Research on a Dual Problem Solution Strategy under Supply-hub Mode

Abstract: As the supply chain node integrates different suppliers' component inventories, a supply-hub is facing the problems of limited storage capacity and unallowed stock-out. This article takes the minimum total supply chain costs as the target to solve these two problems. By considering the dual replenishment methods for space constraint and urgent replenishment, it establishes a multi-varieties inventory control model under random demands to get the optimal inventory strategies for different components. Finally, through simulation analysis, this paper finds that changes in the total storage area, lead time for components and holding cost per unit time for components will significantly affect both inventory strategies for different components and total supply chain costs. Increasing unit purchase costs for urgent replenishment has a remarkable effect on total supply chain costs, but the effect is not significant on inventory strategies for different components.

Keywords: Supply-hub; inventory space constraint; dual sourcing; (R,Q) continuous review inventory strategy

1. Introduction

Modern enterprises are facing an increasingly complex competitive environment. The competition between enterprises has evolved into the competition between the supply chains where core enterprises are located. Inventory is the link between companies in the supply chain. Most core manufacturers adopt the vendor-managed inventory (VMI) model in assembly manufacturing in order to reduce inventory costs. To meet the needs of core manufacturers, suppliers set up their warehouses which are close to the manufacturers. In this mode, suppliers will have an additional level of component management, which increases their inventory costs and management difficulties. In addition, core manufacturers need to directly negotiate and communicate with a number of suppliers at the same time. This raises their coordination difficulties and operating costs. In order to solve the above problems, Barnes E et al. (2000) proposed a supply-hub mode based on the VMI model. A supply-hub can also be named as a distribution center. It refers to a place near a manufacturing factory for storing that from different suppliers, and it is usually managed by a third-party logistics. For example, Burlington is a global company responsible for logistics and distribution businesses for IT companies such as Apple and Dell in Southeast Asia. China United Logistics provides Jiangling Engines Factory with just-in-time collection and distribution services for the required components.

The supply-hub conducts central management to components inventories of multiple suppliers and then delivers complete sets of components to production stations on time so as to support the just-in-time production of core manufacturing enterprise based on the material demand plans. This requires that the supply-hub's location is close to the manufacturing company. However, the supply-hub cannot provide sufficient storage spaces to hold the components from different suppliers due to the limitation of land space. Market demand uncertainty leads to the manufacturer’s uncertain need for components (Uncertain demand from the market leads to the uncertain need of components from the manufacturer), and the manufacturer may face stock shortage risk within the lead time for component replenishment. Therefore, how to reasonably allocate storage resources in a limited inventory space while avoiding component shortage within the replenishment lead time is a dual problem which the supply-hub mode needs to solve during inventory management.

This paper presents a dual sourcing method in the framework of emergency supply besides considerations to storage space limitations in order to solve the dual problem faced by the supply-hub mode. In order to solve the dual problem faced by supply-hub mode, this paper presents a dual sourcing method in the framework of emergency supply in consideration of storage space limitation. When manufacture faces the stock-out risk within a regular supplier's replenishment lead time, the excessive demands are quickly supplied by nearby suppliers to present the components shortage from leading to production suspension.

It is of great practical significance to study in the supply-hub mode on how to solve the problems of space constraints and dual sourcing in inventory management, optimize inventory strategies for different components, and minimum supply chain costs.

2. Literature Review

How to reduce inventory costs and improve inventory management has been a key issue for global scholars in the field of supply chain inventory management to study. The VMI mode was proposed for the purpose of reducing the impact of market demand uncertainties on inventory decisions of every supply chain node and reducing inventory costs of each member in the supply chain. Thereafter, scholars conducted many researches on the implementation of this model mainly in terms of inventory strategies (Lee and Cho 2018; Braglia, Castellano, and Frosolini 2014; Chen et al. 2012), contract design(Cai et al. 2017; Zhong et al. 2017; Guan and Zhao 2010)and supply chain performance evaluation(Yu et al. 2015; Sari 2008) .

However, suppliers often cannot make timely adjustments to match market demand changes. Beyond that, core manufacturers' coordination costs are relatively high under the VMI model. So, Barnes et al. presented a supply-hub mode based on the VMI model. Scholars of both domestic and abroad have subsequently carried out a series of relevant theoretical studies on this new supply logistics management pattern. In the early development stage of this model, scholars mainly conducted qualitative researches aiming to reflect the importance of the supply-hub mode. Zuckerman (2000) and Lee and Hau (2002) discussed the supply-hub mode concept and the advantages of introducing it to the supply chain. They believe that the supply-hub mode can reduce the supply chain's operational and management costs by improving the responsiveness.

Since the supply-hub mode is utilized more extensively in enterprises, scholars have made further researches on various aspects of its operation. Most of these researches focused on distribution strategies, inventory management strategies, and operation forms. In terms of distribution strategies, Ma and Gui (2010) studied batch decisions of the supply-hub under different pickup methods when a number of suppliers provide components to a manufacturer. Ma and Lv (2014) reviewed the impact of demand uncertainty on production and distribution synergy under the supply-hub mode. Yan and Liu (2015) considered the relationship between transportation costs and volume to study the transportation volume problem under different transportation modes. In terms of inventory management, Shah and Gohand (2006) formulated a demand-determined inventory strategy with upper and lower inventory levels. A supplier has to bear corresponding penalty costs when the inventory level is either above the maximum or below the minimum level. Ma and Huang (2011) compared suppliers' collaborative replenishment strategy and decentralized replenishment strategy in the supply-hub and concluded that collaborative replenishment strategy can effectively reduce the costs of the supply chain system. Gui and Ma (2012) studied inventory management strategies under different operating entities or different decision-making methods. Some scholars took advantage of system dynamics methods and tools to build dynamic simulation models for this mode (Li, Wang, and Wang 2013;Shi et al. 2015) on operational form aspects. Some scholars have applied the operation forms of the supply-hub into other fields. Qiu and Huang (2013) presented the concept of SHIP (supply-hub in an industrial park). He believed that the establishment of a supply-hub in an industrial park can take concentrative storage and integrated goods transport for multiple companies (Qiu and Huang 2013, 2016). Enterprises within the industrial park can employ the supply-hub to share warehousing and other logistics services.

