BIG DATA MANAGEMENT

CZ4123

MEMORY HIERARCHY (PART III)

CACHE CONSCIOUS DATA-PROCESSING

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OVERVIEW

- In previous lectures, we show the basic concepts and analysis of memory hierarchy.

- In this lecture, we show some design principles to make use of memory hierarchy for processing big data. We will discuss following two cases.
  - Array access patterns
    - Spatial locality designs
    - Temporal locality designs

  - Big data sorting

MOTIVATING QUESTION

Suppose we have a 10000x10000 matrix of integers, which is stored in a 2-dimentional array A[10000][10000]. We want to set all of its value to 1. Cache size = 5000 integers. How would you write your code?

Solution X

```
for (int i=0;i<10000;i++)
    for(int j=0;j<10000;j++)
        A[i][j]=1;
```

Solution Y

```
for (int j=0;j<10000;j++)
    for(int i=0;i<10000;i++)
        A[i][j]=1;
```

Which one is better, X or Y?

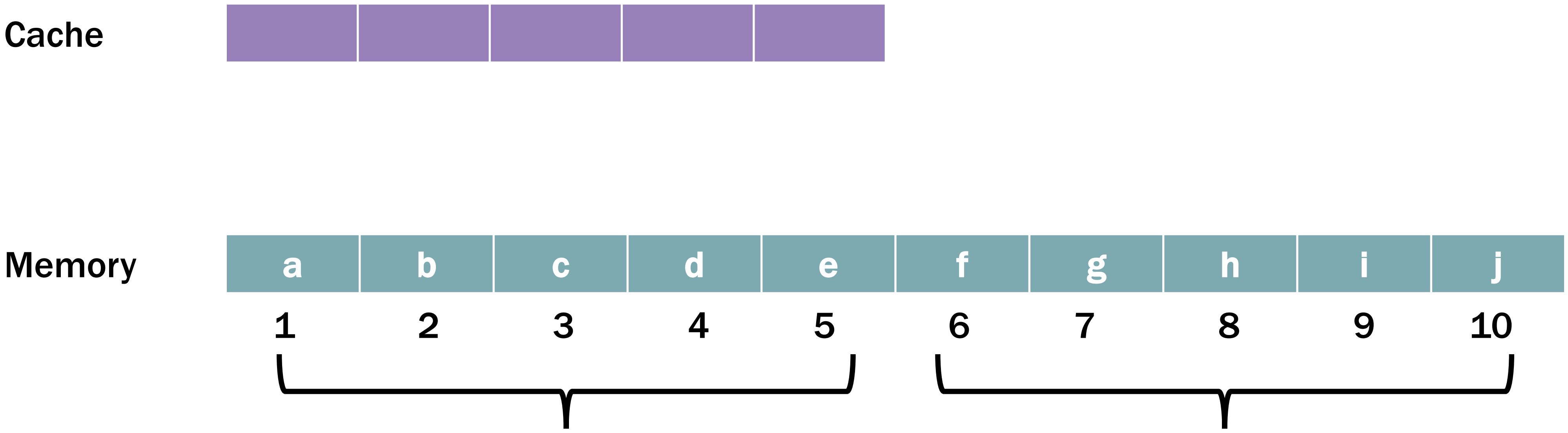
ACCESSING AN ARRAY

- We may access an array with different access patterns
  - Access pattern means the position sequence of accessing an array.
  - Different access patterns may impact the utility of cache

ACCESS PATTERN 1

We have a 10-integer array stored in main memory,
cache size = 5 integers, transfer size (cache line)= 5 integers

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10



ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache

a	b	c	d	e

Cache Miss: 1
Cache Hit: 0

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache

a	b	c	d	e

Cache Miss: 1
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a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

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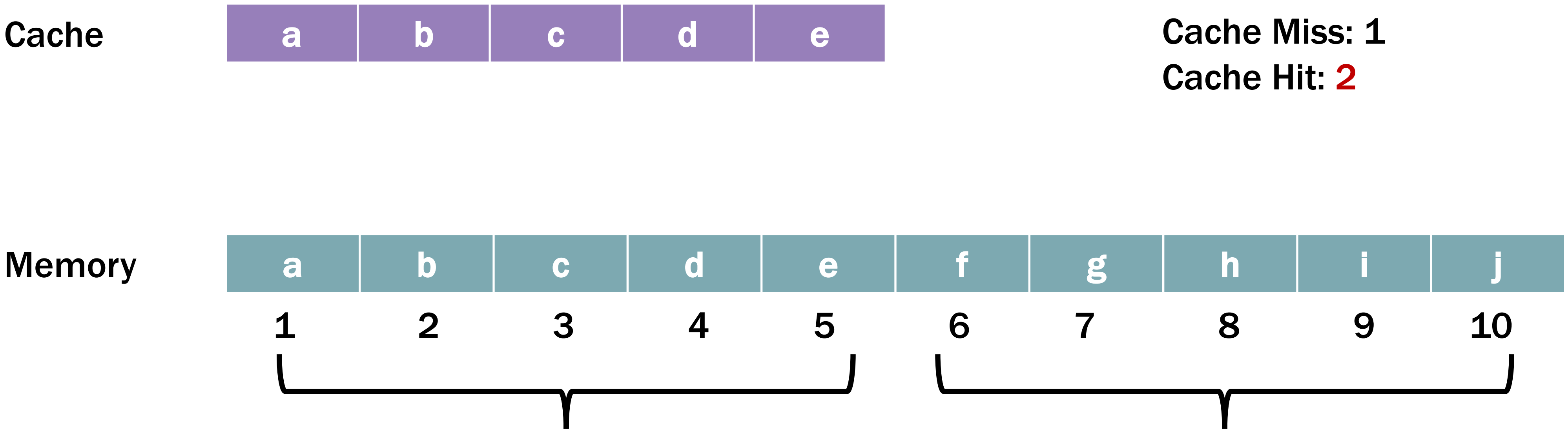


