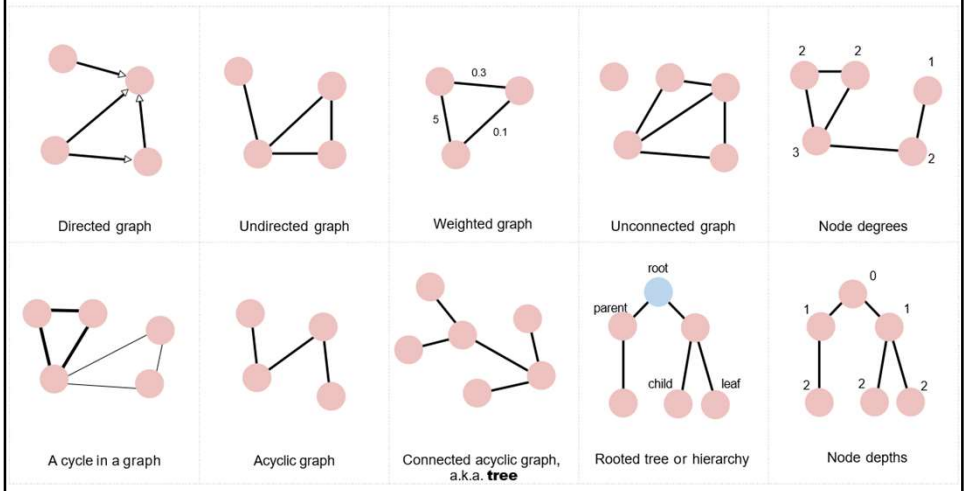
Common Graph Vocabulary



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Outline

- What is a graph/network?
  - Real world networks
  - Common graph vocabulary
- Graph Visualization Methods
  - Node-link diagram
  - Adjacency matrix
  - Others
- Packages and tools for graph visualization

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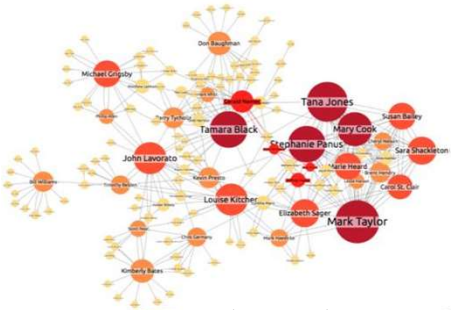
Graph Visualization Methods

- Graph visualization (a.k.a, network visualization) is concerned with visual representations of graph or network data
- Effective graph visualization reveals graph structures and help users understand and analyze the network data



Enron Email Network
https://cambridge-intelligence.com/using-social-network-analysis-measures/

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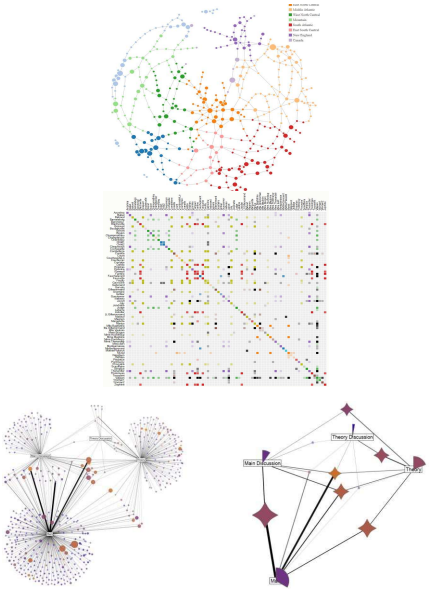
Graph Visualization Methods

- Node-link diagram
  - Force-directed layout
  - MDS layout
- Adjacency matrix
- Others
  - Hybrid layout
  - Graph simplification methods



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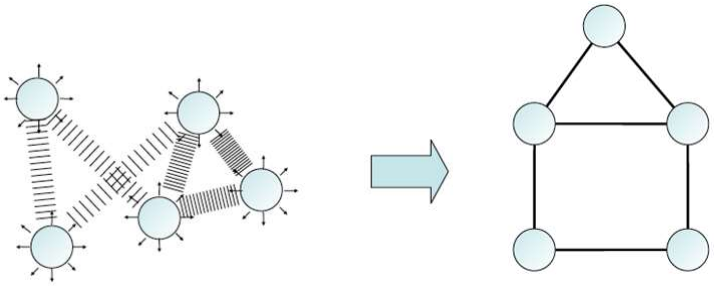
Node-link diagram: Force-Directed Layout

- What about graphs without an intrinsic order?
- Physical model:
  - edge → spring
  - node → mass point



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Node-link diagram: Force-Directed Layout

- It assumes that there is a spring between each pair of nodes, which leads to attractive force and repulsive force as follows:

$$f_a(d) = d^2/k$$

$$f_r(d) = -k^2/d$$

where d is the distance between the two nodes and k is a constant.

