NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

CE4042/CZ4042 – NEURAL NETWORKS

Nov/Dec 2017 Time Allowed: 2hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer **ALL** questions.
3. This is an open-book examination.
4. All questions carry equal marks.

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1. (a) Consider a layer of K neurons learning using the Hebbian learning rule. The Hebbian rule states that a weight of a neuron is changed proportional to the product of the input and the output of the neuron. Let $\boldsymbol{w}_k$ denote the weight of the kth neuron and $\boldsymbol{W} = (\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots \boldsymbol{w}_K)$ be the weight matrix connected to the layer.

   (i) If the output of the kth neuron for an input $\boldsymbol{x}$ is y_k, write Hebbian learning equations for weight vector $\boldsymbol{w}_k$ and weight matrix $\boldsymbol{W}$.

   (6 marks)

   (ii) Given a set of input patterns $\{\boldsymbol{x}_p\}_{p=1}^{P}$, write the Hebbian batch learning equation for weight matrix $\boldsymbol{W}$.

   (4 marks)

Note: Question No. 1 continues on Page 2

(b) A three-layer discrete perceptron network receives 2-dimensional inputs $(x_1, x_2)^T \in \boldsymbol{R^2}$ and has four hidden neurons and one output neuron. The shaded region of Figure Q1 shows the input space for which the output of the neuron, $y = 1$. Draw the perceptron network, clearly indicating the values of weights and biases.

(15 marks)

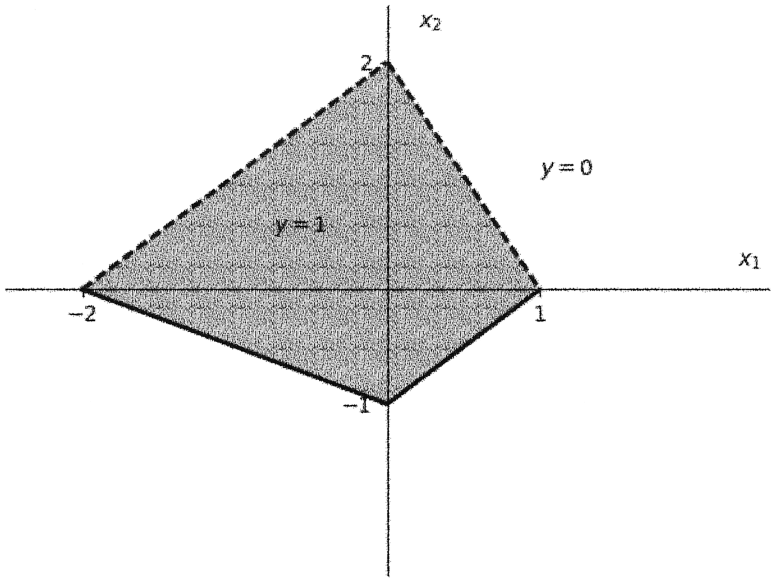

Figure Q1

2. The three-layer feedforward network shown in Figure Q2 receives 2-dimensional inputs $\boldsymbol{x} = (x_1, x_2)^T \in \boldsymbol{R^2}$ and produces an output label $y \in \{0, 1\}$. The hidden-layer is a perceptron layer and the output-layer is a softmax layer, and each layer has two neurons. The weights and biases of the network are initialized as indicated in the figure.

The network is to train to produce outputs $y_1 = 0$ and $y_2 = 1$ for input patterns $\boldsymbol{x_1} = \begin{pmatrix} 1.5 \\ 2.0 \end{pmatrix}$ and $\boldsymbol{x_2} = \begin{pmatrix} -2.0 \\ -0.5 \end{pmatrix}$, respectively, using gradient descent learning. The learning factor $\alpha = 0.4$.

Note: Question No. 2 continues on Page 3

For one iteration of batch gradient descent learning:

(a) Write the initial weight matrices and bias vectors connected to the hidden and output layers.

(4 marks)

(b) Write the input data matrix and the output target vector.

(2 marks)

(c) Find the synaptic inputs and activations of the hidden and output layers.

(6 marks)

(d) Find the error matrices, Δs, at the hidden and output layers.

(8 marks)

(e) Find new weight matrices and bias vectors.

