Solving the Equity Risk Premium Puzzle

and

Inching Towards a Theory of Everything

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Contents

1 Abstract

The equity risk premium puzzle is that the return on equities has far exceeded the average return on short-term risk-free debt and cannot be explained by conventional representative-agent consumption based equilibrium models. We review a few attempts done over the years to explain this anomaly:

1. Inclusion of highly unlikely events with low probability (Ugly state along with Good and Bad), or market crashes, recently also termed as Black Swans.
2. Slow moving habit, or time-varying subsistence level, added to the basic power utility function.
3. Allowing for a separation of the inter-temporal elasticity of substitution and risk aversion, combined with consumption and dividend growth rates modeled as containing a small persistent expected growth rate component and a fluctuating volatility which captures time varying economic uncertainty.

We explore whether a fusion of the above approaches supplemented with better methods to handle the below reservations would provide a more realistic and yet tractable framework to tackle the various conundrums in the social sciences:

1. Unlimited ability of individuals to invest as compared to their ability to consume.
2. Lack of an objective measuring stick of value which gives rise to heterogeneous preferences and beliefs.
3. Unintended consequences due to the dynamic nature of social systems, where changes can be observed and decisions effected by participants to influence the system.
4. Relaxation of the transversality condition to avoid the formation of asset price bubbles.
5. How durable is durable? Since nothing lasts forever, accounting for durable goods to create a comprehensive measure of consumption volatility.

The world we live in produces fascinating phenomenon despite (or perhaps, due to) being a hotchpotch of varying doses of the above elements. The rationale for a unified theory is that beauty can emerge from chaos since the best test for a stew is its taste.

Many long standing puzzles seem to have been resolved using different techniques. The various explanations need to stand the test of time before acceptance; but then unexpected outcomes set in and new puzzles emerge. As real analysis and limits tell us: we are getting Closer and Closer; Yet it seems we are still Far Far Away...

2 Notation and Terminology for Key Results

Unless explicitly specified or respecified for a section, all symbols apply throughout the entire paper. Please consult corresponding sections including the appendix for more details.

2.1 Equity Premium Puzzle

- $R_t^i, R_t^e, R_t^f, R_t^{mv}$, are the returns on any security i, equity, risk free security and any portfolio on the mean variance frontier at time t. We use smaller case to denote the natural logarithm of the corresponding returns, $r^f = \ln R^f$ and so on.

- m, is the discount factor.

- $E(X); \mathrm{Var}(X); \sigma(X)$, are the mean, variance and standard deviation of random variable X. $\mathrm{Cov}(X, Y)$; $\rho_{X,Y}$ are the covariance and the correlation between random variables X and Y.

- (α, β), are the risk aversion coefficient and the subjective time discount factor. $\delta = -\ln \beta$ is also the subjective discount factor.

2.1.1 Hansen and Jagannathan Bound in Appendix 8.1

- α also measures the curvature of the utility function; to start with, we assume a power utility function of the constant relative risk aversion class of the form, $U(C_t, \alpha) = \frac{C_t^{1-\alpha}-1}{1-\alpha}$.

2.1.2 Consumption Growth and Interest Rates in Appendix 8.2

- C_t is the consumption at time t. We also set, $\Delta c_{t+1} \equiv \ln C_{t+1} - \ln C_t$.

2.1.3 Mehra and Prescott variation with Markov Process in Appendix 8.3

- $\pi \in R^n$, is the vector of stationary probabilities for the ergodic Markov process governing consumption growth.

- p^e ; p^f , are the prices of the equity and risk free securities.

- y_t is the firm's dividend payment in the period t. The firm's output is constrained to be less than or equal to y_t.

- $x_{t+1} \in \{\lambda_1, \ldots, \lambda_n\}$ is the growth rate of the dividend payment y_t.

- $\{\phi_{ji}\}$, is the transition probability between states i and j.

- $(\alpha, \beta)$and (μ, ϕ, γ) are parameters that define preferences and technology respectively.

2.2 Highly Unlikely Events

- η, is the low crash probability.

- ψ is a fraction or a combination of the other parameters such that $\lambda_1 > \lambda_2 > \lambda_3$.

2.3 Force of Habit

- X_t ; S_t, denote the level of habits and the surplus consumption ratio included in the utility function. Also, $s_{t+1} \equiv \ln S_{t+1}$.

- $\lambda(s_t)$, is the sensitivity function.

- η_t, is the local curvature when the utility function is modified to include the level of habits and the surplus consumption ratio.

- ϱ, g, $\bar{s}$ are parameters defined in the heteroskedastic AR(1) process for the log surplus consumption ratio.

2.4 Long Run Risks and Survivors

- $\varphi \geq 0$ is the parameter for Inter-temporal Elasticity of Substitution (IES).

- G_{t+1} is the aggregate growth rate of consumption.

- $R_{i,t+1}$ is the gross return on asset i.

- $R_{a,t+1}$ is the unobservable gross return on an asset that delivers aggregate consumption as its dividend each period.

- $R_{m,t+1}$ is the observable return on the market portfolio and the return on the aggregate dividend claim.

- q_t is a small persistent predictable component in the consumption and dividend growth rates.

- g_{t+1} and $g_{d,t+1}$ are the growth rates on consumption and dividends.

- σ_{t+1}, represents the time-varying economic uncertainty incorporated in consumption growth rate and σ^2 is its unconditional mean.
- $r_{f,t}$ and $E_t\left(r_{m,t+1} - r_{f,t}\right)$ are the risk free rate and the equity premium in the presence of time-varying economic uncertainty.
- $\beta_{m,e}$, $\lambda_{m,e}$, $\beta_{m,w}$, $\lambda_{m,w}$ are combinations of other parameters.

2.5 Heterogeneous Agents

- C_{it+1} is individual consumption growth, determined by an independent idiosyncratic shock η_{it}, such that, $\ln\left(\frac{C_{it+1}}{C_{it}}\right) = \eta_{it+1}b_{t+1} - \frac{b_{t+1}^2}{2} \quad ;\eta_{it} \sim N\left(0,1\right)$.
- b_{t+1} is the cross-sectional standard deviation of consumption growth.

3 Equity Premium Puzzle

The equity risk premium puzzle is that the return on equities has far exceeded the average return on short-term risk-free debt and cannot be explained by conventional representative-agent consumption based equilibrium models.

(Mehra and Prescott 1985) study a class of competitive pure exchange economies for which the equilibrium growth rate process on consumption and equilibrium asset returns are stationary. Attention is restricted to economies for which the elasticity of substitution for the composite consumption good between the year t and year $t+1$ is consistent with findings in micro, macro and international economics. In addition, the economies are constructed to display equilibrium consumption growth rates with the same mean, variance and serial correlation as those observed for the U.S. economy in the 1889-1978 period. They find that for such economies, the average real annual yield on equity is a maximum of four-tenths of a percent higher than that on short-term debt, in sharp contrast to the six percent premium observed. Their results are robust to non-stationarities in the means and variances of the economies growth processes.

Historically the average return on equity has far exceeded the average return on short-term virtually default-free debt. Over the ninety-year period 1889-1978 the average real annual yield on the Standard and Poor 500 Index was seven percent, while the average yield on short-term debt was less than one percent. They address the question whether this large differential in average yields can be accounted for by models that abstract from transactions costs, liquidity constraints and other frictions absent in the Arrow-Debreu set-up (Arrow and Debreu 1954; McKenzie 1954, 1959; Debreu 1987). They conclude that, for the class of economies considered, most likely some equilibrium model with a friction will be the one that successfully accounts for the large average equity premium.

The below bound due to (Hansen and Jagannathan 1991) can be used to understand the equity premium puzzle. Appendix 8.1 has the steps to arrive at this result.

$$\left|\frac{E\left(R^{mv}\right)-R^{f}}{\sigma\left(R^{mv}\right)}\right| \approx \alpha\sigma\left(\Delta c_{t+1}\right)$$

The excess return on equity instruments has been in the seven to nine percent range. The returns just after the second world war are around 9% with a standard deviation of about 16%. The risk free rate has been stable around the 1% level. Aggregate non-durable and services consumption growth had a mean and standard deviation of 1%. To explain these observed results, the risk aversion coefficient, α, needs to be around 50. If we consider the actual correlation between annual returns and non-durables plus services consumption growth, which is around 0.2, α, needs to be around 250.

