[PRINT]

21S1 MH1810
SCS[redacted], 9/2/21 at 8:08:02 PM SGT

Question1: *Score 1/1*

Let $\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$. Find

(a) the magnitude of $\mathbf{u}$, express your answer up to 2 decimal places.

Answer :

Your response	Correct response
4.90	**4.90**

Auto graded **Grade:** 1/1.0

.

(b) the dot product of $\mathbf{u}$ and $\mathbf{v}$.

Answer :

Your response	Correct response
-18	**-18**

Auto graded **Grade:** 1/1.0

.

(c) the angle between $\mathbf{u}$ and $\mathbf{v}$, give your answer in radians, up to 2 decimal places.

Answer :

Your response	Correct response
2.32	**2.32**

Auto graded **Grade:** 1/1.0

.

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Question2: *Score 1/1*

Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \cdot \mathbf{b} = 1/2$, $\mathbf{b} \cdot \mathbf{c} = 1/7$ and $\mathbf{a} \cdot \mathbf{c} = 1/8$. Evaluate (write in the exact form)

- $||8\mathbf{a}||$ =

Your response	Correct response
8	**8**

Auto graded **Grade:** 1/1.0

- $3\mathbf{a} \cdot 6\mathbf{b} =$

Your response	Correct response
9	**9**

Auto graded **Grade:** 1/1.0

- $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) =$

Your response	Correct response
$\frac{3}{8}$	3/8

Auto graded Grade: 1/1.0

- $(\mathbf{a}+\mathbf{b}+\mathbf{c})\cdot(\mathbf{a}-\mathbf{b})=$

Your response	Correct response
$-\frac{1}{56}$	-1/56

Auto graded Grade: 1/1.0

Total grade: 1.0×1/4 + 1.0×1/4 + 1.0×1/4 + 1.0×1/4 = 25% + 25% + 25% + 25%

Question3: *Score 1/1*

If $||\mathbf{u}||=3$, $||\mathbf{v}||=4$ and $\mathbf{u}\cdot\mathbf{v}=2$, find $||\mathbf{u}+\mathbf{v}||$, express your answer up to 2 decimal places.

Answer :

Your response	Correct response
5.39	5.39

Auto graded Grade: 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question4: *Score 1/1*

Let $\mathbf{u}$ and $\mathbf{v}$ be vectors where $||\mathbf{u}+\mathbf{v}||=4$ and $||\mathbf{u}-\mathbf{v}||=8$. Find $\mathbf{u}\cdot\mathbf{v}$.

Answer :

Your response	Correct response
-12	-12

Auto graded Grade: 1/1.0

Total grade: 1.0×1/1 = 100%

Question5: *Score 1/1*

Consider two vectors $\mathbf{a}=-2i-j$ and $\mathbf{b}=2i+j+k$. If $\mathbf{c}=xi+yj+kz, x>0$ is the unit vector that is perpendicular to both a and b, find x, y and z.

Answer :

$x=$

Your response	Correct response
$\frac{1}{5}\sqrt{5}$	1/5*5^(1/2)

Auto graded Grade: 1/1.0

$y =$

Your response	Correct response
$\frac{-2}{5}\sqrt{5}$	**-2/5*5^(1/2)**

Auto graded Grade: 1/1.0

$z =$

Your response	Correct response
0	**0**

Auto graded Grade: 1/1.0

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Question6: *Score 1/1*

Find the distance D from the point $S(4,4,1)$ to the line $\ell : \mathbf{r} = (0,1,2) + t(1,0,1), t \in \mathbb{R}$. Give your answer in 2 decimal places.

Answer : $D =$

Your response	Correct response
4.64	**4.64**

Auto graded Grade: 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question7: *Score 1/1*

Consider the line $\ell : \mathbf{r}(t) = (6,5,5) + t(9,3,2), t \in \mathbb{R}$. Let $P(x,y,z)$ be the point on the line ℓ that is nearest to the origin. Find x, y and z. Express your solution in **exact** form.

