In [1]:
```
import numpy as np
import scipy.linalg as linalg
import matplotlib.pyplot as plt
import scipy.sparse as sparse
from scipy.sparse.linalg import spsolve
import time
import timeit

```

Lab Book 01

In [2]:
```
x = np.linspace(1, 5, 5)
print(x)
A = (np.linspace(1, 9, 9)).reshape((3, 3))
print(A)

```

```
[1. 2. 3. 4. 5.]
[[1. 2. 3.]
 [4. 5. 6.]
 [7. 8. 9.]]
```

In [3]:
```
print(
    f"||x||_1 = {np.linalg.norm(x, ord=1)}\n||x||_2 = {np.linal
print(f"||x|| defult = {np.linalg.norm(x)}")
print(f"||A||_1 = {np.linalg.norm(A, ord=1)} \n||A||_2 = {np.li
print(f"||A||_frobenius = {np.linalg.norm(A, ord='fro')}")
print(f"||A|| defult = {np.linalg.norm(A)}")

```

```
||x||_1 = 15.0
||x||_2 = 7.416198487095663
||x|| defult = 7.416198487095663
||A||_1 = 18.0
||A||_2 = 16.84810335261421
||A||_inf = 24.0
||A||_frobenius = 16.881943016134134
||A|| defult = 16.881943016134134
```

The default choice for vector's norm is its l2 norm. The default chioce for matrix's norm is its frobenius norm.

Lab Book 02

In [4]:
```
# Create an example 6*6 linear system (related to elastic membr
n = 6
A = np.diag(np.ones(n-1), -1) - 2*np.diag(np.ones(n)) + \
    np.diag(np.ones(n-1), 1)
b = np.ones(n)
# Calculate LU factorisation
lu, piv = linalg.lu_factor(A)
# Solve a linear system using a previously computed LU factoris
x = linalg.lu_solve((lu, piv), b)
print(f"The solution x is {x}")
print(f"LU\n{lu}\nP\n{piv}")

```

```
The solution x is [-3. -5. -6. -6. -5. -3.]
LU
[[-2.          1.          0.          0.          0.          0.
]
 [-0.5        -1.5         1.          0.          0.          0.
]
 [-0.         -0.66666667 -1.33333333  1.          0.          0.
]
 [-0.         -0.         -0.75       -1.25        1.          0.
]
 [-0.         -0.         -0.         -0.8        -1.2         1.
]
 [-0.         -0.         -0.         -0.         -0.83333333 -1.1
6666667]]
P
[0 1 2 3 4 5]
```

In [5]:
```
residual = np.linalg.norm((A@x - b), ord=2)
print(f"The residual is {residual}")

```

```
The residual is 1.7763568394002505e-15
```

LU is the combination of L and U, where the upper triangular part of LU including the diagonal is U and the lower triangular matrix of LU with diagonal of 1s is L. Since LU can store all the information of both L and U without loss. We don't need to save them seperately each time.

P is stored in piv which can be read as the index of pivot in each row, like piv:[0,1,2] stands for P: [[1,0,0],[0,1,0],[0,0,1]].

Lab Book 03

In [6]:
```python
def geneHilbert(size):
    A = np.zeros([size, size])
    for i in range(size):
        for j in range(size):
            A[i][j] = 1 / (i + j + 1)
    return A

```

In [7]:
```python
n = 15

A = geneHilbert(15)
xtrue = np.ones((n,))
b = A @ xtrue               # RHS of Ax=b with known solution xt
print(b)
x = np.linalg.solve(A, b)  # Gaussian Elimination with partial 
print(x)
print("Relative error of np.linalg.solve =",
      np.linalg.norm(x - xtrue) / np.linalg.norm(xtrue))
lu, piv = linalg.lu_factor(A)
xdash = linalg.lu_solve((lu, piv), b)
print(xdash)
print(
    f"the determinant of A is {np.linalg.det(A)}, which is cols

```

```
[3.31822899 2.38072899 1.93955252 1.66177474 1.46440632 1.31440632
 1.1953587  1.09795611 1.01643437 0.94698992 0.88698992 0.83454237
 0.78824608 0.74703728 0.71009147]
[  1.00000005   0.99999302   1.00024845   0.99635976   1.02582299
   0.91929014   0.94139796   2.48103462  -5.07361829  14.857887
 -19.1097392   20.00938864 -10.37747099   4.9248948    0.40451104]
Relative error of np.linalg.solve = 8.725818712797583
[  1.00000005   0.99999302   1.00024845   0.99635976   1.02582299
   0.91929014   0.94139796   2.48103462  -5.07361829  14.857887
 -19.1097392   20.00938864 -10.37747099   4.9248948    0.40451104]
the determinant of A is -1.2321076338246708e-120, which is colse t
o 0.
Therefore it is almost a singular matrix.
```

