Solution Guide Online Assignment Limits

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1) $f(x) = \begin{cases} x^2+1 & \text{if } x \le 1 \\ x+3 & \text{if } x > 1 \end{cases}$ $\quad g(x) = 2x$

(a) $f(-1) = 2$ (b) $f(3) = 6$ (c) $f(g(0.5)) = f(1)$

$= 2$

(d) $\lim_{x \to 0.5^-} f(g(x)) = f(1)$ $\quad$ (e) $\lim_{x \to 0.5^+} f(g(x)) = \lim_{x \to 1^+} f(x)$

$= 2$ $\quad$ $= 4$

(2) $\lim_{x \to 9} \frac{x^3 - 9^3}{\sqrt{x} - 3} = \lim_{x \to 9} \frac{x^3 - 9^3}{\sqrt{x} - 3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$ $\quad (a^3 - b^3) = (a-b)(a^2+ab+b^2)$

$= \lim_{x \to 9} \frac{(x^3 - 9^3)(\sqrt{x}+3)}{x-9}$

$= \lim_{x \to 9} (x^2 + 9x + 81)(\sqrt{x}+3)$

$= [3(81)](6)$

$= 1458$

(3) $\lim_{x \to 5} \frac{\sqrt{x+15} - 2\sqrt{x}}{\sqrt{3x+5} - \sqrt{10+2x}} = \lim_{x \to 5} \frac{\sqrt{x+15} - 2\sqrt{x}}{\sqrt{3x+5} - \sqrt{10+2x}} \cdot \frac{\sqrt{3x+5} + \sqrt{10+2x}}{\sqrt{3x+5} + \sqrt{10+2x}} \cdot \frac{\sqrt{x+15} + 2\sqrt{x}}{\sqrt{x+15} + 2\sqrt{x}}$

$= \lim_{x \to 5} \frac{x+15-4x}{3x+5-10-2x} \cdot \frac{\sqrt{3x+5} + \sqrt{10+2x}}{\sqrt{x+15} + 2\sqrt{x}}$

$= \lim_{x \to 5} \frac{-3x+15}{x-5} \cdot \frac{\sqrt{3x+5} + \sqrt{10+2x}}{\sqrt{x+15} + 2\sqrt{x}}$

$= \lim_{x \to 5} -3 \cdot \frac{\sqrt{3x+5} + \sqrt{10+2x}}{\sqrt{x+15} + 2\sqrt{x}}$

$= -3 \cdot \frac{\sqrt{20} + \sqrt{20}}{\sqrt{20} + 2\sqrt{5}}$

$= -3$

(4) $\lim_{x \to -\infty} \sqrt{x^2+8x+22} + x = \lim_{x \to -\infty} \sqrt{x^2+8x+22} + x \cdot \frac{\sqrt{x^2+8x+22} - x}{\sqrt{x^2+8x+22} - x}$

$= \lim_{x \to -\infty} \frac{8x+22}{\sqrt{x^2+8x+22} - x}$

$= \lim_{x \to -\infty} \frac{8 + \frac{22}{x}}{-\sqrt{1 + \frac{8}{x} + \frac{22}{x^2}} - 1}$

$= \frac{8}{-\sqrt{1} - 1}$

$= -4$

(5) $\lim_{x \to \frac{\pi}{4}} \frac{\cos^8(2x)}{2^8\left(\sqrt{\cos(x)} - \sqrt{\sin(x)}\right)^8} = \lim_{x \to \frac{\pi}{4}} \left(\frac{\cos(2x)}{2\left(\sqrt{\cos(x)} - \sqrt{\sin(x)}\right)}\right)^8$

$= \lim_{x \to \frac{\pi}{4}} \left[\frac{\left[\cos^2(x) - \sin^2(x)\right]\left(\sqrt{\cos(x)} + \sqrt{\sin(x)}\right)}{2(\cos x - \sin x)}\right]^8$

$= \lim_{x \to \frac{\pi}{4}} \left(\frac{\left[\cos(x) + \sin(x)\right]\left[\sqrt{\cos x} + \sqrt{\sin x}\right)}{2}\right)^8$

$= \left(\frac{\sqrt{2}\left(\sqrt{\frac{1}{\sqrt{2}}} + \sqrt{\frac{1}{\sqrt{2}}}\right)}{2}\right)^8$

$= 4$

(6) $\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} \frac{f(x)}{x^2} \cdot x$

$= \lim_{x \to 0} \frac{f(x)}{x^2} \cdot \lim_{x \to 0} x$

$= 11 \cdot 0$

$= 0$

(7) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{f(x)} = \lim_{x \to 4} \frac{\sqrt{x} - 2}{f(x)} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$ $\quad \frac{2x+b}{x-4}(4) = 12$

