Eiklīda telpas

Henrik Gabrielyan

the 24th of May 2019

Eiklīda telpa

$R \quad (x, y) = x_1 y_1 + x_2 y_2 + x_n y_n$
$e_1, \ldots, e_n \quad i \neq j \rightarrow e_i \perp e_j \quad \forall_i (|e_i| = 1)$

what is A?
$\neg A \qquad A'$
$A \vee B \quad A \cup B \quad L_1 + L_2$
$A \wedge B \quad A \cap B \quad L_1 \cap L_2$

<u>**Def.**</u> Ja E ir Eiklīda telpa, M $\subseteq$ E, tad par kopas M **ortogonālo papildinājumu** sauc:

$M^{\perp} = x \in E | \forall_{y \in M} (x \perp y)$

$M^{\perp}$ īpašības :

$M^{\perp}$ ir lineāra apakštelpa

Ja $M = y^{(1)}, y^{(2)}, \dots, y^{(k)},$ tad $M^{\perp}$ ir atrisinājumu kopa HLVS

$$\begin{cases} y_1^{(1)} x_1 + \ldots + y_n^{(1)} x_n = 0 \\ \ldots\ldots\ldots\ldots\ldots \\ y_1^{(k)} x_1 + \ldots + y_n^{(k)} x_n = 0 \end{cases}$$

L_1, L_2 - lineāras apakštelpas

$L_1 \oplus L_1^{\perp} = E$

$(L_1^{\perp})^{\perp} = L_1$

$(L_1 + L_2)^{\perp} = L_1^{\perp} \cap L_2^{\perp}$

$(L_1 \cap L_2)^{\perp} = L_1^{\perp} + L_2^{\perp}$

Def. Ja E - Eiklīda telpa, L_1 - lin. apakštelpa, tad jebkuram

$x \in E : x = x_1 + x_2, x_1 \in L_1, x_2 \in L_1^{\perp}$

x_1 sauc par x **ortogonālu projekciju** uz L_1, $\quad x_1 = pr_{L_1} x$

x_2 sauc par x **ortogonālo komponenti** pret L_1, $\quad x_2 = ort_{L_1} x$

Grama - Šmidta process

Ieejā: $u_1, u_2, \ldots, u_k$
Izejā: $w_1, w_2, \ldots, w_L$ - ortonormēta bāze $span(u_1, u_2, \ldots, u_k)$

$v_1 = u_1$how to prove that $(v_1, v_2) = 0$ and $v_2 = u_2 \ldots$?

$$v_2 = u_2 - \frac{(u_2, v_1)}{(v_1, v_1)} \cdot v_1$$

$$v_3 = u_3 - \frac{(u_3, v_1)}{(v_1, v_1)} \cdot v_1 - \frac{(u_3, v_2)}{(v_2, v_2)} \cdot v_2$$

$$\frac{(u_2, u_1)}{|u_1|} \cdot \frac{u_1}{|u_1|} = \frac{(u_2, u_1)}{|u_1|^2} \cdot u_1 = \frac{(u_2, u_1)}{u_1, u_1} \cdot v_1$$

$$v_k = u_k - \frac{(u_k, v_1)}{(v_1, v_1)} \cdot v_1 - \ldots - \frac{(u_k, v_{k-1})}{(v_{k-1}, v_{k-1})} \cdot v_{k-1}$$

Normē visus neizmestos vektorus:

$$\begin{cases} w_1 = \frac{1}{|v_1|} \cdot v_1 \\ \ldots\ldots\ldots \\ w_k = \frac{1}{v_k} \cdot v_k \end{cases}$$

1362.

$u_1 = (1; 1; -1; -2)$
$u_2 = (5; 8; -2; -3)$
$u_3 = (3; 9; 3; 8)$

$v_1 = (1; 1; -1; -2)$

$$v_2 = (5; 8 : -2; -3) - \frac{5 \cdot 1 + 8 \cdot 1 + (-2) \cdot (-1) + (-3) \cdot (-2)}{1^2 + 1^2 + (-1)^2 + (-2)^2}(1; 1; -1; -2) =$$

$$= (5; 8 : -2; -3) - \frac{21}{7}(1; 1; -1; -2) = (5; 8 : -2; -3) - (3; 3; -3; -6) = (2; 5; 1; 3)$$

$$v_3 = (3; 9; 3; 8) - \frac{3 \cdot 1 + 9 \cdot 1 + 3 \cdot (-1) + 8(-2)}{7} \cdot (1; 1; -1; -2) -$$

$$- \frac{3 \cdot 2 + 9 \cdot 5 + 3 \cdot 1 + 8 \cdot 3}{2^2 + 5^2 + 1^2 + 3^2} \cdot (2; 5; 1; 3) = (3; 9; 3; 8) - \frac{-7}{7} \cdot (1; 1; -1; -2) - \frac{78}{39}(2; 5; 1; 3) =$$

$$= (3; 9; 3; 8) + (1; 1; -1; -2) - (4; 10; 2; 6) = (0; 0; 0; 0)$$

Ortogonāli operatori

<u>**Def.**</u> Ja E ir Eiklīda telpa, $A \in End(E)$ $?End(E)$, tad saka, ka A ir ortogonāls operators $\Leftrightarrow \forall_{x,y \in E} ((x,y) = (A(x), A(y)))$

Īpašības:

A ir ortogonāls $\Leftrightarrow \forall_{x,y \in E} (|x| = |A(x)|)$
$\Leftrightarrow$ A ortonormētu bāzi attēlo par ortonormētu bāzi

$e_1, \dots, e_n$ - ortonormēta bāze $\quad x = x_1 e_1 + \dots + x_n e_n$
$A(e_1), \dots A(e_n)$ - ortonormēta bāze $\quad A(x) = x_1 A(e_1) + \dots + x_n A(e_n)$
$(x,y) = x_1 y_1 + \dots + x_n y_n = (A(x), A(y))_{A(e)}$

A ir ortogonāls $\Rightarrow \forall_{x,y \in E} (\ \angle(x,y) = \angle(A(x), A(y)\ )) \Rightarrow$
$\Rightarrow \exists_\alpha (\alpha \cdot A$ ir ortogonāls $)$what is α?

<u>**Def.**</u> Ortogonāla operatora matricu ortonormētā bāzē sauc par **ortogonālu matricu**.

Īpašības: A ir ortogonāla

$\Leftrightarrow A \cdot A^\intercal = E$
$\Leftrightarrow A^\intercal \cdot A = E$
$\Leftrightarrow A^{-1} = A^\intercal$
$\Leftrightarrow$ A rindiņas veido ortonormētu bāzi
$\Leftrightarrow$ A stabiņi veido ortonormētu bāzi

(1)