MATH3511/6111 Scientific Computing
Semester 1, 2022 — Lindon Roberts (MSI)

Assignment 1 (MATH3511 non-HPO)
Due date: 9am Tuesday 15 March (Week 4)

Please show all relevant working and present your solutions clearly: do not expect full marks for a correct answer without working, or where your reasoning is hard to follow.

Question 1 (3 marks)**.** Use the bisection method to find one of the places where the graphs of $f(x) = e^{x-2}$ and $g(x) = 1/(x+1)^2$ intersect by finding a root of $f(x) - g(x)$ correct to two decimal digits. Take $a = 0.5$ and $b = 1$ as your starting values. Use a hand calculator. Produce a table similar to that given in page 16 of the rootfinding lecture slides. Figure 1 shows a plot of $f(x)$ and $g(x)$.

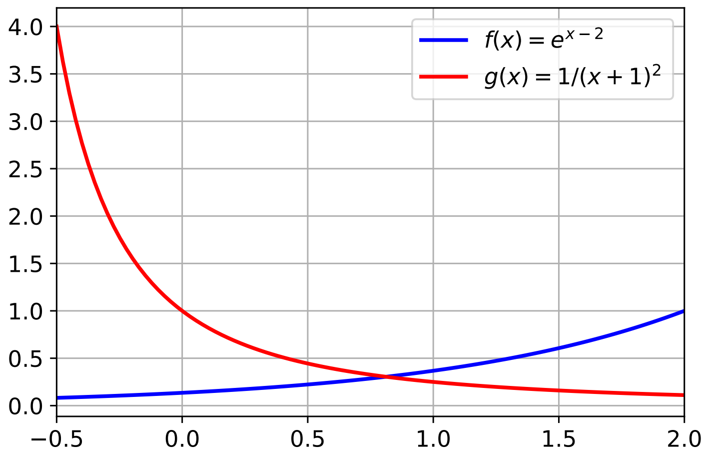

Figure 1: Plot of $f(x)$ and $g(x)$ for $x \in [-0.5, 2]$.

Question 2 (5 marks)**.** In this question, we will approximate the function $f(x) = e^{2x}$ using Taylor polynomials.

(a) Calculate the cubic (i.e. $k = 3$) Taylor polynomial for $f(x)$ based at $a = 0$. [1 mark]

(b) Calculate an upper bound on the Taylor error in (a) valid for all $x \in [-1, 1]$.[1] [2 marks]

(c) Find a value of k such that k-order Taylor error is at most 10^{-5} for all $x \in [-1, 1]$. You do not have to write down the corresponding polynomial. [2 marks]

Question 3 (4 marks)**.** We know from lectures that Newton's method has a quadratic convergence rate, $e_{n+1} \approx Ce_n^2$ for some $C > 0$ whenever $e_n := r - x_n$ is sufficiently small and $f'(r) \neq 0$. However, when we checked this rate in lectures (slide 45) we looked at how quickly $f(x_n) \to 0$, not how quickly $e_n \to 0$.

Show, under the same assumptions as the analysis in lectures, that $f(x_n)$ converges to zero quadratically provided x_n is sufficiently close to r.

Question 4 (4 marks)**.** Consider two new algorithms for rootfinding:

- Bisection2: given an interval $[a_n, b_n]$ containing a root with estimate $m_n = (a_n + b_n)/2$, run two iterations of the regular bisection method to get a new interval $[a_{n+1}, b_{n+1}]$ with root estimate $m_{n+1} = (a_{n+1} + b_{n+1})/2$.
- Newton2: given a root estimate x_n, calculate a new estimate x_{n+1} by running two iterations of Newton's method.

[1] i.e. find a number E such that $|f(x) - p(x)| \leq E$ for all $x \in [-1, 1]$, where $p(x)$ is the Taylor polynomial from (a).

Under suitable assumptions on the problem, calculate the order of convergence of both methods and compare to the corresponding regular method from lectures.

Question 5 (4 marks)**.** Suppose we are implementing our own calculator and we need to use Newton's method to evaluate the division operator. Specifically, suppose we wish to calculate $1/c$ for some number $c > 0$. Show that Newton's method using $f(x) = cx - 1$ and $f(x) = \frac{1}{x} - c$ gives two different formulae. Which formula is more useful and why?