CZ3005 Artificial Intelligence

Week 12a – Fuzzy Logic

Yu Han

han.yu@ntu.edu.sg

Nanyang Assistant Professor
School of Computer Science and Engineering
Nanyang Technological University

Learning Goals

Understanding the:

- Basic definitions and terminology
- Set-theoretic operations
- Membership Function (MF) formulation
  - MFs parameterization
  - Linguistic modifier/hedges

Example: Safe Autonomous Vehicles

- Autonomous Cars implement Duty of Care
  - an individual should exercise “reasonable care” while performing acts that could harm others

- “On a Formal Model of Safe and Scalable Self-driving Cars”, by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374
  - Responsibility Sensitive Safety – mathematical safety assurance
  - System design that adheres to the mathematical model

Example: Safe Autonomous Vehicles

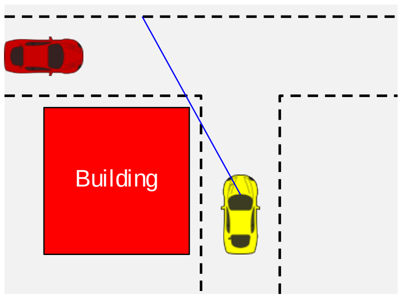
- Responsibility Sensitive Safety (RSS)
  - Do not hit someone from behind
  - Do not cut-in recklessly
  - Right-of-way is given, not taken
  - Be careful of areas with limited visibility
  - If you can avoid an accident without causing another one, you must do so

Example: Safe Autonomous Vehicles

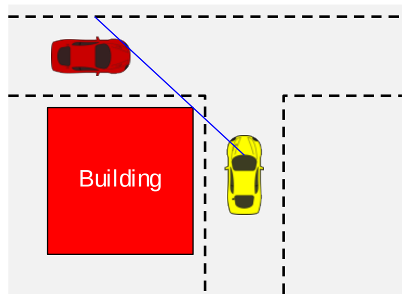
- Responsibility Sensitive Safety (RSS)
  - Do not hit someone from behind
    - Even if not your fault?
  - Do not cut-in recklessly
  - Right-of-way is given, not taken
    - How to resolve polite deadlocks?
  - Be careful of areas with limited visibility
  - If you can avoid an accident without causing another one, you must do so
    - Emergency breaking can cause whiplash

Example: Safe Autonomous Vehicles

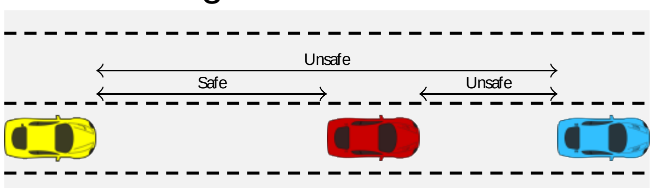
Danger Threshold time



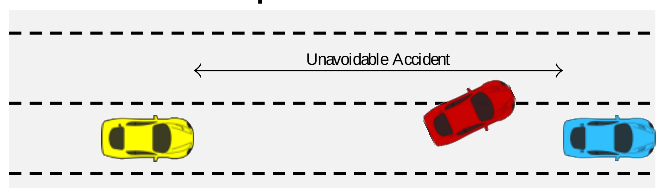
Exposure Time



Danger Threshold time



Exposure Time



"On a Formal Model of Safe and Scalable Self-driving Cars",
by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374

Example: Safe Autonomous Vehicles

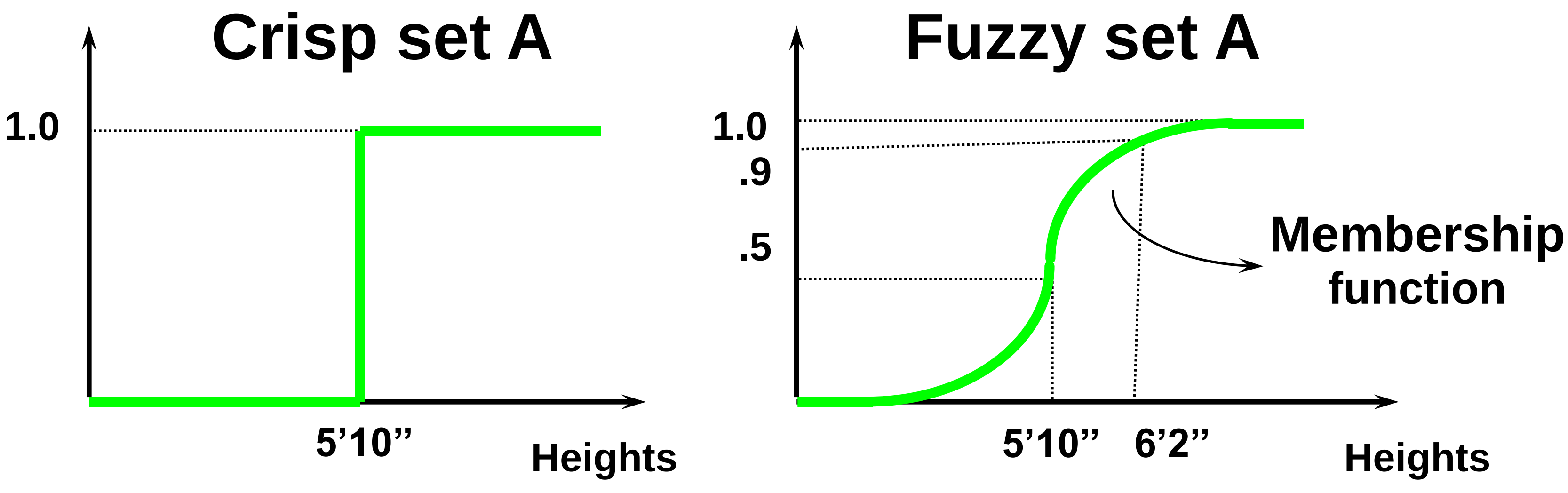
- Use of Semantic Action Space
  - Not "drive for 5.33 kilometers, then reduce speed at the rate of 1 m/s^2"
  - Slow down as you approach red light to stop at the line.
    - IF approach red light, THEN slow down and stop

