数字图像处理　Problem1

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PPT43

a. 求期望

当 $X \sim N(\mu, \sigma^2)$, 换元, 令 $t = \frac{x-\mu}{\sigma}$

$$E(X) = \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{(x-\mu)^2}{2\sigma^2}\} \mathrm{d}x$$

$$= \int_{-\infty}^{+\infty} (\sigma t + \mu) \frac{1}{\sqrt{2\pi}} exp\{-\frac{t^2}{2}\} \mathrm{d}t$$

$$= \sigma \int_{-\infty}^{+\infty} \frac{t}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \mathrm{d}t + \mu \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \mathrm{d}t$$

分析:

由于$\frac{t}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ 是奇函数，在对称区间上积分值为0；

同时，$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ 为标准正态分布，即$\mu = 0, \sigma = 1$时的正态分布概率函数的结果，所以其在$-\infty$到$+\infty$上积分值为1。

故上式最终结果为 μ，　即　　$E(X) = \mu$

b. 求方差

由数学期望的性质，可以得到

$$D(x) = E[x - E(x)]^2$$

$$= E\{X^2 - 2XE(X) + [E(x)]^2\}$$

$$= E(X^2) - 2[E(X)]^2 + [E(x)]^2$$

$$= E(X^2) - (E(X))^2$$

由（A）内的推导过程，可以知道，当$X \sim N(\mu, \sigma^2)$时，$E(X) = \mu$

又因为

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}\mathrm{d}x$$

$$= \int_{-\infty}^{+\infty} (\sigma t + \mu)^2 \frac{1}{\sqrt{2\pi}} exp\{-\frac{t^2}{2}\}\mathrm{d}t$$

$$= \sigma^2 \int_{-\infty}^{+\infty} \frac{t^2}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\mathrm{d}t + \mu^2$$

$$= \sigma^2 + \mu^2$$

因此，正态分布的方差为

$$D(X) = (\sigma^2 + \mu^2) - \mu^2 = \sigma^2$$

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❖result image:

Analysis:

The algorithm of function **adaptiveMedianFilter** is:

- Get the padding-width according to the cornel window size
- Get the size of the noise picture
- Fill pixels at the edge of the original image,which is assured by padding-width
- Get the temp window area Sxy, and calculate Zemd, Zmin, Zmax of this area
- For each pixel in the original figure, do the following operations:

  Stage A:

  A1 = Zmed – Zmin

  A2 = Zmed – Zmax

  If A1 >0 and A2 <0

  Go to stage B

  Else

  increase the window size

  If window size ≤ Smax

  repeat stage A

  Else output Zmed

Stage B:

B1 = Zxy – Zmin

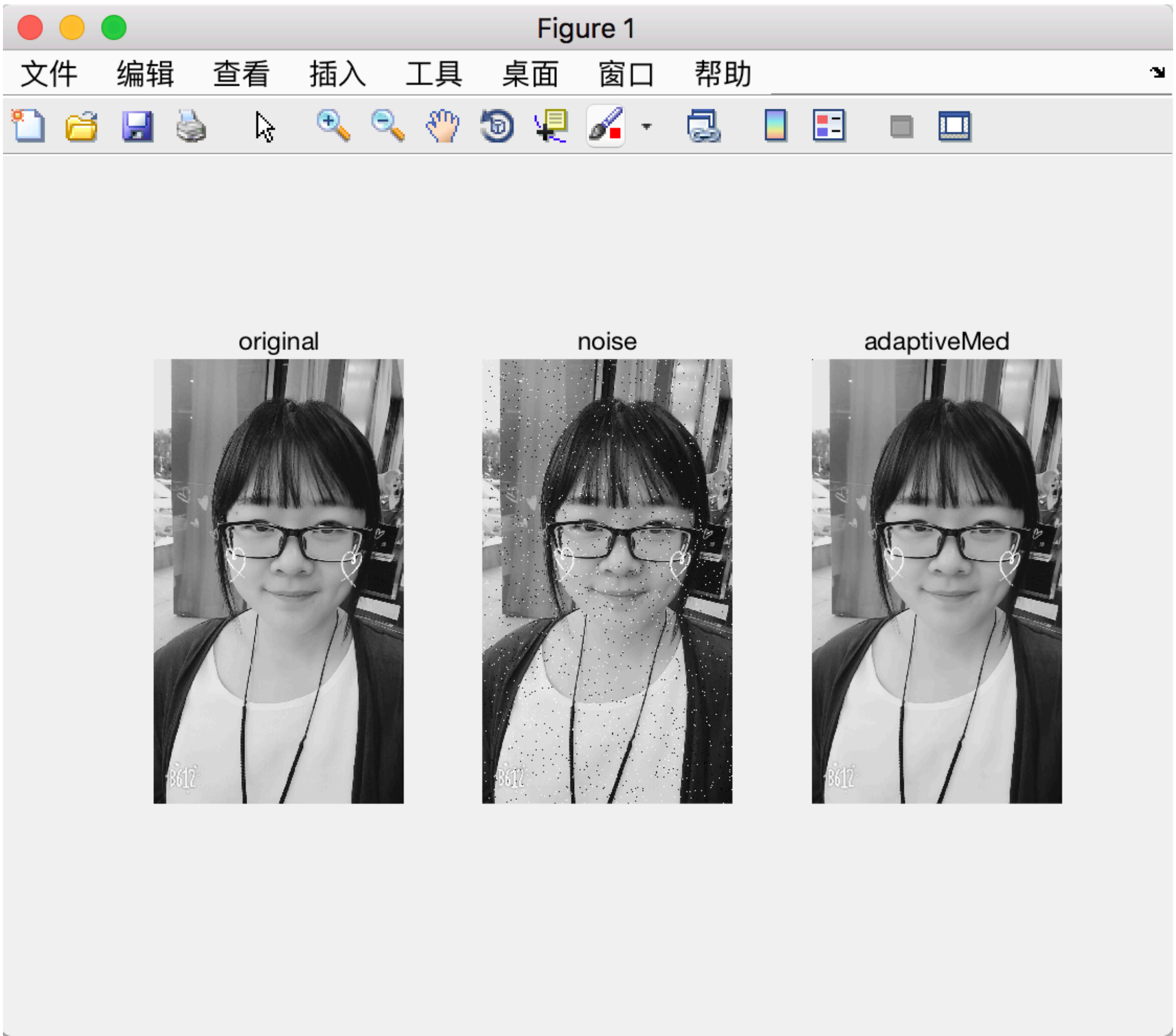
B2 = Zxy – Zmax

If B1 > 0 and B2 < 0

output Zxy

Else output Zmed

- If noise pixels in the window were more than half of the useful pixels, then it means you need to increase the size of the filter window Sxy, the increasing method is to make the window size + 2. For example, if the original filter window size is 3 * 3, so after increasing the value of the window, the filter window size changed into 5 x5, if Sxy < Smax, continue this operation, otherwise the gray levels of the pixel is Zmed. By analogy, ***the adaptive control of filter window size is formed***.
- ***The maximum size of filter window Smax*** should adjust with the spacial density of noise. Generally speaking, if the spacial density of the noise is big, we should choose a bigger Smax; if spacial density of the noise is small, Smax can be set a little smaller. Here we choose a fixed value, 11.



code:

```
clear
```

```
img = imread('cui.jpg');
img_gray = rgb2gray(img);
img_noi = imnoise(img_gray,'salt & pepper',0.02);
img_adaptive = adaptiveMedianFilter(img_noi,11);

figure;
% original image
subplot(1,3,1);
imshow(img_gray);
title('original')

% noised image
subplot(1,3,2);
imshow(img_noi);
title('noise')

% adaptive median filter image
subplot(1,3,3);
imshow(img_adaptive);
title('adaptiveMed')

function[img_result] = adaptiveMedianFilter(img_noi,max_window)

padding_width = (max_window-1)/2;
[m ,n] = size(img_noi);
% Initialize a image equal size with img_noi to receive pixels after
Adaptive Median Filter
img_result = img_noi;
% Fill pixels at the edge of the original image
img_noi = padarray(img_noi,[padding_width padding_width],0);

for i = padding_width+1:padding_width+m
    for j = padding_width+1:padding_width+n
        Zxy = img_noi(i,j);
        window = 3;
        while(window <= max_window)
            %Take the window of the filter plate
            tmp = img_noi(i-(window-1)/2:i+(window-1)/2,...
            j-(window-1)/2:j+(window-1)/2);
            Zmin = min(min(tmp));
            Zmax = max(max(tmp));
            Zmed = median(tmp(:));
            if((Zmed > Zmin) && (Zmed < Zmax))
                if((Zxy > Zmin) && (Zxy < Zmax))
                    img_result(i-padding_width,j-padding_width) = Zxy;
                else
                    img_result(i-padding_width,j-padding_width) = Zmed;
                end
                break;
```

```
            end
            window = window+2;
        end
        img_result(i-padding_width,j-padding_width) = Zmed;
    end
end
end
```