Homework 4

Exercise 1: Q6D.1

$$
\begin{aligned}
|\vec{S}| &= I\omega = \frac{2}{5}mr^2\omega = \frac{1}{2}\hbar \\
\omega &= \frac{5\hbar}{4mr^2} \\
v &= \omega * r = \frac{5\hbar}{4mr} \\
&= 1.449 * 10^{11} m/s
\end{aligned}
$$

Because the speed of light is $3 * 10^8 m/s$, $1.449 * 10^{11} m/s$ is much faster than the speed of light.

Exercise 2: R6R.1

$$I\omega = \frac{1}{2}\frac{M}{m_{Fe}} \cdot N_A \hbar$$

where as

$$
\begin{aligned}
\omega &= \frac{1}{2}\frac{M}{m} \cdot N_A \hbar \cdot \frac{2}{Mr^2} \\
&= \frac{N_A \hbar}{mr^2} \\
&= \frac{(6.02 * 10^2 3 * 1.055 * 10^- 34)}{56 * 10^{-3} * 0.01^2} \\
&= 1.134 * 10^{-5} s^{-1}
\end{aligned}
$$

so the period is

$$
\begin{aligned}
T &= \frac{2\pi}{\omega} \\
&= 554073\ s
\end{aligned}
$$

Exercise 3: Q7R.2

assume that $|\psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$

According to the question, we can get following equation:

$$
\begin{cases}
|\langle +\theta \mid \psi\rangle|^2 = \frac{16}{25} \\
|\langle -\theta \mid \psi\rangle|^2 = \frac{9}{25}
\end{cases}
$$

By solving this equation, we can easily get that

$$|\psi\rangle = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \pm 24/25 \\ \pm 7/25 \end{bmatrix}$$

So it has two distinct p values.

By sending the electron to the SG_x, we can get another equation.

$$|\langle +x \mid \psi\rangle|^2 = 0.77$$

According to this set of equations, we can get know that

$$\frac{1}{2}(a^2+b^2) = 0.77$$
$$ab = 0.27$$

Because $1 \cdot 0 = 0$, $\frac{24}{25} \cdot \frac{7}{25} = 0.27$, we can conclude that $a = \pm 24/25, b = \pm 7/25$

Exercise 4: Q7R.3

For each path

$$\begin{aligned}&\langle +x \,|\, -x\rangle\langle -x \,|\, +z\rangle\langle +z \,|\, +x\rangle\\
=&\left(\begin{bmatrix}\sqrt{1/2} & \sqrt{1/2}\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ -\sqrt{1/2}\end{bmatrix}\right)\left(\begin{bmatrix}\sqrt{1/2} & -\sqrt{1/2}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\right)\left(\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\\
=&0\end{aligned}$$

$$\begin{aligned}&\langle +x \,|\, +x\rangle\langle +x \,|\, -z\rangle\langle -z \,|\, +x\rangle\\
=&\left(\begin{bmatrix}\sqrt{1/2} & \sqrt{1/2}\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\left(\begin{bmatrix}\sqrt{1/2} & \sqrt{1/2}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\\
=&\frac{1}{2}\end{aligned}$$

$$\begin{aligned}&\langle -x \,|\, +x\rangle\langle +x \,|\, -z\rangle\langle -z \,|\, +x\rangle\\
=&\left(\begin{bmatrix}\sqrt{1/2} & -\sqrt{1/2}\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\left(\begin{bmatrix}\sqrt{1/2} & \sqrt{1/2}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\\
=&0\end{aligned}$$

$$\begin{aligned}&\langle -x \,|\, -x\rangle\langle -x \,|\, -z\rangle\langle -z \,|\, +x\rangle\\
=&\left(\begin{bmatrix}\sqrt{1/2} & -\sqrt{1/2}\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ -\sqrt{1/2}\end{bmatrix}\right)\left(\begin{bmatrix}\sqrt{1/2} & -\sqrt{1/2}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\sqrt{1/2}\\ \sqrt{1/2}\end{bmatrix}\right)\\
=&\frac{1}{2}\end{aligned}$$

the total amplitude of $|+x\rangle$ is $\frac{1}{2} + 0 = \frac{1}{2}$, and the total amplitude of $|-x\rangle$ is $\frac{1}{2} + 0 = \frac{1}{2}$.

The two wave,which are not orthogonal interference with each other, which cause the elimination of part of amplitude. So the total probability should be less than 1. It just likes the double slit interference of light. After go through the slits, the total intensity of light is less than before.

