

# *Algorithm Engineering: Presentation 1*

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# *Our approach*

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- Programming language: Java
- Standard Java, Streams, no external libraries
- Python for creating plots
- Object-oriented implementation:
  - Every node is an Object
  - Each node holds the information about it's outgoing neighbors

```
class Node
{
    String label;
    List<Node> outNeighbors;

    boolean deleted;
    int visitIndex;
}
```

# *Our approach*

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- Solver class that executes the main algorithm
- One class for each algorithm:
  - Preprocessing
  - Is the graph a DAG?
  - Find first cycle
  - Log class for printing the result and debug information
- All of these classes offer static methods

# *Our approach*

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→ Nodes are not actually deleted, only labeled:

```
for(Node node: cycle)
{
    node.delete();
    List<Node> S = dfvsBranch(graph, k - 1);
    node.unDelete();
    if(S != null)
    {
        S.add(node);
        return S;
    }
}
```

# *Finding the next cycle*

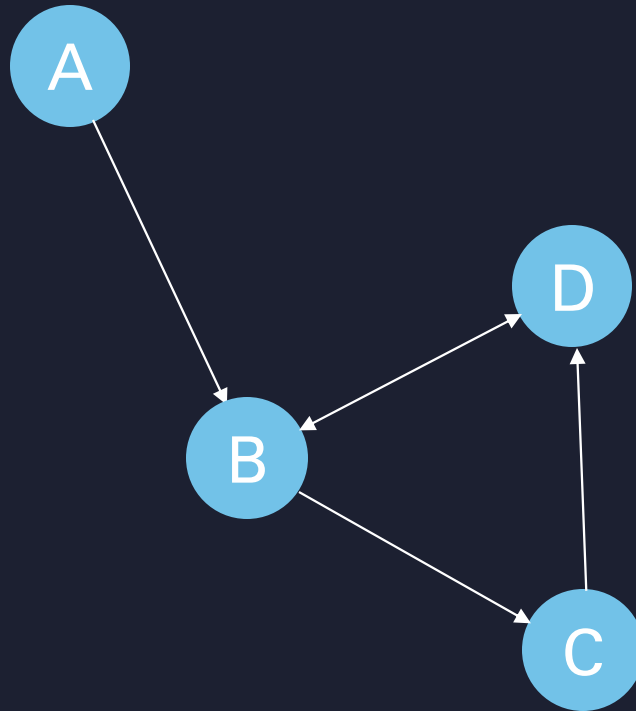
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- Algorithm traverses the graph recursively (DFS)
- Visited nodes get marked with an index
- If a new visited node is already marked, a cycle is found
- Running time is  $O(|V|)$  in the worst case, when each node is visited once
- Will be replaced by BFS in the future



# *Finding the next cycle*

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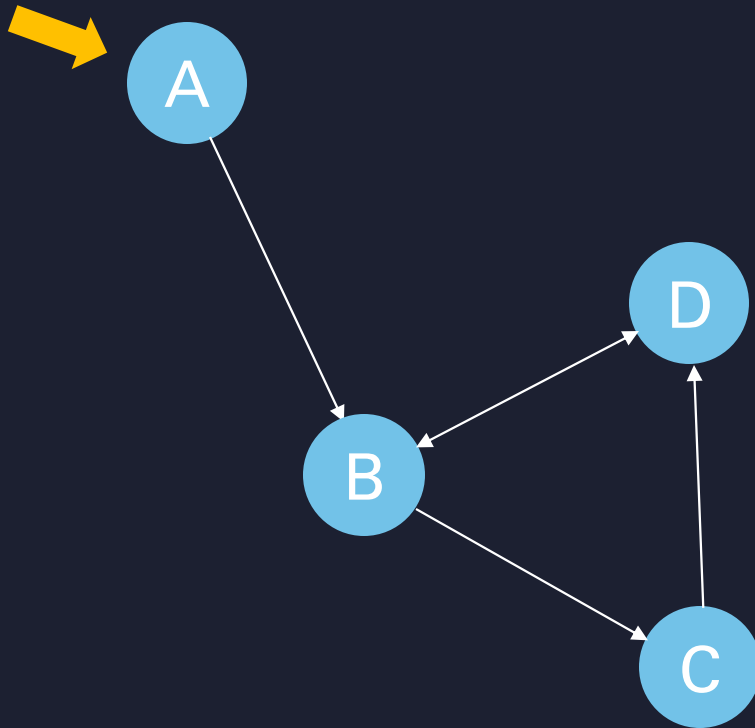


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

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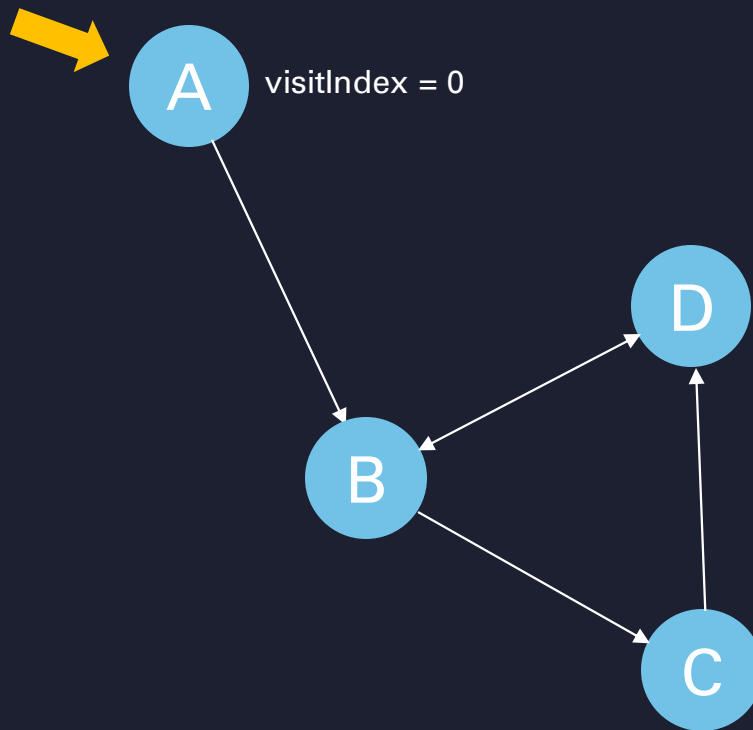


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

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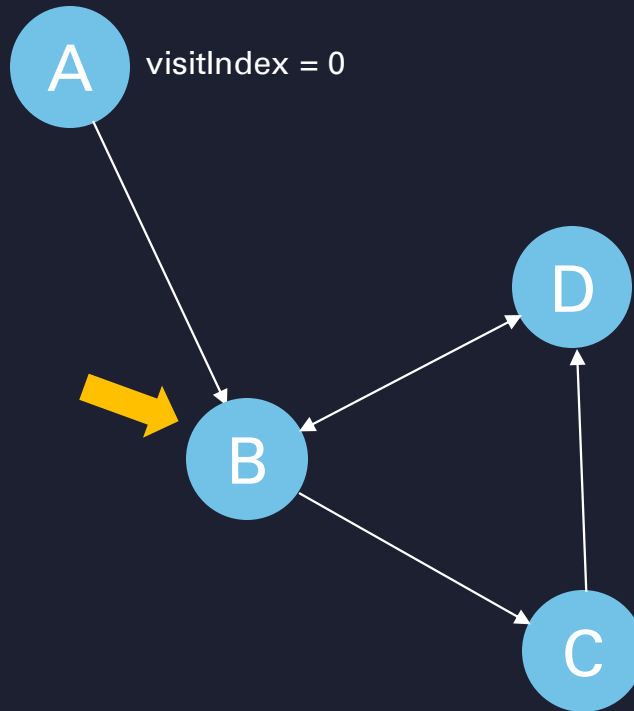
cycleStartIndex: -1

cycle:  $\emptyset$



# *Finding the next cycle*

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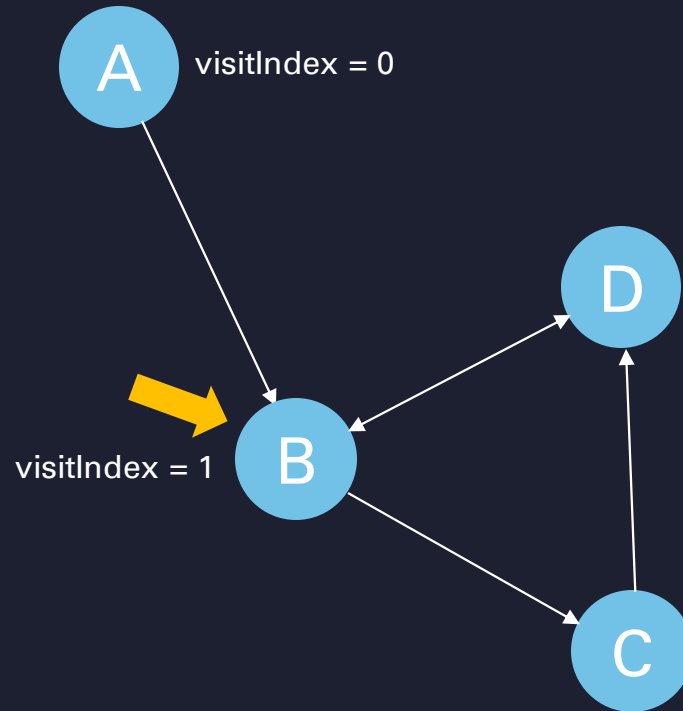


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

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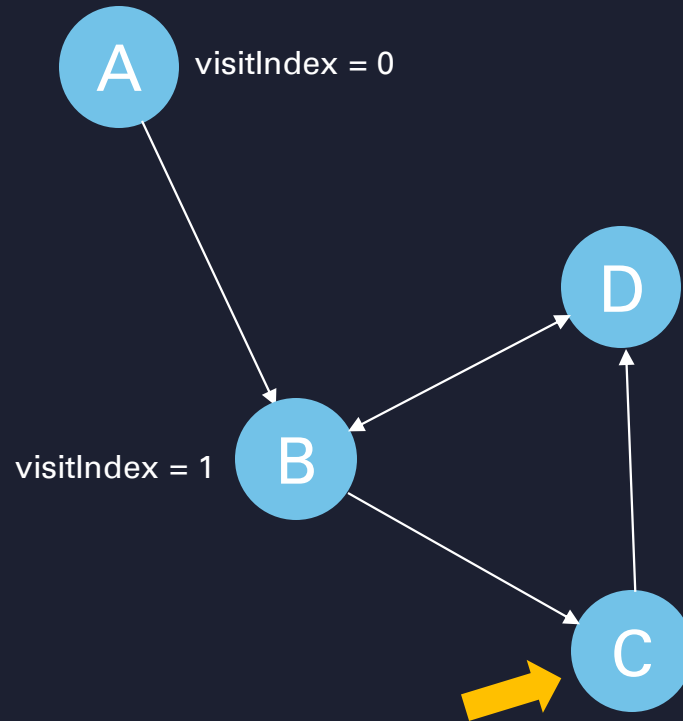


