NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

2020S1 PH1012: Physics A
1D Kinematics

Dr Ho Shen Yong
Lecturer, School of Physical and Mathematical Sciences
Nanyang Technological University

Week 2-3

"The man who does not read good books has no advantage over the man who cannot read them."

- Mark Twain (1835-1910))

Outline

Some basic considerations when solving a mechanics problem:

1. Particle or extended object?
2. 1-dimensional, 2-dimensional or 3-dimensional?
3. Is acceleration / (Net force)
   a. Zero
   b. Constant
   c. Not constant?

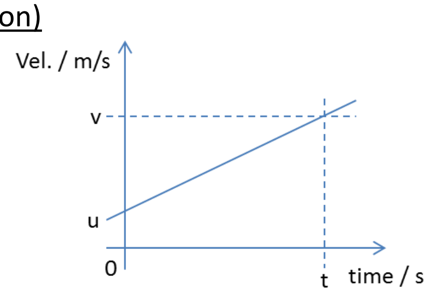
Distinguishing speed vs velocity; average velocity (acceleration) vs instantaneous velocity (acceleration). Recognizing $v = \frac{dx}{dt}; a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ and $x = \int v\,dt + C; v = \int a\,dt + C.$

Kinematic Equations of Motions (Constant acceleration)

$$v_f = v_i + at;$$

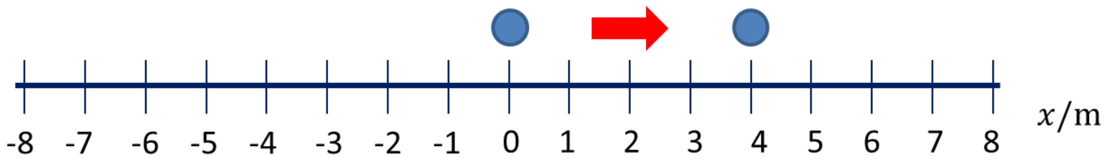
$$s = \frac{1}{2}(v_i + v_f)t;$$

$$s = v_i t + \frac{1}{2}at^2; v_f^2 = v_i^2 + 2as$$



Signs and Conventions

$t_i = 0.0\ s$ $\quad$ $t_f = 2.0\ s$



$x_i = 0.0\ m$ $\quad$ $x_f = 4.0\ m$

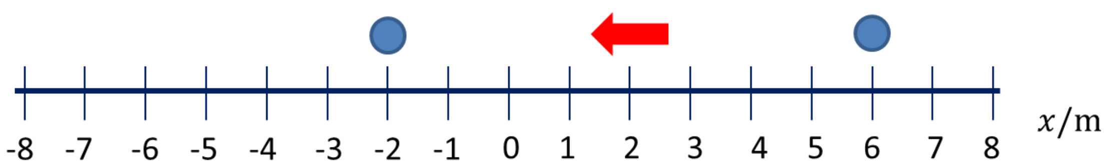
The initial position of the object is $x_i =$
The final position of the object is $x_f =$

The displacement of the object is $\Delta x = x_f - x_i = (4.0 - 0.0)\ \mathrm{m} = 4.0$
The distance travelled by the object is also $4\ \mathrm{m}$.

The average velocity of the object $v_{avg} = \dfrac{total\ displacement}{total\ time} = \dfrac{4.0\ \mathrm{m}}{2.0\ \mathrm{s}} =$

The average speed of object is also $2.0\ \mathrm{m/s}$.

$t_f = 2.0\ s$ $\quad$ $t_i = 0.0\ s$



$x_i = 6.0\ m$ $\quad$ $x_f = -2.0\ m$

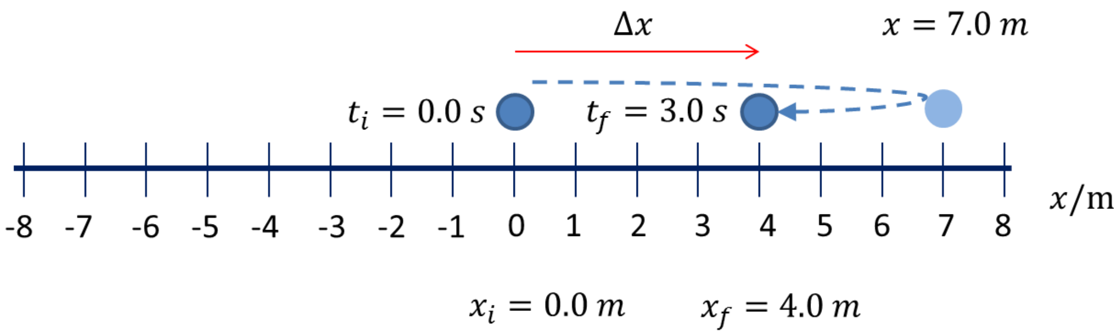
The initial position of the object is $x_i =$
The final position of the object is $x_f =$.

The displacement of the object is $\Delta x = x_f - x_i =$
The distance travelled by the object is $8\ \mathrm{m}$.

The average velocity of the object $v_{avg} =$

The average speed of object is

Signs and Conventions



The initial position of the object is $x_i =$
The final position of the object is $x_f =$

The displacement of the object is $\Delta x = x_f - x_i =$
The distance travelled by the object is

The average velocity of the object $v_{avg} =$

The average speed of object is

One Dimensional Motion
(Constant Acceleration)

Special Case: Zero Acceleration

A car is travelling at a constant speed of 72 km/h. What distance does it cover in 10 minutes?