(Mehra and Prescott 1985) restrict the value of α to be a maximum of ten based on evidence from other studies, "The parameter α, which measures people's willingness to substitute consumption between successive yearly time periods is an important one in many fields of economics. (Arrow 1971) summarizes a number of studies and concludes that relative risk aversion with respect to wealth is almost constant. He further argues on theoretical grounds that α should be approximately one. (Friend and Blume 1975) present evidence based upon the portfolio holdings of individuals that α is larger, with their estimates being in the range of two. (Kydland and Prescott 1982), in their study of aggregate fluctuations, found that they needed a value between one and two to mimic the observed relative variability of consumption and investment. (Altug 1983), using a closely related model and formal econometric techniques, estimates the parameter to be near zero. (Kehoe and Richardson 1984), studying the response of small countries balance of trade to terms of trade shocks, obtained estimates near one, the value posited by Arrow. (Hildreth and Knowles 1982) in their study of the behavior of farmers also obtain estimates between one and two. (Tobin and Dolde 1971), studying life cycle savings behavior with borrowing constraints, use a value of 1.5 to fit the observed life cycle savings patterns."

Looking at this from another angle we get another set of inconsistencies. A high value of risk aversion, $\alpha =$ 50 to 250 implies a very high risk free rate of 50-250% as seen from the below relation between consumption growth and interest rates (Appendix 8.2 has the steps).

$$r^{f}=\ln R^{f}=\delta+\alpha E\left(\Delta c_{t+1}\right)-\frac{\alpha^{2}}{2}\sigma^{2}\left(\Delta c_{t+1}\right)$$

To get a reasonable interest rate (usually around 1%), we need a subjective discount factor of $\delta =$ -0.5 to -2.5 or -50% to -250% (or, $\beta=e^{-\delta}>1$), which seems unreasonable since people prefer earlier utility.

(Mehra and Prescott 1985) start with a pure exchange model, (Lucas 1978) and include a variation (Mehra 1988) such that the growth rate of consumption follows a Markov process in contrast to the Lucas tree economy where the consumption level follows a Markov process. There is one productive unit or firm producing the perishable consumption good and there is one equity share that is competitively traded. Since only one productive unit is considered, the return on this share of equity of equity is also the return on the market.

With two states, the Markov process growth rates and transition probabilities are (see Appendix 8.3 for the steps),

$$\lambda_1 = 1 + \mu + \gamma, \quad \lambda_2 = 1 + \mu - \gamma$$

$$\phi_{11} = \phi_{22} = \phi, \quad \phi_{12} = \phi_{21} = 1 - \phi$$

The above parameters (α, β) and $\{(\mu, \phi, \gamma) \equiv$ elements of $[\phi_{ij}]$ and $[\lambda_i]\}$, define preferences and technology respectively; they are estimated using method of moments by matching the mean, variance and first order auto-correlation of the growth rate of per-capita consumption. Based on the estimated parameters, the maximum value of the equity premium is 0.35 percent.

4 Deep Dive into Possible Explanations

4.1 Highly Unlikely Events

(Rietz 1988) is one of the, if not the, earliest attempt to resolve the equity premium puzzle. Their departure from the Mehra Prescott specification is mainly in the assumption of three possible growth rates in three states of the world. We term them the good, bad and ugly states. The good and bad states are the same as before, but when things get really bad or the market crashes, we end up in the ugly state or in a depression like episode. Equity returns vary little from the norm in good and bad times, but there are rare or low probability events or crashes when consumption falls drastically and equity returns are far below average. It is worth noting that in their original paper, Mehra and Prescott consider a four state Markov process (though the growth rates can only be either good, poor or average), the probability of the states are not significantly different, with average times twice as likely as either poor or good, and the maximum premium explained in this case is only 0.39 percent.

The growth rates in the three states and the transition probability matrix with a disaster scenario are,

$$\lambda_1 = 1 + \mu + \gamma, \quad \lambda_2 = 1 + \mu - \gamma, \quad \lambda_3 = \psi\,(1 + \mu)$$

$$\Phi = \begin{bmatrix} \phi & 1-\phi-\eta & \eta \\ 1-\phi-\eta & \phi & \eta \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Here, ψ is a fraction or a combination of the other parameters such that $\lambda_1 > \lambda_2 > \lambda_3$. A crash will only follow state 1 or 2 and never occurs twice in a row and happens with the low crash probability, η. The estimation of the parameters proceeds in a similar fashion as (Mehra and Prescott 1985) for given crash probabilities; (Reitz 1988) takes $\eta \in [0.0001, 0.2]$; $\alpha \in (0\ ,\ 10]$; $\beta \in (0, 1)$ and the equity premium is explained successfully with this setup. (Mehra and Prescott 1988) comment on the validity of the this unlikely state assumption, especially about the value of the risk aversion parameter, α, being set to 10 and the plausibility of the occurrence of real disaster scenarios.

(Barro 2006) acknowledges that the work of Rietz has been under appreciated and calibrates disaster probabilities, especially the sharp contractions associated with the the two world wars and the great depression. His empirical analysis measures the size and frequency of economic disasters and estimates a disaster probability of 1.5 to 2 percent per year with a distribution of declines in per-capita GDP ranging between 15 percent to 64 percent. He also points out that considering capital formation; the duration of disasters; disaster scenarios and real estate prices; and depreciation and creation of Lucas trees, would be useful extensions.

The recent work of (Taleb 2007) and the occurrence of Black Swans (which are unexpected events and cannot be probabilistically anticipated but they still happen catching us off guard) is a also fillip in this line of thinking. (Weitzman 2007) offers a single unified theory for the equity premium puzzle and two other asset return puzzles: risk-free rate and equity-volatility puzzles, based on the idea that what is learn-able about the future stochastic consumption-growth process from any number of past empirical observations must fall far short of full structural knowledge.

The equity-volatility puzzle refers to the empirical fact that actual returns on a representative stock market index have a variance some two orders of magnitude larger than the variance of any consumption-dividend-like fundamental in the real economy that might possibly be driving them or that might be relevant for welfare calibration. The risk-free-rate puzzle refers to the five percentage points or so discrepancy between the interest rate that is predicted by any standard rational expectations equilibrium (REE) formula and what is actually observed. The key characteristic of REE (defining it as a proper subset of the set of all rationally formed Bayesian equilibria) is the imposed extra assumption that the subjective probability distribution of outcomes believed by agents within an economic system equals the objective frequency distribution actually generated by the system itself. It is effectively a dynamic stochastic general equilibrium where all reduced-form structural parameters of the data-generating process are known, presumably because they have already been learned previously as some kind of an ergodic limit from a sufficiently large sample.

4.2 Force of Habit

The predictability of returns from prices and dividends can be explained based on the idea that people get less risk averse when wealth or consumption increases and more risk averse as wealth decreases. The level of consumption and wealth increases over time, while the equity premia have not declined. So tying risk aversion to the level of consumption or wealth, relative to a historical trend or some previous benchmark, might hold some potential solutions.

Marshall (1920), probably one of the earliest works on this topic, discussed the notion that tastes can be cultivated and that they are affected by past consumption. (Duesenberry 1949) proposed an individual consumption function that depended on the current consumption of other people. Also known as the relative income hypothesis, this was an attempt to rationalize the well established differences between cross-sectional and time-series properties of consumption data. (Pollak 1970) considers dynamic utility functions by allowing some or all of its parameters to depend on past consumption and notes that the dominant assumption upto that point in time was that an individual's utility function depends on his own consumption, but not on the consumption of others. Hence, by allowing some or all of the parameters of an individual's utility function to depend on the consumption of others, interdependence can be incorporated into the theory of consumer behavior.

