In [1]:

```
import numpy as np
import cmath
import time
import matplotlib.pyplot as plt
import scipy.io.wavfile as wavfile
import scipy.integrate as integrate
# import random
```

Lab Book 01

In [2]:

```
def FFMatrix(n):
    F = np.zeros((n, n), dtype=complex)
    omega = np.exp(-2*cmath.pi/n*1j)
    for i in range(n):
        for j in range(n):
            F[i][j] = pow(omega, i*j)
    return F

```

In [3]:

```
n = 10
Fn = FFMatrix(n)
print(f"Check F{n} is symmetric by F{n} = F{n}.T:\n{Fn == Fn.T}
nI = n*np.eye(n)
Fn_conj = np.conj(Fn)
# print(f"F{n}:\n{Fn}")
# print(f"F{n}·F{n}_conj = \n{Fn@Fn_conj}")
# print(f"{n}·I = \n{nI}")
print(
    f"Check ||F{n}·F{n}_conj|| = ||{n}·I||:\n{np.isclose(np.lin

```

```
Check F10 is symmetric by F10 = F10.T:
[[ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]
 [ True  True  True  True  True  True  True  True  True  True]]
Check ||F10·F10_conj|| = ||10·I||:
True
```

Lab Book 02

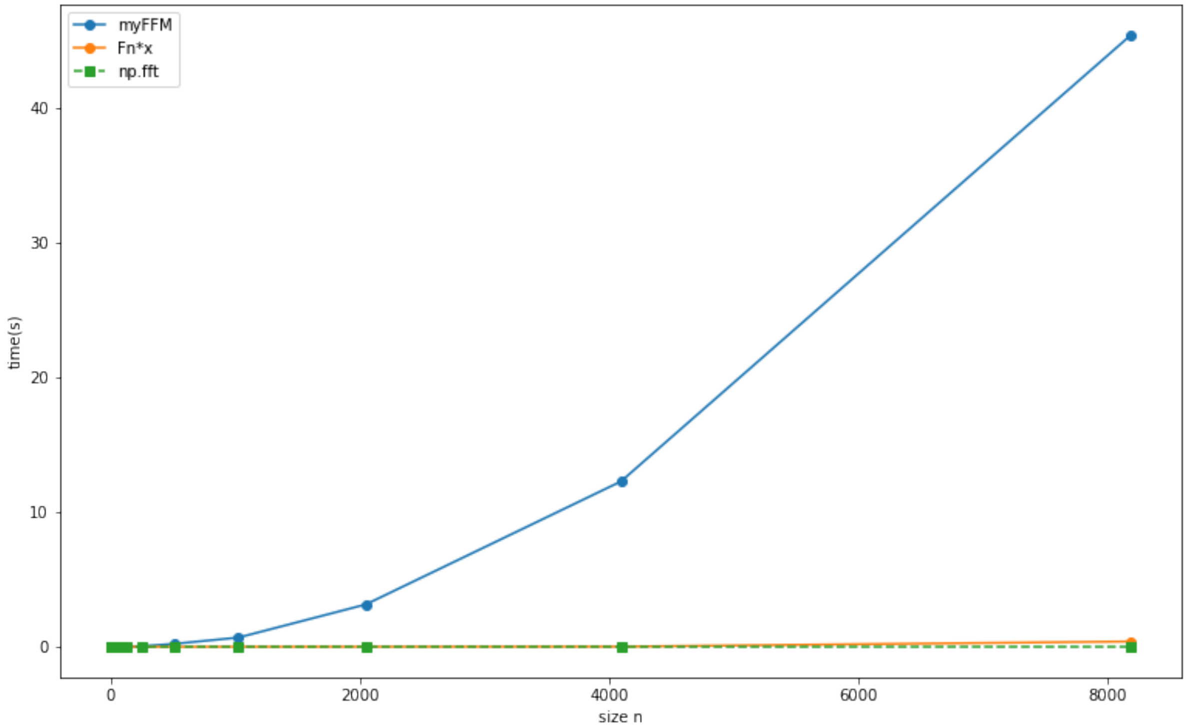
In [4]:

```python
ns = [pow(2, i) for i in range(14)]  # 12/14
FFM_runtime = np.zeros((len(ns), 3))
print("{0:^20}{1:^20}{2:^20}".format(
    "Build Fn", "Calculate DFT", "Use np.fft"))
for i in range(len(ns)):
    xs = np.arange(1, ns[i]+1)
    time_start = time.time()
    # Build matrix Fn
    Fn = FFMatrix(ns[i])
    time1 = time.time()
    # Calculate DFT
    x_hat = Fn @ xs
    time2 = time.time()
    # Calculate the same DFT with np.fft
    Fn2 = np.fft.fft(xs)
    time3 = time.time()
    FFM_runtime[i][0] = time1-time_start
    FFM_runtime[i][1] = time2-time1
    FFM_runtime[i][2] = time3-time2
    print(
        f'{FFM_runtime[i][0]:^20.16f}{FFM_runtime[i][1]:^20.16f

```

```
      Build Fn          Calculate DFT          Use np.fft
 0.0000371932983398  0.0006787776947021  0.0001871585845947
 0.0000460147857666  0.0020492076873779  0.0002048015594482
 0.0000450611114502  0.0001018047332764  0.0000228881835938
 0.0000460147857666  0.0000090599060059  0.0000100135803223
 0.0004668235778809  0.0000169277191162  0.0000150203704834
 0.0006151199340820  0.0000281333923340  0.0000298023223877
 0.0024969577789307  0.0000498294830322  0.0002212524414062
 0.0109620094299316  0.0009467601776123  0.0004591941833496
 0.0499477386474609  0.0001351833343506  0.0000789165496826
 0.2213809490203857  0.0002779960632324  0.0000419616699219
 0.6824581623077393  0.0008959770202637  0.0000710487365723
 3.1462609767913818  0.0034582614898682  0.0001409053802490
12.2830846309661865  0.0112092494964600  0.0002660751342773
45.4605779647827148  0.3940839767456055  0.0006079673767090
```

In [5]:

```
plt.figure(figsize=(13, 8))
plt.clf()
plt.plot(ns, FFM_runtime[:, 0], 'o-', label='myFFM',)
plt.plot(ns, FFM_runtime[:, 1], 'o-', markersize=6, label='Fn*x
plt.plot(ns, FFM_runtime[:, 2], 's--', label='np.fft')
plt.xlabel('size n')
plt.ylabel('time(s)')
plt.legend(loc='best')
plt.show()
```



Due to the double for loops of size n used in my DFT, the time complexity of it is O(n^2), therefore when the size of matrix is pretty big, the time consumed is increasing at a rate of n^2. Compared to the O(n*logn) time complexity of FFT, the DFT method is really time consuming and low efficient when n is big.