A supply-hub operation is often limited by land resources because the hub needs to be close to core manufacturing enterprises. In the above-mentioned studies, few scholars have considered the impact of limited supply-hub storage space on their inventory strategies owing to limited land resources. In the field of inventory management research, the storage space used for goods keeping will inevitably have an impact on inventory strategies (Xu and Leung 2009; Hariga 2010). Among them, Xu and Leung (2009) studied the periodic inventory control strategy of retail stores with limited shelf space under the VMI model. Hariga (2010) had a study on continuous review inventory strategies with limited storage space. Few scholars considered how the supply-hub works out a reasonable inventory strategy for a variety of goods from different suppliers in limited storage space by combining the supply-hub mode. In addition, the market demand that enterprises are facing in the actual operation process is often uncertain. In the study of the supply-hub inventory strategy, most studies allow out-of-stock for situations where the demand is uncertain. Usually, the suppliers have to compensate clients for losses due to stock-outs. But sometimes the shortage will interrupt companies' current production, so the losses thereof are difficult to be compensated from fines. This paper studies the implementation of a rapid response method of a secondary urgent replenishment from the near supplier to avoid stock-out.

The second urgent replenishment is a type of dual sourcing, it is a replenishment method adopted by enterprises to prevent shortage when they face uncertain demands (Tiwari et al. 2018; Chung, Flynn, and Kirca 2008; Cattani, Dahan, and Schmidt 2008; Arıkan and Werner 2014; Choi and Sethi, 2010; Khouja, and M. 1996; Agrawal et al. 2000; Gallego et al. 1993). Currently, there are few types of research on applying dual sourcing to the supply-hub mode. Ma, Huang, and Hong (2011) studied the use of a supply-hub by two suppliers offering the same components through mathematical modeling analysis. This is a collaborative replenishment strategy with horizontal sharing of inventory information. It is a dual sourcing method in the supply-hub mode in order to deal with the supply interruption of a single supplier. No scholars have studied the use of the secondary urgent replenishment in the centralized supply-hub mode to avoid shortages caused by uncertain demands.

This paper considers the centralized supply-hub mode which consists of one supply-hub, one manufacturer and multiple suppliers. It studies the influence of storage capacity restriction and dual sourcing which is in the framework of urgent nearby replenishment to supply-hub inventory strategies.

3. Mathematical model formulation

3.1 Problem statement

In order to solve the problems existing in the operation of supply-hub, this paper builds a three-echelon supply chain system under centralized supply-hub control. The three-echelon supply chain system is composed of one supply-hub, one manufacturer and multiple suppliers. The operation mode of the research problems in this article is shown in Figure 1:

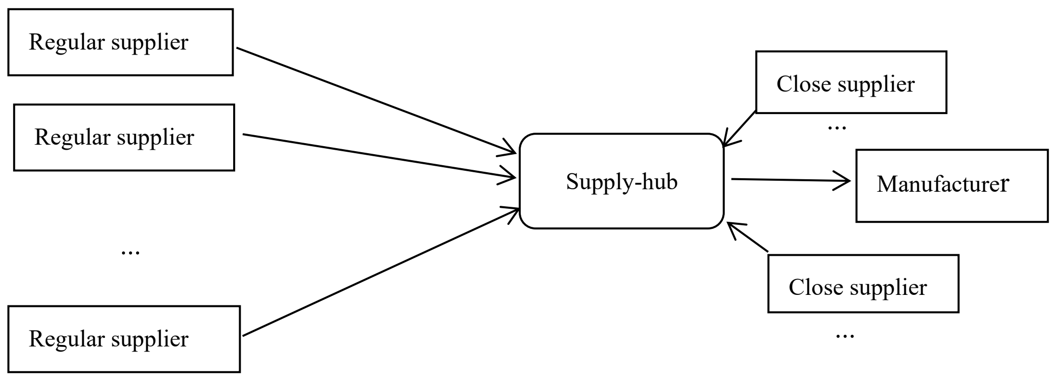


Figure 1 Schematic Diagram of Supply-hub Based

Multi-Manufacturer-Single-Manufacturer Supply Chain

The supply-hub takes a continuous review inventory (R_i, Q_i) control policy for multiple components, all of which are replaceable. In order to prevent the components from being out of stock within the replenishment lead time, the hub adopts a dual sourcing method of urgent nearby replenishment. Every component is supplied by two suppliers. One is a regular supplier with a low price but longer replenishment lead time, and the other has a high price and shorter replenishment lead time which is near to the hub. During every replenishment cycle, the supply-hub usually takes orders from the regular supplier first. When the inventory level of components reaches R_i, the supply-hub gives notice to the regular supplier, asking it to deliver the Q_i quantity of components to the supply-hub for storage. Then the supply-hub sends the components to the production line according to the manufacturer's production plan. The hub makes a secondary urgent order when the supply hub is facing the risk of stock-outs within the lead time of the regular supplier. It means the hub informs the nearby supplier for second urgent replenishment so as to meet excessive demands.

The supply-hub manager aims to minimize the total costs of the expected supply chain operation. With limited supply-hub storage space and urgent nearby replenishment, it considers different regular suppliers' replenishment lead times, transportation costs, inventory expenses, and urgent nearby replenishment charges. The hub works together with all supply chain parties to determine order quantities and reorder points and determine the reasonable stock areas allocated to different components from different regular suppliers.

