CSCE 3110
Data Structures and Algorithms

Hashing

Reading: Weiss, chap. 5

Credits of some slides go to Y. Huang at UNT.

Dictionary ADT

- Data structure with just three basic operations:
  - findItem (i): find item with key (identifier) i
  - insert (i): insert i into the dictionary
  - remove (i): delete i
  - Just like words in a Dictionary

- Where do we use them:
  - Symbol tables for compiler
  - Customer records (access by name)
  - Games (positions, configurations)
  - Spell checkers,etc.

How to Implement a Dictionary?

- Sequences
  - ordered
  - unordered
- Binary Search Trees
- Hashtables

Hashing

- Another important and widely useful technique for implementing dictionaries
- Constant time per operation (on the average)
- Worst case time proportional to the size of the set for each operation (just like array and chain implementation)

Basic Idea

- Use hash function to map keys into positions in a hash table
  Constant time per operation (on the average)

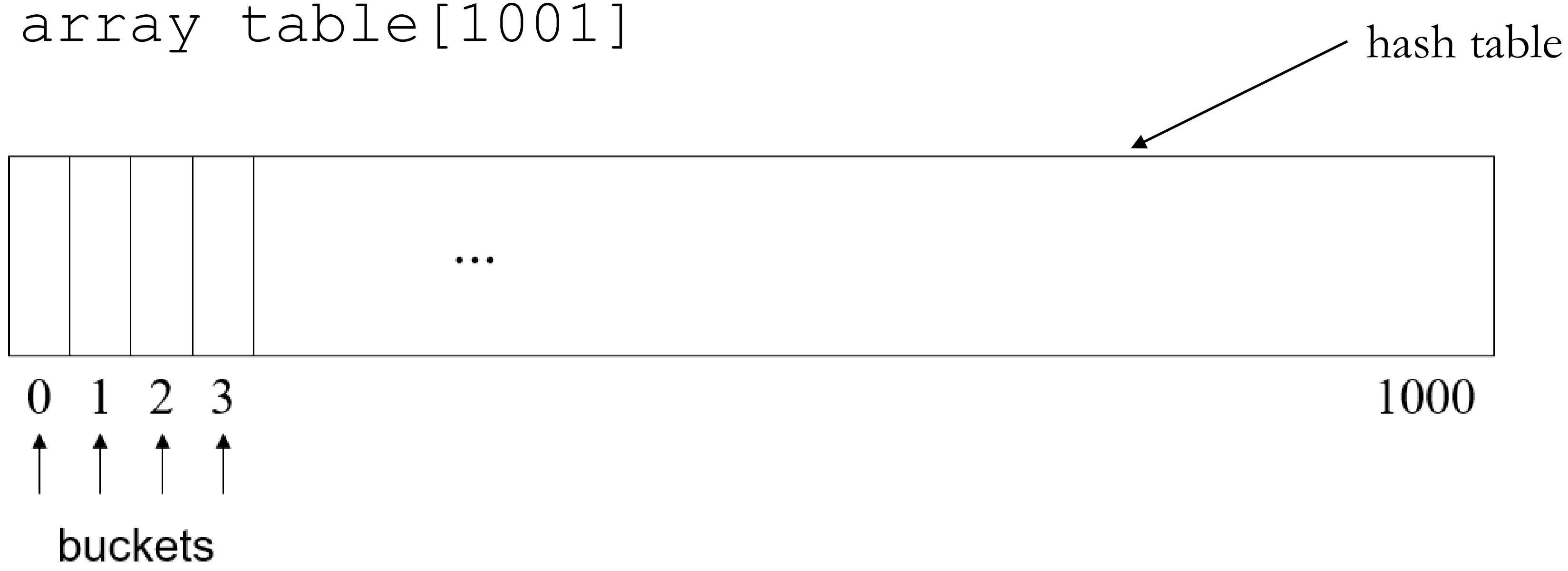
<u>Ideally</u>

- If element e has key k and h is hash function, then e is stored in position h(k) of table
- To search for e, compute h(k) to locate position. If no element, dictionary does not contain e.

