CZ3005 Artificial Intelligence

Week 11b – Default Logic (Process)

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Learning Goals

Understanding the:

- Improved variant of Reiter's Default Logic (RDL)

Recap

- A default rule can be applied to a theory
  - if its precondition is entailed by the theory; and
  - its justifications are all consistent with the theory.
- The application of a default rule leads to the addition of its consequence to the theory.
- Other default rules may then be applied to the resulting theory.
- When the theory is such that no other default can be applied, the theory is called an <u>extension</u> of the default theory.
- The default rules may be applied in <u>different orders</u>, and this may lead to <u>different extensions</u>.

Makinson Approach

- Order ***ground*** instances of defaults in Δ: $d_1, d_2, \ldots$
- Initialize beliefs $\Xi_0 = \Phi$ and used defaults set $\Delta_0 = \emptyset$
- Define Ξ_{n+1} from Ξ_n,
  - Find $\mathrm{d} = \frac{\alpha(c):\ \beta_1(c),\ldots,\beta_n(c)}{\gamma(c)} \notin \Delta_n$ such that
    - Triggered?: $\Xi_n \vdash \alpha(c)$
    - Justified?: Ξ_n is consistent with $\beta_1(c), \ldots, \beta_m(c)$
  - If $\Xi_n \cup \{\gamma(c)\}$ is consistent with each $\beta'(c')$ in $\Delta_n \cup \{d\}$
    - $\Xi_{n+1} = \Xi_n \cup \{\gamma(c)\}, and\ \Delta_{n+1} = \Delta_n \cup \{d\}$
  - else abort -- no extension **for this order of defaults**
- The extension is $\Xi = \bigcup_{i\geq 0} \Xi_i$

Makinson Approach

- No extension guessing
  - **Choose** the order of defaults in Δ: $d_1, d_2, \ldots$

- There still may be more than one possible extension
  - **Different orders** of defaults can lead to different Ξ

- We get the same extensions as in Reiter's approach
  - If they exist at all

Remember …

Delta: Set of Defaults

$$\langle \Delta, \Phi \rangle$$

Phi: Set of “Facts”

$$\mathrm{Cn}()$$

Cn means applying any known inference rules to expand the KB

Operational Semantics

Given a default theory $T = \langle \Delta, \Phi \rangle$, let $\Pi = (\delta_0, \delta_1, \ldots)$ be (a finite or infinite) sequence of (closed) defaults from Δ without multiple occurrences.

$\Pi[k]$ denotes the initial segment of sequence Π with length k.

Model of the world

Each sequence Π is associated with two sets:

- $\mathrm{In}(\Pi) = Cn(\Phi \cup \{consequence(\delta) | \delta \ occurs \ in \ \Pi\})$
- $\mathrm{Out}(\Pi) = \{\neg\phi | \phi \in justifications(\delta) \ for \ some \ \delta \ \mathrm{in} \ \Pi\}$

Everything I have denied the existence of as a result of my assumptions (i.e., the negation of all the justifications)

Example

Consider $\mathrm{T} = \langle \Delta, \Phi \rangle$ with $\Phi = \{\alpha\}$ and defaults from Δ:

$$\delta_1 = \frac{\alpha \ : \neg\beta}{\neg\beta}, \qquad \delta_2 = \frac{\beta \ : \ \gamma}{\gamma}$$

For $\Pi_\mathrm{a} = (\delta_1)$ we have

$$\mathrm{In}(\Pi_a) = Cn(\{\alpha, \neg\beta\}), \mathrm{Out}(\Pi_a) = \{\beta\}$$

For $\Pi_\mathrm{b} = (\delta_2, \delta_1)$ we have

$$\mathrm{In}(\Pi_b) = Cn(\{\alpha, \neg\beta\}), \mathrm{Out}(\Pi_b) = \{\beta\}$$

Process, Successful, Closed

Π is a process of $\mathrm{T} = \langle \Delta, \Phi \rangle$ iff default δ_k is applicable to $In(\Pi[k])$ for every k such that δ_k occurs in Π.

Let Π be a process. We define:

- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$ (*Nothing in the out set can be inferred from the in set*); Otherwise, it **fails**.
- Π is **closed** iff every $\delta \in \Delta$ that is applicable to $In(\Pi)$ already occurs in Π.

Example

Consider $\mathrm{T} = \langle \Delta, \Phi \rangle$ with $\Phi = \{\alpha\}$ and defaults from Δ:

$$\delta_1 = \frac{\alpha \ : \neg\beta}{\eta}, \qquad \delta_2 = \frac{true \ : \ \gamma}{\beta}$$

$\Pi_1 = (\delta_1)$ is successful,

$In(\Pi_1) = Cn(\alpha, \eta)$ and $Out(\Pi_1) = \{\beta\}$

but not closed, since δ_2 is applicable, too.

$\Pi_2 = (\delta_1, \delta_2)$ is closed, but not successful

$In(\Pi_2) = Cn(\alpha, \eta, \beta)$ and $Out(\Pi_2) = \{\beta, \neg\gamma\}$,

$In(\Pi_2) \cap Out(\Pi_2) = \beta$

$\Pi_3 = (\delta_2)$ is a closed and successful process T

$In(\Pi_3) = Cn(\alpha, \beta)$ and $Out(\Pi_3) = \{\neg\gamma\}$,

$In(\Pi_3) \cap Out(\Pi_3) = \emptyset$

Extension

- Let $\mathrm{T} = \langle \Delta, \Phi \rangle$ be a default theory. A set of formulae Ξ is an **extension** of T iff there is some ***closed and successful*** Π such that $\Xi = In(\Pi)$.

- To **find a successful** process: generate a process Π, test whether $\mathrm{in}(\Pi) \cap Out(\Pi) = \emptyset$. If not, then backtrack (try another process).

