



# SC2001/ CX2101: Algorithm Design and Analysis

## Week 12

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Please log into Teams as we would like to conduct 2 polls in this lecture.

# Pseudo-polynomial time

- (Slide 77): An algorithm is polynomial time if it is a polynomial function of the size of the input.
  - All sorting algorithms we covered: E.g MergeSort, Complexity:  $O(n \lg n)$ . Size of input:  $n+1$
  - Graph algorithms: E.g. Dijkstra's, complexity:  $O(n^2)$ , Size of input:  $1+n+n^2$
- (Slide 77): An algorithm is pseudo-polynomial time if it is a polynomial function of the value of the input.
  - DP for Knapsack: complexity:  $O(nC)$ . Size of input:  $2+2n$ . Complexity function is a function of both  $n$  and  $C$ , where  $C$  is the value of the 2<sup>nd</sup> input number

# Pseudo-polynomial time

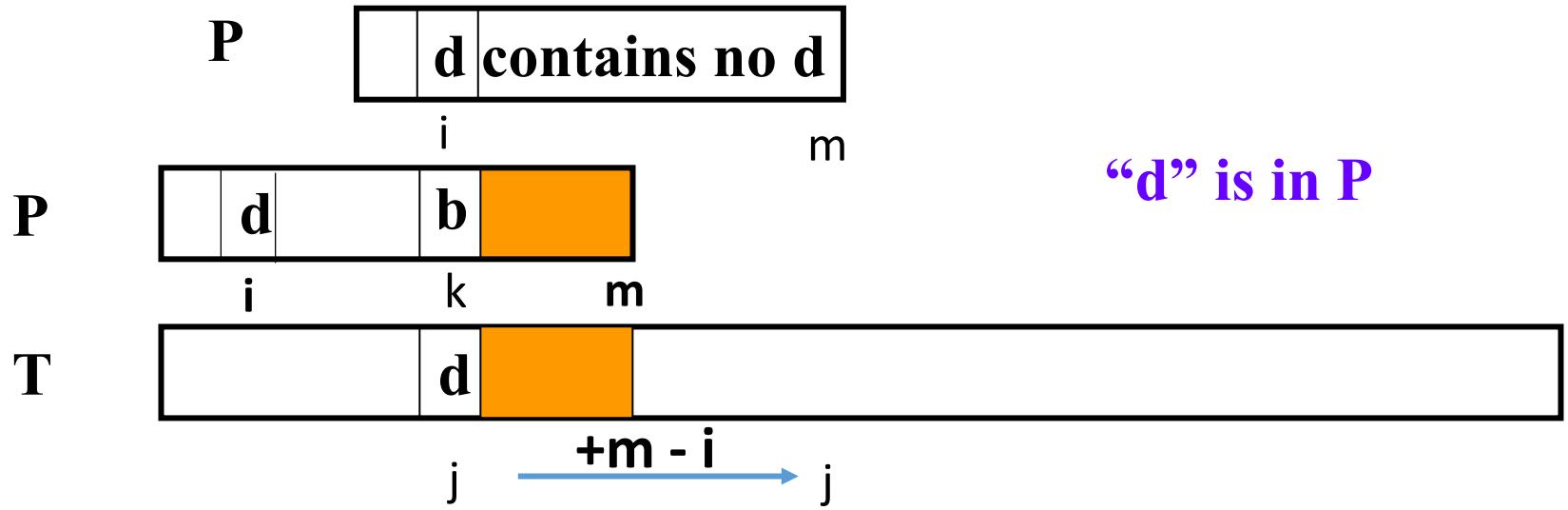
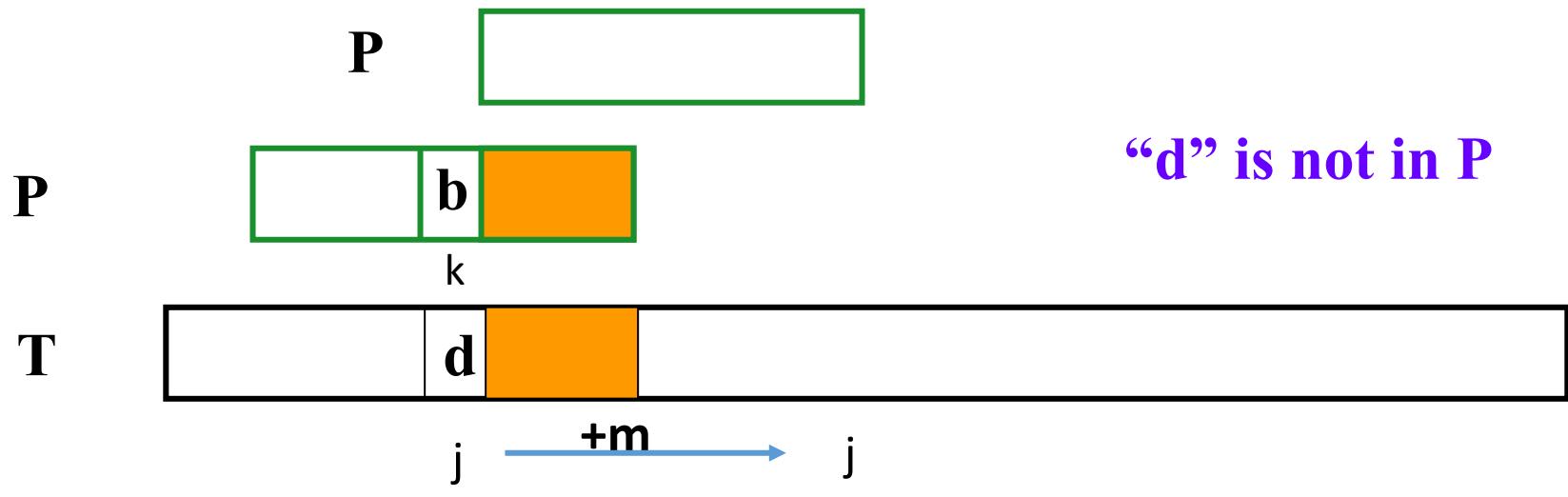
- Poll 1  
DP for chain matrix multiplication: complexity:  $O(n^3)$ . Is the algorithm polynomial or pseudo-polynomial?  

