

SCHOOL

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$$\sum \text{Sum of all even num} = n(n+1)$$

$$\sum \text{Sum of all odd num} = n^2$$

$$\sum 1+2+\dots+(n-1) = \frac{n(n-1)}{2}$$

$$\sum 1+2+\dots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{k-1} a^i = a^0 + a^1 + \dots + a^{k-1} = \frac{a^k - 1}{a - 1}$$

$$a^{\log_b n} = n^{\log_b a}$$

## DI Theorem

$$w(p) = d[CY] + w(Y, Z)$$

$$w(p') = d[Z_{k-1}] + w(Z_{k-1}, Z_k) + \underset{\text{from } Z_k \text{ to } Z}{\text{distance}}$$

$$d[Z_{k-1}] + w(Z_{k-1}, Z_k) \geq d[CY] + w(Y, Z)$$

Limit L'Hopital's rule  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

master

$$f(n) < O(n^{\log_b a - \epsilon}) \quad \epsilon > 0$$

$$w(n) = \Theta(n^{\log_b a})$$

$$f(n) = \Theta(n^{\log_b a / \log_b n}) \quad w(n) = \Theta(n^{\log_b a / \log_b n})$$

$$f(n) = \Omega(n^{\log_b a + \epsilon}) \quad \epsilon > 0$$

$$af(n/b) \leq c f(n) \quad c \leq 1$$

$$w(n) = \Theta(f(n))$$

best = descending order

get max()  $\Rightarrow$  put at end  $\leftarrow$  1st

fix leaf  $\log n$   
 delete Max  $\log n$   
 heapify  $n$   
 construct leaf  $n$   
 max heap  $n$   
 $\log n$

Dynamic programming  
 (top down)  $O(n)$

(bottom up)  $O(n)$

longest common Subsequence  $O(nm)$

1 longest, - seq,  $\lambda$  cost  $O(n+m)$

matrix optimal order  $O(n^3)$

Initialization

QuickFind

QuickUnion

Weighted QuickUnion

union

QuickFind

QuickUnion

Weighted QuickUnion

Find

QuickFind

QuickUnion

Weighted QuickUnion

connected

QuickFind

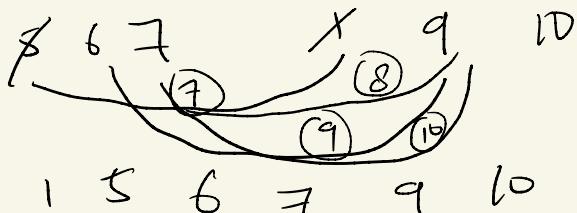
QuickUnion

Weighted QuickUnion

	Size	Complexity
Mergesort	$1+n$	$O(n \lg n)$
Dijkstra	$1+n+n^2$	$O(n^2)$
Knapsack	$2+2n$	$O(nc)$
DP for chain matrix	.	$O(n^3)$
DP for Fibonacci	.	$O(n)$
bayer-Moore		
Simple hash	worst $O(nm)$	$\frac{m \cdot \text{pattern}}{n \cdot \text{text}}$
Robin-Karp	hash = $\Theta(m)$ worst = $\Theta((n-m+1)m)$	convert to number hash = $(\text{new\_shift} - \text{old\_shift} * d^{m-1}) * d + \text{LSB}$
bayer-moore		$O( E  + m)$
mergesort		$O(n \log n)$
insertion sort		best: $O(n)$ worst: $\frac{n-1}{2}$
quick sort		$O(n \log n)$
heap		$O(n \log n)$
Dijkstra		worst $O( V ^2)$
Kruskal		$O( E  \log  E )$

Polynomial time		Exponential Time	
$n$	-	Linear Search	$2^n$
$\log n$	-	Binary Search	$2^n$
$n^2$	-	Insertion Sort	$2^n$
$n \log n$	-	Merge Sort	$2^n$
$n^3$	-	Matrix Multiplication	$2^n$
$\approx$		DJKR	- 0/1 knapsack - Travelling SP - Sum of Subsets - Graph Coloring - Hamilton Cycle

756(1091), 9



77 15 96 8d 42 80 35 04 93 06

42 15 16 89 77 80 35 04 93 06  
1 2 3 4 5 6 7 8 9 10

42 15 35 04 06 80 96 89 93 77

06 15 35 04 42 80 96 89 93 77

15 06 35 04 42 89 96 80 93 77  
10 1 2 14 15 6 17

06 04 15 35 42 89 80 77 93 96  
3 13

04 06 15 35 42 77 80 89 93 96  
11 19

04 06 15 35 42 77 80 89 93 96

TUT 3

1) Worst Case ( $n^2$ )
 $n, n-1, n-2, \dots, 2$   
 $1, n, n-1, n-2, \dots, 2$ 
2) divide and conquer  $n=2^k$ 

$w(n) = 0 \quad w(n_1) = 2w(n_2) + 2$

$w(n_1) = 0 \quad w(n_2) = 2w(n_4) + 2$

$w(n_2) = 1$

$w(n) = 2w(n_2) + 2$

$w(n) = 2(2w(n_4) + 2) + 2$

$w(n) = 2^2 w(n_4) + 2^2 + 2$

$w(n) \leq 2^k (2w(n_4) + 2) + 2^{k-2} + 2$

$w(n) = 2^3 w(n_4) + 2^3 + 2^{k-2} + 2$

$2^{k-1} w(n_{2^{k-1}}) + 2^{k-1} + 2^{k-2} + \dots + 2$

$2^{k-1} + 2^{k-1} + 2^{k-2} + \dots + 2$

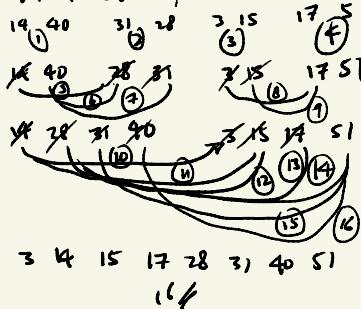
$2^{k-1} + 2(2^{k-2} + 2^{k-3} + \dots + 1)$

$2^{k-1} + 2 \left( \frac{2^{k-1}-1}{2-1} \right)$

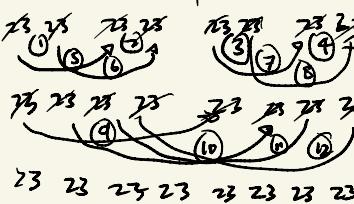
$n/2 + 2(n/2 - 1) = \frac{n}{2} + n - 2 = \frac{3n}{2} - 2$

