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Optimal heating of large block of flats

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ABSTRACT

District heating is used in many urban areas in Sweden. Almost always, the district heating utility is owned by the municipality and the municipality naturally encourages proprietors to connect their buildings to the grid, even if they cannot really force them to do so. The building owners are free to choose the best system, i.e. the cheapest one, for their need. Unfortunately, it is not always so easy to find the best solution. Mixed integer linear programming (MLIP) models might here come to help. By such computer programs it is possible to find the absolutely cheapest system of available alternatives, or even combinations among them. This paper shows how to design such a model and further how to closely depict the district heating, and electricity tariff. This is of course very important because the only interface between the proprietor of the building and the utility is found in this bureaucratic instrument. If the tariff is too high the building owners will choose other heating systems than district heating, or even worse, combining district heating with alternative base load sources. In Sweden, this has been of interest because ground-water coupled heat pumps can be profitable, operated by use of the relatively low electricity prices. In this paper we show that dual-fuel, and sometimes even triple-fuel systems, are of interest when the proprietor aims at minimising the cost for space and domestic hot-water heating.

1. Introduction

District heating systems have been built all around the world. Many times the heating source is based on geothermal energy and in [1] some 75 countries are listed which use this type of energy and hence, also have district heating grids for distribution of the heat. Depending on how you rank these countries they will show up differently in these lists but China, U.S.A., Iceland, Turkey [2] are examples on countries that use much geothermal heat in district heating systems. Other times, the district heating utility is supplied by heat from a combined heat and power (CHP) plant. Denmark, Sweden, Finland have many such power generation companies. See for instance [3–6]. Also in Korea and China district heating is used, see e.g. [7,8]. Even if papers on district heating are published every now and then, it seems as if tariff structures, and the optimal way to adopt your behaviour to them, have not been of the same academic interest. In this paper the focus is set on the building owner who must adopt a strategy for getting as small a cost as possible for the purchased heat. The district heating utility is not studied at all except for the tariff which it uses for billing the building owner.

1.1. District heating in Sweden

Large number of block of flats is today often connected to municipal district heating grids. Such systems became very popular in Sweden some 50 years ago. The reason for this was that cheap low-quality oil was abundant on the energy market but normal building owners could not use it in their own low-cost oil-fired boilers. They had to use better and more expensive oil for their heating purposes. In a district heating plant low-quality cheap oil could be burnt in a sophisticated, but expensive, boiler. Such a plant was also large enough to afford investments in other equipment, e.g. for sulphur reduction. Further, the municipalities saw their chance to get rid of many other sources of heat, such as coal and wood, which polluted the air for many inhabitants. It was better with one high and large chimney than thousands of small. During many years heavy oil was the dominant fuel in our district heating plants. Unfortunately, the use of oil made the trade balance of Sweden problematic and the country vulnerable to fluctuations on the energy market. The oil-crises during the 1970s made the situation even worse. Sweden had to get rid of the dependence of oil and district heating based on other fuels, or even electricity, were available alternatives. Environmental hazards, high prices and the obligation to reduce greenhouse gas emissions have led to modernisation of the plants and nowadays, a number of energy sources are in use, many of them with very competitive prices. Waste, garbage, worn out rubber tyres, demolished wooden

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buildings are used as fuels today and in many municipalities ground, or even sewage, water coupled heat pumps are used. There are however drawbacks. Boilers and equipment for waste incineration are expensive devices and it is many times not possible to cover the total heat demand by use of garbage, etc., as the only sources. The amount of waste might also be too small. Sometimes coal and oil must be used during peak conditions but taxes and emission allowances make such fuels expensive and the utilities try to do their best in order to avoid such fossil heat sources. If it was possible to reduce the demand when peak conditions emerge, fossil fuels could be avoided. Up to now, normal Swedish district heating tariffs were not thought to encourage such a behaviour, but as this study shows, the cheapest solution for a proprietor is many times to abandon district heating during the winter and use alternative solutions. The utilities of course want to sell district heat also during the winter but if the building owners want to reduce their costs as much as possible the district heating tariff tells them to use heat from the utility only during summer.

2. Optimisation basics

Optimisation, i.e. to find the maximum or minimum value for a mathematic expression, is a very time-consuming activity for real world problems. Traditional calculus can be used where the *derivatum* of a function is set to 0. This, however, is only possible if our function is continuous which is not common practice. This calamity can be solved by combining a mix of derivative and trial-and-error methods, see [9] for an example. Even if that effort was a good start, the program described could not test all possible combinations. Only a few, more or less, traditional alternatives were examined. There are, fortunately, other methods coming into rescue. Linear programming (LP) has been used for many years, see for example Ref. [10] which is one of the earliest we have found where this technique is used for buildings and optimisation. LP has significant drawbacks, because it is impossible to deal with "not-linear" functions but the development of so-called mixed integer linear programming (MILP) partly solved this problem. Initially MILP models were very time-consuming to solve but they have now found widespread use among researchers because of cheaper and better computers. The basis for these methods could be found in ordinary text books for university students, see e.g. [11,12] for two examples, but such sources only describe problems suitable for "hand" calculations. Problems closer to reality could of course also be solved. Two examples can be found in [13,14]. It is not possible, or even worthwhile, to go into deeper theoretic discussions on how these methods work but some details are perhaps of interest. Traditionally, LP models are described as found in Ref. [11], p. 13:

$$\begin{array}{ll} \text{Maximize:} & c_1x_1 + c_2x_2 + \cdots + c_nx_n = x_0 \\ \text{Subject to:} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\ & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\ & \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m \\ & x_j \ge 0,\ j = 1, 2, \ldots, n. \end{array},$$

The first line above shows the expression we want to maximise, or minimise. This expression is many times called the objective function. The next lines are called constraints. The letters *a*, *b* and *c* are constants while *x* are those variables we want values on, in order to achieve an optimal solution. Sometimes the *x* variables can only assume the values 0 or 1, which gives an integer programming problem. Problems like the one sketched above are solved by using the so-called Simplex algorithm. There are many computer programs which can be used for this task. We have used a free such program, called GLPSOL, which can be found on the Internet. This program can read input data in the form of a so-called MPS-file which contains ordinary text describing the mathematic problem to solve. It is possible to write this file in an ordinary text editor, such as EMACS, but for larger scale problems you must use the computer also for this. In our case this latter program has been written in C for the Linux "gcc" compiler.