- Poll 2  
DP for Fibonacci function  $F(n)$ : complexity:  $O(n)$ . Is the algorithm polynomial or pseudo-polynomial?  


# Recap on Boyer-Moore Algorithm

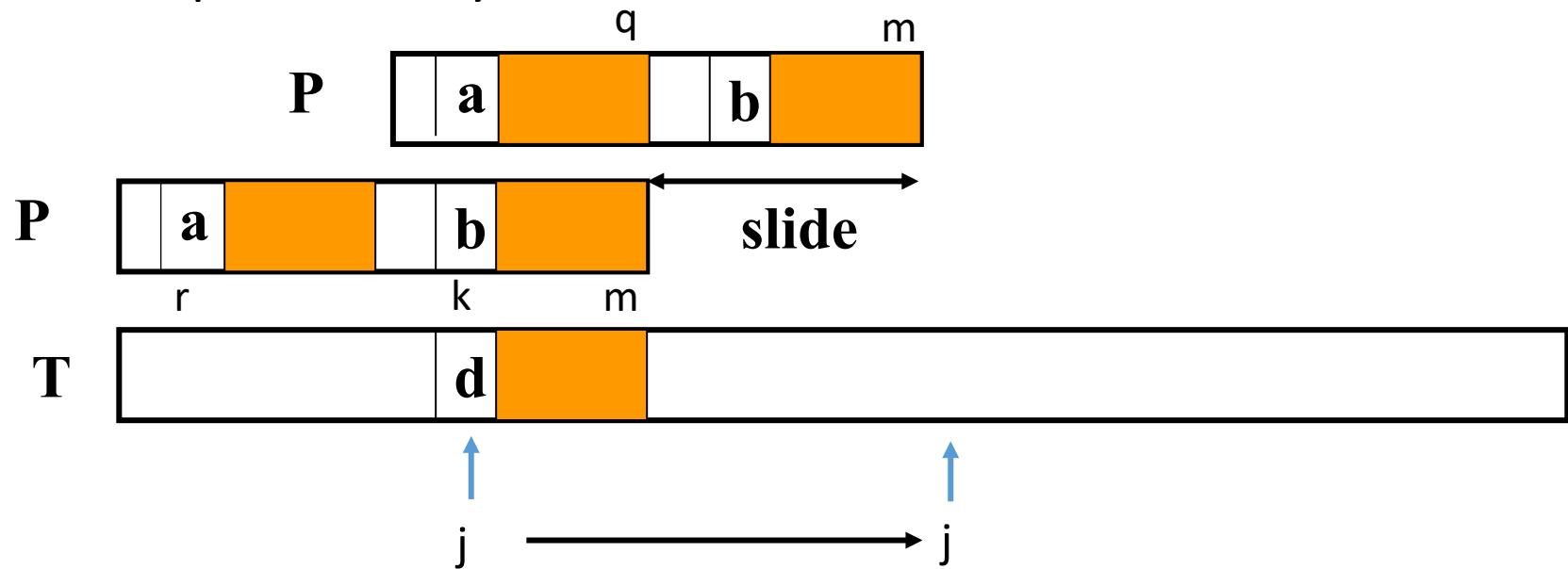
- It is a very efficient algorithm for string searching
- The text being scanned is  $T$  with  $n$  characters
- The pattern we are looking for is  $P$  with  $m$  characters
- Process the text  $T[1..n]$  from left to right
- Scan the pattern  $P[1..m]$  **from right to left**
- Preprocessing to generate two tables based on which to slide the pattern as much as possible after a mismatch
- It performs even better with long patterns