Example

Dictionary Student Records

- Keys are ID numbers (951000 - 952000), no more than 1000 students
- Hash function: h(k) = k-951000 maps ID into distinct table positions 0-1000
- `array table[1001]`



Analysis (Ideal Case)

- O(b) time to initialize hash table (b number of positions or buckets in hash table)
- O(1) time to perform `insert, remove, search`

Ideal Case is Unrealistic

- Works for implementing dictionaries, but many applications have key ranges that are too large to have 1-1 mapping between buckets and keys!

<u>Example</u>

- Suppose key can take on values from 0 .. 65,535 (2 byte unsigned int)
- Expect $\approx$ 1,000 records at any given time
- Impractical to use hash table with 65,536 slots!

Hash Functions

- If key range too large, use hash table with fewer buckets and a hash function which maps multiple keys to same bucket:

  $h(k_1) = \beta = h(k_2)$: k_1 and k_2 have collision at slot β

- Popular hash functions: hashing by division

  $h(k) = k\%D$, where D number of buckets in hash table

- Example: hash table with 11 buckets

  $h(k) = k\%11$

  $80 \rightarrow 3$ ($80\%11 = 3$),

  $40 \rightarrow 7$,

  $65 \rightarrow 10$

  $58 \rightarrow 3$ **collision!**

Collision Resolution Policies

- Two classes:
  - (1) Open hashing, a.k.a. separate chaining
  - (2) Closed hashing, a.k.a. open addressing

- Difference has to do with whether collisions are stored outside the table (open hashing) or whether collisions result in storing one of the records at another slot in the table (closed hashing)

Closed Hashing

- Associated with closed hashing is a rehash strategy:

  "If we try to place x in bucket h(x) and find it occupied, find alternative location $h_1(x)$, $h_2(x)$, etc. Try each in order, if none empty table is full,"

- h(x) is called home bucket
- Simplest rehash strategy is called linear hashing

$$hi(x) = (h(x) + i) \% D$$

- In general, our collision resolution strategy is to generate a sequence of hash table slots (probe sequence) that can hold the record; test each slot until find empty one (probing)

Example Linear (Closed) Hashing

- $D=8$, keys a, b, c, d have hash values $h(a)=3$, $h(b)=0$, $h(c)=4$, $h(d)=3$

Example Linear (Closed) Hashing

- D=8, keys a, b, c, d have hash values h(a)=3, h(b)=0, h(c)=4, h(d)=3

0	b
1	
2	
3	a
4	c
5	
6	
7	

Example Linear (Closed) Hashing

- D=8, keys a, b, c, d have hash values h(a)=3, h(b)=0, h(c)=4, h(d)=3
- Where do we insert d? 3 already filled

0	b
1	
2	
3	a
4	c
5	
6	
7	

Example Linear (Closed) Hashing

- D=8, keys a, b, c, d have hash values h(a)=3, h(b)=0, h(c)=4, h(d)=3
- Where do we insert d? 3 already filled
- Probe sequence using linear hashing:

  h1(d) = (h(d)+1)%8 = 4%8 = 4

  h2(d) = (h(d)+2)%8 = 5%8 = **5***

  h3(d) = (h(d)+3)%8 = 6%8 = 6

  etc.

  7, 0, 1, 2
- Wraps around the beginning of the table!

0	b
1	
2	
3	a
4	c
5	
6	
7	

Example Linear (Closed) Hashing

- $D=8$, keys a, b, c, d have hash values $h(a)=3$, $h(b)=0$, $h(c)=4$, $h(d)=3$
- Where do we insert d? 3 already filled
- Probe sequence using linear hashing:

  $h1(d) = (h(d)+1)\%8 = 4\%8 = 4$

  $h2(d) = (h(d)+2)\%8 = 5\%8 =$ **5***

  $h3(d) = (h(d)+3)\%8 = 6\%8 = 6$

  etc.

  7, 0, 1, 2
- Wraps around the beginning of the table!

