How the Invisible Hand Pushes a Market to Equilibrium:

An Agent-Based Search Model

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1. Intro

In this paper the traditional microeconomic models are put to a test by simulating an exchange economy, with two goods[1]. Comparing the observations from the simulation with the predictions of the model help us to evaluate the accuracy of the predictions. The simulation is an agent-based search model, meaning that independent people (agents) and how they interact (trade) are created in a computer simulation. The “search” part means that agents cannot instantly find a trading partner, and therefore have to spend time searching for one before they can trade.

The changes observed in society due to the interactions of the numerous agents are the result from the model, which is compared to the traditional microeconomic models. Additionally, how this result is reached can be observed- something that is not possible to do with traditional microeconomic models. Before agent-based models, such as the one in this paper, there was no better explanation that the invisible hand. (Smith, 1776) The simulated society is of course greatly simplified compared to the real world, but it will still give insight as to the accuracy of traditional models and how markets actually reach equilibrium.

[1] Two goods are enough, as we can let one good 1 be itself and the good 2 can represent everything else. Thus, we can see how much a consumer wants to spend on good 1 compared to everything else that consumer wants to spend resources on. (Varian, 2010, p. 21)

2. Traditional Microeconomic Models

There are two principles that the traditional microeconomic models are built upon. The optimization principle and the equilibrium principle. The optimization principle is that people are rational and choose what is best for them. This is self-explanatory; people who are given two options will pick what they want the most. Naturally there are exceptions, and people do not always act rational. However, microeconomics typically assumes that people act rational. (Varian, 2010, p. 3)

Then there is the equilibrium principle: in a market, prices adjust until demand is equal to supply. Once a market is in equilibrium then demand and supply will not change, unless there is some external influence. (Varian, 2010, p. 7) This requires understanding supply and demand and how they are built up. These two things are explained below.

Demand is what a consumer is willing to pay for something. The law of demand states that as the price of a good increases the quantity demanded decreases. This inverse relationship means that the demand curve will be downwards sloping. Each consumer has their own individual quantity demanded at different prices. The combined demand of every consumer at each price is the market demand, or aggregate demand. (McConnell, 2009, p. 47) This is shown in Figure 1.

Note that there are two exceptions to the law of demand, Giffen and Veblen goods. A Giffen good is an inferior[2] good that is so inferior that when the price decreases people buy less of it.

[2] Inferior goods are goods that are cheaper, but also worse than some other good. For example, a used car is inferior to a new car.

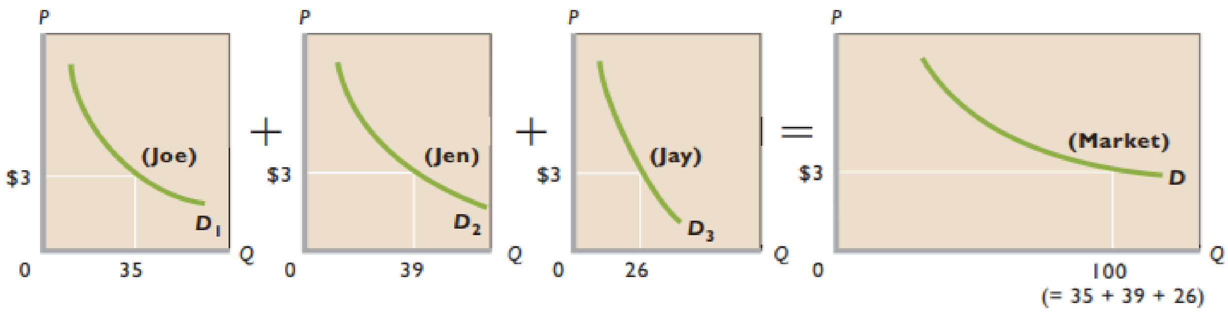
Should the price increase then consumption goes up, since the person who is buying the Giffen good can no longer afford other items. For example, a person may have to consume more gruel as opposed to milk if the price of gruel goes up. (Varian, 2010, p. 104)

The second exception is Veblen goods. Those are luxury goods that are not intrinsically superior to a budget version, but consumers buy them to advertise their wealth. If the price decreased then the demand would go down, as the good is bought primarily due to its high price. (Bagwell & Bernheim, 1996) Neither Veblen nor Giffen goods are present in the agent model this paper is based on, therefore the law of demand can be assumed to always be true in the model.

Figure 1

Market Demand for Corn, Three Buyers

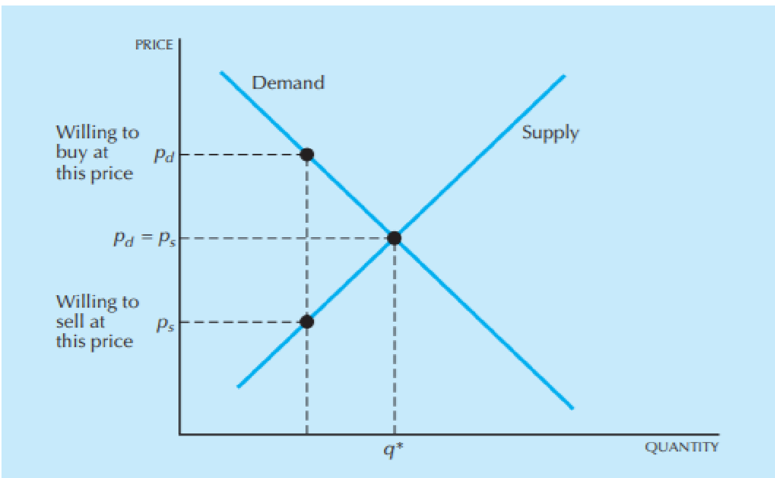
Price per Bushel	Quantity Demanded: Joe		Jen		Jay		Total Quantity Demanded per Week
$5	10	+	12	+	8	=	30
4	20	+	23	+	17	=	60
3	35	+	39	+	26	=	100
2	55	+	60	+	39	=	154
1	80	+	87	+	54	=	221



The market demand curve D is the horizontal summation of the individual demand curves (D_1, D_2, D_3) of all the consumers in the market. At the price of $3, for example, the three individual curves yield a total quantity demanded of 100 bushels. (McConnell, 2009, p. 48)

The law of supply states that an increase in price leads to an increase in quantity supplied. Similar to how the demand curve is made, the supply curve of a market is made by adding together the supply of each producer at each price level. Then by looking at where the supply and demand line intersects the equilibrium can be found. Figure 2 shows this concept.

Figure 2



(Varian, 2010, p. 311)

In the graph above, which shows a competitive market, the optimal quantity (q*) and price can be derived from where the supply intersects the demand curve. Should the quantity that is produced be to the left of q* then the market is not efficient, and producers would increase their production- quantity produced would go up until the market is in equilibrium again. If the quantity produced is to the right of q* then the opposite would happen, consumers don't buy enough of the goods so producers start producing less until the quantity produced is at q* and the market is in equilibrium.

To sum up, a traditional microeconomic model of the market shows the supply and demand line of the producers and consumers of some good. This model tells us where the market would be most efficient, the point where there can be no further gains from trade, and what the price and quantity produced would be at that point. This is the point where the supply curve intersects with the demand curve. The curves are constructed by aggregating the supply and demand in a market with many buyers and sellers, and are frequently illustrated using two lines.

3. What are Agent-Based Models

An agent-based model is a more complex way of reaching answers of how a society behaves, and is a far newer approach than the traditional microeconomic models, with some of the earliest models only being 25 years old. (Gode & Sunder, 1993) The models are created by carefully designing and programing a simulation where free agents interact in a society. The program is then run on a computer, and possible patterns in how the agents interact can be observed. This way by creating rules for interactions on a small scale, the large-scale behavior is visible. (Hamill & Nigel, 2016)

In agent-based computational economics there is seven principles that when followed effectively create a laboratory on a computer where various experiments can be tested. In the agent-based model I created, these principles are followed.

- Agent Definition: An agent is a software entity within a computationally constructed world capable of acting over time on the basis of its own state, i.e., its own internal data, attributes, and methods
- Agent Scope: Agents can represent individuals, social groupings, institutions, biological entities, and/or physical entities.

- Agent Local Constructivity: The action of an agent at any given time is determined as a function of the agent's own state at that time.
- Agent Autonomy: Coordination of agent interactions cannot be externally imposed by means of free-floating restrictions, i.e., restrictions not embodied within agent states.
- System Constructivity: The state of the modeled system at any given time is determined by the ensemble of agent states at that time.
- System Historicity: Given initial agent states, all subsequent events in the modeled system are determined solely by agent interactions
- Modeler as Culture-Dish Experimenter: The role of the modeler is limited to the setting of initial agent states and to the non-perturbational observation, analysis, and reporting of model outcomes.

These principles are taken directly from the same source. (Tesfatsion, Modeling economic systems as locally-constructive sequential games, 2017)

The idea behind using an agent-based model is to not just go straight to equilibrium by doing the math, but observe how (a simulated) society reaches that equilibrium and if it has the same end results as what the theory would predict. This also has the advantage that variables can be easily changed, and the impact of changing them can be observed. Rather than aggregating the supply and demand, we have one agent for each consumer (demand) and producer (supply), and impose rules on how they interact to mimic the market. If the traditional microeconomic theory is correct then the agent-based model should reflect the predictions that a traditional model would make. (Tesfatsion, Agent-bsed Computation Economics (ACE), 2019)

Agent-based models have many strengths. Firstly, compared to mathematical theorizing, an agent model is easily adaptable as the behavior of the agents is trivially controlled and there is no need for representative agents.[3] Additionally, as the model is solved the entire history of the solving process is available so it is not necessary to focus on an optimal point or equilibrium the way a mathematical approach would. Lastly, it is difficult to mathematically model physical space (and social networks) while in an agent-based model it is easily doable to have agents interact based on proximity (for example have agents only interact with neighbors). One disadvantage with agent-based models is that it is necessary with multiple runs varying initial conditions to determine the robustness for the results. (Axtell, 2000)

One of the most emergent uses of agent-based modeling, is to visualize and examine things that previously were impossible.

> *"The most important example of emergent behaviour in economics is Adam Smith's metaphor of the invisible hand: how the self-interested actions of real agents in the economy combine to produce socially optimal outcomes. One of the strengths of agent-based modelling is that this invisible hand is made visible and its workings may be examined."* (Turrell, 2016)

In other words, an agent-based model may be able to give insights that are impossible with other means. To create and fully understand what an agent-based model does and the results it gives, however, requires some programming skills. That is of course a notable challenge and a disadvantage with agent-based models.

[3] A representative agent is the typical agent of a certain type. For example, the typical consumer and typical producer where the agent behave differently based on their type. In contrast to a homogenous agent model (such as the model used in this paper) where all the agents follow the same behavior rules.

4. Utility

The concept of utility started out as a way to show a person's overall well-being, or in other words, a way to measure a person's happiness numerically. The one catch is that there is no way to measure utility. (Varian, 2010, p. 54) However, exactly what utility is, or how it should be measured is not vital for the usefulness of utility. What matters is that utility is a very useful tool for some economic analysis. Alchian explained it the following way:

> *"For analytical convenience it is customary to postulate that an individual seeks to maximize something subject to some constraints. The thing -or numerical measure of the "thing"- which he seeks to maximize is called "utility". Whether or not utility is of some kind glow or warmth, or happiness, is here irrelevant; all that counts is that we can assign numbers to entities or conditions which a person can strive to realize. Then we say the individual seeks to maximize some function of those numbers."* (Alchian, 1953)

The concept of utility has changed over time; today however, "the theory of consumer preferences, and utility is seen only as a way to describe preferences" (Varian, 2010, p. 54). Utility simply shows what a consumer prefers. Suppose a consumer can choose between one apple, and one orange. If the consumer chooses the apple then that means the apple gives the consumer higher utility than the orange.

There is two different ways utility can be described. First there is Ordinal utility, where the utility of a person can be described by saying one bundle of goods is preferred over another, but without specifying how much more. Cardinal utility expands on that, by specifying how much utility a bundle of goods gives numerically. That way how much more utility one bundle of goods gives over another can be calculated.

Ordinal utility

Ordinal utility is ranking goods based on which one is preferred. Suppose there are three goods A, B, and C. If a person prefers A over B and B over C the goods could be ranked after their preference. A is 1st, B is 2nd and C is 3rd. This ranking can be used to predict behavior: a person might trade C for A because the person prefers A. However, these rankings have no intensity- there is no way to know if the person would trade A for both B and C, except of course observe what the person chooses given the choice. That is because there is no way to measure how much a person prefers one good over the other, everything that can be observed is what good is preferred (or if a person is indifferent). (Barnett, 2003)

Cardinal Utility

Cardinal utility avoids the problem of predicting behavior that ordinal utility has by using numbers, which lets us do math to correctly predict behavior. In the example above, if it is known that the person prefers A twice as much as B, and B twice as much as C then more predictions can be made. Suppose the utility of A is 4, B is 2, and C is 1. Would the person trade A for B and C? No, since A have a higher utility. However, the numbers have no meaning by themselves, A could be 100, B 50, and C 25- the answer would be the same. The problem is that these numbers are arbitrary decided based on an Ordinal ranking as in reality there is no way to measure how strongly a person prefers one good over the other. (Barnett, 2003)

In the simulation that this paper is based upon cardinal utility is used. That way a utility function can be used to calculate how much utility a given bundle of goods gives. The numbers that represents the utility of the agents are used to facilitate trade and to observe changes in the overall utility. This is done because it is simply necessary to have numbers for the program to determine what the agents chose. In other words, the numbers are needed for the agents in simulation to compare two different bundles of goods.

5. Overview of my Simulation

Suppose an economy with many agents, who have individual preferences, and two goods (an exchange economy). The agents will trade between each other such that they each increase their individual utility. They will not make a trade that would decrease their utility, and they will not trade if they have no more goods to trade. The total utility in this society would increase as the agents increase their individual utility by trading. This effect where the utility of the society increases through the forces of the market is known as the invisible hand. (Smith, 1776) That is the theory, but is that what will happen in the simulation? The simulation will be a way to see what happens in model markets, and the predictions of traditional microeconomics can be tested.

In the world of the agents there are some simple rules. The agents are randomly given an initial endowment of goods as well as preferences for them. The number of goods and each of the agent's preferences can either be random or set by the user. The agents move in a random direction, and each time two agents collide they attempt to trade and bounce away from each other.