3. Modelling the building

In this paper we want to show how to build a mathematical model of a large block of flats. This model is then to be used for finding the minimum cost for space and domestic hot-water heating in such a building. It is important to design the model so it closely depicts real conditions and also so it is possible to solve the problem within reasonable time. The cost is, of course, dependent of how much heat is used and because of this we need heat demand data for a suitable building. Fortunately, the utilities collect such data, it is the basis for the bill, and therefore it is possible to examine how much heat that is used, hour by hour, for very long periods of time. Because of the tariff structure, which is described in close detail below, we need data for one full year and because of this our data set shows the demand in the form of 8760 values. In our computerised world these values are easily achieved and they have been plotted in a graph in order to depict the situation, see Fig. 1. The data are plotted in chronological order so January is in the left part of the graph, December in the right part and June is in the middle. It is obvious that the beginning of the year was colder than the end because the demand has a maximum sometime in January. During the summer, heat is used for warming domestic water and the demand can be as large as 200 kW just for this purpose. In order to clarify the situation even further we have sorted these values in descending order. Hence, consider the so-called duration graph in Fig. 2. For a start it is worthwhile to study the overall shape of this graph. Every hour during one full year is represented by its demand. Just from the graph we see that the maximum demand was about 1000 kW while the lowest demand is almost 0. It is also obvious that there is a profound peak in the left part of the graph. By examination of the data file it was found that the maximum demand for heat was 1110 kW. The peak in Fig. 2 is very narrow and thin. Just as an example, a closer look at the data set shows that the demand is larger than 800 kW for 176 out of 8760 h and this peak contains 18,420 kWh out of the total 2,807,650 kWh present in the graph. In order to build a model in the same way as sketched above, it is practical to use the heat demand values in chronological order. In our data file this first value is valid for January 1 between 00.00 and 01.00 a.m., and it equals 610 kWh. The next hour shows the same value while for the

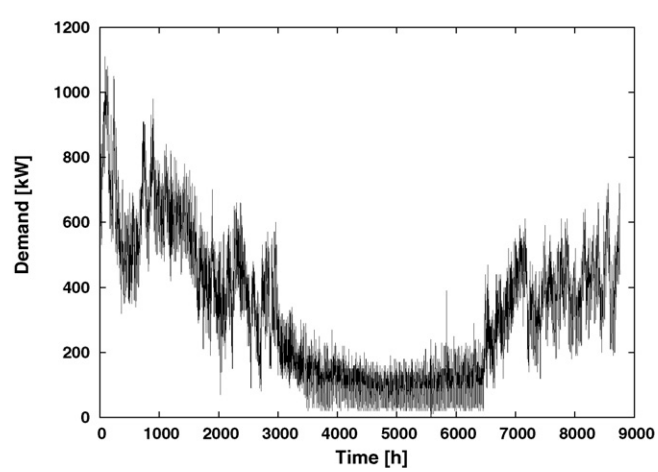

Fig. 1. Graph over the district heating demand for a block-of-flats sited in the Stocholm area, Sweden.

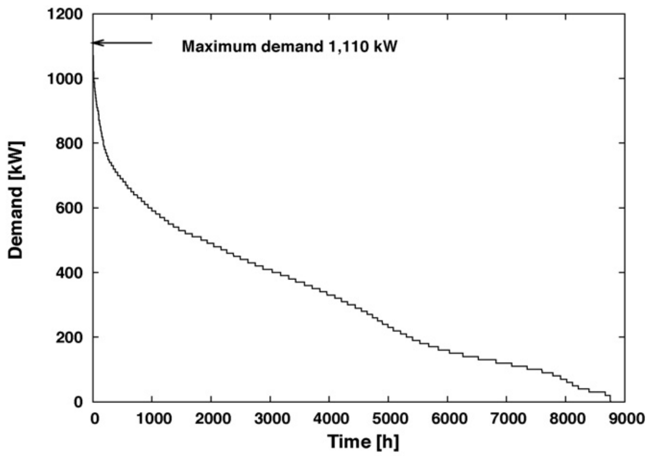

Fig. 2. Duration graph, showing the heat demand in descending order.

last hour of the year a demand of 440 kWh was monitored. All these values show how much heat was used in the building hour by hour. By using a technique found in [15] we can now start to build a model in form of an objective and its accompanying constraints. A heating system, or a combination of heating systems, must be able to provide more than or equal the amounts in the data file, i.e. we have a first constraint in our model. The first data file value corresponds to the b_1 value, the second to b_2 and so on. We do not know if it is optimal to use solely the district heating system or if some other heat source is better or if they should be combined. The x-variables show the demand for different sources but for practical reasons they are renamed. We set the demand of district heat (dh) during hour number 1 to x_{dh1}, i.e. our first variable. Suppose now that we also want to check if a bio-fuelled (bf) boiler could be of interest. The heat coming from that device, will be found in a variable x_{bf1}. Assume for a start that these two heating systems were the only options. We must now find values of x_{dh1} and x_{bf1} so that they can cover the demand of 660 kW. This is the same as

$$\begin{aligned} x_{\mathrm{dh1}} \times 1 + x_{\mathrm{bf1}} \times 1 &\geq 610 \\ x_{\mathrm{dh2}} \times 1 + x_{\mathrm{bf2}} \times 1 &\geq 610 \\ &\vdots \\ x_{\mathrm{dh8760}} \times 1 + x_{\mathrm{bf8760}} \times 1 &\geq 440 \end{aligned}$$

The demand in kW in each hour is multiplied by 1 h in order to achieve kWh on both sides of the $\geq$-sign. Note also that the right hand side always is a constant. By adding more heating systems possibilities, e.g. a ground-water coupled heat pump, an oil-fired boiler, a natural-gas fired boiler and so forth, the demand for heat can be covered in a number of ways, hour by hour, during one full year. The cost for a boiler depends many times on its thermal size. It is therefore necessary to find out the maximum used thermal power for all boilers, etc. among all hours. A new constraint is needed, or actually one constraint for each type of boiler that is included in the model. For the bio-fuelled boiler these constraints are constructed as

$$\begin{aligned} -x_{\mathrm{bf1}} + e_{\mathrm{bf}} &\geq 0 \\ -x_{\mathrm{bf2}} + e_{\mathrm{bf}} &\geq 0 \\ &\vdots \\ -x_{\mathrm{bf8760}} + e_{\mathrm{bf}} &\geq 0 \end{aligned}$$

The variable e_{bf} will then assume the maximum value of all the calculated hourly bio-fuel values. One possible way to ascertain that the constraints are true, is to set e_{bf} to a very large value, say 1500 kW which is larger than the monitored demand, 1110 kW, see Fig. 2. Larger than necessary boilers are expensive so this "problem" is taking care of by adding the cost for the boiler to the objective. This expression shows the total cost for our system and hence we want to minimise its value. This minimisation will therefore ascertain that e_{bf} assumes its lowest possible value. Other costs must also be added to the objective. Suppose that energy in the form of bio-fuel costs 0.30 SEK/kWh and that efficiency for that boiler is 0.7. (One Euro is about 9 Swedish Kronor, i.e. SEK.) The cost for district heat during winter conditions is set to 0.406 SEK/kWh, *vide infra*. It should be noted here that we use the actual monitored demand for district heat so we do not have to modify this price with a value for the efficiency. This is not the case for the bio-fuelled boiler and we must also add the cost for the bio-fuelled boiler itself because there is no such boiler in the existing building. The energy cost emerges year after year while the costs for the actual boilers only comes up when there is a need for replacing them. This calls for some present value (PV) calculations, i.e. future costs must be transferred to present time. This is done by use of two formulae:

$$PV_{\mathrm{s}} = C_{\mathrm{s}}(1+r)^{-n}, \qquad PV_{\mathrm{a}} = C_{\mathrm{a}}\frac{1-(1+r)^{-m}}{r}$$