Cache Miss: 1
Cache Hit: 2

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache	a	b	c	d	e

Cache Miss: 1
Cache Hit: 3

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache

a	b	c	d	e

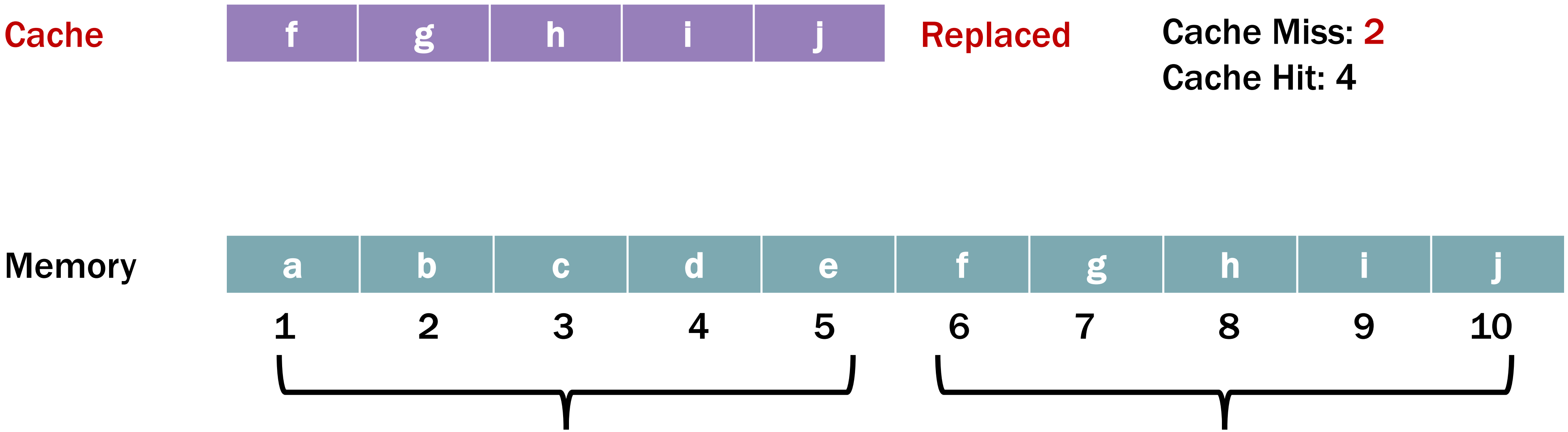
Cache Miss: 1
Cache Hit: 4

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10



ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache					
	f	g	h	i	j

Cache Miss: 2
Cache Hit: 5

Memory										
	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache	f	g	h	i	j

Cache Miss: 2
Cache Hit: 6

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache

f	g	h	i	j

Cache Miss: 2
Cache Hit: 7

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 1

Access pattern: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Cache

f	g	h	i	j

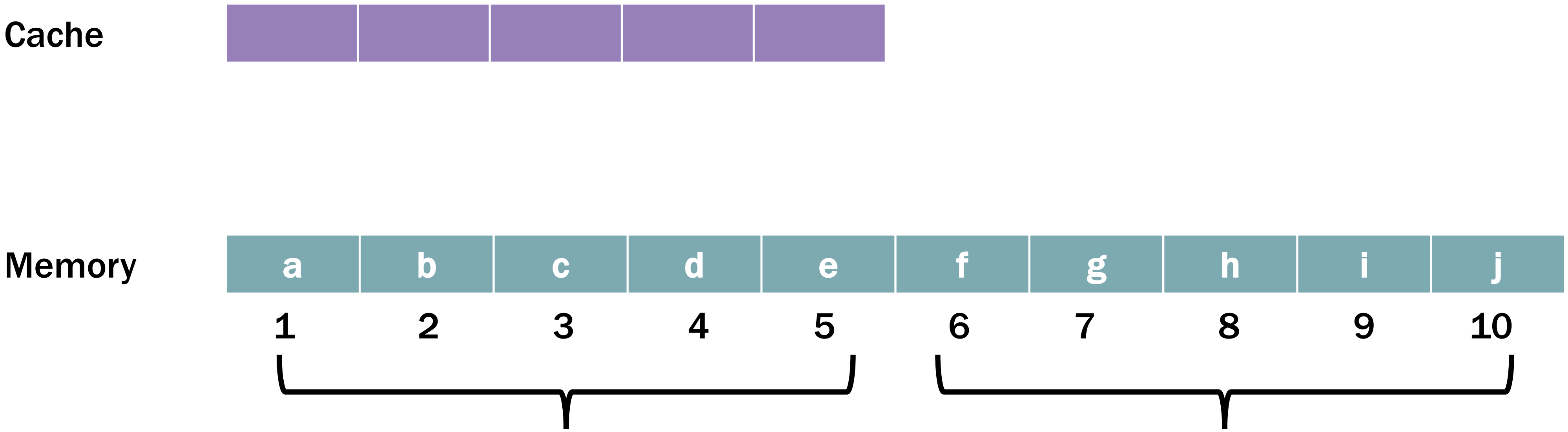
Cache Miss: 2
Cache Hit: 8

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10



ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache

a	b	c	d	e

Cache Miss: 1
Cache Hit: 0

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache

f	g	h	i	j

Replaced

Cache Miss: 2
Cache Hit: 0

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache	a	b	c	d	e	Replaced

Cache Miss: 3
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache

f	g	h	i	j

Replaced

Cache Miss: 4
Cache Hit: 0

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache	a	b	c	d	e	Replaced

Cache Miss: 5
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache

f	g	h	i	j

Replaced

Cache Miss: 6
Cache Hit: 0

Memory

a	b	c	d	e	f	g	h	i	j
1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache	a	b	c	d	e	Replaced

Cache Miss: 7
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache	f	g	h	i	j	Replaced

