Algorithm Engineering: Presentation 1

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Our approach

→ Programming language: Java

→ Standard Java, Streams, no external libraries

→ Python for creating plots

→ Object-oriented implementation:

  → Every node is an Object

  → Each node holds the information about it's outgoing neighbors

```
class Node
{
    String label;
    List<Node> outNeighbors;
}
```

Our approach

- → Solver class that executes the main algorithm
- → One class for each algorithm:
    - → Is the graph a DAG?
    - → Find first cycle
    - → Preprocessing
- → Log class for printing the result and debug information
- → All of these classes offer static methods

Our approach

→ Nodes are not actually deleted, only labeled:

```
for(Node node: cycle)
{
	node.delete();
	List<Node> S = dfvsBranch(graph, k  - 1);
	node.unDelete();
	if(S != null)
	{
		S.add(node);
		return S;
	}
}
```

Finding the next cycle

→ Algorithm traverses the graph recursively

→ Visited nodes get marked with an index

→ If a new visited node is already marked, a cycle is found

→ Running time is $O(|V|^2)$ in the worst case, when the graph is an acyclic line

→ Can easily be improved to $O(|V|)$, so that each node is visited only once

Finding the next cycle

→ findFirstCycle():

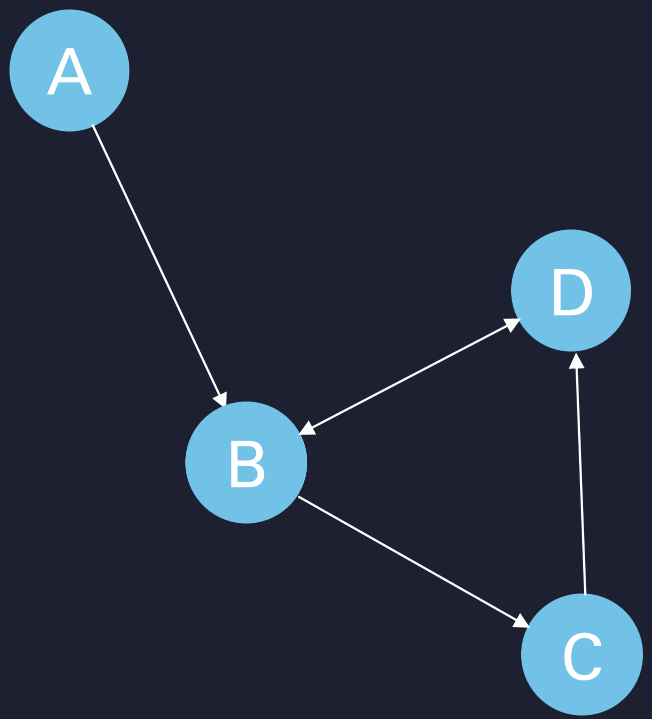
```
int index = 0;
for(Node node: nodes)
{
	List<Node> cycle = visitNode(node);
	if(cycle != null)
	{
		return cycle;
	}
}
```

Finding the next cycle

→ visitNode():

```
if(node.visitIndex !=  -1)
{
        Node[] cycle = [];
        cycle.add(node);
        cycleStartIndex = node.visitIndex;
        return cycle;
} else {
        node.visitIndex = index;
        index++;
        for(Node neighbor: outNeighbors)
        {
                List<Node> cycle = visitNode(neighbor);
                if(cycle)
                {
                        if(node.visitIndex > cycleStartIndex) cycle.add(node);
                        return cycle;
                }
        }
        return null;
}
```

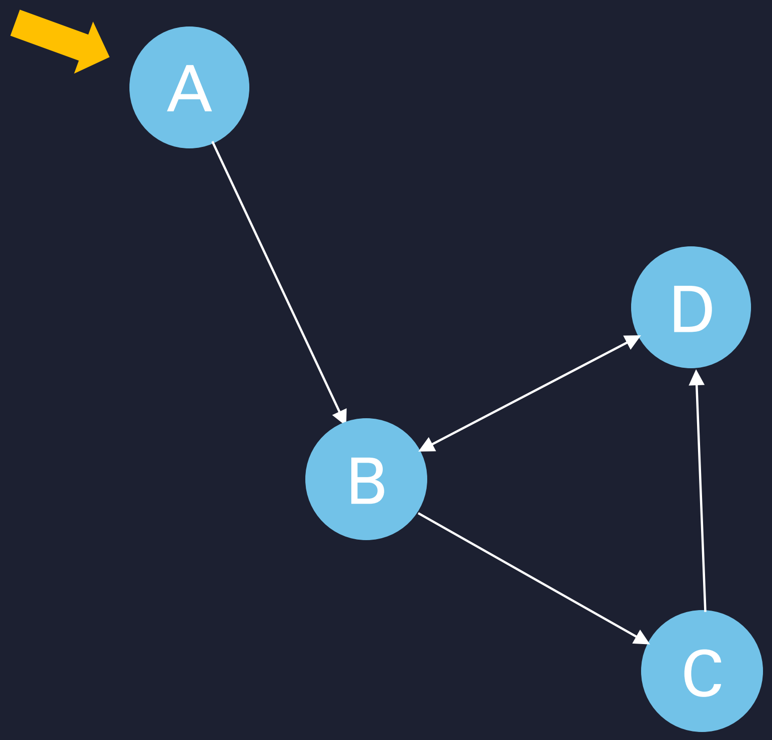
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

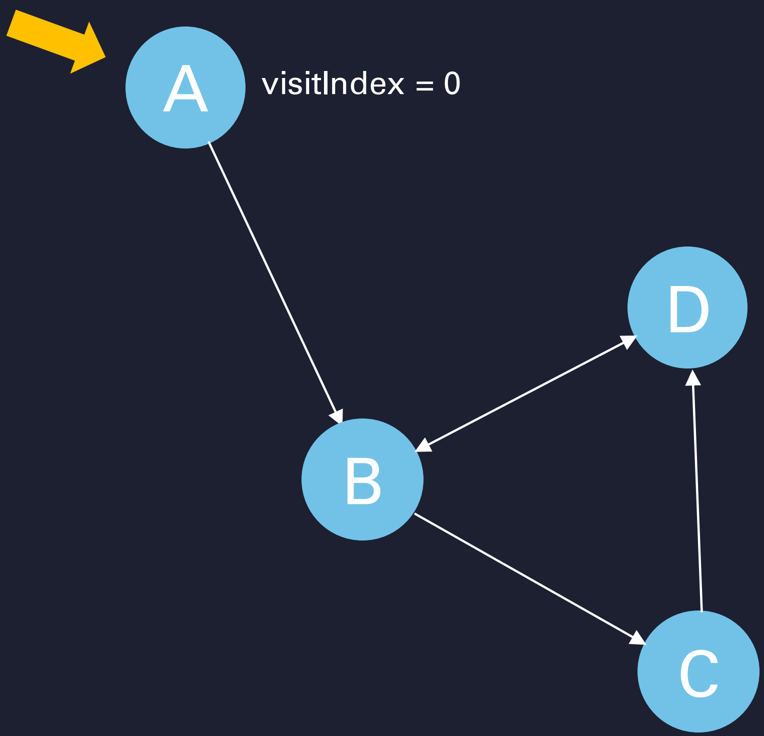
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

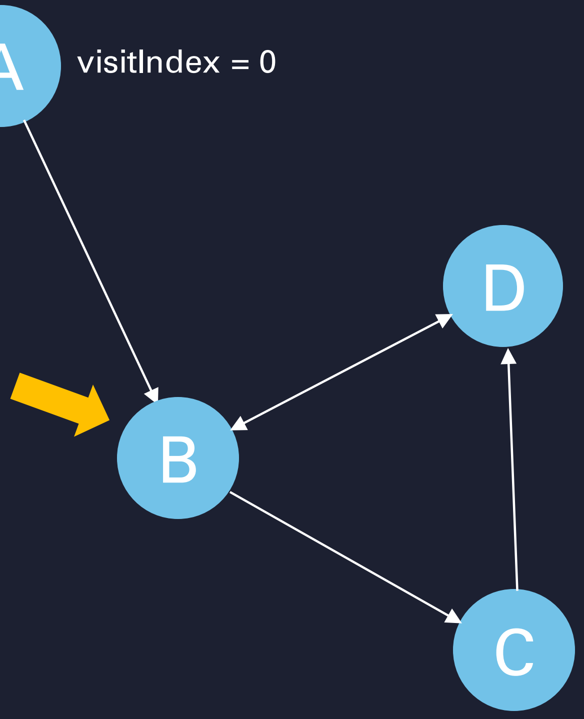
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

