NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MH1810 – MATHEMATICS 1

April 2017 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. Reference materials permitted in this exam are limited in volume to a single two-sided sheet of A4 paper.

5. Candidates may use calculators. Nevertheless, they should write down systematically the steps in their workings.

**QUESTION 1.** **(10 marks)**

Find all values of the variable $x$ for which the matrix

$$\begin{bmatrix} 1 & -3 & 1 \\ 3 & -1 & x \\ 1 & x & 3 \end{bmatrix}$$

is invertible.

**QUESTION 2.** **(10 marks)**

(i) Find all complex third roots of $-8$.

(ii) Express the complex number $1+i$ in exponential form. Hence express

$$\frac{8}{(1+i)^6}$$

in the form $x+iy$ with $x$ and $y$ real.

**QUESTION 3.** **(20 marks)**

Given be the points $A = (0, -1, -1)$, $B = (1, 0, 1)$, $C = (6, 2, 4)$ and $D = (0, 2, 3)$. Consider the plane $\Pi$ passing through the points $A$, $B$ and $C$.

(i) Find an equation describing $\Pi$.

(ii) Find the acute angle between $\Pi$ and the line segment $AD$.

(iii) Find an equation of the line passing through $D$ and perpendicular to $\Pi$.

**QUESTION 4.** **(15 marks)**

Consider the function $f$ defined by

$$f(x) = \begin{cases} x^2 + ax + 4, & \text{when } x < 1, \\ b, & \text{when } x = 1, \\ ax^2 + 6x + c, & \text{when } x > 1, \end{cases}$$

where $a$, $b$ and $c$ are real constants.

(i) Find conditions that must be satisfied by $a$, $b$ and $c$ so that $\lim_{x \to 1} f(x)$ exists.

(ii) Find conditions that must be satisfied by $a$, $b$ and $c$ so that $f$ is continuous at 1.

(iii) Find conditions that must be satisfied by $a$, $b$ and $c$ so that $f$ is differentiable at 1.

## QUESTION 5. (20 marks)

(i) Evaluate the indefinite integral $\displaystyle\int \frac{e^{\tan x}}{\cos^2 x}\,dx$.

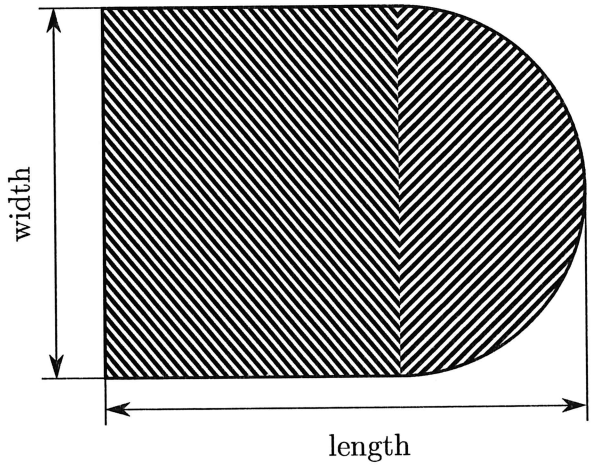
(ii) The region in the $xy$-plane bounded by the curves $x = 0$, $x = \pi$, $y = 0$ and $y = 1 + \sin x$ is revolved about the $x$-axis. Calculate the exact volume of the resulting solid.

(iii) Calculate the derivative $\displaystyle\frac{d}{dx}\int_{\ln x}^{2017} t^3 e^t\,dt$.

## QUESTION 6. (25 marks)

(i) Use linearization to find the approximate value of $\sqrt{16.1}$.

(ii) A garden is being designed in the shape of a rectangle with a semicircle attached to one of its sides, as seen in the figure below.




The garden is required to have a total area of $800\,\mathrm{m}^2$ and to be completely enclosed by a fence. Find the dimensions of the garden for which the total length of fencing required is the smallest.

**END OF PAPER**

# MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.