In [1]:
```python
import numpy as np
import math
import scipy.integrate
import matplotlib.pyplot as plt
```

In [2]:
```python
# Get floating-point information for default float data type
float_info = np.finfo(float)
# Machine epsilon
print(float_info.eps)
```

```
2.220446049250313e-16
```

Lab Book 01

In [3]:
```python
# a)
print("the largest positive finite machine number is", float_in
# b)
print("the largest absolute value of negative finite machine nu
# c)
print("smallest positive machine number is", float_info.tiny)
```

```
the largest positive finite machine number is 1.7976931348623157e+
308
the largest absolute value of negative finite machine number is 1.
7976931348623157e+308
smallest positive machine number is 2.2250738585072014e-308
```

In [4]:

```
# Generate np.nan
x = np.arange(5)
print(x / x)
# Generate np.inf
y = np.ones((5,))
print(y / x)
# Is infinity equal to itself?
print(np.inf == np.inf)
# Is nan equal to itself?
print(np.nan == np.nan)
# Compare nan to other values
print(np.nan < 0, np.nan == 0, np.nan > 0)

```

```
[nan  1.  1.  1.  1.]
[       inf 1.         0.5        0.33333333 0.25      ]
True
False
False False False

/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/ipykernel_lau
ncher.py:3: RuntimeWarning: invalid value encountered in true_divi
de
  This is separate from the ipykernel package so we can avoid doin
g imports until
/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/ipykernel_lau
ncher.py:6: RuntimeWarning: divide by zero encountered in true_div
ide
```

In [5]:

```
print(1+2==3)
print(0.1+0.2==0.3)
print(0.1+0.2+0.7==1)
```

```
True
False
True
```

Lab Book 02

In [6]:

```
print("sin(pi) = ",math.sin(math.pi))
print("cos(pi/2) = ",math.cos(0.5*math.pi))
print("sin(pi/2) = ",math.sin(0.5*math.pi))
print("cos(0) = ",math.cos(0))
print("cos(pi) = ",math.cos(math.pi))
```

```
sin(pi) =  1.2246467991473532e-16
cos(pi/2) =  6.123233995736766e-17
sin(pi/2) =  1.0
cos(0) =  1.0
cos(pi) =  -1.0
```

When the value of sin or cos should be 0, python can't return the exact value but can only return extremely small number. When the value should be +-1, python can give the exactly right answer.

Lab Book 03

In [7]:
```
def f(x):
    return (np.cos(2*x)-1) / pow(x,2)
def taylorF(x):
    return 2 * pow(x,2)/3 - 2
def alterF(x):
    return -2 * pow(np.sin(x),2) / pow(x,2)
```

In [8]:
```python
x=np.array([1e-2,1e-3,1e-4,1e-5,1e-6,1e-7,1e-8,1e-9,1e-10])
for item in x:
    print("x = ", item)
    print("f(x) = ", f(item))
    print("result of Talor series = ", taylorF(item), ", the er
    print("improved result = ", alterF(item), "the difference w
```

```
x =  0.01
f(x) =  -1.9999333342224368
result of Talor series =  -1.9999333333333333 , the error is  8.89
1034575242429e-10
improved result =  -1.9999333342222159 the difference with Taylor
series is  8.888825231423425e-10
x =  0.001
f(x) =  -1.999999333368585
result of Talor series =  -1.9999993333333332 , the error is  3.52
5180147789797e-11
improved result =  -1.9999993333334223 the difference with Taylor
series is  8.903988657493755e-14
x =  0.0001
f(x) =  -1.999999987845058
result of Talor series =  -1.9999999933333332 , the error is  5.48
8275167664369e-09
improved result =  -1.9999999933333334 the difference with Taylor
series is  2.220446049250313e-16
x =  1e-05
f(x) =  -2.0000001654807416
result of Talor series =  -1.9999999999333333 , the error is  1.65
54740822627423e-07
improved result =  -1.999999999933333 the difference with Taylor s
eries is  2.220446049250313e-16
x =  1e-06
f(x) =  -1.999955756559757
result of Talor series =  -1.9999999999993334 , the error is  4.42
4343957643018e-05
improved result =  -1.9999999999993332 the difference with Taylor
series is  2.220446049250313e-16
x =  1e-07
f(x) =  -1.998401444325282
result of Talor series =  -1.9999999999999933 , the error is  0.00
15985556747113439
improved result =  -1.9999999999999933 the difference with Taylor
series is  0.0
x =  1e-08
f(x) =  -2.2204460492503126
result of Talor series =  -2.0 , the error is  0.22044604925031264
improved result =  -2.0 the difference with Taylor series is  0.0
x =  1e-09
f(x) =  0.0
result of Talor series =  -2.0 , the error is  2.0
improved result =  -2.0 the difference with Taylor series is  0.0
x =  1e-10
f(x) =  0.0
result of Talor series =  -2.0 , the error is  2.0
improved result =  -2.0 the difference with Taylor series is  0.0
```

Python has an bigger error when x is smaller when calculating f(x) because of the subtraction in numerator will lead to catastrophic cancellation when two values are close to 0 and therefore leads to big error.