The simulation will act as a greatly simplified version of the human world, where each human is their own individual and has their own preferences. However, in the simulation, unlike real life, many different scenarios can be tested. Though by adjusting variables the impact of different factors can be determined. What would be the impact of the density of agents? With very low-density trade might not happen at all due do the low chances of one agent meeting another agent, and with too high density perhaps trade will be limited as agents keep meeting the same trade partners. What is the effect of increasing the endowments of a given good or increasing the preference for a good? These, and more, questions can be answered by testing different scenarios in the simulation. Where possible, the results can be compared to traditional microeconomic theory.

6. Predictions

In the simulated world of agents moving around and trading we can make some predictions about what will happen to this simulated society based on microeconomic theory. If microeconomic theory is correct about the behavior of society one would expect the predictions to be accurate also in this agent-based world.

Prediction 1

> *Over time the utility of (the simulated) society will go up, until society is at a Pareto[4] efficient point.*

[4] A pareto efficient point is where the utility of a society cannot be increased trough trading goods without making at least one person worse of. (Varian, 2010, p. 15)

The first and perhaps most obvious prediction, Prediction 1 is always the case with rational agents. That is because trade does not happen if it would decrease the utility of an agent, so utility can only increase until society is Pareto efficient. Therefore, the agents will trade, and move a little bit closer to a Pareto efficient allocation, and then trade again, and again. They keep trading until no more trades are possible, and the society is at a Pareto efficient point. (Varian, 2010, p. 585)

Furthermore, as the utility in the society increases and the agents allocate the goods between them it will be a decreasing chance of trade occurring simply due to the finite amount of goods that each agent initially starts with. Therefore, it would seem logical to expect that the trades will eventually stop occurring as the agents reach equilibrium and the society is at a Pareto efficient point- it is not possible to make one agent better off without making at least one other agent worse off (Varian, 2010, p. 15). Thus Prediction 2 can be made.

Prediction 2

> *As the allocation of apples and oranges (goods) approaches Pareto efficiency, the frequency of successful trades slow down.*

Increasing the initial endowments of goods would therefore let trade happen for a longer amount of time because it would take more time steps before each agent has traded away their goods, but the same trend would be expected— as the allocation approaches Pareto efficiency trading slows down and eventually stops. This is partly due to how to simulation is designed, the agent’s always trade 1 apple for some number of oranges. The number of oranges is the lowest number that makes the agent that gives away 1 apple at least as happy as it was before the trade. In the real world it is of course possible to trade more than 1 apple at a time, so therefore this Prediction 3 only apply to this simulation.

Prediction 3

With higher initial endowments the time needed to reach Pareto efficiency increases.

Suppose, the agents are divided into two or more regions. Each region would reach a Pareto efficient point within itself. What then happens if these regions are able to trade with each other? Recall that the equilibrium principle mentioned in the section about Traditional Microeconomic Models state that once the market is in equilibrium it will not change, unless there is some external influence. (Varian, 2010, p. 7) In this case these markets being able to trade means they are an external influence on each other. In essence, that means all the regions that had reached a Pareto optimal point now have more trading partners- and unless every region stopped at the same price, which is unlikely, the new single region opens up for more trading.

Prediction 4

Markets can reach a higher utility if they can trade with other markets.

Prediction 4 crosses into the part of economics that study international trade. The inability to trade with other markets is in a sense a complete trade barrier. Additionally, note that especially with an uneven distribution of resources (Imagine that one market has mostly apples, the other mostly oranges) international trade is mutually beneficial. Tough there is much more to international trade than that, but even in this simulation we can get an idea of why international increases overall utility. (McConnell, 2009, pp. 743-761)

However, there are some things that microeconomic theory cannot predict. In my simulation it is possible to manipulate the radius of the agents, but how that affects behavior of the markets is not something that traditional microeconomic models can answers. Therefore, it is necessary to just run the simulation and see what happens.

Prediction 5

Changing the radius of the agents have an unknown effect.

Further predictions based on traditional microeconomics vary between different types of preferences, which are described by different types of utility functions. For example, the allocation of goods can be predicted based on the utility function. Since the simulated world includes more than one utility function the predictions for those are in the sub-sections below.

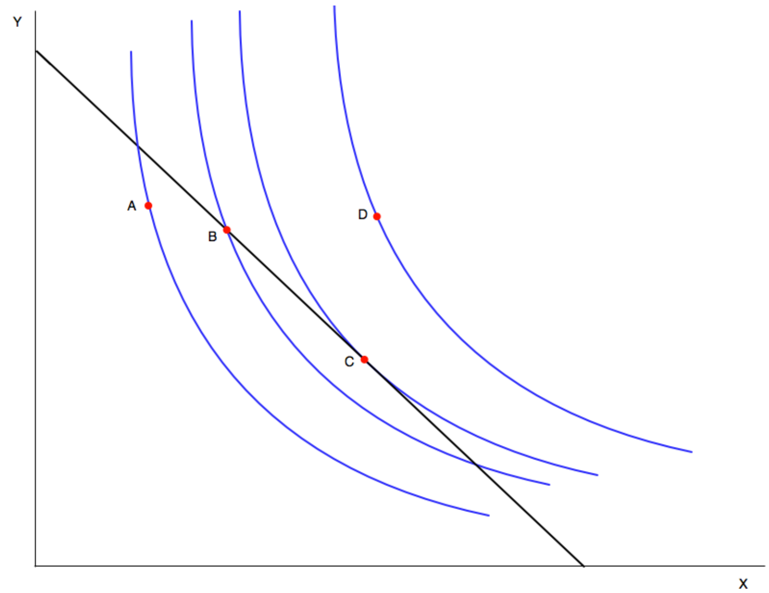
However, before those can be analyzed some assumptions about all preferences must be made. There are three main assumptions economists make about preferences. First, they must be complete, which simply means that any bundle of goods can be compared to any other bundle of goods. Additionally, preferences are assumed to be reflexive. Reflexivity simply means that any bundle is at least as good as itself. Lastly, preferences are transitive. That means that if there is three different bundles of goods A, B and C where A is preferred over B and B is preferred over C. Then A is at least as good as C. (Varian, 2010, p. 35)

Cobb-Douglas Utility Function (normal preferences)

Since Cobb-Douglass is a commonly used utility function I have chosen to include it in my program. Cobb-Douglass represents normal preferences, which have one more assumption in addition to the three assumptions mentioned above. This extra assumption is that more is better, which means that the bundle (3x, 3y) gives a higher utility than (2x, 2y). The simulation uses apples and oranges instead of x's and y's, but is otherwise the same as the equation in Figure 4.

To visualize the preferences, a graph with indifference curves for different levels of utility can be used. An indifference curve shows all the different combinations of goods that give the same utility. The higher indifference curve has more utility, for example in Figure 3 the optimal choice is the bundle of X and Y that gives point C. The bundle D is at a higher indifference curve, but outside the budget line (there is a finite amount of resources, which can be spent entirely on Y, X or on some combination). Bundle B is also on the budget line, but on a lower indifference curve than C. (preferences and indifference curves, 2019)

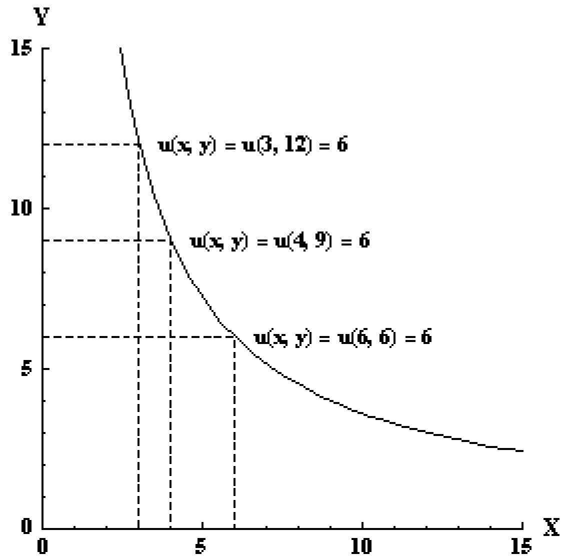
Figure 3



(preferences and indifference curves, 2019)

The slope of the indifference curve at a given point is the rate that one consumer is willing to trade one good for another. The consumer is willing to trade as long as the same level (or higher) of utility is maintained, trading on the slope means the consumer moves to a different point on the same indifference curve. This rate is called the marginal rate of substitution (MRS). (the marginal rate of substitution and gains from trade, 2019)

Figure 4



(EconPort, 2019)

In Figure 4 the indifference curve of the utility function u(x, y) = $x^{0.5}$ $y^{0.5}$ = 6 is drawn, and we can see three bundles of goods that give the same amount of utility (and so would any point on the indifference curve). Therefore, an agent with the bundle u(3,12) would be willing to give up 3 of y to get one more x, which would give the bundle u(4,9). The marginal rate of substitution is $\frac{1}{3}$ = 3 in this case. However, from the new bundle u(4,9) an agent would be willing to give up 3 of y for 2 of x to get bundle u(6,6) which also have the same utility. However, in this case the marginal rate of substitution is $\frac{2}{3}$ = 1.5, which is less than it was previously.

In other words, the utility gained by one more x compared to the utility gained by one more y is reduced for each additional x an agent has. This important property of a Cobb-Douglass utility function is called diminishing marginal rate of substitution. (EconPort, 2019).

In Equation 1, A is the number of apples and O is the number of oranges. α and β represent the respective preferences for the two goods (Varian, 2010, p. 64).

Equation 1

$$U(A, O) = A^{\alpha} * O^{\beta}$$

A very useful characteristic of the Cobb-Douglas utility function is that there is always a monotonic transformation[5] that will make the sum of the exponents equal to 1. Therefore, instead of β we can use $1 - \alpha$ and let α be a number between 0 and 1 (Varian, 2010, p. 65).

Equation 2

$$U(A, O) = A^{\alpha} * O^{1-\alpha}$$

The implementation of Cobb-Douglass utility function in the simulation uses Equation 2 to calculate the utility, because it is easy to use this variation of the Cobb-Douglass utility function. Additionally, the results would be valid for any other Cobb-Douglass utility function where the exponents do not sum to 1, as they can always be transformed into Equation 2.

The reason that it is so useful the exponents sum to 1 in a Cobb-Douglass utility function is that the exponents then also tell what fraction of apples or oranges an agent should have to maximize its utility. Suppose that α is 0.3, then the agent will spend 30% of its resources on apples and 70% on oranges when maximizing its utility.

With this there is enough for a trade to happen between agents. Suppose there are two agents, agent A and B, which have 15 apples and 15 oranges each. A have an α value of 0.7 and B have

[5] "A monotonic transformation is a way of transforming one set of numbers into another set of numbers in a way that preserves the order of the numbers. We typically represent a monotonic transformation by a function f(u) that transforms each number u into some other number f(u), in a way that preserves the order of the numbers in the sense that u1 > u2 implies f(u1) > f(u2)." (Varian, 2010, p. 56)

a α value of 0.5. The utility of A and B can be described by Equation 3 and Equation 4 respectively.

Equation 3

$$\mathrm{U}_{\mathrm{Agent\ A}}(\mathrm{A},\mathrm{O}) = A^{0.7} * O^{1-0.3} = 15^{0.7} * 15^{0.3} = 15$$

Equation 4

$$\mathrm{U}_{\mathrm{Agent\ B}}(\mathrm{A},\mathrm{O}) = A^{0.5} * O^{1-0.5} = 15^{0.5} * 15^{0.5} = 15$$

The marginal rate of substitution with respect to apples is given by Equation 5 and Equation 6. Note that A has a higher marginal rate of substitution (A want's oranges more than B), so in this trade B will give away 1 apple.

Equation 5

$$\mathrm{MRS}_{\mathrm{Agent\ A}} = \frac{\alpha * O}{(1-\alpha) * A} = \frac{0.7 * 15}{0.3 * 15} = 2.33$$

Equation 6

$$\mathrm{MRS}_{\mathrm{Agent\ B}} = \frac{\alpha * O}{(1-\alpha) * A} = \frac{0.5 * 15}{0.5 * 15} = 1$$

In order for B to be willing to trade the number of oranges that B gets from A must be enough that B's utility does not decrease. In this case the price is two oranges, and after the trade both agents have higher utility.

Equation 7

$$\mathrm{U}_{\mathrm{Agent\ A}}(\mathrm{A},\mathrm{O}) = A^{0.7} * O^{1-0.3} = 16^{0.7} * 13^{0.3} = 15.03$$

Equation 8

$$\mathrm{U}_{\mathrm{Agent\ B}}(\mathrm{A},\mathrm{O}) = A^{0.4} * O^{1-0.4} = 14^{0.5} * 17^{0.5} = 15.43$$

However, now the agents cannot trade further without making one of them worse of (It is possible to trade more by not trading whole units, but in this simulation only whole units are traded). This is because they have allocated the goods in a Pareto optimal way. Notice that the goods are allocated depending on the preferences of the agents.

Therefore, when α is a random value (between 0 and 1) then the agents eventually end up with having bundles that consist of many different percentages of apples. Naturally, that is assuming they all begin with the same number of goods.

Prediction 6

> *With random preferences the agents end up having percentages of apples ranging from 0-100% evenly distributed.*

Should the preferences not be random, but the same for all the agents then all the agents will try to have the same percentage of goods. Specifically, if α is 0.5 for all the agents then they will trade such that they all move towards having 50% apples and 50% oranges. However, if both the number apples and oranges and the preference is the same then no trading will occur since the agents have the same MRS. Thus, in this case the agents would have to start with different amounts of apples and oranges.

Prediction 7

> *If all the agents have the same preference they will trade such that they end up with the same percentage of apples.*

With specific equations for each agent's utility function the equilibrium price can be calculated (Varian, 2010, pp. 593-595). However, without knowing the equations the equilibrium price cannot be calculated and all that can be said is that there is an equilibrium price- even though the price is unknown. (Varian, 2010, p. 595) In my simulation we can see the prices change over time, and this should give more insight than what can be done with math. Once the agent has traded at the equilibrium price then the allocation of resources is Pareto efficient. (Varian, 2010, p. 598) Thus, once the market has reached equilibrium there should be no more trades, and no more points on the price graph in my simulation should appear.