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

---

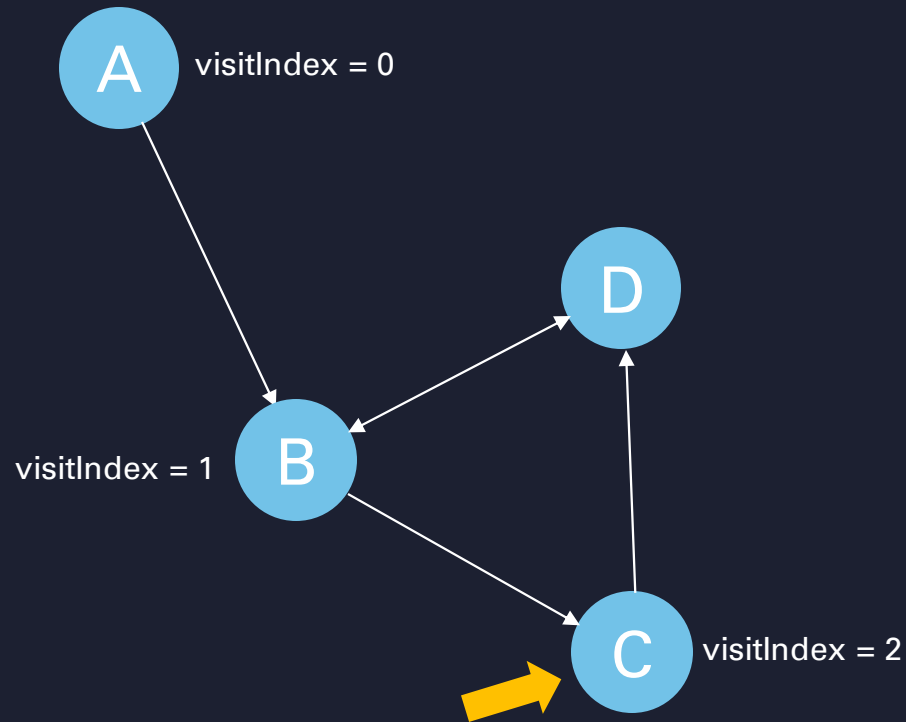


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

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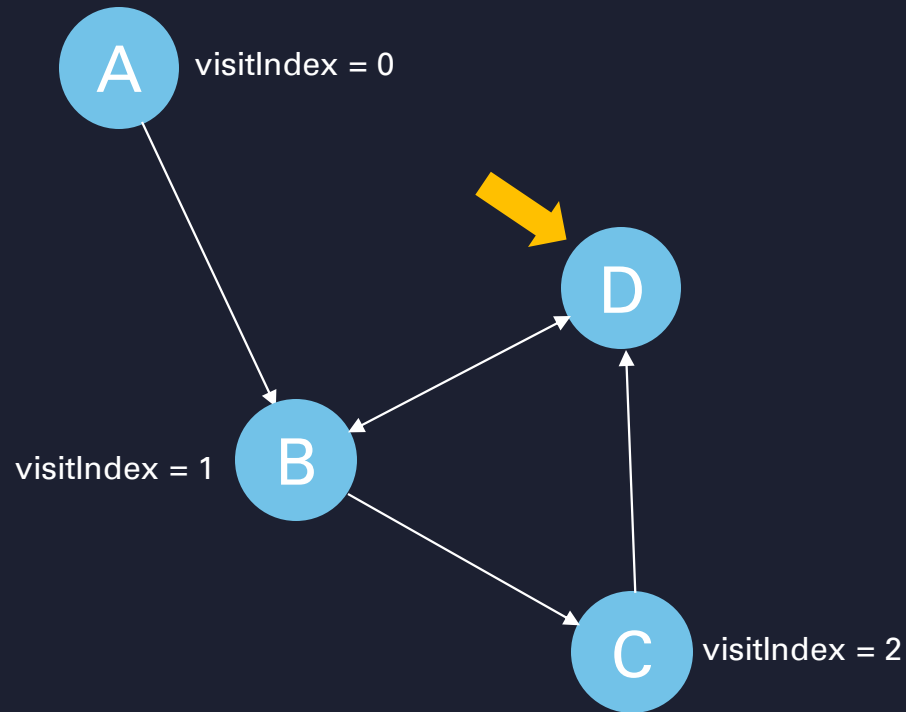


`cycleStartIndex: -1`

`cycle:  $\emptyset$`

# *Finding the next cycle*

---

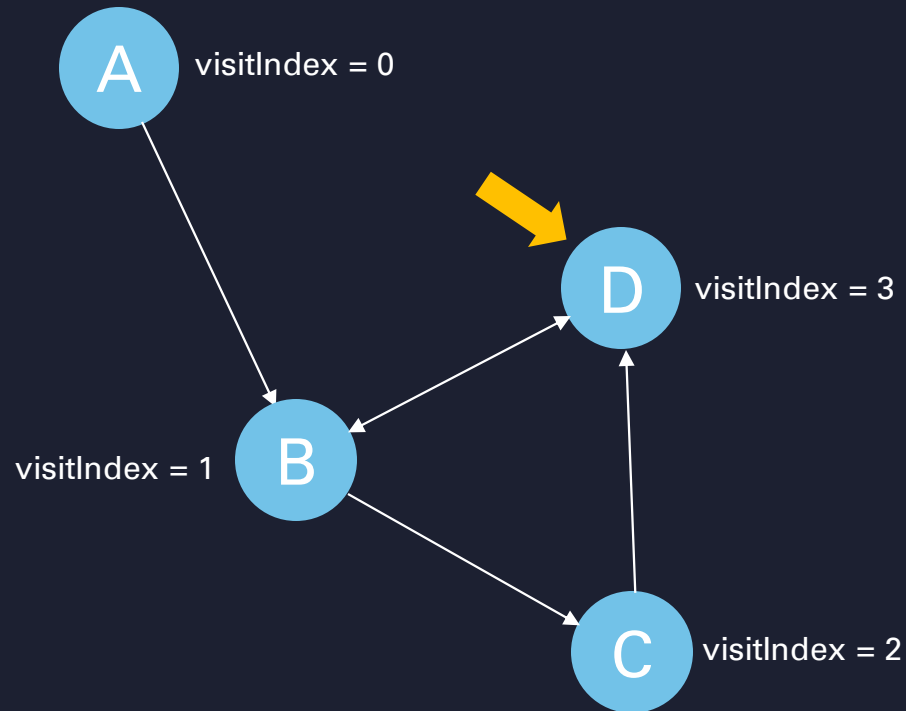


`cycleStartIndex: -1`

`cycle:  $\emptyset$`

# *Finding the next cycle*

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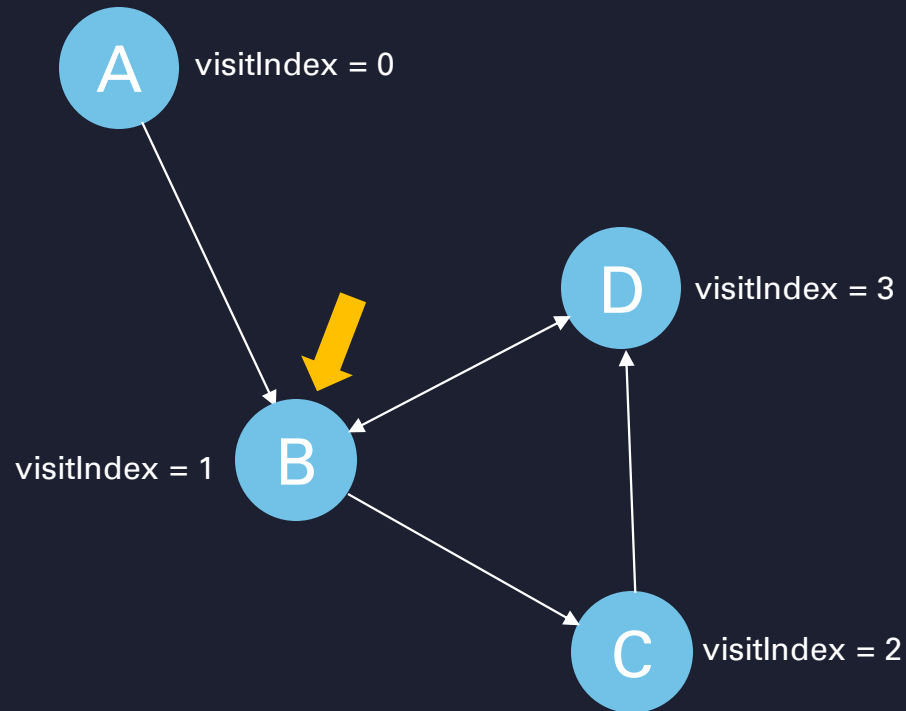


cycleStartIndex: -1

cycle:  $\emptyset$

# *Finding the next cycle*

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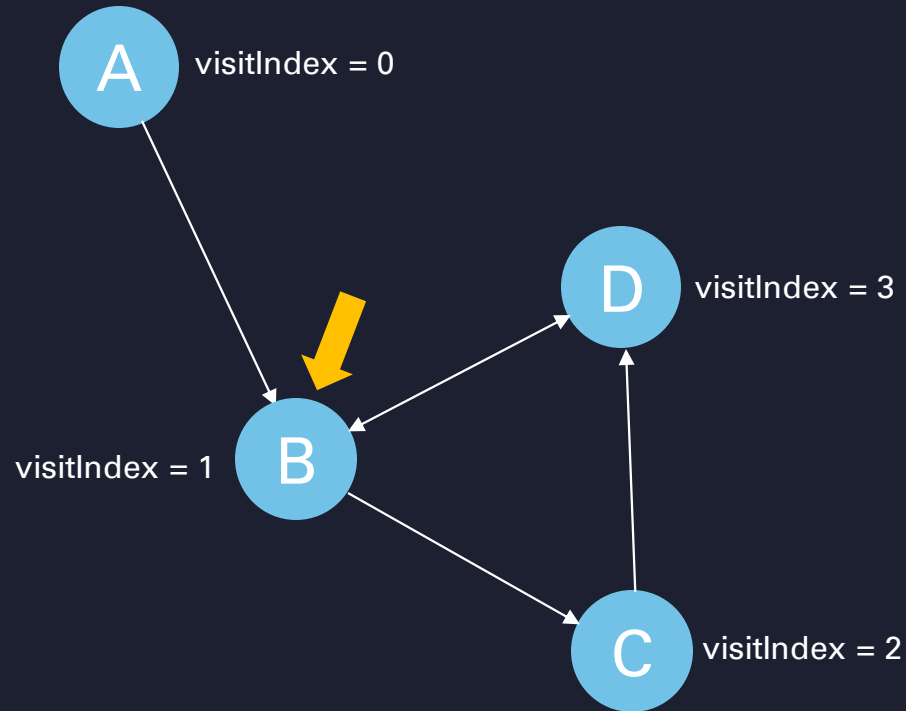
cycleStartIndex: -1

cycle:  $\emptyset$



# *Finding the next cycle*

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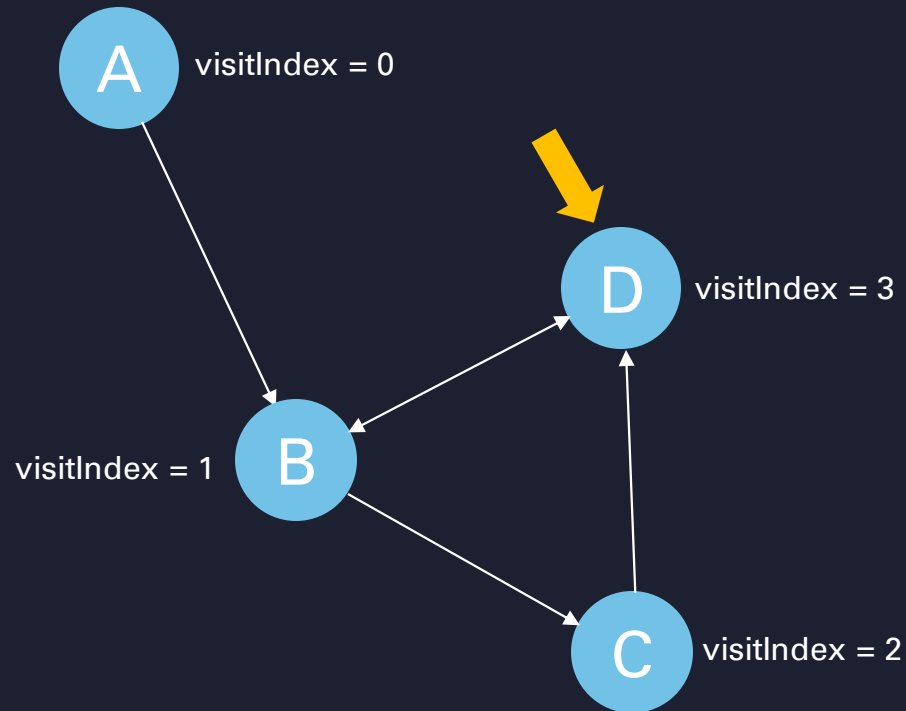