(Constantinides 1990) is an important theoretical work on the subject of habit formation. (Campbell and Cochrane 1999) specify that people slowly develop habits for higher or lower consumption. We have an endowment economy with i.i.d. consumption growth. We modify the utility function to include a term for the level of habits, X_t. The sensitivity function, $\lambda(s_t)$ is specified to satisfy three conditions, 1) Risk free rate is constants; 2) habit is predetermined at the steady state $(s_t - \bar{s})$; and 3) habit moves negatively with consumption everywhere or equivalently, habit is predetermined near the steady state. Local curvature, η_t, depends on how far consumption is above the habit, as well as α.

As consumption falls toward habit, people become much less willing to tolerate further falls in consumption; they become very risk averse. Thus, a low power coefficient α can still mean a high and time-varying curvature. High curvature means that the model can explain the equity premium, and curvature that varies over time as consumption rises in booms and falls toward habit in recessions, means that the model can explain a time-varying and counter-cyclical (high in recessions, low in booms) Sharpe ratio, despite constant consumption volatility $\sigma_t(\Delta c)$ and correlation $\operatorname{corr}(\Delta c, r)$. But higher curvature implies high and time-varying interest rates. This model gets around interest rate problems with precautionary saving. Suppose we are in a bad time, in which consumption is low relative to habit. People want to borrow against future higher consumption, and this force should drive up interest rates. (Habit models tend to have very volatile interest rates.) However, people are also much more risk averse when consumption is low. This consideration induces them to save more, in order to build up assets against the event that tomorrow might be even worse. This precautionary desire to save drives down interest rates. The sensitivity function specification, $\lambda(s_t)$, makes these two forces exactly offset, leading to constant real rates (see Appendix 8.4 for the steps to derive the below result).

$$\left|\frac{E_t\left(R^e_{t+1}\right)}{\sigma_t\left(R^e_{t+1}\right)}\right| = \sqrt{\left\{e^{\alpha^2\sigma^2[1+\lambda(s_t)]^2}-1\right\}} \approx \alpha\sigma\left[1+\lambda(s_t)\right]$$

The mean and standard deviation of log consumption growth are set to match consumption data. The serial correlation parameter ϱ is chosen to match the serial correlation of the logarithm of the ratio of price divided by dividend. β or the subjective discount factor is chosen to match the risk free rate with the average return on real treasury bills. The risk aversion parameter α is then searched so that the returns on the consumption claim matches the ratio of the unconditional mean and unconditional standard deviation of excess returns.

In contrast to the Reitz model of a small probability of a very large negative consumption shock, investors fear stocks because they do badly in occasional serious recessions unrelated to the risks of long-run average consumption growth.

4.3 Long Run Risks and Survivors

(Bansal and Yaron 2004) allow for a separation of the inter-temporal elasticity of substitution (IES) and risk aversion, combined with consumption and dividend growth rates modeled as containing a small persistent expected growth rate component and a fluctuating volatility which captures time varying economic uncertainty. (Epstein and Zin 1989; Weil 1989; Weil 1990; Chen, Favilukis and Ludvigson 2013) present estimates of key preference parameters in the recursive utility model, evaluate the model's ability to fit asset return data relative to other asset pricing models and investigate the implications of such estimates for the unobservable aggregate wealth return. Building on the recursive or non-expected utility preferences, which distinguish attitudes toward risk from behavior toward inter-temporal substitution, we can arrive at the following expressions for the risk free rate, $r_{f,t}$, and the equity premium, $E_t\left(r_{m,t+1}-r_{f,t}\right)$, in the presence of time-varying economic uncertainty (Appendix 8.5 has more details),

$$r_{f,t} = -\theta\ln\beta + \frac{\theta}{\varphi}E_t\left[g_{t+1}\right] + (1-\theta)\,E_t\left[r_{a,t+1}\right] - \frac{1}{2}\mathrm{var}_t\left[\frac{\theta}{\varphi}g_{t+1} + (1-\theta)\,r_{a,t+1}\right]$$

$$E_t\left(r_{m,t+1}-r_{f,t}\right) = \beta_{m,e}\lambda_{m,e}\sigma_t^2 + \beta_{m,w}\lambda_{m,w}\sigma_w^2 - 0.5\mathrm{var}_t\left(r_{m,t+1}\right)$$

$\varphi \geq 0$ is the IES parameter. G_{t+1} is the aggregate growth rate of consumption; $R_{a,t+1}$ is the unobservable gross return on an asset that delivers aggregate consumption as its dividend each period; $R_{m,t+1}$ is the observable return on the market portfolio and the return on the aggregate dividend claim; $g_{t+1}, r_{a,t+1}, r_{m,t+1}$are the logarithms of the variables just mentioned. q_t is a small persistent predictable component in the consumption and dividend growth rates. The growth rates on consumption g_{t+1} and dividends $g_{d,t+1}$ are modeled as shown in appendix 8.5, (following Campbell and Shiller 1988, who use similar log linear approximations to show that price-dividend ratios seem to predict long-horizon equity returns), σ_{t+1}, represents the time-varying economic uncertainty incorporated in consumption growth rate and σ^2 is its unconditional mean. There is an assumption that the shocks are uncorrelated, and allow for only one source of economic uncertainty to affect consumption and dividends. $\beta_{m,e}$, $\lambda_{m,e}$, $\beta_{m,w}$, $\lambda_{m,w}$ are combinations of other parameters.

A simpler specification can set $g_{t+1} = \mu + q_t + \sigma\eta_{t+1}$. But since the economic uncertainty, σ, is constant, the conditional risk premium and the conditional volatility of the market portfolio is constant and hence their ratio, the Sharpe ratio is also constant. The long run risk or time varying uncertainty gives a large value for the equity premium while the separation between the IES parameter and risk aversion ensures that the risk free rate remains small.

(Bansal, Kiku and Yaron 2010) is a generalized long run risks model incorporating a cyclical component in aggregate consumption and dividends and Poisson jumps in the processes. (Drechsler and Yaron 2011) demonstrate conditions under which the variance premium, defined as the difference between the squared Chicago Board Options Excange volatility index (VIX) and the expected realized variance, displays significant time variation and return predictability. They show that a calibrated, generalized long-run risks model generates a variance premium with time variation and return predictability that is consistent with the data, while simultaneously matching the levels and volatilities of the market return and risk-free rate. Using book-to-market, momentum, and size-sorted portfolios, (Bansal, Dittmar and Lundblad 2005) show that economic risks in cash flows, measured via the cash flow beta (larger cash flow beta implies higher aggregate consumption risk), can account for a significant portion of differences in risk premia across assets. (Jagannathan and Marakani 2015) show that the dependence of several asset pricing models on long-run risks, implies that the state of the economy can be captured by factors derived from the price-dividend ratios of stock portfolios. They relate the Fama-French model and the Bansal-Yaron and Merton inter-temporal asset pricing models by using two factors with small growth and large value minus small growth tilts. [Fama and French (1993) interpret the SMB (Small Minus Big) and HML (High Minus Low) factors, constructed from six size / book-to-market portfolios, as innovations to state variables in the Merton (1973) inter-temporal capital asset pricing model; these papers are among the foundations of asset pricing].

4.3.1 Market Survivor Bias

Another thread of explanation is based on the market survivor bias argument of (Brown, Goetzmann, and Ross, 1995). Empirical analysis of rates of return, implicitly condition on the security surviving into the sample. Such conditioning can induce a spurious relationship between observed return and total risk for those securities that survive to be included in the sample. The average return for a market that survives many potentially cataclysmic challenges is likely to be higher than the expected return. This suggests that past average growth rates are, if anything, upward biased estimates of future growth. (Fama and French 2002) estimate the equity premium using dividend and earnings growth rates to measure the expected rate of capital gain. Their estimates for 1951 to 2000, 2.55 percent and 4.32 percent, are much lower than the equity premium produced by the average stock return, 7.43 percent, suggesting that the high average return for 1951 to 2000 is due to a decline in discount rates that produces a large unexpected capital gain. They conclude that the average stock return of the last half-century is a lot higher than expected.