Index 's' stands for single events while index 'a' stands for annual events, C shows the cost, r the interest rate and n and m show the number of years. Suppose that the applicable rate is 5% and that 30 years are considered. The cost C_{a} must therefore be multiplied with 15.37 in order to find the present value. Hence, the first part of the objective can be written as

$$\left(\begin{array}{c} [(x_{\mathrm{dh1}} + x_{\mathrm{dh2}} + \cdots + x_{\mathrm{dh8760}}) \times 1 \times 0.406] + \cdots \\ + \left[(x_{\mathrm{bf1}} + \cdots + x_{\mathrm{bf8760}}) \times 1 \times \dfrac{0.30}{0.7}\right] \cdots \end{array}\right) \times 15.37 + \cdots$$

It is not necessary to use the same district heating price for all hours. In our case the price is in fact 0.406 SEK/kWh for December to February, but much lower from June to August and between those values the rest of the year, see below. We must also add the cost for all other available energy forms included in the model and of course, also consider the efficiency of all other boiler types. The actual boilers costs must also be included. A large boiler is almost always more expensive than a small one. Different price lists can be used to find the cost for various equipment, installation and so on. The costs for several different sizes can be examined and put into a diagram. By using the method of least square, this data set can be transformed into a mathematically defined line. However, the cost is not always totally linear because it will many times start with a distinct step, see Fig. 3.

Assume that this line, for a bio-fuelled boiler (bf) has been calculated to

$$C_{\mathrm{bf}} = g + c \times e_{\mathrm{bf}}$$

In our case we use $g = 100,000$ and $c = 300$. We must now add this cost to the objective, perhaps with some adjustments for present value calculations and efficiency. The fixed cost g must however only be present in the objective if a bio-fuelled boiler is chosen by the optimisation. Such a behaviour is achieved by introducing variables that can only assume the value 0 or 1. Consider the following constraint, where z is such a 0/1 variable and M is a constant with a large value, i.e. larger than e_{bf} can ever become

$$-e_{\mathrm{bf}} + M \times z \geq 0$$

If e_{bf} is present in the chosen system and therefore has a size larger than 0, z must be equal to 1. The expression is then true. If e_{bf} does not exist, i.e. its size is 0, then z can equal both 0 and 1 and the expression still will be true. The z variable must now be multiplied with the "step" cost, i.e. the constant value of 100,000 SEK, and further added to the objective. Because of the minimisation z "wants" to be 0 but

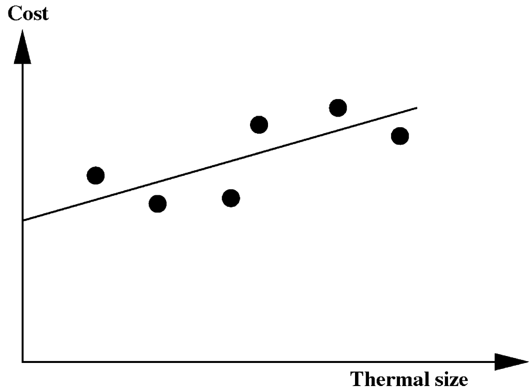

Fig. 3. Costs for boilers and other equipment often show non-linear parts.

this is only possible if also e_{bf} is 0. Hence, if no bio-fuelled boiler is present in the optimal solution, z assumes the value 0, and no cost is added to the objective. If the boiler is present in the solution, z assumes the value 1 and the "step" is added. The size of the constant M can without any hazard be set to e.g. 1200 because the maximum demand in the building was found to be 1110 kW. Because of the minimisation of the cost e_{bf} can never be larger than that value.

3.1. The district heating tariff

The cost for district heat is, of course, dependent on the used tariff. Here, we have used some tariffs, found on the Internet. The traditional Swedish tariff is split in several parts. First you must pay a demand fee which is calculated by use of a value, y^{tot}. This value is elaborated as the ratio between the normal total district heating demand for one full year and a so-called category number which equals 2200 for residences. The thermal size of the building will therefore be important for the demand cost, see Table 1. We do not know in advance how much district heat that is used, this depends on the optimal solution, and thus a constraint must be introduced that will find the value of y^{tot}. As before we multiply each value for the thermal demand with 1 h so we go from kW to kWh. We divide the total district heating demand value by the category number:

$$y^{tot} = \frac{x_{dh1} \times 1 + x_{dh2} \times 1 + \cdots + x_{dh8760} \times 1}{2200}$$

We must now add the cost to the objective. The problem is that the demand fees in Table 1 differ depending on the value of y^{tot} and so do the fixed fees. The desired behaviour can perhaps be solved in a number of ways but here the objective is added with:

$$\cdots + 370w_1 + 340y_1 + 1650w_2 + 308y_2 + 5250w_3 + 272y_3 + \cdots$$

w_1, w_2, etc. are 0/1 variables so it is only if w_1 equals 1 the value 370 is added to the objective. It is also important to multiply all these values with the present value factor because they emerge annually. Only one of the w-variables can be equal to 1 at the same time and likewise important is that this one must only equal 1 if district heating is chosen in the optimal solution. If this is not the case all the w-variables should assume the value 0. By, first, adding a constraint and a 0/1 variable called z it is possible to check if district heating is optimal, or not:

$$y^{tot} - Mz \le 0$$

and then introduce a constraint:

$$w_1 + w_2 + w_3 + \cdots - z \le 0$$

this is solved. y_1, Ey_2 and so on must be equal to y^{tot} if the applicable interval is chosen but 0 if this is not the case. The first interval is between 0 and 40 kW so

$$y_1 \ge 0.0$$

It is also necessary to ascertain that the variable w_1 equals 1 if this interval is chosen. This is achieved by

$$y_1 \le 40.0 - 40(1 - w_1)$$

If y_1 is between 0 and 40, w_1 should equal 1. If y_1 equals 0 w_1 must also be 0. A value on y_1 which is larger than 40 is impossible no matter the value on w_1. If this expression is written in the same form as before it will become

$$y_1 - 40w_1 \le 0.0$$

The other intervals are dealt with like

$$-40(1 - w_2) + 40 \le y_2 \le 100 - 100(1 - w_2)$$

and

$$-100(1 - w_3) + 100 \le y_3 \le 500 - 500(1 - w_3)$$

and so on. y_1, y_2, etc. could also all be 0 but by setting:

$$y^{tot} = y_1 + y_2 + y_3 + \cdots$$

this is prohibited. There is also a cost for the water flow through the district heating pipes. A high flow will result in a very small difference between the inlet and the outlet temperatures of the heat exchanger. In a combined heat and power (CHP) station it is important that the cooling medium in the plant is not too warm. This because it is desirable to have a big difference in steam pressure between the inlet and the outlet of the power generation turbine. Hence, the utility has a fee on the water flow. In this case study it is 1.80 SEK for each m^3 that passes the meter from September to May. The amount of heat that can be stored in water depends on the temperature difference. During the winter a high inlet temperature is used, sometimes higher than 100 °C, but in the springtime this temperature can be lower. Fortunately, these temperatures are monitored because the utility calculates the bill according to these temperatures. The inlet temperature cannot be changed by the building owner. The outlet temperature, on the other hand, will of course depend on the water flow and the amount of heat that is utilised in the heat exchanger, but for the sake of simplicity we have assumed that the amount of heat in 1 m^3 of water is the same for that specific hour no matter the demand. In January 1, between 00.00 and 01.00 a.m., the heat demand was 660 kWh. The water amount passing the meter was measured to 9.30 m^3 and therefore $9.3/660 = 0.014$ m^3 passed for each kWh. It is now possible to calculate how much water which must pass the meter if less, or more, heat is used and the cost for this flow. The cost must be added to the objective in the same way as the "normal" cost for each kWh. The actual heating cost, i.e. the cost per kWh is 0.406 from December to March, 0.306 in April, May and September–November, while it is 0.226 SEK/kWh during June–August, VAT excluded. We have also tried to find district heating tariffs for other countries. Unfortunately, all details are not shown in easily available published documents or web sites. For Denmark, and Copenhagen, two large companies seems to be active, VEKS, i.e. Vestegnens Kraftvarmeselskab I/S and CTR, Centralkommunernes Transmissionsselskab I/S. A closer look at their web pages will not tell us the level of the tariffs but for VEKS you

Table 1
Demand tariff for district heat

Heating power y^{tot} (kW)	Fixed fee (SEK)	Demand fee (SEK/kW)
0–40	370	340E
41–100	1,650	308E
101–500	5,250	272E
501–1000	31,250	220E
1001–3000	36,250	215E
3001–7000	144,250	179E
7001–	*N*	*N*

N = negotiations.

see the principal design. VEKS has one fixed fee, one varying fee, one reward "fee" for low returning water temperature and one reward "fee" for a non-varying heat flow. In Finland, we have for instance Helsingin Energia. They publish their connection fees that depends on the water flow, and the length of the connecting pipe. The heat prices are also shown, but only for the time passed. It is not possible to see the prices for the coming year. For summer 2007 the price was about 20 Euro per MWh while it was 40 Euro per MWh for the winter. There is also a water flow fee, e.g. it is 13,500 Euro for $10\,\mathrm{m}^3/\mathrm{h}$ but there are about 100 alternative values. Other interesting countries to study would be China and Korea, e.g. the Korea District Heating Corp., but unfortunately their tariffs could not be found in the detail needed here.

3.2. The electricity tariff

During many years electricity prices in Sweden were very low. Because of this, heating systems operating on solely electricity is still rather common in the existing building stock. Even such low Swedish prices can many times be too high for the inhabitants and especially during our cold winters the cost was substantial due to a high demand for electricity. Because of the monopolic conditions on the electricity market the utilities could set the prices on their own. About 10 years ago, however, the authorities decided to change things and the utilities had to split in two parts. One part owned the grid and one part only sold the electricity. The grid owners had to distribute electricity from all electricity selling companies and, hence, competition was supposed to increase. In the initial stage this was also the case and electricity prices fell significantly. After a few years prices went up again perhaps because of an increasing European market. The production companies could sell electricity on the Nord Pool exchange and small consumers had to accept the price level set there. If capacity in the transnational electricity grid is increased future prices will probably be almost the same in all European countries. Because of the de-regulated market it is nowadays difficult to say how much electricity will cost even in the close future. If a fixed price for 2 years is chosen, 0.407 SEK/kWh applies just for the electricity itself according to a web-page, owned by our biggest producer. There is also a special electricity tax which must be paid, equalling 0.255 SEK/kWh. A small cost, 0.03 SEK/kWh covering so-called electricity certificates also applies. The electricity therefore costs 0.692 SEK/kWh, VAT excluded. This price is valid each hour all the year around. There is also a cost for the distribution and the access to the grid. This cost varies according to the time of day and season. During working days, 06–22, under the winter, i.e. November–March, the cost is 0.14 SEK/kWh, while it is 0.04 SEK/kWh other times. There is also a demand tariff. Each month the utility charges 10 SEK/kW but during the winter months this fee goes up with an extra 50 SEK/kW and this demand cost is applied each month. Because of this it is important that the model can check the maximum demand each month and add the applicable cost to the objective. For January, hour number 1, the following applies:

$$\frac{p_1 - x_{\mathrm{vp1}}}{2.5} \geq 0.0$$

We need 744, which is the number of hours in January, such constraints and p_1 is therefore set to the largest of the heat pump variables divided by 2.5 which is the efficiency used for the heat pump. For February, which starts at hour number 745, we need 672 constraints and so on.

$$\frac{p_2 - x_{\mathrm{vp745}}}{2.5} \geq 0.0$$

Further, the present values of the demand costs for the different months must be added to the objective:

$$15.37[50x_{e1} + 50x_{e2} + 50x_{e3} + 10x_{e4} + \cdots + 50x_{e12}]$$

4. The model in more general terms

Above we have tried to use a non-mathematical language in order to make the model, and the design of it, easy to understand for the normal engineer. Sometimes, however, it is important to show the model in a more strict mathematical sense, e.g. if the reader wants to design an identical model. There is also a standard, in the field of operational research, where certain letters are to be used. We also add an index j representing the system. Hence, use the following sets:

- T = set of time periods $(\{1, \ldots, 8760\})$.
- J = set of systems $(\{\mathrm{dh}, \mathrm{bf}\})$.
- M = set of months $(\{1, 2, \ldots, 12\})$.
- T_m = set of time periods in month, m.
- L_j = set of tariff levels for system j $(L_{\mathrm{dh}} = \{1, 2, \ldots, 7\}$, $L_{\mathrm{bf}} = 1\})$.

The fixed parameters we need are as follows:

- b_t = energy demand (kW) in time period t.
- g_j = fixed cost for system j.
- k_j = efficiency number for system j.
- l_j = category number for system j.
- c_j = unit cost for system size j.
- o_{jt} = unit operating cost for system j in time period t.
- f_{jl} = fixed fee for system j using tariff level l.
- d_{jl} = demand fee for system j using tariff level l.
- $\overline{h}_{jl}$ = upper power level for system j using tariff level l.
- $\underline{h}_{jl}$ = lower power level for system j using tariff level l.
- q_{jm} = distribution cost (per kWh) for system j in month m.