Kinematics

Displacement is given by the change in position of the object, i.e. how far the object is from the starting point:

$$\boxed{Displacement = Final\ position\ - Initial\ Position}$$

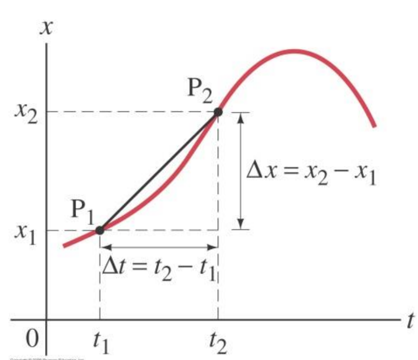
$$\boxed{Average\ velocity = \frac{displacement}{time\ elapsed}}$$

$$\boxed{Average\ speed = \frac{distance\ travelled}{time\ elapsed}}$$

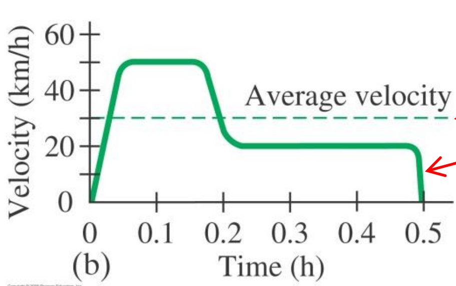
Displacement and velocity are vector quantities and they contain information on magnitude and direction. Distance and speed are scalar quantities and contain information only on magnitude.

So we can write for average velocity

$$\boxed{Average\ velocity, \bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$



Giancoli Fig 2.10: Average velocity (from x-t)

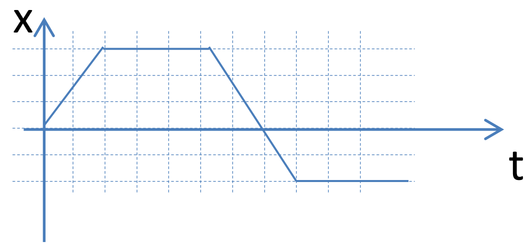


Same area (displacement) under avg vel and vel lines over same time interval.

Giancoli Fig 2.9b: Average velocity

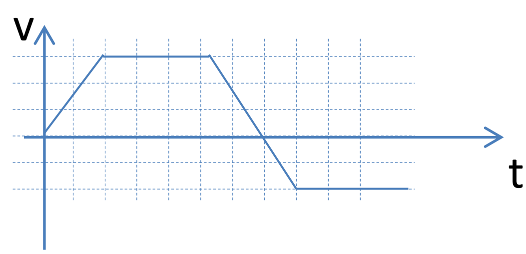
Some Simple Scenarios to introduce kinematics

A ball is at position x = 0 initially. The position of the ball along the x direction is plotted in the graph. Do a simple sketch to indicate the position, velocity and acceleration of the ball.



x = 0

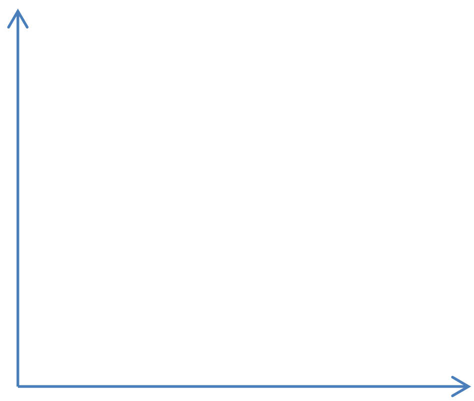
A ball is at position x = 0 initially. The <u>velocity</u> of the ball along the x direction is plotted in the graph. Do a simple sketch to indicate the position, velocity and acceleration of the ball.



x = 0

Some Simple Scenarios to introduce kinematics

A bus moves off from rest from a traffic junction and the speed (velocity) increases uniformly. After 5s, the speed of the bus is 10 m/s. What is the rate of increase of the speed, (velocity) i.e. acceleration? Do a sketch using the axes below to illustrate how the velocity of the bus changes with time. If the bus continues to accelerate uniformly, what is the velocity 3s later?



A car moves past at 10 m/s when the bus just started to move. If the car has the same acceleration as the bus, what will its speed be after 8 s? Sketch on the same axes, how the velocity of the car varies with time. How much further did the car travel?

One Dimensional Motion II
(Constant Acceleration)

Acceleration is defined as the rate of change of velocity.

If acceleration a is constant, we can write

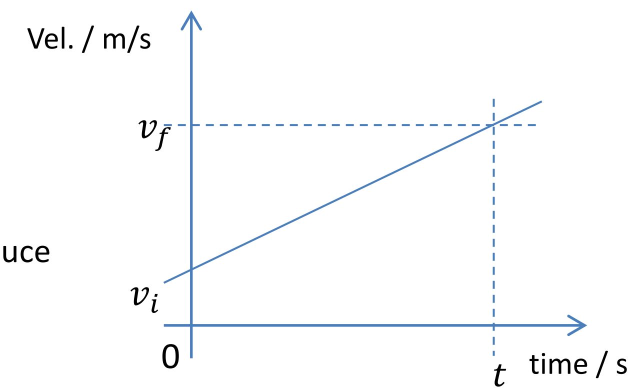
$$a = \frac{v_f - v_i}{t}$$

where v_f is the final velocity and v_i is the initial velocity and t is the time taken for the change.
We can sketch a graph of velocity against time. In the previous example, the velocity time graph for the car looks like the graph below.

From the expression

$$a = \frac{v_f - v_i}{t}$$

and the velocity time graph for constant acceleration, we can deduce some more information.



Giancoli Prob 2.20

A sports car accelerates from rest to 95 km/h in 4.5 s. What is its average acceleration in m/s^2 ?