Since the determinant of Hilbert matrix is extremly small, itself is an ill-conditioned matrix. Therefore, when we are computing the solution by np.linalg.solve, Hilbert matrix will possibly lead to numeric errors, especially when we try to find its inverse, there will be a large floating point number error. There will also have a lot of rounding errors due to the feature of Hilbert matrix. The error also changes with the size of the matrix, the larger the size is, the larger the error is.

Lab Book 04

In [8]:
```python
def lu_factorisation(A):

    n = A.shape[0]  # dimension of A

    P = np.eye(n, dtype='float64')   # P starts as the identity
    L = np.eye(n, dtype='float64')   # L starts as the identity
    U = A.copy().astype('float64')  # U starts as A (we will pu
    for k in range(n-1):    # column of U where we are adding z
        max = np.argmax(abs(U[k:, k:(k+1)])) + k
        U[[k, max], :] = U[[max, k], :]
        P[[k, max], :] = P[[max, k], :]
        for i in range(k+1, n):  # row of U to add zero
            L[[k, max], :(k)] = L[[max, k], :(k)]   # Turn E to
            L[i, k] = U[i, k] / U[k, k]   # entry of L to zero
            for j in range(k, n):   # apply row operation to no
                U[i, j] = U[i, j] - L[i, k]*U[k, j]
    return P, L, U

```

In [9]:
```python
A = np.array([[1, 1, 1], [2, 2, 5], [4, 6, 8]])
# A = np.array([[2,4,-2]
#              ,[4,9,-3]
#              ,[-2,-3,7]])
b = np.linspace(1, 3, 3)
# b = np.linspace(2,8,10)
print(f"A\n{A}\nb\n{b}")

```

```
A
[[1 1 1]
 [2 2 5]
 [4 6 8]]
b
[1. 2. 3.]
```

```
In [10]: P1, L1, U1 = lu_factorisation(A)
         print(f"My solver:\nP\n{P1}\nL\n{L1}\nU\n{U1}")
         P2, L2, U2 = linalg.lu(A)
         print(f"\nscipy.linalg.lu:\nP\n{linalg.inv(P2)}\nL\n{L2}\nU\n{U
         print(
             f"\nCheck:\nis P equal?\n{P1==P2}\nis L equal?\n{L1==L2}\ni

```

```
My solver:
P
[[0. 0. 1.]
 [0. 1. 0.]
 [1. 0. 0.]]
L
[[1.   0.   0.  ]
 [0.5  1.   0.  ]
 [0.25 0.5  1.  ]]
U
[[ 4.   6.   8. ]
 [ 0.  -1.   1. ]
 [ 0.   0.  -1.5]]

scipy.linalg.lu:
P
[[-0.  0.  1.]
 [-0.  1.  0.]
 [ 1.  0.  0.]]
L
[[1.   0.   0.  ]
 [0.5  1.   0.  ]
 [0.25 0.5  1.  ]]
U
[[ 4.   6.   8. ]
 [ 0.  -1.   1. ]
 [ 0.   0.  -1.5]]

Check:
is P equal?
[[ True  True  True]
 [ True  True  True]
 [ True  True  True]]
is L equal?
[[ True  True  True]
 [ True  True  True]
 [ True  True  True]]
is U equal?
[[ True  True  True]
 [ True  True  True]
 [ True  True  True]]
```

```
In [11]:
n = 6
A = np.diag(np.ones(n-1), -1) - 2*np.diag(np.ones(n)) + \
    np.diag(np.ones(n-1), 1)
lu_factorisation(A)
x = max(range(1, 6))
print(x)
A = np.array([[1, 1, 1], [2, 2, 5], [4, 6, 8]])
print(A)
b = np.linspace(1, 3, 3)
```

```
5
[[1 1 1]
 [2 2 5]
 [4 6 8]]
```