cycleStartIndex: 1

cycle: B

# *Finding the next cycle*

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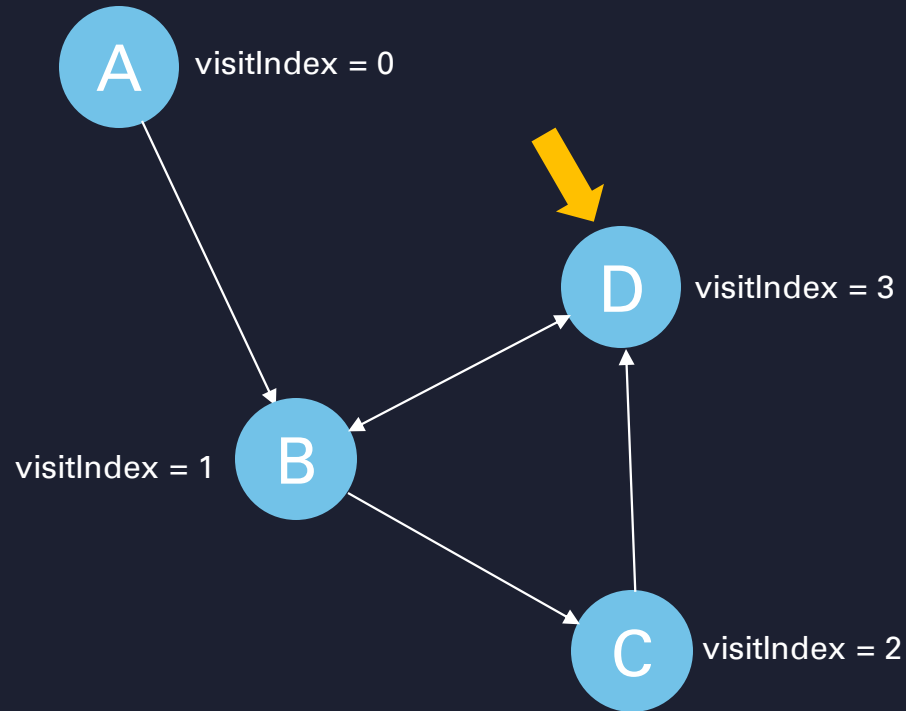


cycleStartIndex: 1

cycle: B

# *Finding the next cycle*

---

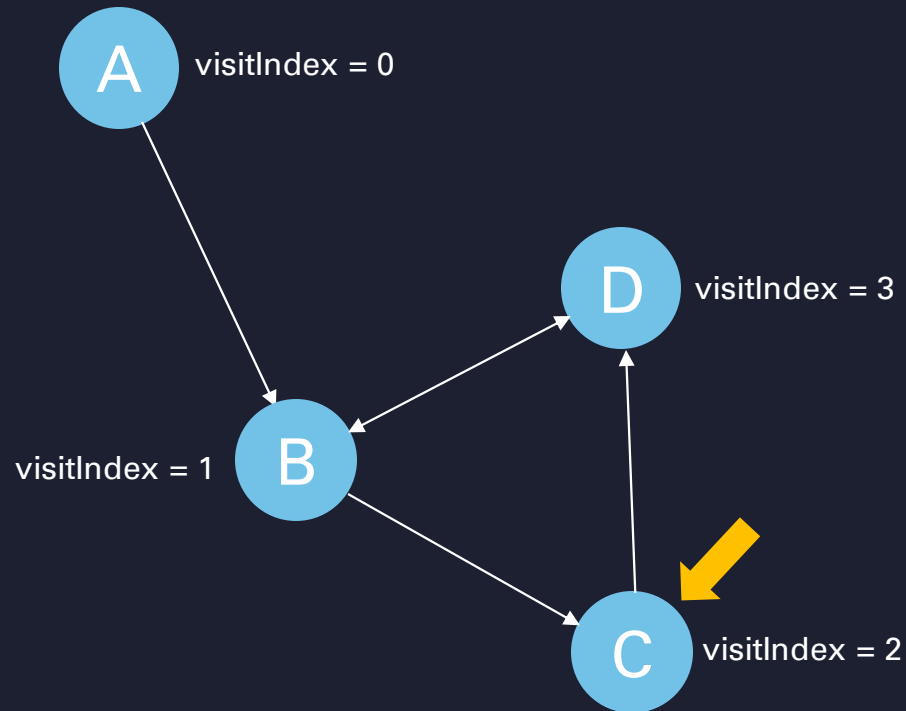


cycleStartIndex: 1

cycle: B, D

# *Finding the next cycle*

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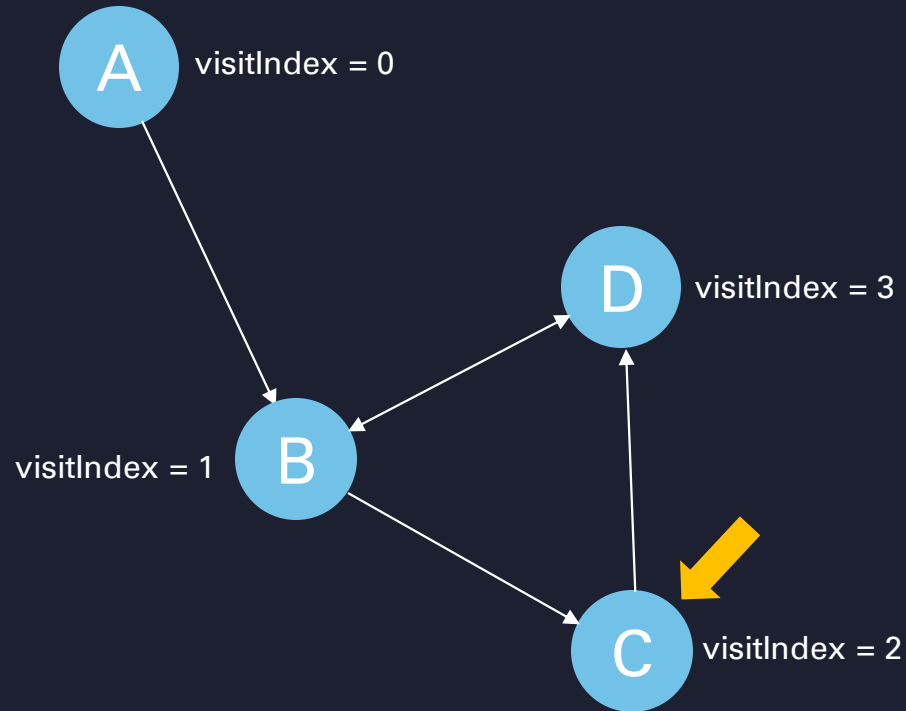