"On a Formal Model of Safe and Scalable Self-driving Cars", by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374

Fuzzy Sets

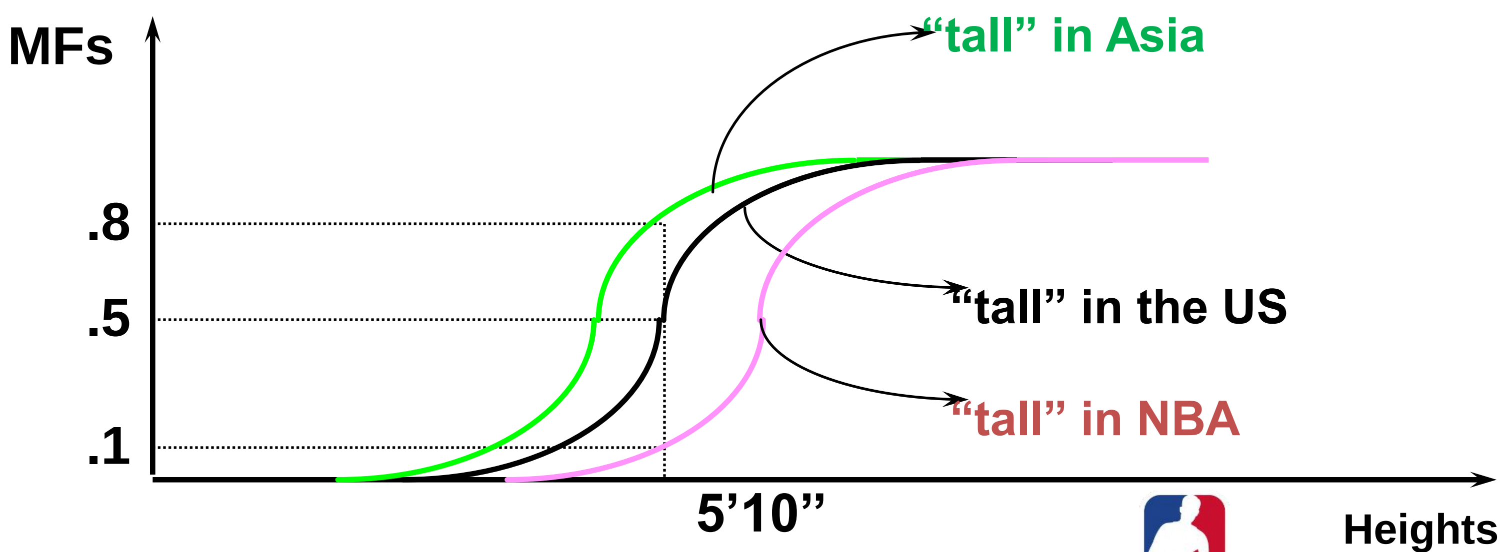
- Sets with fuzzy boundaries

A = Set of tall people



Membership Functions (MFs)

- Characteristics of MFs:
  - Subjective measures
  - Not probability functions



Fuzzy Sets

Formal definition:

A fuzzy set *A* in *X* is expressed as a set of ordered pairs:

$$A = \{(\overbrace{x}^{\text{Crisp input}}, \underbrace{\mu_A(x)}_{\text{Membership function (MF)}}) \mid x \in X\}$$

Fuzzy set

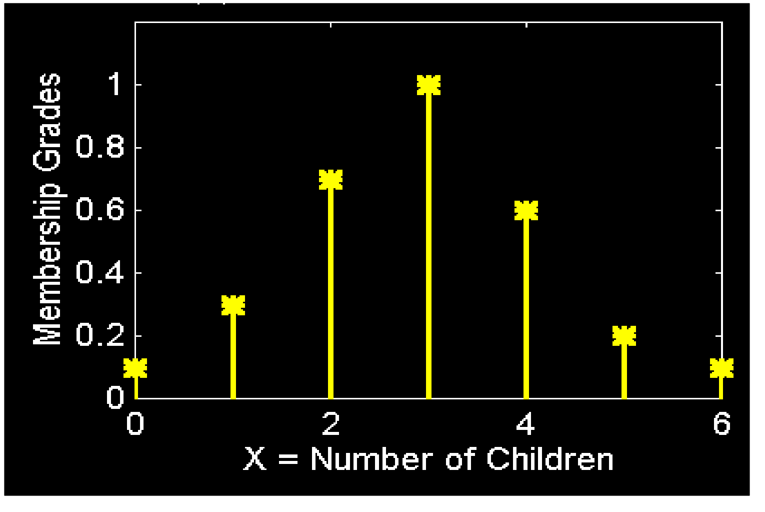
Universe or universe of discourse

A fuzzy set is totally characterized by a membership function (MF).