Giancoli Prob 2.70 (simplified)

The fugitive starts from rest on a empty train box car and accelerates at $a = 2.0\ m/s^2$ to a maximum speed of 8.0 m/s. (a) How long does it take him to reach maximum speed? (b) What is the distance travelled in this time?

Giancoli Prob 2.32

A light plane must reach a speed of 32 m/s for takeoff. How long a runway is needed if the (constant) acceleration is 3.0 m/s^2 ?

Giancoli Prob 2.36

An inattentive driver is traveling 18.0 m/s when he notices a red light ahead. His car is capable of slowing down at a rate of 3.65 m/s^2. If it takes him 0.20 s to get the brakes on and he is 20.0 m from the intersection when he sees the light, will he be able to stop in time?

Free Falling

A 1 kg object and a 1 g object are released from the same height at the same time in a vacuum. Which object will hit the ground first?

a. 1 kg
b. 1 g
c. At the same time

https://www.youtube.com/watch?v=E43-CfukEgs

(a) (b)

In the absence of air resistance, all objects fall with the same acceleration, although this may be tricky to tell by testing in an environment where there is air resistance. We will prove this fact after we have discussed Newton's second law.

https://www.youtube.com/watch?v=E 43-CfukEgs&t=2s

Free Falling

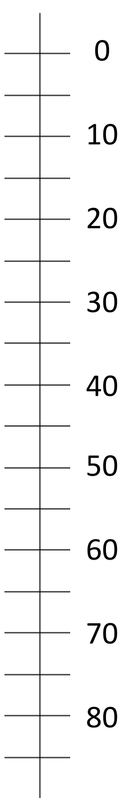
For simplicity, take g = 10 m/s^2 and *neglect air resistance*

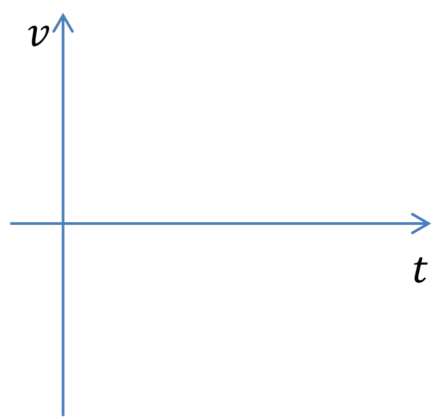
An object at rest is released from a height of $80\ m$.

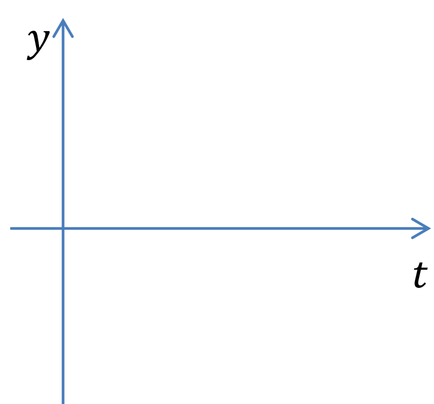
a) What is the velocity of the object when it hits the ground?

b) How much time does it take to hit the ground?

c) Sketch a graph to show how the velocity and position of the object changes with time.







https://www.youtube.com/watch?v=x_Q4znShlK5A

HOSY@ntu.edu.sg

Free Falling (Object thrown upwards)

For simplicity, take g = 10 m/s^2 and $neglect\ air\ resistance$

An object at rest is thrown vertically upwards at a velocity of 12 m/s.

a) What is the greatest height reached by the object?

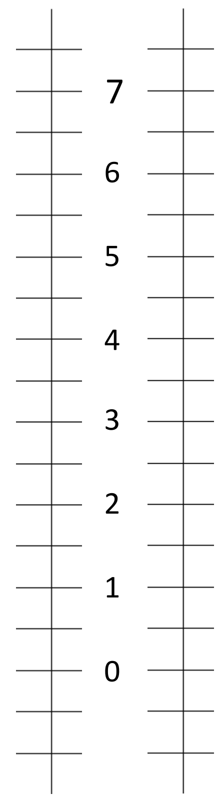
b) How much time does it take to reach the greatest height?

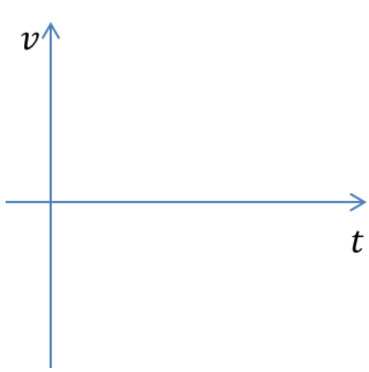
b) What is the velocity of the object when it is back to the hand.

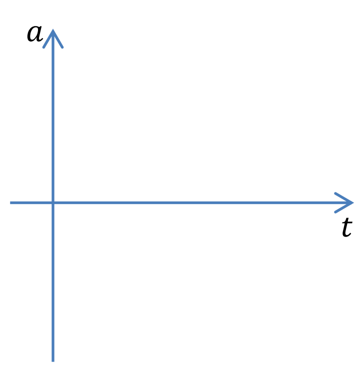
c) How much time does stay in the air before the hitting the ground?

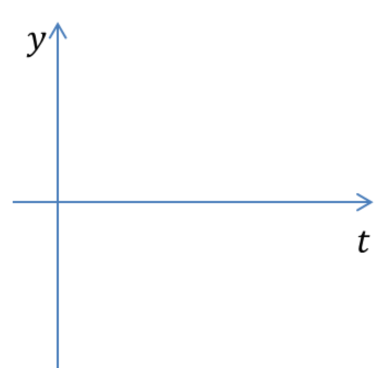
d) Sketch a graph to show how the velocity, acceleration and position of the object changes with time. How would the graph change, velocity and time become if the object hits the ground?