(5 marks)

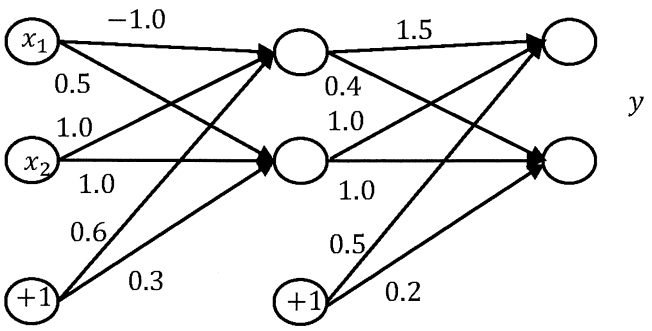

Figure Q2

3. (a) The first layer of a convolutional neural network (CNN) consists of a convolution layer of neurons having weights $\boldsymbol{w} = \begin{pmatrix} 0.2 & 1.0 \\ 0.1 & -0.2 \end{pmatrix}$ and rectified linear (ReLU) activation functions, and a max pooling layer having a pooling window of 2×2 size. The bias connected to the convolution layer is 0.3.

An input image $\boldsymbol{I}$ is applied to the input layer of the CNN:

$$\boldsymbol{I} = \begin{pmatrix} 0.4 & 0.1 & -0.2 & 0.3 & -0.5 \\ -0.8 & -0.2 & 0.5 & 0.4 & 0.1 \\ 0.0 & 0.2 & 1.0 & -0.3 & 0.2 \end{pmatrix}$$

Find the feature maps at the first convolution layer and pooling layer.

(10 marks)

(b) State how an autoencoder is trained in order to

(i) remove noise in data.

(4 marks)

(ii) extract sparse features of data.

(4 marks)

(c) An autoencoder with a single hidden-layer attempts to reconstruct 3-dimensional input patterns $\boldsymbol{x} \in \boldsymbol{R}^3$. The hidden-layer has two neurons with activation functions $g(u) = \frac{1}{1+e^{-u}}$ and the output layer has three neurons with activation functions $f(u) = \frac{1-e^{u}}{1+e^{u}}$. The weight matrix $\boldsymbol{W}$ connected to the hidden-layer, the bias vector $\boldsymbol{b}$ of the hidden-layer, and bias vector $\boldsymbol{b}'$ of the output layer are given by

$$\boldsymbol{W} = \begin{pmatrix} 1.8 & -2.3 \\ -1.2 & 2.9 \\ -3.2 & -2.2 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} -0.7 \\ 0.0 \end{pmatrix} \text{ and } \boldsymbol{b}' = \begin{pmatrix} 1.0 \\ -0.8 \\ 2.6 \end{pmatrix}.$$

For input patterns $\boldsymbol{x}_1 = \begin{pmatrix} 0.5 \\ 0.2 \\ -0.8 \end{pmatrix}$ and $\boldsymbol{x}_2 = \begin{pmatrix} 0.6 \\ -0.4 \\ 0.7 \end{pmatrix}$, find

Note: Question No. 3 continues on Page 4

(i) the hidden-layer representations.

(3 marks)

(ii) the reconstruction errors.

(4 marks)

4. (a) A recurrent neural network (RNN) receives 2-dimensional input patterns $\boldsymbol{x} \in \boldsymbol{R}^2$ and has one hidden layer with recurrent connections. The RNN has three neurons in the hidden layer and one neuron in the output layer. All neurons have logistic activation functions.

The weight matrices $\boldsymbol{U}$ connecting the input to the hidden layer, $\boldsymbol{W}$ connecting the previous hidden state to the next hidden state, and $\boldsymbol{V}$ connecting the hidden output to the output layer are given by

$$\boldsymbol{U} = \begin{pmatrix} -1.0 & 0.5 & 0.2 \\ 0.5 & 0.1 & -2.0 \end{pmatrix}, \boldsymbol{W} = \begin{pmatrix} 2.0 & 1.3 & -1.0 \\ 1.5 & 0.0 & -0.5 \\ -0.2 & 1.5 & 0.4 \end{pmatrix} \text{ and } \boldsymbol{V} = \begin{pmatrix} 2.0 \\ -1.5 \\ 0.2 \end{pmatrix}$$

All bias connections to neurons are set to 0.1 and the hidden-layer activations are initialized to zeros.

Find the output of the network for a sequence $(\boldsymbol{x}(1), \boldsymbol{x}(2), \boldsymbol{x}(3))$ of input patterns:

$$\boldsymbol{x}(1) = \begin{pmatrix} 1.0 \\ 2.0 \end{pmatrix}, \boldsymbol{x}(2) = \begin{pmatrix} -1.0 \\ 1.0 \end{pmatrix}, \text{ and } \boldsymbol{x}(3) = \begin{pmatrix} 0.0 \\ 3.0 \end{pmatrix}.$$

(15 marks)

(b) Briefly explain

(i) why recurrent neural networks are difficult to train to learn long-term dependencies.

(5 marks)

(ii) how long short-term memory (LSTM) networks are able to learn long-term interactions.

(5 marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.