Lab Book 03

In [6]:

```
data = np.loadtxt('lab5_piano_data.csv', delimiter=',')
time = np.linspace(0.0, 1.0, len(data))  # data represents 1 se
print("CSV has a vector of size =", data.shape)
np.random.seed(0)
noisy_data = data + 0.005 * np.random.randn(len(data))
```

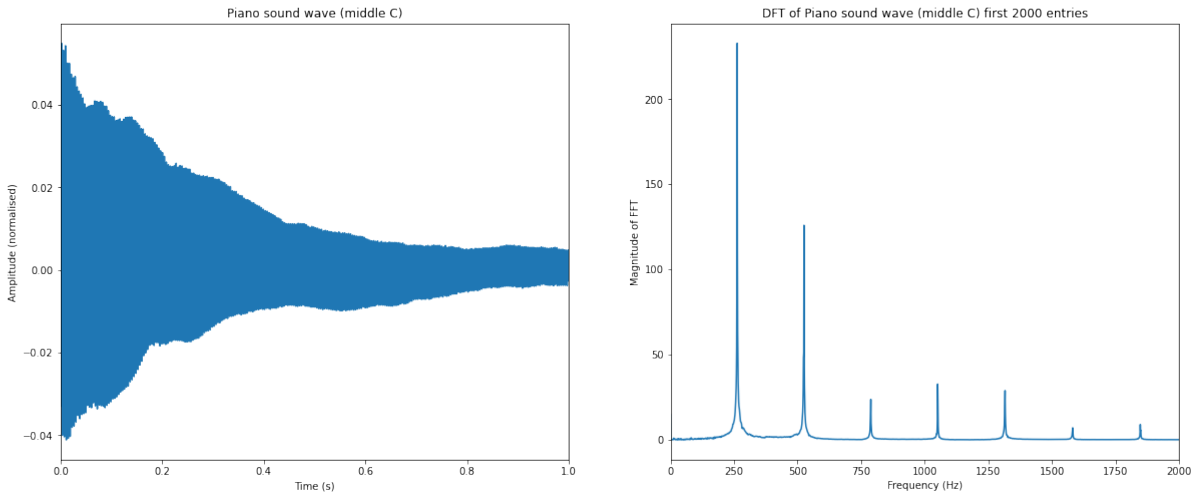
```
CSV has a vector of size = (44100,)
```

In [7]:

```
# Save data as playable files
samplerate = 44100  # samples per second in the audio
# Note: after an inverse DFT you usually get complex values wit
# (of size machine epsilon), which we need to remove before sav
wavfile.write('my_audio.wav', samplerate, np.real(data))
wavfile.write('my_audio_noise.wav', samplerate, np.real(noisy_d
```

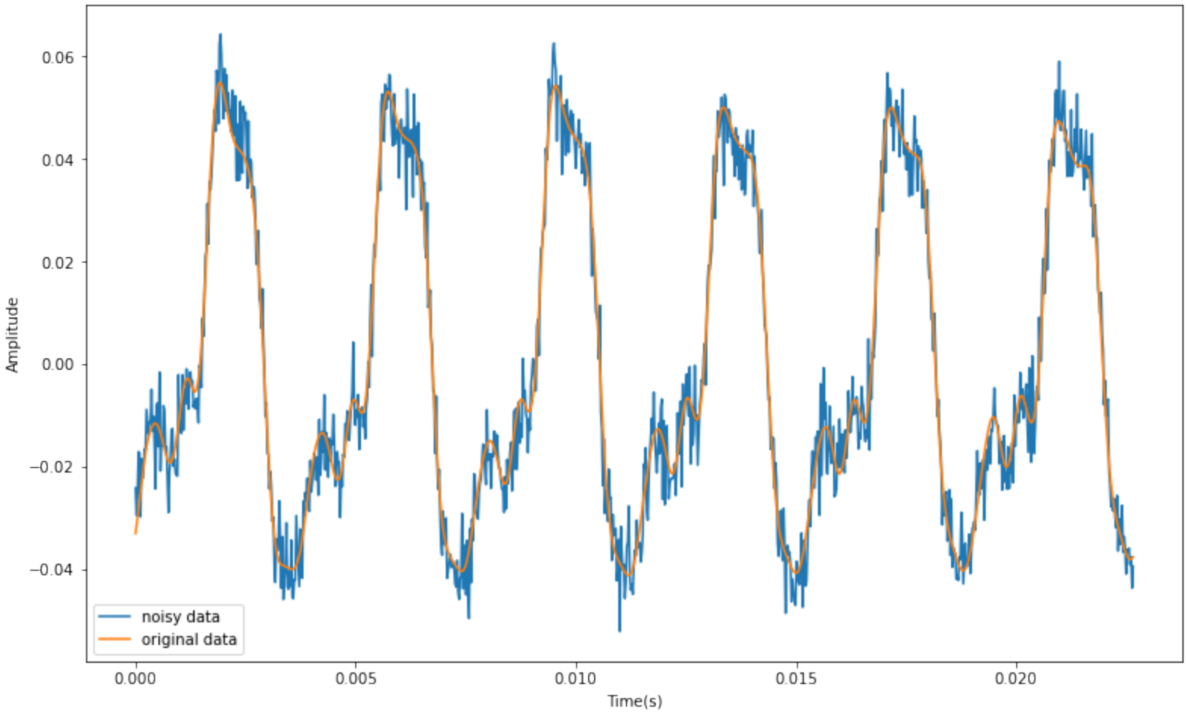
In [8]:

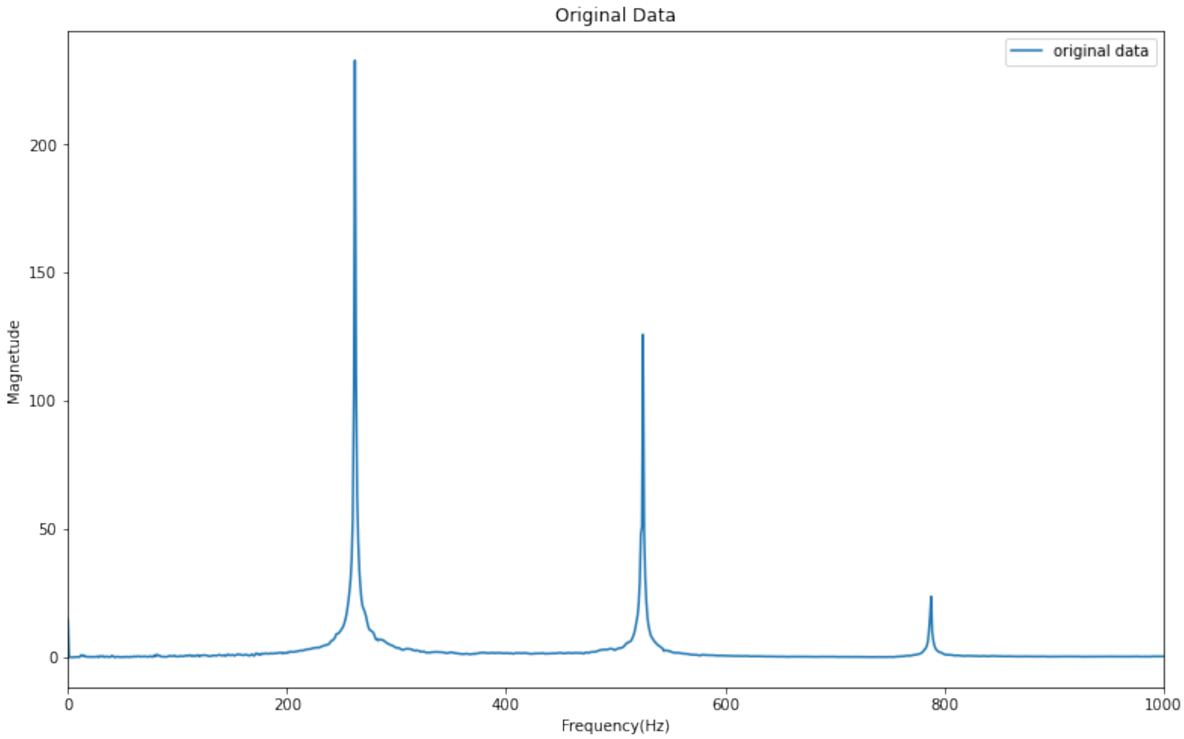
```
plt. figure(figsize= (20,8))
plt. subplot(1,2,1)
plt. plot(time,data)
plt.title('Piano sound wave (middle C)')
plt.xlabel('Time (s)')
plt.ylabel('Amplitude (normalised)')
plt. xlim(0.0,1.0)

plt. subplot(1,2,2)
plt. plot(np. arange(len(data)),abs(np. fft. fft(data[:])))
plt.title('DFT of Piano sound wave (middle C) first 2000 entrie
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude of FFT')
plt. xlim(0.0,2000)

plt.show()
```

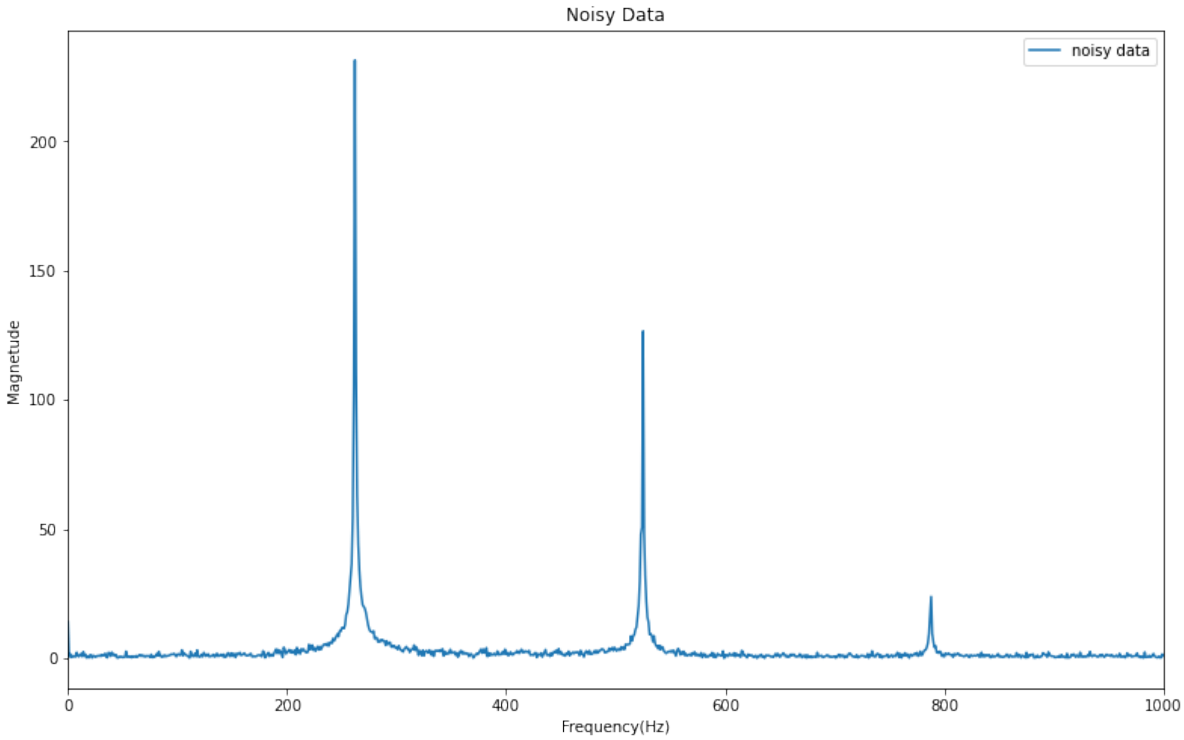


In [9]:

```
frequency = data.shape[0]
time = np.linspace(0, 1, data.shape[0])
fs = np.arange(0, frequency)
# first 2000 elements
n = 1000

plt.figure(figsize=[13, 8])
plt.plot(time[:n], noisy_data[:n], label='noisy data')
plt.plot(time[:n], data[:n], label='original data')
plt.xlabel('Time(s)')
plt.ylabel('Amplitude')
plt.legend(loc='best')
plt.show()

plt.figure(figsize=[13, 8])
# plt.plot(fs, np.fft.fftshift(
#     abs(np.fft.fft(np.real(noisy_data)))), label='noisy data'
plt.plot(fs, abs(np.fft.fft(np.real(data))), label='original da
plt.xlabel('Frequency(Hz)')
plt.ylabel('Magnetude')
plt.title('Original Data')
plt.legend(loc='best')
plt.xlim(0, n)
plt.show()

noisy_data_hat = np.fft.fft(noisy_data)
plt.figure(figsize=[13, 8])
plt.plot(fs, abs(noisy_data_hat), label='noisy data')
# plt.plot(fs, np.fft.fftshift(
#     abs(np.fft.fft(np.real(data)))), label='original data')
plt.xlabel('Frequency(Hz)')
plt.ylabel('Magnetude')
plt.title('Noisy Data')
plt.legend(loc='best')
plt.xlim(0, n)
plt.show()
```