Spreadsheet exercise









Three Scenarios:

An object at a height 20 m above the ground is

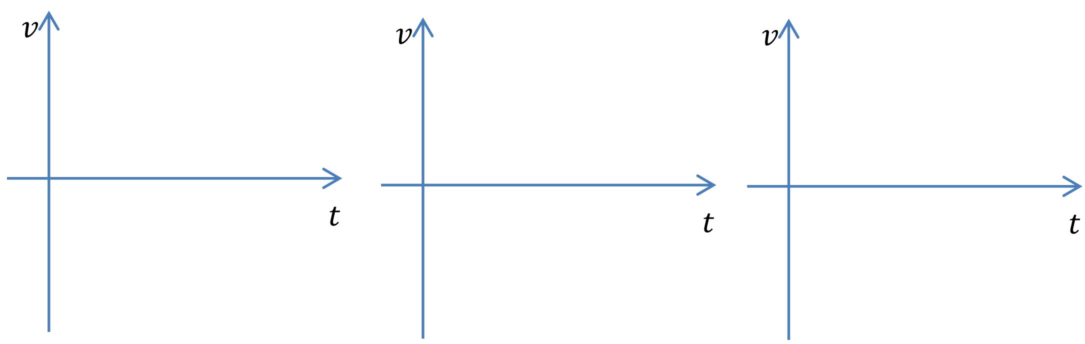
a) released from rest;

b) thrown vertically downwards with an initial velocity of $12\ m\ s^{-1}$.

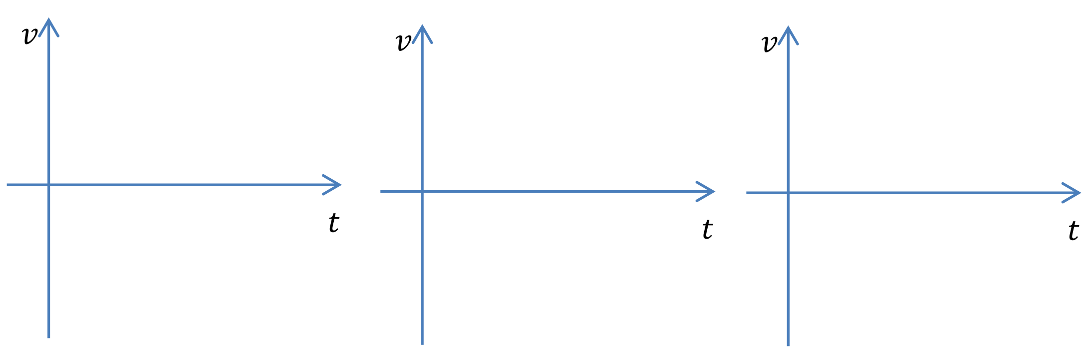
c) thrown vertically upwards with an initial velocity of $12\ m\ s^{-1}$

Sketch the velocity-time graphs.

Taking upwards as +ve



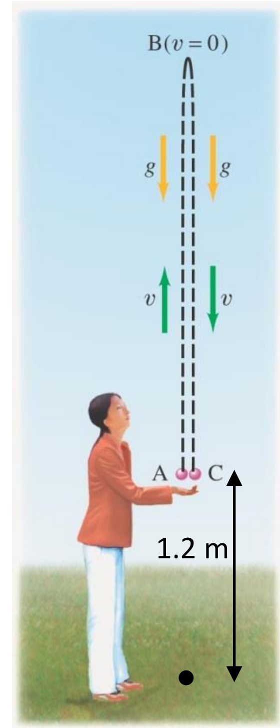
Taking downwards as +ve



(Modified) Giancoli pg 36 Example 2-16

A person throws a ball upward into the air with an initial velocity of 15.0 m/s. His hand is 1.2m above the ground. Take g = 9.80 m/s^2 . Ignore air resistance.

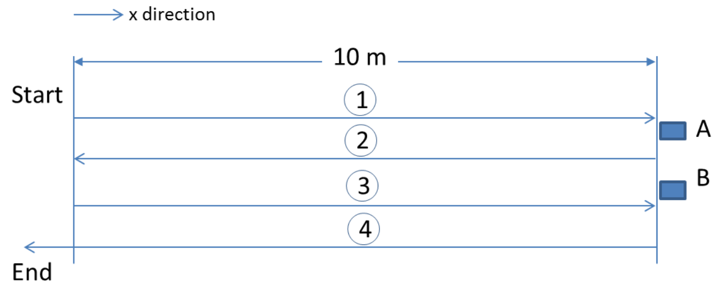
a) What is the acceleration at the highest point (B)?
b) Calculate the maximum height of the ball.
c) Calculate how long the ball comes back to the hand.
d) Calculate at what time *t* the ball passes a point 8.00 m above the person's hand.
e) If he missed catching the ball, how long does the ball take to get from point A to the ground?