3.2 Assumptions and Notations

We make assumptions as follows:

(1) A production line has random demands for components. The demand x_i for different components in the lead time follows the same distribution. The expected annual demand is D_i;

(2) The replenishment lead time from the regular supplier is L_i. The distribution function of daily demand for components during the replenishment lead time distribution function is $F\left(x_{d_i}\right)$. The demand within lead time is $x_i = L_i \times x_{d_i}$ with a mean of $\mu_{x_i} = L_i \times \mu_{d_i}$ and standard demand deviation of $\sigma_{x_i} = \sqrt{L_i \times \sigma_{d_i}{}^2}$;

(3) Stock-out is not allowed. The replenishment lead time for a nearby supplier is 0. The distribution costs of a nearby supplier are not considered;

(4) Distribution costs and operating expenses incurred in the process of delivering components from the hub to production stations are not considered;

(5) The storage area allocated to every kind of component is determined by reorder points and order quantities with the function expression of $S_i = a_i\left(Q_i + R_i - \mu_{x_i}\right)$;

(6) There is no crossover of orders;

(7) The relationship between the market's expected annual demand for final products, D, is proportional to the production line's annual demand for every component D_i, which is, $D : D_1 : D_2 : .... : D_n = 1 : k_1 : k_2 : ... : k_n$, $x_d : x_{d_1} : x_{d_2} : ..... : x_{d_n} = 1 : k_1 : k_2 : ... : k_n$. So the demand distribution function of different components within the lead time is $F\left(L_i k_i \mu_d, L_i k_i \sigma_d{}^2\right)$:

The notations used throughout the paper are summarized in Tabel 1.

Tabel 1

Symbol	Meaning
i	The i^{th} component
D_i	The manufacturer's annual expected demand to different components i
L_i	Regular supplier's replenishment lead time for Component i
h_i	Holding cost per unit time for Component i in supply-hub
a_i	Storage area of per unit Component i
c_i	Unit costs for urgent nearby purchase of Component i, $c_i = c_{i0} + \Delta c_i$, where c_{i0} marks the unit purchase costs by the regular supplier and $\Delta c_i \left(\Delta c_i > 0\right)$ is the increased unit purchase costs for purchase nearby
x_i	Demand quantity of component i during lead time. The demand density function is $f\left(x_i\right)$ with the mean of μ_{x_i} and the variance of $\sigma_{x_i}{}^2$
x_{d_i}	Daily demand of Component i within the lead time. The demand density function is $f\left(x_{d_i}\right)$ with the mean of $\mu_{x_{d_i}}$ and the variance of $\sigma_{x_{di}}{}^2$
R_i	The reordering points of component i
Q_i	Order quantity for Component i
A_i	Order start-up expenses for Component i
B_i	Fixed trunk line transport costs of Supplier i to deliver Component i to the supply-hub every time
W	Total storage area of the supply-hub to hold all components
S_i	Storage area which the supply-hub distributes to component i, $S_i = a_i(Q_i + R_i - \mu_i)$

3.3 Mathematical models

When making decisions, supply-hub managers need to give comprehensive considerations to the costs related to both suppliers and the manufacturer so as to minimize the entire supply chain costs. The expected excessive demand during the lead time of the regular supplier is $\int_{R_i}^{\infty}(x_i - R_i) f(x_i) dx_i$ which is satisfied through the second emergency order. The second urgent replenishment costs consist of two parts: purchase costs and components stock holding costs in the supply-hub. A regular supplier and a manufacturer sign a contract to share urgent nearby replenishment costs based on a proportional factor $\beta_i (0 < \beta_i < 1)$, meaning that supplier i bears β_i and the manufacturer takes $1-\beta_i$ （Jun-Yeon Lee et al. ,2016）.

The holding costs of components in the supply-hub, the ordering cost of preparing orders from regular suppliers to the supply-hub and the mainline transport costs from the regular suppliers delivering components to the supply-hub are covered by regular suppliers. Thus, these models' function expression of supplier i's total annual expected costs is as follows:

$$T_{S_i} = h\left(\frac{Q_i}{2} + R_i - \mu_{xi}\right) + A_i \times \frac{D_i}{Q_i} + B_i \times \frac{D_i}{Q_i} + \beta_i \left[\frac{D_i}{Q_i}(c_{i0} + \Delta c_i) + \frac{h_i}{2}\right] \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i$$

All items in the above equation refer to annual approximated stock holding costs of components, annual ordering cost, trunk line costs of delivering components to the supply-hub, and part of expected urgent nearby replenishment costs.

The function of a number of suppliers' costs is:

$$T_S = \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{xi}\right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \beta_i \frac{D_i}{Q_i}(c_{i0} + \Delta c_i) \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i$$

The manufacturer bears another part of urgent nearby replenishment cost. The manufacturer's cost function is:

$$T_M = \sum_{i=1}^{n} (1-\beta_i) \left[\frac{D_i}{Q_i}(c_{i0} + \Delta c_i) + \frac{h_i}{2}\right] \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i$$

The formula for the total expected annual costs of the supply chain is:

$$\begin{aligned} T_{SC} &= T_S + T_M \\ &= \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} \left(c_{i0} + \Delta c_i \right) + \frac{h_i}{2} \right] \int_{R_i}^{\infty} \left(x_i - R_i \right) f\left(x_i \right) dx_i \end{aligned}$$

When there's limited supply-hub storage space, this question's mathematical model is:

$$\begin{aligned} \min T_{SC} &= T_S + T_M \\ &= \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} \left(c_{i0} + \Delta c_i \right) + \frac{h_i}{2} \right] \int_{R_i}^{\infty} \left(x_i - R_i \right) f\left(x_i \right) dx_i \end{aligned} \quad (1)$$

$$\sum_{i=1}^{n} a_i \left(Q_i + R_i - \mu_{x_i} \right) \le W \quad (2)$$

$$Q_i \ge 0 \,, R_i \ge 0 : \; i = 1, 2, , , n \quad (3)$$

The above model is composed of Equation (1), (2), and (3), which is defined as Model (P). Equation (1) represents the objective function, i.e., the minimum total supply chain costs. Equation (2) marks the total storage area constraint, representing that the total storage area occupied by all components in the supply-hub cannot exceed their maximum total storage areas. Equation (3) is the value constraint on variables.

4. Solution

The expected urgent order quantity to nearby supplier within every cycle $UOQ(R_{\mathrm{i}})$ defined for simplified calculations:

$$UOQ\left(R_i \right) = \int_{R_i}^{\infty} \left(x_i - R_i \right) f\left(x_i \right) dx_i \quad （4）$$

Proposition 1. The above nonlinear programming model (P) is a convex optimization, there must be an optimal solution $\left(Q_i^*, R_i^*\right)$ to minimize total costs of the supply chain.