The decision variables used in the model are as follows:

- x_{jt} = energy usage (kW) with system j in time period t.
- p_{jm} = max energy usage (kW) with system j in month m.
- e_j = max energy usage (kW) with system j.
- y_{jl} = energy usage (kW) with system j in tariff level l.
- y_j^{tot} = total energy usage (kW) with system j.
- $z_j = \begin{cases} 1, & \text{if system } j \text{ is used} \\ 0, & \text{otherwise.} \end{cases}$
- $w_{jl} = \begin{cases} 1, & \text{if system } j \text{ is used in tariff level } l \\ 0, & \text{otherwise.} \end{cases}$

The optimization model can now be formulated as follows:

$$\min \quad z = \sum_{j\in J} g_j z_j + \sum_{j\in J} c_j e_j + \sum_{j\in J}\sum_{m\in M} q_{jm} p_{jm} + \sum_{j\in J}\sum_{l\in L_j} d_{jl} y_{jl} + \sum_{j\in J}\sum_{l\in L_j} f_{jl} w_{jl} + \sum_{j\in J}\sum_{t\in T} o_{jt} x_{jt}$$

$$\text{subject to} \quad \sum_{j\in J} x_{jt} \geq b_t, \quad t\in T \tag{1}$$

$$x_{jt}/k_j \leq p_{jm}, \quad j\in J,\ m\in M, \quad t\in T_m \tag{2}$$

$$p_{jm} \leq e_j, \quad j\in J,\ m\in M \tag{3}$$

$$e_j \leq M z_j, \quad j\in J \tag{4}$$

$$\sum_{l\in L_j} y_{jl} = \frac{\sum_{t\in T} x_{jt}}{l_j}, \quad j\in J \tag{5}$$

$$\sum_{l\in L_j} y_{jl} = y_j^{\mathrm{tot}}, \quad j\in J \tag{6}$$

$$\sum_{j\in J} w_{jl} \leq z_j, \quad j\in J \tag{7}$$

$$y_{jl} \le \overline{h}_{jl} - \overline{h}_{jl}(1 - w_{jl}), \quad j \in J, \ l \in L_j \tag{8}$$

$$y_{jl} \ge \underline{h}_{jl} - \underline{h}_{jl}(1 - w_{jl}), \quad j \in J, \ l \in L_j \tag{9}$$

$$z_j, w_{jl} \in \{0, 1\}, \quad j \in J, \ l \in L_j \tag{10}$$

$$x_{jt}, p_{jm}, e_j, y_{jl}, y_j^{\text{tot}} \ge 0, \quad j \in J, \ l \in L_j, \ t \in T \tag{11}$$

The objective function consists of different parts. The two first parts are associated with the cost of the system (fixed + size related). The third is associated with the maximum monthly usage. Parts four and five are associated with the tariff levels and the last coupled with hourly fees. Each constraint can be described in text as follows, see Table 2.

The model deals with each and every hour during one full year and therefore several thousands of constraints and even more variables are dealt with. The model contains 43,818 rows, 26,310 columns and 126,331 non-zero elements. However, modern software, e.g. GLPSOL, can deal with this without any problems even if it takes about 10 min to find the optimal solution on a common laptop computer.

5. Results

The optimisation shows that a complicated pattern should be used in order to minimise the heating cost for the building. The pattern is the result of, for example, the

- tariff for electricity which has parts based on time-of-use, but also all other details in the tariff such as the level, fixed costs, etc.,
- tariff for district heat with its level, cost segments and so forth, and
- alternative heating equipment costs,
- costs for alternative energy sources.

This makes it very difficult to grasp the situation only by use of the human mind. By using the model, however, it is possible to calculate the situation where a number of things are fixed, e.g. the tariffs which normally are the same for one full year. If the tariffs change in the middle of a year the original calculated strategy might, of course, be wrong but then you can always use the model again with new input data. The climate will of course always change but it is now possible to see for which heat demand a special device, or a combination of them, should be used. It is also important to note that we do not calculate the tariff. Instead the tariff is used as input data. It is, however, a simple task to study the solution for different such tariffs as long as they are not changed in their overall structure. Below we show the result from four different cases. These cases are more examples on how the model can be used than showing the absolute truth on how to provide a large block of flat with heat. In short the four cases examine the following:

- The first optimisation shows the result for the model "as is". This because it is normally a very useful strategy just to see what happens without any pre-determined visions on the result. Of course it is very important to study the solution in close detail in order to understand if the result is logical or not. The main result from this first examination of the model is that district heating prices are so high that it should almost be abandoned.
- The second study just examines what happens if the price of district heat is set to lower values. The tariff is split in different segments and we just lowered the energy price during the "summer segment" and studied the result from this. The main result was that the price is important but not to the grade we had expected.
- Experience from other studies in this field made us suspect that the optimal solution might "bang-bang" from one main strategy to another, i.e. if input data were changed. In our case we expected that the district heating system would "return" as the one-and-only system if we could find a tariff with significantly lower costs.
- Our last study in this paper was also aimed at testing the "flipping" behaviour of the model. By making the initial step cost for one of the systems significantly higher, here we made it 10 times higher, we could see that one of the systems was abandoned but less expected was that district heat only should be used during the summer months.

5.1. First optimisation

The main result of this first optimisation test is that the district heating system should be almost entirely avoided. From a total demand of 1110 kW only 13 should be used. The rest is covered by heat from the heat-pump and the bio-fuelled boiler. The demand and the optimal heat sources for the first 24 h of the year are shown in Table 3. The capacity of the heat pump is chosen, by the optimisation, to be 410 kW while the bio-fuelled boiler is 340 kW,

Table 2
Constraints in the model

(1)	Demand each time period (h)
(2)	Identify the maximum energy usage for each month and system
(3)	Identify the maximum energy usage over the year for each system
(4)	Energy is limited to 0 if the system is not in use
(5)	Energy usage in each system must equal the energy used in the tariff levels
(6)	Energy usage in each system must equal the energy used in the tariff levels
(7)	A tariff level in a system can be used only if the system is used
(8)	Restricting the energy level for each system to its correct upper bound in the tariff levels
(9)	Restricting the energy level for each system to its correct lower bound in the tariff levels
(10)	Restrictions on binary variables
(11)	Restrictions on continuous variables

Table 3
Heat demand and optimal sources according to the first result

Time	Total demand	District heat	Heat pump	Bio-fuel
01	610	0	410	200
02	610	0	410	200
03	600	0	410	190
04	540	0	410	130
05	540	0	410	130
06	570	0	410	160
07	550	0	410	140
08	550	0	410	140
09	550	0	410	140
10	590	0	410	180
11	640	0	410	230
12	660	0	410	250
13	740	0	410	330
14	740	0	410	330
15	810	60	410	340
16	710	0	410	300
17	780	30	410	340
18	780	30	410	340
19	740	0	410	330
20	840	90	410	340
21	720	0	410	310
22	700	0	410	290
23	630	0	410	220
24	630	0	410	220