$= 1.5n - 2$

3) a) 14 40 31 28 | 3 15 17 51



b) 23 23 23 23 | 23 23 23 23



A A B A A C A A B A B A

0	1	2	3	4	5	6
8	9	10	11	12	13	14
1	5	6	7	16	17	
" 13	" 15	" 14	" 11	" 12	" 16	

A S T R A C A S T R A

0	1	2	3	4	5	6
4	10	6	6	6	6	6
1	5	12	13	14	11	12

week 4

4) Quick Sort best  $n^{\log n}$   
worse  $n^2$

worst case  
 $E[F] = 100$   
 $E[L] = 98$   
 $E[F+L/2] = 99$   
other elements  $> 100$

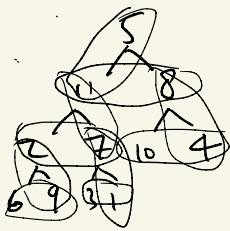
5)

6)

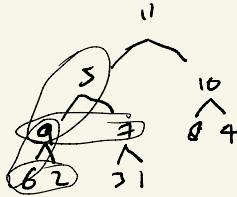
week 5

?) 1 Jan, 30 Jan, 22 Mar, 22 Dec, 30 May, 24 Feb, 3 Nov, 7 Jun, 21 Feb, 21 Nov, 30 Dec

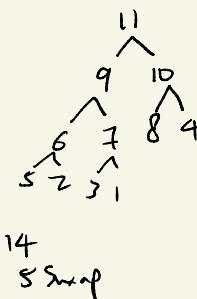
5 11 8 2 7 10 4 6 9 3 1



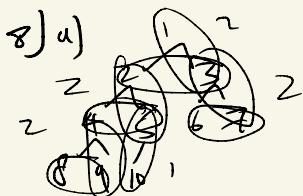
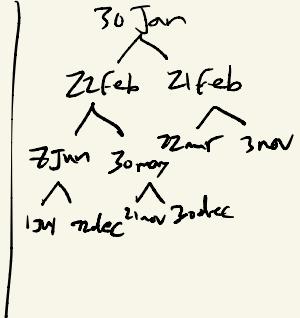
$$5 \cdot 2 = 10$$
$$\text{swap} = 1 + 1 + 1 = 3$$



$$4 \\ 1+1$$



$$14 \\ 5 \text{ swap}$$



$$4 \cdot 2 + 1 = 9$$

$$b) n-1$$

c) best

a) size = K  $O(n \log_2 K)$  ↘  
 $K \geq 2$  max-heap =  $O(K)$  ↗  
 $\Sigma = n$  fix heap =  $O(\log_2 K)$

Week 6

	1	2	3	4	5	1	2	3	4	5
i=0	d 0	$\infty$	0	$\infty$	0	i=4	d 0	4	2	3
	p <sub>i</sub>	-	-	-	-		p <sub>i</sub>	-	1	3
	s	0	0	0	0		s	1	1	1
i=1	d 0	4	2	6	8	i=5	d 0	4	2	3
	p <sub>i</sub>	-	1	1	1		p <sub>i</sub>	-	1	3
	s	1	0	0	0		s	1	1	1
i=2	d 0	4	2	3	8		s	1	1	1
	p <sub>i</sub>	-	1	1	3					
	s	1	0	1	0					
i=3	d 0	4	2	3	6					
	p <sub>i</sub>	-	1	1	3					
	s	1	0	1	0					

2) add <sup>array</sup> Count Variable at end of loop  
 initiate Count = 0 at source = 1

Count[v] = Count[u]  
 else if (d[v] == d[u] + w[u,v]) {  
 Count[v] += Count[u]  
 }

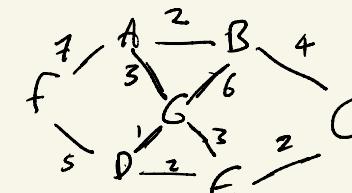
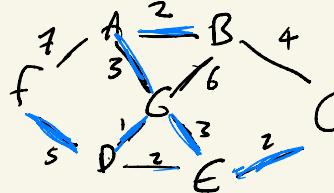
3) assume of non-negative weights used in proof D1  
 $w(p) = d(Y) + w(Y, Z)$   
 $w(p') = d[Z_{k-1}] + w(Z_{k-1}, Z_k) + \text{distance from } Z_k \text{ to } Z$

$$d[Z_{k-1}] + w(Z_{k-1}, Z_k) \geq d(Y) + w(Y, Z)$$

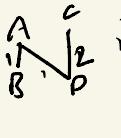
Week 7

- C NULL

	A	B	C	D	E	F	G	
0	S	0	0	0	0	0	1	
d	0	0	0	0	0	0	0	
Pi	-	-	-	-	-	-	-	
1	S	0	0	0	0	0	1	
d	3	6	0	1	3	0	0	
Pi	G	G	-	G	G	-	-	
2	S	0	0	0	1	0	1	
d	3	6	0	1	2	5	0	
Pi	G	G	-	G	G	D	-	
3	S	1	0	0	1	0	1	
d	3	2	0	1	2	S	0	
Pi	G	A	-	G	G	D	-	
4	S	1	0	0	1	1	0	1
d	3	2	2	1	2	S	0	
Pi	G	A	E	G	G	D	-	
5	S	1	1	0	1	1	0	1
d	3	2	2	1	2	S	0	
Pi	G	A	E	G	G	D	-	
6	S	1	1	1	1	1	0	1
d	3	2	2	1	2	S	0	
Pi	G	A	E	G	G	D	-	
7	S	1	1	1	1	1	1	0
d	3	2	2	1	2	S	0	
Pi	G	A	E	G	G	D	-	



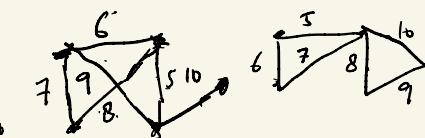
5) False, given graph



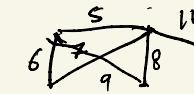
is minimum Spanning tree  
but  $A + C = 3$   
while  $A + C$  can be 2