Prediction 8

> *Once the market is in equilibrium there should be no more trades shown on the price graph*

Linear Utility Function (perfect substitutes)

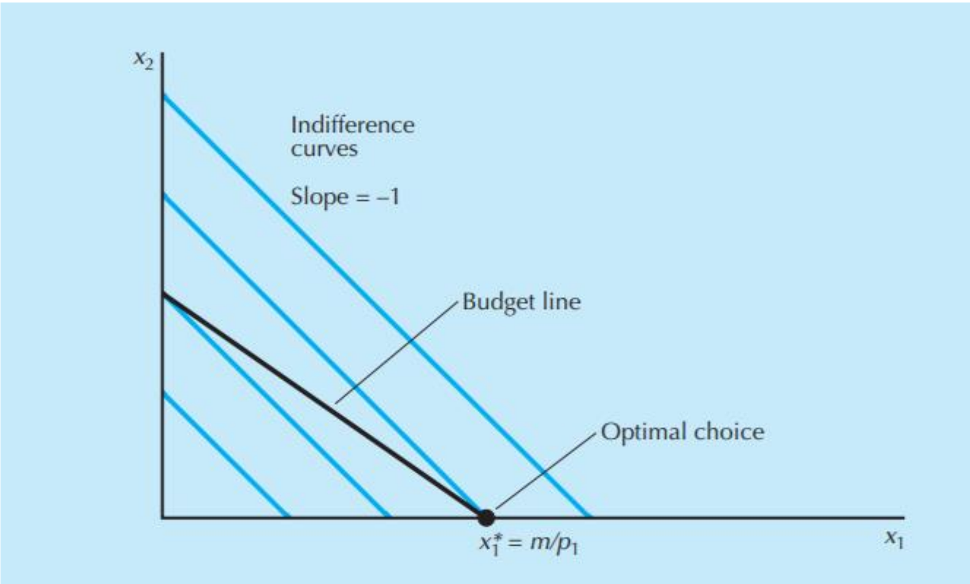
Perfect substitutes are a special type of normal preferences. Two goods are perfect substitutes if an agent is willing to substitute one good for the other at a constant rate. (Varian, 2010, p. 38) Suppose an agent is always willing to give up 1 apple for at least 1 orange, for this agent apples and oranges would be perfect substitutes. Keep in mind this constant rate does not need to be 1 to 1. For example, an agent always being willing to trade 1 apple for at least 2 oranges would also mean apples and oranges are perfect substitutes for that agent.

Agents with perfect substitutes will always trade such that they gain more of the good they prefer, because that will increase their utility. The exception is the agents that prefer both goods equally, since the utility of those agents will only depend on the number of goods they have and not how those goods are allocated. When one agent has a linear utility function, there will be a constant rate where the agent is willing to substitute one good for another. Furthermore, since there is a constant rate then that means a linear utility function describes perfect substitutes. For an agent with perfect substitutes which goods the agents have when maximizing its utility can be predicted.

Prediction 9

> *Agents with a linear utility function will usually end up with a corner solution (they only have apples or only have oranges) after enough time have passed.*

Figure 5



Optimal choice with perfect substitutes. If the goods are perfect substitutes, the optimal choice will usually be on the boundary.

(Varian, 2010, p. 79)

Suppose a world with two goods---apples and oranges---and they are perfect substitutes. For example, agent A has 2 apples (A) and 4 oranges (O). A has a preference value of 6 for apples and 4 for oranges. The preference value for apples and oranges are denoted α and β, respectively. The total utility for A would be then be:

Equation 9

$$U_{Agent\ A}(A, O) = \alpha A + \beta O = 6 * 2 + 4 * 4 = 28$$

A's marginal rate of substitution for apples would be:

Equation 10

$$\frac{\alpha}{\beta} = \frac{6}{4} = \frac{3}{2}$$

That means A is willing to give up 3 oranges if at least 2 apples are given in return. After this exchange A will be as happy as before if A gets exactly 2 apples in return, and happier if more than 2 apples are given in return. In other words, A will after a trade always be at least as happy or happier than before the trade. That can be checked by calculating the utility again.

Equation 11

$$U_{Agent\ A}(A, O) = \alpha A + \beta O = 6 * 4 + 4 * 1 = 28$$

In this case A have the same utility before the trade, calculated in Equation 9, as A does after trading- calculated in Equation 11.

The same concept works the other way around, A is willing to trade away apples if enough oranges is given in return. For example, suppose A gave away 2 apples and got 3 oranges in return. The utility would then be

Equation 12

$$U_{Agent\ A}(A, O) = \alpha A + \beta O = 6 * 0 + 4 * 7 = 28$$

Again, the utility at the end of this trade is the same as in Equation 9 and in Equation 11. That is because the trade is completed on A's margin, in other words A's utility does not change.

However, for a trade to happen there needs to be two sides. Let's say agent B have 6 apples with a preference of 2 and 8 oranges with a preference of 4.

Equation 13

$$U_{Agent\ B}(A, O) = \alpha A + \beta O = 8 * 6 + 2 * 8 = 64$$

B's marginal rate of substitution for apples would be: $\frac{\alpha}{\beta} = \frac{8}{2} = \frac{4}{1}$

Note that A and B have different marginal rates of substitution, if they were the same then trade would not happen since neither agent would be better off without the other being worse off.

B have a greater preference for apples than A. Therefore, if they were to trade A would give up apples in exchange for oranges. The exchange ratio, or price, would be somewhere from A's margin of $\frac{3}{2}$ to B's margin of $\frac{4}{1}$ of apples to oranges. Exactly what the price is does not matter that much, as long it is within that range, so let's suppose a trade on A's margin. A gives up 2 apples to B and get 3 oranges in return. That would leave A with the same utility as before as is calculated above. However, B's utility would go up and so would the total utility in this mini society.

Equation 14

$$U_{Agent\ B}(A, O) = \alpha A + \beta O = 8 * 8 + 2 * 5 = 74$$

The two agents, A and B, would keep trading until one of them had no more goods to trade. In this case A is left with only oranges and B with more apples. In this example the agents have linear preferences. So, the agents will keep trading until they only have one good, since that is where their utility is maximized. Of course, that assumes the agents have trading partners.

Therefore, when an agent has a linear utility preference we can see that Prediction 9 is logical in theory, the agent should end up with only one type of goods.

The price can also be predicted; because the goods are perfect substitutes, the price each individual agent is willing to pay will not change throughout the simulation. The price will always be equal to or less than an agent's marginal rate of substitution. Therefore, the overall price should be relatively constant.

Prediction 10

The price will be relatively constant during the simulation

7. Simulation

Before the observations and results from the simulation can be discussed it is important to understand the logic behind the simulation. In pseudocode this is how it works:

At the start:

All the agents are given endowments and assigned preferences for each good.

Agents are put on a random location and given a random direction.

Each time step:

Agents move in their current direction.

Agents bounce when they hit the border of their region

Agents that collide try to trade goods.

Trade occurs if they have different marginal rates of substitution and enough goods to trade.

Agents that collide get a new direction bouncing away from each other

The big question is when the agents do trade, how much do they trade of each good. In other words, what is the price? The price will be somewhere from the marginal rate of substitution, which is calculated by dividing one good by the other, of one agent to the marginal rate of substitution of the other agent. In this simulation the agents trade on the margin. That means after a trade the utility of one agent is unchanged while the utility of the other agent goes up. In each trade which agent that gives away 1 apple is the same, so the simulation is completely deterministic, from a saved position the same outcome will always happen. This can be avoided by adding randomness, and is a setting that may be implemented in a future update.

8. Implementation

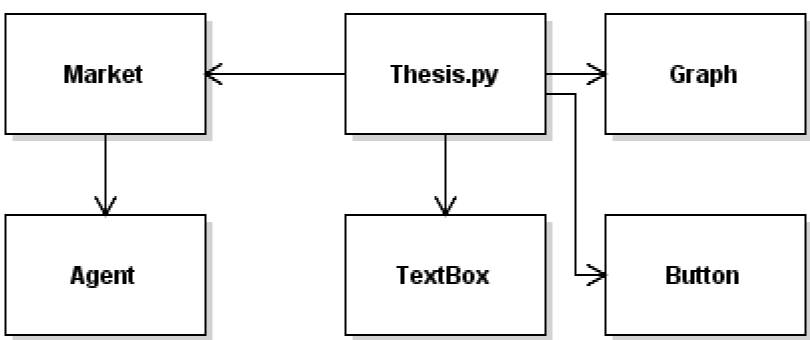
The first part of implementing something is to decide what exactly we are trying to do, so it is natural to begin with the requirements of the simulation. The simulation needs to have agents, which have some number of two different goods- here we use apples and oranges. Furthermore, the agents need to have a utility function that gives a utility based on the number of apples and oranges. With that the agents are now able to trade, trading happens between two agents. One agent gives up x number of oranges to receive one apple, and the other gives up one apple to receive x number of oranges. Trade only happens if it makes one agent better off, and the other agent is no worse off[6]. In order to make things simple, the agents can only trade whole units of apples or oranges.

[6] The other agent may be better off, or may just move to a different point on the same indifference curve.

The simulation is implemented in python, and utilizes the open source library pygame (Shinners, 2019) in addition to other standard libraries. The simulation itself is designed to be flexible and have many different settings. This enables us to easily run many different scenarios by adjusting the settings. For example, multiple regions, varying number of agents, different types of preferences, and different initial conditions are all adjustable. Since the settings for individual markets are adjustable different scenarios can be started and compared simultaneously by having multiple markets.

The simulation is designed with an object-oriented approach and is split into 6 different main parts, with thesis.py being the main file. Figure 6 shows how these sections depend on each other. The purpose of each section is explained below.

Figure 6

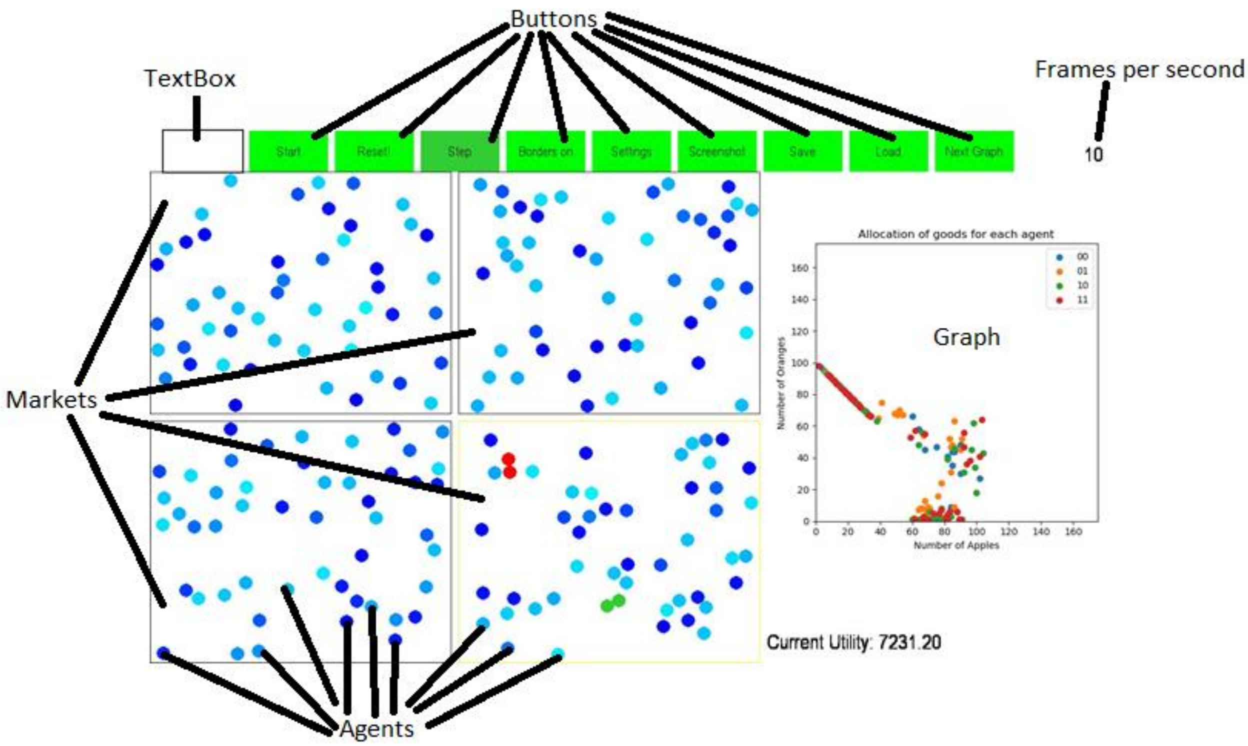


User Interface Design

The user interface is how the user will interact with the application; in the application I have created a graphical user interface. The interface is designed to be intuitive and easy to use, with the goal that a user would not need any instructions to use the application.

Figure 7 shows how the different components of the application are put together to create everything that is visible on the screen. There is a lot of information here, but a user who is familiar with computers should not have a problem navigating the program.