together 750 kW. If an even higher demand is present it will be covered by district heat. The district heating system therefore acts as a peak load system because of its cost. During January the district heating energy cost is 0.406 SEK/kWh, but then we must also add the water flow cost of 1.8 SEK/m^3. By checking the input data file it is found that 8.9 $m^3$was used for the 610 kWh during the first hour of the year which gives us 0.03 SEK/kWh. The costs in Table 1 must also be included. In order to calculate the cost we must assume that district heat is the only available heat source. In total 2,807,430 kWh was used during the year and dividing this sum with the category number 2200 gives us an y^{tot} of 1276.1. The cost from Table 1 is therefore 36,250 + 215 $\times$1276.1 = 310,612 SEK or 0.11 SEK/kWh. All these cost elements give a total fee of 0.55 SEK/kWh. This must be compared with 0.30/0.7 = 0.43 SEK/kWh for bio-fuel and perhaps about 0.4 SEK/kWh for heat from the heat pump. The heat-pump cost depends to a part on the demand fees which is a little more complicated to depict. From Table 3 it is obvious that the heat-pump should be used first, and second in rank is the bio-fuelled boiler. It is interesting to examine if this behaviour is valid the whole year through. In June the district heating tariff is much lower, i.e. 0.226 SEK/kWh. A closer look at the output data file shows that the heat-pump should be used, 190 kW, but also a small amount of district heat, 10 kW. No bio-fuel is used. The reason for the heat-pump is probably due to the fact that also the electricity prices go down during summer, but why the limit 190 kW is present, is still to be revealed. The electricity cost from the producer is 0.692 SEK/kWh but now the distribution cost is only 0.04 SEK/kWh and the demand fee equals a low 10 SEK/kW. If the heat pump coefficient of performance (COP) is 2.5 the heat coming from the heat-pump costs about 0.3 SEK/kWh which is slightly higher than the cost for heat from the district heating system. The optimisation shows, however, that the heat pump should be used nonetheless perhaps because of the district heating demand fees. In Fig. 4 the use of the heat pump is shown. From the graph it is obvious that the heat pump should be used all around the year, and the amount of heat delivered has been calculated to 2337 MWh, i.e. about 83% of the total demand. A similar graph can be drawn for the bio-fuelled boiler, see Fig. 5. The boiler is used whenever the heat demand is larger than 410 kW and up to 750 kW. 438 MWh is used or 16% of the total demand. The system is used during 2907 h. When both the heat pump and the bio-fuel fired boiler are insufficient district heating comes into rescue, see Fig. 6. The peak demand for district heat is now 360 kW while the amount of district heat is 32 MWh or $\approx$1% of the total use of heat. The system is used during 370 h. For the district heating utility this is a poor situation especially if the heat comes from a CHP plant. This because heat can only be sold to the net during peak conditions. One very efficient way to change this is to decrease the price for district heat.

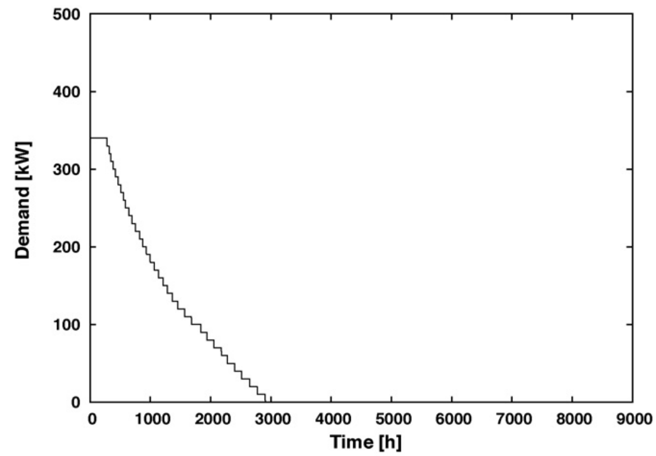

Fig. 5. Duration graph for bio-fuelled boiler. First optimisation.

5.2. Second optimisation

Just for a start, let the summer price for district heat, be 0.15 instead of 0.226 SEK/kWh. The summer starts in June and ends in August, and the output data file from the new optimisation shows that the heat pump is entirely abandoned during those hours, see Fig. 7 which should be compared with Fig. 4. Heat from the heat-pump now adds up to 2113 MWh or 75% of the total. The system operates under 6552 h. Bio-fuel heated heat, 320 kW, to a total amount of 433 MWh should be used for 2907 h. This is almost the same as in the first optimisation so Fig. 5 will not change very much. Instead of the heat-pump, district heating is used. The demand for district heating increased to 380 kW, the amount to 261 MWh, i.e. from 1 to 9%, and the hours from 370 to 2521, see Fig. 8 which should be compared with Fig. 6. It is obvious that the heat pump still will be the major supplier of heat to the building in spite of a lower summer price for district heat. This can also be found in our third experiment with the model.

5.3. Third optimisation

Some district heating utilities have found out that there are competitors in the surrounding world and have also tried to

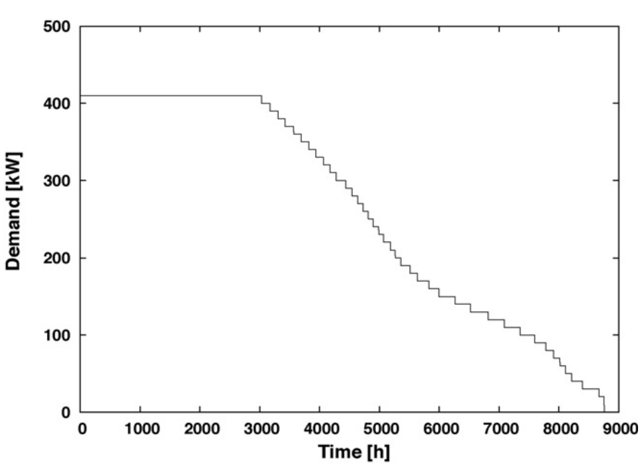

Fig. 4. Duration graph for heat pump. First optimisation.

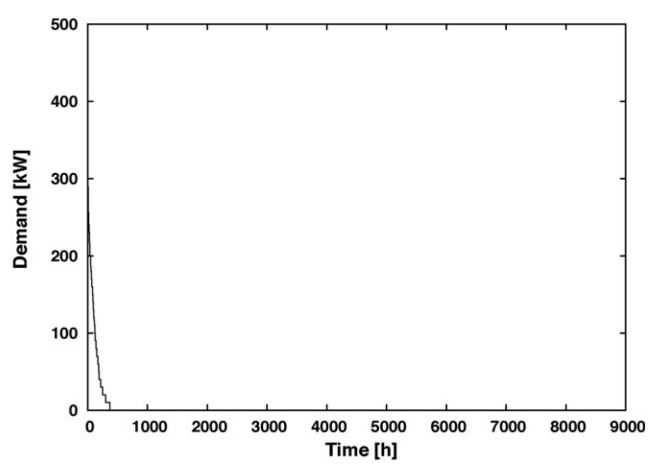

Fig. 6. Duration graph for district heating. First optimisation.