In [10]:

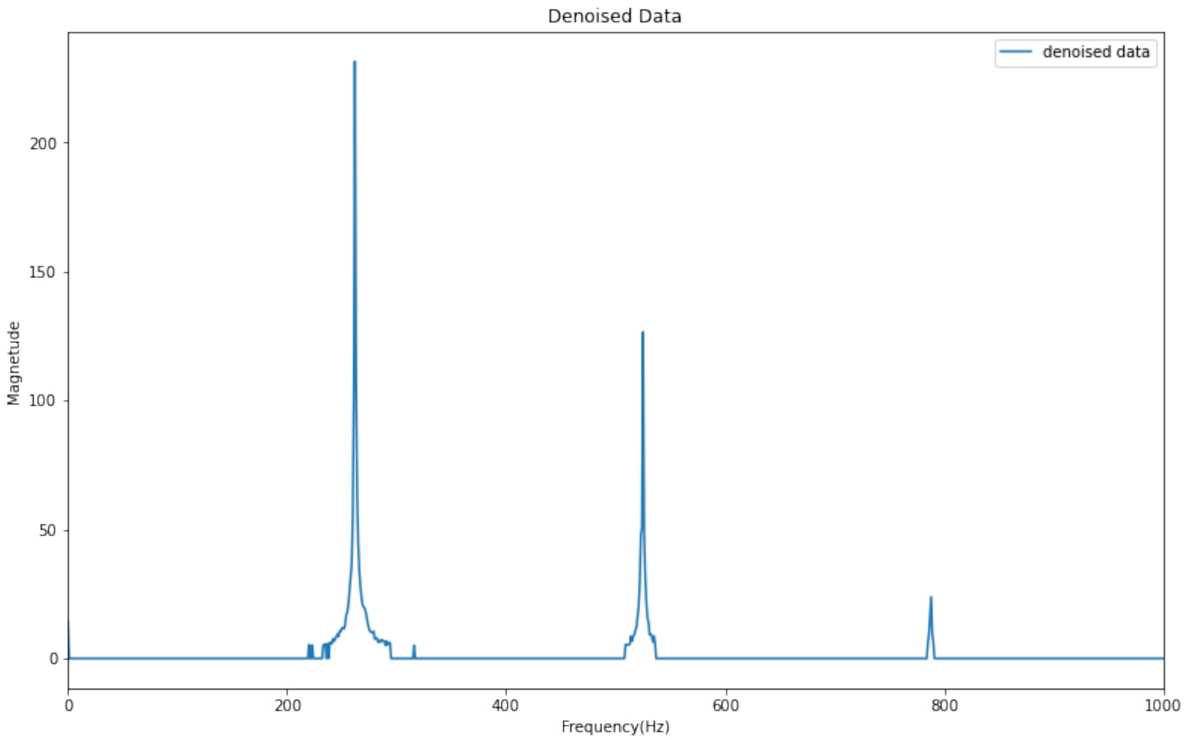
```
def remove_magnitude(x,lim):
    if abs(x) < lim:
        return 0
    else:
        return x
```

In [11]:

```
denoised_hat = list(map(remove_magnitude,noisy_data_hat,[5]*fre
```

In [12]:

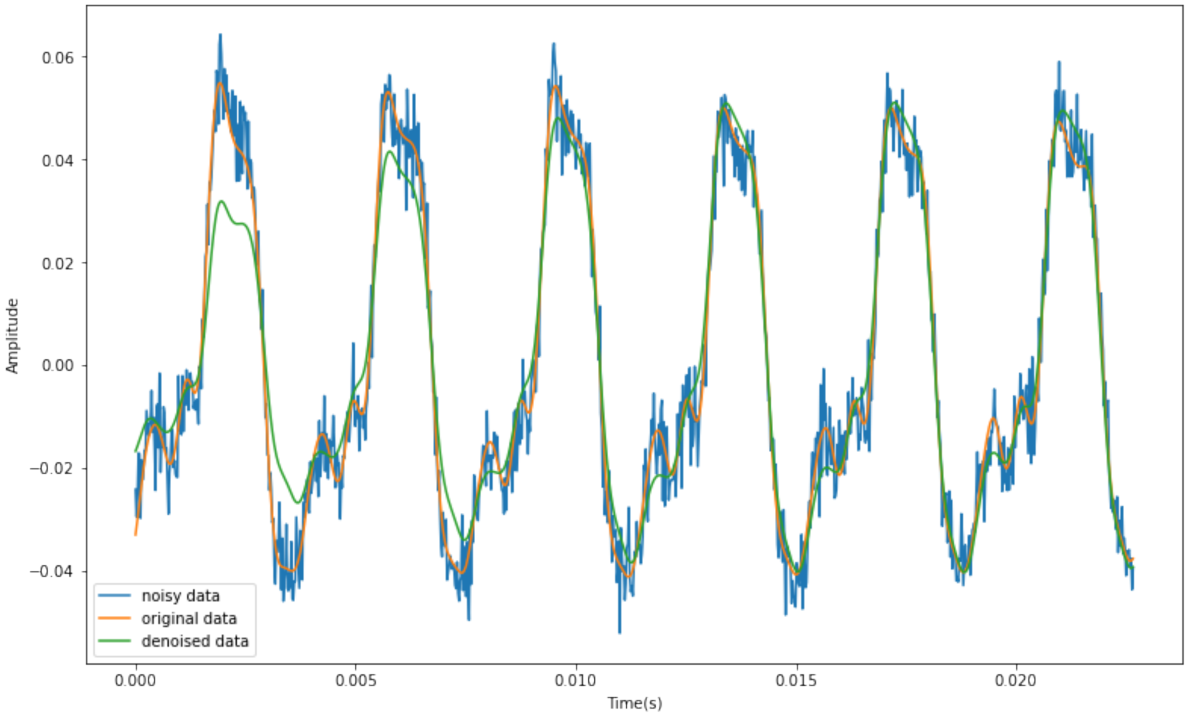
```
plt.figure(figsize=[13, 8])
plt.plot(fs, list(map(abs,denoised_hat)), label='denoised data'
plt.xlabel('Frequency(Hz)')
plt.ylabel('Magnetude')
plt.legend(loc='best')
plt.title('Denoised Data')
plt.xlim(0, n)
plt.show()
```