5 Possibilities for a Deeper Dive into a Theory of Everything

Each of the elegant solutions considered thus far can claim some success in explaining the equity puzzle. All these models are an artifact of having many parameters in the model, so that some parameters can be calibrated to explain particular facets of a phenomenon and other parameters can be set to explain related but different facets of the same phenomenon. We need to be wary that the forces in each solution are not acting in isolation: unlikely events are likely to happen independent of how consumers modify their behavior or develop any longer term habits, and long run risks would still persist (though some dependencies cannot be ruled out, for example, consumption habits could change based on unlikely events, but they could also change based on individual health or lifestyle choices). Hence, a consistent and complete theory needs to combine elements of all the above solutions and also be able to explain a few other fundamental observations.

As a first step, we recognize that one possible categorization of different fields can be done by the set of questions a particular field attempts to answer. The answers to the questions posed by any field can come from anywhere or from phenomenon studied under a combination of many other fields. Hence, we need to keep in mind that the answers to the questions posed under the realm of economics can come from diverse fields such as physics, biology, mathematics, chemistry, and so on.

Hence, before we consider the scope and components of a Theory of Everything for Economics, let us review similar attempts that have been going on for many decades in physics. The "Theory of Everything" (TOE) is a term for the ultimate theory of the universe (Tegmark 1998; Laughlin and Pines 2000), a set of equations capable of describing all phenomena that have been observed, or that will ever be observed. This would be an all-embracing and self-consistent physical theory that summarizes everything that there is to know about the workings of the physical world. (Tegmark 1998; 2008) We can divide TOEs into two categories depending on their answer to the following question: Is the physical world purely mathematical, or is mathematics merely a useful tool that approximately describes certain aspects of the physical world? More formally, is the physical world isomorphic to some mathematical structure?

(Cao, Cao and Qiang 2015) discuss why none of the existing theories (the Theory of Relativity, the Big-Bang, or the Standard Model) can truly serve as the foundation of physics, because they cannot answer the fundamental questions, such as why positive and negative charges exist, why quantum numbers exist, and why electron has mass and never decays. They admit that, the fundamental questions are ignored because they are simply too hard to answer. (Cao and Cao 2013) propose a framework, "Unified Field Theory", that attempts to provide a real foundation and to answer the fundamental questions by starting from space-time-energy-force, the common root for everything, conceptual or physical, to explain and predict the motion, interaction and configuration of matter. (Barrow 1991, 2007) are excellent discussions on the essential components that a successful theory of everything should possess and recent research into the quest for this holy grail.

To find such a common root in the social sciences, we use an existing definition of economics, which calls it the social science of satisfying unlimited wants with limited resources (Endnote 1). This omnipresent and omnipotent scarcity implies that agents will endeavor to get more from less. Coupling this fundamental motivation with the lack of an objective measuring stick of value, leads to an exchange or a trade (perhaps, only a trade-off sometimes); which is one of the cornerstones of economics. A trade requires a decision and it is common to estimate the future value of the item to be traded or a prediction is made to guide this decision.

$$\text{(Trying to get More from Less)} \quad \& \quad \text{(Difference in Assessment of Value)} \implies \text{(The Need for a Trade)}$$

$$\text{(Prediction)} \quad \& \quad \text{(Decision)} \iff \text{(Trade)}$$

We can then draw the following parallel (Table 1) to the common roots in the physical world and in the social sciences,

Physics	**Economics**
Space	*Scarcity*
Time	*Subjectivity*
Energy	*Predictions*
Force	*Decisions*

Table 1: Roots of Physics and Economics

The elements we discuss can be categorized into these buckets, though it should be clear that these prongs are overlapping,

1. Scarcity

(a) Force of habit

(b) How Durable is Durable?

2. Subjectivity

(a) Consumption versus investment ability

(b) Heterogeneous agents

3. Predictions

(a) Highly unlikely events

(b) Long run risks

4. Decisions

(a) Unintended consequences

(b) Transversality condition

We need to explore further whether a fusion of the solutions discussed in section 4 supplemented with better methods to handle the below reservations would provide a more realistic and yet tractable framework to tackle the various conundrums in the social sciences. The world we live in produces fascinating phenomenon despite (or perhaps, due to) being a hotchpotch of varying doses of all these elements. The rationale for a unified theory is that beauty can emerge from chaos since the best test for a stew is its taste.

5.1 Consumption versus Investment Ability

Despite the several advances in the social sciences and in particular economic and financial theory, we have yet to discover an objective measuring stick of value, a so called, True Value Theory. While some would compare the search for such a theory, to the medieval alchemist's obsession with turning everything into gold, for our present purposes, the lack of such an objective measure means that the difference in value as assessed by different participants can effect a transfer of wealth. This forms the core principle that governs all commerce that is not for immediate consumption in general, and also applies specifically to all investment related traffic, which forms a great portion of the financial services industry and hence the mainstay of asset pricing. (Kashyap 2014) looks at the use of a feedback loop to aid in the market making of financial instruments.

Although, some of this is true for consumption assets; because the consumption ability of individuals and organizations is limited and their investment ability is not, the lack of an objective measure of value affects investment assets in a greater way and hence investment assets and related transactions form a much greater proportion of the financial services industry. Consumption assets do not get bought and sold, to an inordinate extent, due to fluctuating prices, whereas investment assets will (Hull 2010 has a description of consumption and investment assets, specific to the price determination of futures and forwards; Kashyap 2014 has a more general discussion).

We can pose two questions based on figures 1 and 2.

1. What is the value of this car in USD?

2. What will be the next closing price for this time series of security prices, assuming we are on the last date of the time series?

From the different answers that different people come up with (and also from the different questions that people ask in order to answer these two questions), it should be evident that most measures of value are subjective. It should also be clear that the price of the security (investment asset) fluctuates more than the price of the car (consumption asset) even after a value is agreed upon. We need to devise appropriate measures to capture how big consumption ability is when compared to investment ability. In essence, what we are comparing is the relative size or the cardinality of two infinite sets, a routine question from real analysis. (Courant and Robbins 1996; Rudin 1964; Royden and Fitzpatrick 1988)

Figure 1: Value of Car

Prices

Date	Open	High	Low	Close	Volume	Adj Close*
Apr 8, 2016	45.45	46.00	45.05	46.00	20,757,500	46.00
Apr 7, 2016	45.80	46.10	45.65	45.80	24,693,500	45.80
Apr 6, 2016	46.10	46.15	45.70	45.95	42,227,300	45.95
Apr 5, 2016	47.45	47.50	46.65	46.75	48,264,800	46.75
Apr 4, 2016	47.95	47.95	47.95	47.95	0	47.95
Apr 1, 2016	48.45	48.50	47.85	47.95	29,888,700	47.95
Mar 31, 2016	48.50	48.85	48.35	48.40	26,596,900	48.40
Mar 30, 2016	48.50	49.25	48.35	49.05	29,039,900	49.05
Mar 29, 2016	48.50	48.70	48.15	48.50	27,907,800	48.50
Mar 28, 2016	48.80	48.80	48.80	48.80	0	48.80
Mar 25, 2016	48.80	48.80	48.80	48.80	0	48.80
Mar 24, 2016	49.00	49.15	48.65	48.80	27,542,200	48.80
Mar 23, 2016	49.50	49.60	49.20	49.50	17,432,500	49.50
Mar 22, 2016	50.50	50.50	49.95	50.00	12,380,700	50.00
Mar 21, 2016	50.40	50.65	50.15	50.30	11,813,300	50.30
Mar 18, 2016	50.20	50.35	49.90	50.00	14,658,100	50.00
Mar 17, 2016	50.50	50.70	50.20	50.45	11,538,700	50.45
Mar 16, 2016	50.25	50.40	49.80	49.85	13,396,100	49.85
Mar 15, 2016	50.50	50.50	50.10	50.20	15,635,200	50.20
Mar 14, 2016	50.50	51.00	50.50	50.70	35,826,700	50.70
Mar 11, 2016	49.80	50.60	49.80	50.30	47,828,900	50.30
Mar 10, 2016	50.15	50.65	49.80	50.20	24,931,200	50.20
Mar 9, 2016	49.40	50.50	49.40	50.05	31,118,000	50.05
Mar 8, 2016	49.55	49.75	49.15	49.25	19,506,000	49.25