Proof: please see Appendix A.

Since the objective function is a convex function of $\left(Q_i, R_i\right)$ and the constraint condition is a linear constraint at the same time. Therefore, this issue is a convex optimization. Moreover, the feasible region is a convex set. Thus, there exists a unique optimal solution. The $\left(Q_i, R_i\right)$ satisfying K-K-T conditions is the globally optimal solution. We establish a Lagrange function:

$$L\left(T_{SC}, \lambda\right) = \sum_{i=1}^{n} h_i\left(\frac{Q_i}{2} + R_i - \mu_{x_i}\right) + A_i \times \frac{D_i}{Q_i} + B \times \frac{D_i}{Q_i} + \sum_{i=1}^{n}\left[\frac{D_i}{Q_i}\left(c_{i0} + \Delta c_i\right) + \frac{h_i}{2}\right]\int_{R_i}^{\infty}\left(x_i - R_i\right) f\left(x_i\right) dx_i$$
$$+ \lambda\left[\sum_{i=1}^{n} a_i\left(Q_i + R_i - \mu_{x_i}\right) - W\right] \qquad (5)$$

The variable λ is the multiplier for the area constraint. The optimal solution can be obtained via K-K-T conditions.

From $\dfrac{\partial L\left(T_{SC}, \lambda\right)}{\partial Q_i} = 0$

We obtain

$$\frac{h_i}{2} - \frac{\left(A_i + B_i\right) D_i}{Q_i^2} - \frac{D_i}{Q_i^2}\left(c_{i0} + \Delta c_i\right)\int_{R_i}^{\infty}\left(x_i - R_i\right) f\left(x_i\right) dx_i + \lambda a_i = 0 \qquad (6)$$

The calculation result is $Q_i = \sqrt{\dfrac{2D_i\left[A_i + B_i + c_i \times UOQ\left(R_i\right)\right]}{h_i + 2\lambda a_i}}$ （7）

From $\dfrac{\partial L(T_{SC}, \lambda)}{\partial R_i} = 0$, we have:

$$h_i - \left[\frac{D_i}{Q_i}\left(c_{i0} + \Delta c_i\right) + \frac{h_i}{2} \right] \int_{R_i}^{\infty} f\left(x_i\right) dx_i + \lambda a_i = 0 \; ; \tag{8}$$

Then we get the following equation after consolidation:

$$\int_{R_i}^{\infty} f\left(x_i\right) dx_i = \frac{\left(h_i + \lambda a_i\right) 2Q_i}{2D_i \mathrm{c}_i + h_i Q_i} \tag{9}$$

At the same time $$\frac{\partial L\left(T_{SC}, \lambda\right)}{\partial \lambda} = a_i \left(Q_i + R_i - \mu_{X_i} \right) - W \le 0 \tag{10}$$

$$R_i, \quad Q_i, \quad \lambda \ge 0 \;\; . \;\; \forall i \tag{11}$$

The specific analytical solution is unavailable from the function distribution $f(x_i)$ in the above model. This paper simplifies this model by minimizing maximum costs through Lemma 1.

Lemma 1. （Kundu and Chakrabarti 2012）

$$\int_{R_i}^{\infty} \left(x_i - R_i\right) f\left(x_i\right) dx_i \le \frac{\left[\sqrt{\sigma_{x_i}^{\;2} + \left(R_i - \mu_{x_i}\right)^2} - \left(R_i - \mu_{x_i}\right) \right]}{2}$$

Proof: please see Appendix B.

Let $UOQ\left(R_i\right) = \frac{\left[\sqrt{\sigma_{x_i}^{\;2} + \left(R_i - \mu_{x_i}\right)^2} - \left(R_i - \mu_{x_i}\right) \right]}{2}$, i.e., the maximum expected amount of urgent nearby purchase of every component.

Thus, the original cost function can be expressed as:

$$T_{SC} \le \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i}\left(c_{i0} + \Delta c_i\right) + \frac{h_i}{2} \right] \times UOQ\left(R_i\right) \tag{12}$$

Let

$$\overline{T_{SC}} = \max T_{SC}$$
$$= \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} \left(c_{i0} + \Delta c_i \right) + \frac{h_i}{2} \right] \times UOQ\left(R_i\right) \tag{13}$$

We take the method of minimizing maximum costs to turn the problem model (P) into Eqs. (14), (15) and (16). The method of minimizing the maximum cost value is consistent with management decisions because supply-hub managers consider reducing maximum supply chain costs as much as possible during the inventory management process.

$$\overline{T_{SC}} = \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} \left(c_{i0} + \Delta c_i \right) + \frac{h_i}{2} \right] \times UOQ\left(R_i\right) \quad (14)$$

s.t. $$\sum_{i=1}^{n} a_i \left(Q_i + R_i - \mu_{x_i} \right) \le W \quad (15)$$

$$Q_i \ge 0, \quad R_i \ge 0 \ : \quad i = 1, 2, , , n \quad (16)$$

The model’s optimal solution meets the needs of KKT conditions;

Establishment of Lagrange multiplier equation;

$$L\left(\overline{T_{SC}}, \lambda\right) = \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} \left(c_{i0} + \Delta c_i \right) + \frac{h}{2} \right] \times UOQ\left(R_i\right)$$
$$+ \lambda \left[\sum_{i=1}^{n} a_i \left(Q_i + R_i - \mu_{x_i} \right) - W \right] \quad (17)$$

From $\frac{\partial L\left(T_{SC}, \lambda\right)}{\partial Q_i} = 0$, $\frac{\partial L(T_{SC}, \lambda)}{\partial R_i} = 0$, and $\frac{\partial L\left(T_{SC}, \lambda\right)}{\partial \lambda} = 0$, we get

$$Q_i = \sqrt{\frac{2D_i \left[A_i + B_i + c_i \times UOQ\left(R_i\right) \right]}{h_i + 2\lambda a_i}}, \quad (18)$$

$$h+\frac{1}{2}\left[\frac{D_i}{Q_i}\times c_i+\frac{h_i}{2}\right]\left[\frac{R_i-\mu_{xi}}{\sqrt{\sigma_i^{\,2}+(R_i-\mu_{xi})^2}}-1\right]+\lambda a_i=0 \qquad (19)$$

and $$a_i\left(Q_i+R_i-\mu_{x_i}\right)-W=0 \qquad (20)$$

As Eqs. (18), (19), and (20) are complex expressions and difficult to calculate with traditional methods, this paper will use Matlab in simulation analysis to calculate the optimal order quantity Q^* and optimal reorder point R^* before drawing more conclusions after relevant parameter analysis.