- With the effects of **attractive force and repulsive force**, nodes far away will be dragged near and nodes that are overlapped will be pushed away, and finally reach a stable balance after **iterations.**

T. M. Fruchterman and E. M. Reingold. Graph drawing by force-directed placement. Software: Practice and Experience, 21(11):1129–1164, 1991.

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Node-link diagram: Force-Directed Layout

- Starting positions: random or initial **configuration**
- Loop:
  - Compute **the attractive and repulsive** forces for every pair of nodes
  - Accumulate the **force** (vector) for every node
  - Update the position of each node **step by step** according to their forces
- The loop stops until the layout is "good enough"

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Results of Force-directed Layout

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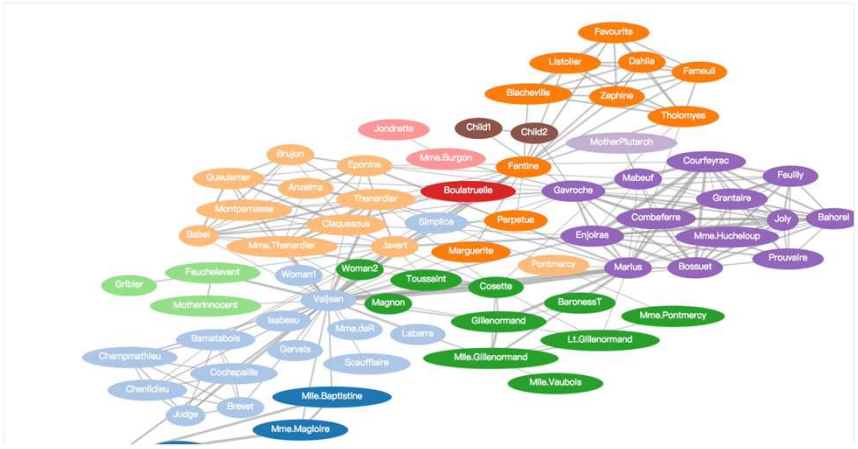
Force-directed graph with elliptic forces



https://bl.ocks.org/jpurma/6dd2081cf25a5d2dfcdcab1a4868f237

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Results of Force-directed Layout

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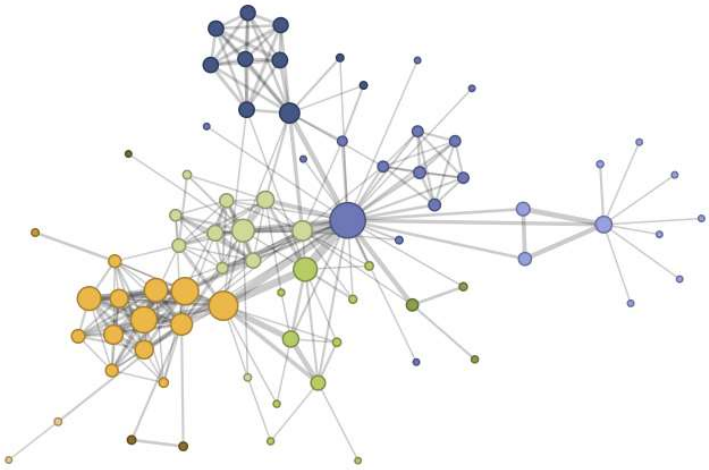

Character relations in Les Miserables

http://hci.stanford.edu/jheer/files/zoo/

https://homes.cs.washington.edu/~jheer//files/zoo/ex/networks/force.html

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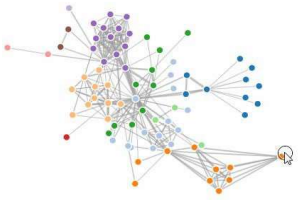
Pros and Cons of Force-directed Layout

- Pros
  - Very flexible for any type of graphs
  - Forces can be customized
  - Easy to implement



- Cons
  - **Local optimal**
  - **Initial configuration** is important
  - **Computation complexity** of iterative algorithm