Figure 2: Time Series of Close Prices

5.2 Heterogeneous Agents

The lack of an objective measuring stick of value also gives rise to heterogeneous preferences and beliefs. (Constantinides and Duffie 1996) provide a clever and simple model with relatively standard preferences, in which idiosyncratic risk can be tailored to generate any pattern of aggregate consumption and asset prices. Idiosyncratic risk stories face two severe challenges. First, the basic pricing equation applies to each individual. If we are to have low risk aversion and power utility, the required huge volatility of consumption is implausible for any individual. Second, if you add idiosyncratic risk uncorrelated with asset returns, it has no effect on pricing implications. {(Constantinides 1982) looks at other issues that come up with consumer heterogeneity}. Say, agent A gets more income when the market is high, and agent B gets more income when it is low. But then A will short the market, B will go long, and they will trade away any component of the shock that is correlated with the returns on available assets. Shocks uncorrelated with asset returns have no effect on asset pricing, and shocks correlated with asset returns are quickly traded away.

The way around this problem is to make the idiosyncratic shocks permanent. We can give individuals idiosyncratic income shocks that are correlated with the market but are uncorrelated with returns. We can give people income shocks that are uncorrelated with returns, so they cannot be traded away. Then we exploit the non-linearity of marginal utility. Then we have a nonlinear marginal utility function turn these shocks into marginal utility shocks that are correlated with asset returns, and hence can affect pricing implications. This is why Constantinides and Duffie specify that the variance of idiosyncratic risk rises when the market declines. If marginal utility were linear, an increase in variance would have no effect on the average level of marginal utility. Therefore, Constantinides and Duffie specify power utility, and the interaction of nonlinear marginal utility and changing conditional variance produces an equity premium.

Each consumer i has power utility and a simple model can be specified wherein, individual consumption growth C_{it+1} is determined by an independent idiosyncratic shock η_{it} (Appendix 8.6 has the model specification and related details). The cross-sectional standard deviation of consumption growth is specified so that people suffer a high cross-sectional variance of consumption growth on dates of a low market return. The excess return, R^e_{t+1}, can be written, after aggregating across all consumers as,

$$0 = E_t \left[\left(e^{-\alpha E_N[\Delta c_{it+1}] + \frac{\alpha^2}{2} \sigma^2_N [\Delta c_{it+1}]} \right) R^e_{t+1} \right]$$

From this we see that the economy displays more risk aversion than would a representative agent with aggregate consumption, $\triangle c^a_{t+1} = E_N \triangle c_{it+1}$. If σ_N, the aggregate standard deviation of consumption growth over all consumers N, varies over time, the risk aversion can also vary over time and this variation can generate risk premia.

5.3 Unintended Consequences

Due to the dynamic nature of social systems, changes can be observed and decisions effected by participants to influence the system. In the social sciences, as soon as any generalization and its set of conditions becomes common knowledge, the entry of many participants shifts the equilibrium or the dynamics, such that the generalization no longer applies to the known set of conditions. As long as participants are free to observe the results and modify their actions, this effect will persist and the varying behavior of participants in a social system will give rise to unintended consequences. (Kashyap 2015; 2016) discuss recent examples in the financial markets where unintended consequences set in.

All attempts at prediction, including both the physical and the social sciences, are like driving cars with the front windows blackened out. The Uncertainty Principle of the Social Sciences can be stated as, "Any generalization in the social sciences cannot be both popular and continue to yield accurate predictions or in other words, the more popular a particular generalization, the less accurate will be the predictions it yields". An artifact of this is unintended consequences. Many long standing puzzles seem to have been resolved using different techniques. The various explanations are still to be tested over time before acceptance; but then unexpected outcomes set in and new puzzles emerge. As real analysis and limits tell us (Rosenlicht 1968; Schumacher 2008): We are getting Closer and Closer; Yet it seems we are still Far Far Away...

(McManus and Hastings 2005) clarify the wide range of uncertainties that affect complex engineering systems and present a framework to understand the risks (and opportunities) they create and the strategies system designers can use to mitigate or take advantage of them. (Simon 1962) points out that any attempt to seek properties common to many sorts of complex systems (physical, biological or social), would lead to a theory of hierarchy since a large proportion of complex systems observed in nature exhibit hierarchic structure. (Lawson 1985) argues that the Keynesian view on uncertainty (that it is generally impossible, even in probabilistic terms, to evaluate the future outcomes of all possible current actions; Keynes 1937; 1971; 1973), far from being innocuous or destructive of economic analysis in general, can give rise to research programs incorporating, amongst other things, a view of rational behavior under uncertainty, which could be potentially fruitful. These viewpoints hold many lessons for policy designers in the social sciences and could be instructive for researchers looking at ways to understand and contend with complex systems, keeping in mind the caveats of dynamic social systems.

Another under-appreciated problem in empirical results on asset pricing is that a large part of the U.S. postwar average stock return may represent good luck rather than ex ante expected return. The standard deviation of stock returns is so high that standard errors are surprisingly large that an eight percent return is not statistically different from zero. (Siegel 1992a, b) extend the U.S. data on real stock and bond returns back to 1802 and find that early stock returns did not exceed fixed income returns by nearly the same magnitude they did in more recent data. The equity premium is a puzzle because the measured risk associated with equity returns is not high enough to justify the observed high returns. (Poterba and Summers 1988) show that the standard deviation of stock returns actually decreases more quickly than it would if returns were a random walk because stock returns display mean reversion. (Siegel and Thaler 1997) highlight that asset returns deviate from a random walk, which implies that for long-horizon investors, the risk of holding stocks is less than one would expect by just looking at the annual standard deviation of returns.

5.4 Transversality Condition

$$\lim_{t\to\infty} E_t\left[m_{t,t+j}p_{t+j}\right] = 0$$

This innocuous assumption is made to rule out the formation of asset pricing bubbles, so that prices grow so fast that people will buy now just to resell at higher prices later, even if there are no dividends. The reality of financial markets makes it clear that participants trade primarily to benefit from temporary bubbles or to capitalize from a jump in prices. Hence, we need to consider if there are any alternatives to this assumption or can this be relaxed under any situations?

We need to consider the extent of trading in stocks as compared to other assets, or the risk free asset. The equity premium could be due to the possibility that stocks are available for trading by a larger segment of the population and there is a possibility that the equity market can harbor price bubbles more than any other asset class. If the expectations of investors change in such a way that they believe they will be able to sell an asset for a higher price in the future than they had been expecting, then the current price of the asset will rise (Stiglitz 1990). If the reason that the price is high today is only because investors believe that the selling price will be high tomorrow-when "fundamental" factors do not seem to justify such a price-then a bubble exists. If the asset price increases more slowly than the discount factor, eventually the terminal price becomes of negligible importance as viewed from today. Under such circumstances, the value of the asset has to be just equal to the discounted value of the stream of returns it generates, and no bubbles can exist. But as long as no one in the economy has an infinite planning horizon, there is nothing to ensure that this condition on prices (called the transversality condition) will be satisfied.