Proposition 2.

Assume that $Q_i\wedge$ and $R_i\wedge$ are optimal order quantities and reorder point of every component without storage space constraints, then the following theorems are set up:

（1）If $\sum_{i=1}^{n} a_i\left(Q_i^{\wedge}+R_i^{\wedge}-\mu_{x_i}\right)\le W$, $Q_i^*=Q_i^{\wedge}$, $R_i^*=R_i^{\wedge}$;

（2）In case that $\sum_{i=1}^{n} a_i\left(Q_i^{\wedge}+R_i^{\wedge}-\mu_{x_i}\right)> W$, $\sum_{i=1}^{n} a_i\left(Q_i^{*}+R_i^{*}-\mu_{x_i}\right)= W$;

Proof:

(1) If $\sum_{i=1}^{n} a_i\left(Q_i^{\wedge}+R_i^{\wedge}-\mu_{x_i}\right)\le W$, $\lambda=0$, then $Q_i^*=Q_i^{\wedge}$, and $R_i^*=R_i^{\wedge}$;

(2) In the case that $\sum_{i=1}^{n} a_i\left(Q_i^{\wedge}+R_i^{\wedge}-\mu_{x_i}\right)> W$, the constraint of the total storage will play a role in the optimal solution because TC_{SC} is a strict convex-function about Q_i

and R_i. Due to this constraint is becoming a tight constraint, the optimal solution will satisfy $\sum_{i=1}^{n} a_i \left(Q_i^* + R_i^* - \mu_{x_i} \right) = W$.

Proposition 2 shows that the supply chain's total storage space will affect inventory decisions for every component. The storage area of the supply-hub will be fully utilized when the storage area is limited and plays a restrictive role in the model. While the total storage area is not bound to the optimal solution, both the optimal order quantity and the reorder point of each component will reach a peak and remain unchanged.

Proposition 3.

The optimal reorder point R_i^* of every component increases with the increase of Δc_i and L_i.

This Theorem stands to reason. A decision-maker needs to increase the reorder point of components and improve the safety stock when there's an increase of either the nearby purchase costs or the lead time of regular suppliers. The purpose is to lower stock-out risk within the lead time period and reduce urgent nearby replenishment costs.

5. Numerical simulation

5.1 Example Calculation

Assume that the supply-hub needs to develop inventory strategies for three components that are fungible. The total storage area is W=5000. Regular suppliers deliver them into the supply hub, these three components will be put to the production with the proportion of 1:1:1. The annually expected demand is D_i = 4800. Their lead times L_i are $L_1 = 7$, $L_2 = 5$ and $L_3 = 6$ respectively. The cumulative distribution of daily demand during the lead time is normally distributed as $x_{d_i} : N(13, 4)$. The other basic parameters are shown in Table 2. Above parameters are put into the established model through Matlab to get the optimal inventory decisions $\left(Q_i^*, R_i^*\right)$, storage area S_i, urgent nearby replenishment costs NRC_i and total supply costs T_{SC} of these three components, which are shown in Table 2 :

Table 2 Basic Information of All Components and optimal solutions

Component No.	A_i	B_i	c_{i_0}	Δc_i	h_i	a_i	Q_i^*	R_i^*	S_i	NRC_i	T_{SC}
1	85	30	8	2	6	6	261	100	1620	134.60	
2	85	20	9	2.5	4	7	250	74	1813	116.96	8939
3	87	26	6	1.5	7	6	251	85	1548	113.43	

5.2 Sensitivity Analysis

We change the value of different parameters. We allow parameter values to change within a certain scope. These parameters are the total storage area W, the replenishment lead time L_i of the regular supplier, the increased unit purchasing cost Δc_i from the urgent nearby replenishment for every component and the unit stock holding cost in the supply-hub. Then we calculate optimal solutions of every component under different conditions, analyze the effects of different parameters to storage strategies of different components, distributed storage area, and total supply chain costs.

(1) Effect of total storage area W:

Table 3 Example Calculation Results with W Changes

W	Q_1^*	R_1^*	Q_2^*	R_2^*	Q_3^*	R_3^*	NRC_1	NRC_2	NRC_3
2000	104	96	93	70	102	82	529.60	508.66	414.28
3000	157	98	143	72	153	83	273.99	253.49	238.79
4000	209	99	195	73	203	84	185.16	166.08	157.87
5000	261	100	250	74	251	85	134.60	116.96	113.43
6000	312	101	310	75	295	85	103.03	85.93	96.91
7000	360	101	375	75	337	85	89.55	71.20	85.17
8000	405	101	447	75	374	85	79.82	59.89	77.01
9000	440	101	513	76	402	85	73.63	47.91	71.84
10000	440	101	513	76	402	85	73.63	47.91	71.84