6) 5 6 7 8 9 10

$$\begin{aligned} Z_8 &= 5+6+7+10^{8,9} \\ &= 5+6+8+9^{7,10} \end{aligned}$$

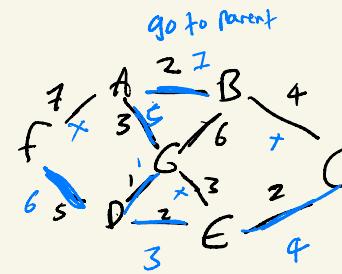


$$\begin{aligned} Z_9 &= 8+10+6+5^{7,9} \\ &= 8+9+7+5^{6,10} \end{aligned}$$



week 8

7) i	A B C D E F G
0 id	A B C D E F G
SZ	1 1 1 1 1 1 1
1 id	A B C D E F D
SZ	1 1 1 2 1 1 1
2 id	A B C D D F D
SZ	1 1 1 3 1 1 1
3 id	A A C D D F D
SZ	2 1 1 3 1 1 1
4 id	A A D D D F D
SZ	2 1 1 4 1 1 1
5 id	D A D D D F D
SZ	2 1 1 6 1 1 1
6 id	D A D D D D D
SZ	2 1 1 7 1 1 1



g)



g)



Week 9

$$\begin{aligned} \textcircled{1} \quad T(1) &= 1 \quad n \geq 2 \\ k=1 \quad T(n) &= 3T(n-1) + 2 \quad T(n-1) = 3T(n-2) + 2 \\ k=2 \quad T(n) &= 3(3T(n-2) + 2) + 2 = 3^2T(n-2) + 2 \cdot 3 + 2 \\ &= 3^2(3T(n-3) + 2) + 2 \cdot 3 + 2 \\ k=3 \quad T(n) &= 3^3T(n-3) + 2 \cdot 3^2 + 2 \cdot 3 + 2 \\ &\leq 3^k T(n-k) + 2\left(3^{k-1} + 3^{k-2} + \dots + 1\right) \\ &= 3^{n-1} T(1) + 2\left(3^{n-2} + 3^{n-3} + \dots + 1\right) \\ &= 3^{n-1} + 2\left(\frac{3^{n-1} - 1}{3 - 1}\right) \\ &= 2 \cdot 3^{n-1} - 1 \leq 3^n \end{aligned}$$

$$\begin{matrix} n-k=1 \\ k=n-1 \end{matrix}$$

$$O(3^n)$$

$$\begin{aligned} \textcircled{2} \quad T(1) &= 1 \quad n \geq 2 \\ T(n) &= 2T(n/2) + 6n \quad T(n/2) = 2T(n/4) + 6n/2 \\ &= 2\left(2T(n/4) + \frac{6n}{2}\right) + 6n \quad T(n/4) = 2T(n/8) + \frac{6n}{4} \\ &= 2^2 T\left(\frac{n}{2^2}\right) + 6n + 6n \\ &= 2^2\left(2T\left(\frac{n}{2^3}\right) + \frac{6n}{2^2}\right) + 2 \cdot 6n \quad \frac{n}{2^k} = 1 \\ &= 2^3 T\left(\frac{n}{2^3}\right) + 3 \cdot 6n \quad n = 2^k \\ &= 2^k T\left(\frac{n}{2^k}\right) + k \cdot 6n \quad n = 2^k \cdot \ln_2 k \\ &\leq 2^{\log_2 n} T(1) + \log_2 n \cdot 6n \quad k = \frac{\log n}{\ln 2} \\ &= n + 6n \log n \leq O(n \log n) \quad k = \log n \end{aligned}$$

2)  $\textcircled{1} \quad T(1) = 1 \quad n \geq 2$

$$\begin{aligned} T(n) &= 3T(n-1) + 2 \quad \text{inductive} \\ \text{guess: } T(n) &\leq C \cdot 3^n \rightarrow T(n+1) \leq C \cdot 3^{n+1} \end{aligned}$$

base case

$$T(1) = 1 \leq 3^1 \text{ true}$$

$$\begin{aligned} T(n+1) &= 3T(n) + 2 \\ &\leq 3T(n) + 2 \\ &\leq 3 \cdot 3^n + 2 \\ &\leq 3^{n+1} + 2 \leq \textcircled{1}(3^{n+1}) \end{aligned}$$

$\textcircled{2} \quad T(1) = 1 \quad \text{guess: } O(8n \log n) \quad n = 2^k$

$$\begin{aligned} \text{base case: } T(n) &\leq O(8n \log n) \quad T(2^k) \leq 8 \cdot 2^k \cdot k \\ T(2) &= 14 < 16 \end{aligned}$$

Inductive Step  $T(2^{k+1}) \leq 8 \cdot 2^{k+1} (k+1)$

$$\begin{aligned} T(2^{k+1}) &= 2T(2^{\frac{k+1}{2}}) + 6 \cdot 2^{k+1} = 2^{k+1}(8k+6) \\ &= 2T(2^k) + 2^k \cdot 6 \cdot 2 = 2^{k+1}(k+\frac{1}{4}) \leq 8 \cdot 2^{k+1}(k+1) \\ &\leq 2(8 \cdot 2^k \cdot k) + 2^k \cdot 6 \cdot 2 \quad \checkmark \end{aligned}$$

$$3) \text{ a) } w(n) < w(n/3) + 5 \quad f(n) = 5$$

$$\begin{array}{l} a=1 \quad b=3 \\ n^{\log_3 1} = 1 \leq 5 \end{array}$$

$$w(n) = \Theta(n^{\log_3 1} \log n) \quad f(n) = \Theta(n) \quad \text{2nd}$$

$$= \Theta(\log n)$$

$$\text{b) } T(n) = 2T(n/2) + \frac{n}{4}$$

$$n^{\log_2 n} = n^{\log_2 2} = n \leq \frac{n}{4} \quad \text{2nd}$$

$$T(n) = \Theta(\log n)$$

$$\text{c) } w(n) = 2w(n/4) + n^{\frac{3}{2}}$$

$$\begin{array}{l} f(n) = n^{\frac{3}{2}} \\ n^{\log_4 2} = n^{\frac{1}{2}} \quad n^2 \geq n^{\frac{3}{2}} \quad \text{3rd} \end{array}$$