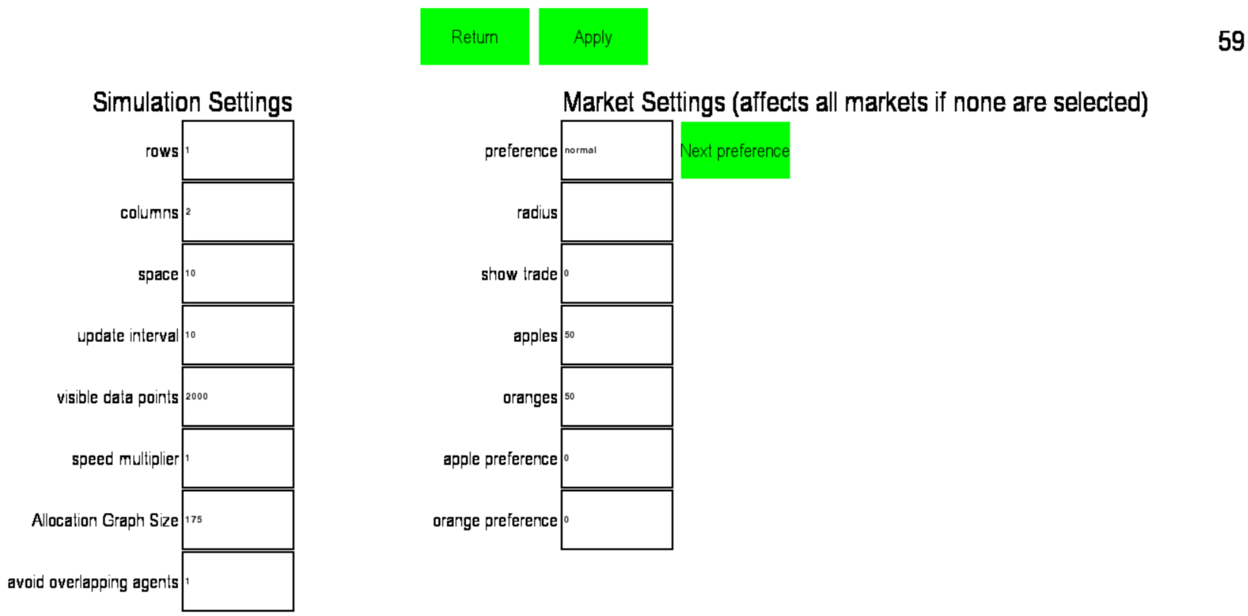
Figure 7



TextBox is the input boxes, where the user can enter some input. This is used to let the user enter many agents at once by typing in a number and then pressing the enter key. Of course, this will only create new agents in selected markets (or all markets if no markets are selected).

The Buttons are on the top of the simulation and each have a different function when executed (the button is clicked on). What each button does is indicated by its name. From a user interface perspective, the most interesting button is the Settings button. That button enables the user to change the settings, and after clicking it the screen will now look like Figure 8. The TextBoxes show the current settings, and the user can simply click a box to enter a different value. If a box under the Market Settings is blank then multiple markets with different values for that box are selected. In Figure 8 the value for the radius is different for the two markets, but both the markets are selected (so the settings for both markets are being changed) therefore the box is empty. The preference has a button next to it so that the user can go through all available preferences and to make it easier to change (less typing).

Figure 8



When changing settings, it's important that only valid settings are possible. Part of ensuring that is done by only accepting digits in all the boxes except the preference. For example, if a user attempts to type a letter into the box next to rows then nothing will happen. However, if the user enters a value that is too big or too small (such as 0 rows, or too big radius) then it must be clear that the value entered by the user is out of bounds.

When the user clicks apply usually the application will return to the screen seen in Figure 7. However, if a setting is out of bounds then it will remain on the same screen. The settings that are out of bounds will be changed to the closest value that is valid, thus making the user understand what is wrong. The boxes of the settings that are changed are also outlined in red, to make it very clear where the problem is. Figure 9 shows what this looks like.

Figure 9

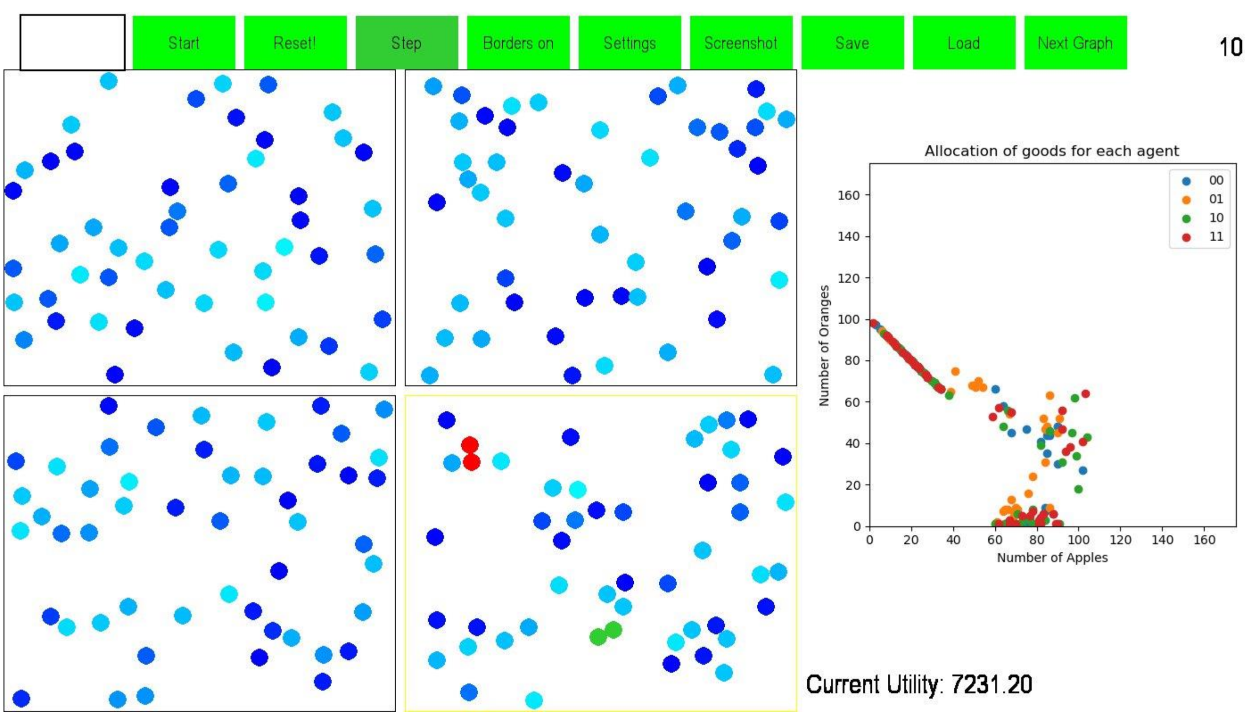
Markets

Each Market contains a list of agents, and facilitates trading between them. The simulation has the capability to have multiple markets at once. Figure 10 shows 4 markets, where the bottom right market is selected. The selected market is in column 1 and row 1 (begins at 0), so it is the red dots on the graph. In essence each market is its own simulation, and they can have different settings from each other.

However, the borders between markets can be turned on or off. That way all the different markets can be enabled to trade with each other. This is similar to having many countries in the real world. The countries can then begin to trade with each other, thus simulating international trade. Should the borders be turned on after they have been turned off, which one can imagine is comparable to a country imposing an embargo, the agents return to their respective markets before they resume trading.

Next, there is the issue of how deciding who is trading (if possible) with who. Since this simulation is particularly meant to gain insight it is important to visualize what happens. The agents are represented by circles in a market, and when two agents collide (their circles touch) then they attempt to trade. Visually the agents that successfully trade are indicated by the agents flashing green for one frame, while red blinks happen when a trade is not possible. This way it can be observed if trades are happening or not, but of course this can be turned on or off in the settings. Figure 10 shows how this looks, green agents successfully completed a trade and red agents are not able to trade. Note that it is only in the selected market that the setting for displaying when trades happen is turned on.

Figure 10



Modeling Details

One of the important reasons to make this program is to gain insight by visually being able to see what happens in an economy. The plot, which updates in real time, and the colors of the agents changing to indicate their allocation of apples and oranges both aid the user in seeing what happens. For humans it is helpful to see what happens, and that along with an understanding of the math behind microeconomics can give a greater understanding of why markets work the way they do.

The agents in Figure 10 have different colors, that is because the color of each agent depends on its allocation of goods. The colors scale from blue, or in RGB[7] (0,0,255), to cyan (0,255,255). Therefore, the only color that needs to be changed is green. To do that the percentage of apples is calculated, and then that is used to scale the intensity of the green for each agent. The code for this is shown below in Figure 11. An agent with only apples will have scalar value equal to 1. This will lead to the agent's color containing no green, as shown in Equation 15. Therefore, agents that only have apples will be blue. Should do agent have no apples, then they would be cyan.

Equation 15

$$255 * (1 - 1) = 255 * 0 = 0 = g$$

Figure 11

```
def update_color(self):
    scalar = self.apples / (self.apples + self.oranges)
    r = 0
    g = int(255 * (1 - scalar))
    b = 255
    self.color = (r, g, b)
```

[7] RGB, short for "red, green blue" is how colors are represented on a digital screen. Each color has an intensity from 0 to 255. Every color that can be shown on a screen is a mix of red, green and blue. For example, red is just (255,0,0).

Collision Detection

The most important part is of course moving the agents, and the trading of goods between colliding agents. The code shown in Figure 12 is a snippet of the complete implementation, and shows the logic behind moving the agents. Note that this matches the pseudocode in the Simulation section.

Figure 12

```
for row in range(len(market_list)):
    for column in range(len(market_list[row])):
        for agent in market_list[row][column].agents:
            r,c = agent.box
            agent.move()

            for other_agent in get_nearby_agents(r,c):
                if agent.collision(other_agent):
                    agent.bounce(other_agent)
                    market_list[row][column].trade(agent, other_agent)
                    #only collide with one agent each timestep
                    break
            find_new_box(agent)
```

Computationally speaking there is one big bottleneck in this simulation, detecting if agents collide with each other. The basic approach of moving an agent and then checking if that agent is colliding with any other agents work, but it is not very fast. With n number of agents, this algorithm would lead to a worst-case scenario of $n*n = n^2$ times the program would have to check if two agents collide. That is bad, as the number of agents increase the execution time quadratically. For example, 10 agents mean 100 collision checks is needed. 100 agents mean that 10 000 collision checks are needed[8]. To handle a large number of agents a better algorithm is needed.

[8] 10*10 = 100
100*100 = 10 000

The execution time can be cut in half by the following process: Each time the agents move imagine that the market is empty- there is no agents anywhere. The first agent to move would not need to check for collision, since there are no other agents to collide with. The second agent to move would only have to check for collision against the first agent, and so on. This way the overall number of operations can be cut in half- rather than n^2 it is now $\frac{n^2}{2}$. This helps, but does not solve the problem with the exponential growth.

One algorithmic approach to this problem is to use a cell list. To avoid checking if agents far apart collide the rectangle (market) can be divided into a grid. With a grid the number of collisions can be drastically reduced. In Figure 14 this is visualized, we can see that there are many agents, which it is no longer necessarily to check for collision against (the ones that are not in the same or neighboring boxes). Figure 13 shows how the boxes are created in my simulation. Of course, adding a grid does also add some computation time because which cell an agent is in needs to be continuously updated. However, with a large number of agents the reduced number of collision checks needed leads to a significant reduction in overall operations needed. In other words, it is much faster even though some time is spent on updating which agent is in which cell.

Figure 13

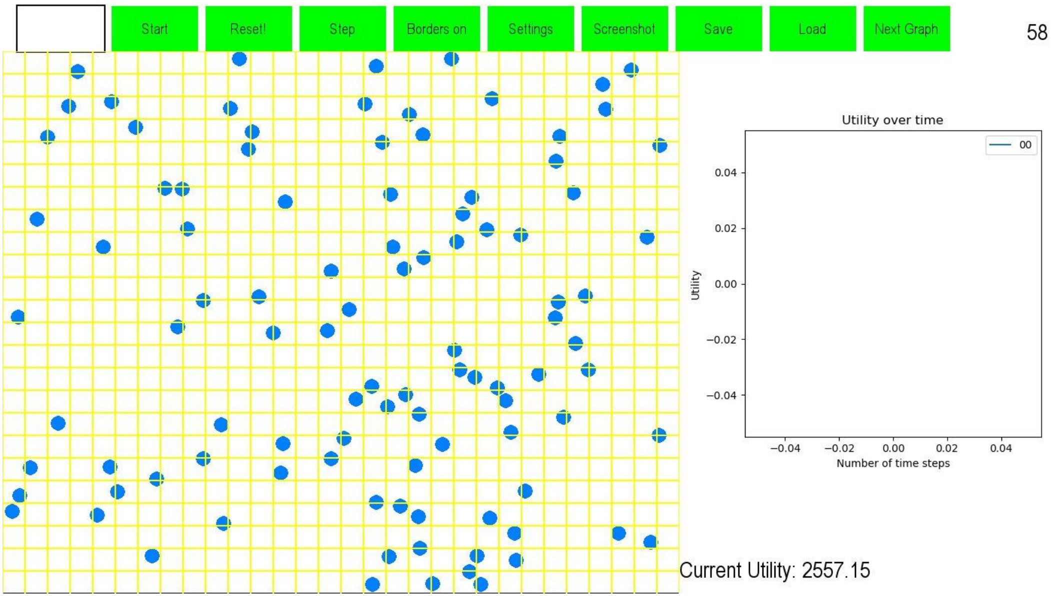
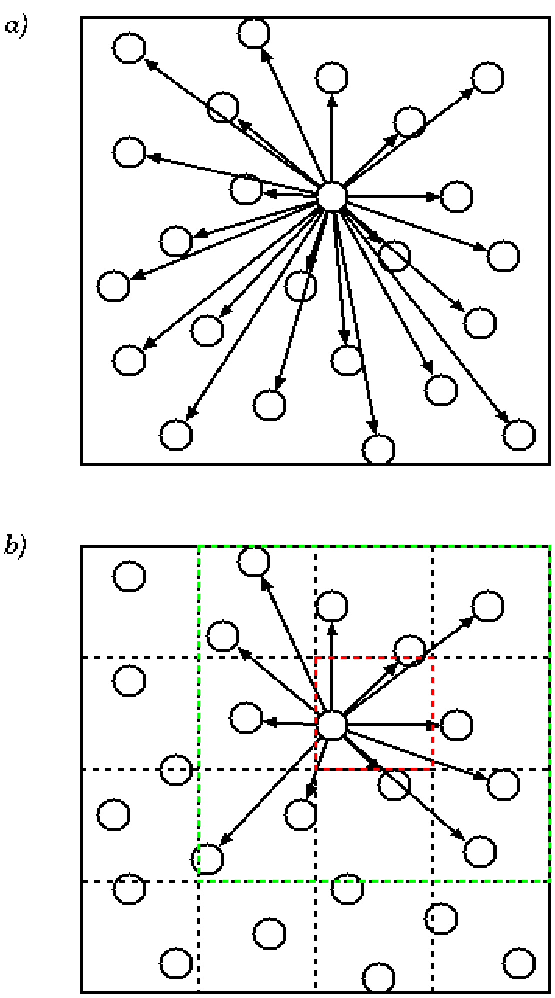