In [13]:

```
denoised = np.fft.ifft(denoised_hat)

plt.figure(figsize=[13, 8])
plt.plot(time[:n], noisy_data[:n], label='noisy data')
plt.plot(time[:n], data[:n], label='original data')
plt.plot(time[:n], denoised[:n], label='denoised data')
plt.xlabel('Time(s)')
plt.ylabel('Amplitude')
plt.legend(loc='best')
plt.show()
```

```
/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/matplotlib/cb
ook/__init__.py:1298: ComplexWarning: Casting complex values to re
al discards the imaginary part
  return np.asarray(x, float)
```



In [14]:

```
err_with_data = np.linalg.norm(denoised-data)
# print(denoised)
# print(data)
err_with_noisy = np.linalg.norm(data-noisy_data)
# print(err_with_data)
# print(err_with_noisy)
print(f"The error of denoised data and noisy data against origi
print(f"The denoised signal is clearly closer to the original s
```

```
The error of denoised data and noisy data against original data ar
e 0.3475626828391964 and 1.0451029075185467 respectively.
The denoised signal is clearly closer to the original signal.
```

Lab Book 04

In [15]:

```
P = 10000
u0 = 1/P  # initial condition
T = 100   # end time
n = 200  # use n+1 equally spaced time steps
ts = np.linspace(0, T, n+1)  # vector of timesteps, tk = ts[k]
h = T / n  # gap between timesteps
```

In [16]:

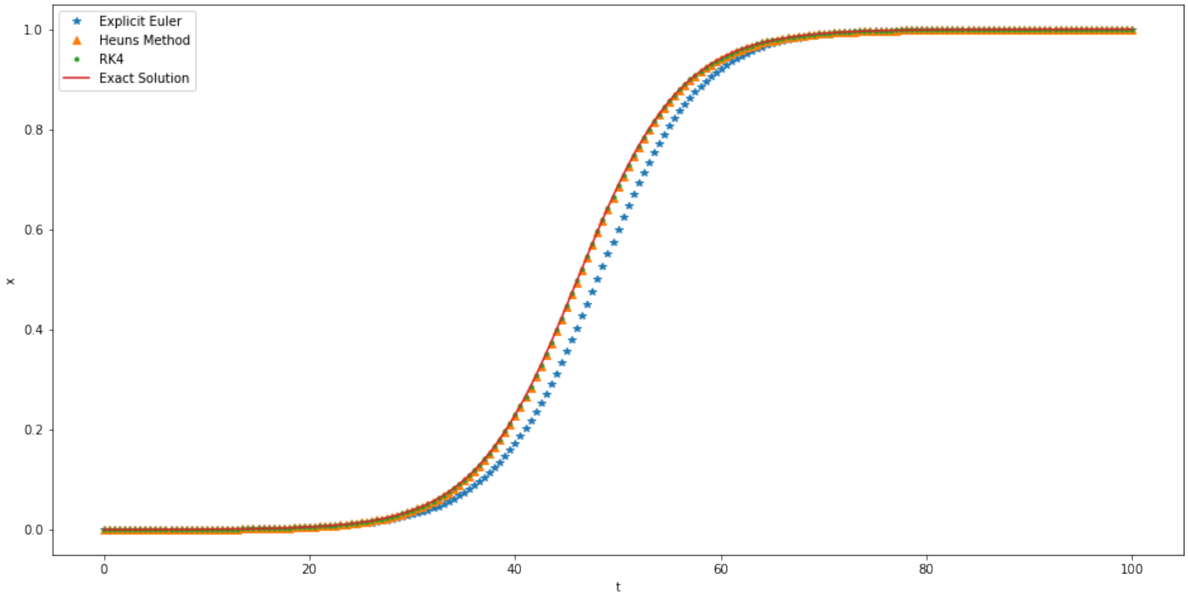
```python
def f(t, u):
    c = 0.2
    return c * u * (1-u)


def u(t):
    P = 10000
    c = 0.2
    return 1/(1+(P-1)*np.exp(-c*t))


def Euler(t0, u0, n):
    u_Euler = np.zeros((n+1,))
    u_Euler[0] = u0
    for k in range(n):
        u_Euler[k+1] = u_Euler[k] + h * f(ts[k], u_Euler[k])
    return u_Euler


def Heun(t0, u0, n):
    u_Heun = np.zeros((n+1,))
    u_Heun[0] = u0
    for k in range(n):
        u_Heun[k+1] = u_Heun[k] + 0.5 * h * \
            (f(ts[k], u_Heun[k]) + f(ts[k]+h, u_Heun[k]+h*f(ts[
    return u_Heun


def RK4(t0, u0, T, n):
    h = T / n
    ts = []
    us = []
    ts.append(t0)
    us.append(u0)
    # i = 0
    # while ts[i]+h <= tmax:
    for i in range(n):
        k1 = f(ts[i], us[i])
        k2 = f(ts[i] + 0.5 * h, us[i] + 0.5 * h * k1)
        k3 = f(ts[i] + 0.5 * h, us[i] + 0.5 * h * k2)
        k4 = f(ts[i] + h, us[i] + h * k3)
        us.append(us[i] + (h / 6)*(k1 + 2 * k2 + 2 * k3 + k4))
        ts.append(ts[i] + h)
        # i += 1
    return ts, us

```

In [17]:

```python
u_Euler = Euler(0, u0, n)
u_Heun = Heun(0, u0, n)
ts_RK4, u_RK4 = RK4(0, u0, T, n)
u_exact = u(ts)
```

In [18]:

```
plt.figure(figsize=[16, 8])
plt.plot(ts, u_Euler, '*', label='Explicit Euler')
plt.plot(ts, u_Heun, '^', label='Heuns Method')
plt.plot(ts_RK4, u_RK4, '.', label='RK4')
plt.plot(ts, u_exact, label='Exact Solution')
plt.xlabel('t')
plt.ylabel('x')
plt.legend(loc='best')
plt.show()

```



The Explicit Euler method has a much lower accuracy conpared to Heuns Method and RK4.

