

Ex 117 p 304 :

1)

$$\sum_{i=1}^n p_i = p_1 + p_2 + \dots + p_n = 1$$

2)

$$Y = aX + b$$

a)

$$y_1 = ax + b, y_2 = ax_2 + b, \dots, y_n = ax_n + b$$

b)

$$q_1 = p_1, q_2 = p_2, \dots, q_n = p_n$$

c)

$$E(Y) = q_1 y_1 + q_2 y_2 + \dots + q_n y_n$$

$$E(aX + b) = p_1(ax_1 + b) + p_2(ax_2 + b) + \dots + p_n(ax_n + b)$$

$$E(aX + b) = a(p_1 x_1 + p_2 x_2 + \dots + p_n x_n) + b(p_1 + p_2 + \dots + p_n)$$

$$\text{D'où } E(aX + b) = aE(X) + b$$

3)

a)

$$(y_i - E(Y))^2 = (ax_i + b - aE(X) - b)^2 = (a(x_i - E(X)))^2 = a^2 (x_i - E(X))^2$$

3)

b)

$$V(Y) = \sum_{i=1}^n q_i (y_i - E(Y))^2$$

$$V(Y) = \sum_{i=1}^n p_i a^2 (x_i - E(X))^2$$

$$V(Y) = a^2 \sum_{i=1}^n p_i (x_i - E(X))^2$$

$$V(Y) = a^2 V(X)$$

3)

c)

$$\sqrt{V(Y)} = \sqrt{a^2 V(X)}$$

$$\text{Donc } \sigma(Y) = |a| \sigma(X)$$

4)

$$E(X) = -0,10 \text{ et } \sigma(X) = 2,7$$

$$Y = 2X$$

$$E(Y) = 2E(X)$$

$$\text{Donc } E(Y) = 2 * (-0,10) = -0,20 \text{ (perte de 0,20€ en moyenne) et } \sigma(Y) = 2 * \sigma(X)$$

$$\sigma(Y) = 2 * 2,7 = 5,4 \text{ €}$$

Ex 118 p 305 :

Pour tout $x \in \mathbb{R}$, $f(x) = \sum_{i=1}^n p_i (x_i - x)^2$

Partie A :

1) $E(X) = 0,80 \cdot 0 + 0,10 \cdot 5 + 0,02 \cdot 100 + 0,08 \cdot 20$

$E(X) = 4,1$

$V(X) = 0,80 \cdot (0 - 4,1)^2 + 0,10 \cdot (5 - 4,1)^2 + 0,02 \cdot (100 - 4,1)^2 + 0,08 \cdot (20 - 4,1)^2$

$V(X) = 217,69$

2) $f(x) = 0,80 \cdot (0 - x)^2 + 0,10 \cdot (5 - x)^2 + 0,02 \cdot (100 - x)^2 + 0,08 \cdot (20 - x)^2$

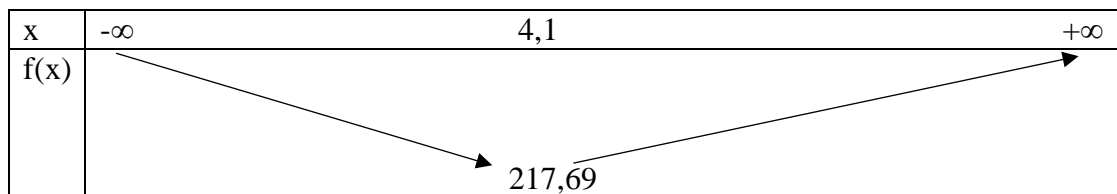
$f(x) = 0,80x^2 + 0,10(25 - 10x + x^2) + 0,02(10\,000 - 200x + x^2) + 0,08(400 - 40x + x^2)$

$f(x) = 0,80x^2 + 2,5 - x + 0,1x^2 + 200 - 4x + 0,02x^2 + 32 - 3,2x + 0,08x^2$

$f(x) = x^2 - 8,2x + 234,5$

3) $f(x) = (x - 4,1)^2 - 4,1^2 + 234,5$

$f(x) = (x - 4,1)^2 + 217,69$

Minimum en $x=4,1$