Lab Book 04

In [9]:
```python
def expTaylor(x):
    term = 1.0
    result = 0.0
    n = 1
    while term + result != result:
        term = pow(x,n) / math.factorial(n)
        result += term
        n += 1
    return result + 1
for item in x:
    print("x = ", item)
    print("exp(x) = ",math.exp(item)-1)
    print("expm1(x) = ",math.expm1(item))
    print("Taylor series = ",expTaylor(item)-1)
```

```
x =  0.01
exp(x) =  0.010050167084167949
expm1(x) =  0.010050167084168058
Taylor series =  0.010050167084167949
x =  0.001
exp(x) =  0.0010005001667083846
expm1(x) =  0.0010005001667083417
Taylor series =  0.0010005001667083846
x =  0.0001
exp(x) =  0.00010000500016667141
expm1(x) =  0.00010000500016667084
Taylor series =  0.00010000500016667141
x =  1e-05
exp(x) =  1.0000050000069649e-05
expm1(x) =  1.0000050000166668e-05
Taylor series =  1.0000050000069649e-05
x =  1e-06
exp(x) =  1.0000004999621837e-06
expm1(x) =  1.0000005000001665e-06
Taylor series =  1.0000004999621837e-06
x =  1e-07
exp(x) =  1.0000000494336803e-07
expm1(x) =  1.0000000500000016e-07
Taylor series =  1.0000000494336803e-07
x =  1e-08
exp(x) =  9.99999993922529e-09
expm1(x) =  1.0000000050000001e-08
Taylor series =  9.99999993922529e-09
x =  1e-09
exp(x) =  1.000000082740371e-09
expm1(x) =  1.0000000005000001e-09
Taylor series =  1.000000082740371e-09
x =  1e-10
exp(x) =  1.000000082740371e-10
expm1(x) =  1.00000000005e-10
Taylor series =  1.000000082740371e-10
```

Since e^x is quite close to 1 when x is close to 0, therefore the digits after 1 can not be stored properly and when calculating e^x -1, the catastrophic cancellation will occur and lead to inaccurate results compared to expm1(x). Similar error occurs when using Taylor series expansion due to the same result.