Lab Book 05

```
In [12]:
def plu_solve(P, L, U, b):
    # Ly = Pb
    y = linalg.solve(L, P @ b)

    # Ux = y
    x = linalg.solve(U, y)

    return x
```

```
In [13]:
ans = linalg.solve(A, b)
P1, L1, U1 = lu_factorisation(A)
x = plu_solve(P1, L1, U1, b)
print(f"computed x = {x}\nans = {ans}")
print(f"Check:\n{ans == x}")
```

```
computed x = [ 1.5 -0.5 -0. ]
ans = [ 1.5 -0.5 -0. ]
Check:
[ True  True  True]
```

Lab Book 06

In [14]:
```
# Create an example 6*6 linear system (related to elastic membr
n = 6
A = np.diag(np.ones(n-1), -1) - 2*np.diag(np.ones(n)) + \
    np.diag(np.ones(n-1), 1)
b = np.ones(n)
# Calculate QR factorisation
Q, R = np.linalg.qr(A)
# Solve a linear system using a QR factorisation
x = linalg.solve_triangular(R, Q.T @ b, lower=False)

```

In [15]:
```
I = np.eye(n)
# check QT·Q = I
residual1 = linalg.norm(Q.T @ Q - I, ord=2)
# check A = Q·R
residual2 = linalg.norm(A - Q @ R, ord=2)
print(
    f"The error between A and Q·R is {residual2}\nThe error bet

```

```
The error between A and Q·R is 7.543868986480381e-16
The error between I and QT·Q is 5.055024574123691e-16
```

Lab Book 07

In [16]:
```
# Create an example 10x6 overdetermined linear system
m = 10
n = 6
A = np.sqrt(np.arange(m*n)).reshape((m, n))
b = np.ones(m)
# Calculate reduced QR factorisation
Q, R = np.linalg.qr(A, mode='reduced')
# Solve a linear system using a QR factorisation
x = linalg.solve_triangular(R, Q.T @ b, lower=False)
print("Residual =", np.linalg.norm(A@x - b))

```

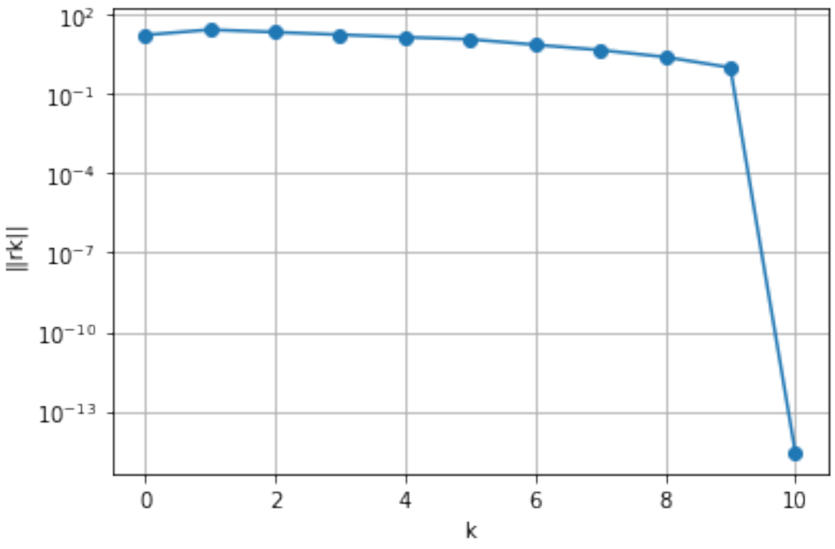
```
Residual = 0.0010262943079517286
```

In [17]:

```python
def conjugate_gradient(A, b, x0, tol):
    # Initialise variables
    x = x0.copy()
    r = b - A @ x
    d = r
    # Run the main CG loop
    k = 0
    residual_norm_history = []
    residual_norm_history.append(np.linalg.norm(r))
    while k < len(b) and np.linalg.norm(r) >= tol:  # stop when
        temp = r.T @ r

        alpha = temp / (d.T @ A @ d)
        x += alpha * d
        r = r - alpha * A @ d

        beta = (r.T @ r) / temp
        d = r + beta * d

        # Complete CG iteration here
        # Store the norm of the current residual
        residual_norm_history.append(np.linalg.norm(r))
        k += 1
    return x, np.array(residual_norm_history)

```

In [18]:
```python
# Create example symmetric positive definite linear system
n = 10
A = -np.diag(np.ones(n-1), -1) + 2*np.diag(np.ones(n)) - \
    np.diag(np.ones(n-1), 1)
b = np.arange(n)
x0 = np.zeros((n,))
# Solve with CG
x, resids = conjugate_gradient(A, b, x0, tol=1e-5)
print("Final residual ||Ax-b|| =", np.linalg.norm(A@x - b))

# Plot decrease in residuals
plt.figure(1)
plt.clf()
plt.semilogy(resids, 'o-')
plt.grid()
plt.xlabel('k')
plt.ylabel('||rk||')
plt.show()
```

```
Final residual ||Ax-b|| = 4.2335469486733446e-14
```



CG converges in 10 iterations and there is a drop in the value of error at 10th iteration.