cycleStartIndex: 1

cycle: B, D

# *Finding the next cycle*

---

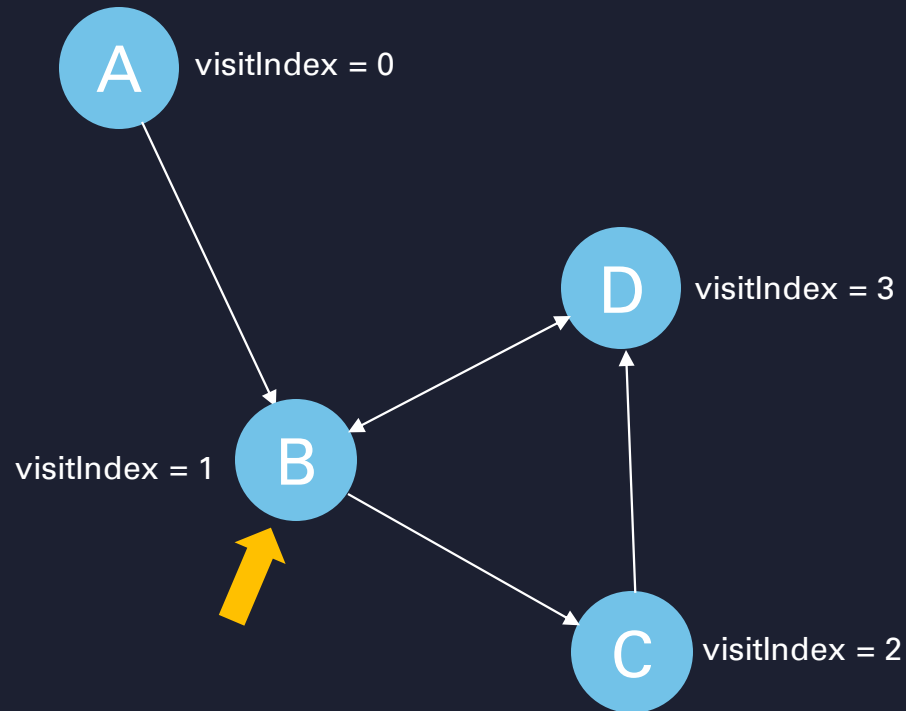


cycleStartIndex: 1

cycle: B, D, C

# *Finding the next cycle*

---

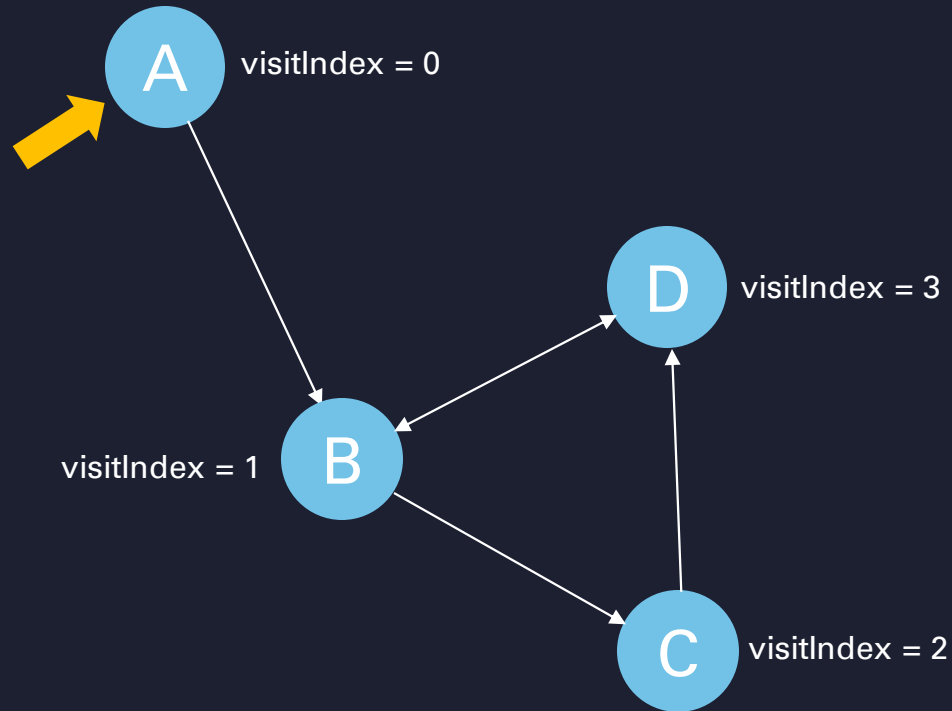


cycleStartIndex: 1

cycle: B, D, C

# *Finding the next cycle*

---



cycleStartIndex: 1

cycle: B, D, C



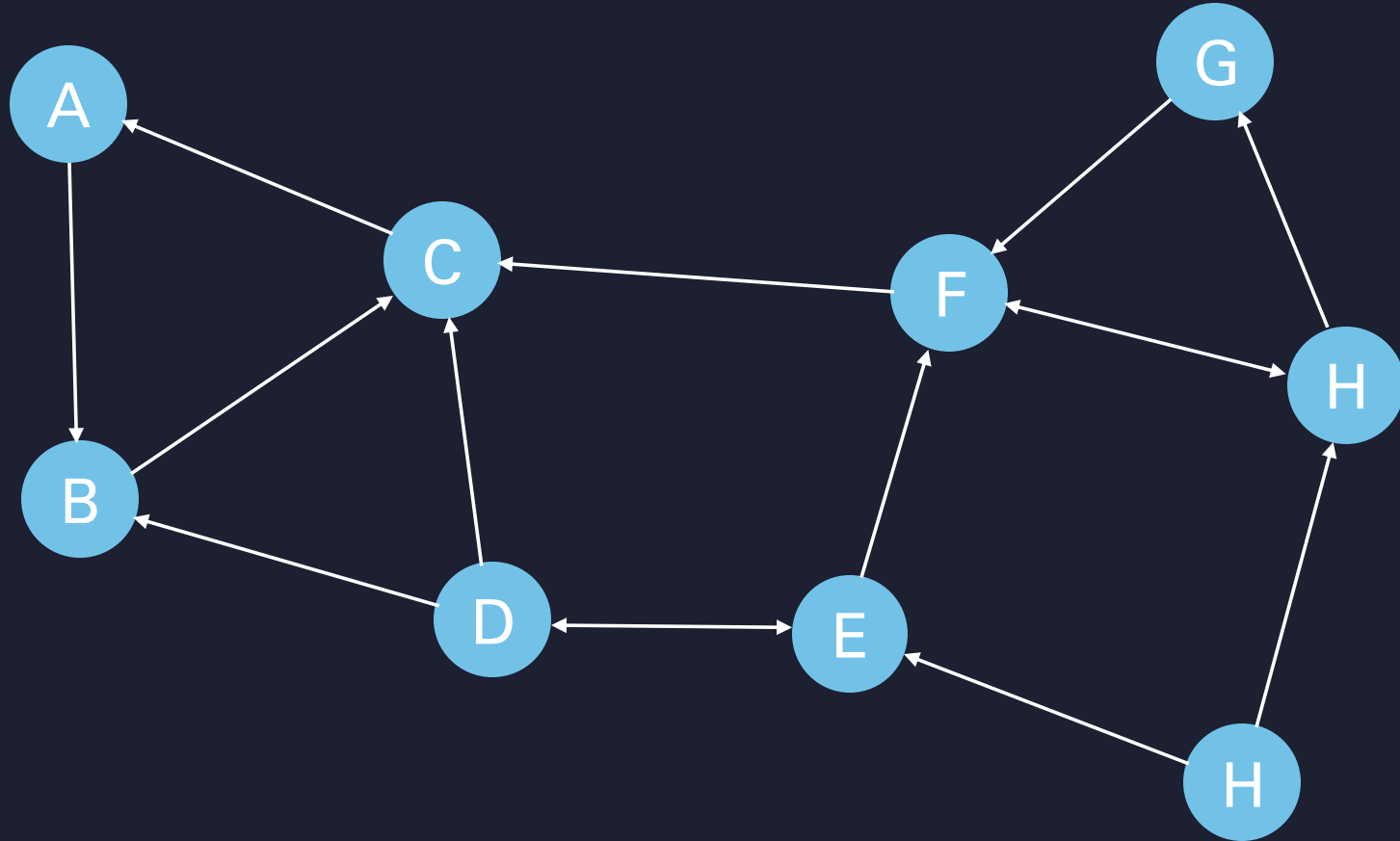
# *Tarjan's Algorithm*

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- Algorithm finds cyclic components in the graph
- Cyclic component: set of nodes, which have any cyclic connection
- Linear running time:  $O(|V| + |E|)$
- We used it once before the main algorithm

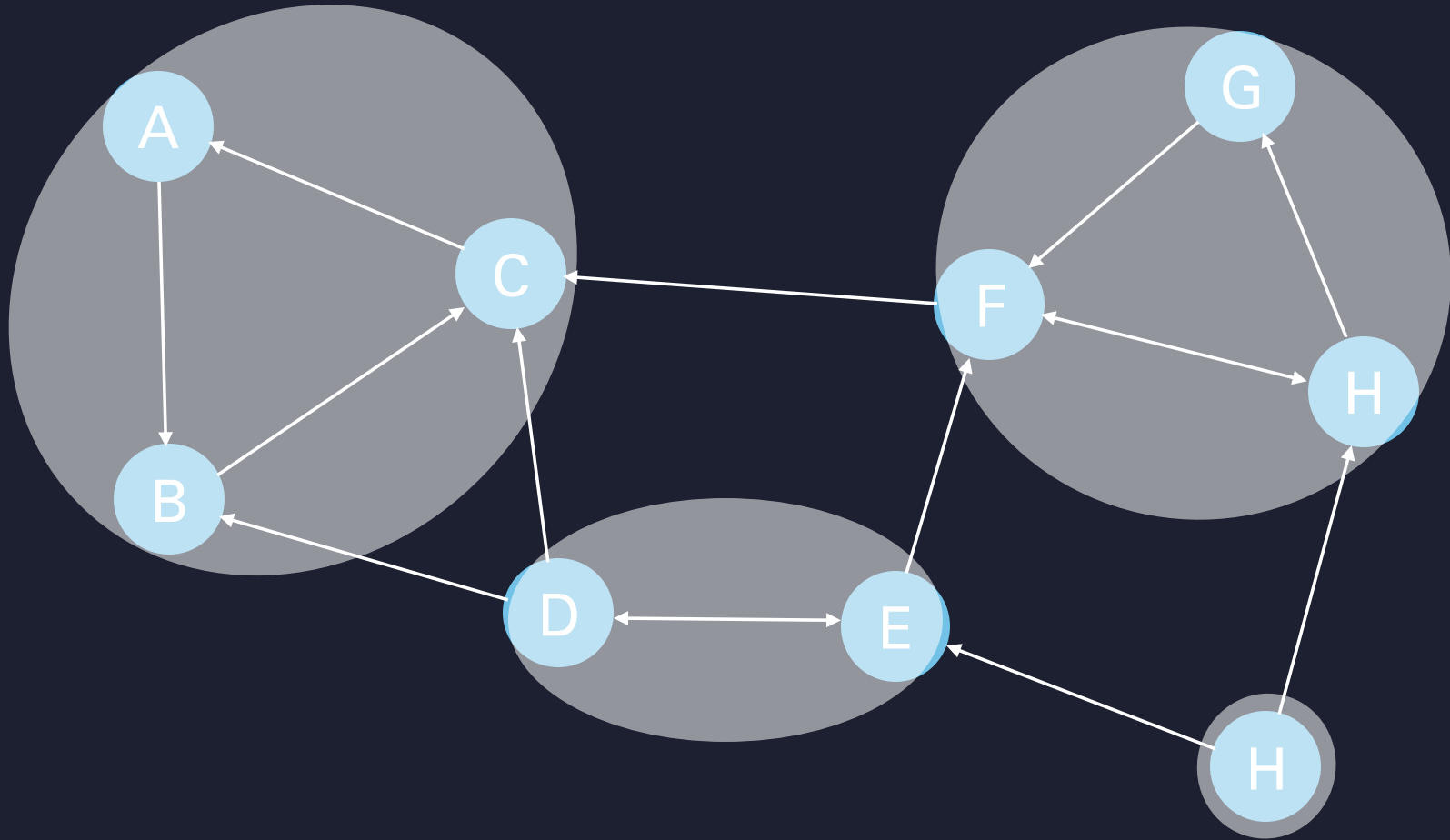
# *Tarjan's Algorithm*

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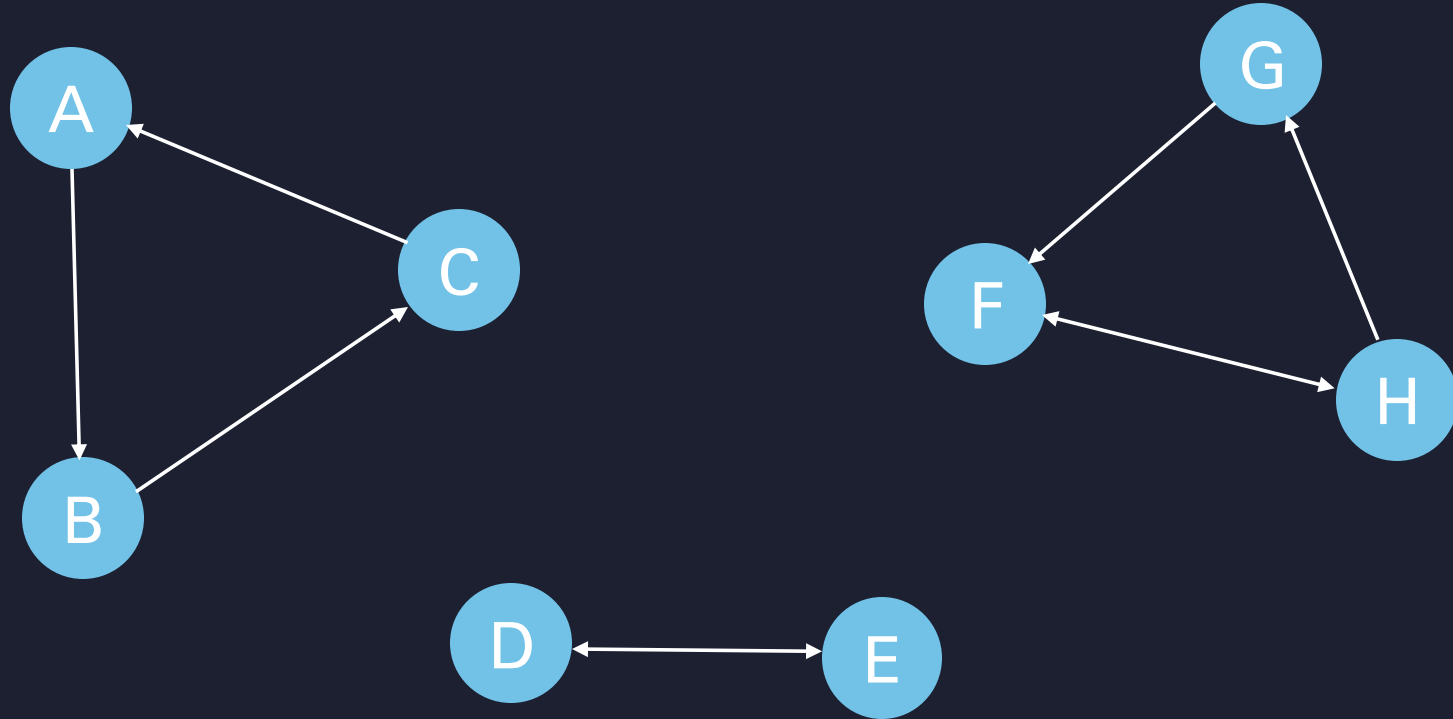
# *Tarjan's Algorithm*