Fuzzy Sets – Discrete Universes

- Fuzzy set C = "desirable city to live in"
  X = {SF, Boston, LA} (discrete and non-ordered)
  C = {(SF, 0.9), (Boston, 0.8), (LA, 0.6)}

- Fuzzy set A = "sensible number of children to have"
  X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
  A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}



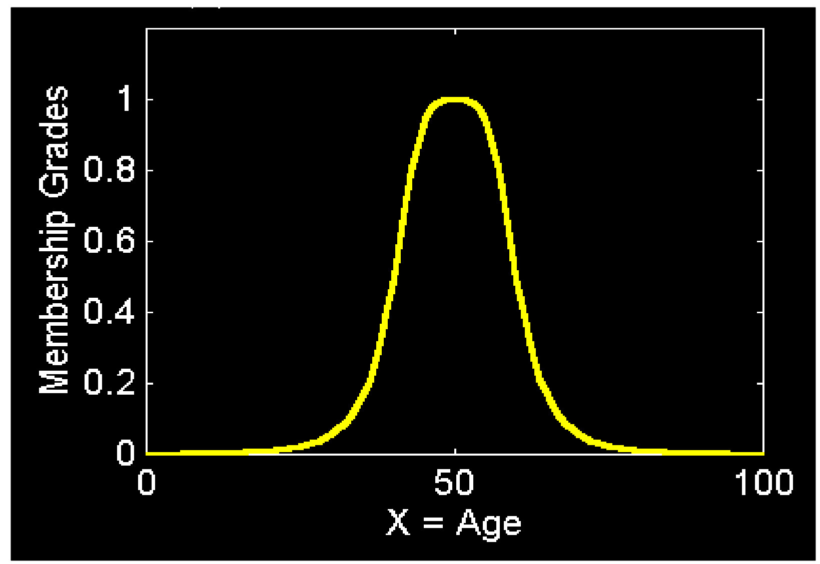
Fuzzy Sets – Continuous Universes

- Fuzzy set B = "about 50 years old"

  X = Set of positive real numbers (continuous)

  B = {(x, μ_B(x)) | x in X}

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x - 50}{10}\right)^2}$$



Alternative Notation

A fuzzy set A can be alternatively denoted as follows:

X is discrete ⇨ $$A = \sum_{x_i \in X} \mu_A(x_i) / x_i$$

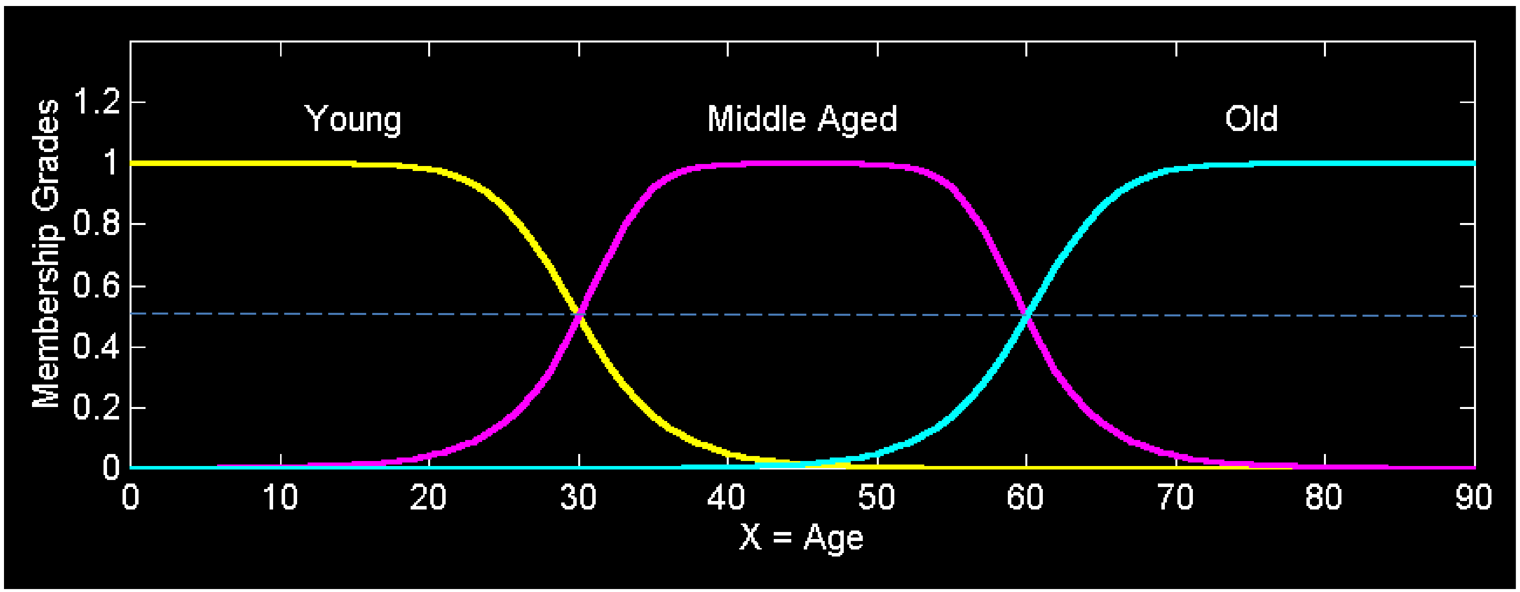
X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}

X is continuous ⇨ $$A = \int_X \mu_A(x) / x$$

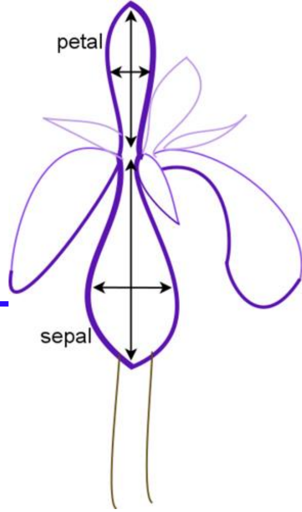
Note that Σ and integral signs stand for the union of membership grades;
"/" stands for a marker and does not imply division.