Cache Miss: 8
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

Cache	a	b	c	d	e

Replaced

Cache Miss: 9
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

ACCESS PATTERN 2

Access pattern: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10

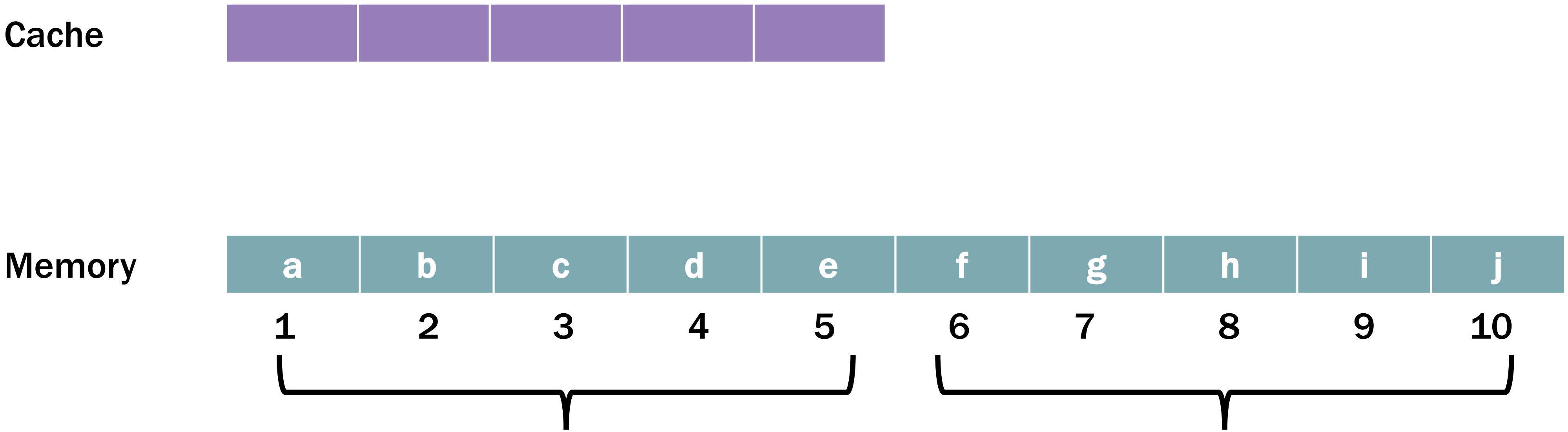
Cache	f	g	h	i	j	Replaced

Cache Miss: 10
Cache Hit: 0

Memory	a	b	c	d	e	f	g	h	i	j
	1	2	3	4	5	6	7	8	9	10

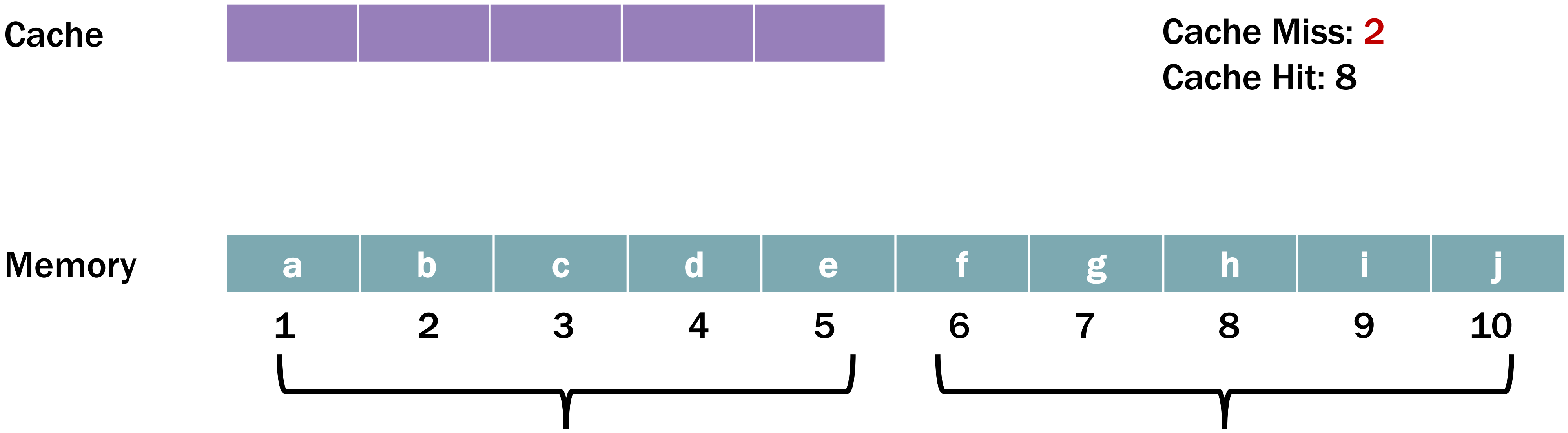
ACCESS PATTERN 3

Access pattern: 1, 6, 8, 7, 7, 8, 9, 9, 6, 10



ACCESS PATTERN 3

Access pattern: 1, 6, 8, 7, 7, 8, 9, 9, 6, 10



Cache Miss: 2
Cache Hit: 8

SUMMARY

- Access pattern with spatial locality (pattern 1) or temporal locality (pattern 3) is cache friendly.

- The adversarial access (pattern 2) may cause the worst case read performance

- When data is big, the impact is very significant

- Whenever possible, we store the data in a way that the access pattern has certain locality.

2D ARRAYS

Suppose we have a 10000x10000 matrix of integers, which is stored in a 2-dimentional array A[10000][10000]. We want to set all of its value to 1. Cache size = 5000 integers. How would you write your code?

Solution X

```
for (int i=0;i<10000;i++)
    for(int j=0;j<10000;j++)
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```

Solution Y

```
for (int j=0;j<10000;j++)
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Which one is better, X or Y?

2D ARRAYS

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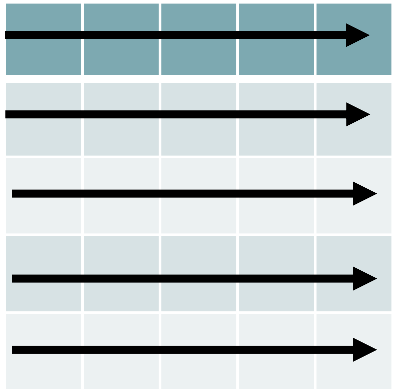
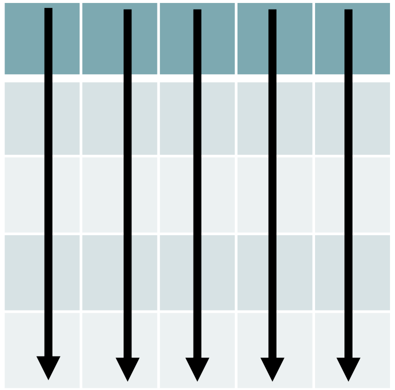


Cache friendly

Solution Y

```
for (int j=0;j<10000;j++)
    for(int i=0;i<10000;i++)
        A[i][j]=1;
```



Always evict cache

Which one is better, X or Y?

SORTING

In the big data era, it happens very often that the data volume is too big to hold in main memory.

Suppose we have 1 TB of integers stored in disk and we want to sort the array. My computer only has 16GB main memory, what should I do?

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Suppose we have 1 TB of integers stored in disk and we want to sort the array. My computer only has 16GB main memory, what should I do?

We have learnt a lot of sorting algorithms, can we simply use them?

-- We can't allocate enough large array in memory when coding.

SORTING

The problem of sorting a big array larger than the main memory is called external memory sorting.

Suppose the size of main memory is M, the array size is 5M.

SORTING

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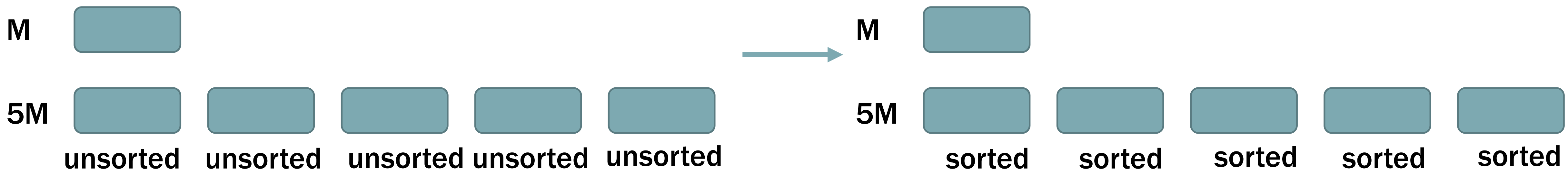

Step 1:

Cut 5M arrays into 5 parts. Each part is put in main memory to sort (using any sorting algorithm we learnt, e.g., quick sort)

SORTING

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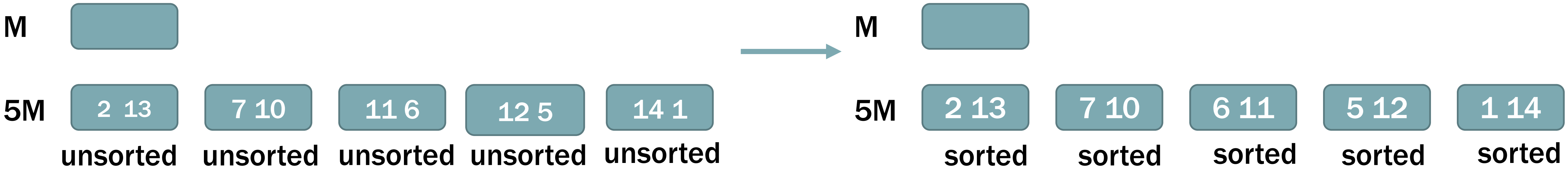