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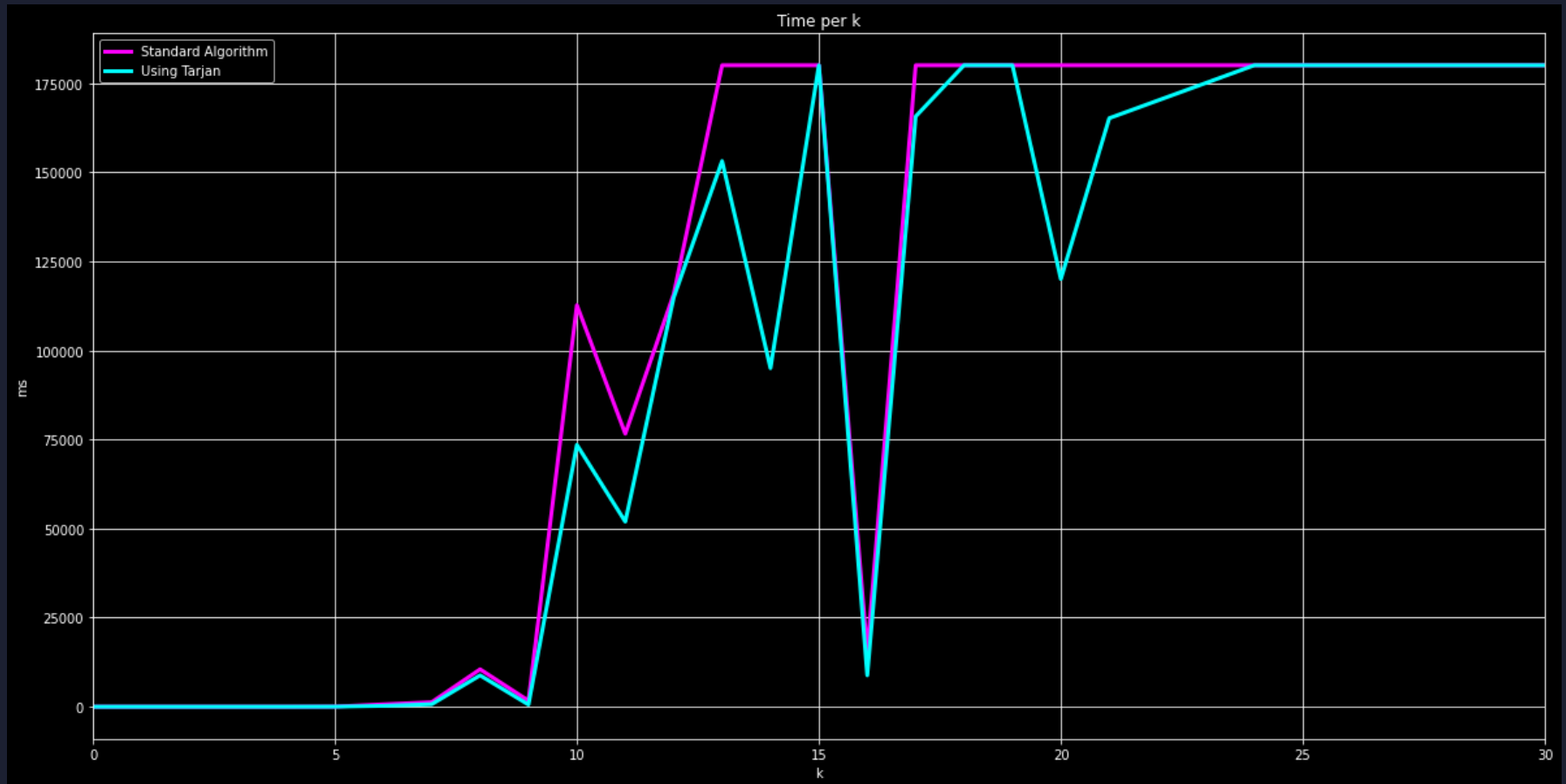


# *Tarjan's Algorithm*

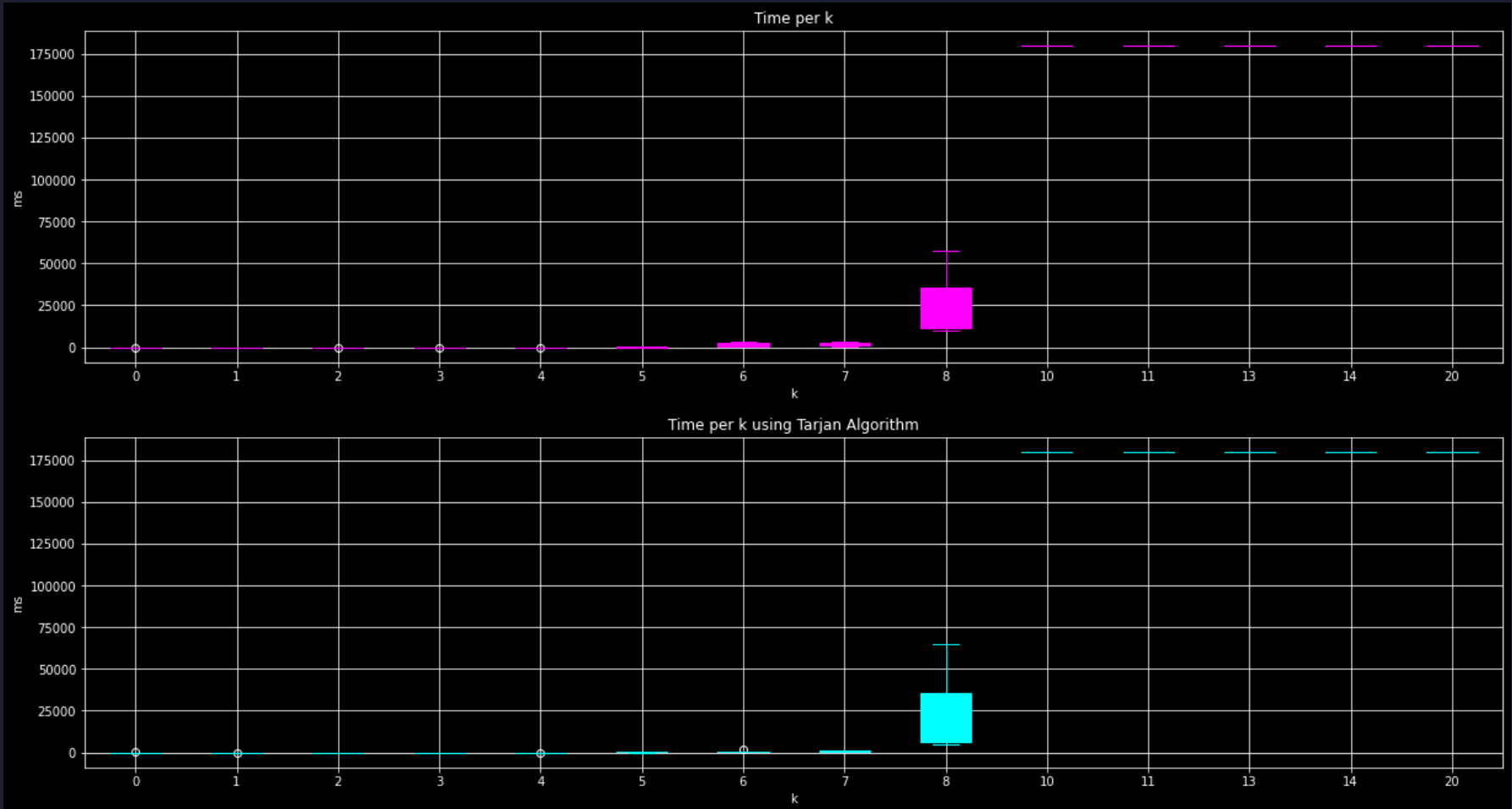
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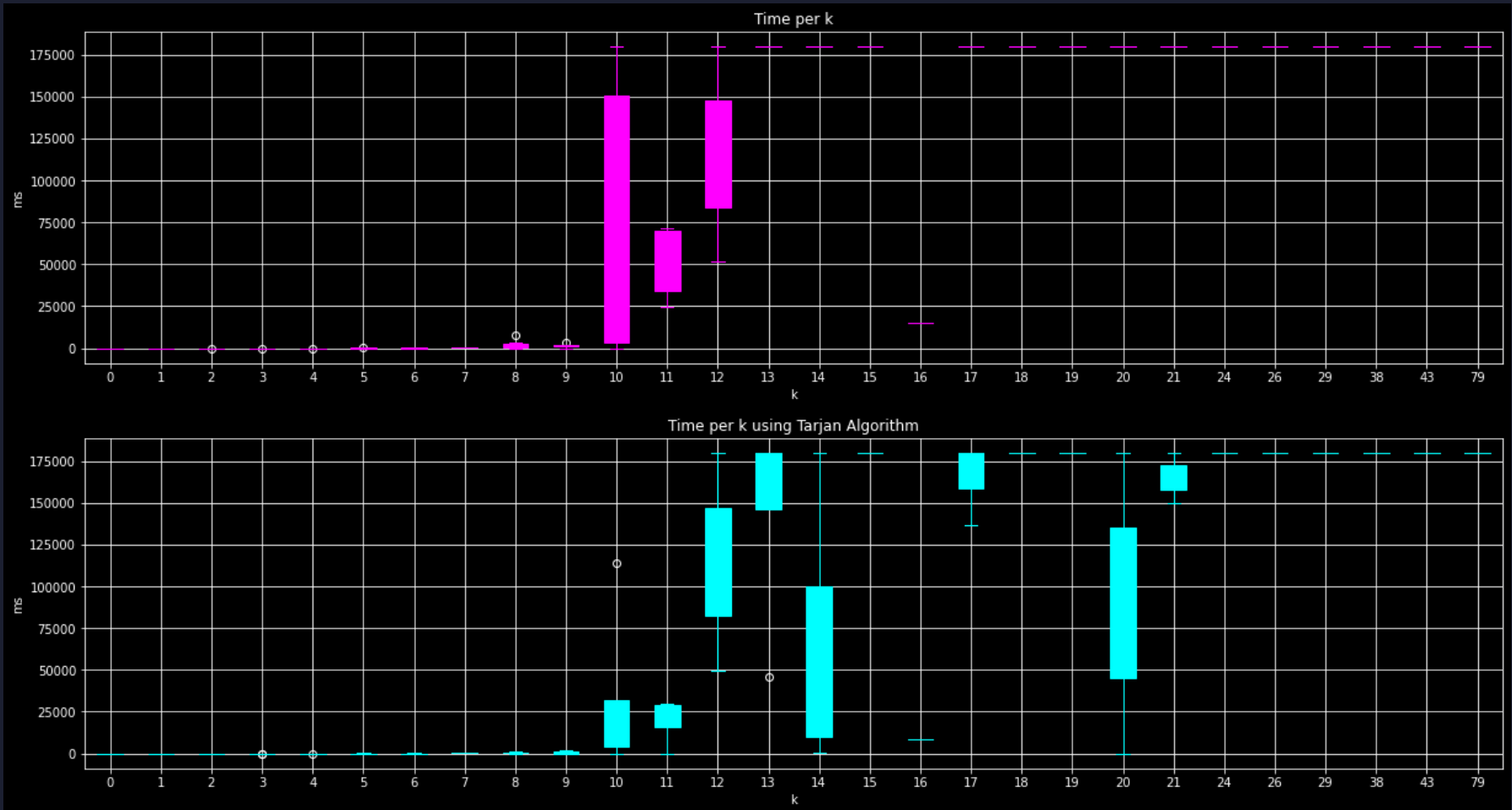
# *Tarjan's Algorithm - Performance*



# Tarjan's Algorithm - Synthetic



# Tarjan's Algorithm - Complex





# Calculation of min $k$

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→ What is the worst case graph?