Figure 14



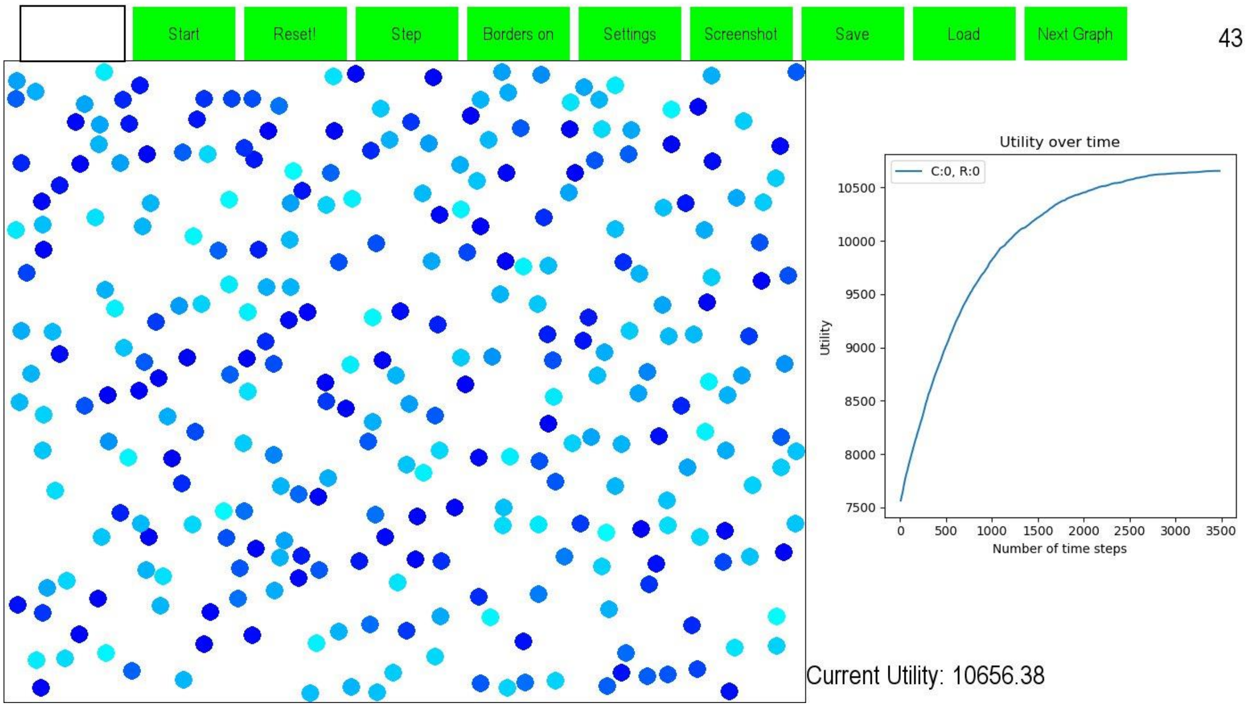
"The pairwise interactions for a single particle can be computed by a) computing the interaction to all other particles or b) by dividing the domain into cells with an edge length of at least the cut-off radius of the interaction potential and computing the interaction between the particle and all particles in the same (red) and in the adjacent (green) cells." (Cell_list, 2019)

The algorithm used in my program combines both these methods, and evidence of this is visible in Figure 12. In my implementation each agent has a variable (agent.box), which stores that agent's current box. The rows and columns of that box are then passed to a function which returns a list of all agents in the same or bordering boxes. This list does not contain agents that have not yet moved. The agent that just moved then get added to the list of moved agents when it's current box is update in the last line "find_new_box(agent)" in Figure 12. All the code is available on GitHub, which is linked in the Appendix.

9. Results

Based on the simulation it can be observed that Prediction 1 is correct, over time the utility of the agent-based society goes up. This simple predication does not give much insight, except perhaps being an indication that the simulation does in fact follow basic economic principles. In Figure 15 the increase in utility is clear on the graph, and I can be observed that Current Utility also increases as the simulation runs.

Figure 15



The next prediction, Prediction 2, is much more interesting as it is a difficult concept to demonstrate mathematically. Remember that the prediction is:

> *As the allocation of apples and oranges (goods) approaches Pareto efficiency, the frequency of successful trades slow down.*

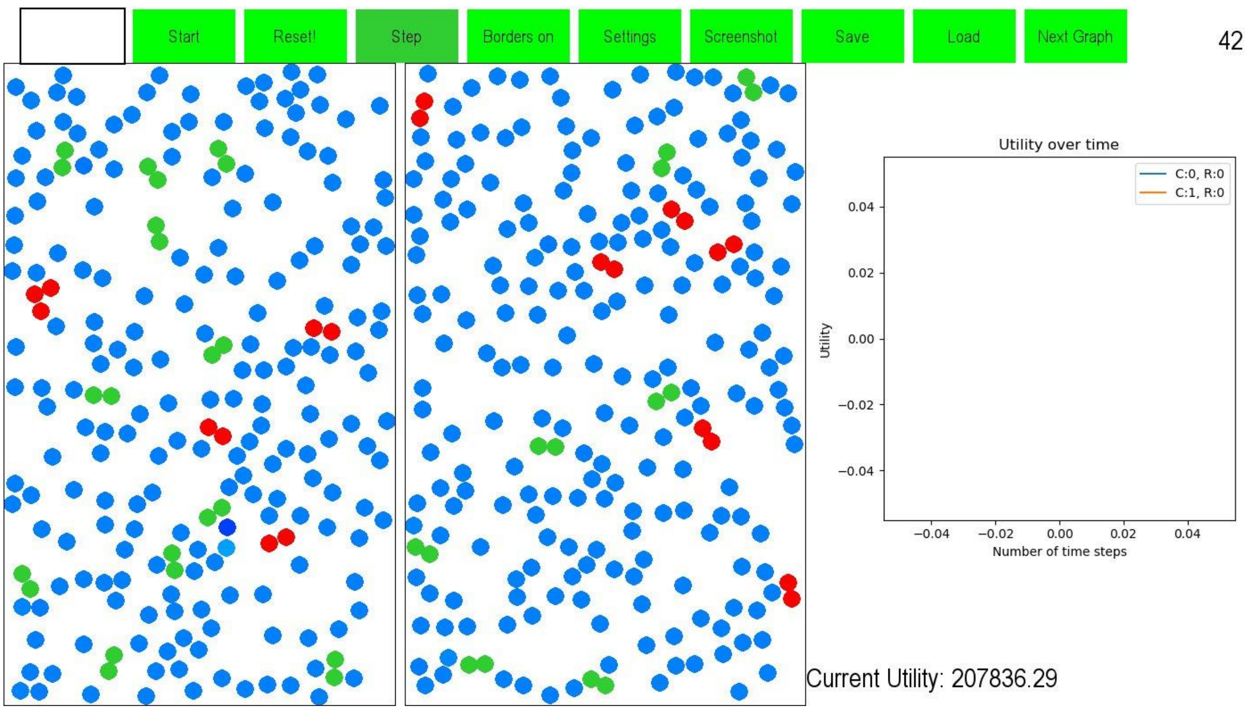
This can be verified by visually looking on the simulation and observe if agent's trade or not. With the "show trade" setting turned on successful trades are visualized by agents blinking green, and unsuccessful trades by agents blinking red. To test this I created two markets, one where the agents have their preferences described by a Cobb-Douglas utility function and one where the preferences are described by a linear utility function.

Early on there is a high frequency of successful trades, as can be seen in Figure 16. With the progression of the simulation the change in frequency of successful trades can be observed.

The number of trades that fail increase and the number of successful trades decrease. This can be seen in

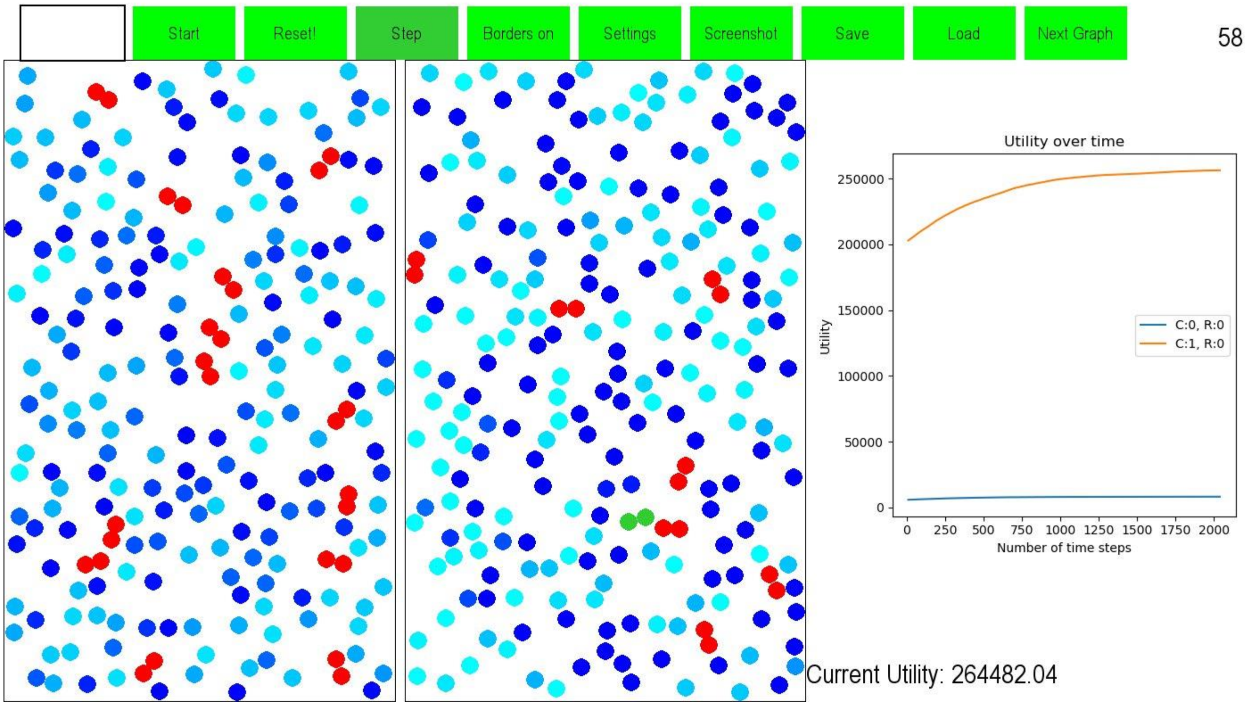
Figure 17. This observation matches the initial prediction, as Pareto efficiency is approached there is less possible trades and naturally that will reduce the number of successful trades.

Figure 16



Showing trades early in the market, many successful trades (green agents). The left market uses a Cobb-Douglass utility function, and the right a linear utility function.

Figure 17



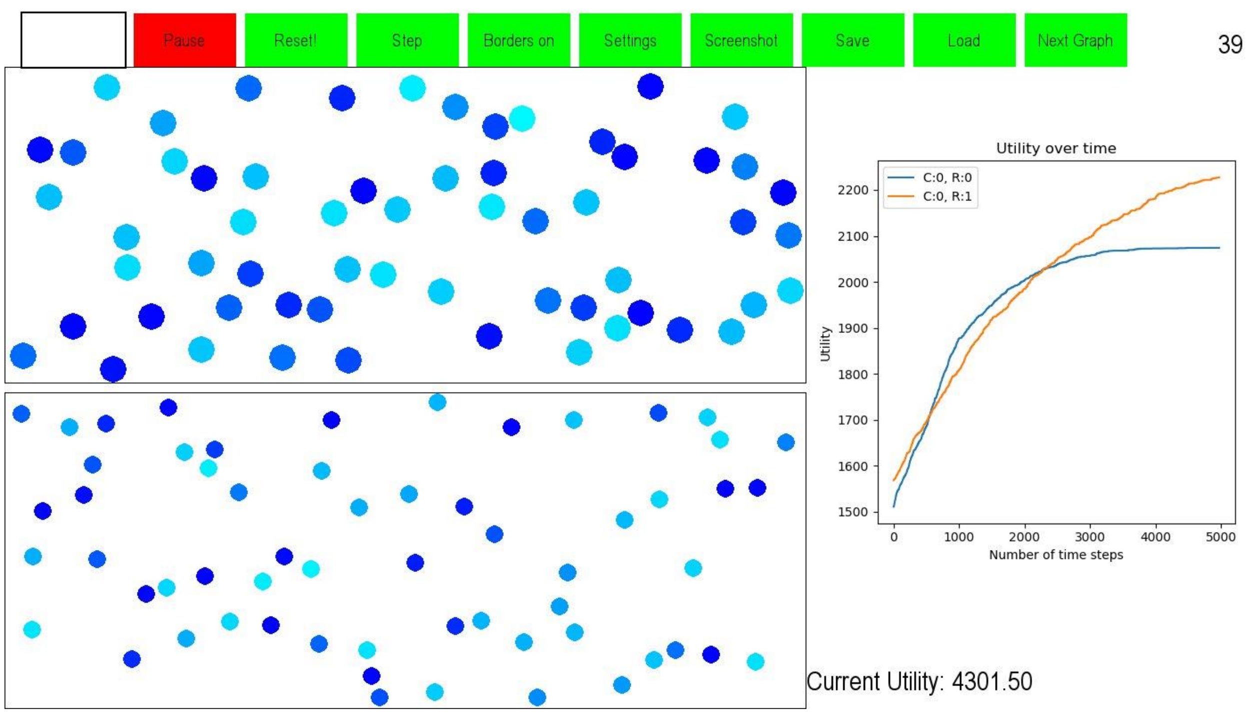
Showing trades as the market approaches Pareto efficiency, many unsuccessful trades (red agents)

The same can be concluded by looking at the data, the graph that displays the utility in the society begins with a steep increase that eventually flattens out as the utility approaches a Pareto efficient point. The reduced growth in utility shows that the society is approaching Pareto efficiency. This leads to another question, does more trading correlate with faster growth in utility?

One trade can only have a small effect on the market, if one of two conditions are true. Either each agent has an individual continues demand function (the utility functions used in this simulation leads to continues demand functions), or there is a large enough number of agents that one trade will have a small effect on the overall market anyway. (Varian, 2010, p. 594) This would imply frequency of successful trades determines the growth of utility and therefore the growth slows down as more and less trades are successful.