$$2((\frac{n}{4})^{\frac{3}{2}}) \leq c n^{\frac{3}{2}} \quad c < 1$$

$$0.25 \frac{7}{8} \sqrt{n^3} \leq c \sqrt{n^3}$$

$$T(n) = \Theta(n^{\frac{3}{2}})$$

$$4) \text{ a) } x^3 - 4x - 5 = 0 \quad 3$$

$$\text{b) } \times$$

$$\text{3) } x^4 - x^3 - 1 = 0 \quad 4$$

$$\text{4) } \times$$

$$\text{5) } \times$$

$$5) \text{ a) } a_n = 7a_{n-1} - 10a_{n-2} \quad n \geq 2 \quad a_0 = 1 \quad a_1 = 0$$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$\begin{array}{l} x = 5 \\ x = 2 \end{array}$$

$$a_n = 5^nc + 2^nD \quad -\frac{2}{3}5^n + \frac{5}{3}2^n$$

$$a_0 = c + D = 1 \quad C = 1 - D$$

$$a_1 = 5c + 2D = 0 \quad D = \frac{5}{3} \quad C = -\frac{2}{3}$$

$$5 - 5D + 2D = 0$$

$$\text{b) } a_n + 4a_{n-2} = \frac{a_{n-1}}{a_{n-2}}$$

$$\begin{array}{l} x^2 - 4 = 0 \\ (x-2)(x+2) = 0 \\ x_1 = 2 \quad x_2 = -2 \end{array}$$

$$\begin{array}{l} x^2 - 2x + 1 = 0 \\ (x-1)^2 = 0 \\ x_3 = 1 \end{array}$$

$$\begin{array}{l} a_0 + 2a_2 = a_1 \\ a_0 + 6 = c + 0 \\ a_0 + 6 = 2c - 2D \\ a_0 + 6 = 2(c - D) \\ a_0 + 6 = 2 \cdot 1 = 2 \end{array}$$

$$\begin{array}{l} a_1 = 2^0 + (-2)^1 \\ a_1 = 1 - 2 = -1 \end{array}$$

$$\begin{array}{l} a_2 = 2^1 + (-2)^2 \\ a_2 = 2 + 4 = 6 \end{array}$$

$$\begin{array}{l} a_3 = 2^2 + (-2)^3 \\ a_3 = 4 - 8 = -4 \end{array}$$

$$\begin{array}{l} a_4 = 2^3 + (-2)^4 \\ a_4 = 8 + 16 = 24 \end{array}$$

$$\begin{array}{l} a_5 = 2^4 + (-2)^5 \\ a_5 = 16 - 32 = -16 \end{array}$$

$$\begin{array}{l} a_6 = 2^5 + (-2)^6 \\ a_6 = 32 + 64 = 96 \end{array}$$

$$\begin{array}{l} a_7 = 2^6 + (-2)^7 \\ a_7 = 64 - 128 = -64 \end{array}$$

$$\begin{array}{l} a_8 = 2^7 + (-2)^8 \\ a_8 = 128 + 256 = 384 \end{array}$$

$$\begin{array}{l} a_9 = 2^8 + (-2)^9 \\ a_9 = 256 - 512 = -256 \end{array}$$

$$\begin{array}{l} a_{10} = 2^9 + (-2)^{10} \\ a_{10} = 512 + 1024 = 1536 \end{array}$$

$$\begin{array}{l} a_{11} = 2^{10} + (-2)^{11} \\ a_{11} = 1024 - 2048 = -1024 \end{array}$$

$$\begin{array}{l} a_{12} = 2^{11} + (-2)^{12} \\ a_{12} = 2048 + 4096 = 6144 \end{array}$$

$$\begin{array}{l} a_{13} = 2^{12} + (-2)^{13} \\ a_{13} = 4096 - 8192 = -4096 \end{array}$$

$$\begin{array}{l} a_{14} = 2^{13} + (-2)^{14} \\ a_{14} = 8192 + 16384 = 24576 \end{array}$$

$$\begin{array}{l} a_{15} = 2^{14} + (-2)^{15} \\ a_{15} = 16384 - 32768 = -16384 \end{array}$$

$$\begin{array}{l} a_{16} = 2^{15} + (-2)^{16} \\ a_{16} = 32768 + 65536 = 98304 \end{array}$$

$$\begin{array}{l} a_{17} = 2^{16} + (-2)^{17} \\ a_{17} = 65536 - 131072 = -65536 \end{array}$$

$$\begin{array}{l} a_{18} = 2^{17} + (-2)^{18} \\ a_{18} = 131072 + 262144 = 393216 \end{array}$$

$$\begin{array}{l} a_{19} = 2^{18} + (-2)^{19} \\ a_{19} = 262144 - 524288 = -262144 \end{array}$$

$$\begin{array}{l} a_{20} = 2^{19} + (-2)^{20} \\ a_{20} = 524288 + 1048576 = 1572864 \end{array}$$

$$\begin{array}{l} a_{21} = 2^{20} + (-2)^{21} \\ a_{21} = 1048576 - 2097152 = -1048576 \end{array}$$

$$\begin{array}{l} a_{22} = 2^{21} + (-2)^{22} \\ a_{22} = 2097152 + 4194304 = 6291456 \end{array}$$

$$\begin{array}{l} a_{23} = 2^{22} + (-2)^{23} \\ a_{23} = 4194304 - 8388608 = -4194304 \end{array}$$

$$\begin{array}{l} a_{24} = 2^{23} + (-2)^{24} \\ a_{24} = 8388608 + 16777216 = 25165824 \end{array}$$

$$\begin{array}{l} a_{25} = 2^{24} + (-2)^{25} \\ a_{25} = 16777216 - 33554432 = -16777216 \end{array}$$

$$\begin{array}{l} a_{26} = 2^{25} + (-2)^{26} \\ a_{26} = 33554432 + 67108864 = 98663392 \end{array}$$

$$\begin{array}{l} a_{27} = 2^{26} + (-2)^{27} \\ a_{27} = 67108864 - 134217728 = -67108864 \end{array}$$

$$\begin{array}{l} a_{28} = 2^{27} + (-2)^{28} \\ a_{28} = 134217728 + 268435456 = 302653184 \end{array}$$

$$\begin{array}{l} a_{29} = 2^{28} + (-2)^{29} \\ a_{29} = 268435456 - 536870912 = -268435456 \end{array}$$

$$\begin{array}{l} a_{30} = 2^{29} + (-2)^{30} \\ a_{30} = 536870912 + 1073741824 = 1610612736 \end{array}$$

$$\begin{array}{l} a_{31} = 2^{30} + (-2)^{31} \\ a_{31} = 1073741824 - 2147483648 = -1073741824 \end{array}$$