Giancoli Fig 2.30
Ball thrown up

Example – Horizontal motion with "U-turns"

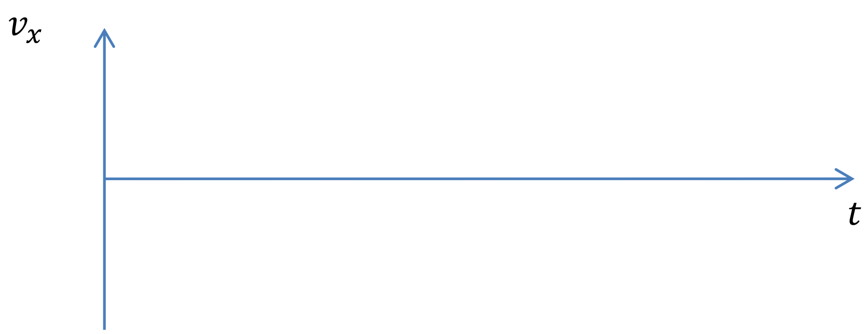
The 4 x 10m shuttle run is one of the items of the Physical Fitness Test. An individual starts running (1) from start line towards block A, picks it up; (2) runs back towards the start line, drops block A; (3) runs towards block B, picks it up; and (4) runs pass the start line in the end. The individual is to complete the 4 x 10 m in the shortest possible time.



Top View

https://www.youtube.com/watch?v=Z_cj_xdwLnNc&t=3s

On the axis provided, sketch how the velocity v_x of the individual in the x direction changes with time t, from the start to the end of the shuttle run.



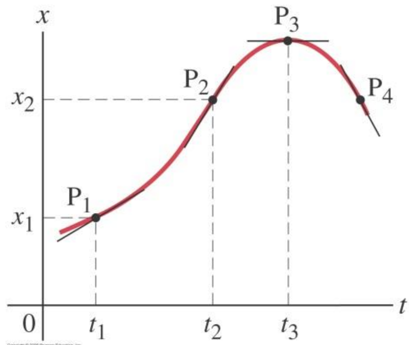
Examples of Non-constant accelerations

a. Oscillatory motions

b. Objects falling through a viscous medium.

c. Moving through large vertical distances where acceleration of free fall g is not constant.

Gradients (Basics)



Giancoli Fig 2.12

Previously, we have dealt with constant acceleration where the v-t graph is a straight line. When we have a curve, we have to talk about the gradient at a specific point. Graphically, we can sketch the tangent (line) at this point and then compute the gradient of this tangent. So essentially we finding the rate of change of x in the limit when $\Delta t \rightarrow 0$ or writing in formal mathematics

$$gradient = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

So we see that at P1 and P2, x is increasing with t so the gradient is positive. In fact, the gradient is increasing from P1 to P2 before it levels off to zero gradient at P3. The value of x is momentarily constant here and is a maximum for x. After that, the gradient is negative and the value of x is decreasing with time t.

If we know the equation of the line, is there a simpler way to obtain the gradient, other can obtaining it graphically?

Gradients and Differentiation (Basics)

We need to know how to compute the quantity

$$gradient = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

to be able to find a gradient at point on a curve. Consider a relation, $x = 5t^2$ and consider two nearby points t_1 and $t_1 + \delta t$ where δt is very small (say 0.0000001 or even smaller).

$$\frac{\Delta x}{\Delta t} = \frac{\{5(t_1 + \delta t)^2 - 5(t_1)^2\}}{\{(t_1 + \delta t) - t_1\}} = \frac{\{5(t_1^2 + 2t_1\delta t + (\delta t)^2) \quad - 5(t_1)^2\}}{\delta t} = 5(2t_1 + \delta t)$$

Thus, at $t = t_1$ and making $\delta t \rightarrow 0$,

$$gradient = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \lim_{\delta t \rightarrow 0} 5(2t_1 + \delta t) = 5\,(2t_1) = 10t_1$$

Gradients (Basics)

Let's see if we can find any pattern for polynomial functions of the form

$$x = kt^n$$

where k is a constant and n is an integer (for the time being).

$$x = kt^3$$

$$\frac{\Delta x}{\Delta t} = \frac{\{k(t+\delta t)^3 - kt^3\}}{\{(t+\delta t) - t\}} = \frac{\{k(t^3 + 3t^2\delta t + 3t(\delta t)^2 + (\delta t)^3) \quad - k(t)^3\}}{\delta t} = \mathrm{k}(3t^2 + 3t\delta t + (\delta t)^2)$$

$$\frac{dx}{dt} = \lim_{\delta t \to 0} \mathrm{k}(3t^2 + 3t\delta t + (\delta t)^2) = k(3t^2) = 3kt^2$$

$$x = kt^4$$

$$x = kt^5$$

We can generalize to

$$x = kt^n \Rightarrow \frac{dx}{dt} = k(nt^{n-1})$$

This finding is even applicable for when n are negative integers, fractions and even irrational numbers such as $\sqrt{2}$ and π. (You can check these out after you learn how to do binomial expansion of non-positive integer exponents.)

Exercise:

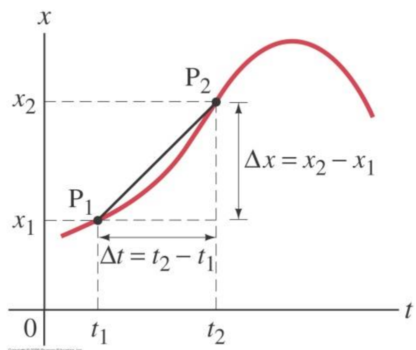
For $y = 4x^4 - 3x^2 + 2$, compute $\frac{dy}{dx}$ and evaluate it at $x = 1.2$.

We will discuss about areas and integration later.