→ A fully connected graph

→ In fact, a fully connected graph is not the worst case:

```
if(m == n * (n - 1))  
{  
    return k = n - 1;  
}
```

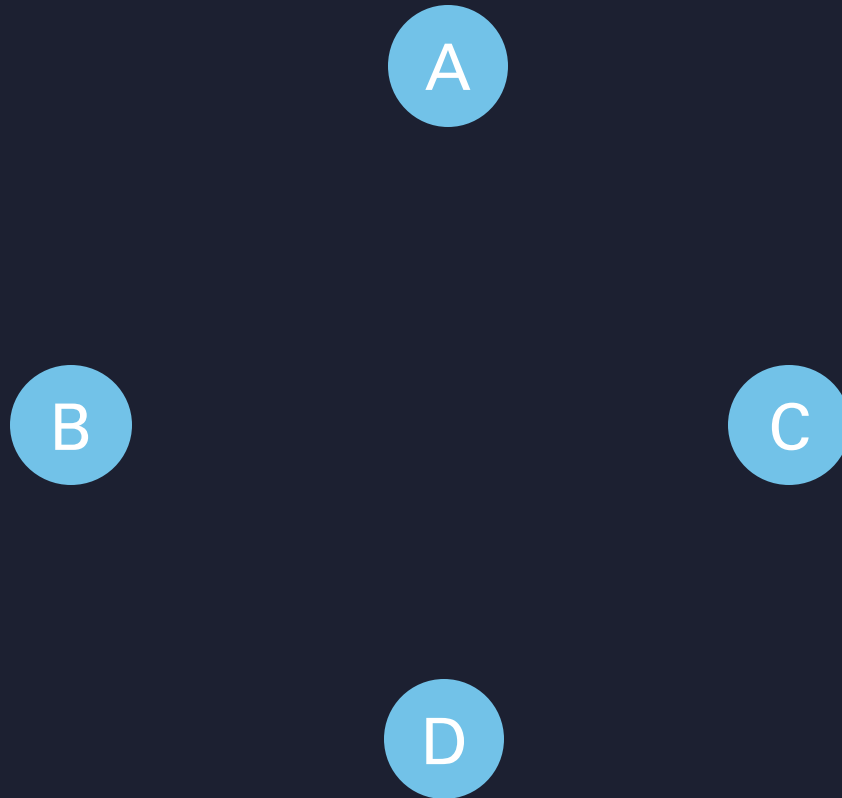
→ Removing a single edge  $(a,b) \Rightarrow k = n - 2$

→ Can we rule out even more cases with this approach?

→ Idea for min  $k$ : add as many edges as possible to a graph without creating a cycle

# Calculation of $\min k$

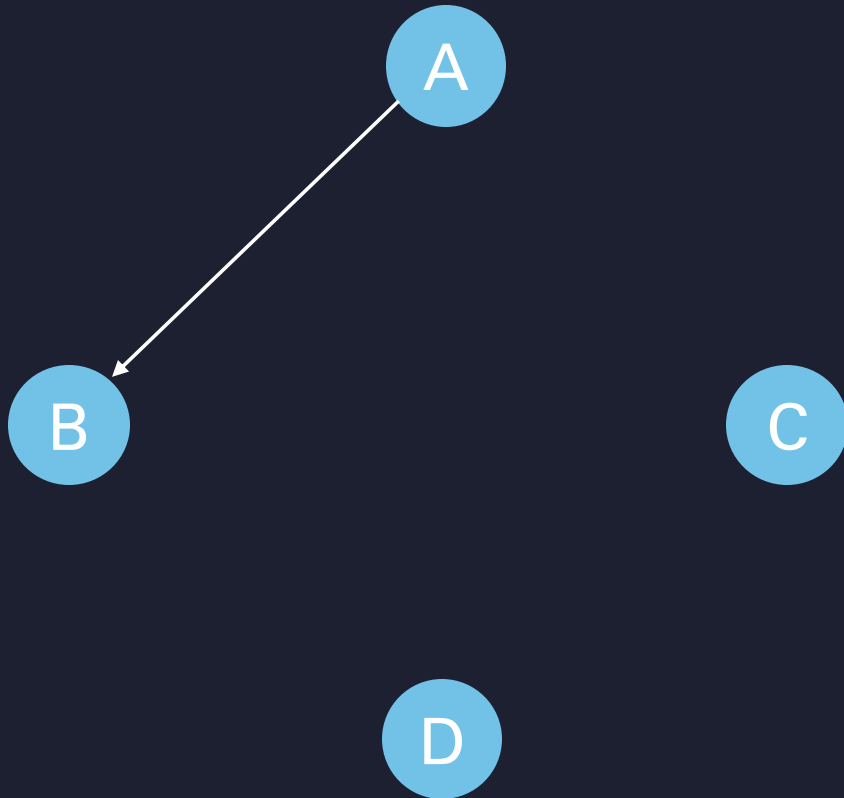
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$|V| = 4$   
 $|E| = 0$   
 $k = 0$