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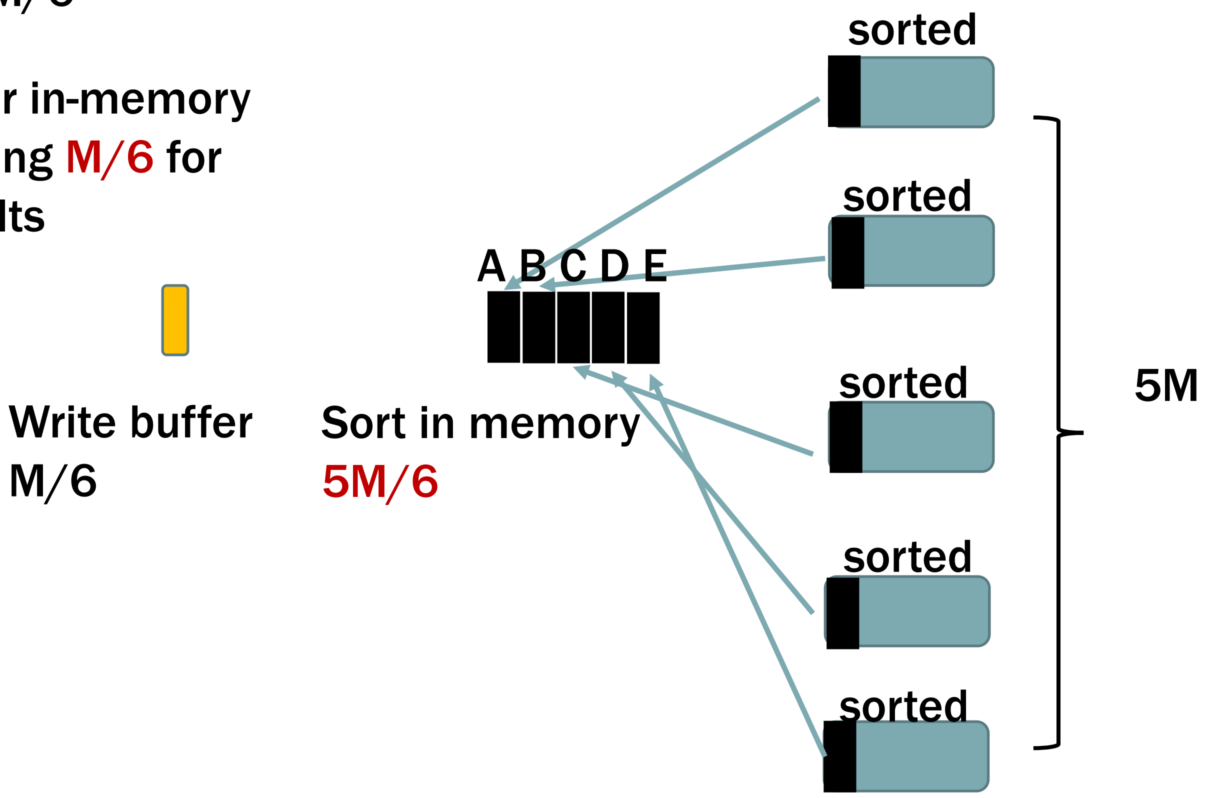
SORTING

5M/6 in total for in-memory sorting, remaining M/6 for writing the results



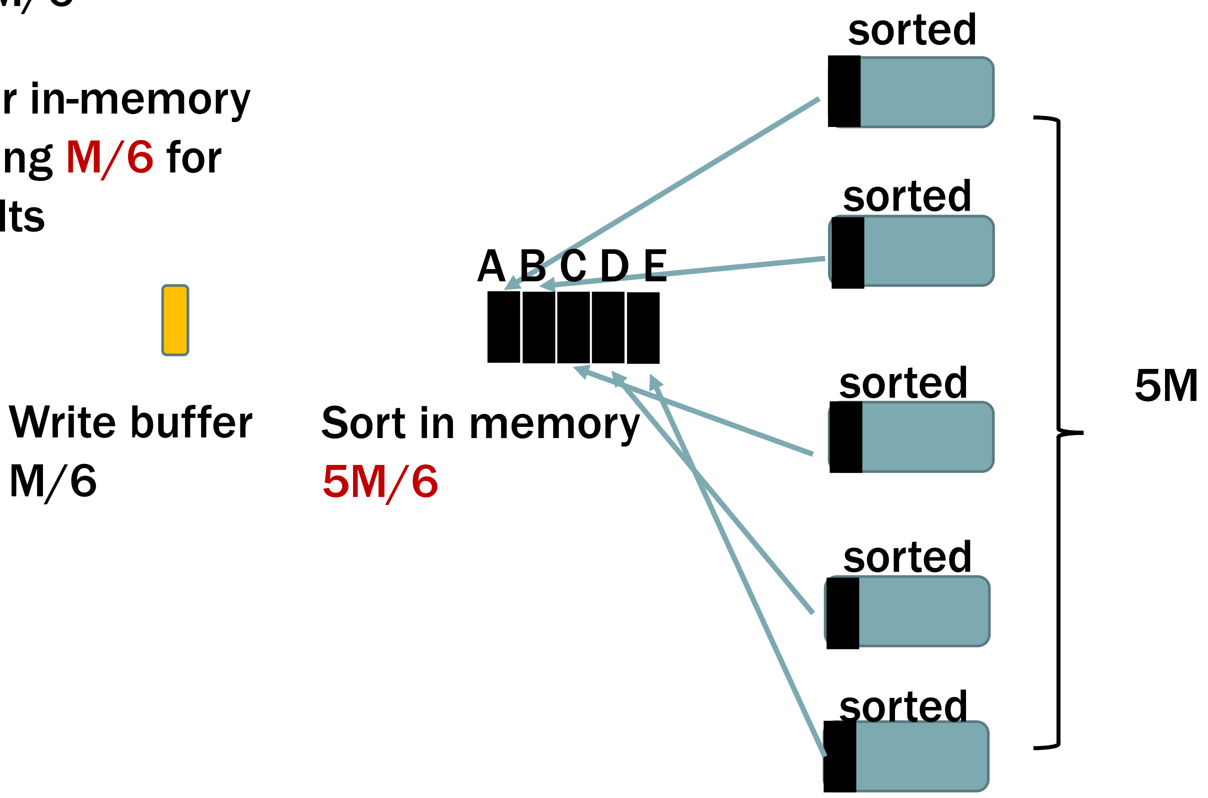
Step 2:

Apply 5-way merge sort to the first M/6 of each sorted part.

SORTING

or size M/6

5M/6 in total for in-memory sorting, remaining M/6 for writing the results



Step 2:

Apply 5-way merge sort to the first M/6 of each sorted part.