Exercise 5: QAM.2

(a)

for each path go through the $|+\theta\rangle$

$$
\begin{aligned}
&\langle +\theta \mid +y\rangle\langle +y \mid +z\rangle \\
=&\left(\begin{bmatrix} cos(\theta/2) & sin(\theta/2) \end{bmatrix} \begin{bmatrix} \sqrt{1/2} \\ \sqrt{1/2} \end{bmatrix} \right) \left(\begin{bmatrix} \sqrt{1/2} & \sqrt{1/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \\
=&\frac{1}{2}(cos(\theta/2) + sin(\theta/2))
\end{aligned}
$$

$$
\begin{aligned}
&\langle +\theta \mid -y\rangle\langle -y \mid +z\rangle \\
=&\left(\begin{bmatrix} cos(\theta/2) & sin(\theta/2) \end{bmatrix} \begin{bmatrix} \sqrt{1/2} \\ -\sqrt{1/2} \end{bmatrix} \right) \left(\begin{bmatrix} \sqrt{1/2} & -\sqrt{1/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \\
=&\frac{1}{2}(cos(\theta/2) - sin(\theta/2))
\end{aligned}
$$

Then we can get the total amplitude:

$$\langle +\theta \mid +y\rangle\langle +y \mid +z\rangle + \langle +\theta \mid -y\rangle\langle -y \mid +z\rangle = cos(\theta/2)$$

So the probability is $cos^2(\theta/2)$

(b) The input was divided in to two part by SG_y, and then add up in the tube. So we can ignore the SG_y because it does nothing. So the probability state $|+\theta\rangle$ can be write as

$$|\langle +\theta \mid +z\rangle|^2 = cos^2(\theta/2)$$

(C)

$$
\begin{aligned}
P &= |\langle +\theta \mid +y\rangle\langle +y \mid +z\rangle|^2 \\
&= \left| \left(\begin{bmatrix} cos(\theta/2) & sin(\theta/2) \end{bmatrix} \begin{bmatrix} \sqrt{1/2} \\ \sqrt{1/2} \end{bmatrix} \right) \left(\begin{bmatrix} \sqrt{1/2} & \sqrt{1/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \right|^2 \\
&= |\frac{1}{2}(cos(\theta/2) + sin(\theta/2))|^2 \\
&= \frac{1}{4}(1 + sin(\theta))
\end{aligned}
$$

(D) Because part of the information was blocked, the total probability or intensity must less than 1, which is agrees the answer we get.

Exercise 6: .

Find the Fourier exponential series (discussed during Lec 8) for the periodic function defined as $f(x) = Ax$ for $L/2 \leqslant x \leqslant L/2$. Check the orthogonality of complex exponential basis functions. Express Fourier exponential series in terms of Fourier trigonometric series (sines and cosines) and see if it agrees with what we showed in Lec 2.

$$\begin{aligned}
C_n &= \frac{1}{L}\int_{-L/2}^{L/2} Axe^{\frac{-2i\pi nx}{L}}\,\mathrm{d}x \\
&= Ae^{\frac{-2i\pi nx}{L}}\left(\frac{L}{4\pi^2n^2}+\frac{ix}{2\pi n}\right)\Bigg|_{-L/2}^{L/2} \\
&= \frac{ALi}{4\pi^2n^2}\left(e^{-i\pi n}-e^{i\pi n}\right)+\frac{ALi}{4\pi n}\left(e^{-i\pi n}+e^{i\pi n}\right) \\
&= \frac{ALi}{2n\pi}(-1)^n \\
\therefore f(x) &= \sum_{n=-\infty}^{\infty}\frac{ALi}{2n\pi}(-1)^n e^{\frac{2i\pi nx}{L}}
\end{aligned}$$

$$\begin{aligned}
&C_ne^{\frac{2i\pi nx}{L}}+C_{-n}e^{-\frac{2i\pi nx}{L}} \\
&= \frac{ALi}{2n\pi}(-1)^n e^{\frac{2i\pi nx}{L}}-\frac{ALi}{2n\pi}(-1)^{-n}e^{-\frac{2i\pi nx}{L}} \\
&= -\frac{AL}{n\pi}(-1)^n sin\left(\frac{2n\pi x}{L}\right)
\end{aligned}$$

Because the Ax goes through (0,0), $C_0 = 0$. Therefore, $f(x) = \sum_{n=1}^{\infty} -\frac{AL}{n\pi}(-1)^n sin\left(\frac{2n\pi x}{L}\right)$, which is agree with what we get in lecture 2.

Check orthogonality:

Because $2\pi/L$ is constant, let $k = 2\pi/L$. So $e^{\frac{2i\pi nx}{L}} = e^{ikn}$. if $m \neq n$, we can get:

$$\begin{aligned}
\left\langle e^{ikn}, e^{ikm}\right\rangle &= \int_{-L/2}^{L/2} e^{-ikn}e^{ikm}\,\mathrm{d}x \\
&= \frac{1}{ikm-ikn}e^{(ikm-ikn)x}\Bigg|_{-L/2}^{L/2} \\
&= \frac{i}{kn-km}2isin(\pi(m-n)) \\
&= 0
\end{aligned}$$

if $m = n$, we can get:

$$\begin{aligned}
\left\langle e^{ikn}, e^{ikn}\right\rangle &= \int_{-L/2}^{L/2} e^{-ikn}e^{ikn}\,\mathrm{d}x \\
&= \int_{-L/2}^{L/2} 1;\mathrm{d}x \\
&= L
\end{aligned}$$

if $n \neq m$, $\left\langle e^{ikn}, e^{ikm}\right\rangle = 0$; if $n = m$, $\left\langle e^{ikn}, e^{ikm}\right\rangle \neq 0$. So e^{ikn} is orthogonal.
$\langle\psi_n|$