- Extensions
  - Barnes-Hut quadtree decomposition
  - FADE、GRIP、FMS、FM^3、GVA

try it interactively at https://observablehq.com/@d3/force-directed-graph

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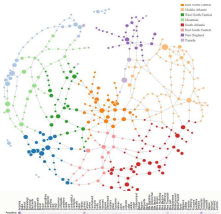
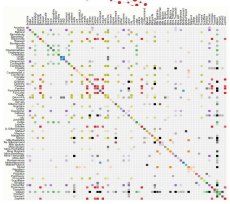
Graph Visualization Methods

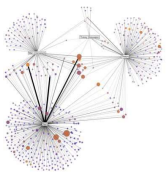
- Node-link diagram
  - Force-directed layout
  - **MDS layout**

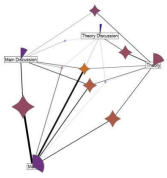


- Adjacency matrix



- Others
  - Hybrid layout
  - Graph simplification methods





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Node-link diagram: MDS (Multidimensional Scaling) Layout

- Focus on addressing the limitations of force-directed layout
- Dimension reduction
  - Keep the consistency of relative distance between nodes
- MDS is a global optimal method
- Optimal function $||x_i - x_j|| \approx d_{ij}$, where d_{ij} is the graph-theoretical distance between Node i and j, and x_i and x_j correspond to their coordinates in the 2D plane

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Solving Optimal Function

- Stress majorization
  - Stress(X) = $\sum_{i,j} w_{ij}(d_{ij} - ||x_i - x_j||)^2$
  - d_{ij} is the graph-theoretical distance between the i-th node and the j-th node
  - $w_{ij} = d_{ij}^q$, usually q = -2

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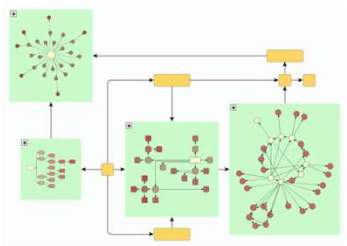
Other Presentations of Node-link Diagrams

- Orthogonal Diagram
  - UML diagram

- Nested ordered
  - Recursively applying nested layout
  - For intrinsic ordered topology
- Arc Diagram



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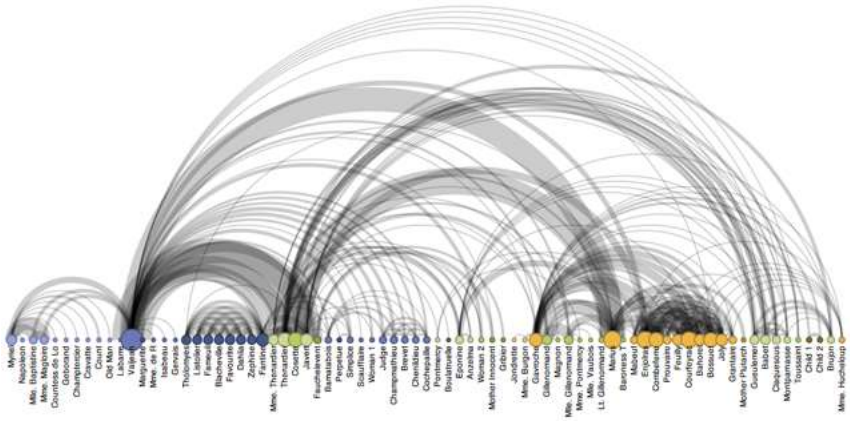
Arc Diagram



Character relations in Les Miserables

http://hci.stanford.edu/jheer/files/zoo/
https://homes.cs.washington.edu/~jheer//files/zoo/ex/networks/arc.html

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Node-Link Diagram Summary

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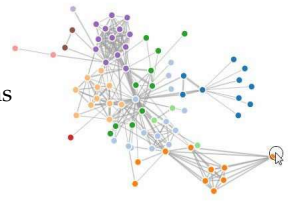
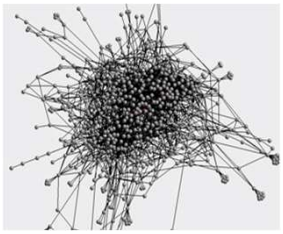
- Pros:
  - Intuitive visual interpretation
  - Good representation of topology, clusters and paths
  - Flexible, many variants



- Cons:
  - Almost for all algorithms, time complexity is a bit high ~ $O(n^2)$
  - Not so good for dense graphs (especially edge cluttered graphs)