Details of merge sort

1. 5 iterators scanning each of {A,B,C,D,E} from left to right;
2. Put the smallest-value (pointed by the 5 iterators) in the write-buffer; forward the corresponding iterator;

A	2, 13
B	7, 10
C	6, 11
D	5, 12
E	1, 14

SORTING

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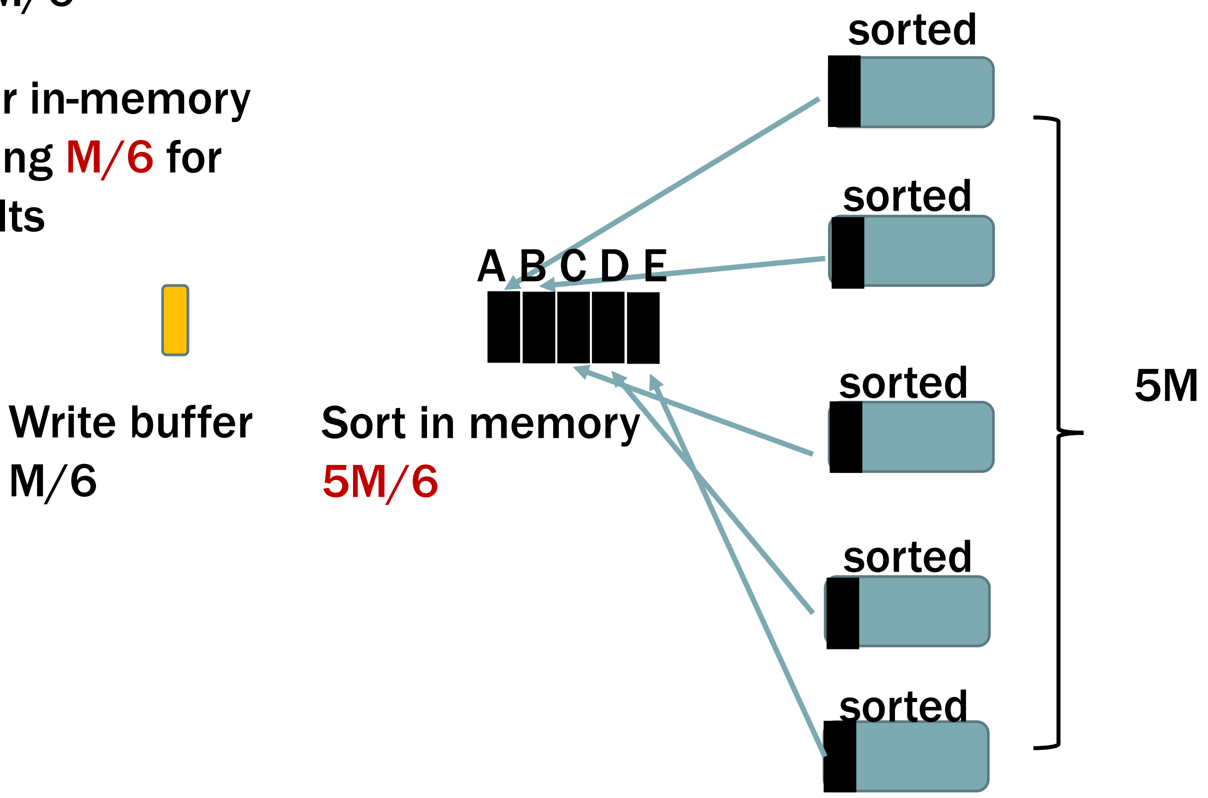

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Step 3:

Whenever one of {A,B,C,D,E} is empty, refill it from the source part.

Whenever the write-buffer is full, write it to the disk.

DETAILED EXAMPLE FOR (STEP 2&3)

Details of merge sort

1. 5 iterators scanning each of {A,B,C,D,E} from left to right;

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A: 2, 13

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C: 6, 11

D: 5, 12

E: 1, 14

Write buffer (size 2)

DETAILED EXAMPLE FOR (STEP 2&3)

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E	14

Write buffer

1

DETAILED EXAMPLE FOR (STEP 2&3)

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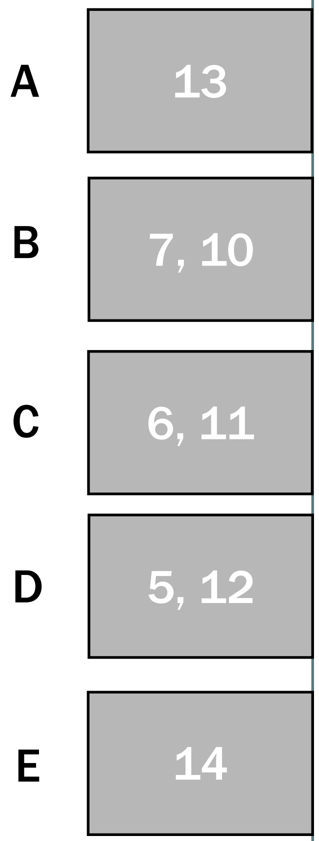

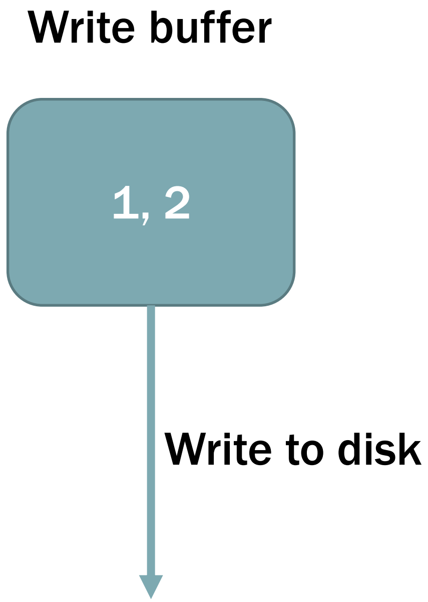

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Write buffer

DETAILED EXAMPLE FOR (STEP 2&3)

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C: 6, 11

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E: 14

Write buffer

5

DETAILED EXAMPLE FOR (STEP 2&3)

Details of merge sort

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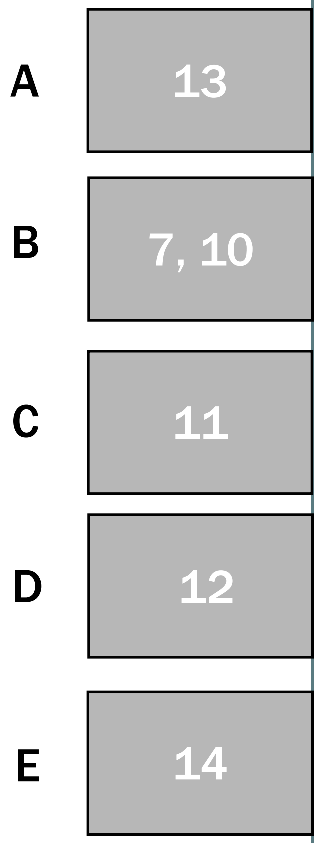

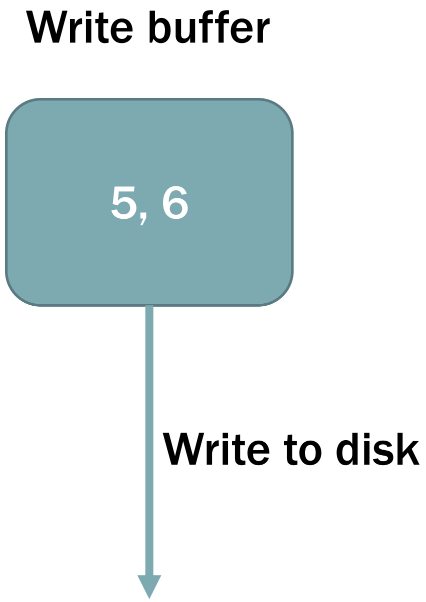

DETAILED EXAMPLE FOR (STEP 2&3)