$$\begin{array}{l} a_{32} = 2^{31} + (-2)^{32} \\ a_{32} = 2147483648 + 4294967296 = 6442450944 \end{array}$$

$$\begin{array}{l} a_{33} = 2^{32} + (-2)^{33} \\ a_{33} = 4294967296 - 8589934592 = -4294967296 \end{array}$$

$$\begin{array}{l} a_{34} = 2^{33} + (-2)^{34} \\ a_{34} = 8589934592 + 17179869184 = 25769733776 \end{array}$$

$$\begin{array}{l} a_{35} = 2^{34} + (-2)^{35} \\ a_{35} = 17179869184 - 34359738368 = -17179869184 \end{array}$$

$$\begin{array}{l} a_{36} = 2^{35} + (-2)^{36} \\ a_{36} = 34359738368 + 68719476736 = 103079215104 \end{array}$$

$$\begin{array}{l} a_{37} = 2^{36} + (-2)^{37} \\ a_{37} = 68719476736 - 137438953472 = -68719476736 \end{array}$$

$$\begin{array}{l} a_{38} = 2^{37} + (-2)^{38} \\ a_{38} = 137438953472 + 274877906944 = 412316850416 \end{array}$$

$$\begin{array}{l} a_{39} = 2^{38} + (-2)^{39} \\ a_{39} = 274877906944 - 549755813888 = -274877906944 \end{array}$$

$$\begin{array}{l} a_{40} = 2^{39} + (-2)^{40} \\ a_{40} = 549755813888 + 1099511627776 = 1649267441664 \end{array}$$

$$\begin{array}{l} a_{41} = 2^{40} + (-2)^{41} \\ a_{41} = 1099511627776 - 2199023255552 = -1099511627776 \end{array}$$

$$\begin{array}{l} a_{42} = 2^{41} + (-2)^{42} \\ a_{42} = 2199023255552 + 4398046511104 = 6597069766656 \end{array}$$

$$\begin{array}{l} a_{43} = 2^{42} + (-2)^{43} \\ a_{43} = 4398046511104 - 8796093022208 = -4398046511104 \end{array}$$

$$\begin{array}{l} a_{44} = 2^{43} + (-2)^{44} \\ a_{44} = 8796093022208 + 17592186044416 = 26388279066624 \end{array}$$

$$\begin{array}{l} a_{45} = 2^{44} + (-2)^{45} \\ a_{45} = 17592186044416 - 35184372088832 = -17592186044416 \end{array}$$

$$\begin{array}{l} a_{46} = 2^{45} + (-2)^{46} \\ a_{46} = 35184372088832 + 70368744177664 = 105553116266496 \end{array}$$

$$\begin{array}{l} a_{47} = 2^{46} + (-2)^{47} \\ a_{47} = 70368744177664 - 140737488355328 = -70368744177664 \end{array}$$

$$\begin{array}{l} a_{48} = 2^{47} + (-2)^{48} \\ a_{48} = 140737488355328 + 281474976710656 = 422212465065984 \end{array}$$

$$\begin{array}{l} a_{49} = 2^{48} + (-2)^{49} \\ a_{49} = 281474976710656 - 562949953421312 = -281474976710656 \end{array}$$

$$\begin{array}{l} a_{50} = 2^{49} + (-2)^{50} \\ a_{50} = 562949953421312 + 1125899906842624 = 1688809850263936 \end{array}$$

$$\begin{array}{l} a_{51} = 2^{50} + (-2)^{51} \\ a_{51} = 1125899906842624 - 2251799813685248 = -1125899906842624 \end{array}$$

$$\begin{array}{l} a_{52} = 2^{51} + (-2)^{52} \\ a_{52} = 2251799813685248 + 4503599627370496 = 6755399438055744 \end{array}$$

$$\begin{array}{l} a_{53} = 2^{52} + (-2)^{53} \\ a_{53} = 4503599627370496 - 9007199254740992 = -4503599627370496 \end{array}$$

$$\begin{array}{l} a_{54} = 2^{53} + (-2)^{54} \\ a_{54} = 9007199254740992 + 18014398509481984 = 27028797018962976 \end{array}$$

$$\begin{array}{l} a_{55} = 2^{54} + (-2)^{55} \\ a_{55} = 18014398509481984 - 36028797018962976 = -18014398509481984 \end{array}$$

$$\begin{array}{l} a_{56} = 2^{55} + (-2)^{56} \\ a_{56} = 36028797018962976 + 72057594037925952 = 108185591075888720 \end{array}$$

$$\begin{array}{l} a_{57} = 2^{56} + (-2)^{57} \\ a_{57} = 72057594037925952 - 144115188075777504 = -72057594037925952 \end{array}$$

$$\begin{array}{l} a_{58} = 2^{57} + (-2)^{58} \\ a_{58} = 144115188075777504 + 288230376151555008 = 432345552303132512 \end{array}$$

$$\begin{array}{l} a_{59} = 2^{58} + (-2)^{59} \\ a_{59} = 288230376151555008 - 576460752303132512 = -288230376151555008 \end{array}$$

$$\begin{array}{l} a_{60} = 2^{59} + (-2)^{60} \\ a_{60} = 576460752303132512 + 1152921504606265024 = 1729382256909397536 \end{array}$$

$$\begin{array}{l} a_{61} = 2^{60} + (-2)^{61} \\ a_{61} = 1152921504606265024 - 2305843009212530048 = -1152921504606265024 \end{array}$$

$$\begin{array}{l} a_{62} = 2^{61} + (-2)^{62} \\ a_{62} = 2305843009212530048 + 4611686018425060096 = 6923332027837590144 \end{array}$$

$$\begin{array}{l} a_{63} = 2^{62} + (-2)^{63} \\ a_{63} = 4611686018425060096 - 9243362036850180192 = -4611686018425060096 \end{array}$$

$$\begin{array}{l} a_{64} = 2^{63} + (-2)^{64} \\ a_{64} = 9243362036850180192 + 18486724073700360384 = 27969488147400720576 \end{array}$$

$$\begin{array}{l} a_{65} = 2^{64} + (-2)^{65} \\ a_{65} = 18486724073700360384 - 36933448147400720576 = -18486724073700360384 \end{array}$$

$$\begin{array}{l} a_{66} = 2^{65} + (-2)^{66} \\ a_{66} = 36933448147400720576 + 73866896294801441152 = 107790384439202161728 \end{array}$$