# Boyer-Moore: Preprocessing to compute charJump



# Boyer-Moore: Preporocessing to compute matchJump

Case 1: The matching suffix occurs earlier in the pattern, but preceded by a different character

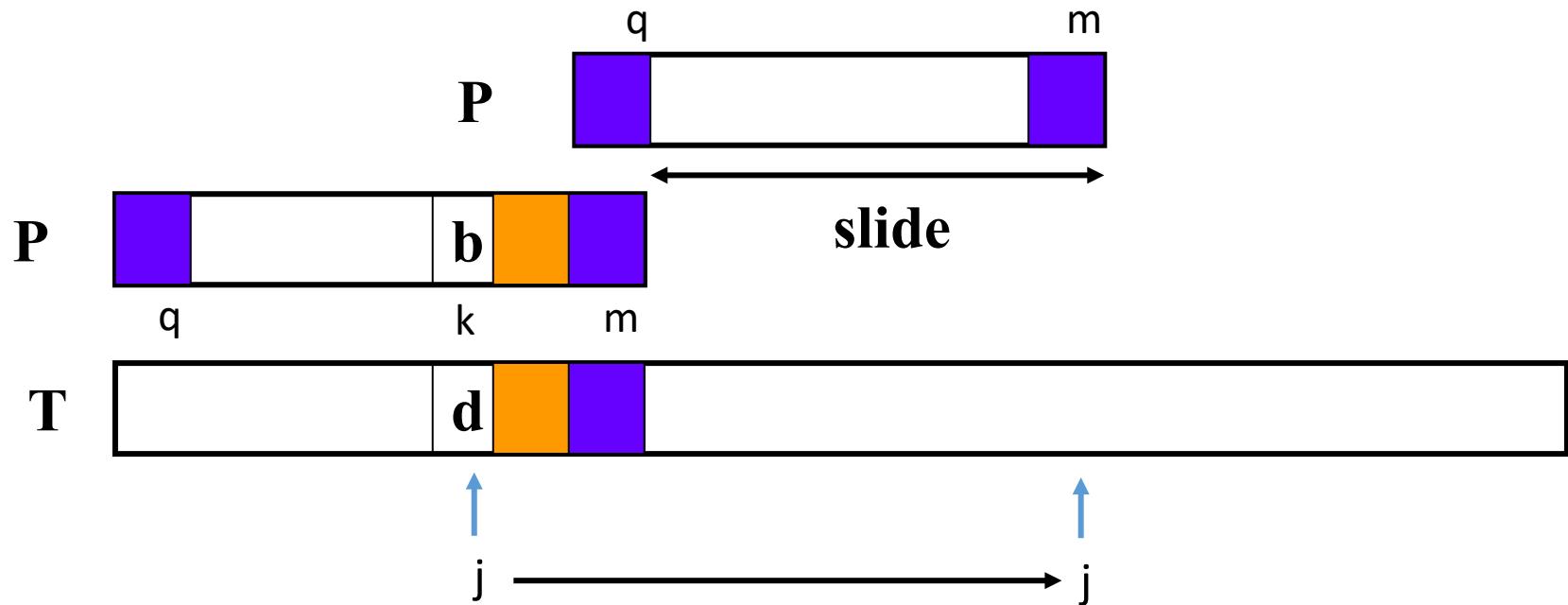


$$\text{matchJump}[k] = m - k + \text{slide}[k]$$

$$\text{slide}[k] = m - q \quad (P[r] \neq P[k])$$

i.e.  $\text{matchJump}[k] = \text{no. of characters matched} + \text{slide}[k]$

Case 2: Only part of the matching suffix occurs at the beginning of the pattern (a prefix).