0	b
1	
2	
3	a
4	c
5	**d**
6	
7	

Operations Using Linear Hashing

- Test for membership: `findItem`
- Examine $h(k)$, $h_1(k)$, $h_2(k)$, …, until we find k or an empty bucket or home bucket
- If no deletions possible, strategy works!
- What if deletions?
- If we reach empty bucket, cannot be sure that k is not somewhere else and empty bucket was occupied when k was inserted
- Need special placeholder `deleted`, to distinguish bucket that was never used from one that once held a value
- May need to reorganize table after many deletions

Performance Analysis - Worst Case

- Initialization: O(b), b# of buckets
- Insert and search: O(n), n number of elements in table; all n key values have same home bucket
- Not better than linear list for maintaining dictionary!

Improved Collision Resolution

- Linear probing: $h_i(x) = (h(x) + i) \% D$
  - all buckets in table will be candidates for inserting a new record before the probe sequence returns to home position
  - clustering of records, leads to long probing sequences

Improved Collision Resolution

- Linear probing: $h_i(x) = (h(x) + i) \% D$
  - all buckets in table will be candidates for inserting a new record before the probe sequence returns to home position
  - clustering of records, leads to long probing sequences
- Linear probing with skipping: $h_i(x) = (h(x) + ic) \% D$
  - c constant other than 1
  - records with adjacent home buckets will not follow same probe sequence

Improved Collision Resolution

- Linear probing: $h_i(x) = (h(x) + i) \% D$
  - all buckets in table will be candidates for inserting a new record before the probe sequence returns to home position
  - clustering of records, leads to long probing sequences
- Linear probing with skipping: $h_i(x) = (h(x) + ic) \% D$
  - c constant other than 1
  - records with adjacent home buckets will not follow same probe sequence
- (Pseudo)Random probing: $h_i(x) = (h(x) + r_i) \% D$
  - r_i is the i^{th} value in a random permutation of numbers from 1 to D-1
  - insertions and searches use the <u>same</u> sequence of "random" numbers

Example

Hash function: $h(k) = k\%11$

- What if the next element has home bucket 0? $\rightarrow$ go to bucket 3

  Same for elements with home bucket 1 or 2!

  Only a record with home position 3 will stay.

  $\Longrightarrow$ probability=4/11 that next record will go to bucket 3

- Similarly, records hashing to 7, 8, 9 will end up in 10

Only records hashing to 4 will end up in 4 (probability=? that next record will go to bucket 4); same for 5 and 6

0	1001
1	9537
2	3016
3	
4	
5	
6	
7	9874
8	2009
9	9875
10	

Example

- insert 1052 (home bucket: 7)

0	1001
1	9537
2	3016
3	
4	
5	
6	
7	9874
8	2009
9	9875
10	

Example

- insert 1052 (home bucket: 7)

0	1001
1	9537
2	3016
3	
4	
5	
6	
7	9874
8	2009
9	9875
10	1052

Example

- insert 1052 (home bucket: 7)
- next element in bucket 3 with probability = ?