$ E $	$k$
0	0
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



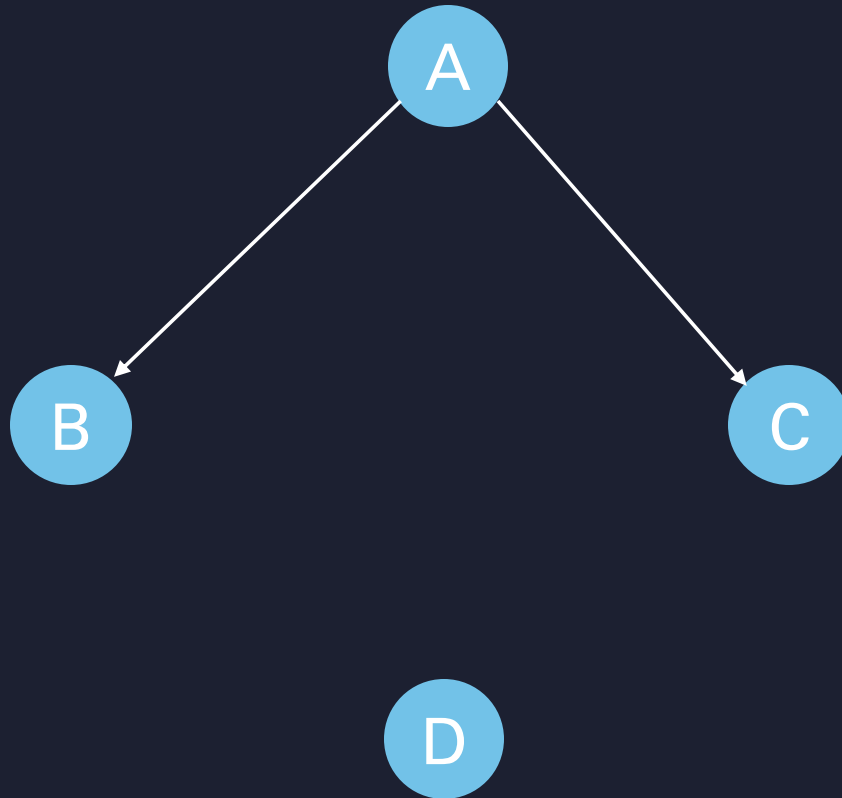
$$|V| = 4$$

$$|E| = 1$$

$$k = 0$$

E	k
0	0
1	0
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



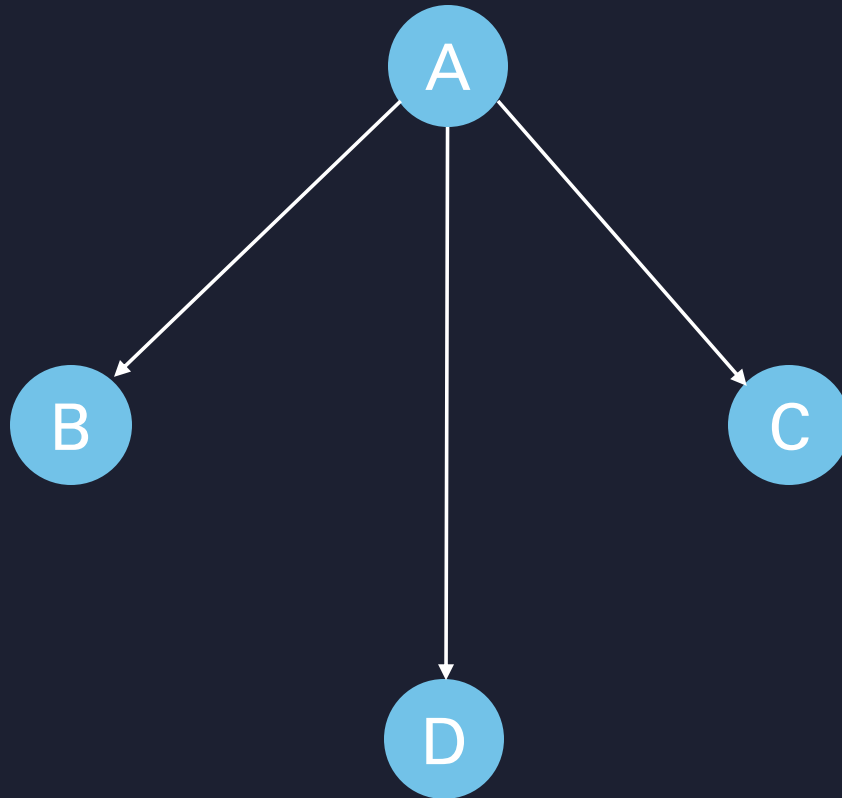
$$|V| = 4$$

$$|E| = 2$$

$$k = 0$$

E	k
0	0
1	0
2	0
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



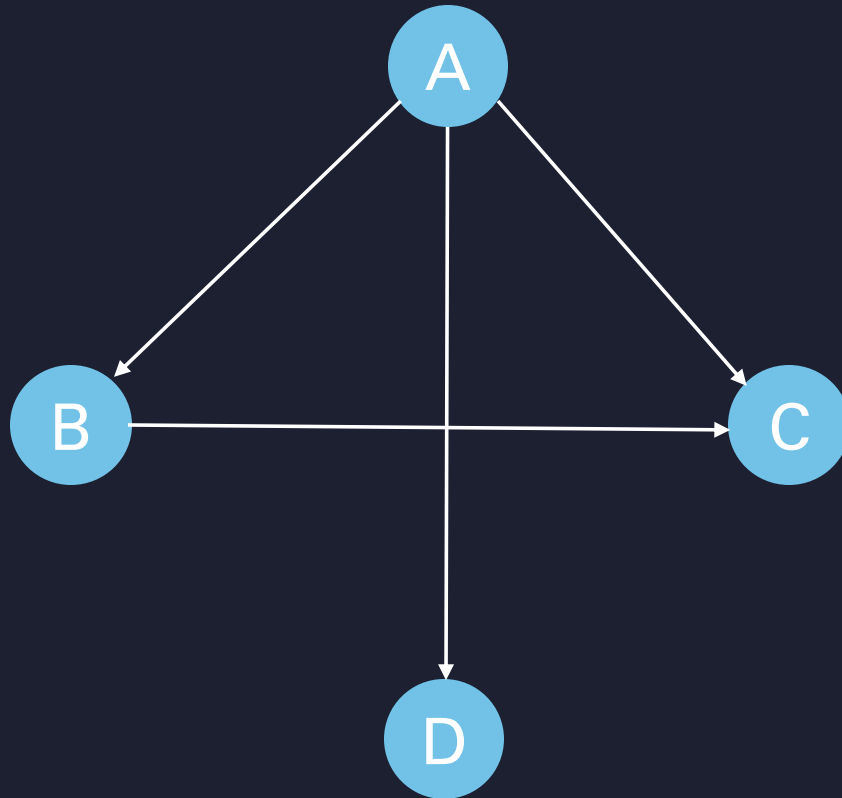
$$|V| = 4$$

$$|E| = 3$$

$$k = 0$$

$ E $	$k$
0	0
1	0
2	0
3	0
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



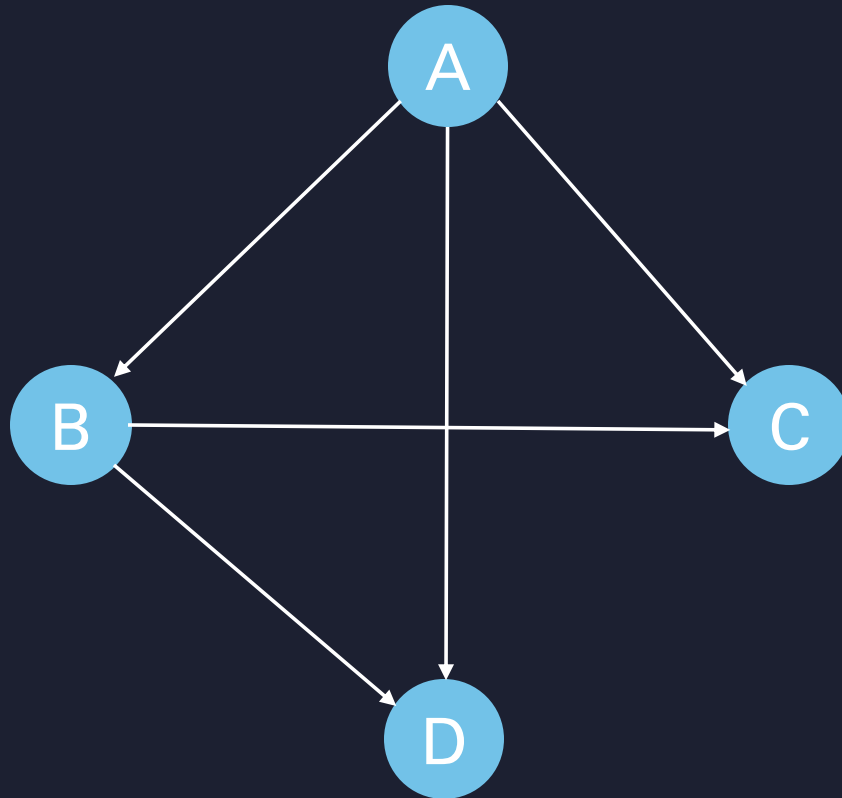
$$|V| = 4$$

$$|E| = 4$$

$$k = 0$$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	
6	
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