Fuzzy Partition

- Fuzzy partitions formed by the linguistic values "young", "middle aged", and "old":



Non-Pseudo Partitioning



Iris flower

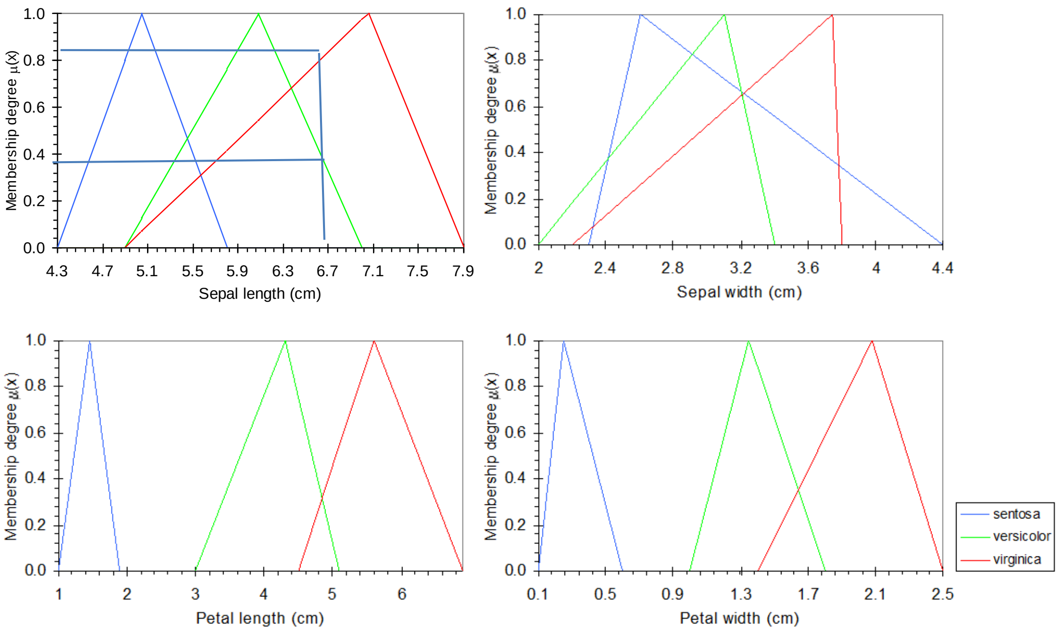
- Let c the set of membership functions that fuzzy partition the space of x.
- This fuzzy space is non-Pseudoly partitioned when:

Each MF value is normal and convex

$$\sup_{x}(\mu_{i,i\in c}(X))=1$$

Summation of MF values at X is NOT 1

$$\sum_{i=1}^{c}\mu_{i,i\in c}(X))\neq 1$$



Pseudo Partitioning

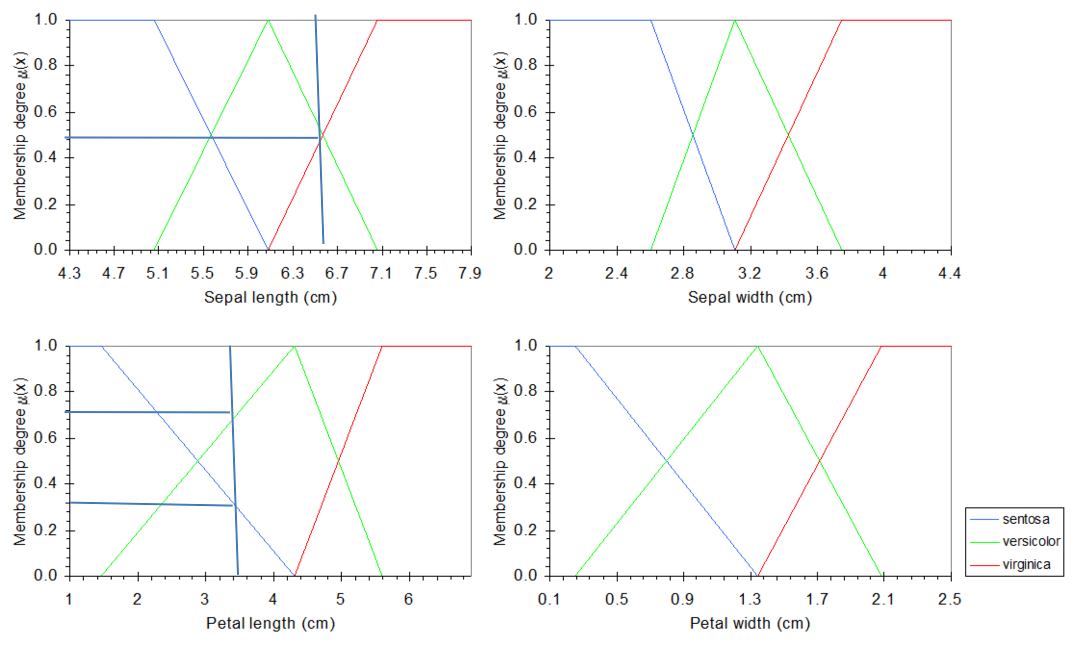
- Let *c* be the set of membership functions that fuzzy partition the space of *x*.
- This fuzzy space is Pseudoly partitioned when:

Each MF value is normal and convex

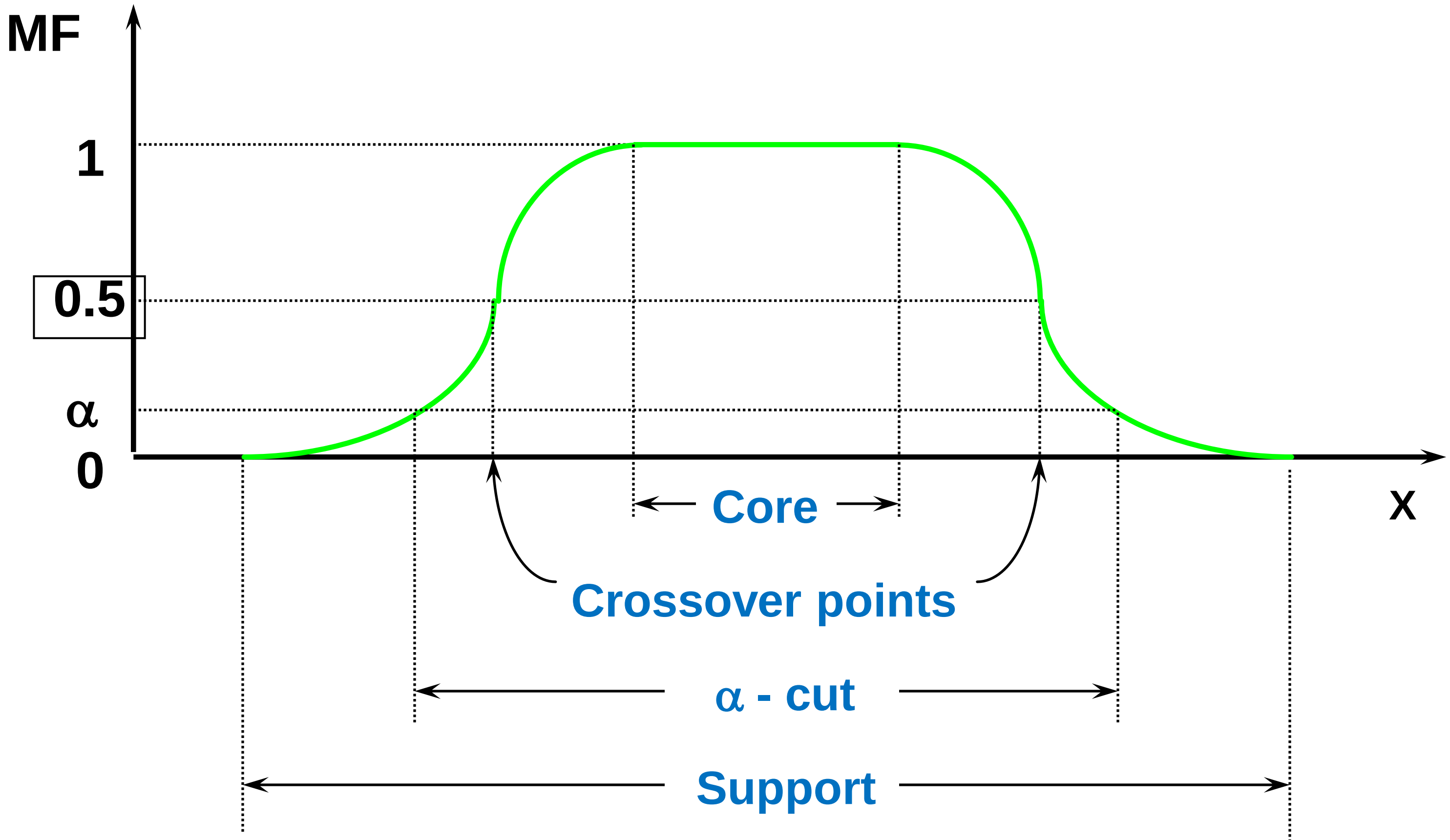
$$\sup_{x}(\mu_{i,i\in c}(X)) = 1$$

Summation of MF values at *X* is 1

$$\sum_{i=1}^{c} \mu_{i,i\in c}(X)) = 1$$



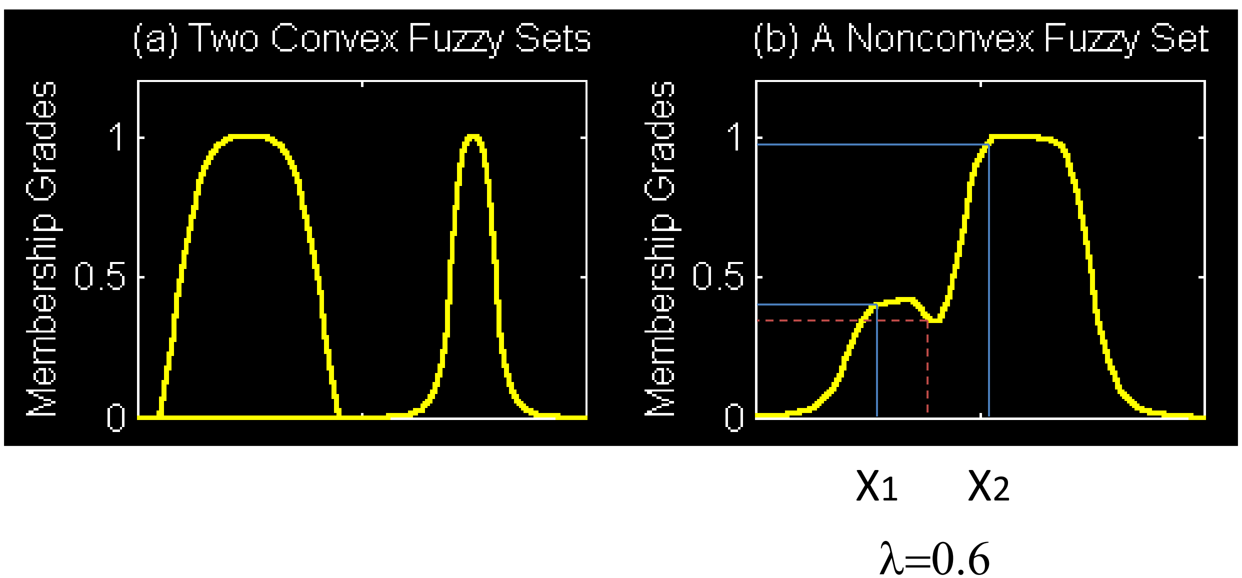
MF Terminology



Convexity of Fuzzy Sets

A fuzzy set *A* is convex if for any λ within [0, 1]:

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$



Alternatively, *A* is convex if all its *α–cuts* are convex.