(Weitzman 1973; Araujo and Scheinkman 1983) derive duality conditions necessary and sufficient for infinite horizon optimality, emphasizing the close connection between duality theory for infinite horizon convex models and dynamic programming, showing that a necessary and sufficient condition for optimality is the existence of support prices such that the limit value of the optimal capital stocks is zero. (Michel 1982; Benveniste and Scheinkman 1982) discuss the assumptions required to set to zero the value of the stocks at the limit. (Michel 1990) studies general concave discrete time infinite horizon optimal control problem and establish necessary and sufficient conditions for optimality. In finite horizon optimal control problems without constraints on the final state, necessary conditions for optimality include the transversality condition: the final value of the shadow price-vector is zero. This means that one more unit of any good at final time gives no additional value to the criterion. Halkin's example (Halkin 1974) shows that this property is not necessarily true in an infinite horizon. In an infinite horizon, one more unit of a good, at any time, changes the whole future, and the zero value of the state becomes a limit property which is not necessarily verified. (Ekeland and Scheinkman 1986) prove the necessity of a standard transversality condition under certain technical conditions; (Kamihigashi 2000) provides a simplification of the same proof with some relaxed assumptions. (Benveniste and Scheinkman 1982) prove the envelope condition to find the derivative of the value function of a recursive optimization problem. (Kamihigashi 2000; 2001; 2002) provide proofs for the necessity of transversality conditions under deterministic scenarios. (Kamihigashi 2003) considers stochastic versions.

5.5 How Durable is Durable? Nothing Lasts Forever...

The equity premium puzzle is based on the consumption growth of non-durable goods and services. (Startz 1989) looks at the time series behavior of consumption and verifies that purchases of non-durable goods follow a random walk while purchases of durable goods require an ARMAX model (X-extension of autoregressive–moving-average, ARMA, models with X-exogenous inputs; See Hamilton 1994), one in which lags of nondurables and services enter on the right-hand side. (Conrad and Schröder 1991) propose an integrated framework for modeling consumer demand for durables and nondurables and employ this approach for measuring the effect of an enforced environmental policy on energy demand and on consumer welfare. (Erceg and Levin 2002) find that a monetary policy innovation has a peak impact on durable expenditures that is several times as large as its impact on non-durable expenditures, and hence a greater interest rate sensitivity.

Looking at the companies listed on the NASDAQ (Endnotes 2 and 3), we see that a huge number of companies are labeled as durable goods producers. The aggregate valuation of durable goods providers is comparable to the aggregate valuation of non-durable producers. As of May 24, 2016 of the companies listed on the NASDAQ, 235 were non-durable producers with 2.5 Trillion USD market capitalization versus 151 durable producers with 400 Billion USD market capitalization; worth noting that the top ten non-durable companies have a combined market cap of 1.4 Trillion USD. The profits and cash flow from these two groups needs to be analyzed further. The change in consumption or spending can be argued to be higher for durable goods than for non-durable goods, since most basic necessities fall under non-durable goods. Do housing prices change more during bad times as compared to the price of toothpaste and milk? If we look at consumption changes in either group separately, then perhaps we need to consider returns from the stock market for each group separately as well.

6 Conclusions

We have discussed the equity premium puzzle and a few well known attempts to resolve it. Additional state variables are the natural route to solving empirical puzzles. The Campbell–Cochrane model is a representative from the literature that attacks the equity premium by modifying the representative agent's preferences. The Constantinides and Duffie model is a representative of the literature that attacks the equity premium by modeling uninsured idiosyncratic risks, market frictions, and limited participation. These models are quite similar in spirit. First, both models make a similar, fundamental change in the description of stock market risk. Consumers do not fear much the loss of wealth of a bad market return per se. They fear that loss of wealth because it tends to come in recessions, in one case defined as times of heightened idiosyncratic labor market risk, and in the other case defined as a fall of consumption relative to its recent past. This recession state variable or risk factor drives most variation in expected returns. The Banson and Yaron model modifies the representative agent's preferences and separates the inter-temporal elasticity of substitution and the risk aversion parameter and introduces variables to capture long term uncertainty. All these models are an artifact of having many parameters in the model so that some parameters can be calibrated to explain particular facets of a phenomenon and other parameters can be set to explain related but different facets of the same phenomenon.

We have discussed some possibilities for future research that might be able to resolve this and other related puzzles in the social sciences. Many long standing puzzles seem to have been resolved using different techniques. The various explanations need to stand the test of time before acceptance; but then unexpected outcomes set in and new puzzles emerge. As real analysis and limits tell us: we are getting Closer and Closer; yet it seems we are still Far Far Away...

7 Notes and References

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3. Consumer Non-Durables Companies on NASDAQ

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8 Appendix: Mathematical Steps

8.1 Hansen and Jagannathan Bound

All assets priced by the discount factor m need to be obey (Cochrane 2009),

$$1 = E\left(mR^i\right)$$

$$1 = E\left(m\right)E\left(R^i\right) + \rho_{m,R^i}\sigma\left(R^i\right)\sigma\left(m\right)$$

$$E\left(R^i\right) = R^f - \rho_{m,R^i}\frac{\sigma\left(m\right)}{E\left(m\right)}\sigma\left(R^i\right) \quad \because \frac{1}{E\left(m\right)} = R^f$$

$$\left|\frac{E\left(R^i\right) - R^f}{\sigma\left(R^i\right)}\right| \leq \frac{\sigma\left(m\right)}{E\left(m\right)} \quad \because \left|\rho_{m,R^i}\right| \leq 1$$

We could also write this as,

$$1 = E\left(mR^i\right) \Rightarrow e^0 = E\left[e^{\ln m + \ln R^i}\right] = e^{E(\ln m) + E\left(\ln R^i\right) + \frac{1}{2}\mathrm{Var}\left(\ln m + \ln R^i\right)}$$

$$0 = E\left(\ln m\right) + E\left(\ln R^i\right) + \frac{1}{2}\mathrm{Var}\left(\ln m\right) + \frac{1}{2}\mathrm{Var}\left(\ln R^i\right) + \mathrm{Cov}\left(\ln m, \ln R^i\right) \quad \because \mathrm{Var}\left(X+Y\right) = \mathrm{Var}\left(X\right) + \mathrm{Var}\left(Y\right) + 2\mathrm{Cov}\left(X,Y\right)$$

$$0 = E\left(\ln R^i\right) + \frac{1}{2}\mathrm{Var}\left(\ln R^i\right) - \ln R^f + \mathrm{Cov}\left(\ln m, \ln R^i\right)$$

$$\left[\because Y = \ln X \sim N\left\{\mu = E\left(Y\right) = E\left(\ln X\right), \sigma^2 = \mathrm{Var}\left(Y\right) = \mathrm{Var}\left(\ln X\right)\right\} \Rightarrow E\left(X\right) = e^{\mu + \frac{1}{2}\sigma^2} \Rightarrow \ln E\left(X\right) = \mu + \frac{1}{2}\sigma^2 \right.$$
$$\left. \text{Above, set } X = m \Rightarrow E\left(\ln m\right) + \frac{1}{2}\mathrm{Var}\left(\ln m\right) = \ln E\left(m\right); \quad \ln E\left(m\right) = \ln\frac{1}{R^f} = -\ln R^f\right]$$

For an excess return these equations become,

$$0 = E\left(mR^e\right)$$

$$0 = E\left(m\right)E\left(R^e\right) + \rho_{m,R^e}\sigma\left(R^e\right)\sigma\left(m\right)$$

$$\left|\frac{E\left(R^e\right)}{\sigma\left(R^e\right)}\right| \leq \frac{\sigma\left(m\right)}{E\left(m\right)}$$

When the correlation $|\rho_{m,R^{mv}}| = 1$ or for assets on the mean variance frontier we have,

$$\left|\frac{E\left(R^{mv}\right) - R^f}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma\left(m\right)}{E\left(m\right)}$$

Assuming a power utility function of the constant relative risk aversion class of the form,

$$U\left(C_t,\alpha\right)=\frac{C_t^{1-\alpha}-1}{1-\alpha}$$

Here, α measures the curvature of the utility function and C_t is the consumption at time t. This specification ensures that the equilibrium return process is stationary. We have from the first order conditions of a utility maximizing representative consumer,

$$m=\beta\frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)}$$

$$\left|\frac{E\left(R^{mv}\right)-R^f}{\sigma\left(R^{mv}\right)}\right|=\frac{\sigma\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]}{E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]}$$

Assuming that consumption growth is log normal, i.e., $\Delta c_{t+1}=\ln C_{t+1}-\ln C_t=\ln\left(\frac{C_{t+1}}{C_t}\right)=x$, say, which is a normally distributed random variable.