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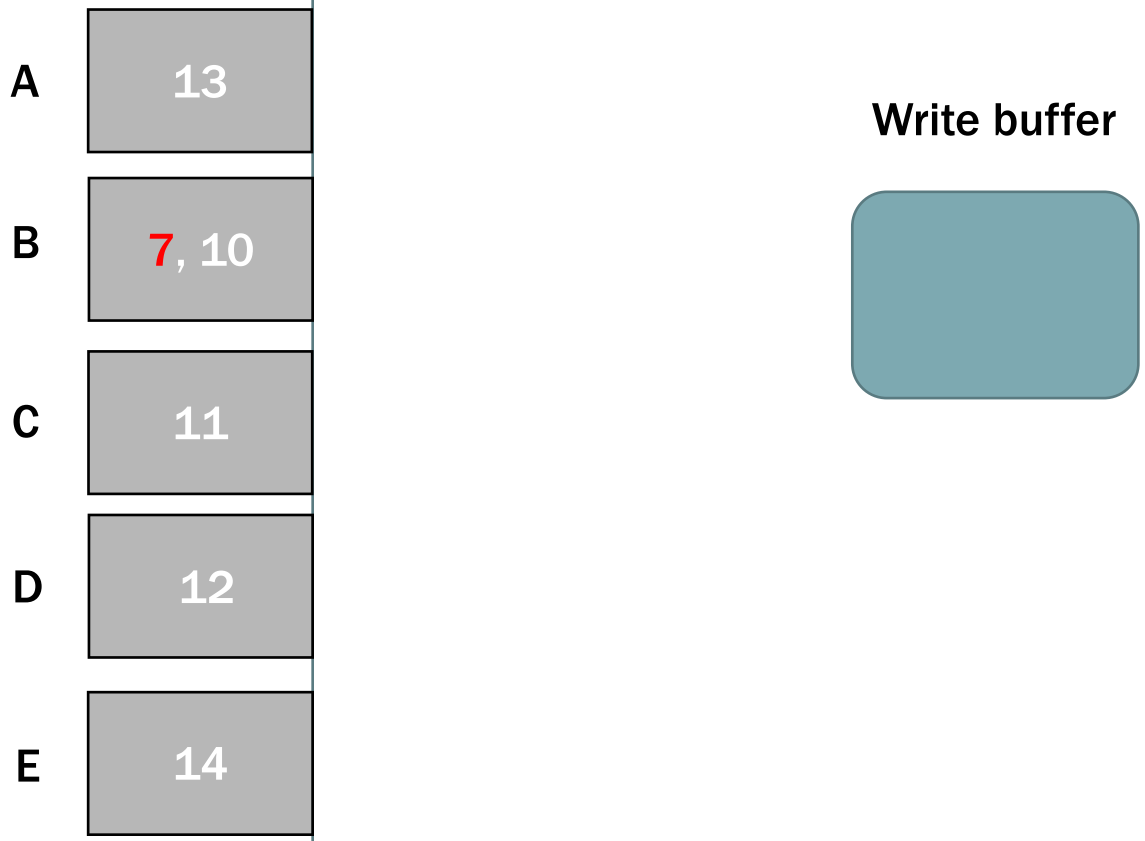

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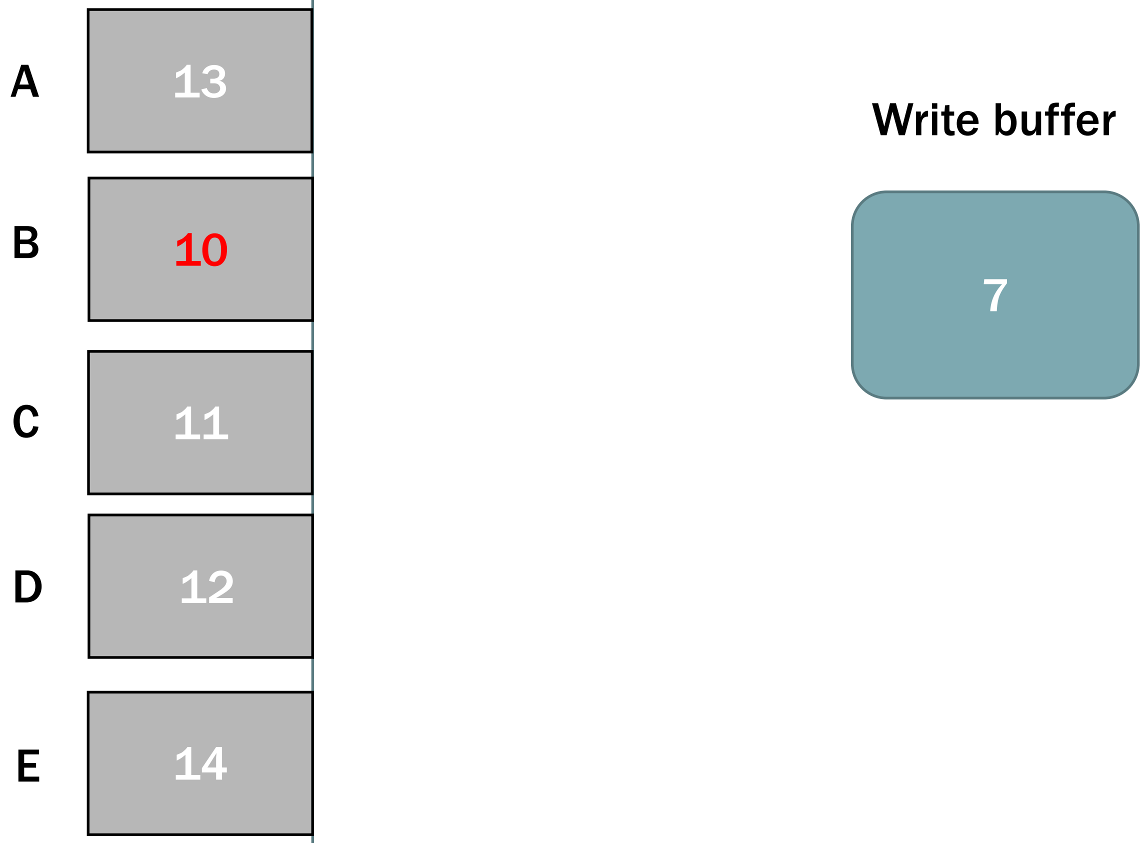