Set-Theoretic Operations

- Subset:

$$A \subseteq B \Leftrightarrow \mu_A \leq \mu_B$$

- Complement:

$$\overline{A} = X - A \Leftrightarrow \mu_{\overline{A}}(x) = 1 - \mu_A(x)$$

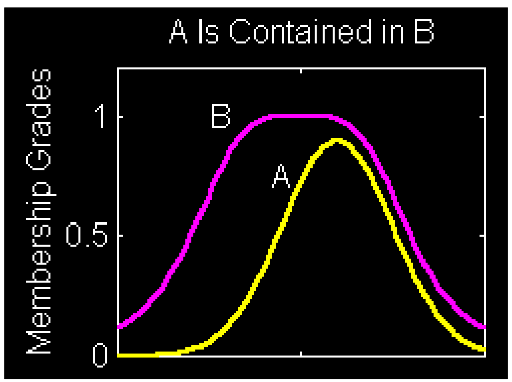
- Union: (OR - Disjunction)

$$C = A \cup B \Leftrightarrow \mu_c(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$$

- Intersection: (AND – Conjunction)

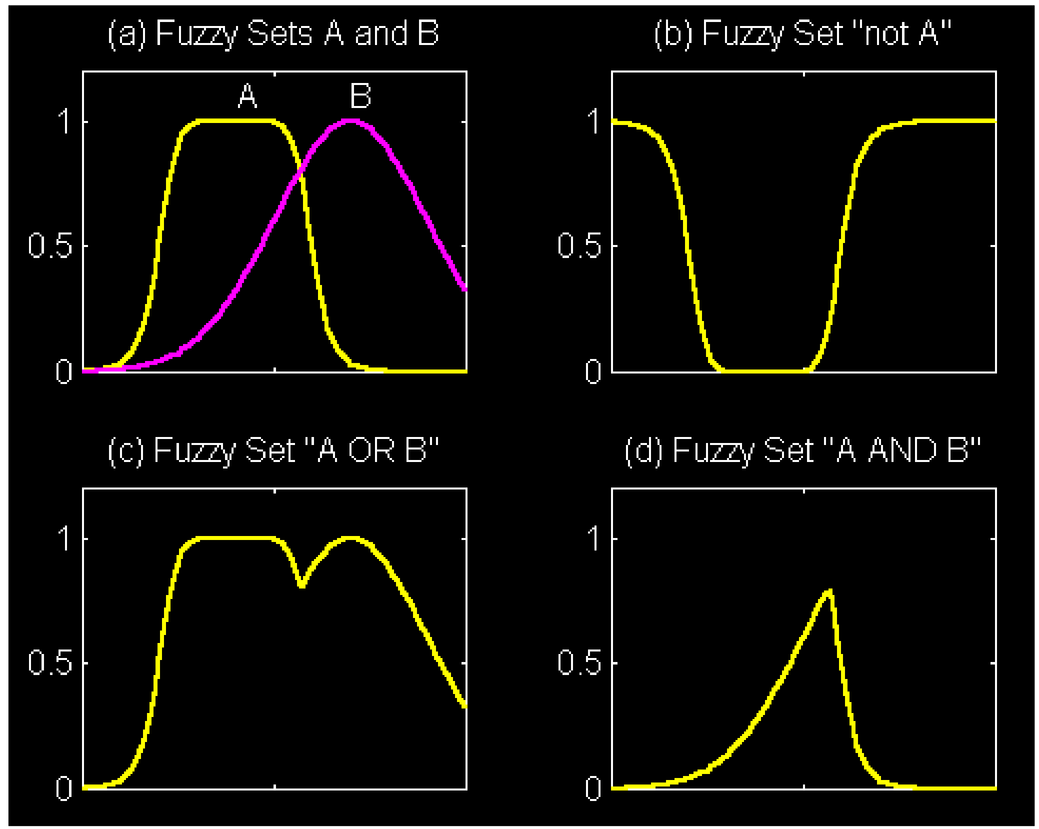
$$C = A \cap B \Leftrightarrow \mu_c(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