$$\begin{array}{l} a_{67} = 2^{66} + (-2)^{67} \\ a_{67} = 73866896294801441152 - 147533792589602282304 = -73866896294801441152 \end{array}$$

$$\begin{array}{l} a_{68} = 2^{67} + (-2)^{68} \\ a_{68} = 147533792589602282304 + 295067585179204564608 = 442595177758806846912 \end{array}$$

$$\begin{array}{l} a_{69} = 2^{68} + (-2)^{69} \\ a_{69} = 295067585179204564608 - 590135170357607733216 = -295067585179204564608 \end{array}$$

$$\begin{array}{l} a_{70} = 2^{69} + (-2)^{70} \\ a_{70} = 590135170357607733216 + 1180270340715215466432 = 1770365511073123199552 \end{array}$$

$$\begin{array}{l} a_{71} = 2^{70} + (-2)^{71} \\ a_{71} = 1180270340715215466432 - 236054068142782723264 = -1180270340715215466432 \end{array}$$

$$\begin{array}{l} a_{72} = 2^{71} + (-2)^{72} \\ a_{72} = 236054068142782723264 + 472108136285565446528 = 708162204430121169792 \end{array}$$

$$\begin{array}{l} a_{73} = 2^{72} + (-2)^{73} \\ a_{73} = 472108136285565446528 - 944216472871130933056 = -472108136285565446528 \end{array}$$

$$\begin{array}{l} a_{74} = 2^{73} + (-2)^{74} \\ a_{74} = 944216472871130933056 + 1888432945742261866112 = 2832649418513392799168 \end{array}$$

$$\begin{array}{l} a_{75} = 2^{74} + (-2)^{75} \\ a_{75} = 1888432945742261866112 - 3776865891484434732224 = -1888432945742261866112 \end{array}$$

$$\begin{array}{l} a_{76} = 2^{75} + (-2)^{76} \\ a_{76} = 3776865891484434732224 + 7553731782968869464448 = 11307463575933213996720 \end{array}$$

$$\begin{array}{l} a_{77} = 2^{76} + (-2)^{77} \\ a_{77} = 7553731782968869464448 - 15107463575933213996720 = -7553731782968869464448 \end{array}$$

$$\begin{array}{l} a_{78} = 2^{77} + (-2)^{78} \\ a_{78} = 15107463575933213996720 + 30214927151866427993440 = 45322386727799431989840 \end{array}$$

$$\begin{array}{l} a_{79} = 2^{78} + (-2)^{79} \\ a_{79} = 30214927151866427993440 - 60449854303732855978880 = -30214927151866427993440 \end{array}$$

$$\begin{array}{l} a_{80} = 2^{79} + (-2)^{80} \\ a_{80} = 60449854303732855978880 + 120899708607465711957760 = 181799417211231467936560 \end{array}$$

$$\begin{array}{l} a_{81} = 2^{80} + (-2)^{81} \\ a_{81} = 120899708607465711957760 - 241599121212432823915520 = -120899708607465711957760 \end{array}$$

$$\begin{array}{l} a_{82} = 2^{81} + (-2)^{82} \\ a_{82} = 241599121212432823915520 + 483198242424865647831040 = 724796363649731295662560 \end{array}$$

$$\begin{array}{l} a_{83} = 2^{82} + (-2)^{83} \\ a_{83} = 483198242424865647831040 - 966396684849462591262560 = -483198242424865647831040 \end{array}$$

$$\begin{array}{l} a_{84} = 2^{83} + (-2)^{84} \\ a_{84} = 966396684849462591262560 + 1932793369698925182525120 = 2899586739397850364545280 \end{array}$$

$$\begin{array}{l} a_{85} = 2^{84} + (-2)^{85} \\ a_{85} = 1932793369698925182525120 - 3869586739397850364545280 = -1932793369698925182525120 \end{array}$$

$$\begin{array}{l} a_{86} = 2^{85} + (-2)^{86} \\ a_{86} = 3869586739397850364545280 + 7739173478795700729090560 = 11578950218193551093635360 \end{array}$$

$$\begin{array}{l} a_{87} = 2^{86} + (-2)^{87} \\ a_{87} = 7739173478795700729090560 - 15157950218193551093635360 = -7739173478795700729090560 \end{array}$$

$$\begin{array}{l} a_{88} = 2^{87} + (-2)^{88} \\ a_{88} = 15157950218193551093635360 + 30315900436387102187270720 = 45474850654574703280362080 \end{array}$$

$$\begin{array}{l} a_{89} = 2^{88} + (-2)^{89} \\ a_{89} = 30315900436387102187270720 - 60749701309149406560724160 = -30315900436387102187270720 \end{array}$$

$$\begin{array}{l} a_{90} = 2^{89} + (-2)^{90} \\ a_{90} = 60749701309149406560724160 + 121499402618298813121448320 = 18309870392744862948216640 \end{array}$$

$$\begin{array}{l} a_{91} = 2^{90} + (-2)^{91} \\ a_{91} = 121499402618298813121448320 - 24219870392744862948216640 = -121499402618298813121448320 \end{array}$$

$$\begin{array}{l} a_{92} = 2^{91} + (-2)^{92} \\ a_{92} = 24219870392744862948216640 + 48439740785489765896433280 = 7265951157823432584566640 \end{array}$$

$$\begin{array}{l} a_{93} = 2^{92} + (-2)^{93} \\ a_{93} = 48439740785489765896433280 - 9687951157823432584566640 = -48439740785489765896433280 \end{array}$$

$$\begin{array}{l} a_{94} = 2^{93} + (-2)^{94} \\ a_{94} = 9687951157823432584566640 + 19375902315646865179133280 = 28751902315646865179133280 \end{array}$$

$$\begin{array}{l} a_{95} = 2^{94} + (-2)^{95} \\ a_{95} = 19375902315646865179133280 - 38751902315646865179133280 = -19375902315646865179133280 \end{array}$$

$$\begin{array}{l} a_{96} = 2^{95} + (-2)^{96} \\ a_{96} = 38751902315646865179133280 + 7750380463129373035826640 = 11600760463129373035826640 \end{array}$$

$$\begin{array}{l} a_{97} = 2^{96} + (-2)^{97} \\ a_{97} = 7750380463129373035826640 - 15300760463129373035826640 = -7750380463129373035826640 \end{array}$$