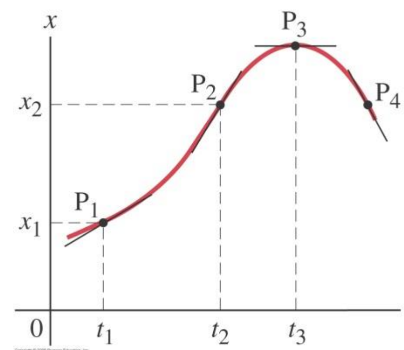
Kinematics

We define the average velocity over an infinitesimally short time as the instantaneous velocity:

$$\boxed{Instantaneous\ velocity, v = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{x}}{\Delta t} = \frac{dx}{dt}}$$



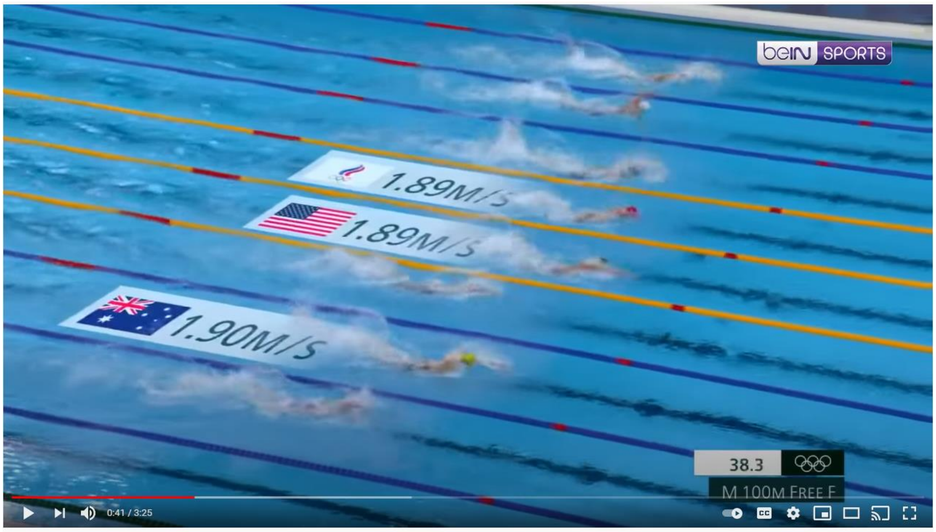
Giancoli Fig 2.10: Average velocity (from x-t)



Giancoli Fig 2.12: Instantaneous velocity

Example [G2.4] A rolling ball moves from to during the time from $x_1 = 3.4\ cm$ to $x_2 = 24.2\ cm$ and during the time from $t_1 = 3.0\ s$ to $t = 5.1\ s$. What is its average velocity?

Instantaneous velocity



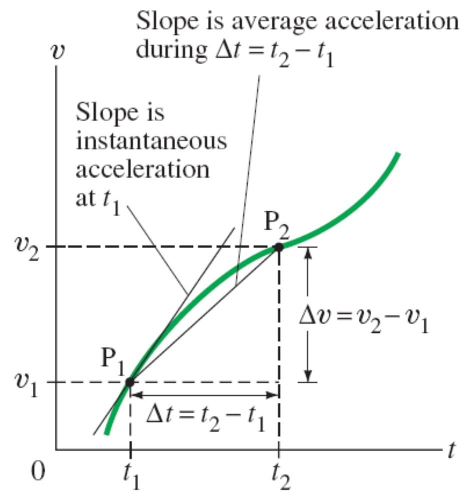
OR and GOLD for Dressel (USA) | Men's 100m Freestyle | Tokyo 2020 Olympics Highlights

World record for men’s 100m is 9.58 s. The record for 4 x 100 m is 36.84s which is much less than four times of 9.58s. How is that possible?

Kinematics

Similarly, we define average acceleration as the change in velocity divided by the time to make this change

$$\boxed{Average\ acceleration, \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}}$$