$$E\left(e^x\right)=e^{E(x)+\frac{1}{2}\sigma^2(x)}$$

$$\sigma^2\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]=E\left\{\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]^2\right\}-\left\{E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]\right\}^2$$

$$\sigma\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]=\sqrt{E\left[e^{-2\alpha x}\right]-\left\{E\left[e^{-\alpha x}\right]\right\}^2}$$

For any real or complex number s, the s-th moment of a log-normally distributed variable $X\equiv e^Y$ is given by,

$$\mathrm{E}[X^s]=e^{sE(Y)+\frac{1}{2}s^2\sigma^2(Y)}$$

$$\sigma\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]=\sqrt{e^{-2\alpha E(x)+2\alpha^2\sigma^2(x)}-\left\{e^{-\alpha E(x)+\frac{1}{2}\alpha^2\sigma^2(x)}\right\}^2}$$

$$=\sqrt{e^{-2\alpha E(x)+\alpha^2\sigma^2(x)}\left[e^{\alpha^2\sigma^2(x)}-1\right]}$$

$$=e^{-\alpha E(x)+\frac{1}{2}\alpha^2\sigma^2(x)}\sqrt{\left[e^{\alpha^2\sigma^2(x)}-1\right]}$$

$$\sigma\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]=E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]\sqrt{\left[e^{\alpha^2\sigma^2(x)}-1\right]}$$

$$\left|\frac{E\left(R^{mv}\right)-R^f}{\sigma\left(R^{mv}\right)}\right|=\sqrt{\left[e^{\alpha^2\sigma^2(x)}-1\right]}$$

If $\alpha^2\sigma^2(x)$ is small, then using the approximation, $e^y \approx 1 + y$, the following holds,

$$\left|\frac{E(R^{mv}) - R^f}{\sigma(R^{mv})}\right| \approx \alpha\sigma(\Delta c_{t+1})$$

8.2 Consumption Growth and Interest Rates

$$R^f = \frac{1}{E(m)} = \frac{1}{E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right]}$$

Similar to before, if we have log normal consumption growth, $\Delta c_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right) = x$ and setting $\beta = e^{-\delta}$,

$$R^f = \left[e^{-\delta}e^{-\alpha E(x)+\frac{1}{2}\alpha^2\sigma^2(x)}\right]^{-1}$$

Taking logarithms,

$$r^f = \ln R^f = \delta + \alpha E(\Delta c_{t+1}) - \frac{\alpha^2}{2}\sigma^2(\Delta c_{t+1})$$

8.3 Mehra and Prescott variation with Markov Process

The firm's output is constrained to be less than or equal to y_t. y_t is the firm's dividend payment in the period t as well. The growth rate in y_t is subject to a Markov chain,

$$y_{t+1} = x_{t+1}y_t$$

Here, $x_{t+1} \in \{\lambda_1, \ldots, \lambda_n\}$ is the growth rate and the transition probability between states i and j, $\Pr\{x_{t+1} = \lambda_j; x_t = \lambda_i\} = \phi_{ij}$. The price of any security in period t with payments given by the process d_s is,

$$P_t = E_t\left\{\sum_{s=t+1}^{\infty}\beta^{s-t}\frac{U'(y_s)\,d_s}{U'(y_t)}\right\}$$

For an equity share of the firm with dividend payment process, $\{y_s\}$

$$P_t^e = P^e(x_t, y_t)$$

$$= E_t\left\{\sum_{s=t+1}^{\infty}\beta^{s-t}\frac{y_t^\alpha y_s}{y_s^\alpha}\,|x_t, y_t\right\}$$

Under equilibrium, since, $y_s = y_t x_{t+1} \ldots x_s$, the price is homogeneous of degree one in y_t, which is the current endowment of the consumption good. The state is fully represented by (x_t, y_t). Recognizing that the equilibrium values are time invariant functions of the state, we can then redefine it as the pair $(c, i) \equiv (y_t, \lambda_i)$. The price of the equity share then satisfies (adopting small letters for the prices and dropping time subscripts due to the time in-variance of the functions under equilibrium),

$$p^e(c,i) = \beta \sum_{j=1}^{n} \phi_{ij} (\lambda_j c)^{-\alpha} \left[p^e(\lambda_j c, j) + \lambda_j c\right] c^{\alpha}$$

Since, $p^e(c,i)$ is homogeneous of degree one in c, we represent this function using a constant w_i as,

$$p^e(c,i) = w_i c$$

$$\Rightarrow \qquad w_i c = \beta \sum_{j=1}^{n} \phi_{ij} (\lambda_j c)^{-\alpha} \left[\lambda_j w_j c + \lambda_j c\right] c^{\alpha}$$

$$\Rightarrow \qquad w_i = \beta \sum_{j=1}^{n} \phi_{ij} (\lambda_j)^{1-\alpha} \left[w_j + 1\right]$$

We then get the period return of the equity security as,

$$r_{ij}^e = \frac{p^e(\lambda_j c, j) + \lambda_j c - p^e(c,i)}{p^e(c,i)} = \frac{\lambda_j (w_j + 1)}{w_i} - 1$$

The expected return denoted by capital letters with the current state i, is,

$$R_i^e = \sum_{j=1}^{n} \phi_{ij} r_{ij}^e$$

Similarly we have for the risk free rate,

$$p_i^f = p^f(c,i) = \beta \sum_{j=1}^{n} \phi_{ij} (\lambda_j)^{-\alpha}$$

$$R_i^f = \frac{1}{p_i^f} - 1$$

From the assumption of an ergodic Markov process, the vector of stationary probabilities, $\pi \in R^n$ on state i is given by the solution of the system of equations,

$$\pi = \phi^T \pi \quad ; \sum_{i=1}^{n} = 1 \quad ; \phi^T = \{\phi_{ji}\}$$

The state independent returns for the equity and risk free security and hence the equity risk premium are given by,

$$R^e = \sum_{i=1}^{n} \pi_i R_i^e \quad ; \quad R^f = \sum_{i=1}^{n} \pi_i R_i^f$$

With two states, the Markov process growth rates and transition probabilities are,

$$\lambda_1 = 1 + \mu + \gamma, \quad \lambda_2 = 1 + \mu - \gamma$$

$$\phi_{11} = \phi_{22} = \phi, \quad \phi_{12} = \phi_{21} = 1 - \phi$$

The parameters (α, β) define preferences and (μ, ϕ, γ) define technology. They are estimated using method of moments by matching the mean, variance and first order auto-correlation of the growth rate of per-capita consumption. Based on the estimated parameters, the maximum value of the equity premium is 0.35 percent.

8.4 Force of Habit

Maximization of utility function now becomes,

$$E \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\alpha} - 1}{1 - \alpha}$$

$$\ln C_{t+1} - \ln C_t \equiv \Delta c_{t+1} = g + \vartheta_{t+1}, \quad \vartheta_{t+1} \sim \text{ i.i.d. } N\left(0, \sigma^2\right)$$

Instead of the habit level, the log surplus consumption ratio, s_t, evolves as a heteroskedastic AR(1) process,

$$\ln S_{t+1} \equiv s_{t+1} = (1 - \varrho)\, \bar{s} + \varrho s_t + \lambda\left(s_t\right)\left(c_{t+1} - c_t - g\right)$$

Here, the surplus consumption ratio is given by,

$$S_t = \frac{C_t - X_t}{C_t}$$

The marginal utility is given by,

$$U_c\left(C_t, X_t\right) = \left(C_t - X_t\right)^{-\alpha} = S_t^{-\alpha} C_t^{-\alpha}$$