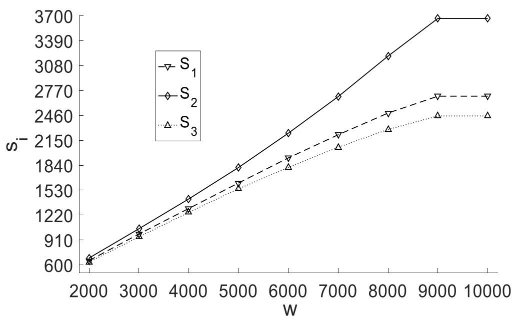


Figure 2. Effect of W on S_i

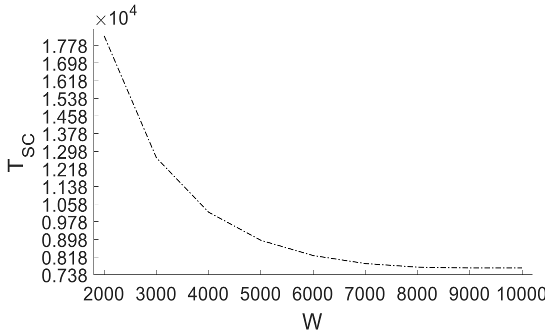


Figure 3. T_{SC} as a function of W

Table 3 demonstrates the changes in inventory strategies for each component and the costs of urgent nearby replenishment under the change of the total supply storage space. We know from Table 3 that every component's order quantity Q_i and reorder point R_i increase with the increase of the total storage area W. We found the order quantity changes significantly and the reorder point varies slightly when the storage area was changed. This is because the manager tends to increase a component's replenishment amount from the regular supplier when there is a larger storage space, thereby reducing distribution frequencies and delivery costs.

Figure 2 reveals changes in the storage area allocated to different components with the alteration of the total supply-hub storage space. It can be seen from Figure 2 that with the increase of the total storage area W, the storage area allocated by different components increase accordingly. The rate of the curve is getting smaller and smaller, which finally keeps stable after reaching its peak. This explains that the total storage area is sufficiently large and fails to play a restrictive role at this time. In the case where the total storage area acts as a restraint, the storage area allocated to every component increases more significantly with the increase of total storage area when the total storage area is small. The larger the total storage area is, the smaller changes in the storage area are allocated to each component. It is also found from Figure 2 that with the increase of the total storage area, the size difference of the storage area allocated to different components increases. This description of Figure 2 illustrates that storage areas allocated to different components have different sensitivities to total storage space. As shown in Figure 2, the second component has the biggest slope, indicating that it has the greatest sensitivity to changes in the total storage area.

Figure 3 displays the impact of total storage area W changes on the total supply chain costs. We can see from Figure 3 that in the presence of multiple components, the total supply chain costs T_{SC} decrease as the total storage area increases, and the slope of the function becomes smaller. This means a remarkable supply chain cost reduction with the total storage area expands when the total storage area is smaller. While the total storage area is bigger, the total cost reduction of the supply chain becomes smaller.

The above analysis illustrates that the supply-hub manager can properly increase the total supply chain storage area within their capabilities, which may reduce the total supply chain costs.

（2）Effect of increased unit purchase costs for emergency nearby replenishment Δc_i

Table 4 Example Calculation Results under Δc_2 Changes

Δc_2	Q_1^*	R_1^*	Q_2^*	R_2^*	Q_3^*	R_3^*	NRC_1	NRC_2	NRC_3
1	262	100	250	73	251	85	134.10	113.02	113.43
1.5	262	100	250	74	251	85	134.10	106.88	113.43
2	261	100	250	74	251	85	134.60	111.92	113.43
2.5	261	100	250	74	251	85	134.60	116.96	113.43
3	261	100	250	74	250	85	134.60	122.00	113.87
3.5	261	100	250	75	250	85	134.60	115.49	113.87
4	261	100	250	75	250	85	134.60	120.07	113.87
4.5	261	100	250	75	250	85	134.60	124.65	113.87
5	261	100	250	75	250	85	134.60	129.23	113.87

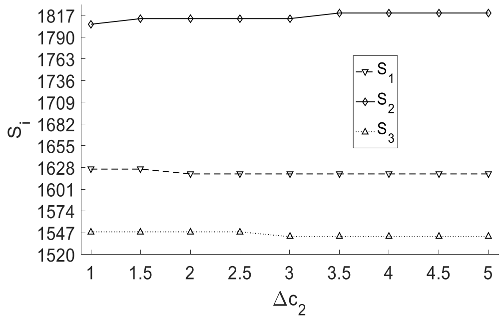


Figure 4　Effect of Δc_2 on S_i

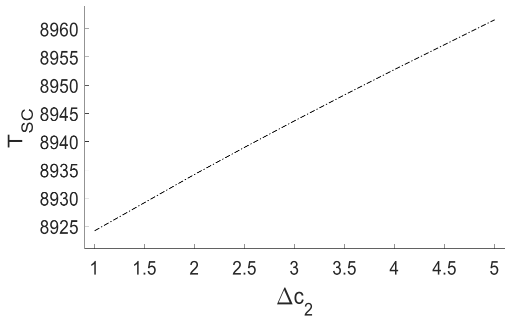


Figure 5　T_{SC} as a function of Δc_2

We take the second component as an example to observe the impact of the increased unit purchase cost Δc_i of nearby procurement on its own and other components' inventory strategies. Table 4 presents the changes in three components' inventory strategies and urgent nearby replenishment costs as Δc_2 increases. It is known from Table 4 that the reorder point for the second component increases with the increase of Δc_2 while its order quantity remains nearly unchanged. The order quantities，reorder points and urgent nearby replenishment costs of other components are almost unchanged, while their urgent nearby replenishment costs increase. This indicates that the changes in one of the components' purchase price for nearby replenishment have little effect on the inventory strategies for both its own and other components when there are multiple components.

Figure 5 reveals the changes in S_i with the increase of Δc_2. We can see that only S_2 increases slightly. This is because the R_2 increases slightly while Q_2 keeps unchanged as shown in Table 4. We can see from Figure 6 that supply chain costs go up with the addition of Δc_2. This interprets that the unit purchase cost changes from nearby replenishment for a component has little effect on the storage area allocated to it but has a great effect on the total costs of the supply chain. The above analysis may offer decision-makers with some storage management suggestions. It means that they do not need to give excessive considerations to urgent nearby replenishment purchase prices for some components when decision-makers are making storage strategies and allocating storage areas for different components. However, core companies should try as much as possible to choose suppliers with lower purchase prices in order to reduce the total costs of the supply chain when selecting suppliers for urgent replenishment.