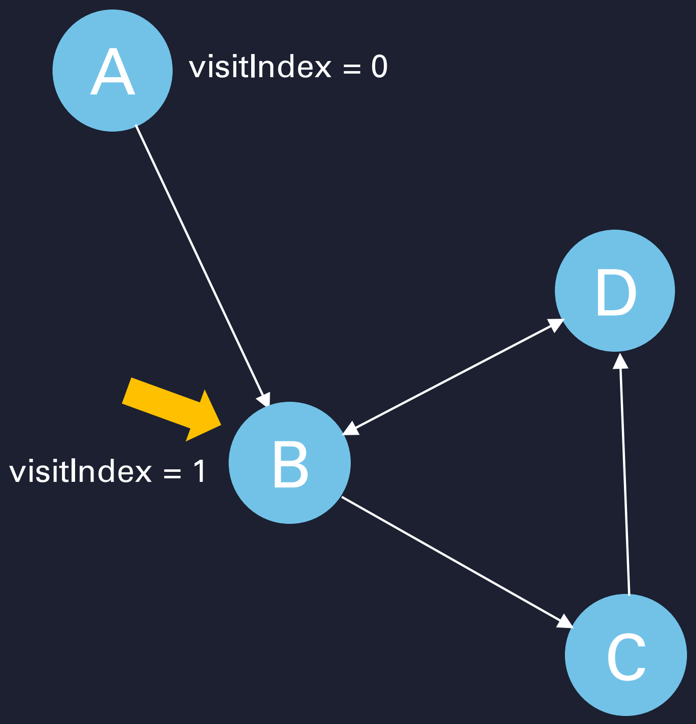
Finding the next cycle



cycleStartIndex: -1

cycle: Ø

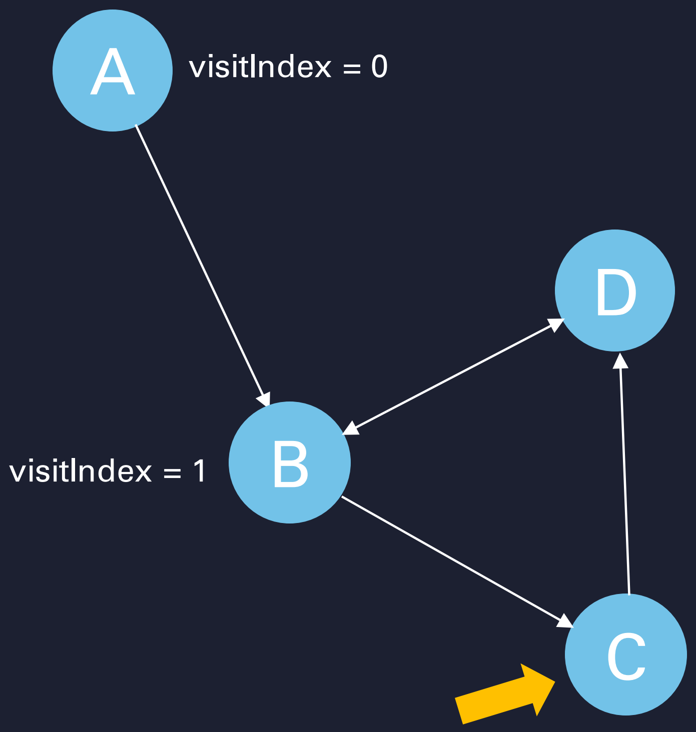
Finding the next cycle



cycleStartIndex: -1

cycle: Ø

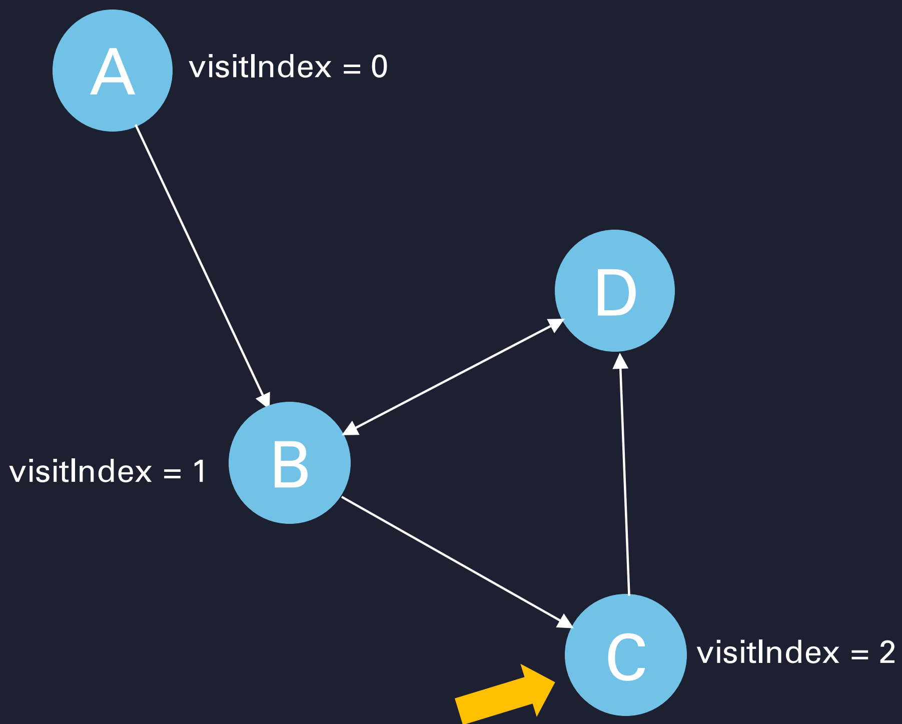
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

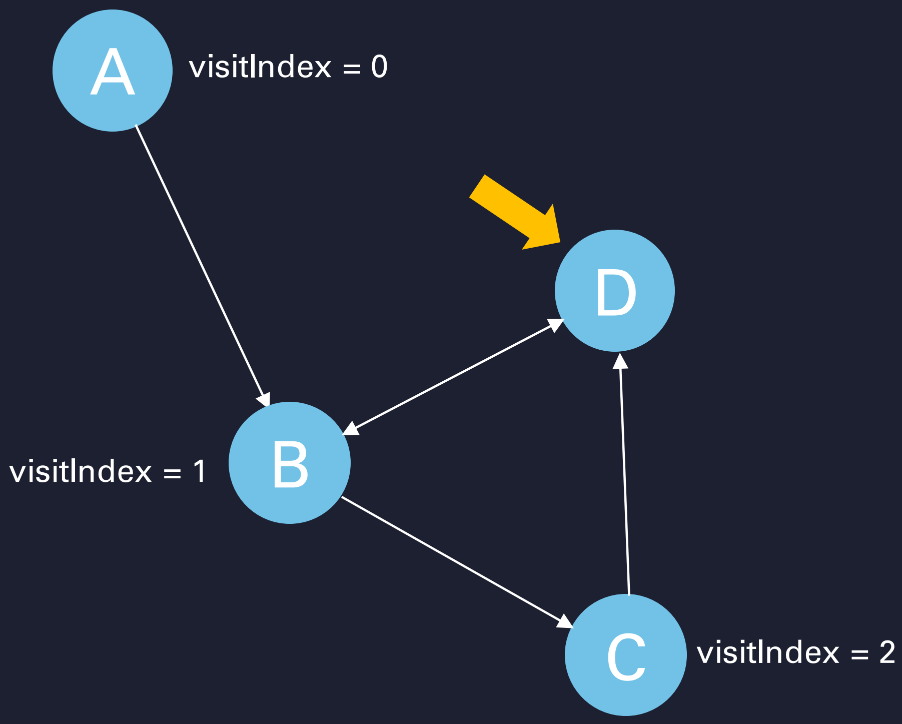
Finding the next cycle



cycleStartIndex: -1

cycle: Ø

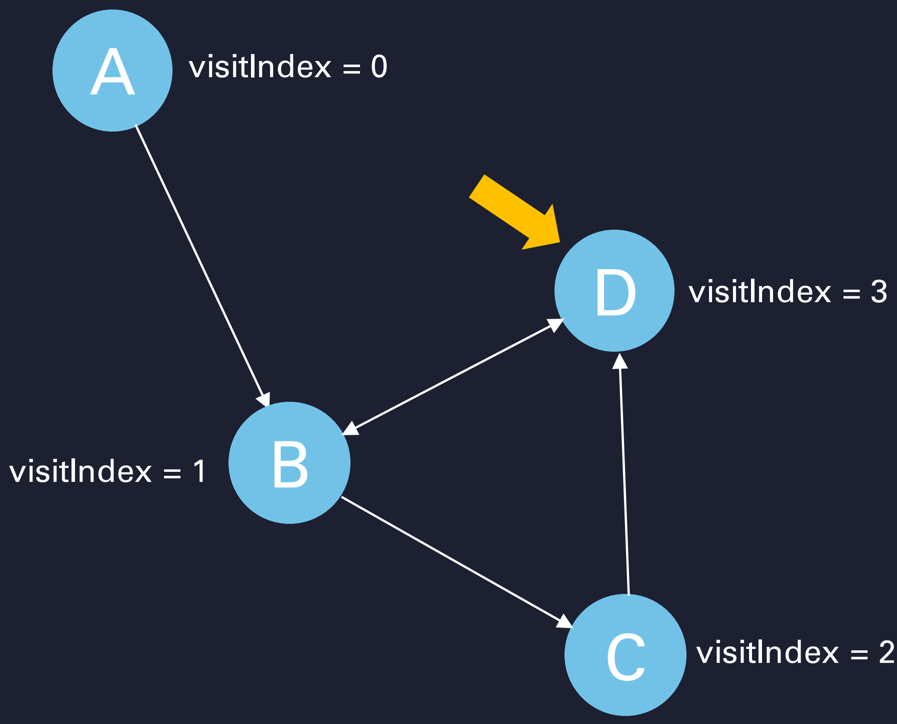
Finding the next cycle



cycleStartIndex: -1

cycle: Ø

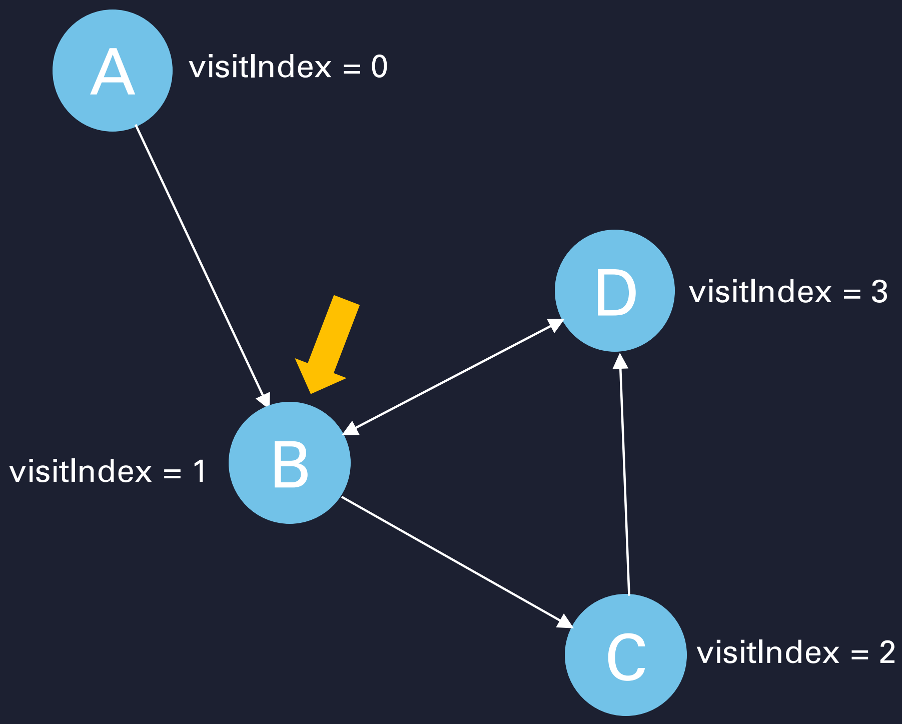
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