Answer :

$x =$

Your response	Correct response
$-\frac{147}{94}$	**-147/94**

Auto graded Grade: 1/1.0

$y =$

Your response	Correct response
$\frac{233}{94}$	**233/94**

Auto graded Grade: 1/1.0

$z =$

Your response	Correct response
$\frac{156}{47}$	**156/47**

Auto graded **Grade:** 1/1.0

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Question8: *Score 1/1*

Find the shortest distance d from the origin to the plane $x + 3y + 3z = 1$. Give your answer in 2 decimal places.

Answer : $d =$

Your response	Correct response
0.23	**.23**

Auto graded **Grade:** 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question9: *Score 1/1*

Find the **acute** angle θ (in radian) between the two planes $4x + 2y + 5z = 1$ and $3x + 5y - 2z = 2$. Give your answer in 2 decimal places.

Answer : $\theta =$

Your response	Correct response
1.28	**1.28**

Auto graded **Grade:** 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question10: *Score 0.66/1*

Suppose the plane $x + ay + bz = c$ contains the point $(1, 2, 3)$ and the line $\ell : \mathbf{r}(t) = (4, 6, 5) + t(1, 1, 1)$, $t \in \mathbb{R}$. Find a, b and c.

Answer :

$a =$

Your response	Correct response

$-\frac{1}{2}$	**-1/2**

Auto graded Grade: 1/1.0

$b =$

Your response	Correct response
$-\frac{1}{2}$	**-1/2**

Auto graded Grade: 1/1.0

$c =$

Your response	Correct response
-3	**-3/2**

Auto graded Grade: 0/1.0

Total grade: 1.0×1/3 + 1.0×1/3 + 0.0×1/3 = 33% + 33% + 0%

Question11: *Score 1/1*

Consider two planes $\pi_1 : x + y + z = 11$ and $\pi_2 : 3x + 8y + 10z = 10$. Suppose the plane $x + ay + bz = c$ is perpendicular to both π_1 and π_2 and contains the point $(2, 4, 6)$. Find a, b and c.

Answer :

$a =$

Your response	Correct response
$-\frac{7}{2}$	**-7/2**

Auto graded Grade: 1/1.0

$b =$

Your response	Correct response
$\frac{5}{2}$	**5/2**

Auto graded Grade: 1/1.0

$c =$

Your response	Correct response
3	**3**

Auto graded Grade: 1/1.0

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Question12: *Score 1/1*

Consider two planes

$$4x + 8y + 5z = 0,$$
$$4x + 8y + 5z = d.$$

Suppose the distance in between the two planes is 12 and $d > 0$.

Find d. Express your answer up to 2 decimal points.

Answer : $d =$

Your response	Correct response
122.96	**122.96**

Auto graded Grade: 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question13: *Score 1/1*

Four points A, B, C and D forms a parallelogram with adjacent sides AB and AC and vertices $A\,(2,0,0)$, $B\,(0,1,0)$ and $C\,(0,0,3)$. Find the coordinates of D.

Answer : $D =($

Your response	Correct response
-2	**-2**

Auto graded Grade: 1/1.0

,

Your response	Correct response
1	**1**

Auto graded Grade: 1/1.0

,

Your response	Correct response
3	**3**

Auto graded Grade: 1/1.0

).

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Question14: *Score 1/1*

Four points A, B, C and D forms a parallelogram with adjacent sides AB and AC and vertices $A\,(7,0,0)$, $B\,(0,8,0)$ and $C\,(0,0,4)$. Find the area of the parallelogram, express your answer up to 2 decimal places.

Answer : The area of the parallelogram =

Your response	Correct response
70.31	**70.31**

Auto graded Grade: 1/1.0

.

Total grade: 1.0×1/1 = 100%

Question15: *Score 1/1*

Find the reflection of the point $(1,2,3)$ in the plane $2x+9y+8z=12$.