Lab Book 08

In [19]:

```python
# Set the random number generator (for reproducibility)
np.random.seed(0)
# Build a random n*n orthogonal matrix of eigenvectors
n = 20
Q1 = np.linalg.qr(np.random.rand(n, n))[0]
Q2 = np.linalg.qr(np.random.rand(n, n))[0]
# Desired set of eigenvalues (need all > 0 for positive definit
evals1 = np.linspace(1, 1e5, n)
# evals2 = np.array([1,1,1,1
#                   ,1e2,1e2,1e2,1e2
#                   ,1e3,1e3,1e3,1e3
#                   ,1e4,1e4,1e4,1e4
#                   ,1e5,1e5,1e5,1e5])
# evals2 = np.array([1, 1e2, 1e3, 1e4, 1e5]*4)
evals2 = np.array([1, 1e5-3, 1e5-2, 1e5-1, 1e5]*4)
# Construct symmetric positive definite linear system with desi
A1 = Q1.T @ np.diag(evals1) @ Q1
A2 = Q1.T @ np.diag(evals2) @ Q1
b = np.arange(n)
x0 = np.zeros((n,))

```

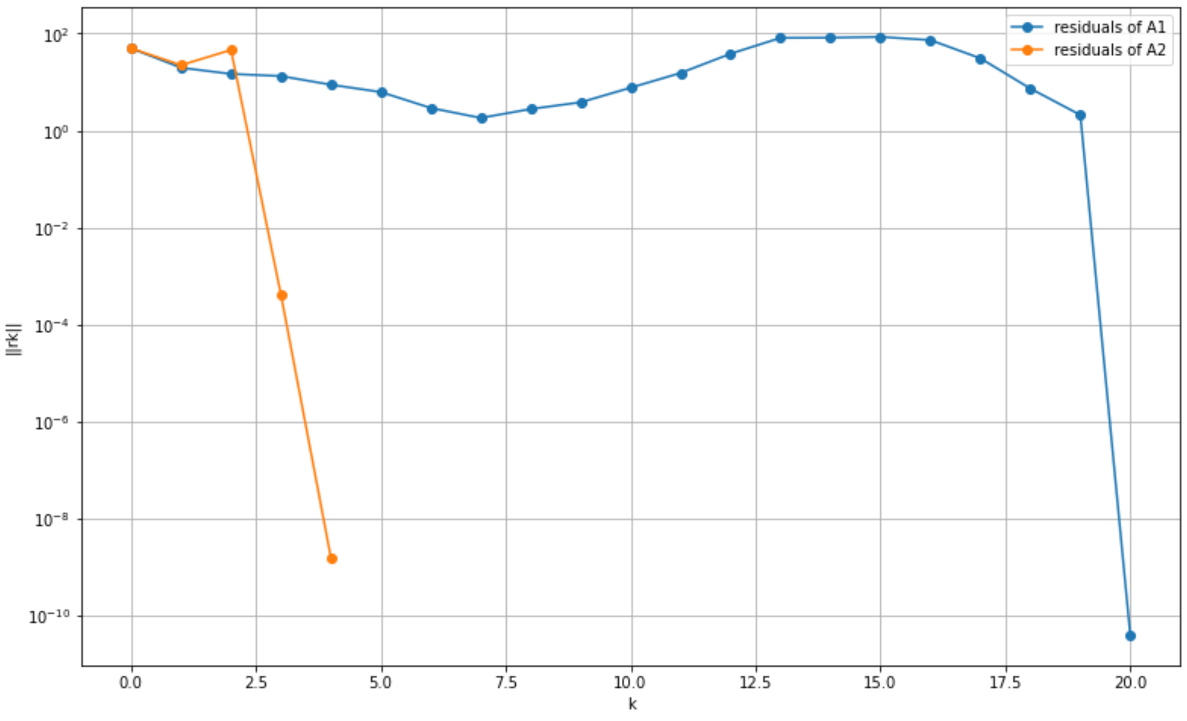
The condition number k(A) of matrix A is defined by the absolute value of maximum eigenvalue divided by the smallest eigenvalue. Therefore, we can find the above two matrix by setting their maximum eigenvalue to 1e5 and minimum to 1. Then construct a square matrix with eigenvalues as its diagonal. By multiplying two othogonal matrices to the square matrix can give us the required matrices.