Set-Theoretic Operations



$\mu_A \leq \mu_B$

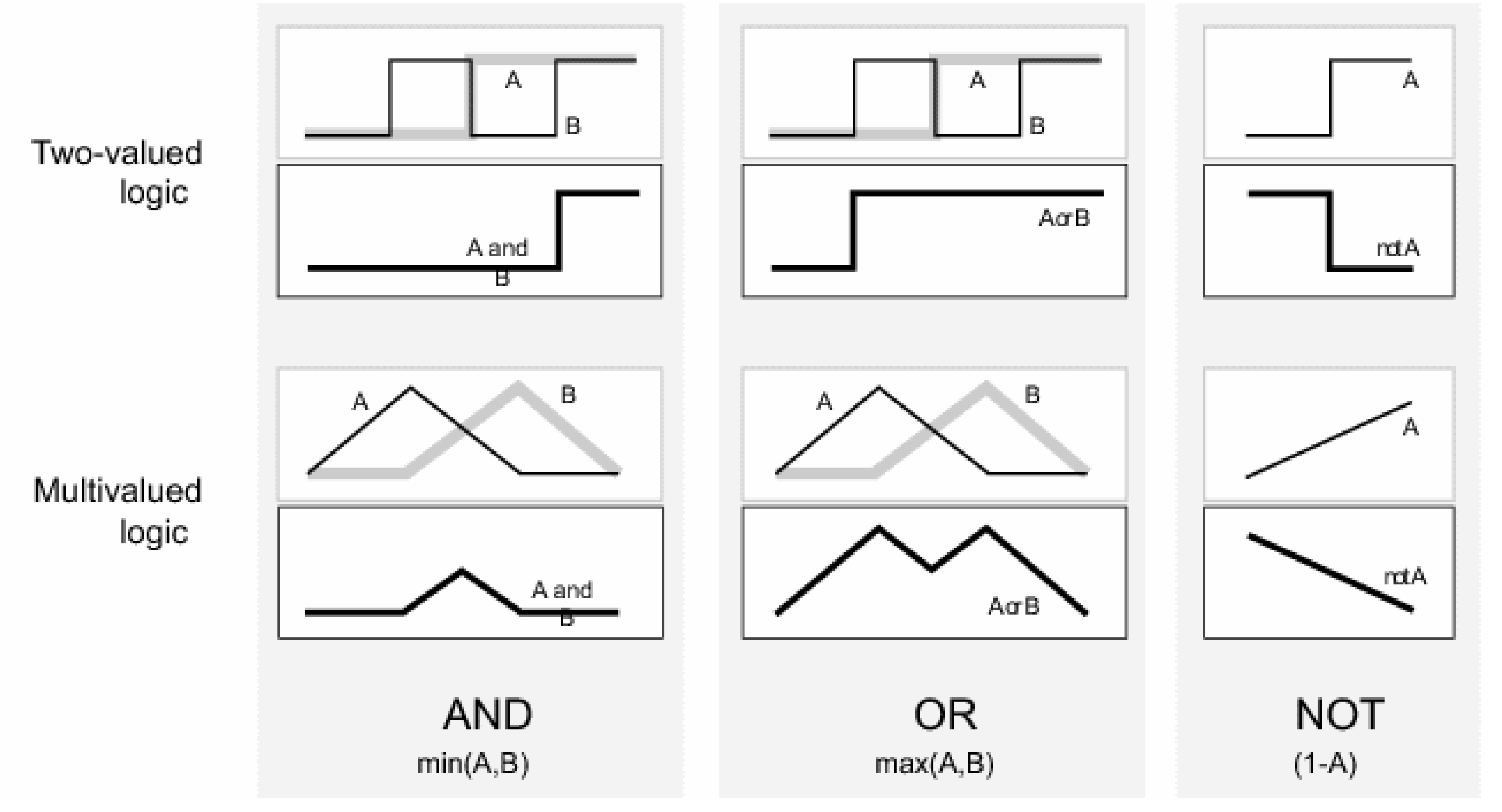
$1 - \mu_A(x)$



$\max(\mu_A(x), \mu_B(x))$

$\min(\mu_A(x), \mu_B(x))$

Fuzzy Logical Operation



T-norm S-norm/T-conorm

MF Formulation

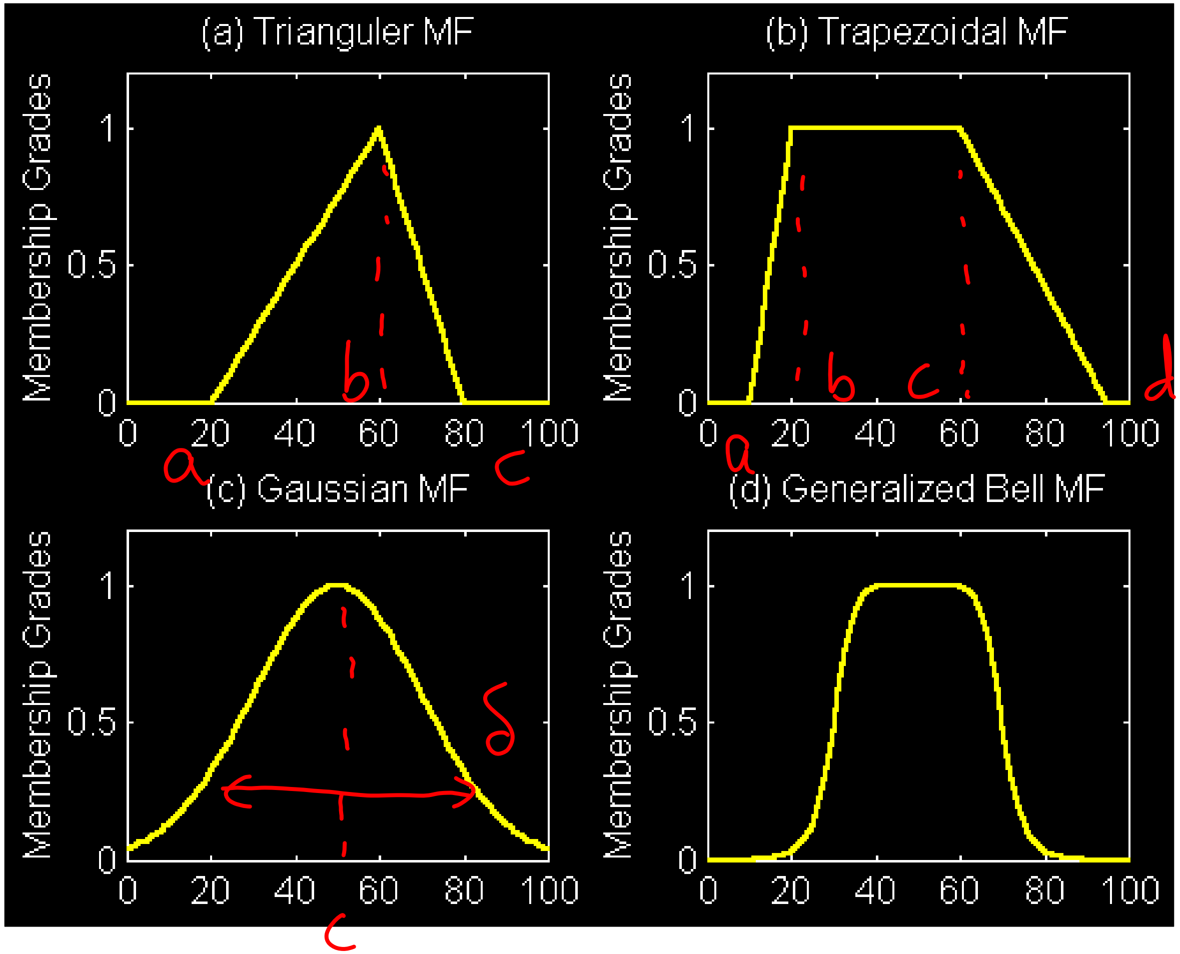
Triangular MF: $$trimf(x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