$$\begin{array}{l} a_{98} = 2^{97} + (-2)^{98} \\ a_{98} = 15300760463129373035826640 + 30601520926258746071653280 = 46902280926258746071653280 \end{array}$$

$$\begin{array}{l} a_{99} = 2^{98} + (-2)^{99} \\ a_{99} = 30601520926258746071653280 - 61203160926258746071653280 = -30601520926258746071653280 \end{array}$$

$$\begin{array}{l} a_{100} = 2^{99} + (-2)^{100} \\ a_{100} = 61203160926258746071653280 + 12240632185251749214330560 = 18480948277477498428664640 \end{array}$$

$$\begin{array}{l} a_{101} = 2^{100} + (-2)^{101} \\ a_{101} = 12240632185251749214330560 - 24481264370503498428664640 = -12240632185251749214330560 \end{array}$$

$$\begin{array}{l} a_{102} = 2^{101} + (-2)^{102} \\ a_{102} = 24481264370503498428664640 + 48962528741006996857329120 = 73923792741006996857329120 \end{array}$$

$$\begin{array}{l} a_{103} = 2^{102} + (-2)^{103} \\ a_{103} = 48962528741006996857329120 - 97925585482013993714658240 = -48962528741006996857329120 \end{array}$$

$$\begin{array}{l} a_{104} = 2^{103} + (-2)^{104} \\ a_{104} = 97925585482013993714658240 + 195851170964027987429316480 = 291702351928055974858632640 \end{array}$$

$$\begin{array}{l} a_{105} = 2^{104} + (-2)^{105} \\ a_{105} = 195851170964027987429316480 - 391702351928055974858632640 = -195851170964027987429316480 \end{array}$$

$$\begin{array}{l} a_{106} = 2^{105} + (-2)^{106} \\ a_{106} = 391702351928055974858632640 + 783404703856111949717265280 = 1176809407712223949717265280 \end{array}$$

$$\begin{array}{l} a_{107} = 2^{106} + (-2)^{107} \\ a_{107} = 783404703856111949717265280 - 1553609407712223949717265280 = -783404703856111949717265280 \end{array}$$

$$\begin{array}{l} a_{108} = 2^{107} + (-2)^{108} \\ a_{108} = 1553609407712223949717265280 + 3107218815434447899434530560 = 4661228815434447899434530560 \end{array}$$

$$\begin{array}{l} a_{109} = 2^{108} + (-2)^{109} \\ a_{109} = 3107218815434447899434530560 - 6214437630868895798869061120 = -3107218815434447899434530560 \end{array}$$

$$\begin{array}{l} a_{110} = 2^{109} + (-2)^{110} \\ a_{110} = 6214437630868895798869061120 + 12428875261737791597738122240 = 18647750523505583195576244480 \end{array}$$

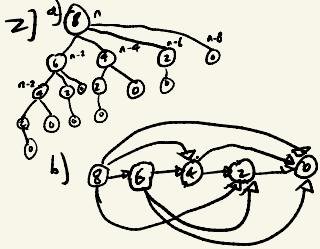
$$\begin{array}{l} a_{111} = 2^{110} + (-2)^{111} \\ a_{111} = 12428875261737791597738122240 - 2485750523505583195576244480 = -12428875261737791597738122240 \end{array}$$

$$\begin{array}{l} a_{112} = 2^{111} + (-2)^{112} \\ a_{112} = 248575052350$$

Week 10/11



CGG



3) a)

b)  $C(5, 3)$

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1, 0 & 1, 1 & 1, 2 & 1, 3 \\ 2 & 2, 0 & 2, 1 & 2, 2 & 2, 3 \\ 3 & 3, 1 & 3, 2 & 3, 3 \\ 4 & 4, 2 & 4, 3 \\ 5 & 5, 3 \end{matrix}$$

4)  $A \quad B \quad C$

$$\begin{aligned} 100 \times 1 & \quad 1 \times 100 \quad 100 \times 1 \\ A(BC) & = 1 \times 100 \times 1 + 100 \times 1 \times 1 \\ & = 100 + 100 = 200 \\ (AB)C & = 100 \times 1 \times 100 + 100 \times 100 \times 1 \\ & = 10000 + 10000 = 20000 \end{aligned}$$

5) d  $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 20 & 2 & 15 & 40 & 4 \end{matrix}$

	0	1	2	3	4	last
cost	0	1	2	3	4	
0	0	300	2800	1600	0	
1	0	1200	1520	1		
2	0	2400	2			
3	0	3				
4	0	4				

$$\begin{aligned} 12 & 24 = 0 + 2400 + 120 \\ 13 & 74 = 1200 + 0 + 320 \end{aligned}$$

6)  $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 20 & 2 & 15 & 40 & 4 \end{matrix}$

0	1	2	3	4
20	2	15	40	4
0	1	2	3	4
0	0	600	2800	1600
1	0	1200	1520	
2	0	2400		

$$(10, 0) \leftarrow (10, 1) \leftarrow (10, 2) \leftarrow (10, 3) \leftarrow (10, 4)$$

0	1	2	3	4
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	50
4	0	0	40	50
5	0	10	40	50
6	0	10	40	50
7	0	10	40	90
8	0	10	40	90
9	0	10	50	90
10	0	10	50	90

0	1	1
0	2	3
0	3	0

$$\begin{aligned} 0112 &= 600 \\ 1223 &= 1200 \\ 2334 &= 2400 \\ 0113 &= 1600 + 0 + 1200 \\ 0223 &= 960 \\ 1224 &= 1200 + 0 + 2400 \\ 1334 &= 1200 + 1200 + 1600 \\ 0114 &= 0 + 1520 + 160 \\ 0224 &= 600 + 1400 + 1600 \\ 0334 &= 2800 + 0 + 3200 \end{aligned}$$

↑  
Check  
↓  
maximal handle

7)



week 12

1)

3)  $P = AAA \dots AB \dots m-1$

$$m=0 \quad m=1 \quad m=m-2$$

$$\text{slide} = 1 \quad \text{slide} = m \quad \text{slide} = m+n-2$$

$$\begin{matrix} x & 1 \\ \downarrow & \end{matrix} \quad \begin{matrix} m+1 \\ \dots \\ m+2 \\ \dots \\ 2m-2 \\ \dots \\ 2m-1 \end{matrix}$$

2)