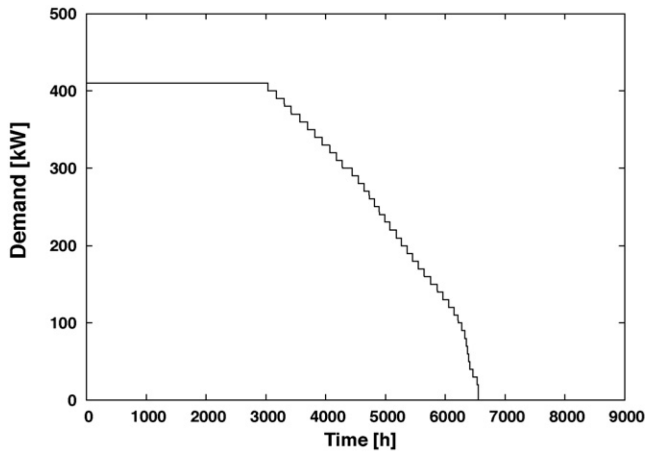

Fig. 7. Duration graph for heat-pump heating. Second optimisation.

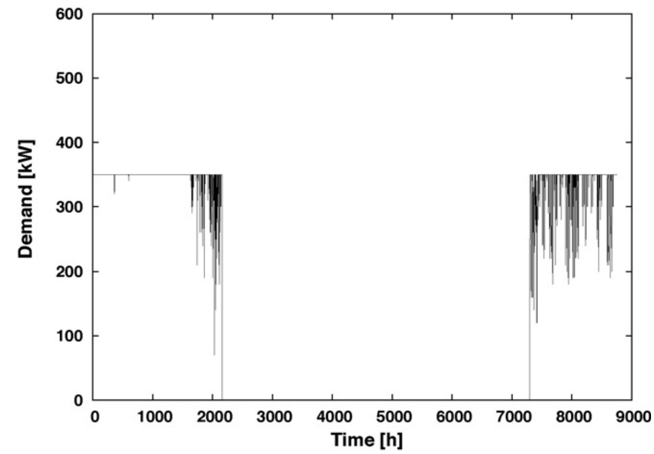

Fig. 9. Optimal use of heat-pump. Third optimisation.

change their tariffs accordingly. One Swedish company sells district heat for 0.07 SEK/kWh from April to October, perhaps in order to make things harder for solar panels, and 0.29 SEK/kWh from November to March. They use a fee for the water flowing through the pipes of 3 SEK/m^{3}for this same period while the demand fees are shown in Table 4. The value of *P* is, for residences, calculated as the energy demand during November–March divided with 1200. The structure of the tariff is very similar to the one in Table 1 but it is not identical. Optimisation shows that a heat pump still is of interest when the district heating price is high, i.e. during November–March. The optimal thermal size is 350 kW and 1224 MWh should be used, i.e. 44% of the total demand. During the cheap district heating hours, i.e. from April to October, heat-pump use should be avoided, see Fig. 9 where the result is shown in chronological order. January is in the left part of the graph while December is in the right part. During the expensive hours the district heating system should be avoided almost entirely, see Fig. 10. Of the total demand 36% or 1007 MWh come from the district heating system. During the winter, November–March, all three systems should be used. If the heat-pump is not sufficient, extra heat is taken from the bio-fuelled boiler and on rare occasions from the district heating system, see the left part of Fig. 10. The *P*-value which is based on the district heat demand between November and March was calculated to 12.48 which is only a few decimals over the lowest interval in Table 4. From April to October district heating is optimal and the maximum demand is found to be 590 kW. Some few hours the demand is larger than that value and then bio-fuel comes into rescue. Bio-fuel is only used during winter conditions, when district heat is expensive, and when the heat pump is not sufficient for covering the demand. In Fig. 11 the situation is depicted. The maximum bio-fuel demand was found to be 470 kW and 576,990 kWh.

5.4. Fourth optimisation

It is obvious that, still, a rather complicated pattern should be used in order to provide the building with heat. Also now the existing district heating system should be abandoned during long periods of time. This depends to a large part on the district heating tariff but also on the cost for alternative boilers and other equipment. Above it was mentioned that these later costs were depicted as a straight line, starting with a step, see Fig. 3. In the case of the heat-pump, this step was set to 100,000 SEK and the slope to 10,000 SEK/kW. It must be noted that this step is important. If the “step” is very low it might be optimal with very small heat-pumps, say only 1 kW, which does not correspond with reality. If, on the other hand, the “step” is too high, the equipment falls out from the optimal solution. If the “step” for the heat-pump is set to 1 MSEK, i.e. 10 times the original value, only district heat, 590 kW, and bio-fuel, 820 kW, should be used. District heating will still be avoided during the high price months, as found in Fig. 10, but instead of the heat-pump, bio-fuel should be used. Also a bio-fuelled boiler might have a large such step, for example because there is a need for building a large chimney. A test shows, however, that the optimal

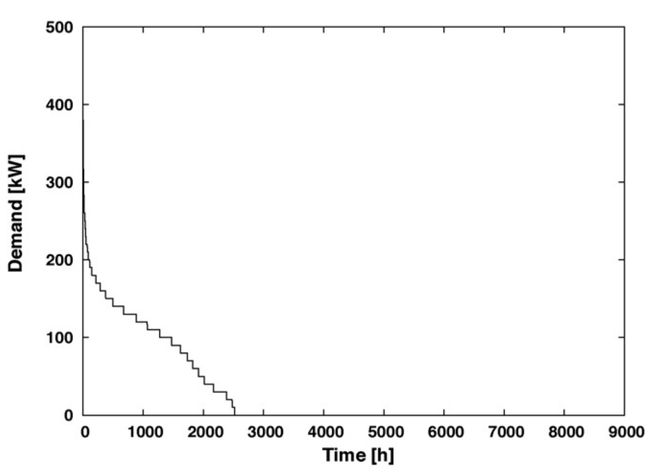

Fig. 8. Duration graph for district heating. Second optimisation.

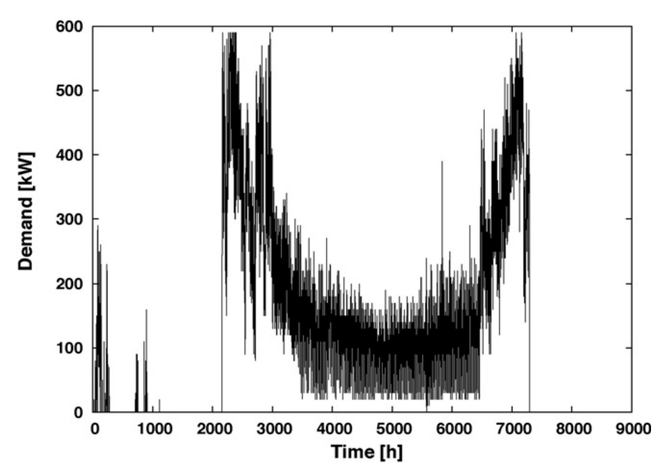

Fig. 10. Optimal use of district heating. Third optimisation.

Table 4
Demand tariff for district heat

Heating power P (kW)	12–100	101–500	501–2000	2000–
Fixed fee (SEK)	655	6,000	21,500	78,800
Demand fee (SEK)	457P	404P	372P	343P

Third optimisation.

situation does not change if the bio-fuelled boiler "step cost" is 10 times larger than in the original study.