0	1001
1	9537
2	3016
3	
4	
5	
6	
7	9874
8	2009
9	9875
10	1052

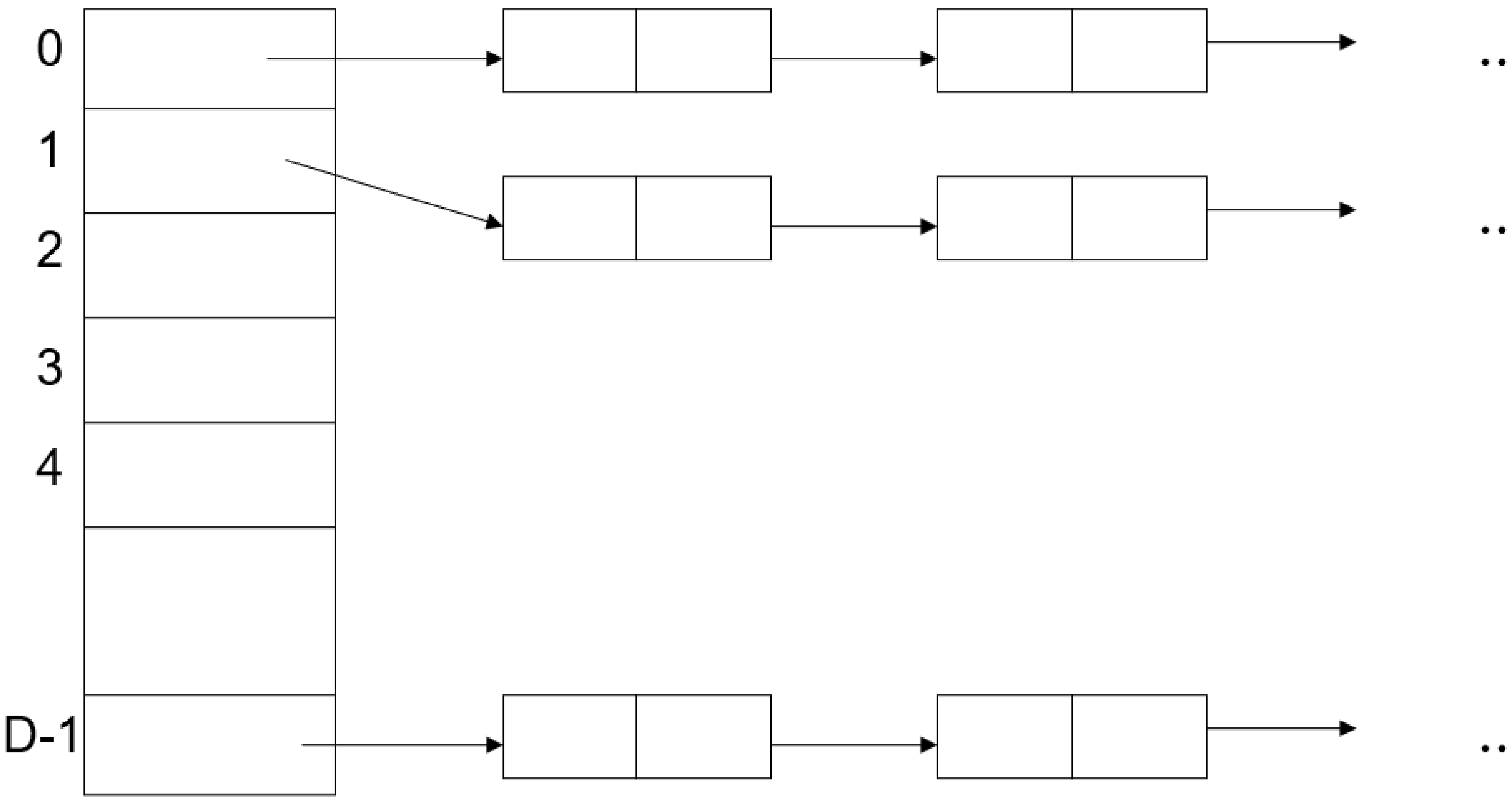
Open Hashing

- Each bucket in the hash table is the head of a linked list

- All elements that hash to a particular bucket are placed on that bucket's linked list

- Records within a bucket can be ordered in several ways
  - by order of insertion
  - by key value order
  - by frequency of access order

Open Hashing Data Organization



Analysis

- Open hashing is most appropriate when the hash table is kept in main memory, implemented with a standard in-memory linked list

- We hope that number of elements per bucket roughly equal in size, so that the lists will be short
- If there are n elements in set, then each bucket will have roughly n/D
- If we can estimate n and choose D to be roughly as large, then the average bucket will have only one or two members

Analysis

<u>Average time per dictionary operation:</u>

- D buckets, n elements in dictionary $\Longrightarrow$ average n/D elements per bucket
- insert, search, remove operation take O(1+n/D) time each
- If we can choose D to be about n, constant time
- Assuming each element is likely to be hashed to any bucket, running time constant, independent of n

Comparison with Closed Hashing

- Worst case performance is O(n) for both

- Number of operations for hashing

  23 6 8 10 23 5 12 4 9 19

  D=9

  h(x) = x % D

Practice

- Draw the 11 entry hashtable for hashing the keys 12, 44, 13, 88, 23, 94, 11, 39, 20 using the function (2i+5) mod 11 with

  (1) closed hashing, linear probing

  (2) open hashing, linked list

Next Class

Sorting

Reading: Weiss, chap. 7