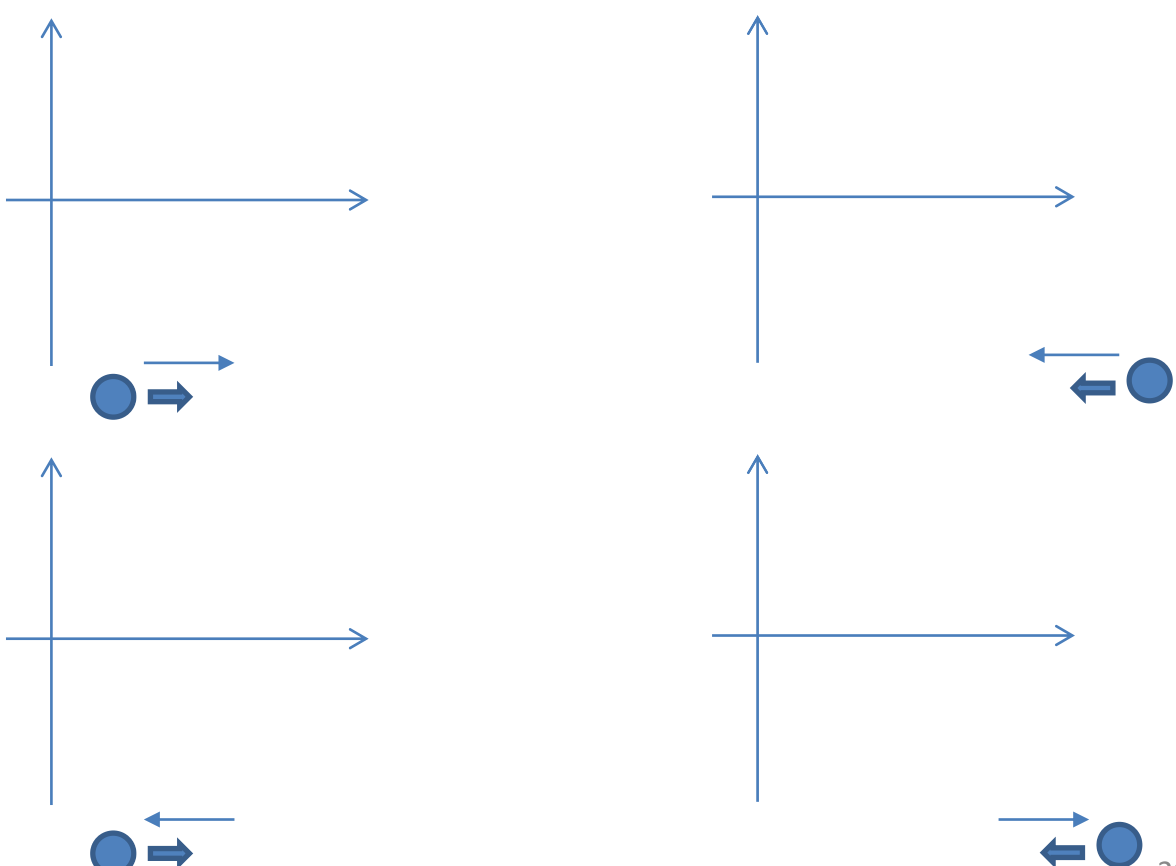


As before, we define the average acceleration over an infinitesimally short time as the instantaneous acceleration

$$\boxed{\begin{gathered} Instantaneous\ acceleration, \\ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \end{gathered}}$$

Velocity & acceleration: same sign => speeding up; opposite sign => slowing down.

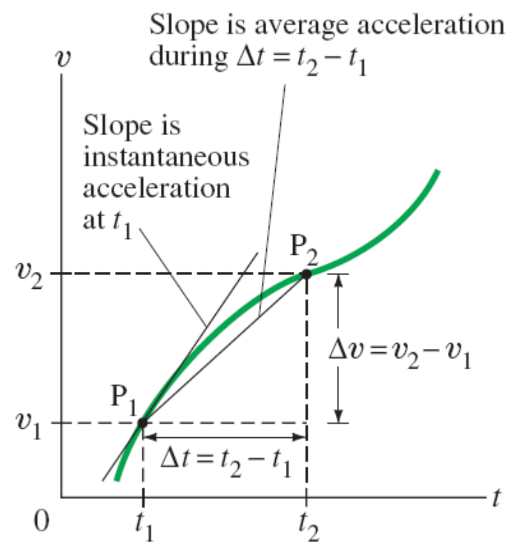
Speeding up and slowing down



Instantaneous / average acceleration

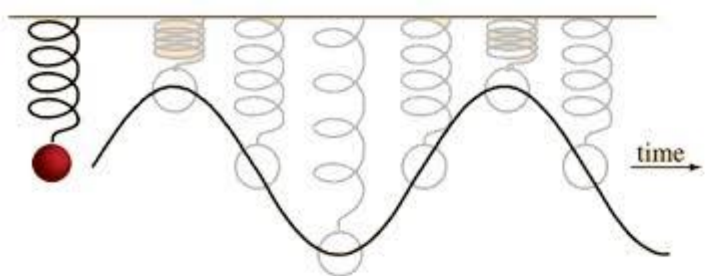
$$\textit{Average acceleration,}$$
$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta \mathrm{v}}{\Delta t}$$

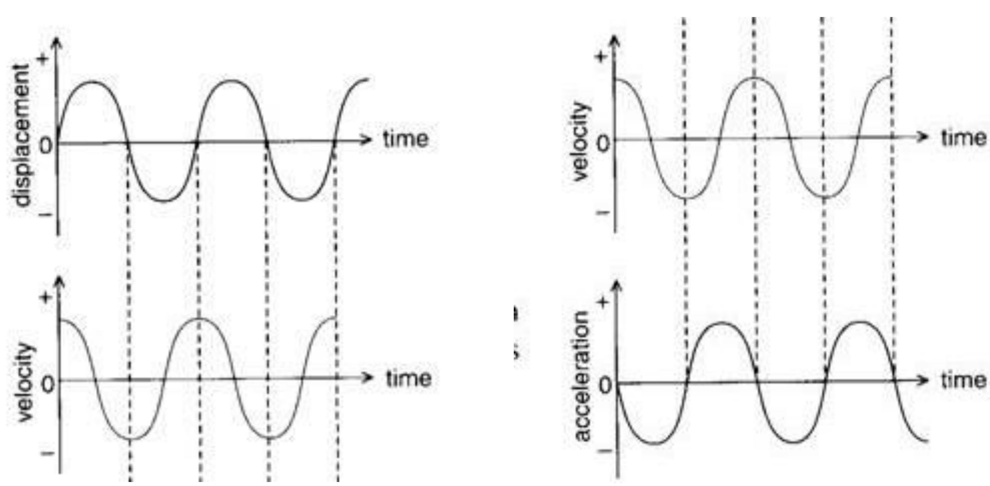
$$\textit{Instantaneous acceleration,}$$
$$a = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{v}}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$



An example of non-constant acceleration is the acceleration of a swinging pendulum. (Talk about trigonometric functions)

displacement





Example:

The position of a particle as it moves along the x axis is given for time $t > 0$ by

$$x = (t^3 - 3t^2 - 6t)\ m,$$

where t is in s.

a) What is the average velocity of the particle from $t = 2.0\ s$ to $t = 3.0\ s$?

b) What is the instantaneous velocity at $t = 2.0\ s$ and $t = 3.0\ s$?