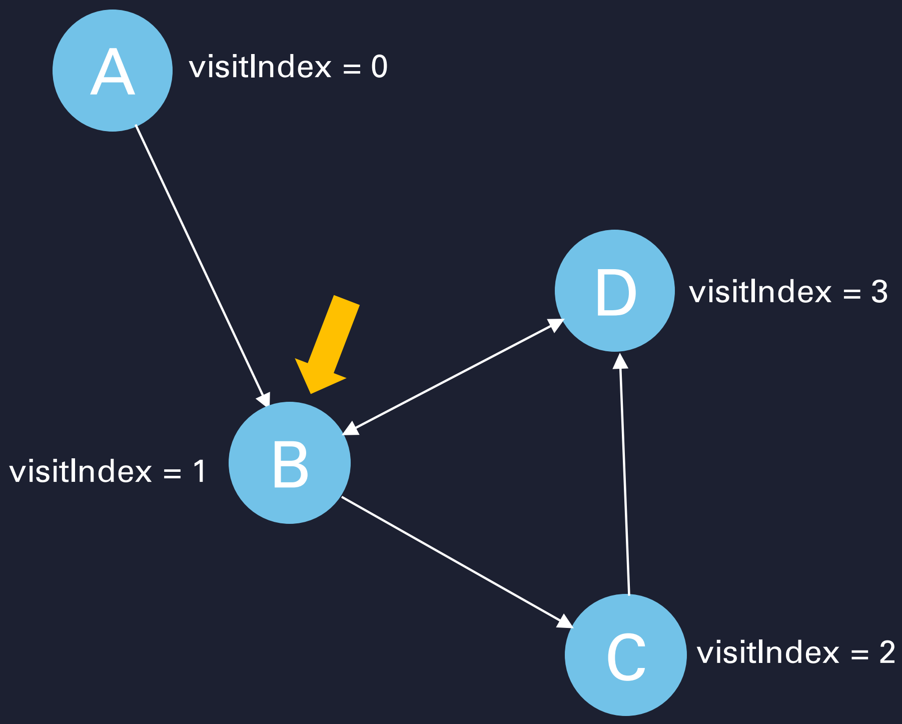
Finding the next cycle



cycleStartIndex: -1

cycle: ∅

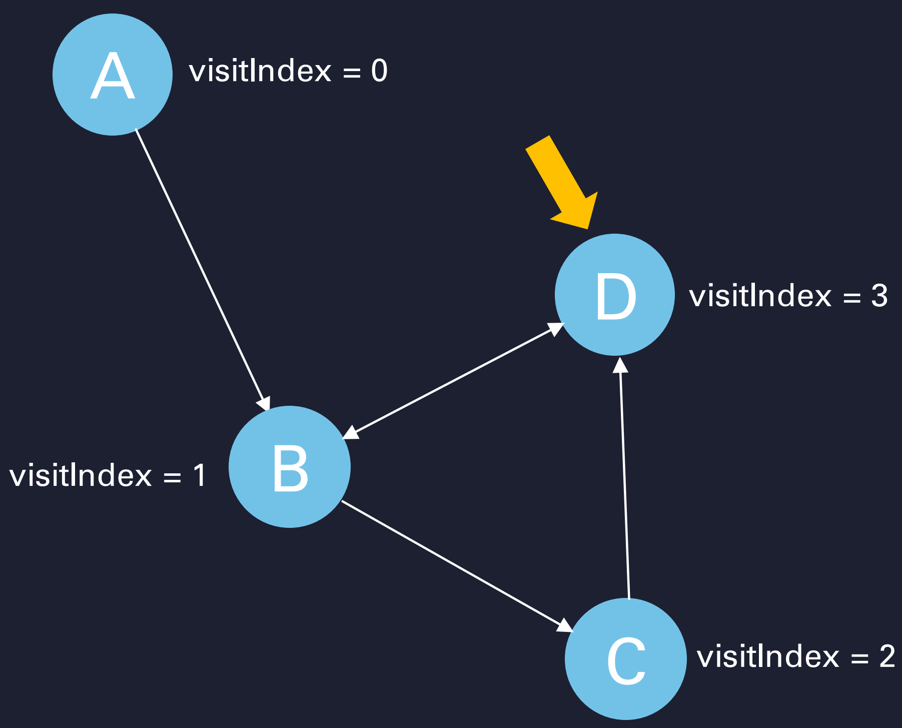
Finding the next cycle



cycleStartIndex: 1

cycle: B

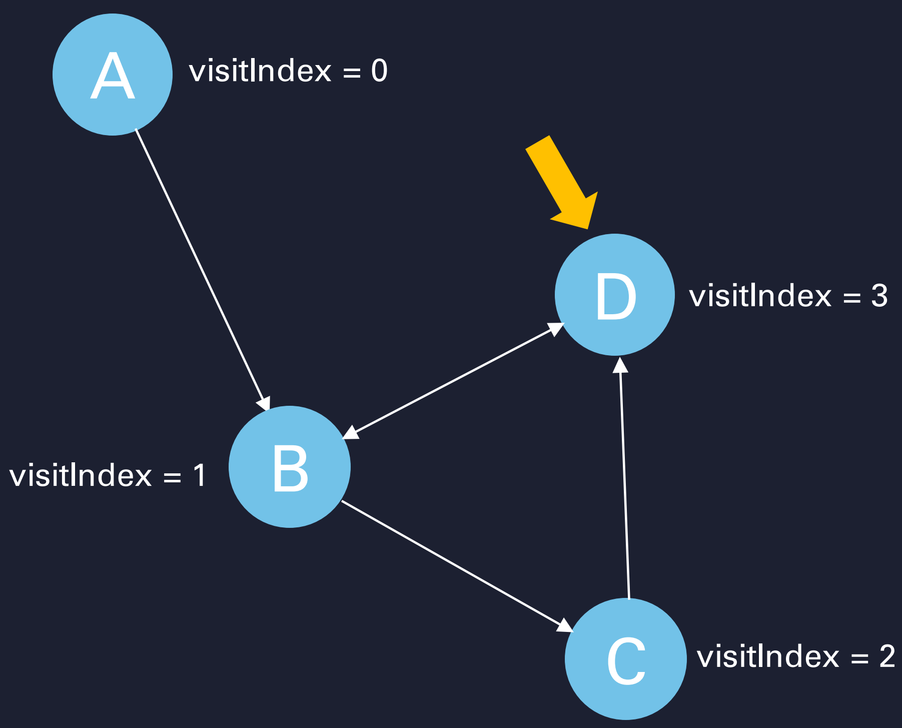
Finding the next cycle



cycleStartIndex: 1

cycle: B

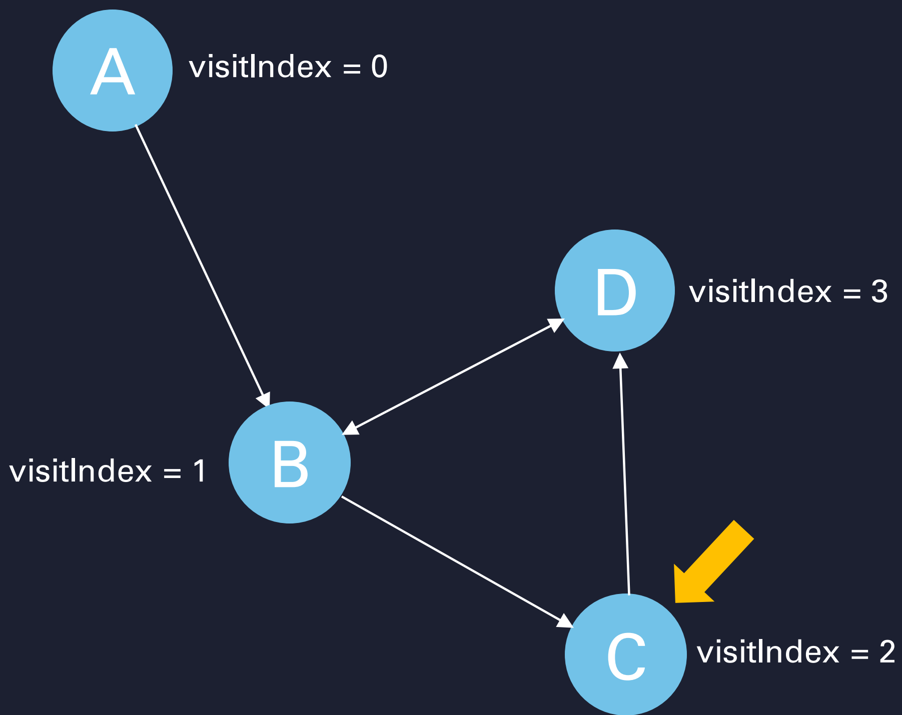
Finding the next cycle



cycleStartIndex: 1

cycle: B, D

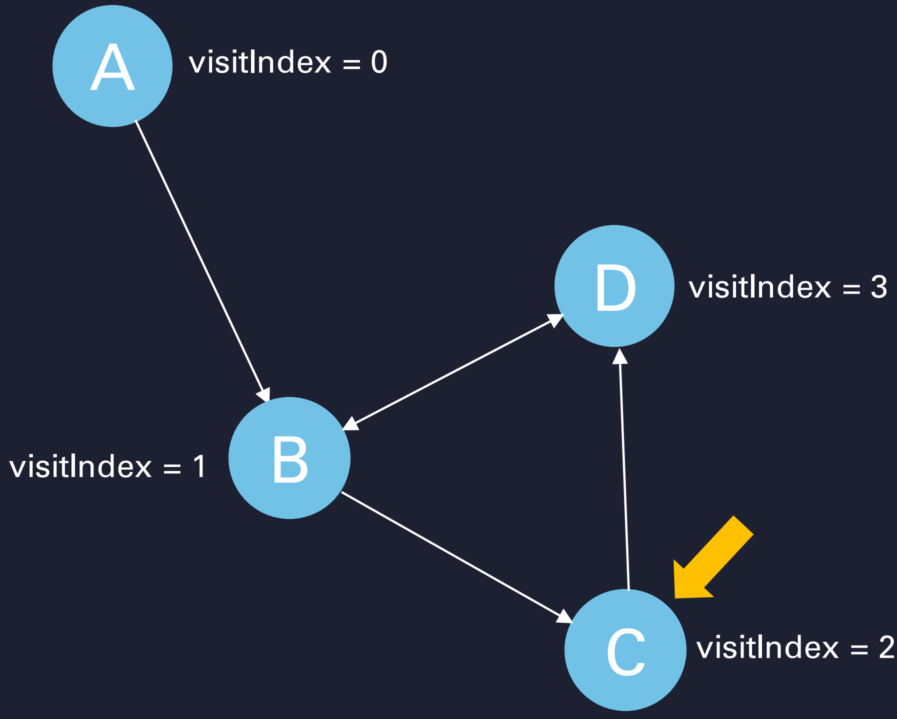
Finding the next cycle



cycleStartIndex: 1

cycle: B, D

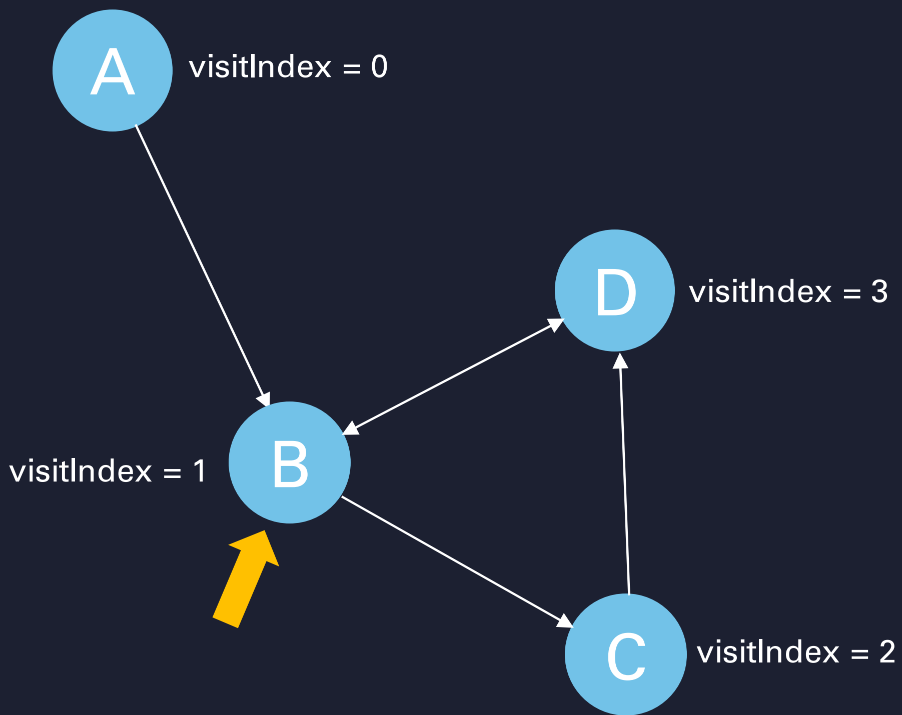
Finding the next cycle