This solution of ODE shows the whole process of the spread of a disease in 100 days. We can see a quick raise in infected population between 40-60 days and about 80% of the population will be infected in this period if no method like quarantine and vaccine.

Lab Book 05

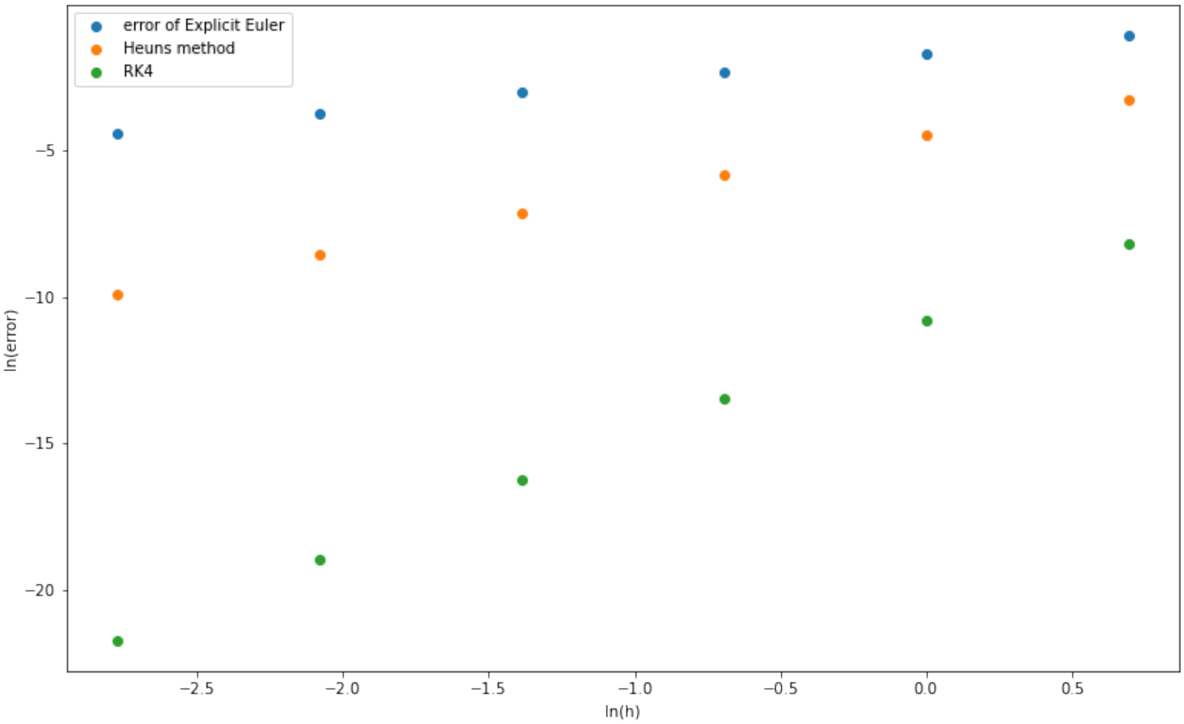
In [19]:

```python
ns = [50, 100, 200, 400, 800, 1600]
E = []
H = []
RK = []
hss = []
print("{0:^7}{1:^20}{2:^20}{3:^20}".format("h", "Euler", "Heun"
for n in ns:
    ts = np.linspace(0, T, n+1)
    h = T / n
    u_Euler = Euler(0, u0, n)
    u_Heun = Heun(0, u0, n)
    ts_RK4, u_RK4 = RK4(0, u0, T, n)
    u_exact = u(ts)
    Euler_max = max(abs(u_Euler-u_exact))
    Heun_max = max(abs(u_Heun-u_exact))
    RK4_max = max(abs(u_RK4-u_exact))
    hss.append(h)
    E.append(Euler_max)
    H.append(Heun_max)
    RK.append(RK4_max)
    print(f'{h:^5.5f}{Euler_max:^20.16f}{Heun_max:^20.16f}{RK4_
```

```
   h           Euler                Heun                 RK4
2.00000 0.3511787828005281  0.0392733327566064  0.0002786116269167
1.00000 0.1876126831731261  0.0112001559813495  0.0000203235920652
0.50000 0.0961272883004560  0.0029997295057085  0.0000013730543685
0.25000 0.0485262270538754  0.0007761598708744  0.0000000892409335
0.12500 0.0243630855173878  0.0001974844529142  0.0000000056883146
0.06250 0.0122042807862079  0.0000498082592145  0.0000000003590142
```

In [20]:

```
plt.figure(figsize=[13, 8])
plt.scatter(np.log(hss), np.log(E), label='error of Explicit Eu
plt.scatter(np.log(hss), np.log(H), label='Heuns method')
plt.scatter(np.log(hss), np.log(RK), label='RK4')
plt.legend()
plt.xlabel('ln(h)')
plt.ylabel('ln(error)')
plt.show()
slope1 = np.polyfit(np.log(hss), np.log(E), 1)[0]
slope2 = np.polyfit(np.log(hss), np.log(H), 1)[0]
slope3 = np.polyfit(np.log(hss), np.log(RK), 1)[0]
print(f"the order of convergence of Explicit Euler method is {s
print(f"the order of convergence of Heun's method is {slope2}")
print(f"the order of convergence of RK4 is {slope3}")

```




```
the order of convergence of Explicit Euler method is 0.97299656124
87067
the order of convergence of Heun's method is 1.9297731741060276
the order of convergence of RK4 is 3.919458729847732
```

Lab Book 06

In [21]:

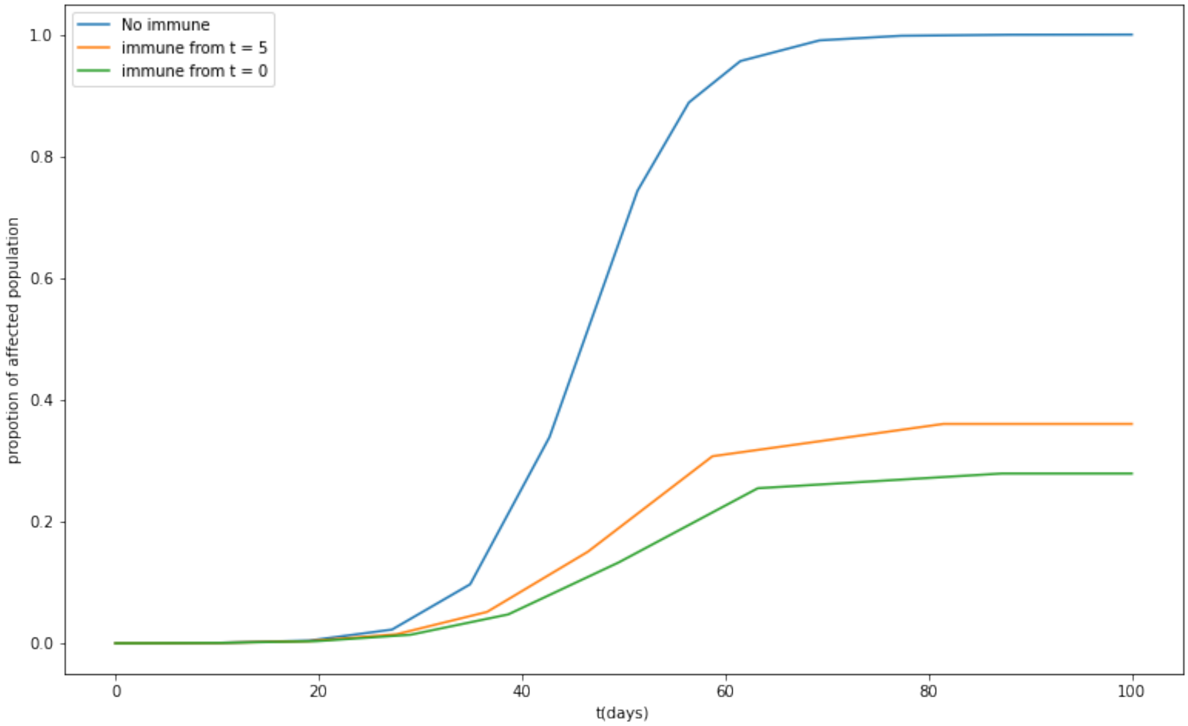
```python
def f_immu(t, u):
    c = 0.2
    return c * u * max(1-u-0.01*max(t-5, 0), 0)


def f_immu0(t, u):
    c = 0.2
    return c * u * max(1-u-0.01*t, 0)

```

In [22]:

```python
P = 10000
u0 = np.array([1/P])
T = 100
sol = integrate.solve_ivp(f, [0, T], u0, method='RK45')
# print(sol.y)
sol_immu = integrate.solve_ivp(f_immu, [0, T], u0, method='RK45
# print(sol_immu.y)
healthy = sol.y * P
healthy_immu = sol_immu.y * P
sol_immu0 = integrate.solve_ivp(f_immu0, [0, T], u0, method='RK
healthy_immu0 = sol_immu0.y * P

```

In [23]:

```
plt.figure(figsize=[13, 8])
plt.plot(sol.t, sol.y[0,:], label='No immune')
plt.plot(sol_immu.t, sol_immu.y[0,:], label='immune from t = 5'
plt.plot(sol_immu0.t, sol_immu0.y[0,:], label='immune from t =
plt.xlabel('t(days)')
plt.ylabel('propotion of affected population')
plt.legend(loc='best')
plt.show()

```



In [24]:

```
print(f"Without immune, the number of healthy people mathmetica
print(f"\nWith vaccinating from the fifth day, the mathmeticall
extra_healthy = abs(sol_immu0.y[0][-1] - sol_immu.y[0][-1])
print(f"\n{extra_healthy*P} (815) more people will be healthy i
```

```
Without immune, the number of healthy people mathmetically is 0.74
88286951184975, which means almost no one is still healthy after 1
00 days.

With vaccinating from the fifth day, the mathmetically number of h
ealthy people after 100 days is 6395.897785341395, which is about
6396 people.

814.9610528256312 (815) more people will be healthy if we start va
ccination from t = 0 after 100 days.
```

Lab Book 07

In [25]:

```
def f(x):
    return np.sin(np.pi*x)


def f_t_ut(a, b, c, n, u):
    delta_x = 1 / n
    u_k = np.zeros(n+1)
    u_k[0] = a
    u_k[-1] = b
    for i in range(1, n):
        u_k[i] = c * (u[i-1] - 2 * u[i] + u[i+1]) / pow(delta_x
    return u_k

```

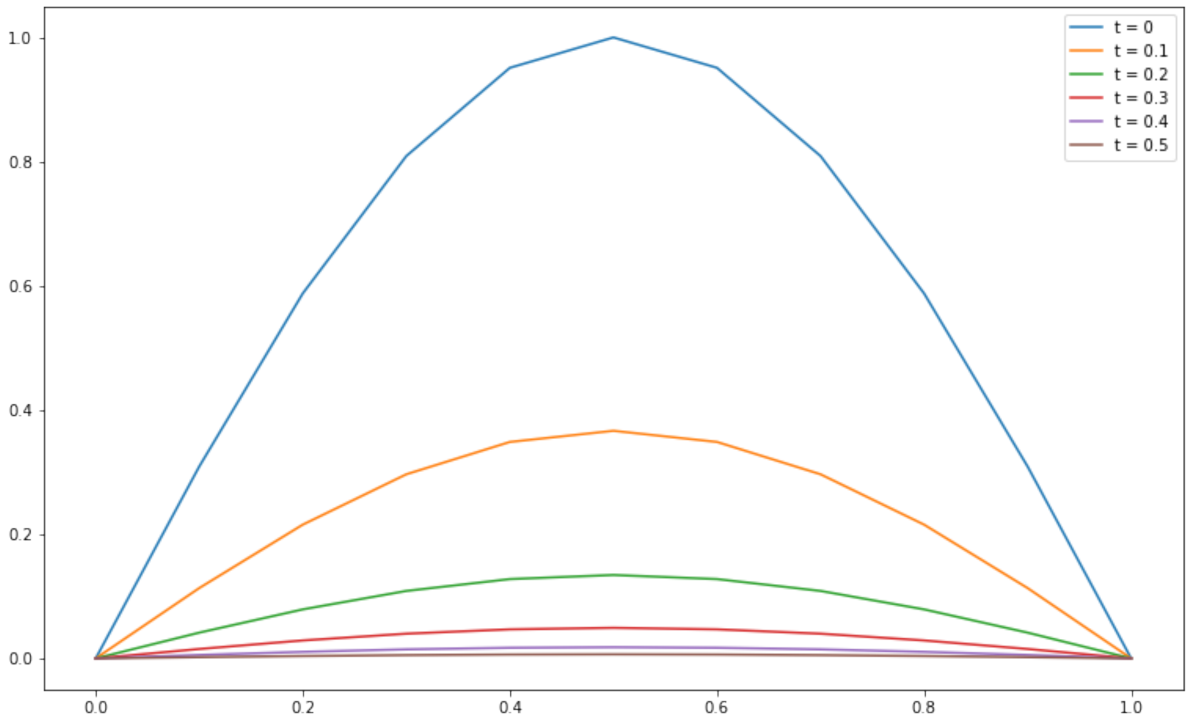
In [26]:

```
def PDE_solver(a, b, c, f, t_max, n):
    delta_t = t_max / (pow(n, 2))
    delta_x = 1 / n
    j = np.arange(n+1)
    xs = j * delta_x
    u = np.zeros((pow(n, 2)+1, n+1))
    # u = np.array([f(xs)])
    u[0] = f(xs)
    # t = 0
    # while t <= t_max:
    #     u_next = u[-1] + [delta_t * f_t_ut(a, b, c, n, u[-1])
    #     u = np.append(u, u_next, axis=0)
    #     t += delta_t
    for i in range(1, u.shape[0]):
        u[i] = u[i-1] + [delta_t * f_t_ut(a, b, c, n, u[i-1])]
    return u