$\frac{1}{12} = \lim_{x \to 4} \frac{x-4}{(2x+b)(\sqrt{x}+2)}$ $\quad 2x + b = 3(x-4)$

$= \lim_{x \to 4} \frac{1}{\left(\frac{2x+b}{x-4}\right)(\sqrt{x}+2)}$ $\quad = 3x - 12$

$\therefore f(x) = 3x - 12,$

$a = 3, \; b = -12$

(8) $\lim_{x \to 0} \frac{x^3 + \sin^2(4x)}{12x^2 + \sin x \tan(-4x)} = \lim_{x \to 0} \frac{x + \left(\frac{\sin(4x)}{x}\right)^2}{12 + \frac{\sin x \tan(-4x)}{x^2}} = \lim_{x \to 0} \frac{x + \left(\frac{\sin(4x)}{x}\right)^2}{12 + \frac{\sin x}{x} \cdot \frac{\tan(-4x)}{x}}$

$\lim_{x \to 0} x + \left(\frac{\sin(4x)}{x}\right)^2 = 0 + (4)^2$

$= 16$

$\lim_{x \to 0} 12 + \frac{\sin x}{x} \cdot -\frac{\sin 4x}{x} \cdot \frac{1}{\cos 4x} = 12 + 1(-4)(1)$

$= 8$

$= \frac{16}{8}$

$= 2$

(9) $\lim_{x \to -\infty} 5^{x + \sin(-4x)} = 5^{\lim_{x \to -\infty} x + \sin(-4x)}$

$-1 \le \sin(-4x) \le 1$

$x - 1 \le x + \sin(-4x) \le x + 1$

$\lim_{x \to -\infty} x + \sin(-4x) = -\infty$

$5^{-\infty} = \frac{1}{5^{\infty}}$

$= 0.$

(10) $\lim_{x \to \infty} \frac{\sin(x) + \sin(5x)}{8x + \sin(3x)} = \lim_{x \to \infty} \frac{\frac{\sin(x)}{x} + \frac{\sin(5x)}{x}}{8 + \frac{\sin(3x)}{x}}$ $\quad \frac{1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$

$= \frac{0+0}{8+0}$

$= 0$

11) $\lim_{x \to 0^+} \frac{\sin(x)\sin\left(\frac{6}{x}\right)}{\sqrt{x}} = \lim_{x \to 0^+} \frac{\sqrt{x}\sin(x)\sin\left(\frac{6}{x}\right)}{x}$

$= \lim_{x \to 0^+} \frac{\sin(x)}{x} \cdot \lim_{x \to 0^+} \sqrt{x}\sin\left(\frac{6}{x}\right)$ $\quad -1 \le \sin\frac{6}{x} \le 1$

$-\sqrt{x} \le \sqrt{x}\sin\frac{6}{x} \le \sqrt{x}$

$= 1 \cdot (0)$ $\quad$ As $x \to 0, -\sqrt{x} \to 0, \sqrt{x} \to 0,$

$\sqrt{x}\sin\frac{6}{x} \to 0$

$= 0$

12) $\lim_{x \to \infty} \left(\frac{1}{6}|\sin 8x + \cos 8x|\right)^x = 0$ (squeeze theorem)

$0 \le |\sin 8x + \cos 8x| \le \sqrt{2}$

$0 \le \frac{1}{6}|\sin 8x + \cos 8x| \le \frac{\sqrt{2}}{6}$

$0 \le \left(\frac{1}{6}|\sin 8x + \cos 8x|\right)^x \le \left(\frac{\sqrt{2}}{6}\right)^x$ $\quad x \to \infty, \left(\frac{\sqrt{2}}{6}\right)^x \to 0$

(13) $f(x) = \frac{1}{2}\sqrt{6x + 6\sqrt{3x}}$

$f(0) = \frac{1}{2}\sqrt{0 + 6(0)}$ $\quad f(3) = \frac{1}{2}\sqrt{6(3) + 6\sqrt{9}}$

$= 0$ $\quad = \frac{1}{2}\sqrt{36}$

$= 3$

(14) $f(x) = \left|1 + |x^2 - 9|\right|, \; f(0) = 10, \; f(6) = 28$

$|x^2 - 9| = 0, \; x^2 = 9, \; x = 3$

$f(3) = 1$

$[1, 28]$

(15) $f(x) = \sin(x) + |5\sin(x)|$

$-1 < \sin x < 1, \quad 0 < |5\sin(x)| < 5$

$0 < \sin x + |5\sin(x)| < 6.$ $\quad$ When $\sin x = -1, \; |5\sin x| = 5,$

$\sin x + |5\sin x|$ min 0