In [20]:

```python
print(f"condition number of A1 is {np.linalg.cond(A1)}")
print(f"condition number of A2 is {np.linalg.cond(A2)}")
```

```
condition number of A1 is 100000.00000022692
condition number of A2 is 100000.00000133809
```

In [21]:
```
x0 = np.zeros((n,))
x1, resids1 = conjugate_gradient(A1, b, x0, tol=1e-5)
x2, resids2 = conjugate_gradient(A2, b, x0, tol=1e-5)
plt.figure(figsize=(13, 8))
plt.clf()
plt.semilogy(resids1, 'o-', label='residuals of A1')
plt.semilogy(resids2, 'o-', label='residuals of A2')
plt.legend(loc='best')
plt.grid()
plt.xlabel('k')
plt.ylabel('||rk||')
plt.show()

```



If we consider eigenvalues with relatively very small difference as a cluster, we can find that with same condition number, the matrix with fewer clusters will converge much faster. A1 has 20 clusters and needs exactly n = 20 itrations of CG to return the result while A2 only needs 5 iterations because there's only two clusters of eigenvalues.

In [22]:
```
print(f"start time: {t_start}\ntime1: {t_stop1}\ntime2: {t_stop
print(time.process_time_ns())
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent c
all last)
<ipython-input-22-b82844792960> in <module>
----> 1 print(f"start time: {t_start}\ntime1: {t_stop1}\ntime2: {t
_stop2}")
      2 print(time.process_time_ns())

NameError: name 't_start' is not defined
```

Lab Book 09

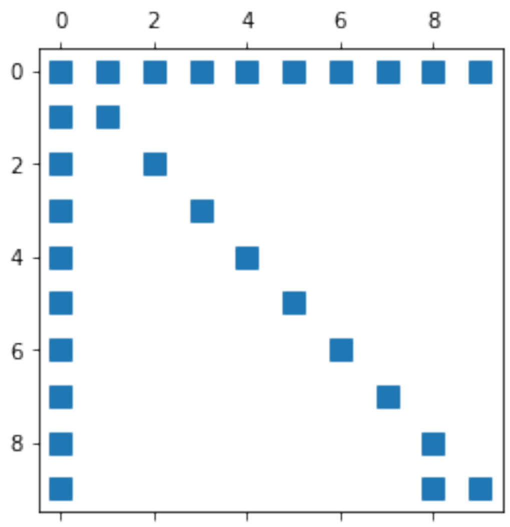
In [23]:
```python
# Column indices of the nonzero values
# They are ordered as: first column (top to bottom), first row
# Note: the '+' operator appends lists: [1,2] + [3,4,5] gives [
col_idx = [0]*10 + list(range(1, 10)) + list(range(1, 10))
print(col_idx)
# Row indices of the nonzero values
row_idx = list(range(10)) + [0]*9 + list(range(1, 10))
# Values of the nonzero entries
values = list(range(1, 11)) + list(range(2, 11)) + list(range(2
print(f"values{values}")
# Form the sparse matrix
nrows, ncols = 10, 10  # dimensions of A
A = sparse.coo_matrix((values, (row_idx, col_idx)),
                      shape=(nrows, ncols), dtype=float)
print(f"A is\n{A}")
# Convert to CSR format for efficient linear algebra (if desire
A = A.tocsr()
print(f"A' is\n{A}")
# Dimensions of a sparse matrix (same as NumPy arrays)
print("Dimensions =", A.shape)
# How many nonzero entries?
print("Number of nonzero entries =", A.nnz)
# The "sparsity" of a matrix is the fraction of entries which a
print("Sparsity =", A.nnz / (A.shape[0] * A.shape[1]))
# Convert to a regular NumPy array
# This is useful to check our code for col_idx, row_idx and val
# WARNING: do not do this for very large matrices (or you will
A_as_dense_matrix = A.toarray()
print(A_as_dense_matrix)
# Create a sparse identity matrix the same size as A
I = sparse.eye(nrows)
A_plus_two_I = A + 2*I
# check answer correct by printing A+I as a dense matrix
print(A_plus_two_I.toarray())
# Matrix-vector multiplication
x = np.arange(nrows)
print("Ax =", A @ x)
print("A^T x =", A.T @ x)
# Matrix-matrix multiplication
A_times_I = A @ I
print(A_times_I.toarray())
print("Entry (1,1) is", A[1, 1])
print("Entry (-1, -2) is", A[-1, -2])
# Change an entry from zero to nonzero
A[-1, -2] = -5.5
print("Entry (-1, -2) is", A[-1, -2])