Answer : The reflection of the point $(1,2,3)$ is the point (a,b,c), where

$a=$

Your response	Correct response
$\frac{21}{149}$	**21/149**

Auto graded Grade: 1/1.0

$b=$

Your response	Correct response
$-\frac{278}{149}$	**-278/149**

Auto graded Grade: 1/1.0

$c=$

Your response	Correct response
$-\frac{65}{149}$	**-65/149**

Auto graded Grade: 1/1.0

Total grade: 1.0×1/3 + 1.0×1/3 + 1.0×1/3 = 33% + 33% + 33%

Solution Guide Online Assignment Vector

Q1: $\underset{\sim}{u} = \begin{pmatrix}2\\4\\2\end{pmatrix}$, $\underset{\sim}{v} = \begin{pmatrix}2\\-4\\3\end{pmatrix}$

(a) $|\underset{\sim}{u}| = \sqrt{4+16+4}$
$= \sqrt{24}$
$= 4.90$

(b) $\underset{\sim}{u}\cdot\underset{\sim}{v} = (4) + (-16) + (-6)$
$= -18$

(c) $-18 = \sqrt{24}\sqrt{4+16+9}\cos\theta$
$\theta = \cos^{-1}\left(\frac{-18}{\sqrt{24}\sqrt{29}}\right)$
$= 2.32$

Q2: (a) $\|\underset{\sim}{a}\| = 8\|\underset{\sim}{a}\| = 8$

(b) $3\underset{\sim}{a}\cdot 6\underset{\sim}{b} = 18(\underset{\sim}{a}\cdot\underset{\sim}{b})$
$= 9$

(c) $\underset{\sim}{a}\cdot(\underset{\sim}{b}-\underset{\sim}{c}) = \underset{\sim}{a}\cdot\underset{\sim}{b} - \underset{\sim}{a}\cdot\underset{\sim}{c}$
$= \frac{1}{2} - \frac{1}{8}$
$= \frac{3}{8}$

(d) $(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = (\underset{\sim}{a}\cdot\underset{\sim}{a}) - (\underset{\sim}{a}\cdot\underset{\sim}{b}) + (\underset{\sim}{c}\cdot\underset{\sim}{a}) - (\underset{\sim}{c}\cdot\underset{\sim}{b}) - (\underset{\sim}{b}\cdot\underset{\sim}{b}) + (\underset{\sim}{b}\cdot\underset{\sim}{a})$
$= \frac{1}{8} - \frac{1}{7}$
$= -\frac{1}{56}$

Q3: $\|\underset{\sim}{u}\| = 3 \quad \|\underset{\sim}{v}\| = 4$

$\underset{\sim}{u}\cdot\underset{\sim}{v} = 2$

$(\underset{\sim}{u}+\underset{\sim}{v})\cdot(\underset{\sim}{u}+\underset{\sim}{v}) = (\underset{\sim}{u}\cdot\underset{\sim}{u}) + 2(\underset{\sim}{u}\cdot\underset{\sim}{v}) + (\underset{\sim}{v}\cdot\underset{\sim}{v})$

$\|\underset{\sim}{u}+\underset{\sim}{v}\|^2 = \|\underset{\sim}{u}\|^2 + 2(\underset{\sim}{u}\cdot\underset{\sim}{v}) + \|\underset{\sim}{v}\|^2$

$\|\underset{\sim}{u}+\underset{\sim}{v}\| = \sqrt{9+16+2(2)}$
$= 5.39$ (2dp)

Q4: $\|\underset{\sim}{u}+\underset{\sim}{v}\| = 4 \quad \|\underset{\sim}{u}-\underset{\sim}{v}\| = 8$

$\|\underset{\sim}{u}+\underset{\sim}{v}\|^2 = \|\underset{\sim}{u}\|^2 + \|\underset{\sim}{v}\|^2 + 2(\underset{\sim}{u}\cdot\underset{\sim}{v})$

$\|\underset{\sim}{u}-\underset{\sim}{v}\|^2 = \|\underset{\sim}{u}\|^2 + \|\underset{\sim}{v}\|^2 - 2(\underset{\sim}{u}\cdot\underset{\sim}{v})$

$-4(\underset{\sim}{u}\cdot\underset{\sim}{v}) = 64 - 16$

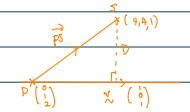
$\underset{\sim}{u}\cdot\underset{\sim}{v} = -12$

Q5: $\underset{\sim}{a} = \begin{pmatrix}-2\\-1\\0\end{pmatrix} \quad \underset{\sim}{b} = \begin{pmatrix}2\\1\\1\end{pmatrix} \quad \underset{\sim}{a}\times\underset{\sim}{b} = \begin{pmatrix}-1\\2\\0\end{pmatrix}$