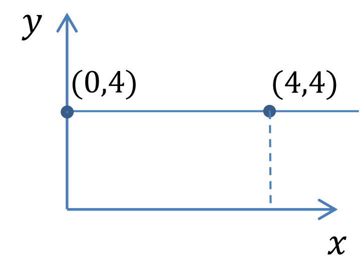
c) What is the average acceleration of the particle from $t = 2.0\ s$ to $t = 3.0\ s$?

d) What is the instantaneous acceleration at $t = 2.0\ s$ to $t = 3.0\ s$?

e) What is the position of the particle when it comes momentarily to rest (after $t = 0$)?

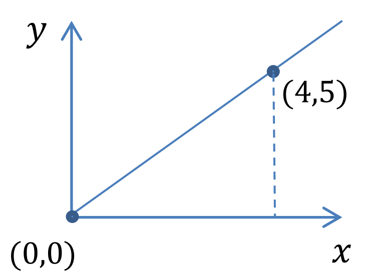
Areas Under Graphs and Integration (Basics) - Video

Let us start from something simple first.



a. What is the equation of this line?

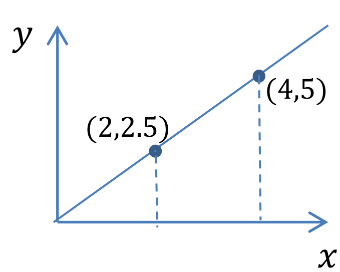
b. Find the area of bounded by the dotted line and the y-axis.



a. What is the equation of this line?

b. Find the area of bounded by the dotted line and the y-axis.

We can make some simple generalizations for areas under straight lines and can even define an “area function” $A(x)$.



Find the area bounded by the two dotted lines.

Areas Under Graphs and Integration (Basics) - Video

Let us do some generalizations. Referring to Appendix 1, we can write

$$If\ y = \int f(x')\, dx' + C, then\ \frac{dy}{dx} = f(x)$$

And using this,

consider $y = x^n$; using what we have learnt before $\frac{dy}{dx} = nx^{n-1}$. Therefore,

$$\int nx^{n-1}\, dx = x^n\ (where\ n \neq 0)$$

Rewriting $m = n - 1; (or\ n = m + 1), we\ get$

$$\int (m+1)x^m\, dx = x^{m+1} + C\ (where\ m \neq -1)$$

Hence

$$\int x^m\, dx = \frac{1}{m+1} x^{m+1}\ (where\ m \neq -1)$$

Giancoli pg 40, Example 2-21

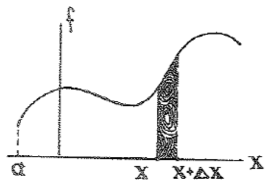
An experimental vehicle starts from rest ($v_o = 0$) at $t = 0$ and accelerates at a rate given by $a = 7.00\ t\ m/s^2$. What is the (a) velocity and (b) displacement after $2.00s$ later?

Appendix 1: Explaining why $If\ y = \int f(x')\,dx' + C, then \dfrac{dy}{dx} = f(x)$

The Indefinite Integral

That the operation of integration is the inverse to the operation of differentiation has been indicated several times in previous discussions. In particular in section A we demonstrated that the slope of the integral (or area) function at a point was equal to the integrand evaluated at that point.

To see this in a slightly different manner consider the definite integral from a to X of f(x)



$$A(X) = \int_a^X f(x)\,dx \qquad (1)$$

Now take the definite integral from a to $(X + \Delta X)$,

$$A(X + \Delta X) = \int_a^{(X+\Delta X)} f(x)\,dx \qquad (2)$$

The area in the small interval ΔX we shall denote as ΔA.

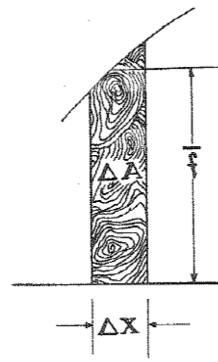
$$\Delta A = A(X + \Delta X) - A(X) = \int_a^{(X+\Delta X)} f(x)\,dx - \int_a^X f(x)\,dx$$

$$= \int_X^{(X+\Delta X)} f(x)\,dx \qquad (3)$$

Note carefully that

$$\int_X^{X+\Delta X} f(x)\,dx$$

is the area in the small trapezoid having a base ΔX.



Therefore the average value of f(x) in the interval ΔX is $\bar{f}$.

$$f(X) \le \bar{f} \le f(X + \Delta X)$$

where

$$\bar{f} = \frac{\text{area}}{\Delta X} = \frac{1}{\Delta X}\int_X^{(X+\Delta X)} f(x)\,dx \qquad (4)$$

We now divide both sides of equation (3) by ΔX.

$$\frac{\Delta A}{\Delta X} = \frac{1}{\Delta X}\int_X^{(X+\Delta X)} f(x)\,dx = \bar{f}$$

In the limit as ΔX approaches zero $\bar{f}$ approaches f(x); thus

$$\lim_{\Delta X \to 0} \frac{\Delta A}{\Delta X} = \frac{dA}{dx} = \lim_{\Delta x \to 0} \bar{f} = f(X)$$

Finally if we denote the integral by a function A(x) then the integrand f(x) is equal to the first derivative of the integral.

This relation provides a convenient method for evaluating integrals.

If

$$A(x) = \int f(x')\,dx'$$

then

$$f(x) = \frac{dA}{dx}(x)$$

To illustrate the usefulness of this equation let

$$A(x) = x^n$$

then

$$\frac{dA}{dx}(x) = nx^{n-1} = f(x)$$

Because of this, the indefinite integral of nx^{n-1} is

$$\int nx^{n-1}\,dx = x^n \text{ (where } n \neq 0)$$

Taken from: Fundamentals of Scientific Mathematics by George E Owen (Dover 2003)