Interestingly, this can be tested by the simulation too. In a market where the agents have a bigger radius the chances of the agent's colliding (and attempting to trade) would be greater than in an equivalent market where the agents have a smaller radius. Prediction 5 is that the radius will have an unknown effect, but perhaps it can be used to give insight towards the effect of frequency of trading.

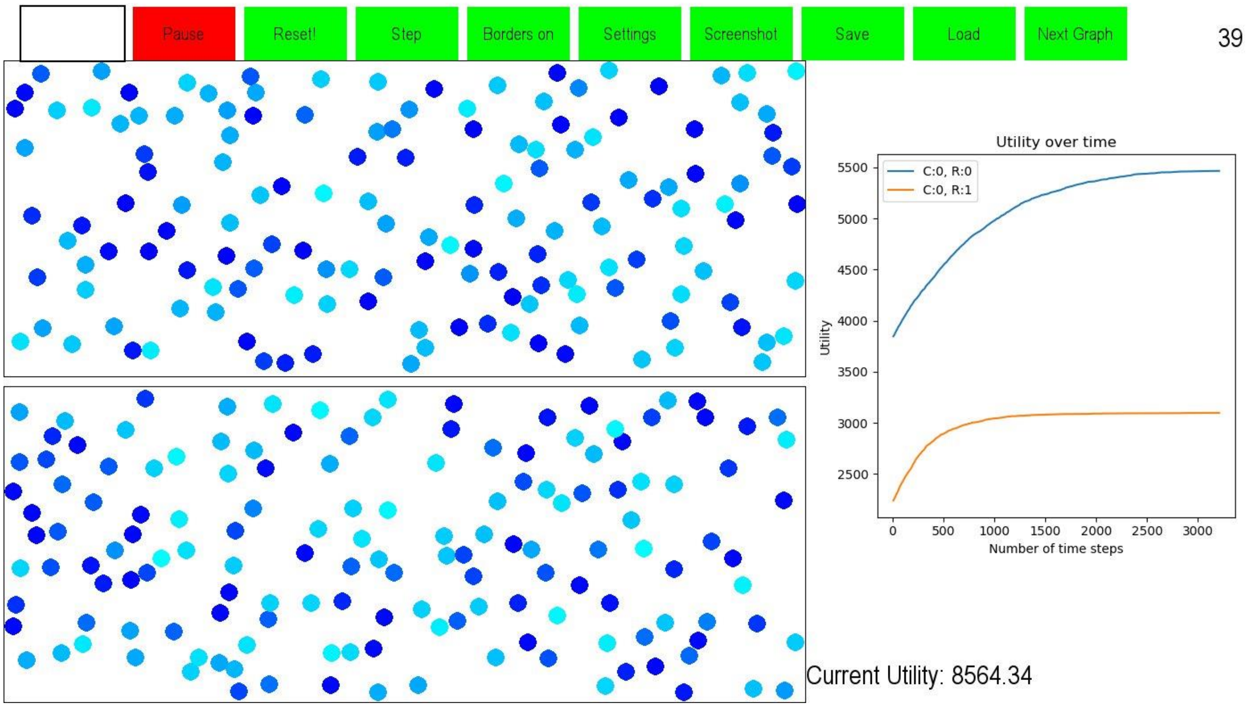
Figure 18



Observe that the market where the agents have a bigger radius does have a much greater growth of utility at the beginning, but over time this growth slows down and eventually stops faster too. At the end it is the market with lower radius, which ends up with a higher utility, even though for a temporary period it has lower utility. Manipulating the radius has an effect, but why it has this effect is not easy to tell. Perhaps, in the market with bigger radius the agents trade more frequently, but also trades more often with the same agents. With the idea that varying the trading partners lead to a higher utility long term. This is something that would require further study to answer.

Prediction 3, that with more endowments it will take more time to reach Pareto efficiency, can be tested by creating two identical markets where one market is given more goods than the other. In Figure 19 the agents in the top market was given twice as many goods as the agent's in the bottom market. The effect is clear, it takes much longer time before the agents in the top market reach Pareto efficiency, and the growth in utility stops. It is important to be mindful that a small change in utility would not necessarily be visible on the "utility over time" graph, but by looking on the "price over time" it can be confirmed that even the last few sporadic trades happen much later in the market that begins with more goods (top market). Therefore, it can be concluded that Prediction 3 is true.

Figure 19



Prediction 4, “markets can reach a higher utility if they can trade with other markets”, requires turning of the borders between two markets after they have each reached their Pareto optimal allocation of goods. In theory, this could be tested with two agents with each agent being the only agent in its market. That would mean the utility would go up if they trade, and this could be generalized to markets that contain more than one agent. However, with my model we can simply test it with many markets with many agents, and observe the result. In Figure 20 there is 9 markets, each containing 5 agents. I let the simulation run until all the markets were done treading, and a Pareto optimal allocation have been reached. That can be shown by the utility no longer increasing over time. Now it is time to see what happens when the markets can trade, Figure 20 is right before the borders are turned off (the simulation is paused, and then I clicked the Borders button). Figure 21 is taken after the markets have been able to trade with each other for some time.

Figure 20

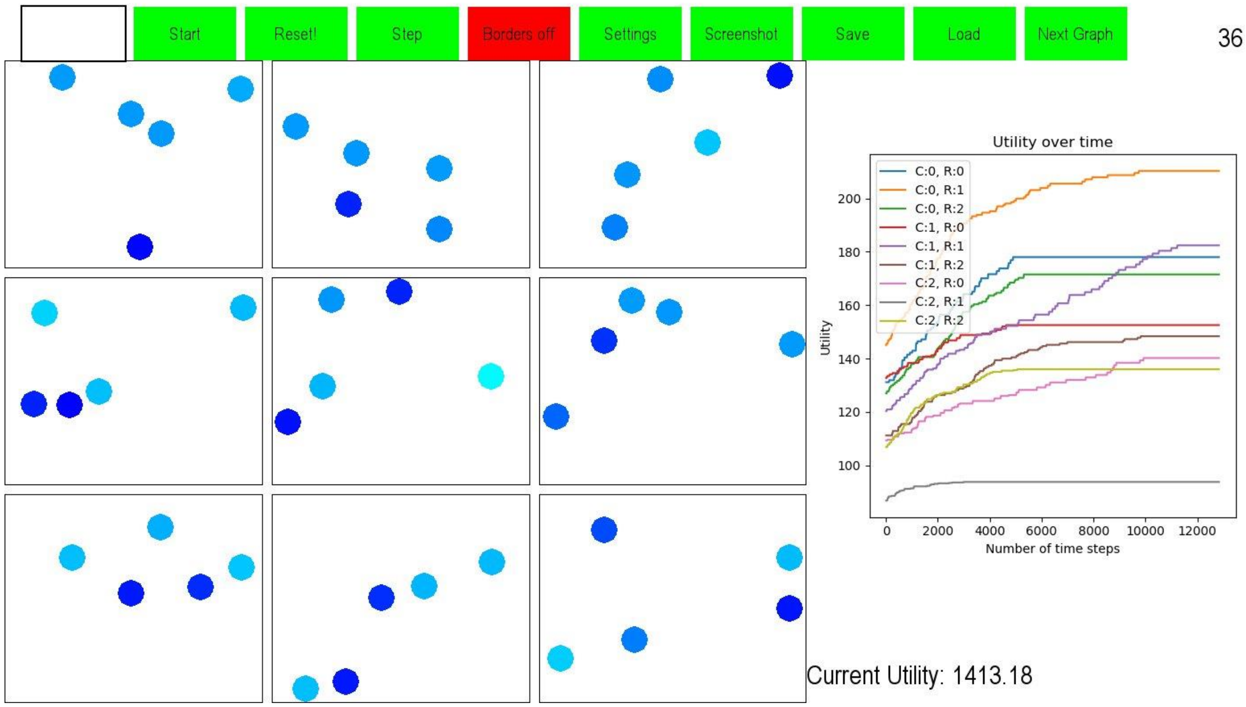
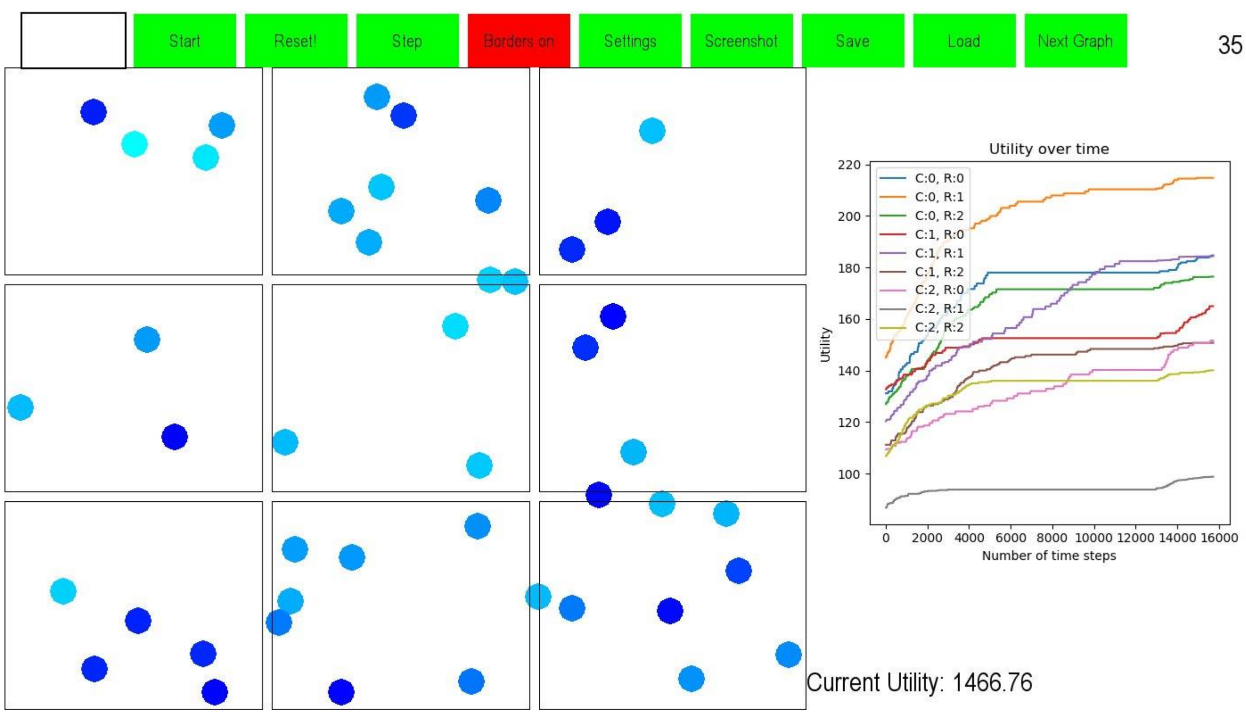


Figure 21

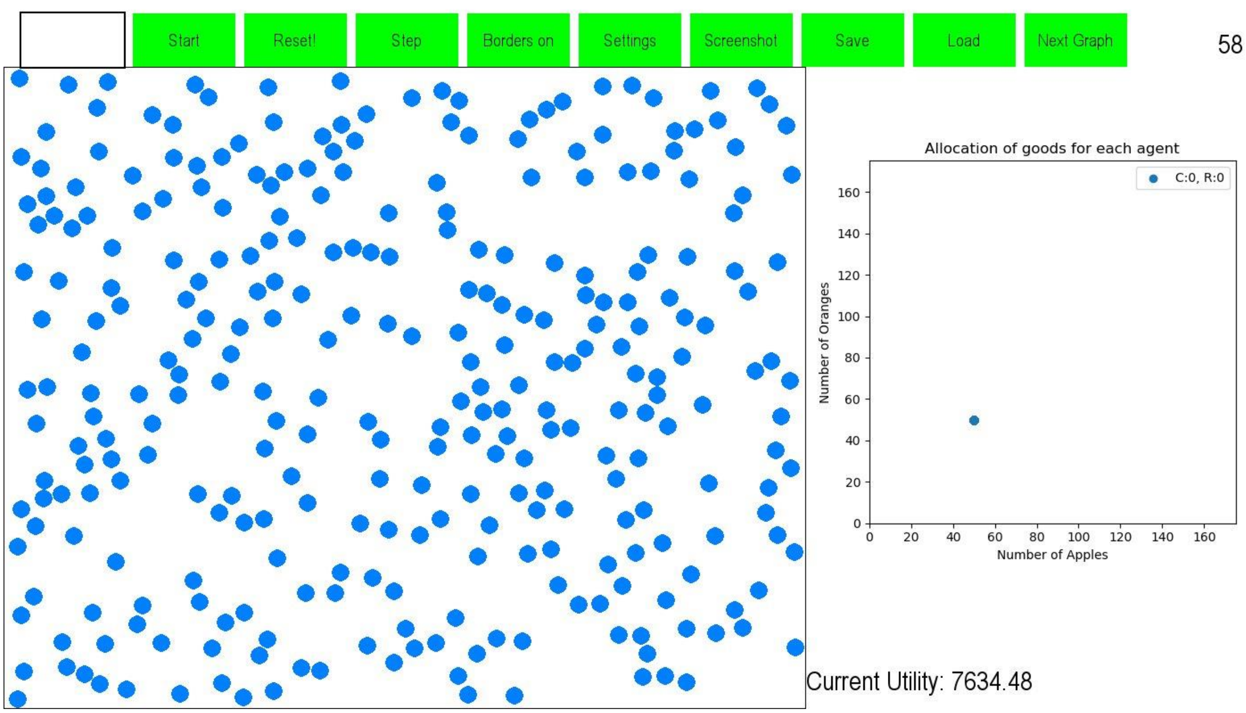


Notice that all the markets have a higher utility after they are able to trade, some benefit more than others, but all of them benefits. The simulation shows that Prediction 4 is correct; markets can reach a higher utility if they are able to trade with other markets.

The next predictions are specifically when preferences are described by the Cobb-Douglas Utility Function (normal preferences). Prediction 6 and Prediction 7 are both about the allocation of apples and oranges.

In Prediction 6 the agents are supposed to end up with many different allocations of apples and oranges, after initially all of them were given the same endowments, due to the agents having random preferences. Figure 22 shows all the agents starting with the same number of goods (50 apples and oranges).