（3）Effect of lead time from regular supplier L_i

Table 5 Example Calculation Results for L_2 Changes

L_2	Q_1^*	R_1^*	Q_2^*	R_2^*	Q_3^*	R_3^*	NRC_1	NRC_2	NRC_3
1	264	100	250	17	253	85	133.10	52.60	112.55
2	263	100	250	32	252	85	133.60	70.54	112.99
3	263	100	250	46	252	85	133.60	90.26	112.99
4	262	100	250	60	251	85	134.10	105.19	113.43
5	261	100	250	74	251	85	134.60	116.96	113.43
6	261	100	250	88	250	85	134.60	126.50	113.87
7	260	100	250	102	250	85	135.11	134.41	113.87
8	260	100	250	116	249	85	135.11	141.09	114.32
9	259	100	250	129	249	85	135.63	157.79	114.32

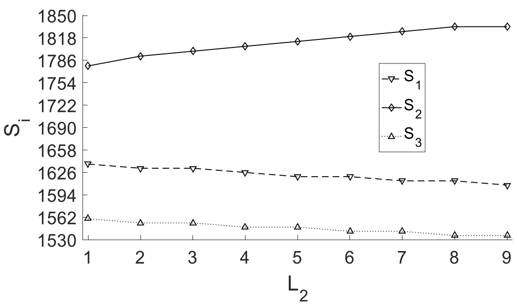


Figure 6 Effect of L_2 on S_i

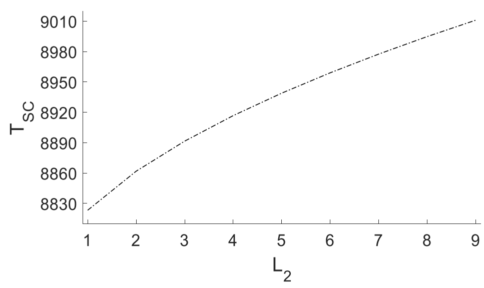


Figure 7 T_{SC} as a function of L_2

We use the second component as an example to observe the impact of L_i on all components' inventory strategies. It can be seen from Table 5 that the second component's order quantity decreases and the reorder point increases when its lead time L_2 is on the rise. This is because decision-makers need to increase the safety stock in order to reduce the stock-out risk during the lead time. The order quantity for other components decrease but there are no significant changes to reorder points. This tells managers that when a component's replenishment lead time is on the rise, they should increase the reorder point for this component and slightly lower other components' order quantities.

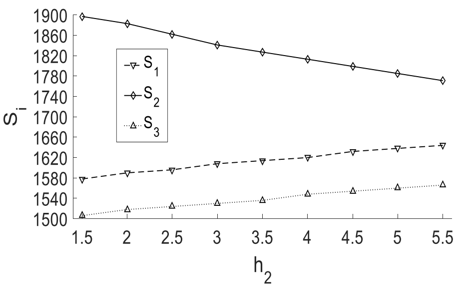
Figure 6 indicates changes in the storage area allocated to every component when there is an increase in the lead time of the second component replenishment. It can be seen from Figure 6 that when L_2 increases, the storage area allocated to the second component becomes larger while the storage area of other components decreases correspondingly. Such change can be explained like this: When the replenishment lead time of a component increases, decision-makers are willing to raise its reorder point for stock out risk reduction. Reorder points for other components are unchanged. Limited by the total storage space, the order quantities for all components will drop accordingly. This leads to changes in the allocated storage space. The allocated storage area for this specific component becomes larger, and other components' storage area becomes smaller. This suggests that when the replenishment lead time of a component increases, decision-makers should appropriately increase the storage area for this component and decrease the storage area allocated to other components.

Figure 7 marks the total supply chain cost changes out of variations of the second component's replenishment lead time from the regular supplier. The above changes make it clear that a company should do its best to select suppliers with shorter replenishment lead time when choosing regular suppliers. Such practice is effective not only for risk reduction but also for lower total supply chain costs.

(4) Effect of the holding cost per unit time h_i.

Table 6 Example Calculation Results of h_2 Changes

h_2	Q_1^*	R_1^*	Q_2^*	R_2^*	Q_3^*	R_3^*	NRC_1	NRC_2	NRC_3
1.5	254	100	261	75	244	85	138.25	101.88	116.60
2	256	100	259	75	246	85	137.19	102.66	115.68
2.5	257	100	256	75	247	85	136.66	103.86	115.22
3	259	100	254	74	248	85	135.63	115.13	114.77
3.5	260	100	252	74	249	85	135.11	116.04	114.32
4	261	100	250	74	251	85	134.60	116.96	113.43
4.5	263	100	248	74	252	85	133.60	117.89	112.99
5	264	100	246	74	253	85	133.10	118.84	112.55
5.5	265	100	244	74	254	85	132.60	119.81	112.12



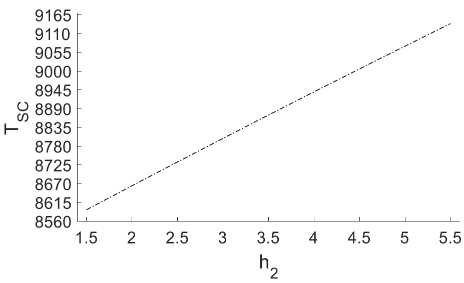


Figure 8 Effect of h_2 on S_i

Figure 9 T_{SC} as a function of h_2

We also use the second component as an example to observe the impact of h_i on all components' inventory strategies. The data in Table 6 expresses that both order quantity and reorder point of the second component are smaller while urgent replenishment costs are higher with the h_2 increases. The reason is that decision-makers will lower the component's order quantity and reorder point so as to further decrease inventory costs when a certain component’s unit stock holding cost becomes higher. However, it raises the stock-out probability of this component within the replenishment lead time. So, the costs for urgent nearby replenishment will be on the rise. The order quantity for the other two components is on the increase correspondingly. However, their reorder points are almost unchanged and their urgent nearby replenishment costs are less.