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Graph Visualization Methods

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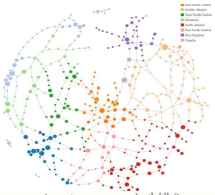
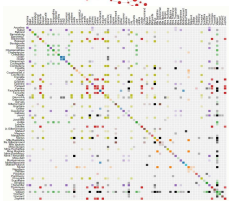
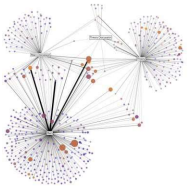
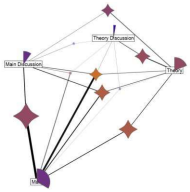
- Node-link diagram
  - Force-directed layout
  - MDS layout
- **Adjacency matrix**
- Others
  - Hybrid layout









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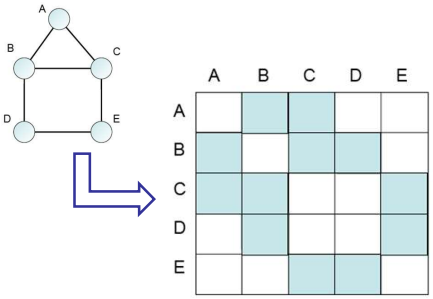
Adjacency Matrix

- $N \times N$ matrix, representing relations among N objects
- Position (i,j) represents the relation between the i-th object and the j-th object
  - Weight
  - Direction
- Related issues
  - Ordering
  - Path finding



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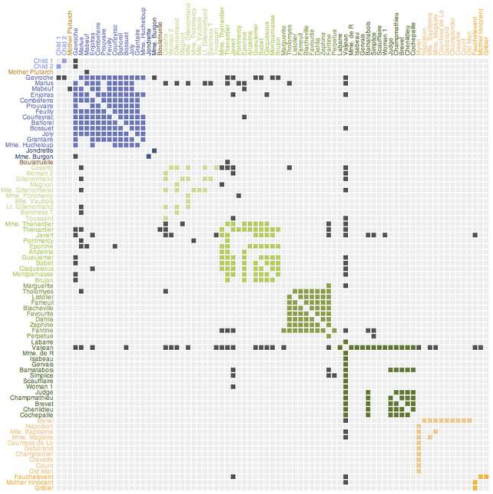
Good Ordering of Adjacency Matrix



Appropriate node ordering is critical for visually identifying the meaningful graph patterns

https://homes.cs.washington.edu/~jheer/files/zoo/ex/networks/matrix.html

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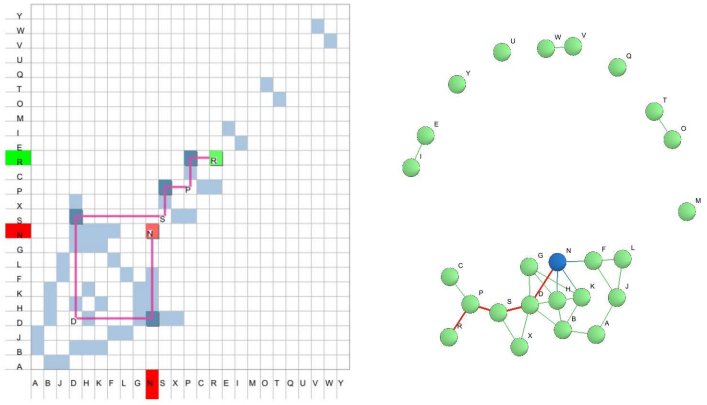
Path Visualization for Adjacency Matrix



Left: Connecting matrix entries and the diagonal.
Right: Path {N;D;S;P;R} is highlighted in the Node-link Diagram.

Compared with node-link diagrams, it is **hard to perform path-finding tasks** in adjacency matrix.

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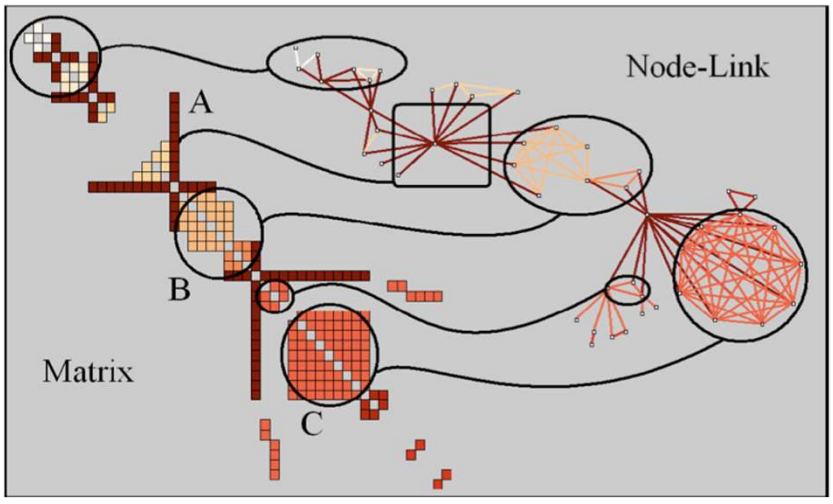
Recognizing the Patterns of Matrix