Figure 22

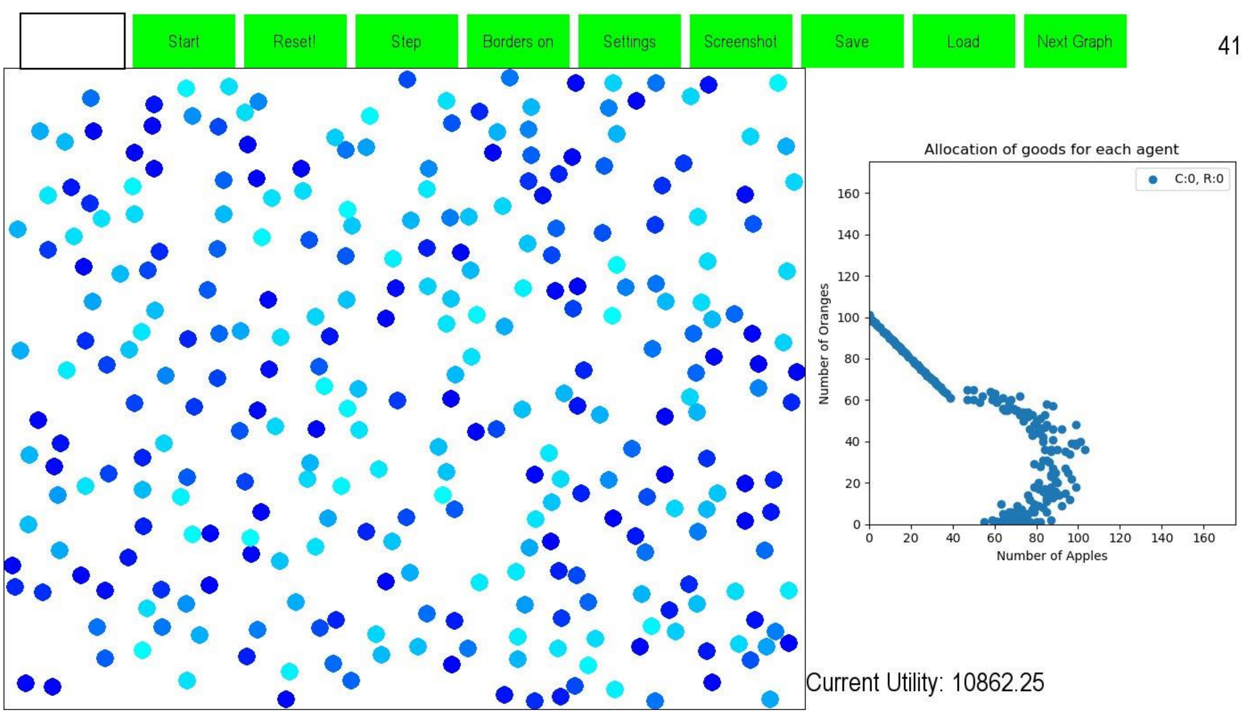


With time the allocation of goods among the agents end up spread around, as shown in Figure 23 below. The many different percentages of apples and oranges are also visible in the colors of the agents, which ranges from cyan to blue.

The agents that prefer oranges tend to form a line, while the agents that prefer apples form a convex arch. That is because in this simulation the agents always trade 1 apple for a variable number of oranges, and therefore the best trade possible for an agent that wants oranges is to give up 1 apple for 1 orange. If trades were to be done the other way (give up 1 orange for x number of apples) too then all the agents would probably form this convex arch.

Nevertheless, Prediction 6 can be confirmed by my simulation. The allocation of goods is clearly spread out in Figure 23.

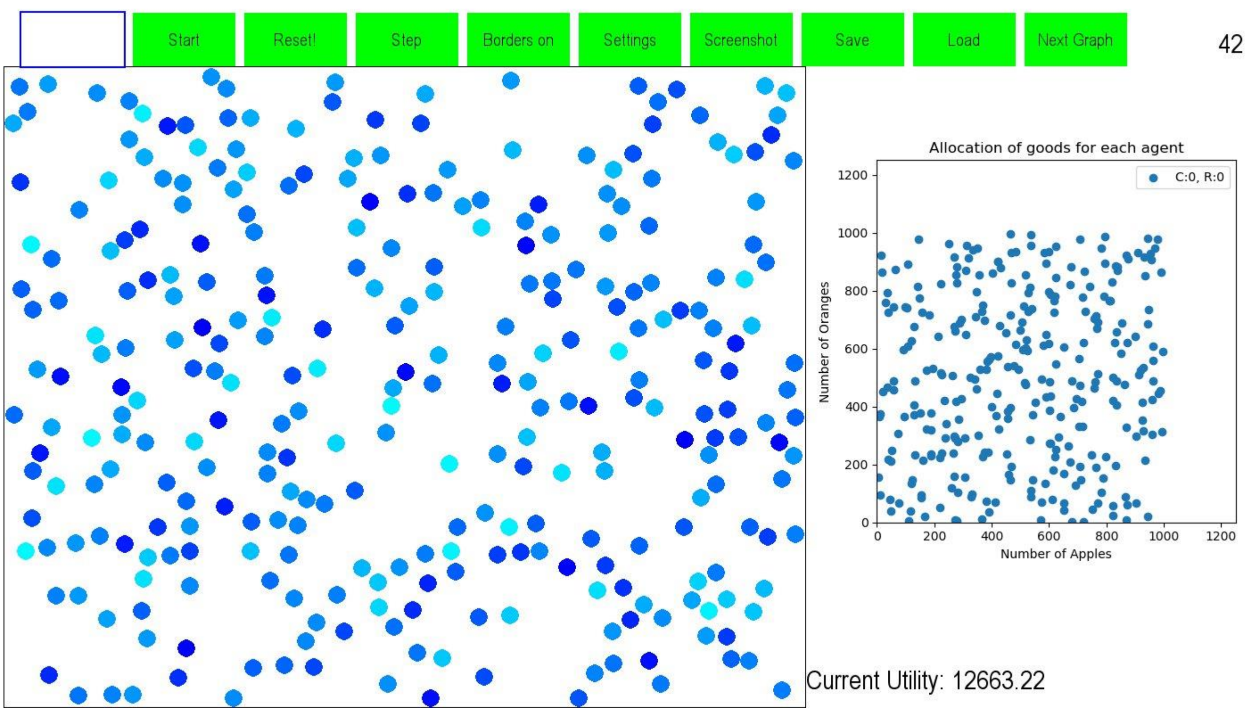
Figure 23



In some sense, the opposite of Prediction 6 is Prediction 7. In Prediction 7 the preferences are all the same, but the initial endowments of goods are random. In this case based on microeconomic theory the agents would all end up with the same percentage of apples.

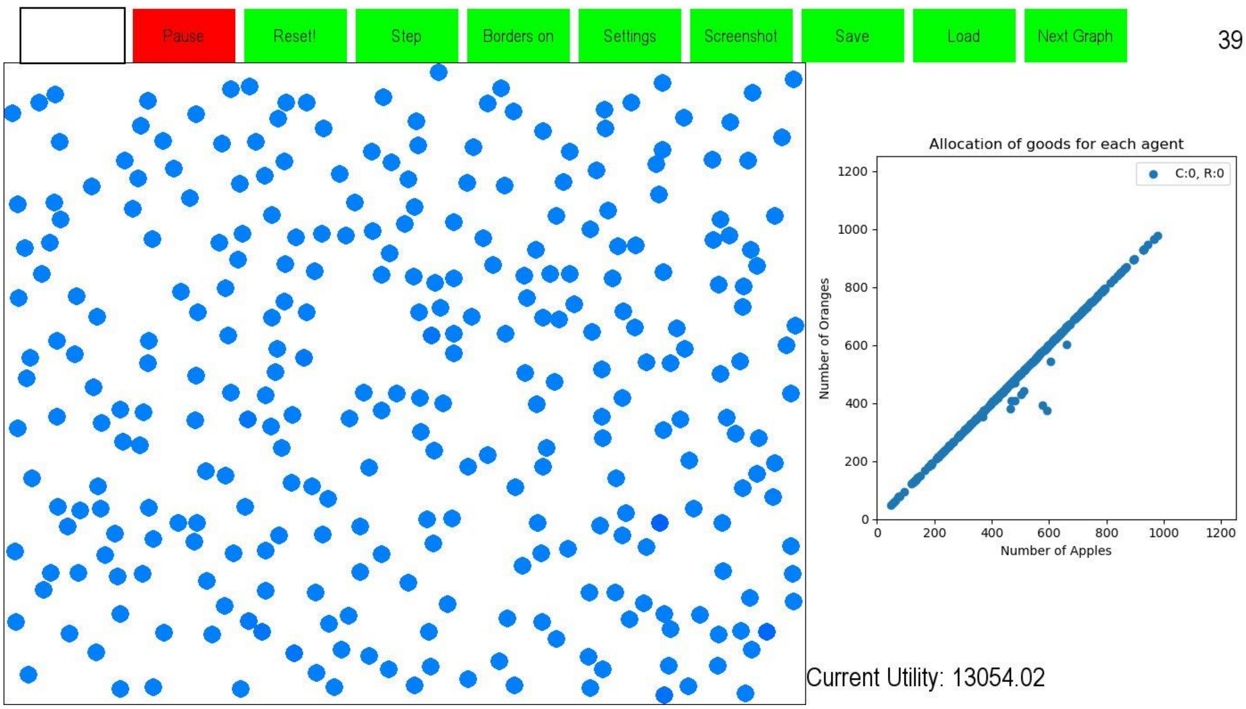
To test this the preferences can be set to be equal, which would give the following utility function: $U(A,O) = A^{0.5} * O^{0.5}$. Thus, all the agents would want to have 50% apples. Figure 24 shows the allocation of goods at the beginning of the simulation, andFigure 25 shows the allocation after running the simulation for some time. Observe that on the graph the agents have converged and form a 1:1 line[9] from the origin. There are some agents that don't end up on the line, but close to it. That is likely an effect of the simulation always trading whole units of apples and oranges, and that trades are only completed by giving away 1 apple for x oranges.

Figure 24



[9] A line where x=y (apples = oranges), sometimes called line of equality

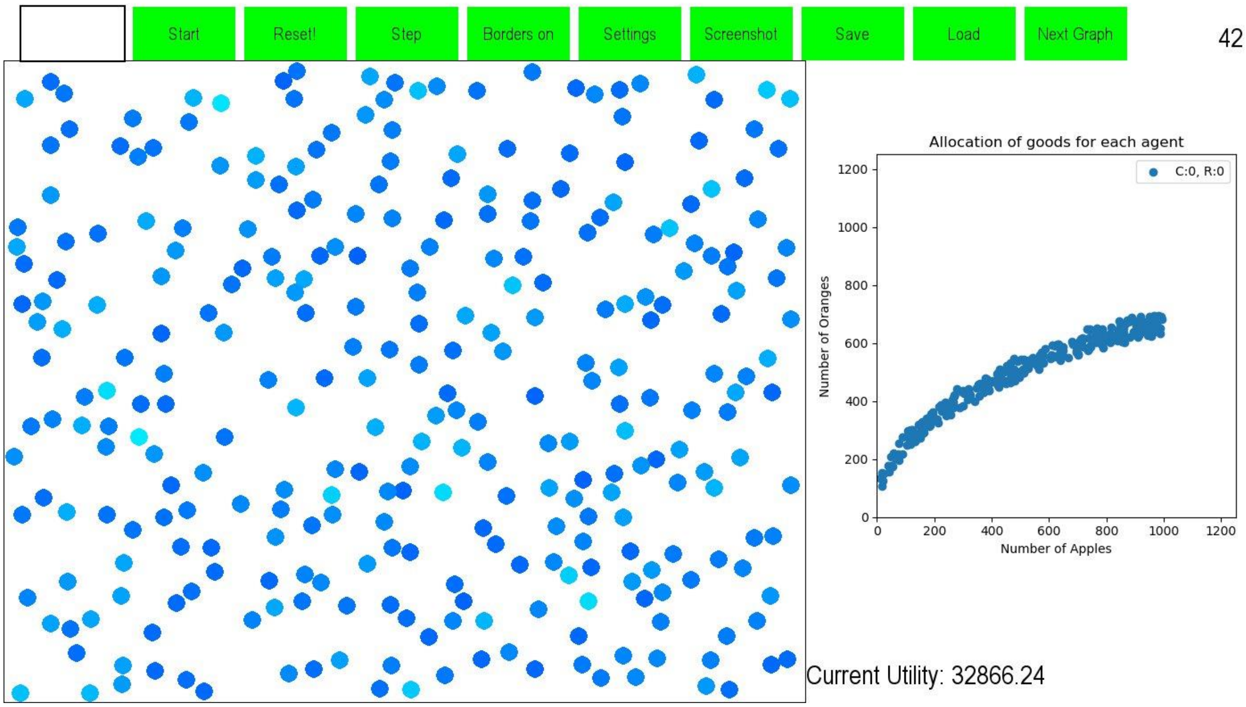
Figure 25



Should the preferences not be equal then the allocation would form a different shape, in Figure 26 the agents have the following utility function; $u(A,O) = A^{0.75} * O^{0.25}$. The curve on the graph is similar to the curve that would be given by $O = A^{0.25}$. The agents get as close as possible to the allocation where they maximize their utility (75% apples, 25% oranges), but can't quite get there. This is due to the same reason some agents are unable get on the line in Figure 25.

To sum up, the simulation provides strong evidence that Prediction 7 is true. The allocations not matching the predictions exactly is likely due to artifacts of the simulation.