cycleStartIndex: 1

cycle: B, D, C

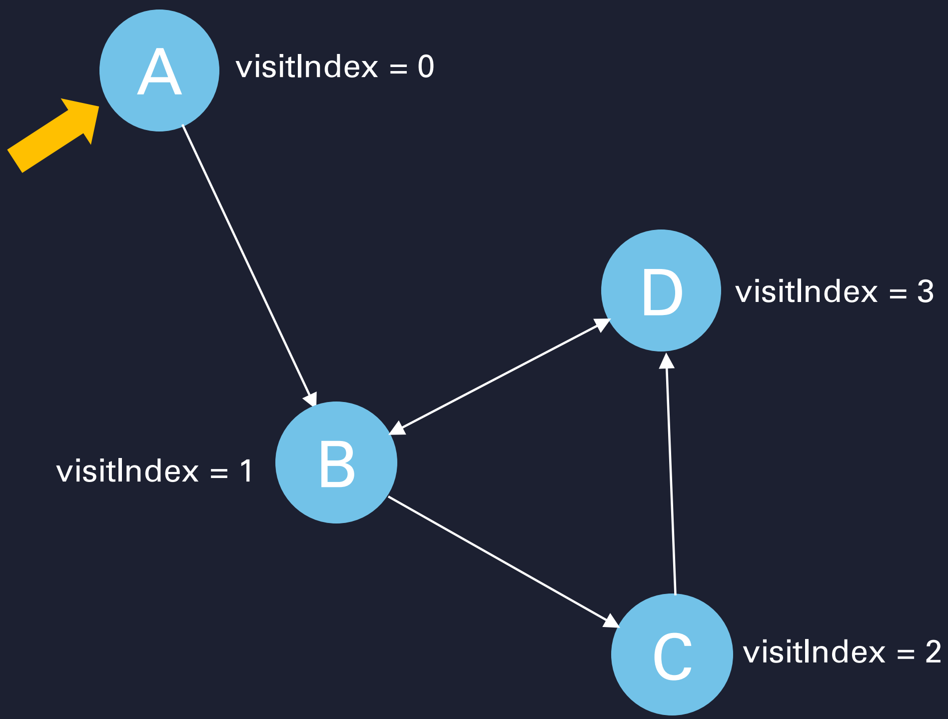
Finding the next cycle



cycleStartIndex: 1

cycle: B, D, C

Finding the next cycle



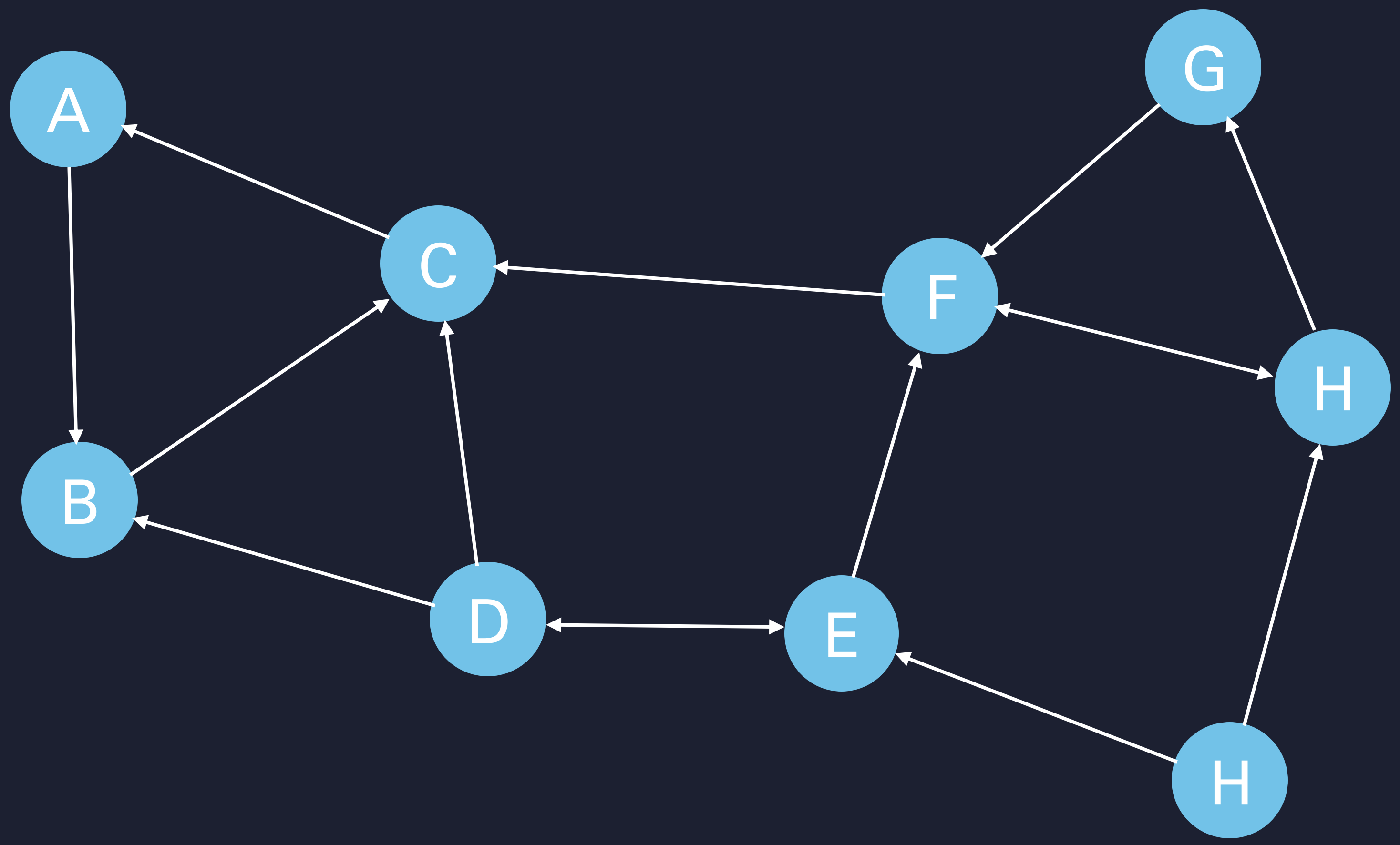
cycleStartIndex: 1

cycle: B, D, C

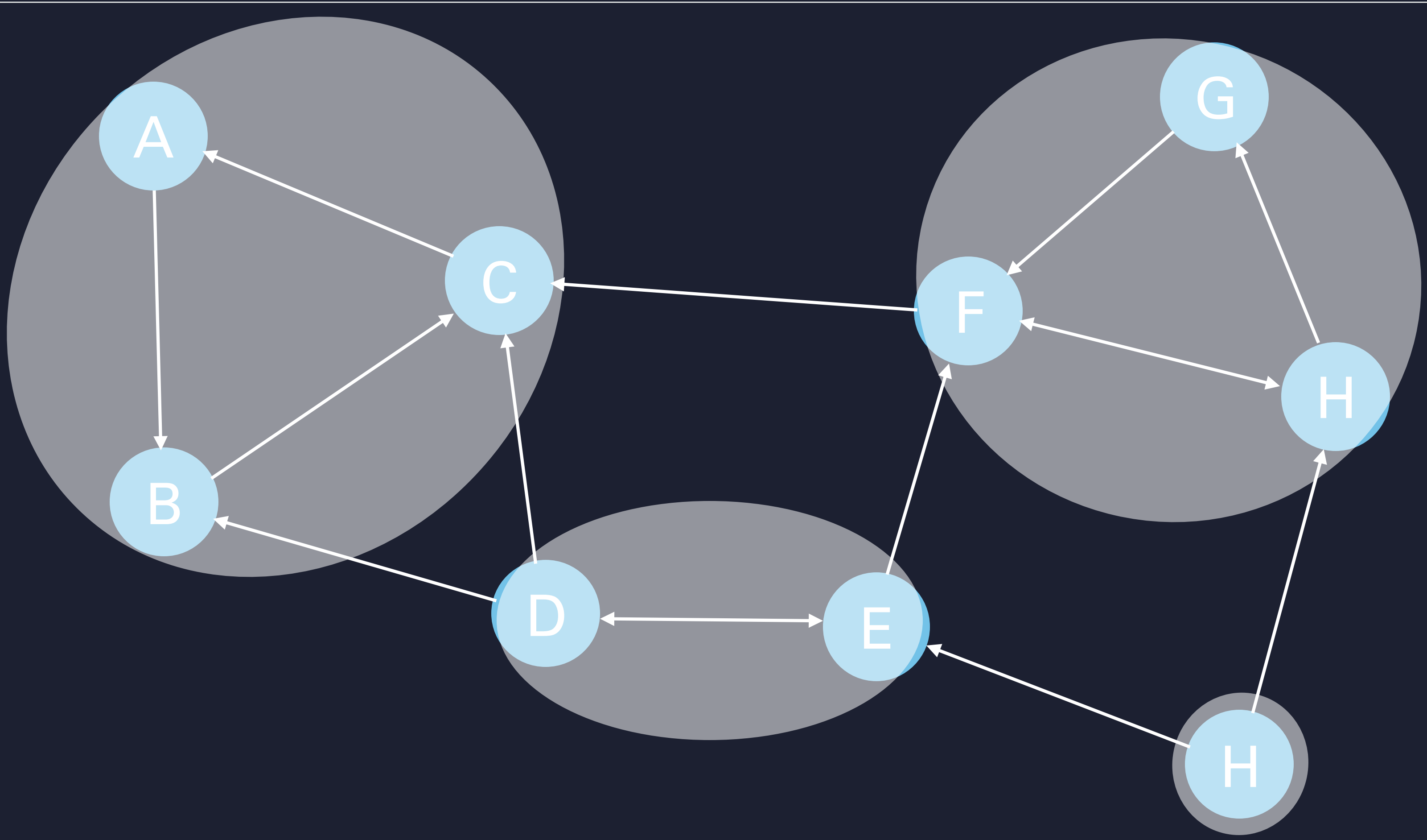
Preprocessing with Tarjan's Algorithm

- → Algorithm finds cyclic components in the graph
- → Cyclic component: all nodes, which have any cyclic connection
- → Linear running time: O(|V| + |E|)
- → The preprocessing allows us to split the graph into cyclic subgraphs