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Adjacency Matrix Summary

- Pros:
  - No edge crossing, good for edge cluttered graph
  - Good visual scalability
  - Good presentation of graph pattern
- Cons:
  - Visualization is abstract to understand
  - Difficult to follow a transitive relation path

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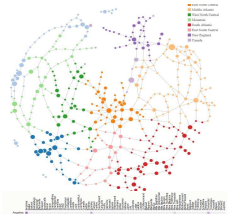
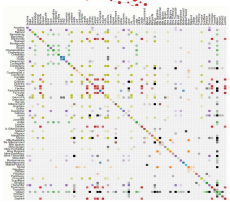
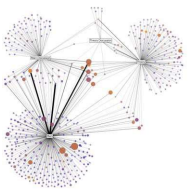
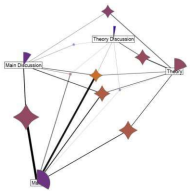
Graph Visualization Methods

- Node-link diagram
  - Force-directed layout
  - MDS layout
- Adjacency matrix
- Others
  - **Hybrid layout**
  - **Graph simplification methods**









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Hybrid Layout

- Adjacency matrix can handle complex and dense edge relations, but suffers from insufficient space usage when there are many nodes
- Node-link diagram can handle relatively more nodes, but can suffer from serious visual clutters when there are dense edge relations
- What if there are a large number of nodes, and some of them have dense edge relations?

-- Combine node-link diagram with matrix

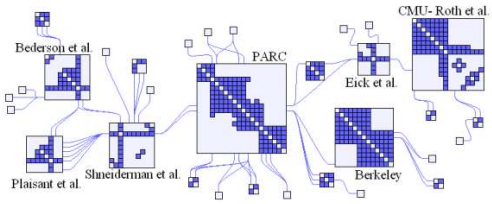

Fig. 1: NodeTrix Representation of the largest component of the Info-Vis Co-authorship Network

Nathalie Henry, et al. NodeTrix: A Hybrid Visualization of Social Networks. TVCG 2007.

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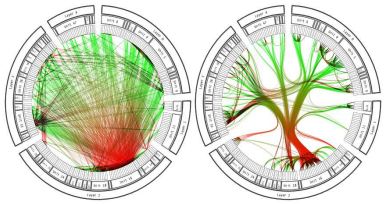
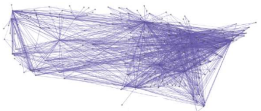
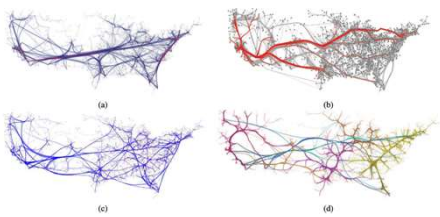
Graph Simplification Methods: - Edge Bundling

- Edge bundling approaches are designed to handle the dense edges between nodes, reducing visual clutters and helping users identify the overall links between nodes





Original Graph Visualization



[1] D. Holten, Hierarchical edge bundles: Visualization of adjacency relations in hierarchical data, TVCG 2006.
[2] Zhou, Hong, et al. "Edge bundling in information visualization." *Tsinghua Science and Technology* 18.2 (2013): 145-156.

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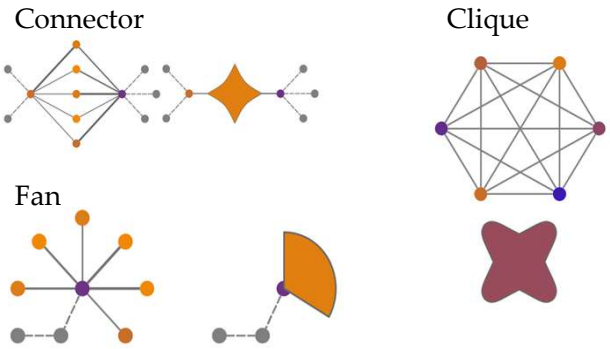
Graph Simplification Methods: - Network Motifs