```

```
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3,
4, 5, 6, 7, 8, 9]
values[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 2, 3, 4, 5, 6, 7, 8, 9, 10,
2, 3, 4, 5, 6, 7, 8, 9, 10]
A is
```

```
    (0, 0)        1.0
    (1, 0)        2.0
    (2, 0)        3.0
    (3, 0)        4.0
    (4, 0)        5.0
    (5, 0)        6.0
    (6, 0)        7.0
    (7, 0)        8.0
    (8, 0)        9.0
    (9, 0)        10.0
    (0, 1)        2.0
    (0, 2)        3.0
    (0, 3)        4.0
    (0, 4)        5.0
    (0, 5)        6.0
    (0, 6)        7.0
    (0, 7)        8.0
    (0, 8)        9.0
    (0, 9)        10.0
    (1, 1)        2.0
    (2, 2)        3.0
    (3, 3)        4.0
    (4, 4)        5.0
    (5, 5)        6.0
    (6, 6)        7.0
    (7, 7)        8.0
    (8, 8)        9.0
    (9, 9)        10.0
A' is
    (0, 0)        1.0
    (0, 1)        2.0
    (0, 2)        3.0
    (0, 3)        4.0
    (0, 4)        5.0
    (0, 5)        6.0
    (0, 6)        7.0
    (0, 7)        8.0
    (0, 8)        9.0
    (0, 9)        10.0
    (1, 0)        2.0
    (1, 1)        2.0
    (2, 0)        3.0
    (2, 2)        3.0
    (3, 0)        4.0
    (3, 3)        4.0
    (4, 0)        5.0
    (4, 4)        5.0
    (5, 0)        6.0
    (5, 5)        6.0
    (6, 0)        7.0
    (6, 6)        7.0
    (7, 0)        8.0
    (7, 7)        8.0
    (8, 0)        9.0
    (8, 8)        9.0
    (9, 0)        10.0
    (9, 9)        10.0
```

```
Dimensions = (10, 10)
Number of nonzero entries = 28
Sparsity = 0.28
[[ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
 [ 2.  2.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 3.  0.  3.  0.  0.  0.  0.  0.  0.  0.]
 [ 4.  0.  0.  4.  0.  0.  0.  0.  0.  0.]
 [ 5.  0.  0.  0.  5.  0.  0.  0.  0.  0.]
 [ 6.  0.  0.  0.  0.  6.  0.  0.  0.  0.]
 [ 7.  0.  0.  0.  0.  0.  7.  0.  0.  0.]
 [ 8.  0.  0.  0.  0.  0.  0.  8.  0.  0.]
 [ 9.  0.  0.  0.  0.  0.  0.  0.  9.  0.]
 [10.  0.  0.  0.  0.  0.  0.  0.  0. 10.]]
[[ 3.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
 [ 2.  4.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 3.  0.  5.  0.  0.  0.  0.  0.  0.  0.]
 [ 4.  0.  0.  6.  0.  0.  0.  0.  0.  0.]
 [ 5.  0.  0.  0.  7.  0.  0.  0.  0.  0.]
 [ 6.  0.  0.  0.  0.  8.  0.  0.  0.  0.]
 [ 7.  0.  0.  0.  0.  0.  9.  0.  0.  0.]
 [ 8.  0.  0.  0.  0.  0.  0. 10.  0.  0.]
 [ 9.  0.  0.  0.  0.  0.  0.  0. 11.  0.]
 [10.  0.  0.  0.  0.  0.  0.  0.  0. 12.]]
Ax = [330.   2.   6.  12.  20.  30.  42.  56.  72.  90.]
A^T x = [330.   2.   6.  12.  20.  30.  42.  56.  72.  90.]
[[ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
 [ 2.  2.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 3.  0.  3.  0.  0.  0.  0.  0.  0.  0.]
 [ 4.  0.  0.  4.  0.  0.  0.  0.  0.  0.]
 [ 5.  0.  0.  0.  5.  0.  0.  0.  0.  0.]
 [ 6.  0.  0.  0.  0.  6.  0.  0.  0.  0.]
 [ 7.  0.  0.  0.  0.  0.  7.  0.  0.  0.]
 [ 8.  0.  0.  0.  0.  0.  0.  8.  0.  0.]
 [ 9.  0.  0.  0.  0.  0.  0.  0.  9.  0.]
 [10.  0.  0.  0.  0.  0.  0.  0.  0. 10.]]
Entry (1,1) is 2.0
Entry (-1, -2) is 0.0
Entry (-1, -2) is -5.5

/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/scipy/sparse/
_index.py:82: SparseEfficiencyWarning: Changing the sparsity struc
ture of a csr_matrix is expensive. lil_matrix is more efficient.
  self._set_intXint(row, col, x.flat[0])
```