$\underset{\sim}{c} = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\-2\\0\end{pmatrix}$ (Flip the vector, $x > 0$)

$x = \frac{\sqrt{5}}{5} \quad y = -\frac{2\sqrt{5}}{5} \quad z = 0$

Q6:



$\overrightarrow{PS} = \begin{pmatrix}4\\4\\1\end{pmatrix} - \begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}4\\3\\-1\end{pmatrix}$

$\underset{\sim}{n} = \|\overrightarrow{PS}\times\hat{\underset{\sim}{v}}\|$
$= \frac{1}{\sqrt{2}}\left\|\begin{pmatrix}4\\3\\-1\end{pmatrix}\times\begin{pmatrix}0\\1\\1\end{pmatrix}\right\|$
$= \frac{1}{\sqrt{2}}\left\|\begin{pmatrix}4\\-1\\-3\end{pmatrix}\right\|$
$= \frac{\sqrt{26}}{2}$
$= 4.64$ (2dp)

Q7:

Let $P = \begin{pmatrix}4+9t\\5+3t\\2+2t\end{pmatrix}$

Since P is closes to origin, $\overrightarrow{OP} \perp \underset{\sim}{v}$, $\overrightarrow{OP}\cdot\underset{\sim}{v} = 0$

$\begin{pmatrix}4+9t\\5+3t\\2+2t\end{pmatrix}\cdot\begin{pmatrix}9\\3\\2\end{pmatrix} = 0$

$(36+81t) + (15+9t) + (10+4t) = 0$

$94t = -61$

$t = -\frac{61}{94}$

$x = -\frac{167}{94} \quad y = \frac{233}{94} \quad z = \frac{154}{47}$

Q8: $\pi : x + 3y + 3z = 1$

$\pi : \underset{\sim}{r}\cdot\begin{pmatrix}1\\3\\3\end{pmatrix} = 1$

Let P be $\begin{pmatrix}0\\0\\0\end{pmatrix}$ on π.

$\overrightarrow{PQ} = \begin{pmatrix}-1\\0\\0\end{pmatrix}$

$d = \frac{1}{\sqrt{19}}\left|\begin{pmatrix}-1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\3\\3\end{pmatrix}\right|$
$= \frac{1}{\sqrt{19}} = 0.23$ (2dp)

Q9 Angle between 2 planes = Angle between their $\underset{\sim}{n}$.

$\pi_1 : \underset{\sim}{r}\cdot\begin{pmatrix}4\\2\\5\end{pmatrix} = 1 \qquad \pi_2 : \underset{\sim}{r}\cdot\begin{pmatrix}3\\5\\-2\end{pmatrix} = 2$

$\begin{pmatrix}4\\2\\5\end{pmatrix}\cdot\begin{pmatrix}3\\5\\-2\end{pmatrix} = \left|\begin{pmatrix}4\\2\\5\end{pmatrix}\right|\left|\begin{pmatrix}3\\5\\-2\end{pmatrix}\right|\cos\theta$

$12 = \sqrt{45}\sqrt{38}\cos\theta$

$\theta = \cos^{-1}\left(\frac{12}{\sqrt{45}\sqrt{38}}\right)$
$= 1.28$ (2dp)

Q10: $\underset{\sim}{r}\cdot\begin{pmatrix}1\\3\\2\end{pmatrix} = c$

Let P be $(1,2,3)$, Q be $(4,6,5)$

$\overrightarrow{PQ} = \begin{pmatrix}3\\4\\2\end{pmatrix}$ lies on the plane. $\overrightarrow{QP} = \begin{pmatrix}-3\\-4\\-2\end{pmatrix}$

$\underset{\sim}{n} = \begin{pmatrix}3\\4\\2\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}$
$= \begin{pmatrix}2\\-1\\-1\end{pmatrix}$
$= 2\begin{pmatrix}1\\-1/2\\-1/2\end{pmatrix}$

$\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}-1/2\\-1/2\\-1/2\end{pmatrix} = -\frac{3}{2}$

$a = -\frac{1}{2}, \quad b = -\frac{1}{2}, \quad c = -\frac{3}{2}$

Q11: $\pi_1 : \underset{\sim}{r}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} = 11 \qquad \pi_2 : \underset{\sim}{r}\cdot\begin{pmatrix}3\\8\\10\end{pmatrix} = 10$

A vector perpendicular to the normal of π_1 and π_2 is parallel to π_1 and π_2. Since π_3 is perpendicular to π_1 and π_2 this vector is parallel to the normal of π_3.