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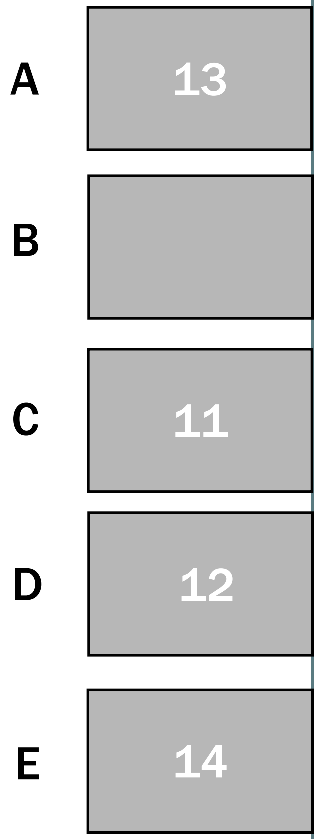

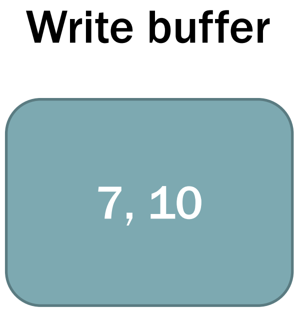

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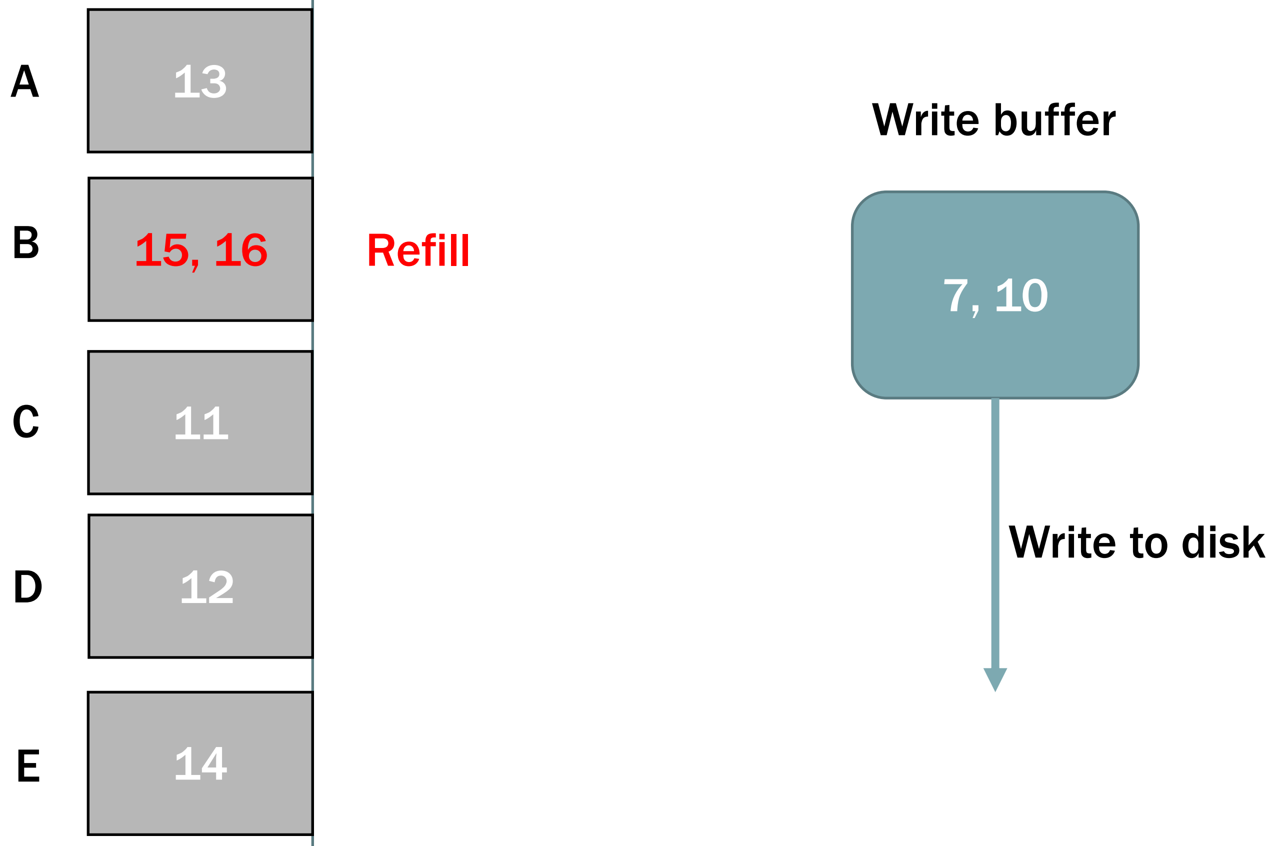

What's the cost?

COST ANALYSIS

Let N be the data size, M be the memory size, and the B be the page (transfer unit) size.

- Step 1: Read all the N items once and write back to the disk once, incurring O(N/B) I/Os.

COST ANALYSIS

Let N be the data size, M be the memory size, and the B be the page (transfer unit) size.

- Step 1: Read all the N items once and write back to the disk once, incurring O(N/B) I/Os.

- Step 2&3: During merge and writeback, each item is read from and written to the disk exactly once, incurring O(N/B) I/Os.

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Does this analysis apply to all situations?

COST ANALYSIS

To make the analysis hold, we should have

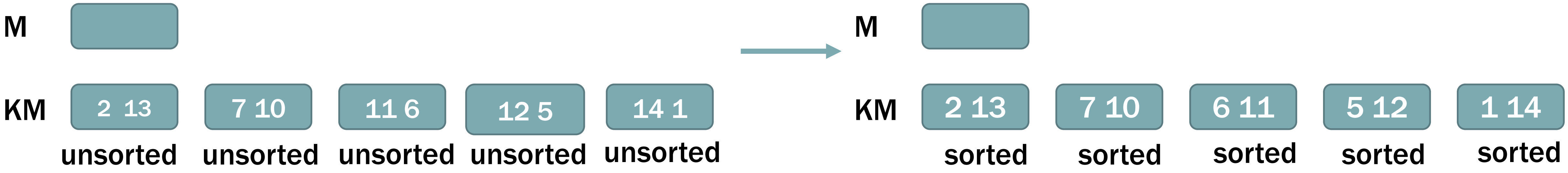
- ❑ The size of write buffer is at least one page size (size B)

- ❑ The process of in-memory merge sort should be fit in main memory.

Does this analysis apply to all situations?

SORTING

Suppose the size of main memory is M, the array size is KM(=N).