In [24]:
```python
plt.figure(1)
plt.clf()
plt.spy(A)
plt.show()

```



In [25]:
```python
def generate_sparse(n):
    base1 = list([7/2]*n)
    base2 = list([-4/3]*(n-1))
    base3 = list([1/12]*(n-2))
    A = sparse.diags([base1, base2, base2, base3, base3],
                     offsets=[0, 1, -1, 2, -2], format='csr', d
    # A = A.tocsr()
    return A

```

In [26]:
```python
n = 5
A = generate_sparse(n)
print(f"The CSR format of A is:\n{A}")
print(f"The {n}*{n} sparse matrix is:\n{A.toarray()}")

```

```
The CSR format of A is:
  (0, 0)	3.5
  (0, 1)	-1.3333333333333333
  (0, 2)	0.08333333333333333
  (1, 1)	3.5
  (1, 2)	-1.3333333333333333
  (1, 0)	-1.3333333333333333
  (1, 3)	0.08333333333333333
  (2, 2)	3.5
  (2, 3)	-1.3333333333333333
  (2, 1)	-1.3333333333333333
  (2, 4)	0.08333333333333333
  (2, 0)	0.08333333333333333
  (3, 3)	3.5
  (3, 4)	-1.3333333333333333
  (3, 2)	-1.3333333333333333
  (3, 1)	0.08333333333333333
  (4, 4)	3.5
  (4, 3)	-1.3333333333333333
  (4, 2)	0.08333333333333333
The 5*5 sparse matrix is:
[[ 3.5        -1.33333333  0.08333333  0.          0.        ]
 [-1.33333333  3.5        -1.33333333  0.08333333  0.        ]
 [ 0.08333333 -1.33333333  3.5        -1.33333333  0.08333333]
 [ 0.          0.08333333 -1.33333333  3.5        -1.33333333]
 [ 0.          0.          0.08333333 -1.33333333  3.5       ]]
```

Lab Book 10

In [27]:
```
n = [10, 100, 1000]
times = np.zeros([len(n), 2])
times2 = np.zeros(len(n))
# niter = 50
# for k in range(niter):
for i in range(len(n)):
    b = np.arange(n[i])
    A = generate_sparse(n[i])
    t_start = time.time()
    x = linalg.solve(A.toarray(), b)
    t_stop1 = time.time()
    x = spsolve(A, b)
    t_stop2 = time.time()
    times[i] = list([t_stop1-t_start, t_stop2-t_stop1])
    print(f"n = {n[i]}\nnp.linalg.solve takes {times[i][0]} secon


```

```
n = 10
np.linalg.solve takes 0.000247955322265625 seconds
scipy.sparse.linalg.spsolve takes 0.0028259754180908203 seconds
n = 100
np.linalg.solve takes 0.0020210742950439453 seconds
scipy.sparse.linalg.spsolve takes 0.0009870529174804688 seconds
n = 1000
np.linalg.solve takes 0.04650712013244629 seconds
scipy.sparse.linalg.spsolve takes 0.0013680458068847656 seconds
```

With the increasing of n, np.linalg.solve clearly takes more time, which takes about 100 times time when n = 1000 compared to n = 10. However, spsolve almost takes similar time when n = 10, 100 and 1000.