NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MH1810 - Mathematics 1

May 2018 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **SEVEN (7)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. Let $z = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

(a) Find the modulus and the principal argument of z. **(7 marks)**

(b) Using part (a) and De Moivre's Theorem, find z^5. Express your answer in the form $x + yi$, where x and y are real numbers. **(8 marks)**

QUESTION 2. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(a) Compute $\mathbf{u} \cdot \mathbf{v}$ **(5 marks)**

(b) Compute $\mathbf{u} \times \mathbf{v}$ **(5 marks)**

QUESTION 3. Let $B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 19 & 6 & 0 \end{pmatrix}$.

(a) Calculate the determinant of B via cofactor expansion along the second row. **(5 marks)**

(b) Decide whether B is invertible or not. Justify your answer. **(5 marks)**

QUESTION 4. Find the following limits

(a) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$ **(8 marks)**

(b) $\lim_{x \to \infty} \dfrac{3x^2 + 1}{1 - x^2}$ **(7 marks)**

QUESTION 5. Use the squeeze theorem to find the following limit:

$$\lim_{x \to 0} \left(e^{x^2} - 1\right) \sin(x).$$

(10 marks)

QUESTION 6. Suppose that a function f is continuous on the closed interval $[0, 3]$ and $0 \leq f(x) \leq 3$ for every $x \in [0, 3]$. Is it true that $f(c) = c$ for some $c \in [0, 3]$? Justify your answer. **(10 marks)**

QUESTION 7. Let $f(x) = (x^2 - 1)^{2/3}$. Find the global maximum and global minimum values of f on the interval $[-3, 3]$. **(10 marks)**

QUESTION 8.

(a) Evaluate the integral $\int \frac{x+4}{x^3 + 3x^2 - 10x}\, dx$. **(6 marks)**

(b) Evaluate the integral $\int \frac{1}{\sqrt{4 + x^2}}\, dx$. **(7 marks)**

(c) Determine whether $\int_2^\infty \frac{1}{\ln x}\, dx$ converges or diverges. Justify your answer. **(7 marks)**

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C$$

$$\int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \; |x| < |a|$$

$$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

END OF PAPER

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.