- Network motifs aims to represent representative subgraphs of a large graph as a series of meaningful motifs to simplify the visualization of large graphs



Dunne and Shneiderman, Motif Simplification: Improving Network Visualization Readability with Fan, Connector, and Clique Glyphs. CHI2013 47

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Graph Simplification Methods: - Network Motifs

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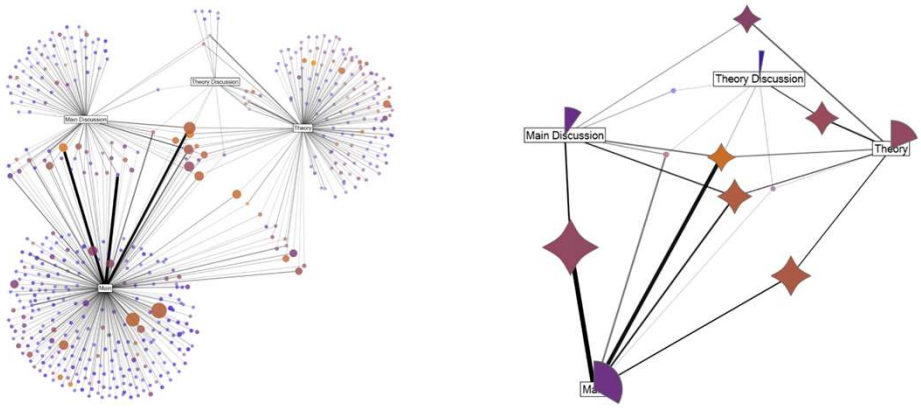

Dunne and Shneiderman, Motif Simplification: Improving Network Visualization Readability with Fan, Connector, and Clique Glyphs. CHI2013 48

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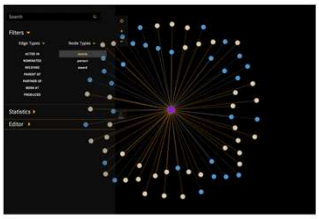
Graph Visualization Tools

- Visualization packages



Alchemy.js
…

Sigma.js
Cytoscape.js
VivaGraphJS
…

D3.js
Pixi.js
…

More and more flexible

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Graph Visualization Tools

- Softwares
  - Gephi
  - Cytoscape
  - Palantir
  - visone
  - …

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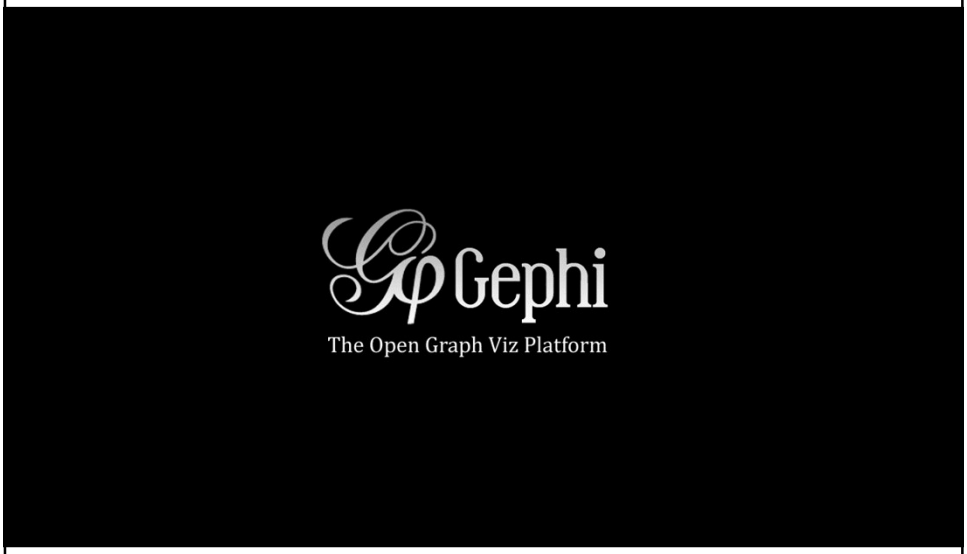
Graph Visualization Tools



https://gephi.org/

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Summary

- Graph visualizations are widely used
- Graph visualization methods
  - Node-link diagram
    - Familiar, but problematic for dense graphs
  - Matrix
    - Abstract, hard to follow paths
  - Graph simplification can help
    - Not always possible, and not always appropriate
- Take-home message: no best solution; graph visualization is still under active research!!!

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Questions?

Thank You!

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