CE/CZ1104 & SC1004 (Semester 2 – AY 21-22): Take home test 1: Version J

Name: Hendy

Matric. ID: U2122559J

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1. (2 points) Find basis of $Span\left\langle \begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -7 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$.

2. (3 points) Find LU factorization of $A = \begin{pmatrix} -4 & -1 & -1 \\ -2 & -5 & -2 \\ -2 & 2 & -3 \end{pmatrix}$.

3. (5 points) Let V be a vector space of all symmetric matrices of the size 2×2. Choose a basis in this space and find the matrix of the linear operator L with respect to this basis if $L(A) = \begin{pmatrix} -1 & 2 \\ 2 & 2 \end{pmatrix} \cdot A + A \cdot \begin{pmatrix} -1 & 2 \\ 2 & 2 \end{pmatrix}$. Find the range and the kernel of the linear operator L. Find the Null space, the column space and the rank of the matrix of the operator L.

Take home test J

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1) Find basis of Span $\left\langle \begin{pmatrix}-2\\-4\\-1\end{pmatrix}, \begin{pmatrix}-1\\-7\\2\end{pmatrix}, \begin{pmatrix}1\\7\\2\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rangle$

Take $X = \begin{pmatrix}-2 & -1 & 1 & 1\\ -4 & -7 & 7 & 1\\ -1 & 2 & 2 & 1\end{pmatrix}$ $\begin{matrix} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - \frac{1}{2}R_1 \end{matrix}$ $\rightarrow \begin{pmatrix}-2 & -1 & 1 & 1\\ 0 & -5 & 5 & -1\\ 0 & 5/2 & 3/2 & 1/2\end{pmatrix} \rightarrow R_3 \leftarrow R_3 + \frac{1}{2}R_2 \rightarrow \begin{pmatrix}-2 & -1 & 1 & 1\\ 0 & -5 & 5 & -1\\ 0 & 0 & 4 & 0\end{pmatrix}$

Basis is linear independent $= \left\{ \begin{pmatrix}-2\\-4\\-1\end{pmatrix}, \begin{pmatrix}-1\\-7\\2\end{pmatrix}, \begin{pmatrix}1\\7\\2\end{pmatrix} \right\}$

2. Find LU Factorization of $A = \begin{pmatrix}-4 & -1 & -1\\ -2 & -5 & -2\\ -2 & 2 & -3\end{pmatrix}$

$\begin{pmatrix}-4 & -1 & -1\\ -2 & -5 & -2\\ -2 & 2 & -3\end{pmatrix}$ $\begin{matrix} R_2 \leftarrow R_2 - \frac{1}{2}R_1 \\ R_3 \leftarrow R_3 - \frac{1}{2}R_1 \end{matrix}$ $\rightarrow \begin{pmatrix}-4 & -1 & -1\\ 0 & -9/2 & -3/2\\ 0 & 5/2 & -5/2\end{pmatrix}$ $R_3 \leftarrow R_3 + \frac{5}{9}R_2 \rightarrow$ $U = \begin{pmatrix}-4 & -1 & -1\\ 0 & -9/2 & -3/2\\ 0 & 0 & -10/3\end{pmatrix}$ //

$L = \begin{pmatrix}1 & 0 & 0\\ 1/2 & 1 & 0\\ 1/2 & -\frac{5}{9} & 1\end{pmatrix}$ //

$LU = \overset{L}{\begin{pmatrix}1 & 0 & 0\\ 1/2 & 1 & 0\\ 1/2 & -5/9 & 1\end{pmatrix}} \overset{U}{\begin{pmatrix}-4 & -1 & -1\\ 0 & -9/2 & -3/2\\ 0 & 0 & -10/3\end{pmatrix}}$ //

3) Find range and kernel of linear operator L, Null Space, Column Space and rank of matrix of operator L

$L(A) = \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} \cdot A + A \cdot \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix}$

take basis $b_1 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$, $b_2 = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$, $b_3 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$L(b_1) = \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} \cdot \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 2 & 0\end{pmatrix} + \begin{pmatrix}-1 & 2\\ 0 & 0\end{pmatrix} = \begin{pmatrix}-2 & 2\\ 2 & 0\end{pmatrix} \rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix}$

$L(b_2) = \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} \cdot \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} = \begin{pmatrix}0 & 2\\ 0 & 2\end{pmatrix} + \begin{pmatrix}0 & 0\\ 2 & 2\end{pmatrix} = \begin{pmatrix}0 & 2\\ 2 & 4\end{pmatrix} \rightarrow \begin{pmatrix}0\\4\\2\end{pmatrix}$

$L(b_3) = \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} \cdot \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \cdot \begin{pmatrix}-1 & 2\\ 2 & 2\end{pmatrix} = \begin{pmatrix}2 & -1\\ 2 & 2\end{pmatrix} + \begin{pmatrix}2 & 2\\ -1 & 2\end{pmatrix} = \begin{pmatrix}4 & 1\\ 1 & 4\end{pmatrix} \rightarrow \begin{pmatrix}4\\4\\1\end{pmatrix}$

Range (L) = Span $\left\langle \begin{pmatrix}-2 & 2\\ 2 & 0\end{pmatrix}, \begin{pmatrix}0 & 2\\ 2 & 4\end{pmatrix}, \begin{pmatrix}4 & 1\\ 1 & 4\end{pmatrix} \right\rangle$

Kernel (L) = $\left\{ \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} \right\}$

$B = \begin{pmatrix}-2 & 0 & 4\\ 0 & 4 & 4\\ 2 & 2 & 1\end{pmatrix}$ $R_3 \leftarrow R_3 + R_1$

$\downarrow$

$\begin{pmatrix}-2 & 0 & 4\\ 0 & 4 & 4\\ 0 & 2 & 5\end{pmatrix}$ $R_3 \leftarrow R_3 - \frac{1}{2}R_2$

$\downarrow$

$\begin{pmatrix}-2 & 0 & 4\\ 0 & 4 & 4\\ 0 & 0 & 3\end{pmatrix}$

Null Space $N(B) = \{0\}$ because B has trivial Solution

Column Space $C(B)$ = Span $\left\langle \begin{pmatrix}-2\\0\\2\end{pmatrix}, \begin{pmatrix}0\\4\\2\end{pmatrix}, \begin{pmatrix}4\\4\\1\end{pmatrix} \right\rangle$

rank (B) = 3