5.5. Conservation measures

It is of course also possible to reduce the heating cost by actually reducing the demand, i.e. conservation measures. Consider for example the possibility to put extra insulation on the walls. In [16] it is shown that the new U-value will become

$$U_{\text{new}} = U_{\text{exi}} \frac{k}{k + U_{\text{exi}} t}$$

where U_{new} is the new U-value, U_{exi} the old or existing U-value, k the thermal conductance and t is the thickness of the extra insulation. By adding insulation to the walls or attic we will get a lower total $\sum UA$ value for the building but as is obvious from the expression above the reduction is not linear. Hence, it is not possible just to add the expression to the model as is. Instead we must introduce a linear expression with 0/1 variables which will reduce the value of b_t in constraint number (1). Also the objective must include costs for the different alternatives. By this method the model can "choose" between different levels of extra insulation, e.g. $5, 10, 15, 20, \ldots,$ cm of mineral wool. The method has been described in detail in [17]. There is also another calamity. Insulation measures take a long time before the costs are paid back. The model we have described is only designed to consider 1 year. If we do not consider longer time spans the insulation will always be unprofitable and fall out of the optimal solution. It is of course possible to increase the time span but it must be noticed that the number of variables included in the model will be very high even for a time span of 10 years. Even the most modern computer will then be choked. The same consideration is valid for new and better windows, see [18]. Because it is very difficult to find linear expressions for window retrofits, 0/1 variables must be introduced also for such retrofits. Further difficulties will occur because a thermally better window will reduce the transmitted solar radiation. Together with long time spans the number of variables will grow very much. These problems have earlier been dealt with by reducing the number of "time steps" in the model and several models use only two such steps for each month. From a mathematical point of view this works very well but we will loose all fine details which we wanted to study in this paper. Important is however to notice that the size and pattern of the thermal load will influence the best way to minimise the cost for the building owner. One solution will not be optimal for all buildings.

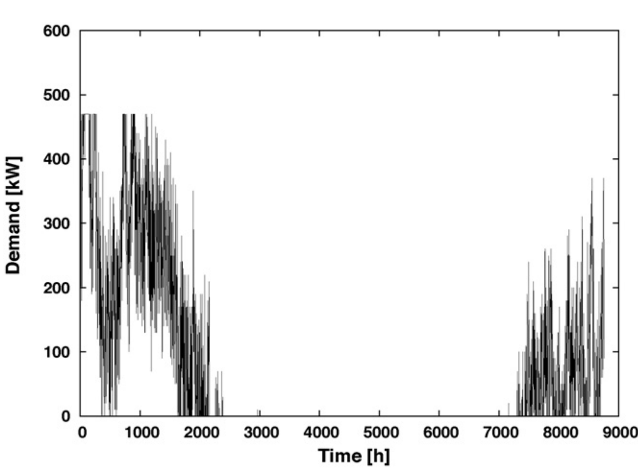

Fig. 11. Optimal use of bio-fuel. Third optimisation.

5.6. Minimising risk

This study shows, as expected, that the tariffs for electricity and district heat are very important for the heating cost. When a building is in its design state certain levels and demand elements of these tariffs apply. Unfortunately, these tariffs are only valid for a very limited time, mostly for 1 year, and all of a sudden both the levels and the tariff design might change. A building is used for very long periods of time, many times 100 years, and hence the original heating system might be a very poor and expensive device. As an example, we can take the large number of buildings in Sweden, Norway and Finland which are heated with electricity. Determination from the authorities to reduce the impact of the society on global warming will perhaps lead to increased taxes on electricity. If the building had been equipped with an alternative heating system, e.g. a wood-fired boiler, it would be easy, and cheap, to change the heat source, and by this make the proprietor immune against higher electricity taxes. At least for larger buildings it must be a good strategy to ascertain space for different boilers and to build devices like chimneys in order to make it possible to change the heating system later. If these things are built at the same time as the rest of the building they will not jeopardize the project. A reconstruction later might however, be impossible, or at least very expensive.

5.7. Models for other countries

Above it was shown that tariff studies are not a very hot topic in the scientific society. We had severe difficulties to find refereed papers where tariffs were shown in such detail as is needed for a model of the type described here. For Denmark, and VEKS, the tariff is somewhat similar to the ones used in Sweden even if we have other means to encourage low return temperatures. In Finland, and the tariff found for Helsinki, a higher emphasis is put on the water flow through the system but this should not be a major obstacle for a MILP model, even if it will take some time to implement the tariff in the "model generator" program.

6. Future work

There are a number of things to test if the model is to be used as an instrument for real-world decisions.

- *Sensitivity* for variations in the thermal load itself. For example, what happens if the building is retrofitted with an added amount of extra insulation, better windows and so forth. To a part this is discussed above but it is not possible to use just a smaller building in order to clarify this. Insulation measures will not influence the thermal pattern during summer, or outside of the heating season, while solar panels are useless during winter nights.
- *Storage* of heat in the building structure. Multi-family blocks-of-flats are very heavy items and can store a lot of heat. If it is of interest for the district heating utility to reduce the peak during certain circumstances it must be possible to use the building as an active heat storage.
- *Tariff* elements can be changed, by the utility, in many ways not dealt with in this paper. What will happen if the intervals and the levels are changed?

- *Real* time optimisation can be of interest if we can use weather forecasts as tools for demand side management.
- *Interest* rates and other economic input data might change things significantly. What will happen if the interest is only 3% or if a shorter life-span is considered.
- *Ventilation* systems are not dealt with at all in this study. How can such devices be included in the form of MILP programming?

7. Conclusions

This study shows that a complicated pattern should be used in order to provide a large building with heat in an optimal way. Optimal is here to be understood as the pattern which gives the proprietor the minimum life-cycle cost. Input data from a real building was used in the form of hourly demand values, i.e. the very same values used by the utility for calculating the district heating bill. Real tariff data are also used and these were found by using the Internet. In fact, one rather expensive tariff has been used and one that is thought to be very competitive. The latter tariff was supposed to show that district heat was a very cheap solution but optimisation revealed that the demand parts in the tariff were so expensive that district heating was to be avoided during the winter, i.e. when these parts of the tariff apply. Everyone knows that it is very hard to predict the future and one important lesson to be drawn from this study is that each new building should be equipped with a number of alternative heating systems. Initial costs for heating systems added later, will many times make it impossible to change a buildings heating system with any profitability. If a chimney is added under the construction phase of the building, this extra cost will probably not be noticeable compared to the total cost for the block of flats. On the other hand, the cost for a new chimney added 20 years after the building was taken into use might make such a system impossible. This study also shows the importance of making models that closely depicts reality. By use of the mixed integer linear programming technique it was possible to adequately address both the district heating, as well as the electricity tariffs. The findings show that the tariff structures have immense influence on the optimal way to heat the building. Without the 0/1 variables, used in MILP, this way had not been revealed.

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