$\underset{\sim}{n}$ of $\pi_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}3\\8\\10\end{pmatrix}$
$= \begin{pmatrix}2\\-7\\5\end{pmatrix} = 2\begin{pmatrix}1\\-7/2\\5/2\end{pmatrix}$

$\pi_3 : \underset{\sim}{r}\cdot\begin{pmatrix}1\\-7/2\\5/2\end{pmatrix}$

$c = \begin{pmatrix}2\\4\\6\end{pmatrix}\cdot\begin{pmatrix}1\\-7/2\\5/2\end{pmatrix} = 3$

$\pi_3 : x - \frac{7}{2}y + \frac{5}{2}z = 3$

$a = -\frac{7}{2}, \quad b = \frac{5}{2}, \quad c = 3$

Q12: $\pi_1 : \underset{\sim}{r}\cdot\begin{pmatrix}4\\8\\5\end{pmatrix} = 0 \qquad \pi_2 : \underset{\sim}{r}\cdot\begin{pmatrix}4\\8\\5\end{pmatrix} = d$

Let $O(0,0,0)$ be on π_1 Let $P(d/4, 0, 0)$ be on π_2

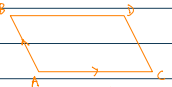
$\overrightarrow{OP} = \begin{pmatrix}d/4\\0\\0\end{pmatrix}$

$12 = \frac{1}{\sqrt{105}}\left[\begin{pmatrix}d/4\\0\\0\end{pmatrix}\cdot\begin{pmatrix}4\\8\\5\end{pmatrix}\right]$

$12 = \frac{d}{\sqrt{105}}$

$d = 12\sqrt{105}$
$= 122.96$ (2dp)

Q13:

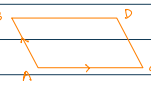


$\overrightarrow{AB} = \begin{pmatrix}-2\\1\\0\end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix}-2\\0\\3\end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix}-2\\1\\0\end{pmatrix} + \begin{pmatrix}-2\\0\\3\end{pmatrix}$
$= \begin{pmatrix}-4\\1\\3\end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix}-4\\1\\3\end{pmatrix} + \begin{pmatrix}2\\0\\0\end{pmatrix}$
$= \begin{pmatrix}-2\\1\\3\end{pmatrix}$

Q14:



$\overrightarrow{AB} = \begin{pmatrix}-7\\8\\0\end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix}-7\\0\\4\end{pmatrix}$

Area $= \|\overrightarrow{AB}\times\overrightarrow{AC}\|$
$= \left\|\begin{pmatrix}-7\\8\\0\end{pmatrix}\times\begin{pmatrix}-7\\0\\4\end{pmatrix}\right\|$
$= \left\|\begin{pmatrix}32\\28\\56\end{pmatrix}\right\|$
$= 70.31$

Q15: $\pi : 2x + 9y + 8z = 12$

$\underset{\sim}{r}\cdot\begin{pmatrix}2\\9\\8\end{pmatrix} = 12$

Reflection of $(1,2,3)$ in plane π

$P(1,2,3)$ and reflection lies on l: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}2\\9\\8\end{pmatrix}$, $t \in \mathbb{R}$.

$\begin{pmatrix}1+2t\\2+9t\\3+8t\end{pmatrix}\cdot\begin{pmatrix}2\\9\\8\end{pmatrix} = 12$

$(2+4t) + (18+81t) + (24+64t) = 12$

$149t = -32$

$t_P = -\frac{32}{149}$

Let R be reflection

$t_R = 2t_P$
$= -\frac{64}{149}$

$R = \begin{pmatrix}1\\2\\3\end{pmatrix} - \frac{64}{149}\begin{pmatrix}2\\9\\8\end{pmatrix}$
$= \left(\frac{21}{149}, -\frac{278}{149}, -\frac{65}{149}\right)$