Process Tree

$\mathrm{T} = \langle \Delta, \Phi \rangle$ be a default theory. A **process tree** is a tree $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ such that all nodes $\mathrm{v} \in V$ are labelled with two sets of formulae:

- an In-set $\mathrm{In}(\mathrm{v})$ and
- an Out-set $\mathrm{Out}(\mathrm{v})$.

The root of G is labelled with $\mathrm{Cn}(\Phi)$ as the In-set and $\emptyset$ as the Out-set. Every $\mathrm{e} \in \mathrm{E}$ denotes a default application and is labelled by it.

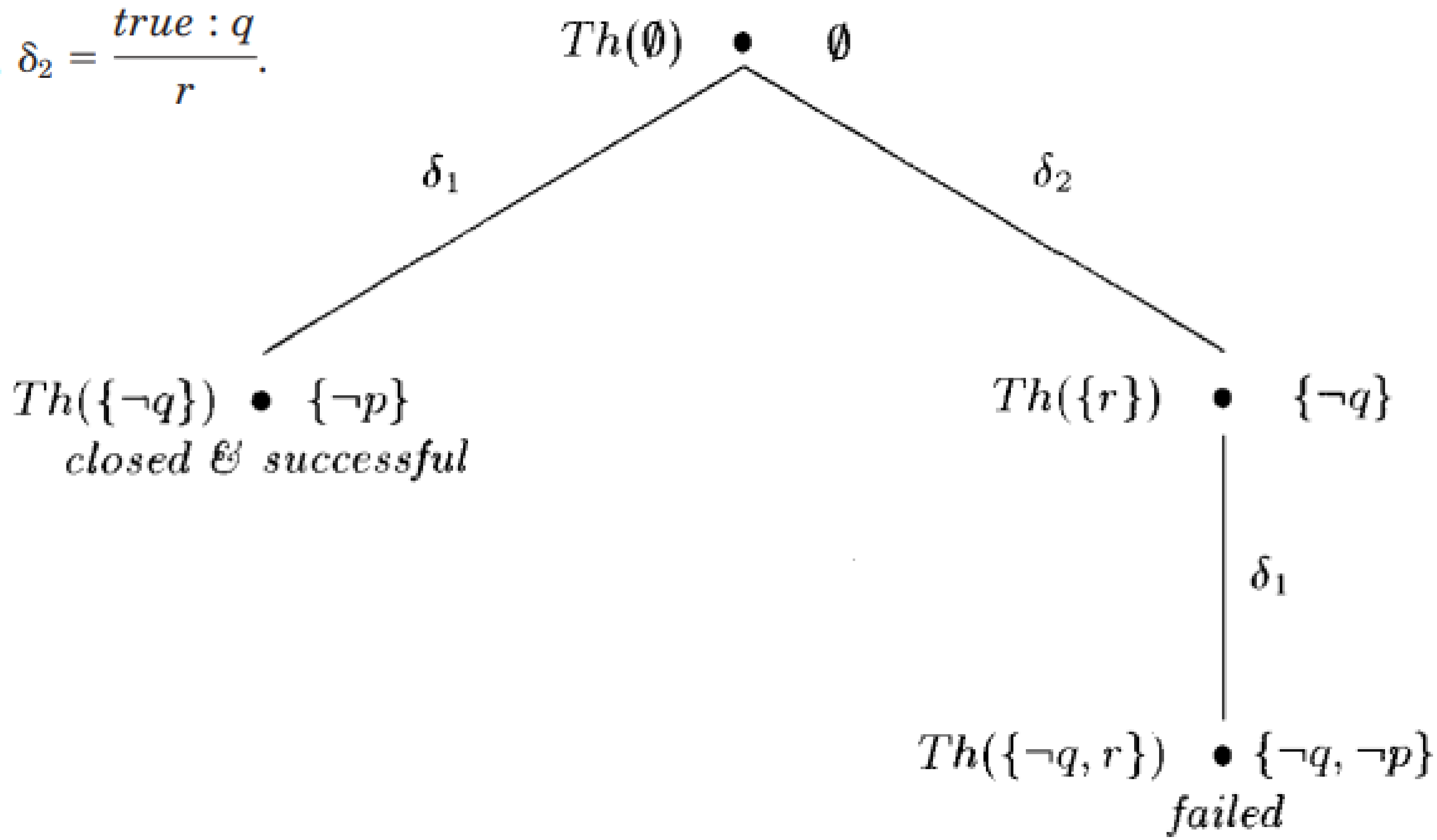
A process is thus a path in G starting from the root.

A node $\mathrm{v} \in V$ is **expanded** if $In(v) \cap Out(v) = \emptyset$. Otherwise, it is a “failed” leaf of the tree.

Process Tree Example

Let $T = (W, D)$ be the default theory with $W = \emptyset$ and $D = \{\delta_1, \delta_2\}$ with

$$\delta_1 = \frac{true : p}{\neg q}, \ \delta_2 = \frac{true : q}{r}.$$



Process Tree: Properties

- A process is thus a path in G starting from root.
- A node v $\in V$ is **expanded** if $In(v) \cap Out(v) = \emptyset$.
- Otherwise, it is a “failed” leaf of the tree.
- Expanded v $\in V$ has a child node, w_δ, for every $\delta = \frac{\alpha :\ \beta_1,\ldots,\beta_n}{\gamma}$
  - w_δ does not appear on the path from the root to v
  - δ is applicable to $In(v)$
  - w_δ connected to v by an edge labelled with δ
  - w_δ is labelled with $In(w_\delta) = Cn(In(v) \cup \{\gamma\})$ and $Out(w_\delta) = Out(v) \cup \{\neg\beta_1, \ldots, \neg\beta_n\}$

Thank you!