The inter-temporal marginal rate of substitution and hence the discount factor are given by,

$$M_{t+1} \equiv \beta \frac{U_c\left(C_{t+1}, X_{t+1}\right)}{U_c\left(C_t, X_t\right)} = \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t}\right)^{-\alpha}$$

$$M_{t+1} = \beta \left[e^{(1-\varrho)(\bar{s}-s_t)+\lambda(s_t)(c_{t+1}-c_t-g)} e^{g+\vartheta_{t+1}} \right]^{-\alpha}$$

$$= \beta G^{-\alpha} \left[e^{-\alpha\{(1-\varrho)(\bar{s}-s_t)+\vartheta_{t+1}[1+\lambda(s_t)]\}} \right]$$

$$r_t^f = -\ln\left[E_t\left(M_{t+1}\right)\right] = -\ln(\beta) + \alpha g - \alpha(1-\varrho)(s_t - \bar{s}) - \frac{\alpha^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2$$

The sensitivity function, $\lambda(s_t)$ is specified to satisfy three conditions, 1) Risk free rate is constant; 2) habit is predetermined at the steady state $(s_t - \bar{s})$; and 3) habit moves negatively with consumption everywhere, or equivalently, habit is predetermined near the steady state. Simplifying gives,

$$\lambda(s_t) = \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1\ ; \quad \bar{S} = \sigma\sqrt{\frac{\alpha}{1-\varrho}}$$

$$r_t^f = -\ln(\beta) + \alpha g - \frac{\alpha}{2}(1-\varrho)$$

Local curvature, η_t, depends on how far consumption is above the habit, as well as α,

$$\eta_t = -\frac{C_t U_{cc}(C_t - X_t)}{U_c(C_t - X_t)} = \frac{\alpha}{S_t}$$

8.5 Long Run Risks and Survivors

The asset pricing restriction for gross return, $R_{i,t+1}$, satisfies,

$$E_t\left[\beta^\theta G_{t+1}^{-\frac{\theta}{\varphi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}\right] = 1, \quad \theta = \frac{1-\alpha}{1-\frac{1}{\varphi}}$$

Recursive preferences are given by,

$$U_t = \left\{(1-\beta)\, C_t^{\frac{1-\alpha}{\theta}} + \beta E_t\left[U_{t+1}^{1-\alpha}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\alpha}}$$

The logarithm of the inter-temporal marginal rate of substitution is,

$$\ln M_{t+1} \equiv m_{t+1} = \theta \ln\beta - \frac{\theta}{\varphi} g_{t+1} + (\theta - 1)\, r_{a,t+1}$$

Asset returns satisfy,

$$E_t\left[e^{m_{t,t+1}+r_{i,t+1}}\right] = 1$$

$\varphi \geq 0$ is the IES parameter. G_{t+1} is the aggregate growth rate of consumption; $R_{a,t+1}$ is the unobservable gross return on an asset that delivers aggregate consumption as its dividend each period; $R_{m,t+1}$ is the observable return on the market portfolio and the return on the aggregate dividend claim; g_{t+1} , $r_{a,t+1}$, $r_{m,t+1}$ are the logarithms of the variables just mentioned. q_t is a small persistent predictable component in the consumption and dividend growth rates. The growth rates on consumption g_{t+1} and dividends $g_{d,t+1}$ are modeled as below, (following Campbell and Shiller 1988, who use similar log linear approximations to show that price-dividend ratios seem to predict long-horizon equity returns),

$$\ln\left(R_{a,t+1}\right) = r_{a,t+1} = a_0 + a_1 z_{t+1} - z_t + g_{t+1}, \quad z_t = \ln\left(\frac{P_t}{C_t}\right)$$

$$\begin{aligned}
q_{t+1} &= \rho q_t + \varphi_e \sigma_t \varepsilon_{t+1} \\
g_{t+1} &= \mu + q_t + \sigma_t \eta_{t+1} \\
g_{d,t+1} &= \mu_d + \varphi q_t + \varphi_d \sigma_t u_{t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu_1\left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1} \\
& \quad \varepsilon_{t+1}, \eta_{t+1}, u_{t+1}, w_{t+1} \sim \text{i.i.d. } N\left(0,1\right)
\end{aligned}$$

Here, σ_{t+1}, represents the time-varying economic uncertainty incorporated in consumption growth rate and σ^2 is its unconditional mean. There is an assumption that the shocks are uncorrelated, and allow for only one source of economic uncertainty to affect consumption and dividends. The risk free rate, $r_{f,t}$, and the equity premium, $E_t\left(r_{m,t+1} - r_{f,t}\right)$, in the presence of time-varying economic uncertainty are,

$$r_{f,t} = -\theta \ln \beta + \frac{\theta}{\varphi} E_t\left[g_{t+1}\right] + \left(1-\theta\right) E_t\left[r_{a,t+1}\right] - \frac{1}{2}\text{var}_t\left[\frac{\theta}{\varphi} g_{t+1} + \left(1-\theta\right) r_{a,t+1}\right]$$

$$E_t\left(r_{m,t+1} - r_{f,t}\right) = \beta_{m,e}\lambda_{m,e}\sigma_t^2 + \beta_{m,w}\lambda_{m,w}\sigma_w^2 - 0.5\text{var}_t\left(r_{m,t+1}\right)$$

$\beta_{m,e}$, $\lambda_{m,e}$, $\beta_{m,w}$, $\lambda_{m,w}$ are combinations of other parameters. A simpler specification can set $g_{t+1} = \mu + q_t + \sigma\eta_{t+1}$. But since the economic uncertainty, σ, is constant, the conditional risk premium and the conditional volatility of the market portfolio is constant and hence their ratio, the Sharpe ratio is also constant. The long run risk or time varying uncertainty gives a large value for the equity premium while the separation between the IES parameter and risk aversion ensures that the risk free rate remains small.

8.6 Heterogeneous Agents

Each consumer i has power utility,

$$U = E\sum_t e^{-\delta t} C_{it}^{1-\alpha}$$

The simple model can be specified such that, individual consumption growth C_{it+1} is determined by an independent idiosyncratic shock η_{it},

$$\ln\left(\frac{C_{it+1}}{C_{it}}\right) = \eta_{it+1} b_{t+1} - \frac{b_{t+1}^2}{2} \quad ; \eta_{it} \sim N(0,1)$$

b_{t+1} is the cross-sectional standard deviation of consumption growth. It is specified so that people suffer a high cross-sectional variance of consumption growth on dates of a low market return R_{t+1}.

$$b_{t+1} = \sigma\left[\ln\left(\frac{C_{it+1}}{C_{it}}\right) | R_{t+1}\right] = \sqrt{\frac{2}{\alpha(\alpha+1)}}\sqrt{\delta - \ln R_{t+1}}$$

The general model is,

$$b_{t+1} = \sqrt{\frac{2}{\alpha(\alpha+1)}}\sqrt{\ln m_{t+1} + \delta + \alpha \ln\frac{C_{t+1}}{C_t}} \quad ; p_t = E_t\left[m_{t+1} x_{t+1}\right] \ \forall x_{t+1} \in \underline{X} \equiv \{\text{Set of Payoffs}\}$$

$$\ln\left(\frac{\nu_{it+1}}{\nu_{it}}\right) = \eta_{it+1} b_{t+1} - \frac{b_{t+1}^2}{2} \quad ; C_{it+1} = \nu_{it+1} C_{it}$$

Using this it is easily shown that,

$$1 = E_t\left[e^{-\delta}\left(\frac{C_{it+1}}{C_{it}}\right)^{-\alpha} R_{t+1}\right]$$

The excess return can be written as,

$$0 = E_t\left[\left(\frac{C_{it+1}}{C_{it}}\right)^{-\alpha} R_{t+1}^e\right]$$

Now aggregating across all consumers by summing over i, $E_N = \frac{1}{N}\sum_{i=1}^N$ and assuming that cross-sectional variation of consumption growth is log-normally distributed gives,

$$0 = E_t\left[E_N\left(\left(\frac{C_{it+1}}{C_{it}}\right)^{-\alpha}\right) R_{t+1}^e\right]$$

$$0 = E_t\left[\left(e^{-\alpha E_N[\Delta c_{it+1}] + \frac{\alpha^2}{2}\sigma_N^2[\Delta c_{it+1}]}\right) R_{t+1}^e\right]$$