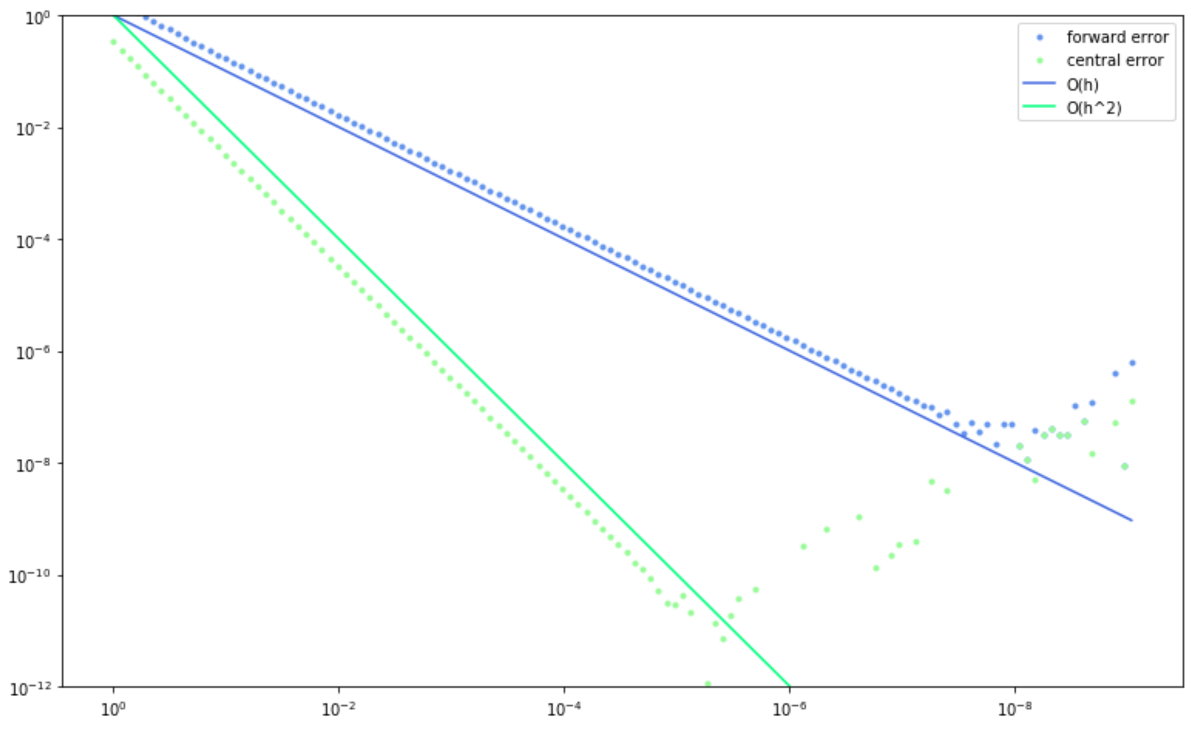
Lab Book 05

In [10]:
```
def f(x):
    return math.exp(x) - math.cos(x)
def df(x):
    return math.exp(x) + math.sin(x)
```

In [11]:
```
# Decreasing sequence of h values
hs = 2**(-np.linspace(0,30,128))
# Make an empty vector of errors
forward_error = np.zeros((len(hs),))
central_error = np.zeros((len(hs),))
for i in range(len(hs)):
    h = hs[i]  # current value of h
    forward_error[i] = (f(1+h) - f(1)) / h - df(1)
    central_error[i] = (f(1+h) - f(1-h)) / (2*h) - df(1)
# The below vectors show the error decrease for particular orde
first_order_rate = hs      # plot hs vs first_order_rate to see
second_order_rate = hs**2  # plot hs vs second_order_rate to se

# Plot hs vs forward_error and central_error (on a log-log plot
plt.figure(figsize = (13,8))
plt.clf()
plt.loglog(hs, forward_error,'.', label = "forward error", colo
plt.loglog(hs, central_error,'.', label = "central error", colo
plt.loglog(hs, first_order_rate, label = "O(h)", color = 'royal
plt.loglog(hs, second_order_rate, label = "O(h^2)", color='spri
plt.legend(loc = 'best')
ax = plt.gca()
ax.invert_xaxis()
plt.ylim(1e-12,1)
plt.show
```

Out[11]:
```
<function matplotlib.pyplot.show(*args, **kw)>
```



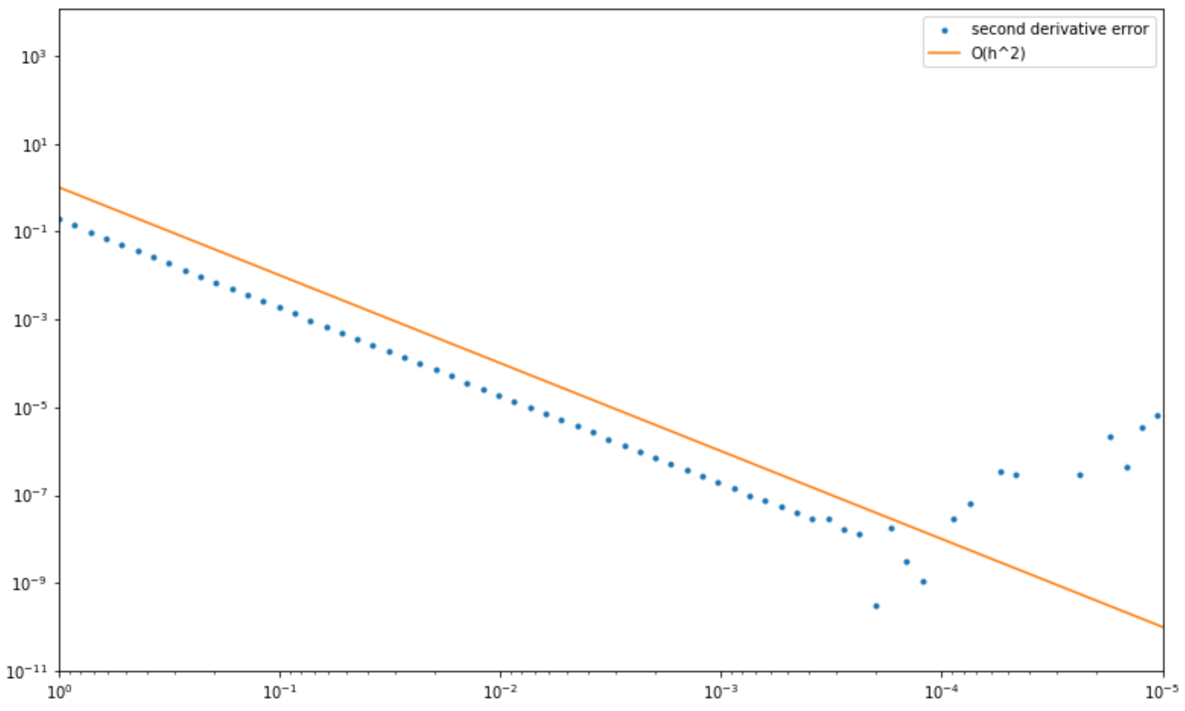
As h approaches 0, the forward error is almost the same as O(h)'s convergence rate before reaching the machine epsilon, which suggests it's order of convergence is 1. Similarly, the central error is almost the same as O(h^2) before reaching the machine epsilon, therefore it has an order of convergence of 2.

