

- using Markdown

## 1D Gaussians [10 pts]

Let  $X$  be a univariate random variable distributed according to a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$

- `md"""`
- `# 1D Gaussians [10 pts]`
- `Let  $X$  be a univariate random variable distributed according to a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$`
- `"""`

Can the probability density function (pdf) of  $X$  ever take values greater than 1?

Answer:

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- `### Can the probability density function (pdf) of  $X$  ever take values greater than 1?`
- `Answer:`
- `"""`

Write the expression for the pdf of a univariate gaussian:

Answer:

Write the code for the function that computes the pdf at  $x$ .

gaussian\_pdf (generic function with 1 method)

- `function gaussian_pdf(x; mean=0., variance=0.01)`
- `#default variables mean and variance`
- `#set with keyword arguments`
- `return #TODO: implement pdf at x`
- `end`

# Test your implementation against a standard implementation

E.g. from a library, e.g. Distributions.jl.

• Enter cell code...

• using Test

• using Distributions: pdf, Normal  
• # Note Normal uses N(mean, stddev) for parameters

Some tests did not pass: 0 passed, 0 failed, 2 errored, 0 broken.

```
1. finish(::Test.DefaultTestSet) @ Test.jl:879
2. top-level scope @ Test.jl:1125
3. top-level scope @ (Local: 2
```

```
• @testset "Implementation of Gaussian pdf" begin
•   x = randn()
•   @test gaussian_pdf(x) ≈ pdf.(Normal(0.,sqrt(0.01)),x)
•   # ≈ is syntax sugar for isapprox, typed with '\approx <TAB>'
•   # or use the full function, like below
•   @test isapprox(gaussian_pdf(x,mean=10., variance=1) , pdf.(Normal(10., sqrt(1)),x))
• end
```

## What is the value of the pdf at $x = 0$ ? What is probability that $x = 0$ ?

```
• md"""
• ### What is the value of the pdf at $x=0$? What is probability that $x=0$?
• """
```

• Enter cell code...

## A Write the transformation that takes $x \sim \mathcal{N}(0., 1.)$ to $z \sim \mathcal{N}(\mu, \sigma^2)$

A Gaussian with mean  $\mu$  and variance  $\sigma^2$  can be written as a simple transformation of the standard Gaussian with mean 0. and variance 1..

Answer:

```
• md"""
• ### A Write the transformation that takes $x \sim \mathcal{N}(0.,1.)$ to $z \sim \mathcal{N}(\mu, \sigma^2)$
•
• A Gaussian with mean $\mu$ and variance $\sigma^2$ can be written as a simple
  transformation of the standard Gaussian with mean $0.$ and variance $1.$
•
• Answer:
• """
```

• Enter cell code...

## Write a code to sample from $\mathcal{N}(\mu, \sigma^2)$

Implement function returning  $n$  independent samples from  $\mathcal{N}(\mu, \sigma^2)$  by transforming  $n$  samples from  $\mathcal{N}(0., 1.)$

```
• function sample_gaussian(n; mean=0., variance=0.01)
•   # n samples from standard gaussian
•   x = #TODO
•
•   # TODO: transform x to sample z from N(mean, variance)
•   z =
•   return z
• end;
```

## Test your implementation by computing statistics on the samples

```
• using Statistics: mean, var
```

```
• @testset "Numerically testing Gaussian Sample Statistics" begin
•   #TODO: choose some values of mean and variance to test
•   #TODO: Sample 100000 samples with sample_gaussian
•   #TODO: Use 'mean' and 'var' to compute statistics
•   #TODO: test statistics against true values
•   # hint: use isapprox with keyword argument atol=1e-2
• end;
```

## Plot pdf and normalized histogram of samples

Sample 10000 samples from a Gaussian with mean 10. and variance 2.0.

1. Plot the **normalized** histogram of these samples.
2. On the same axes plot! the pdf of this distribution.

Confirm that the histogram approximates the pdf.

(Note: with Plots.jl the function plot! will add to the existing axes.)

```
• md"""
• ### Plot pdf and normalized histogram of samples
•
• Sample $10000$ samples from a Gaussian with mean $10.$ and variance $2.0$.
•
• 1. Plot the normalized 'histogram' of these samples.
• 2. On the same axes 'plot!' the pdf of this distribution.
• Confirm that the histogram approximates the pdf.
•
• (Note: with 'Plots.jl' the function 'plot!' will add to the existing axes.)
• """
```

- using **Plots**

- *#histogram() #TODO*
- *#plot!() #TODO*