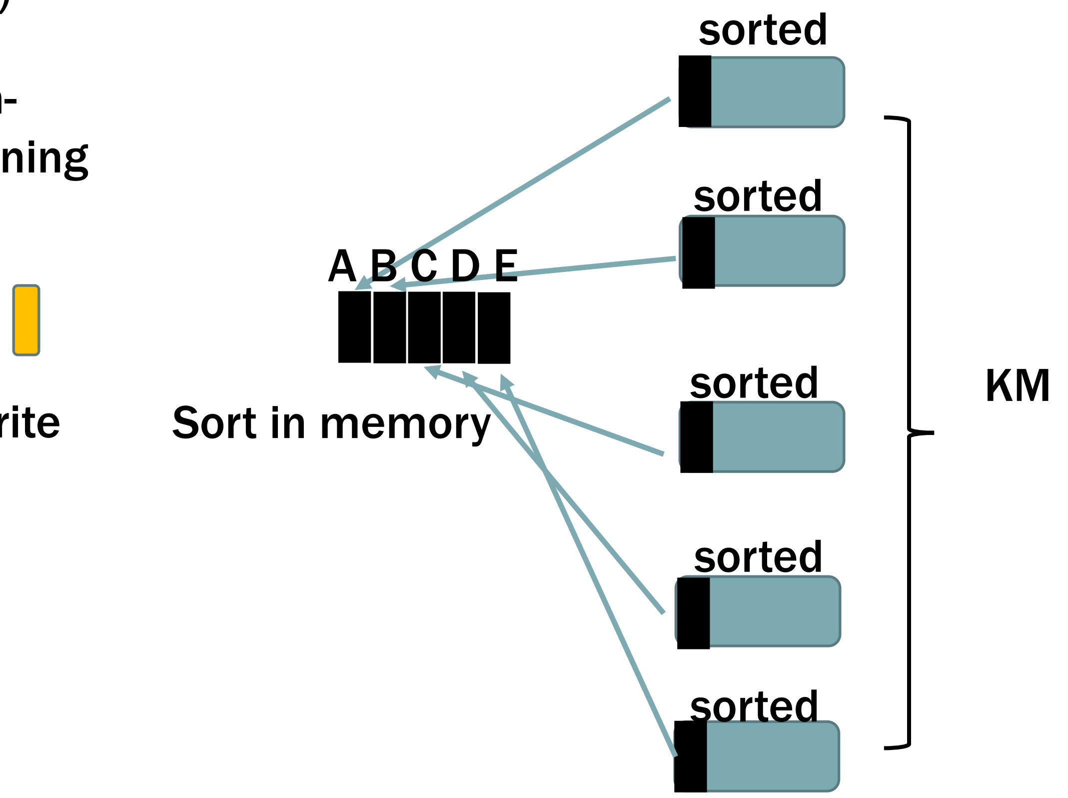
Step 1:

Cut KM arrays into K parts. Each part is put in main memory to sort (using any sorting algorithm we learnt, e.g., quick sort)

SORTING

or size M/(K+1)

KM/(K+1) in total for in-memory sorting, remaining M/(K+1) for writing the results



Step 2:

Apply K-way merge sort to the first M/(K+1) of each sorted part.

THE RANGE OF K

- Condition 1:

A write buffer has at least one page size (denoted by B).

Hence, we have $M/(K+1) >= B$, giving us **$K <= M/B-1$**

- Condition 2:

The process of K-way merge-sort should be fit in remaining memory size (excluding write buffer).

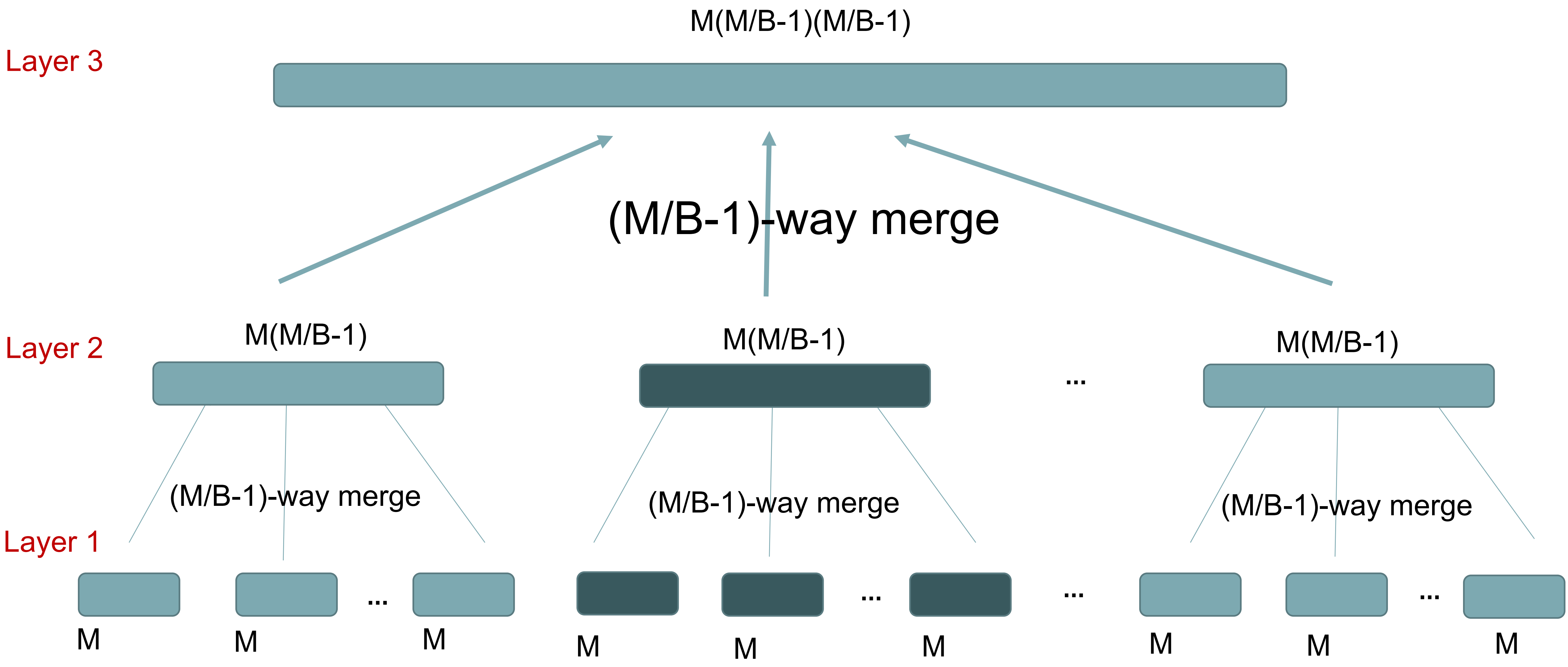
Hence, $K \times B <= KM/(K+1)$, also giving us **$K <= M/B-1$**

EXTEND TO GENERAL CASE

- The cost analysis holds when N is at most (M/B-1)M.

- We can extend the method to general N by recursively applying the (M/B-1)-way merge.

RECURSIVELY APPLY (M/B-1)-WAY MERGE



GENERAL ANALYSIS

- If there are L layers, then $M(M/B-1)^{L-1} = N$ because N items should be sorted, giving

  $L = O(\log_{M/B} N/M)$

$$
\begin{aligned}
M(M/B-1)^{L-1} &= N \\
\Leftrightarrow (M/B-1)^{L-1} &= N/M \\
\Leftrightarrow \log_{M/B-1}(M/B-1)^{L-1} &= \log_{M/B-1} N/M \\
\Leftrightarrow L-1 &= \log_{M/B-1} N/M \\
\Leftrightarrow L &= 1 + \log_{M/B-1} N/M \\
\Leftrightarrow L &= O(\log_{M/B} N/M)
\end{aligned}
$$

- The merges in each layer costs O(N/B) I/Os.
- Hence, the total cost is $O((N/B) \log_{M/B} N/M)$ I/Os.

We finish Memory Hierarchy!

Next lecture: Column Store