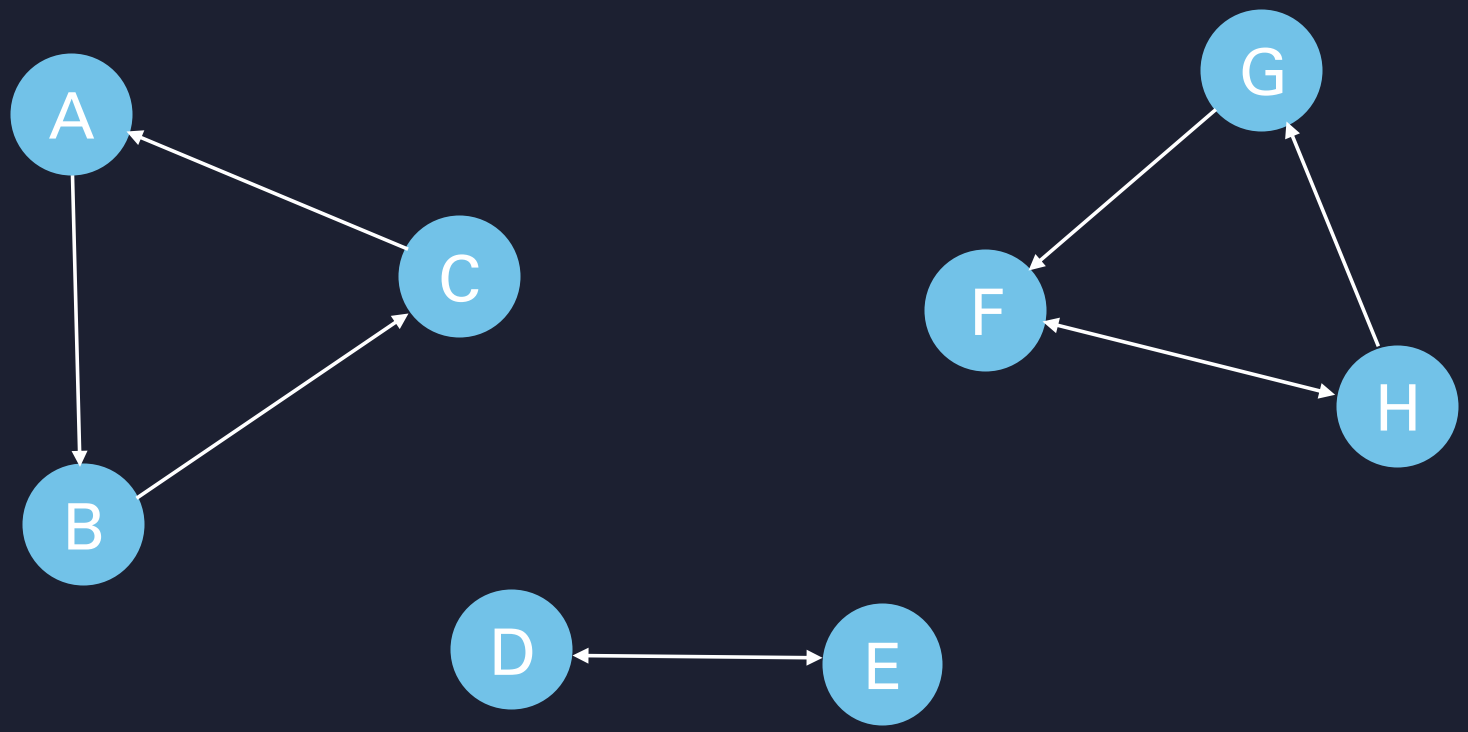
Preprocessing with Tarjan's Algorithm



Preprocessing with Tarjan's Algorithm



Preprocessing with Tarjan's Algorithm



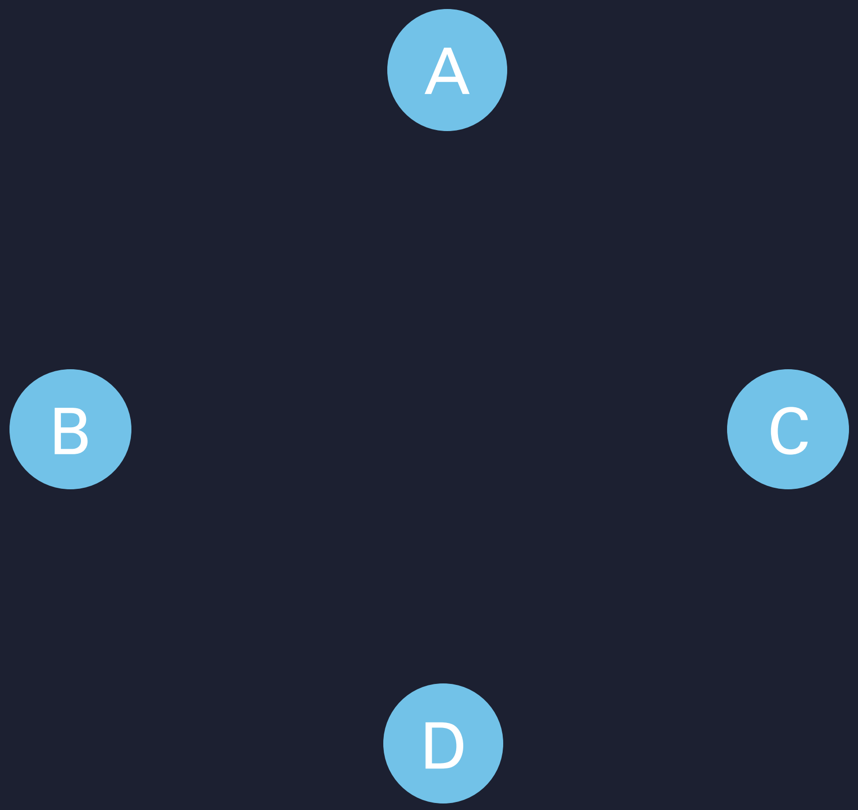
Calculation of min k

- What is the worst case graph?
- A fully connected graph
- In fact, a fully connected graph is not the worst case:

```
if(m == n * (n + 1))
{
        return k = n – 1;
}
```

- Removing a single edge (a,b) => k = n – 2
- Can we rule out even more cases with this approach?
- Idea for min k: add as many edges as possible to a graph without creating a cycle