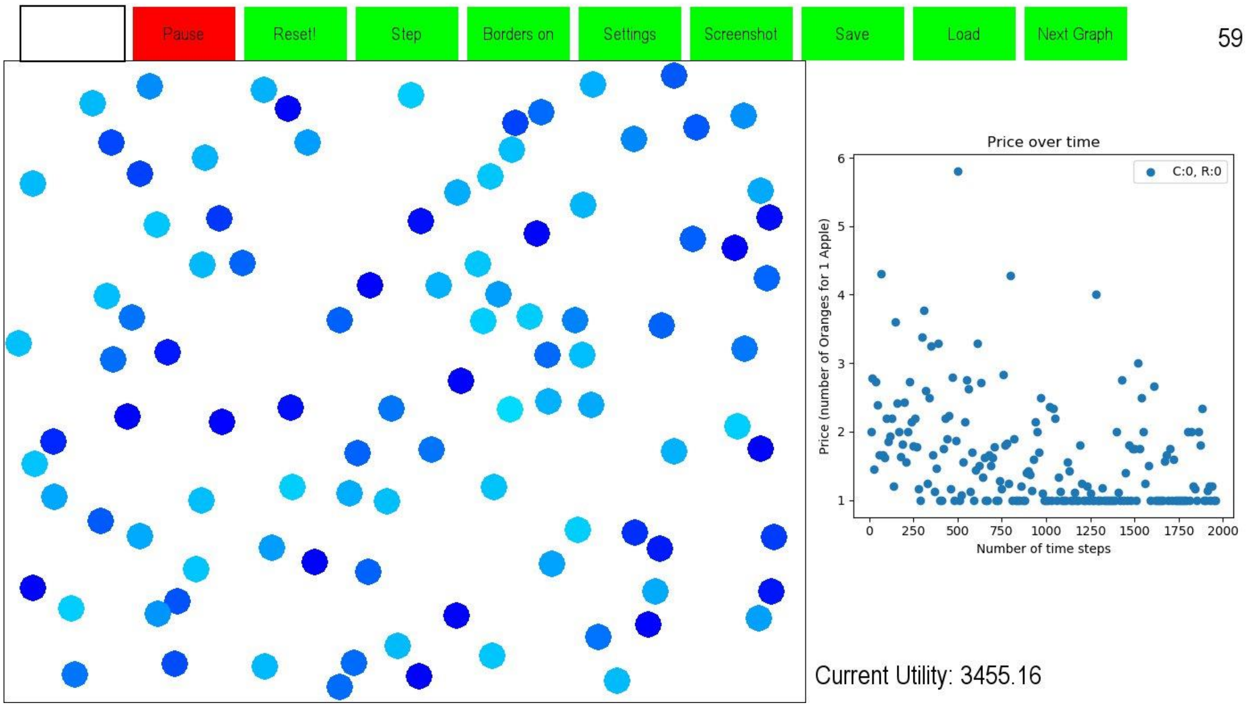
Figure 26



Prediction 8, that once the market is in equilibrium there should be no more trades shown on the price graph, is straightforward to test. Simply make a market and just let the simulation run. However, this will give some insight that traditional microeconomic theory cannot give. This will let us observe how the price in a market changes over time, before equilibrium is reached.

Initially, in Figure 27, the price seems to converge towards 1. That is of course the lowest the price can be in this simulation. Also noteworthy is that there is a wide range of prices in general.

Figure 27



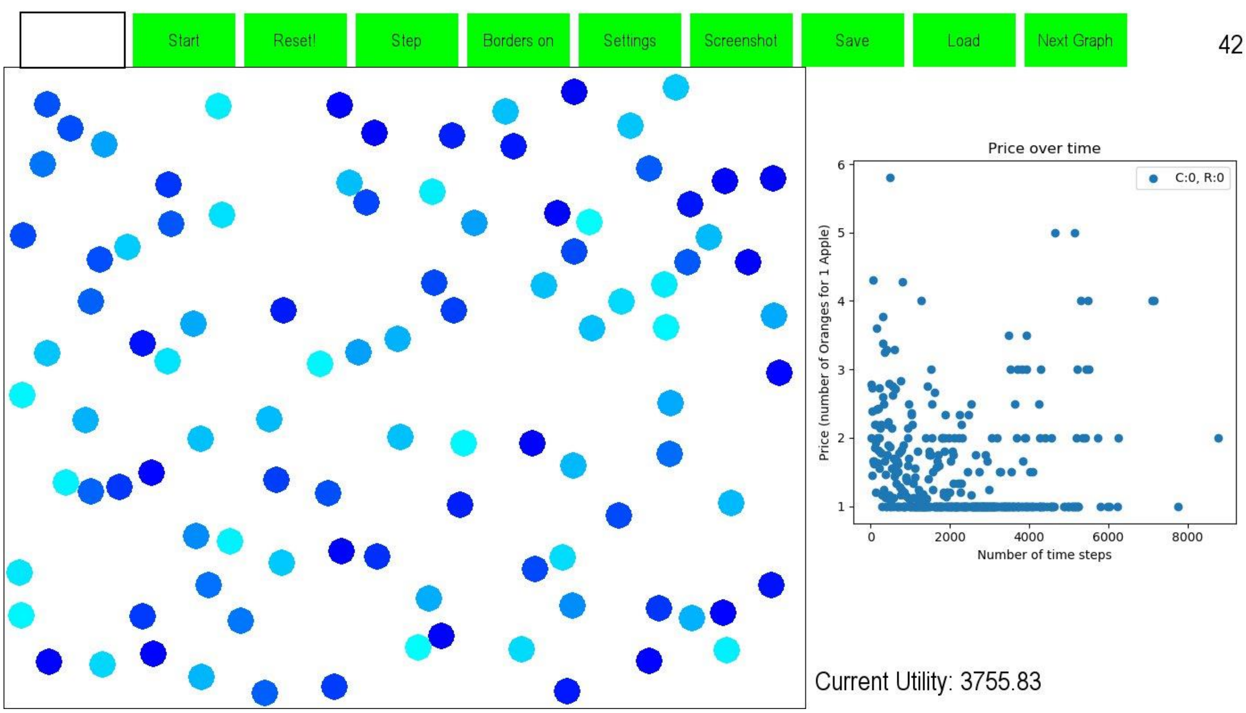
However, over time a different effect can be observed. Figure 28 is from the same simulation as Figure 27, but after more time has passed. Interestingly, there are many points later that have a relatively high price. It's important to be mindful that in this case each point is the average price over 10 timesteps, which is the default setting[10]. However, the trend is the same even if each point just represented the average for 1 timestep. The reason for the seemingly increase in price as the market approaches Pareto efficiency is because at this point many more trades are unsuccessful, and with most of trades being at a price of 1, that would bring the price down initially. However, once there is only one trade that is successful the price will be whatever it was in that trade. Therefore, ignoring these last outlier prices, the price trends toward 1. That seems logical considering the average preference value of apples would be approximately 0.5

[10] This can be adjusted by changing the "update interval" in the settings menu

($\alpha = 0.5$). Then with the same number of apples and oranges available it makes sense that the competitive price would end up at 1, and that the price in the simulation converges towards 1. This convergence of price is something traditional models would be unable to observe, and further investigation into the topic is necessary should anything be concluded about the nature of the prices.

Lastly, there is the result of Prediction 8. Indeed, trades stop occurring once the market has reached equilibrium and no new data is added to the price graph. The simulation in Figure 28 was run until about 18'000 timesteps had passed, yet no new trades happened.

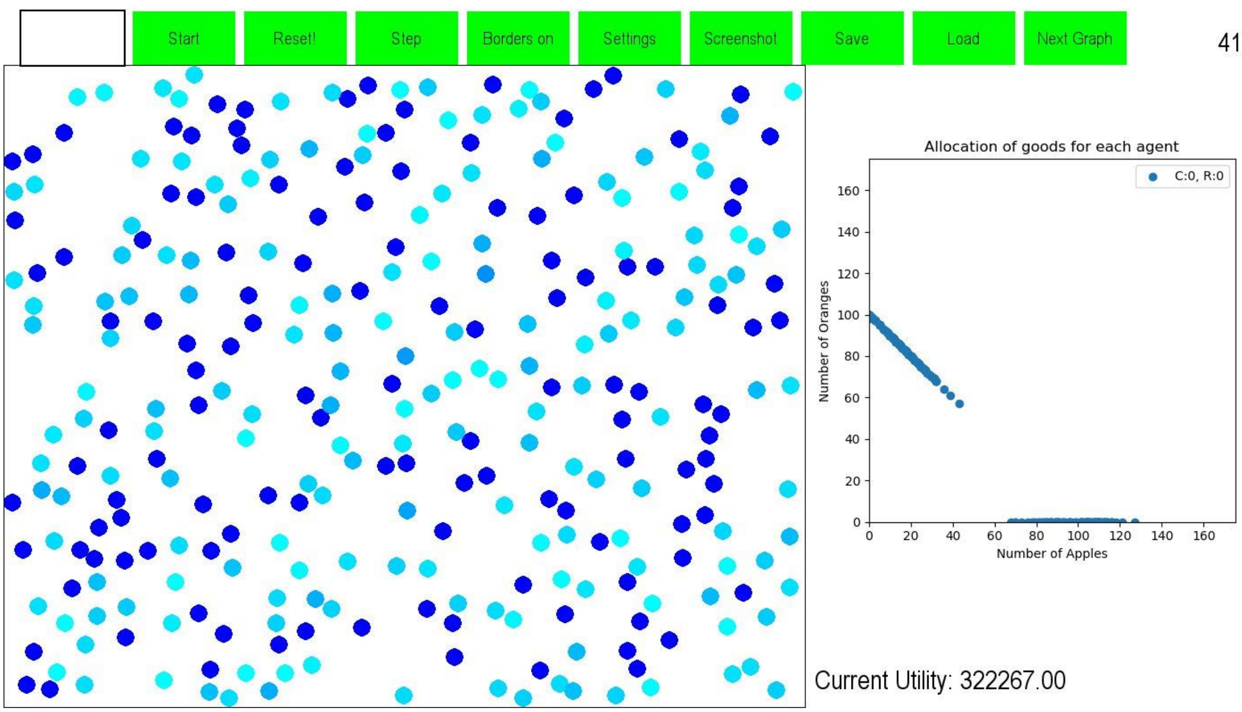
Figure 28



Prediction 9, that with linear utility agents typically end up with a corner solution, can be confirmed two different ways. First by looking on the color of the agents, that changes based on the distribution of goods, it can be observed that the agents tend to end up being cyan or blue (assuming the agents initially have an equal amount of both goods). Secondly, the graph over allocation of goods shows that the agents indeed typically end up with only one good. Initially this will look just like Figure 22, however the end result is very different from Figure 23.

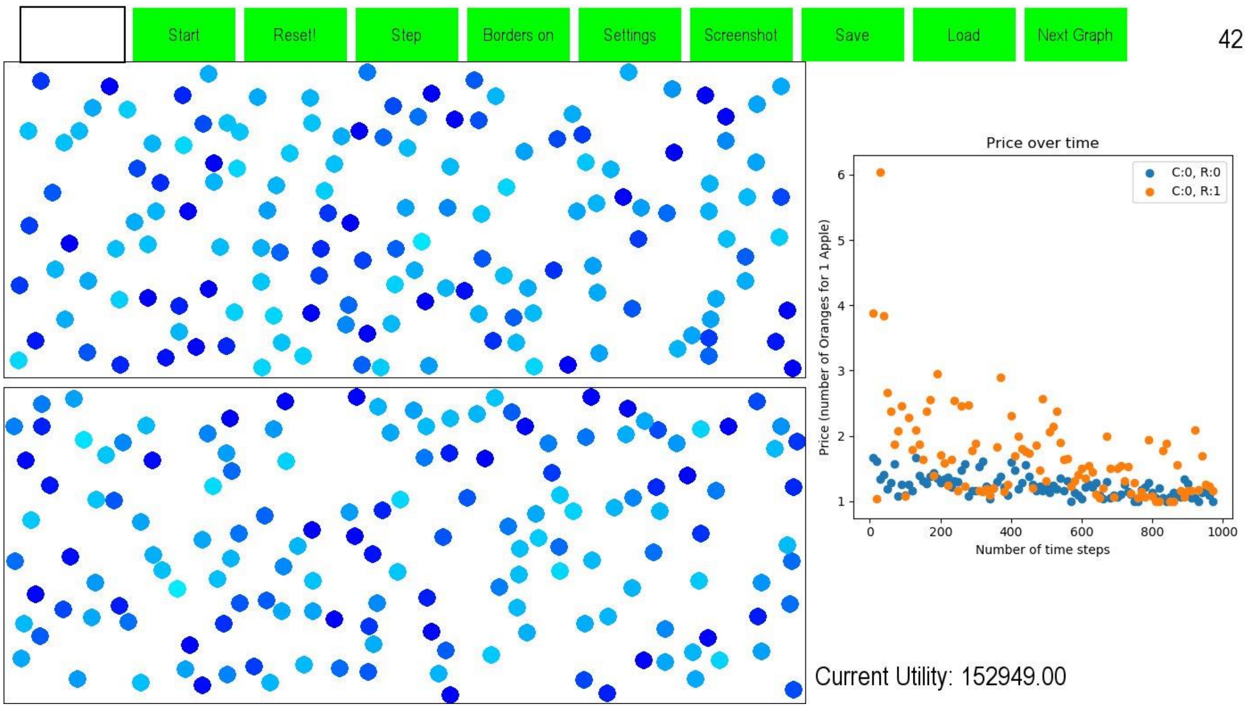
The agents end up with a corner solution, as can be seen in Figure 29. Similar to the case with Cobb-Douglass utility functions there is a line which shows the agents that prefer oranges more than apples, and this line happen for the same reason. If the agents were able to trade both ways then these agents would typically end up with no apples left. In essence, Prediction 9 is correct.

Figure 29



Finally, there is Prediction 10, which states that with linear preferences the price will be relatively constant. To observe this the price can be compared to the prices a market with Cobb-Douglass utility function gave.

Figure 30



In Figure 30, the top market (blue on the graph) has agents with a linear utility functions and the bottom market have agents with Cobb-Douglass utility functions. The prices in the top market are not nearly as volatile as in the bottom one. Thus the 10^{th} and last prediction can also be said to be correct.

10. Conclusion

In this paper the traditional microeconomic models have been tested by an agent-based model. The predictions that can be made with microeconomics are all confirmed by the model I made, which both validates my model as well as what microeconomic theory have been for many years. The agent-based model I have create have visualized the invisible hand, and enables research that can lead to understanding the market forces better.

There are some things that are yet to be tested, for example other types of preferences (bads, perfect complements etc.), which can be done with a future update to my model. Furthermore, there is insight with can be gained from an agent-based model that the traditional models cannot give. That however, would be a potential topic for additional investigation.

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12. Appendix

The complete code for the simulation can be found on GitHub at the link below.

https://github.com/Hennns/Thesis