Trapezoidal MF: $$trapmf(x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

Gaussian MF: $$gaussmf(x;a,b,c) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

Generalized bell MF: $$gbellmf(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{b}\right|^{2b}}$$

MF Formulation



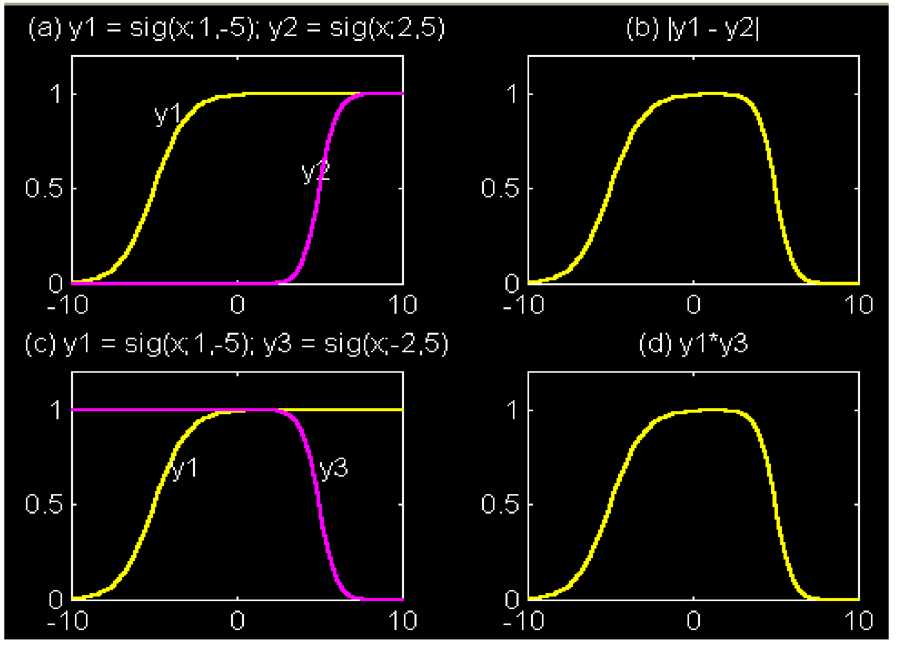
MF Formulation

Sigmoidal MF: $$sigmf(x;a,b,c) = \frac{1}{1+e^{-a(x-c)}}$$

Examples:

Absolute difference of two sig. MFs

Product of two sig. MFs



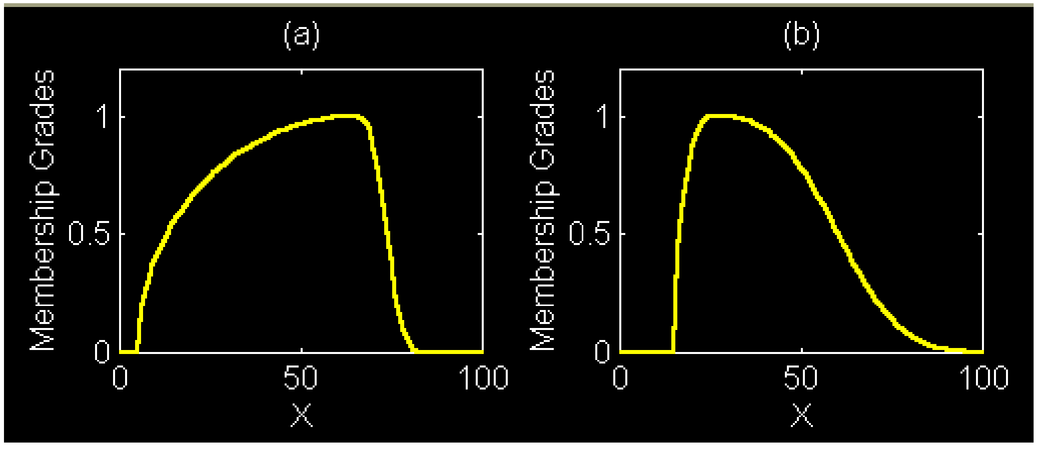
MF Formulation

Left-Right MF:

$$LR(x;c,\alpha,\beta)=\begin{cases}F_L\left(\dfrac{c-x}{\alpha}\right), x<c\\[2ex] F_R\left(\dfrac{x-c}{\beta}\right), x\geq c\end{cases}$$

Example: $F_L(x)=\sqrt{\max(0,1-x^2)}$ $F_R(x)=\exp(-|x|^3)$

c=65
a=60
b=10



c=25
a=10
b=40

Thank you!