```

In [27]:

```python
a = 0
b = 0
c = 1
t_max = 0.5
n = 10
u = PDE_solver(a, b, c, f, t_max, n)
print(u.shape)
xs = np.linspace(0, 1, len(u[0]))
ts = np.linspace(0, t_max, 6)
t_index = (ts*100/t_max)
plt.figure(figsize=[13, 8])
for i in t_index:
    i = int(i)
    plt.plot(xs, u[i])
plt.legend(['t = 0', 't = 0.1', 't = 0.2', 't = 0.3', 't = 0.4'
plt.show()
```

```
(101, 11)
```



Lab Book 08

In [28]:

```python
def f(x):
    return 0.5 - x
```

In [29]:

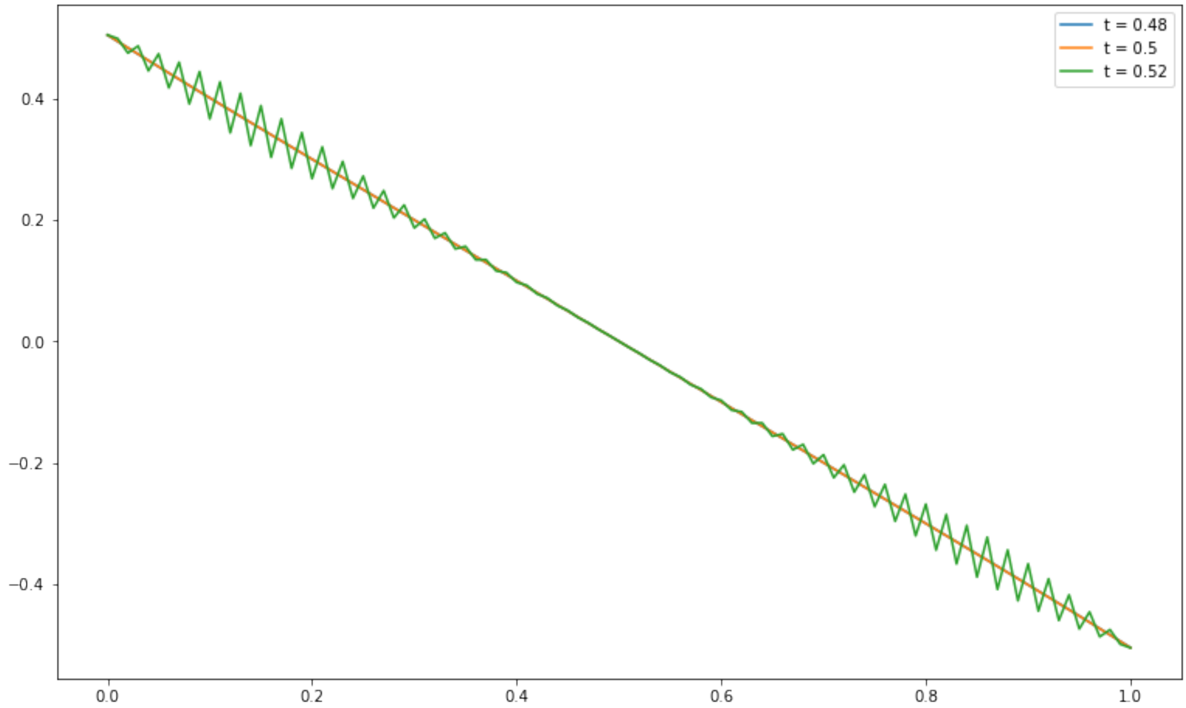
```
alpha1 = 0.48
dt1 = alpha1/pow(n,2)
alpha2 = 0.5
dt2 = alpha2/pow(n,2)
alpha3 = 0.52
dt3 = alpha3/pow(n,2)
n = 100
```

In [30]:

```
u1 = PDE_solver(0.5,-0.5,1,f,alpha1,n)
u2 = PDE_solver(0.5,-0.5,1,f,alpha2,n)
u3 = PDE_solver(0.5,-0.5,1,f,alpha3,n)
```

```
/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/ipykernel_lau
ncher.py:11: RuntimeWarning: overflow encountered in double_scalar
s
  # This is added back by InteractiveShellApp.init_path()
/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/ipykernel_lau
ncher.py:15: RuntimeWarning: invalid value encountered in add
  from ipykernel import kernelapp as app
/Users/x_x/opt/anaconda3/lib/python3.7/site-packages/ipykernel_lau
ncher.py:11: RuntimeWarning: invalid value encountered in double_s
calars
  # This is added back by InteractiveShellApp.init_path()
```

In [31]:
```
xs = np.linspace(0, 1, len(u1[0]))
ts = np.linspace(0, t_max, 6)
t_index = 200
plt.figure(figsize=[13, 8])
plt.plot(xs, u1[t_index], label = 't = 0.48')
plt.plot(xs, u2[t_index],label = 't = 0.5')
plt.plot(xs, u3[t_index],label = 't = 0.52')
plt.legend()
plt.show()
```



Both t = 0.48 and t = 0.5 gives correct solution because dt <= dx^2 / 2c.

However when t = 0.52, dt > dx^2 / 2c, therefore the method we use to find the solution to the heat equation is no longer stable and leads to the fluctuations in solutions.