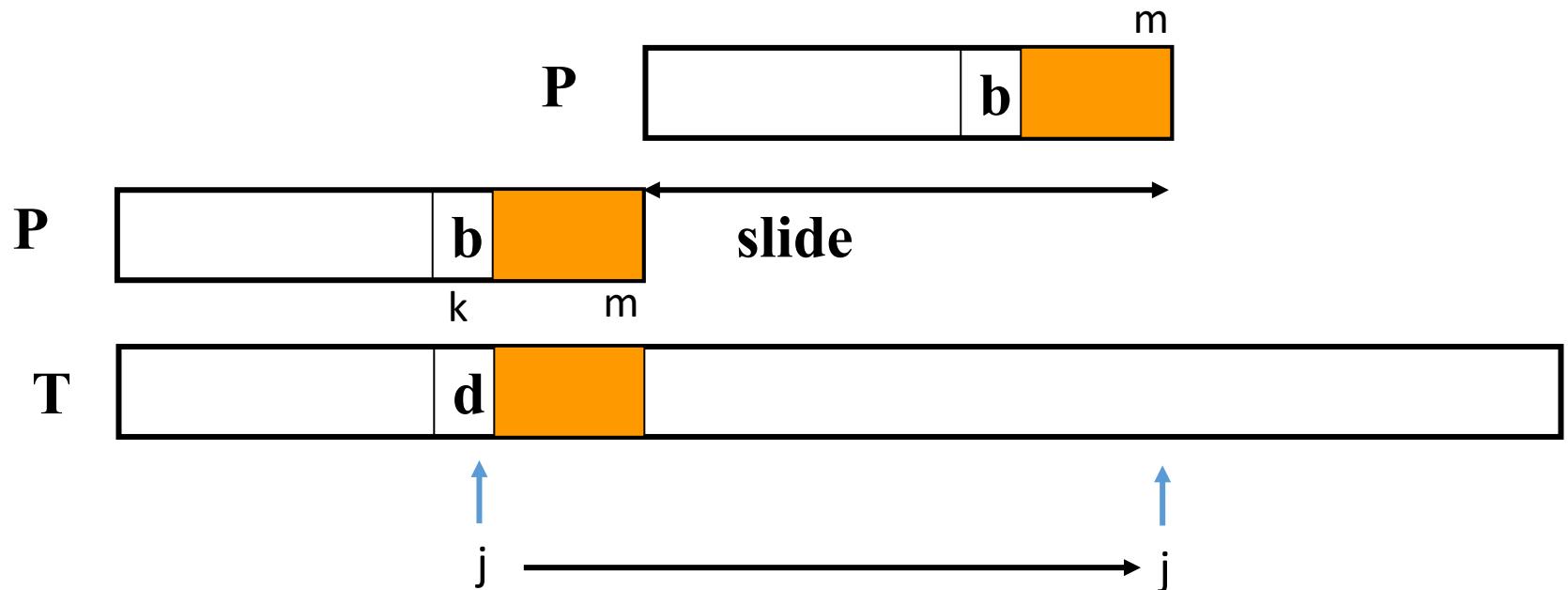


$$\text{matchJump}[k] = m - k + \text{slide}[k]$$

$$\text{slide}[k] = m - q$$

i.e.  $\text{matchJump}[k] = \text{no. of characters matched} + \text{slide}[k]$

Case 3: There is no other occurrence of the matching suffix in the pattern. (Case 1 and Case 2 do not happen)



$$\text{matchJump}[k] = m - k + \text{slide}[k]$$

$$\text{slide}[k] = m \quad (q = 0)$$

i.e.  $\text{matchJump}[k] = \text{no. of characters matched} + \text{slide}[k]$

Note:  $\text{Slide}[m] = 1$

# Example 1 of finding matchJump arrays

List the values in the *charJump* and *matchJump* arrays for the Boyer-Moore Algorithm for the following pattern:

AABAAACAAABABA

charJump[A] = 0  
charJump[B] = 1  
charJump[C] = 6  
charJump[X] = 12

X is any character in the character set not A, B, C

```
void computeJumps(char [] P, int m,  
                  int alpha, int [] charJump)  
{  char ch;  int k;  
    for (ch = 0; ch < alpha; ch++)  
        charJump[ch] = m;  
    for (k = 1; k <= m; k++)  
        charJump[ P[k] ] = m - k;  
}
```

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

											1
--	--	--	--	--	--	--	--	--	--	--	---

Note:  $\text{Slide}[m] = 1$

Matched = 0  
 $\text{Slide}[12] = 1$

*↗ not count as  
 next one is B*

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

											5	1
--	--	--	--	--	--	--	--	--	--	--	---	---



Matched = 1  
 $\text{Slide}[11] = 12 - 8 = 4$

BPA  
PA

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

									13	5	1
--	--	--	--	--	--	--	--	--	----	---	---

Not found

Matched = 2

Slide[10] = 12-1=11

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

								5	13	5	1
--	--	--	--	--	--	--	--	---	----	---	---

+ ABA

Matched = 3

Slide[9] = 12-10=2

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

							15	5	13	5	1
--	--	--	--	--	--	--	----	---	----	---	---

not found

Matched = 4

Slide[8] =  $12 - 1 = 11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

							16	15	5	13	5	1
--	--	--	--	--	--	--	----	----	---	----	---	---

nk

Matched = 5

Slide[7] =  $12 - 1 = 11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

					17	16	15	5	13	5	1
--	--	--	--	--	----	----	----	---	----	---	---

N

Matched = 6

Slide[6] =  $12 - 1 = 11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

				18	17	16	15	5	13	5	1
--	--	--	--	----	----	----	----	---	----	---	---

Matched = 7

Slide[5] =  $12 - 1 = 11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

			19	18	17	16	15	5	13	5	1
--	--	--	----	----	----	----	----	---	----	---	---

Matched = 8

Slide[4] =  $12-1=11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

		20	19	18	17	16	15	5	13	5	1
--	--	----	----	----	----	----	----	---	----	---	---

Matched = 9

Slide[3] =  $12-1=11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

	21	20	19	18	17	16	15	5	13	5	1
--	----	----	----	----	----	----	----	---	----	---	---

Matched = 10

Slide[2] =  $12-1=11$

A	A	B	A	A	C	A	A	B	A	B	A
1	2	3	4	5	6	7	8	9	10	11	12

22	21	20	19	18	17	16	15	5	13	5	1
----	----	----	----	----	----	----	----	---	----	---	---

Matched = 11

Slide[1] =  $12-1=11$

reminder

try out

## Example 2 of finding *matchJump* arrays

List the values in the *charJump* and *matchJump* arrays for the Boyer-Moore Algorithm for the following pattern:

A S T R A C A S T R A

$\text{charJump}[A] = 0$

$\text{charJump}[R] = 1$

$\text{charJump}[T] = 2$

$\text{charJump}[S] = 3$

$\text{charJump}[C] = 5$

$\text{charJump}[X] = 11$

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

										1
--	--	--	--	--	--	--	--	--	--	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

									5	1
--	--	--	--	--	--	--	--	--	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

									12	5	1
--	--	--	--	--	--	--	--	--	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

							13	12	5	1
--	--	--	--	--	--	--	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

						14	13	12	5	1
--	--	--	--	--	--	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1      2      3      4      5      6      7      8      9      10      11

					11	14	13	12	5	1
--	--	--	--	--	----	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

				12	11	14	13	12	5	1
--	--	--	--	----	----	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

			13	12	11	14	13	12	5	1
--	--	--	----	----	----	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

		14	13	12	11	14	13	12	5	1
--	--	----	----	----	----	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

	15	14	13	12	11	14	13	12	5	1
--	----	----	----	----	----	----	----	----	---	---

A	S	T	R	A	C	A	S	T	R	A
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

16	15	14	13	12	11	14	13	12	5	1
----	----	----	----	----	----	----	----	----	---	---

Please complete the online teaching feedback by 11.59pm, this Sunday.

Thank you!