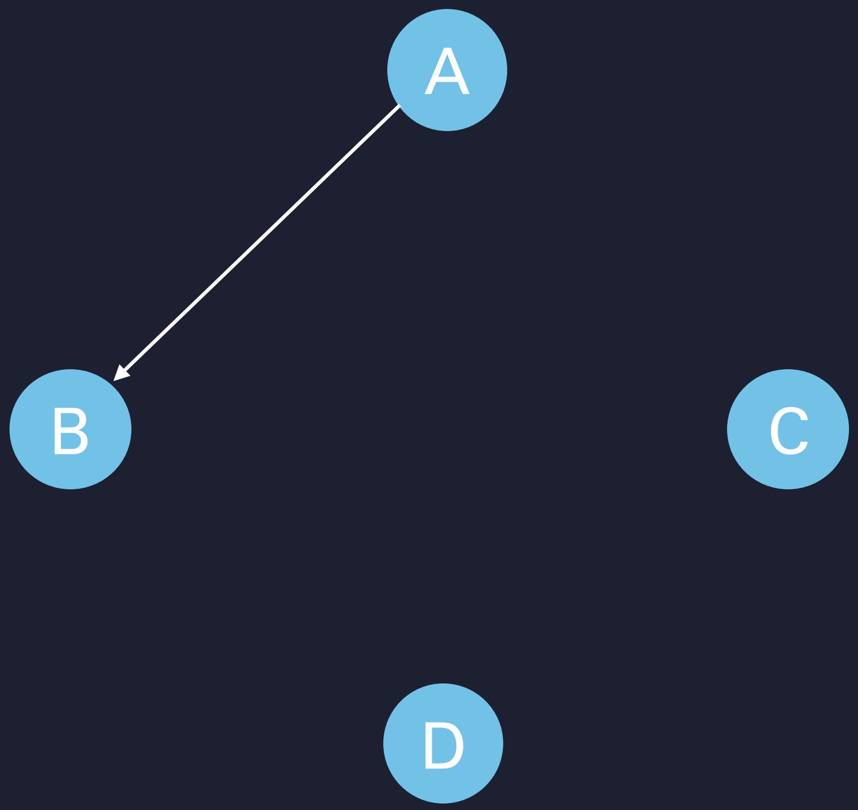
Calculation of min k



$|V| = 4$
$|E| = 0$
$k = 0$

|E|	k
0	0
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

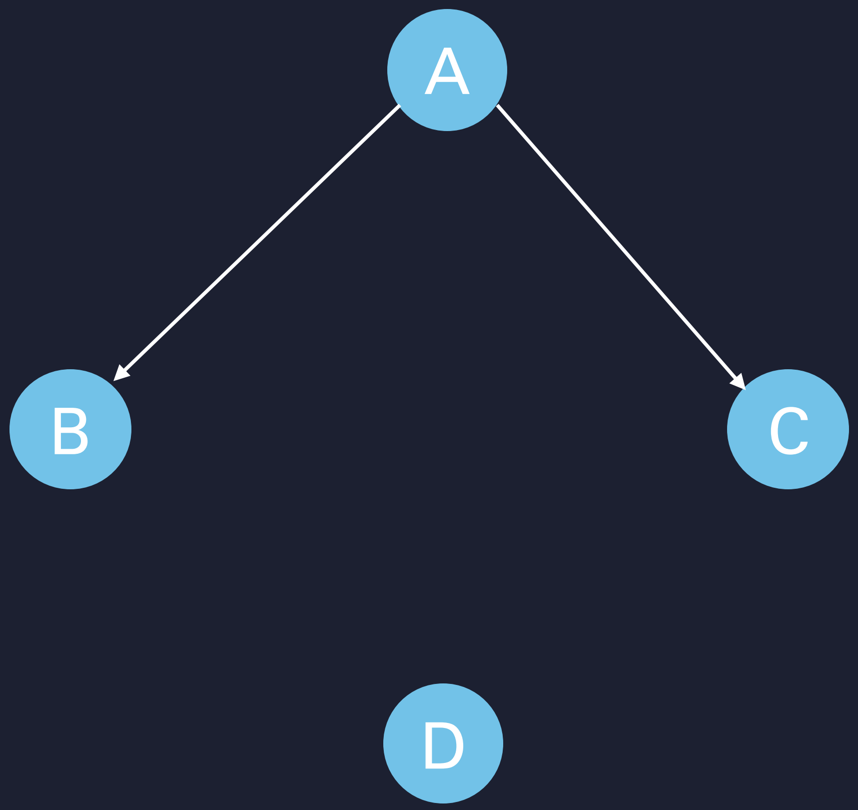
Calculation of min k



$|V| = 4$
$|E| = 1$
$k = 0$

|E|	k
0	0
1	0
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

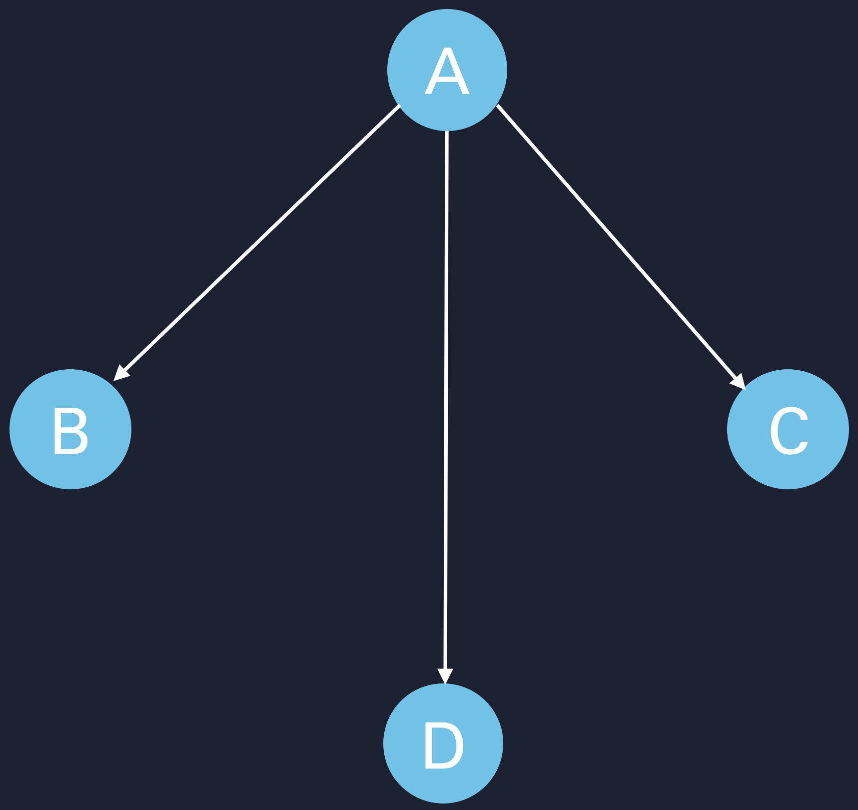
Calculation of min k



|V| = 4
|E| = 2
k = 0

|E|	k
0	0
1	0
2	0
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

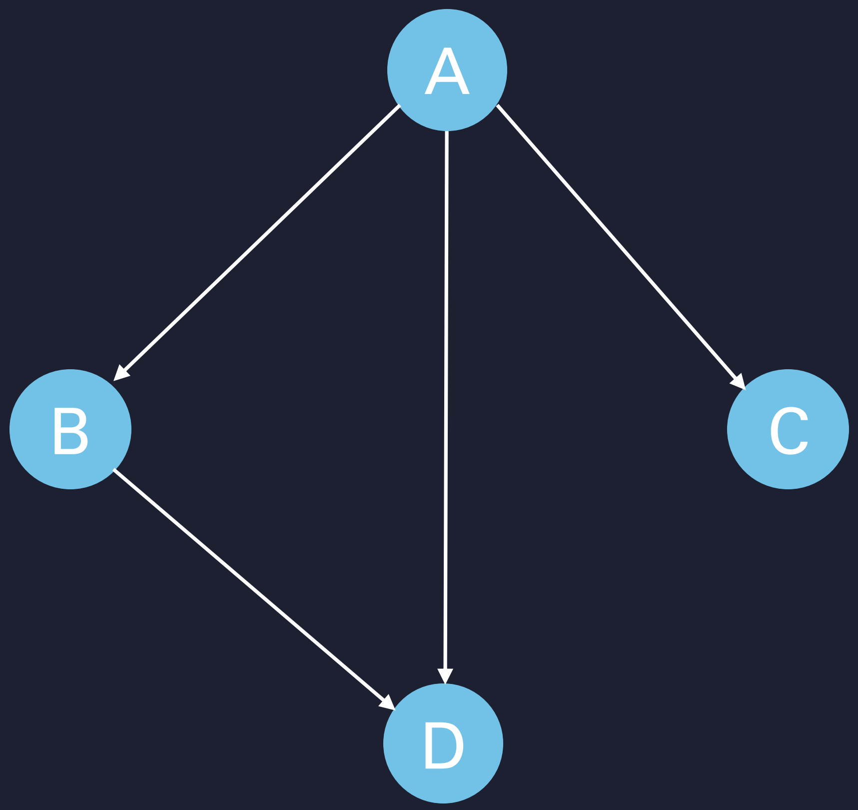
Calculation of min k



|V| = 4
|E| = 3
k = 0

|E|	k
0	0
1	0
2	0
3	0
4	
5	
6	
7	
8	
9	
10	
11	
12	

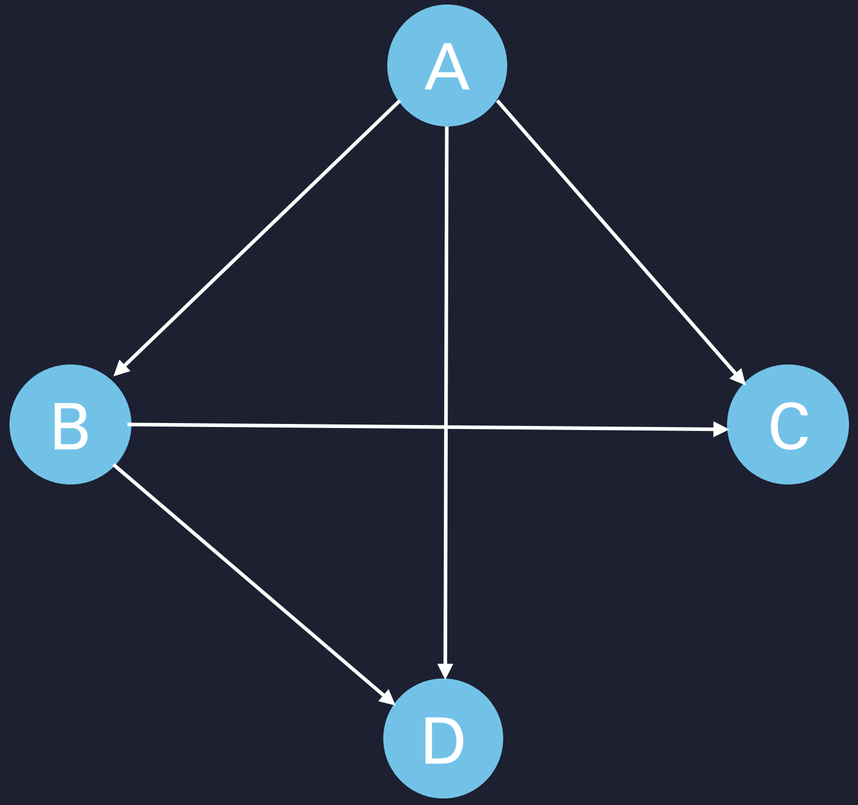
Calculation of min k



|V| = 4
|E| = 4
k = 0

|E|	k
0	0
1	0
2	0
3	0
4	0
5	
6	
7	
8	
9	
10	
11	
12	

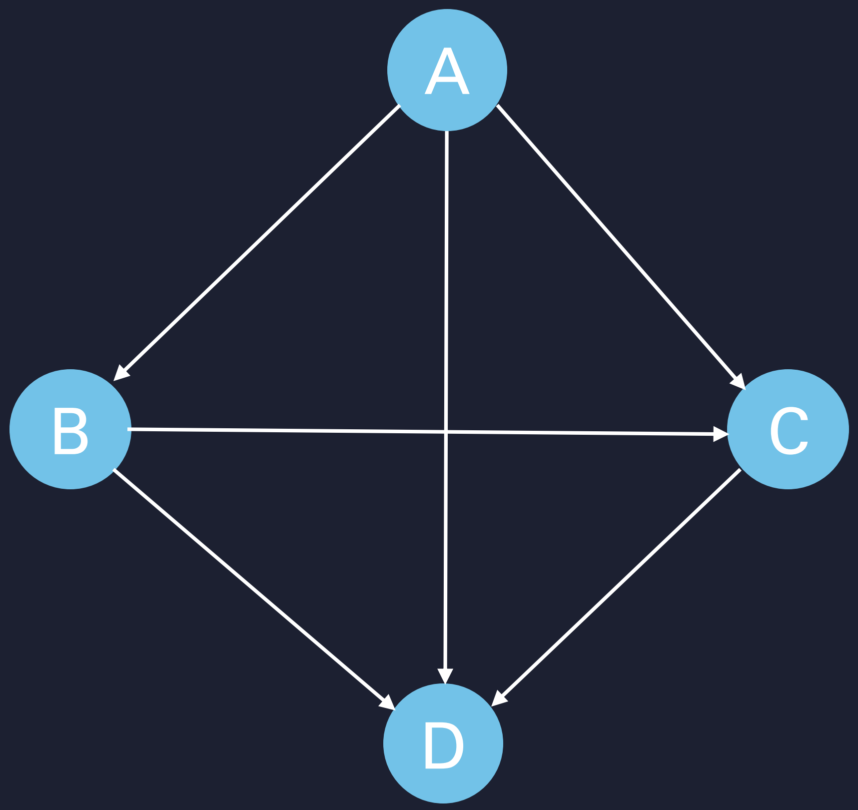
Calculation of min k



|V| = 4
|E| = 5
k = 0

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	
7	
8	
9	
10	
11	
12	

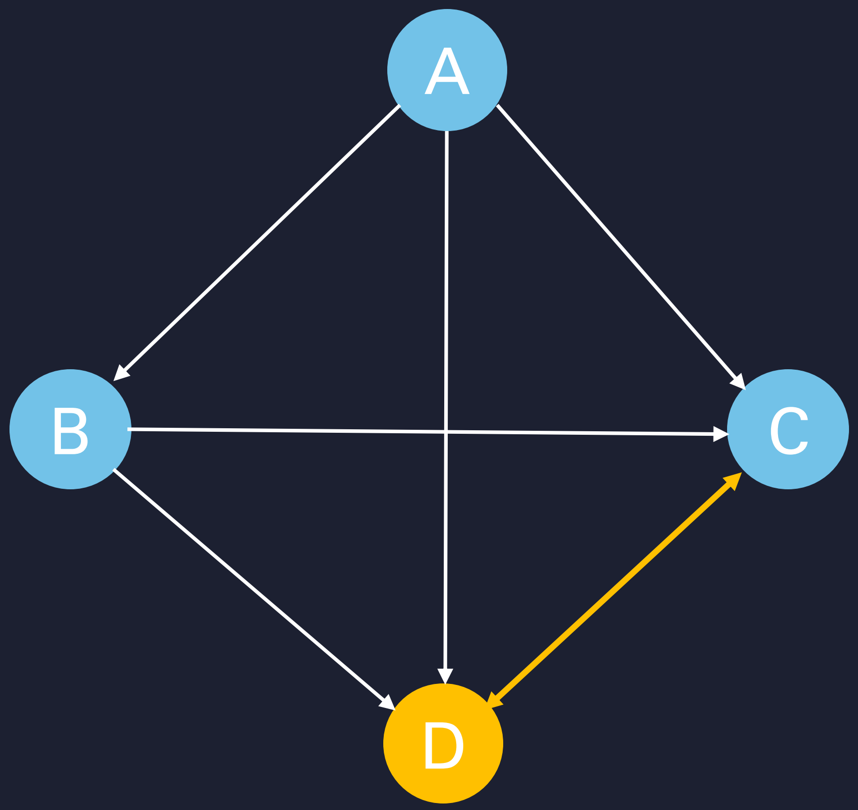
Calculation of min k



$|V| = 4$
$|E| = 6$
$k = 0$

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	
8	
9	
10	
11	
12	