1)	2m-1	2m-2	... m+2	m+1	1
----	------	------	---------	-----	---

$$T = "AAA \dots A" \quad n$$

$$\begin{matrix} w \\ \dots \\ A \\ \dots \\ A \end{matrix}$$

$$l=1 \quad \begin{matrix} A=1 \\ B=0 \\ X=n \end{matrix}$$

$$\begin{matrix} n-(m-1) \\ n-m+1 \end{matrix}$$

4) i) BANANA

$$\begin{matrix} m=0 & m=1 & m=2 & m=3 & m=4 \\ S=1 & S=6-2=4 & S=6 & S=6-4+2 & S=6 \\ mJ=1 & mJ=5 & mJ=8 & mJ=5 & mJ=10 \end{matrix}$$

11	10	5	8	5	1
----	----	---	---	---	---

2) POTATO

$$\begin{matrix} m=1 & m=2 & m=3 & m=4 & m=5 \\ S=6-2=4 & S=6 & S=6-4+2 & S=6 & S=6 \\ mJ=5 & mJ=8 & mJ=9 & mJ=6 & mJ=11 \end{matrix}$$

11	10	9	8	5	1
----	----	---	---	---	---

week 13

1) P

R A T S C A T S

$m = 10$      $m = 1$      $m = 2$      $m = 3$   
slide = 1    slide = 8    slide = 8    slide = 8 - 4 = 4  
 $m_J = 1$      $m_J = 9$      $m_J = 10$      $m_J = 7$

$m = 4$      $m = 5$      $m = 6$      $m = 7$   
slide = 8    slide = 8    slide = 8    slide = 8  
 $m_J = 12$      $m_J = 13$     14    15

2020-2021 S2

2) a)  $w(1) = 0$

$$w(n) = 2w(n-1) + 2$$

$$w(n) \leq 2(2w(n-2) + 2) + 2$$

$$\leq 2^2 w(n-2) + 2^2 + 2$$

$$\leq 2^2(2w(n-3) + 2) + 2^2 + 2$$

$$\leq 2^3 w(n-3) + 2^3 + 2^2 + 2$$

$$\leq 2^k w(n-k) + 2^k + 2^{k-1} + 2$$

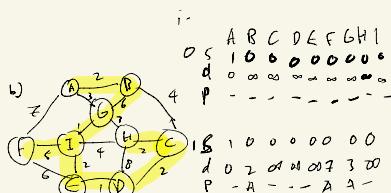
$$\leq 2^{n-1} w(1) + 2^{n-1} + 2^{n-2} + \dots + 2$$

$$\leq 2(2^{n-2} + 2^{n-3} + \dots + 1) - 1$$

$$= 2^{n-1} - 1 \leq 2^n$$

$$n-k=1$$

$$k=n-1$$



$$\boxed{1} \quad 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$\boxed{2} \quad 2 \ 4 \ 1 \ 2 \ 5 \ 3 \ 3 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

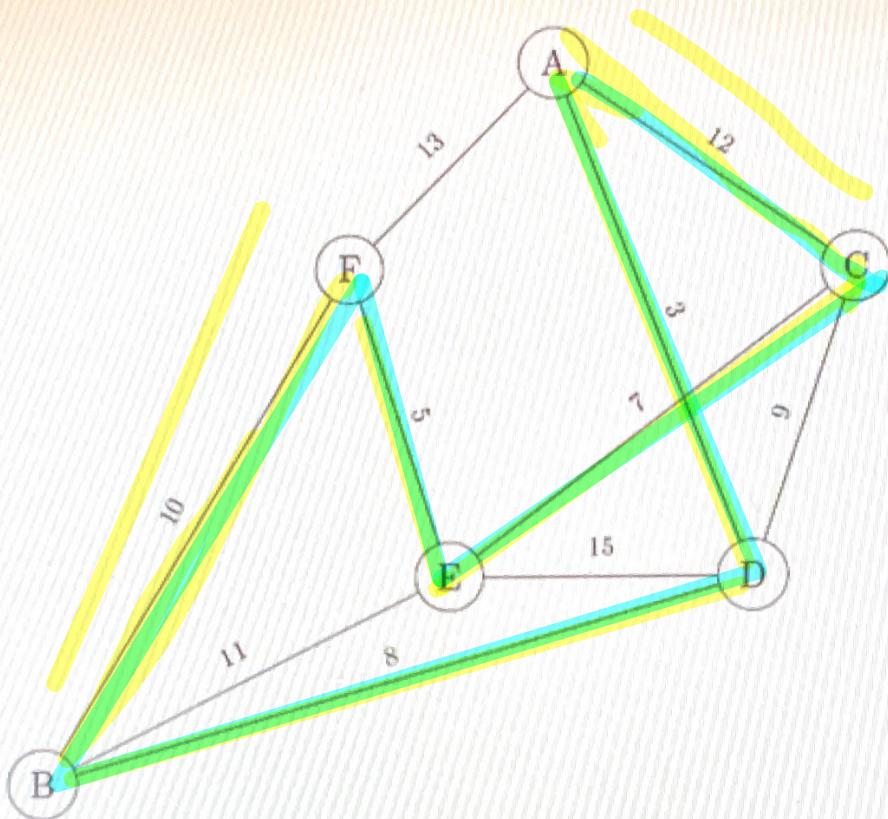
$$\boxed{3} \quad P \quad A \ B \ C \ I \ A \ G \ G \quad P \quad A \ B \quad A \ A$$

$$\boxed{4} \quad P \quad A \ B \ C \ I \ A \ G \ G \quad P \quad A \ B \quad A \ A \ G \ G$$

$$\boxed{5} \quad P \quad A \ B \ C \ I \ A \ C \ G \quad P \quad A \ B \quad I \ I \ A \ G \ G$$

$$\boxed{6} \quad P \quad A \ B \ C \ I \ A \ C \ G \quad P \quad A \ B \quad I \ I \ A \ C \ G$$

4. (a) The Travelling Salesman Problem for the cities is given in Figure Q4.



**Figure Q4**

- (i) Show the total distance and the sequences of the edges of the graph chosen by using the Shortest-link algorithm. (5 marks)
- (ii) Show the total distance and the sequences of the edges of the graph chosen by using the Nearest Neighbour algorithm, starting from node A. (5 marks)

19/20 S1

i) a) i)  $\log_2 N \cdot \log_2 N = O(\log_2 N^2)$

ii)  $T(n) = 2T(n-1) + N$   
 $O(2^n)$

b)