Figure 8 reveals changes in the storage area allocated to all components. We know from Figure 8 that the storage area allocated to the second component becomes smaller with the rise of unit stock holding cost, while the storage area distributed to other components becomes larger. These changes are conspicuous. It is known from Table 6 that order quantities reduction for the second component will naturally lead to the storage area shortage allocated to it. The inventory area given to other components becomes larger in turn. Figure 9 indicates that the total supply chain cost gets higher together with the increase of the second component's unit stock holding cost. The analysis above demonstrates that a component's stock holding cost is higher when there are multiple components for consideration. Decision-makers should adequately cut down this component's order size and allocated storage area by adding other components' order quantities and storage areas.

The description above is the analysis of the supply-hub's total storage area, the replenishment lead time of regular supplier, the purchase cost for urgent nearby replenishment, and the sensitivity to unit stock holding cost. In a supply chain consisting of a number of suppliers, a supply-hub and a manufacturer, the total supply-hub storage area W, stock holding cost h_i and replenishment lead time L_i for different components generate remarkable effects on components' inventory strategies and total supply chain cost. Changes of Δc_i, however, have little effect on the component's strategies. Among them, stock holding cost and replenishment lead time changes have opposite effects on components' inventory strategies. In addition, we have found that there's no correspondence between components' reorder point and their respective allocated storage areas. That is, the storage area given to the component with the most order size may not be the largest. The above analysis can offer valuable reference to the hub manager when they are working out inventory strategies for different components.

6. Conclusion

This article describes a dual problem of limited storage resources and unallowable stock-out faced by the inventory manager based on the supply-hub's centralized control. With the goal of minimizing the total supply chain cost, it establishes a multi-variety continuous review inventory control model covering both storage area constraints and urgent nearby replenishment to set up inventory strategies for different components.

Through numerical analysis, we find that different parameter changes affect components inventory strategies, allocated storage areas, and total supply chain costs. The storage area allocated to every component will increase and total supply chain costs will go down as the total storage area is on the rise. The increasing unit purchase cost of urgent nearby replenishment for a certain component has little effect on component inventory decisions. However, it will lead to a significant increase in the total supply chain costs. The increase in both lead time and unit stock holding cost for a component will increase the total costs of the supply chain. As to storage areas distributed to the components, however, the rise of a certain component's replenishment lead time will result in a larger increase of the storage area allocated to this component and reduce the storage areas of other components. Raising a component's replenishment lead time will make its allocated storage area larger and other components' proportions smaller. The unit stock holding cost increase of a certain component will decrease its allocated storage area and increase the storage areas for other components.

The study mentioned above may provide supply-hub managers with guidance when they face the above-mentioned problems in inventory management. The purpose is to help them draw up a scientific inventory strategy on the basis of different component features so as to reasonably allocate storage resources and avoid shortages.

The main idea about this paper is replenishment strategies for alternative components. Future research could continue to explore replenishment strategies for other types of components. In addition, the paper assumes that component demands follow a normal distribution. Component inventory strategies under complicated needs may also be considered in future studies.

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Appendix A. Proof for Proposition 1

$$\min T_{SC} = T_S + T_M$$
$$= \sum_{i=1}^{n} h_i \left(\frac{Q_i}{2} + R_i - \mu_{x_i} \right) + \sum_{i=1}^{n} A_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} B_i \times \frac{D_i}{Q_i} + \sum_{i=1}^{n} \left[\frac{D_i}{Q_i} (c_{i0} + \Delta c_i) + \frac{h_i}{2} \right] \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i$$

We have

$$\frac{\partial T_{SC}}{\partial Q_i} = \frac{h_i}{2} - \frac{(A_i + B_i) D_i}{Q_i^2} - \frac{D_i}{Q_i^2} (c_i + \Delta c_i) \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i \text{；}$$

$$\frac{\partial T_{SC}}{\partial R_i} = h_i - \frac{D_i}{Q_i} (c_i + \Delta c_i) \int_{R_i}^{\infty} f(x_i) dx_i \text{；}$$

$$\frac{\partial^2 T_{SC}}{\partial Q_i^2} = \frac{2(A_i + B_i) D_i}{Q_i^3} + \frac{2D_i}{Q_i^3} (c_i + \Delta c_i) \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i > 0 \text{；}$$

$$\frac{\partial^2 T_{SC}}{\partial Q_i \partial R_i} = \frac{D_i}{Q_i^2} (c_i + \Delta c_i) \int_{R_i}^{\infty} f(x_i) dx_i > 0 \text{；}$$

$$\frac{\partial^2 T_{SC}}{\partial R_i^2} = \frac{D_i}{Q_i} (c_i + \Delta c_i) f(R_i) > 0 \text{；}$$

$$\frac{\partial T_{SC}{}^2}{\partial R_i \partial Q_i} = \frac{D_i}{Q_i^2} (c_i + \Delta c_i) \int_{R_i}^{\infty} f(x_i) dx_i > 0$$

All second order derivatives are greater than 0 for all non-negative Q_i, R_i, eqs (1) is a strictly convex function about (Q_i, R_i). In addition, the constraint condition is a linear constraint at the same time, thus this issue is a convex programming one.

Appendix B. Proof for Lemma 1

Since $(x_i - R_i)^+ = \dfrac{|x_i - R_i| + (x_i - R_i)}{2}$

The result follows by taking expectations and by using the Cauchy-Schwarz

Inequality

$$
\begin{aligned}
E\left|x_i - R_i\right| &\le \left[E\left(x_i - R_i\right)^2\right]^{\frac{1}{2}} \\
&= \left[E\left\{\left(x_i - \mu_i\right) - \left(R_i - \mu_i\right)\right\}\right]^{\frac{1}{2}} \\
&= \left[E\left\{\left(x_i - \mu_i\right)^2 - 2\left(x_i - \mu_i\right)\left(R_i - \mu_i\right) + \left(R_i - \mu_i\right)^2\right\}\right]^{\frac{1}{2}} \\
&= \left[\sigma^2 + \left(R - \mu\right)^2\right]^{\frac{1}{2}}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
E\left(x_i - R_i\right) &= \frac{E\left(x_i - R_i\right) + E\left(x_i - R_i\right)}{2} \\
&\le \frac{\sqrt{\sigma^2 + \left(R_i - \mu_{x_i}\right)^2} - \left(R_i - \mu_{x_i}\right)}{2}
\end{aligned}
$$