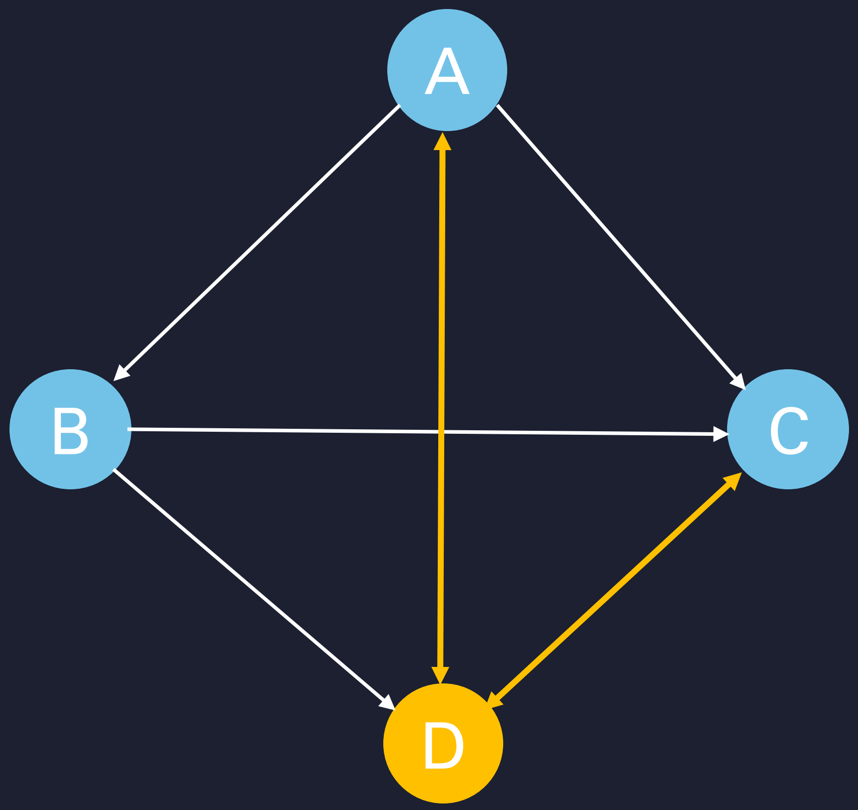
Calculation of min k



$|V| = 4$
$|E| = 7$
$k = 1$

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	
9	
10	
11	
12	

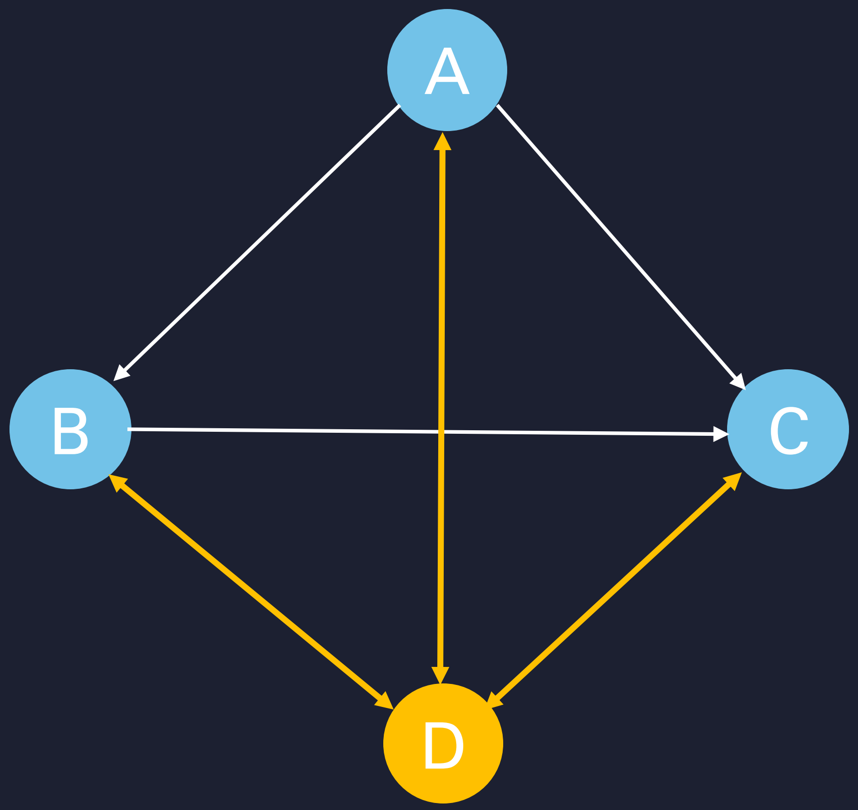
Calculation of min k



|V| = 4
|E| = 8
k = 1

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	
10	
11	
12	

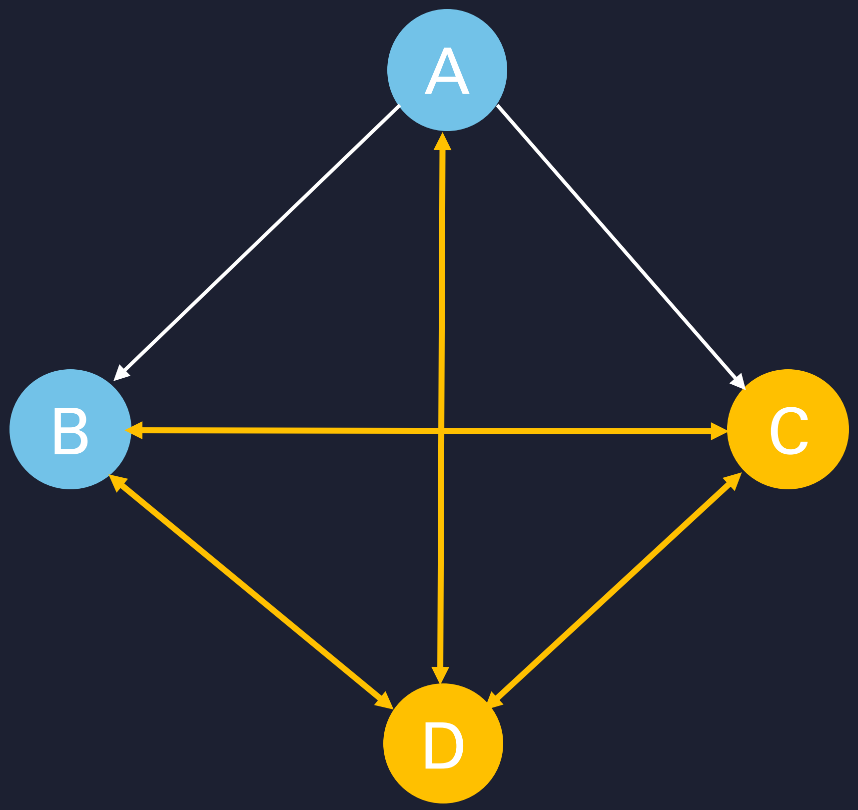
Calculation of min k



|V| = 4
|E| = 9
k = 1

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	
11	
12	

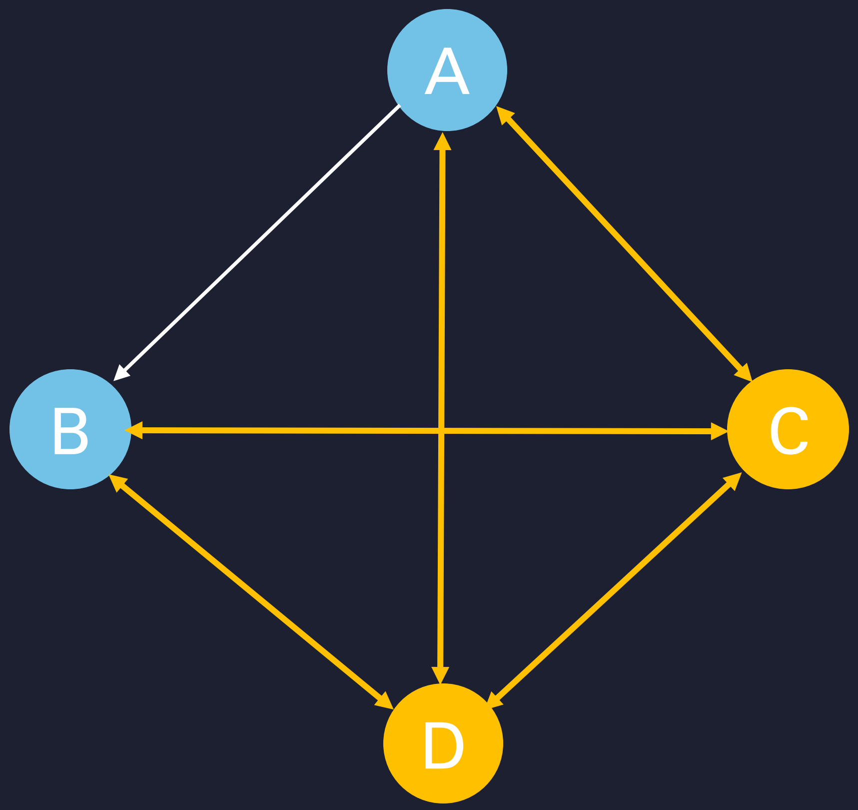
Calculation of min k



$|V| = 4$
$|E| = 10$
$k = 2$

$\|E\|$	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	
12	

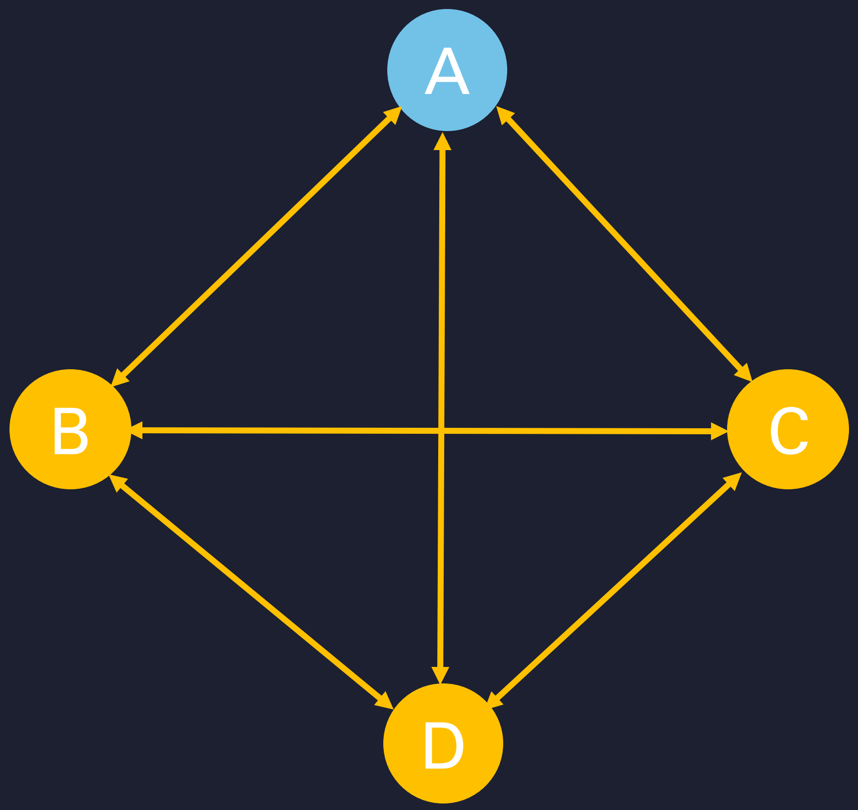
Calculation of min k



$|V| = 4$
$|E| = 11$
$k = 2$

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	

Calculation of min k



|V| = 4
|E| = 12
k = 3

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	3

Calculation of min k

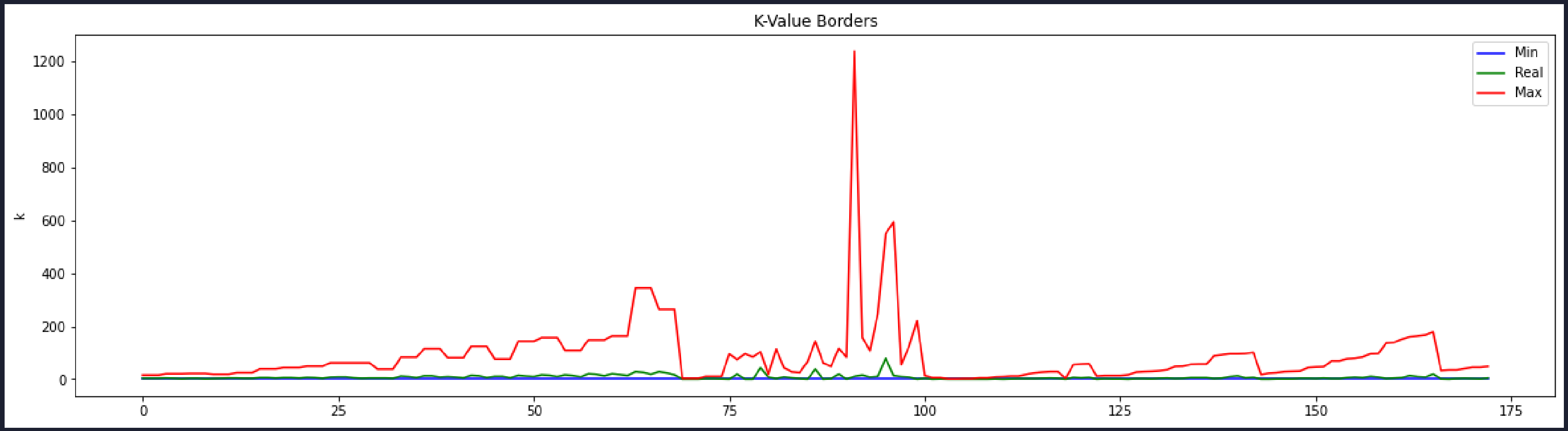
1. max m = n * (n – 1)
2. min k = 0 if m < n * (n – 1) / 2

|E|	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	3

Calculation of max k

→ The max value for k can be calculated:

max k = m / 2



Calculation of max k

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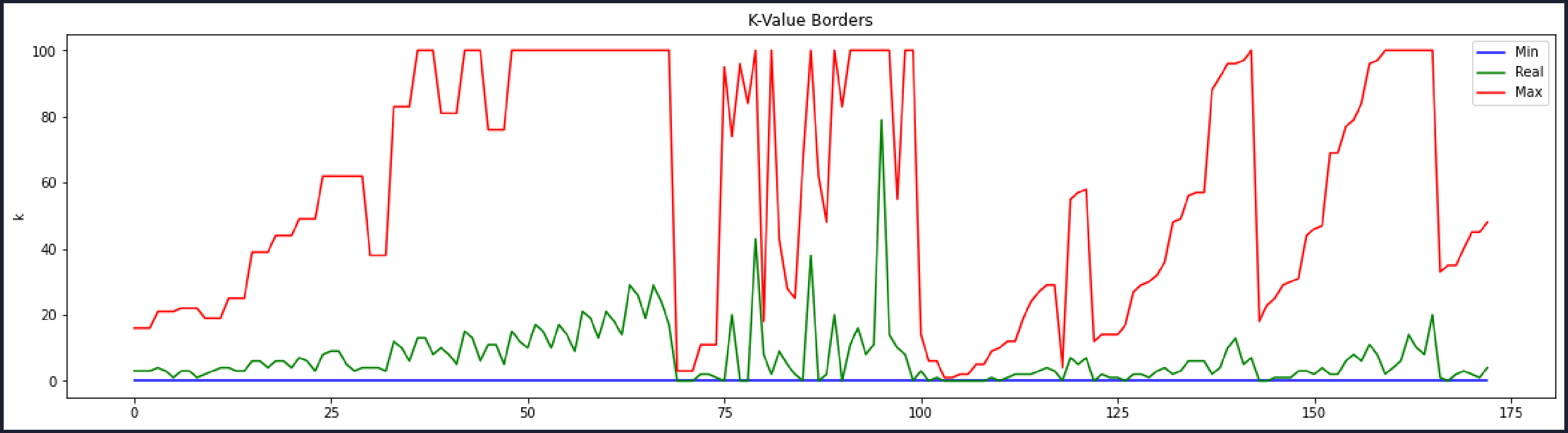
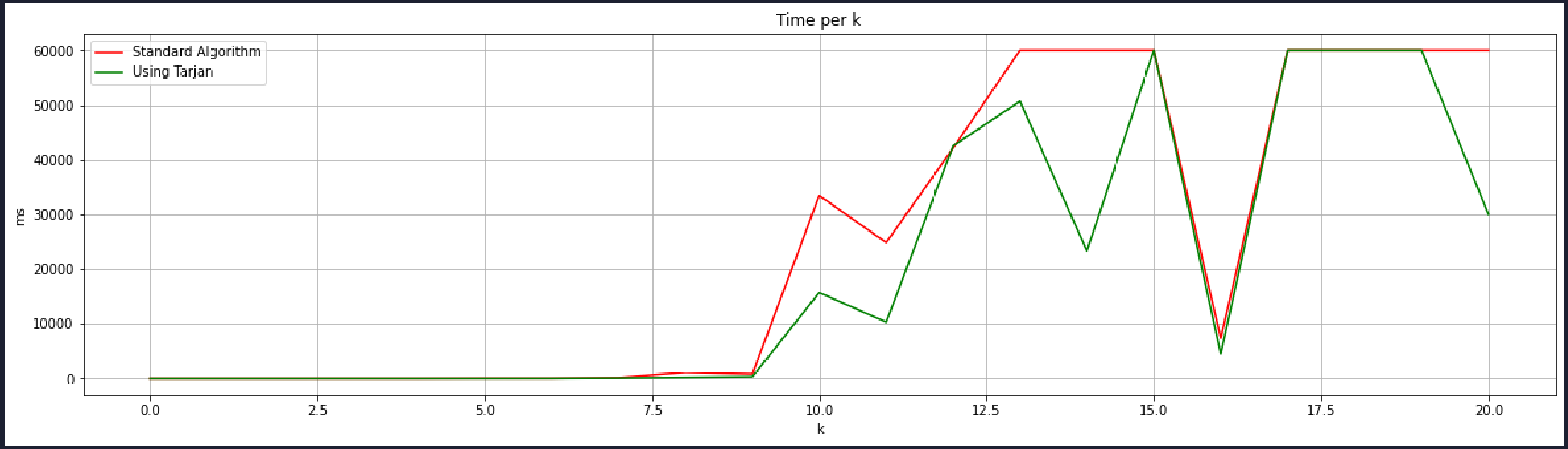
max k = m / 2

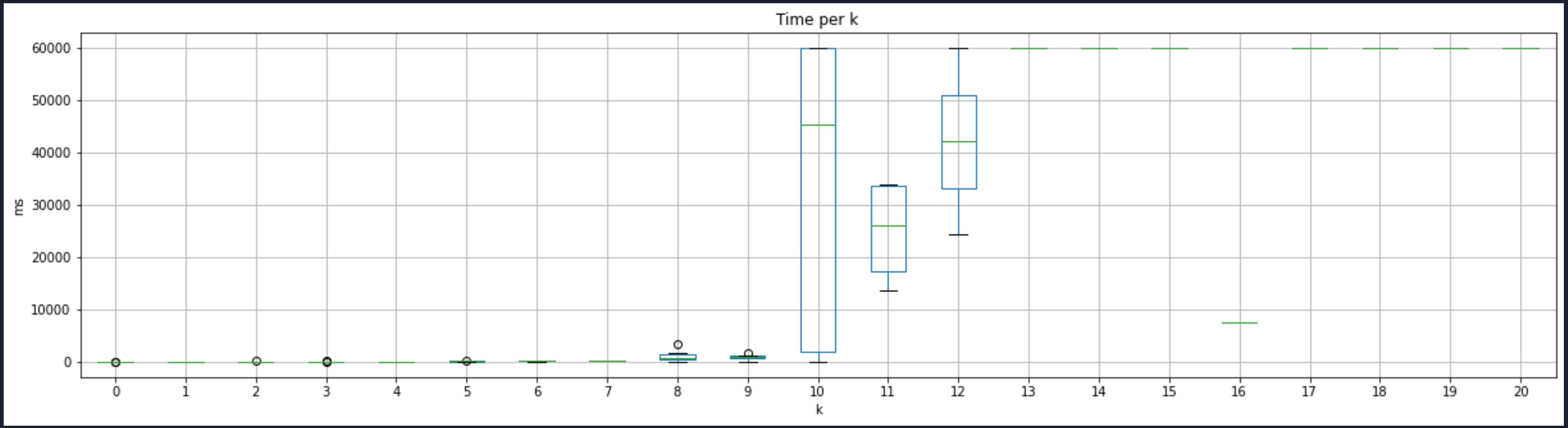


Performance - k

- We set a timeout after 60 seconds for the plots
- Graphs with k > 20 are excluded
- We only plotted the complex cases

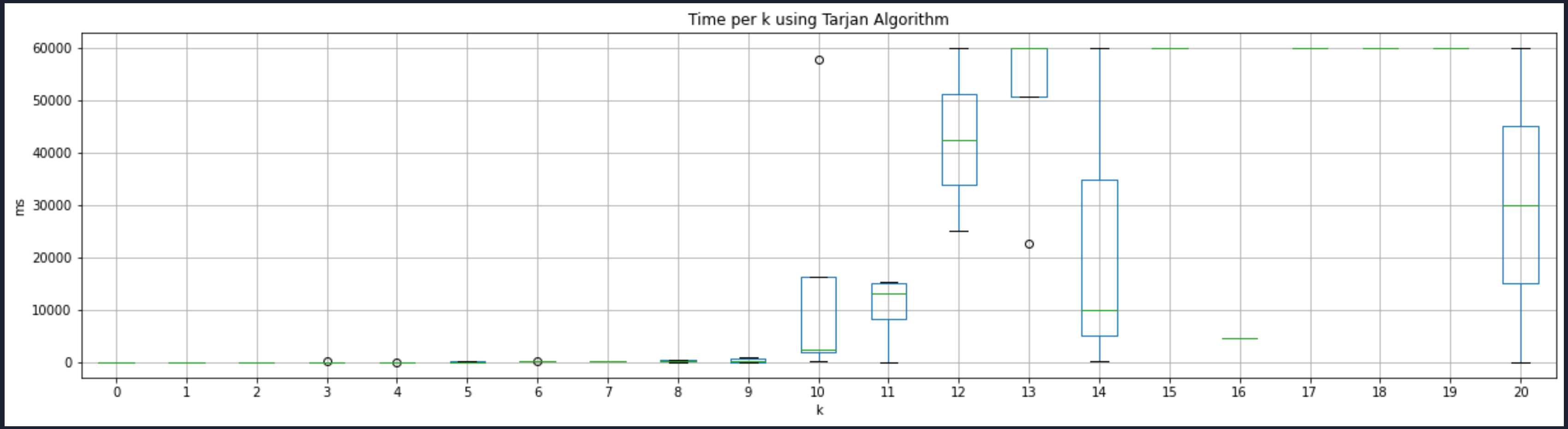


Performance - k



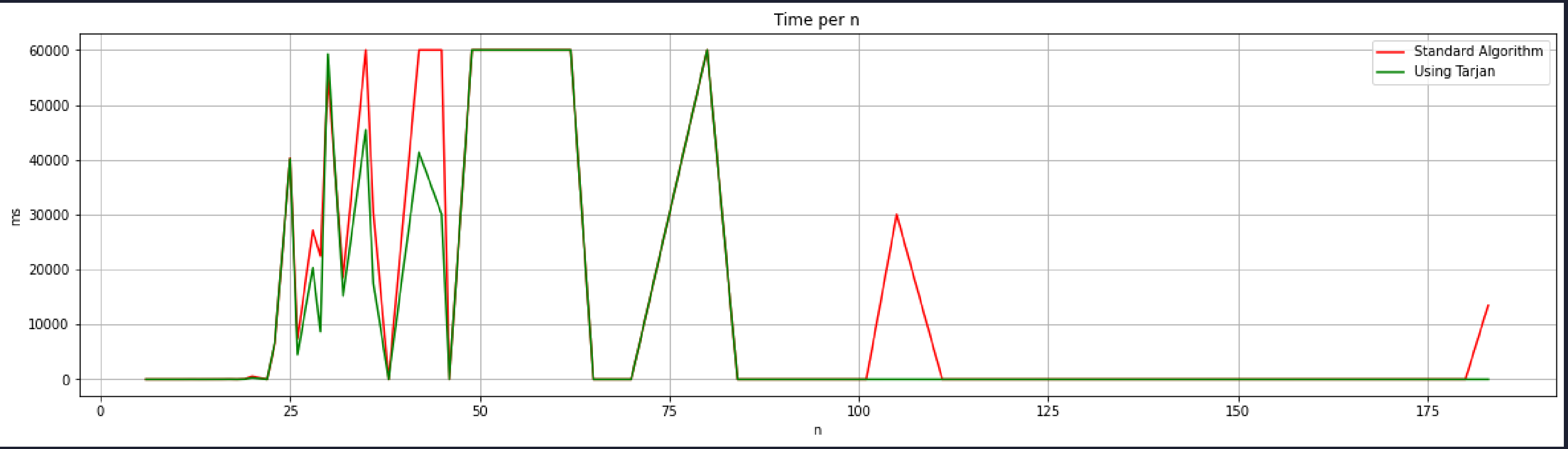
→ Looking at the optimal solution, the algorithm seems to struggle at k = 10 without using the algorithm of Tarjan

Performance - k



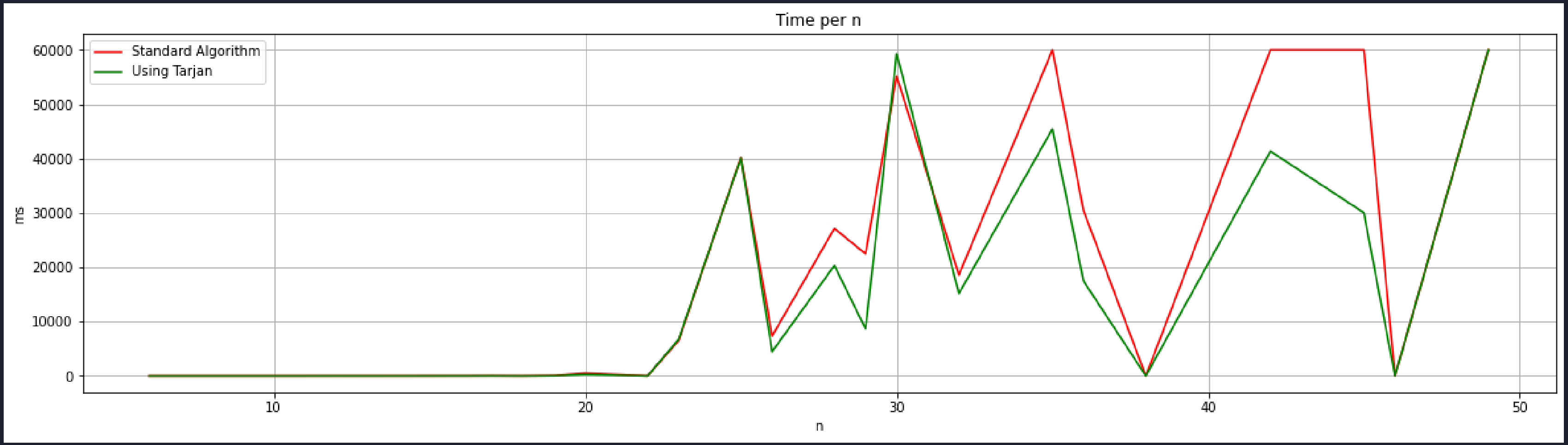
→ With Tarjan's algorithm we also managed to solve some k = 13 graphs

Performance - n



→ The runtime in general seems to increase with higher number of nodes, but there is some variation

Performance - n



→ The runtime in general seems to increase with higher number of nodes, but there is some variation

Do you have any questions?