Lab Book 06

In [12]:
```
def d2f(x):
    return math.exp(x) + math.cos(x)
second_error = np.zeros((len(hs),))
for i in range(len(hs)):
    h = hs[i]
    second_error[i] = (f(1+h) - 2*f(1) + f(1-h)) / pow(h,2) - d

plt.figure(figsize = (13,8))
plt.clf()
plt.loglog(hs, second_error,'.', label = "second derivative err
plt.loglog(hs, second_order_rate, label = "O(h^2)")
plt.legend(loc = 'best')
plt.xlim(1e-5,1)
plt.ylim(1e-11,)
ax = plt.gca()
ax.invert_xaxis()
plt.show
```

Out[12]:
```
<function matplotlib.pyplot.show(*args, **kw)>
```



As h approaches to 0, the error of second order derivative's approximation is almost the same as O(h^2)'s convergence rate before reaching the machine epsilon, this suggests that it's order of convergence is 2.

Lab Book 07

In [13]:
```
def left_riemann_sum(xs, ys):
    n = len(xs)
    if len(xs) != len(ys):
        return np.NaN
    integrals = np.zeros(n)
    x0s = xs[:n-1]
    x1s = xs[1:]
    integrals = (x1s-x0s) * ys[:n-1]
    return sum(integrals)
def f(x):
    return math.exp(-x)
```

In [14]:
```
a = -1
b = 1
n = 100
xs = np.linspace(a, b, n)  # equally spaced points xi
ys = np.zeros(n)
for i in range(n-1):
    ys[i] = f(xs[i])
print("Left Riemann sum =", left_riemann_sum(xs, ys))
print("True value is around 2.350402387287603")
```

```
Left Riemann sum = 2.374223762501847
True value is around 2.350402387287603
```

The left Reimann Sum can roughly approximate the integral and has 2 acutare digits. The accuracy can also be improved by adding more points and make each points closer but in a raletively low convergence rate.

Lab Book 08

In [15]:
```
def trapezoidal_rule(xs, ys):
    n = len(xs)
    if len(xs) != len(ys):
        return np.NaN
    integrals = np.zeros(n)
    x0s = xs[:n-1]
    x1s = xs[1:]
    y0s =  ys[:n-1]
    y1s =ys[1:]
    integrals = (x1s-x0s) * (y0s+y1s) / 2
    return sum(integrals)
```

In [16]:
```
print("Trapezoidal Rule =", trapezoidal_rule(xs, ys))
print("Left Riemann sum =", left_riemann_sum(xs, ys))
print("True value is around 2.350402387287603")
```

```
Trapezoidal Rule = 2.346766370295189
Left Riemann sum = 2.374223762501847
True value is around 2.350402387287603
```

The Trapezoidal Rule clearly has a much more accuracy with same number of points although it also can only give two accurate digits. When incresing the number of points, the Trapezoidal Rule has a faster convergence rate than left Riemann Sum.

Lab Book 09

In [17]:
```python
def runges(x):
    return 1 / (1 + 25*pow(x,2))
```

In [18]:
```python
n = 20
xs = np.linspace(a, b, n)  # equally spaced points xi
ys = runges(xs)
print("Simpson's Rule = ", scipy.integrate.simps(ys, xs))
print("Gaussian Quadrature = ", scipy.integrate.fixed_quad(rung
print("True value is around 0.549360307")
```

```
Simpson's Rule =  0.5493758748195977
Gaussian Quadrature =  0.548997098104952
True value is around 0.549360307
```

Both ways gives three acurate digits compared to the ture value. The Simpson's Rule in python is more accurate than Gaussian Quadrature and have errors at 10^5 and 10^-3 respectively.

Lab Book 10

In [19]:
```python
print("The approximation is ", scipy.integrate.quad(runges, a,
```

```
The approximation is  0.5493603067780066 , the error is  2.8668279
350011863e-09
```

The output is quite accurate compared to the true value and the error is at 10^-9, which is a very small error compared to the above two methods.

In [ ]:
```python

```