$$|V| = 4$$

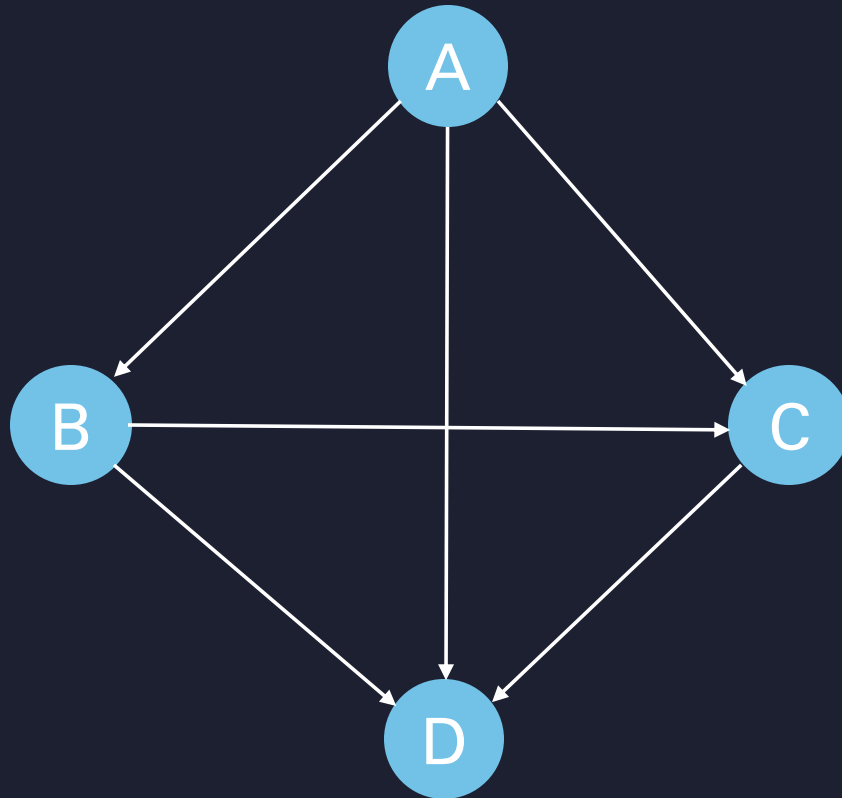
$$|E| = 5$$

$$k = 0$$

E	k
0	0
1	0
2	0
3	0
4	0
5	0
6	
7	
8	
9	
10	
11	
12	



# Calculation of $\min k$



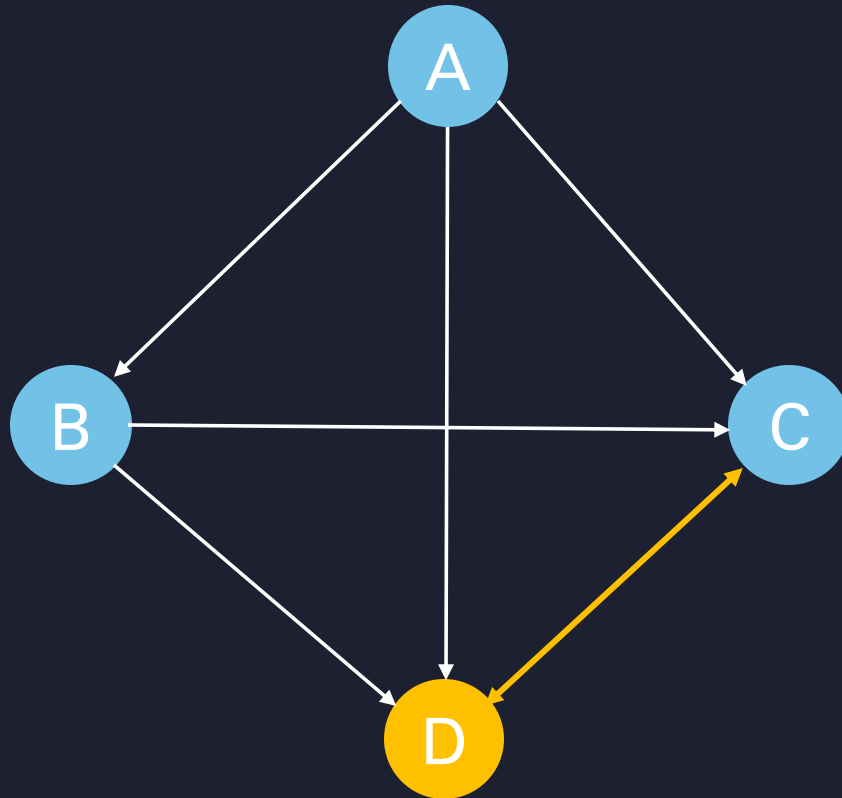
$$|V| = 4$$

$$|E| = 6$$

$$k = 0$$

E	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	
8	
9	
10	
11	
12	

# Calculation of $\min k$



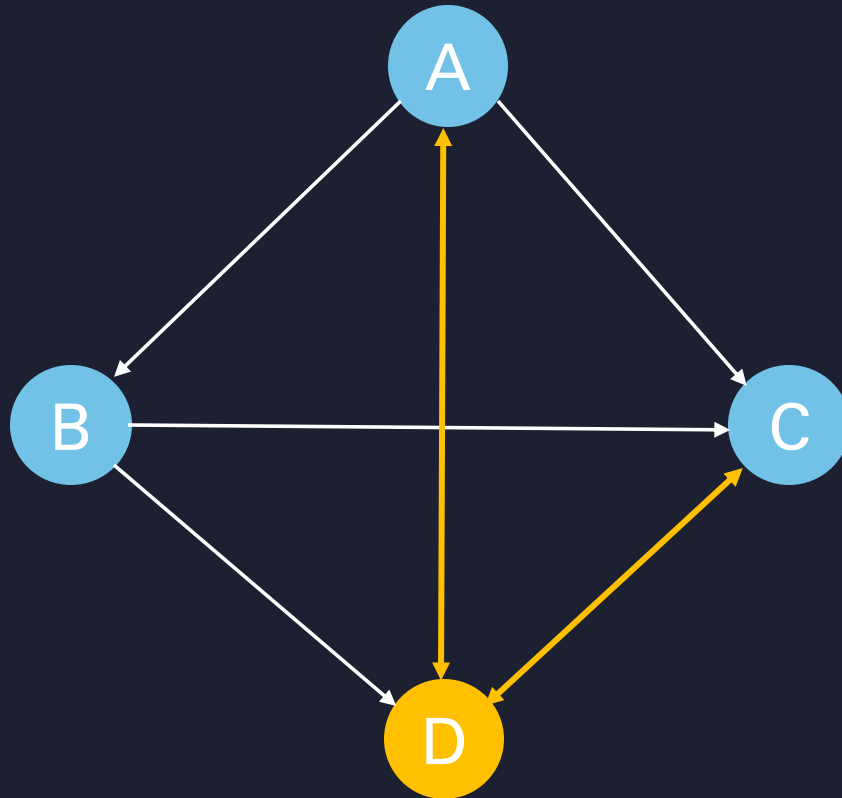
$$|V| = 4$$

$$|E| = 7$$

$$k = 1$$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	
9	
10	
11	
12	

# Calculation of $\min k$



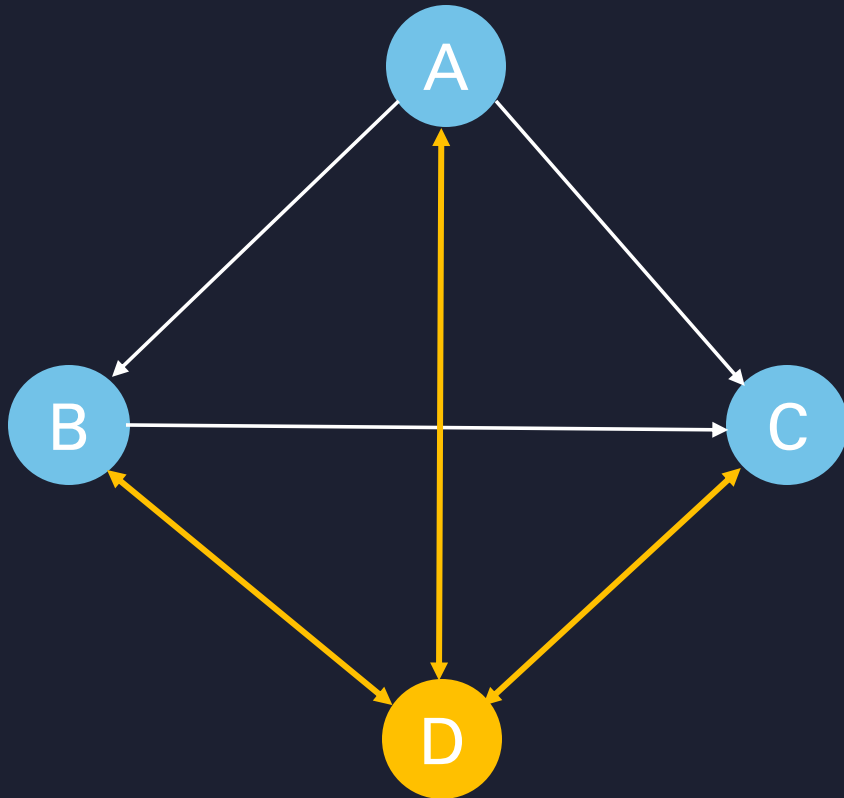
$$|V| = 4$$

$$|E| = 8$$

$$k = 1$$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	
10	
11	
12	

# Calculation of $\min k$



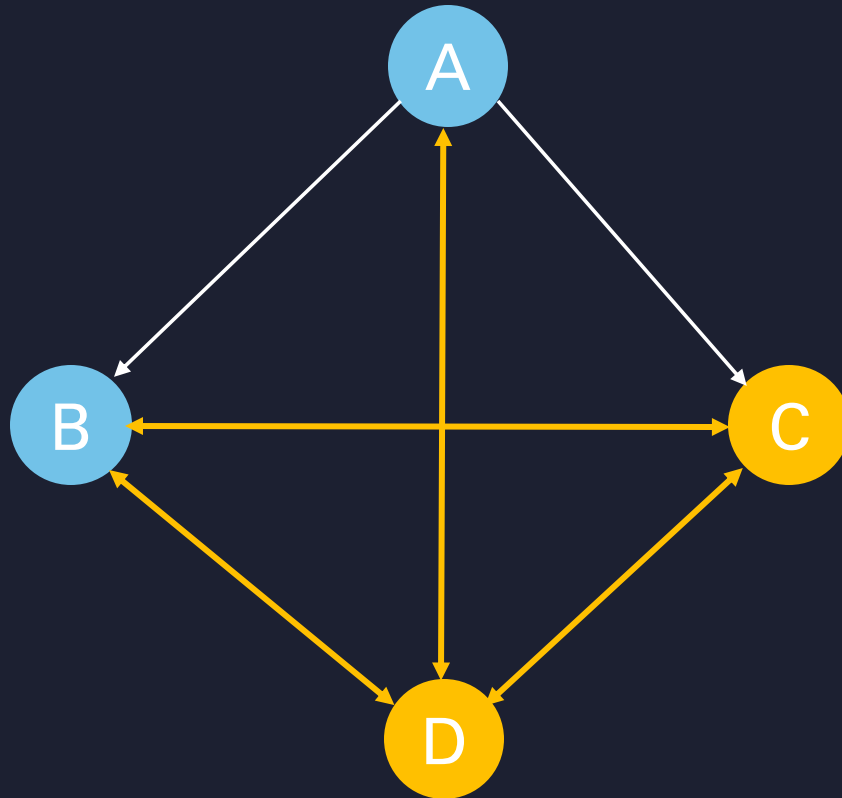
$$|V| = 4$$

$$|E| = 9$$

$$k = 1$$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	
11	
12	

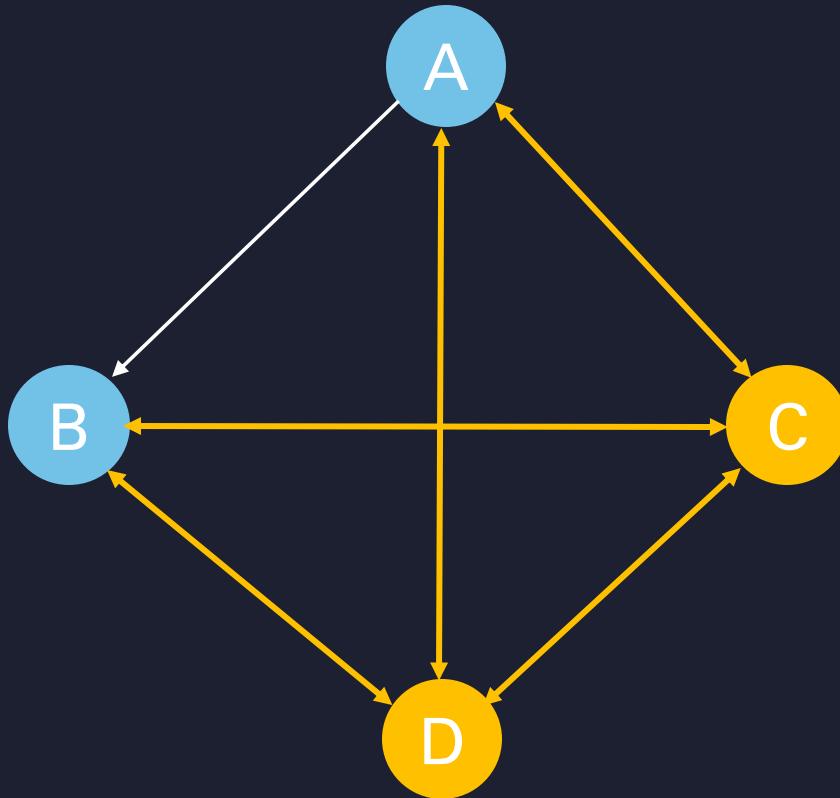
# Calculation of $\min k$



$|V| = 4$   
 $|E| = 10$   
 $k = 2$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	
12	

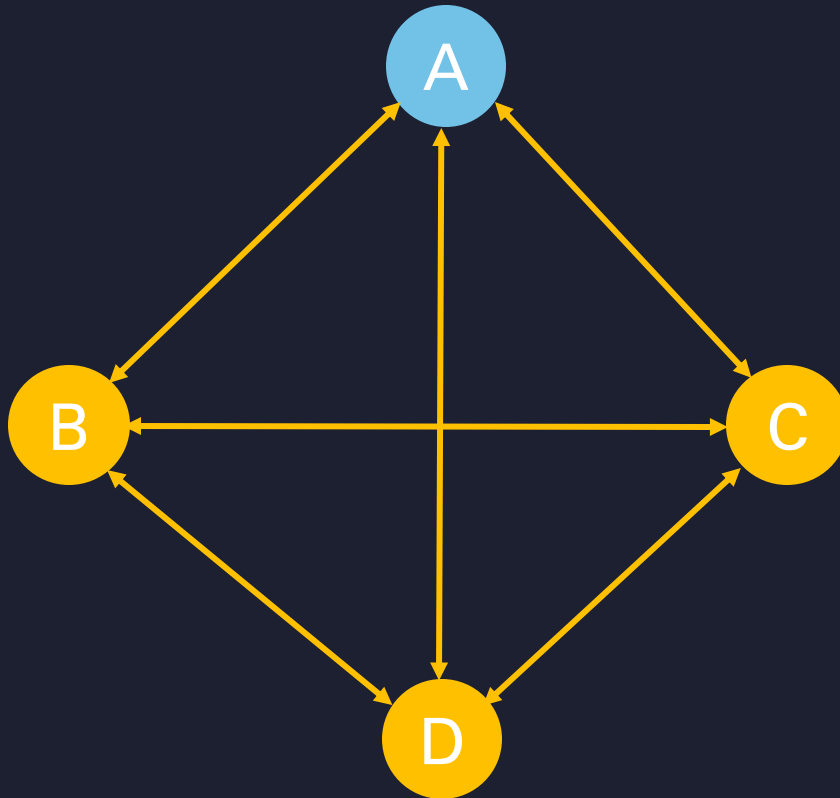
# Calculation of $\min k$



$|V| = 4$   
 $|E| = 11$   
 $k = 2$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	

# Calculation of $\min k$



$|V| = 4$   
 $|E| = 12$   
 $k = 3$

$ E $	$k$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	3

# Calculation of min k

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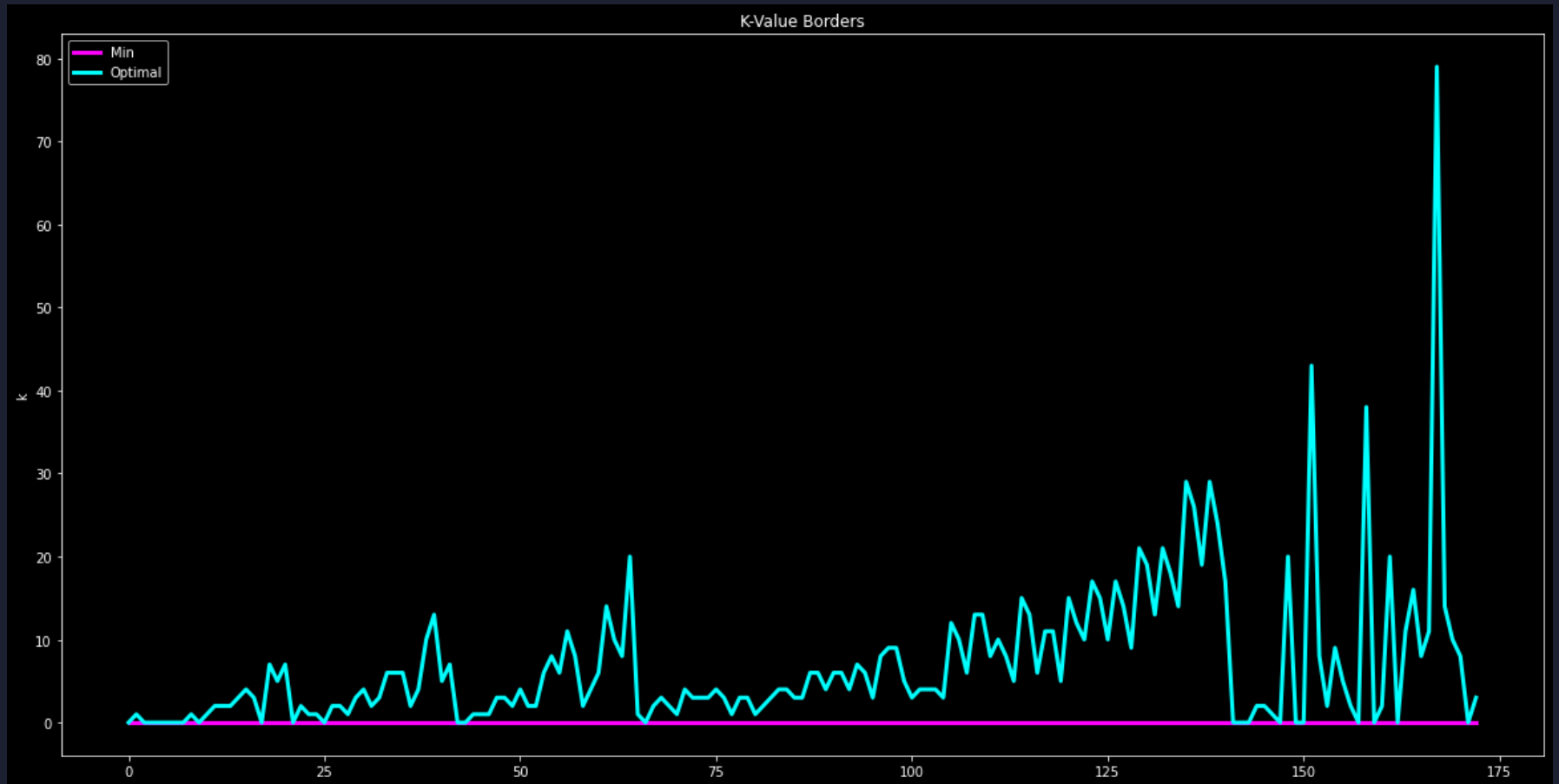
1.  $\max m = n * (n - 1)$

2.  $\min k = 0$  if  $m \leq n * (n - 1) / 2$

E	k
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	3



# *Calculation of $\min k$*

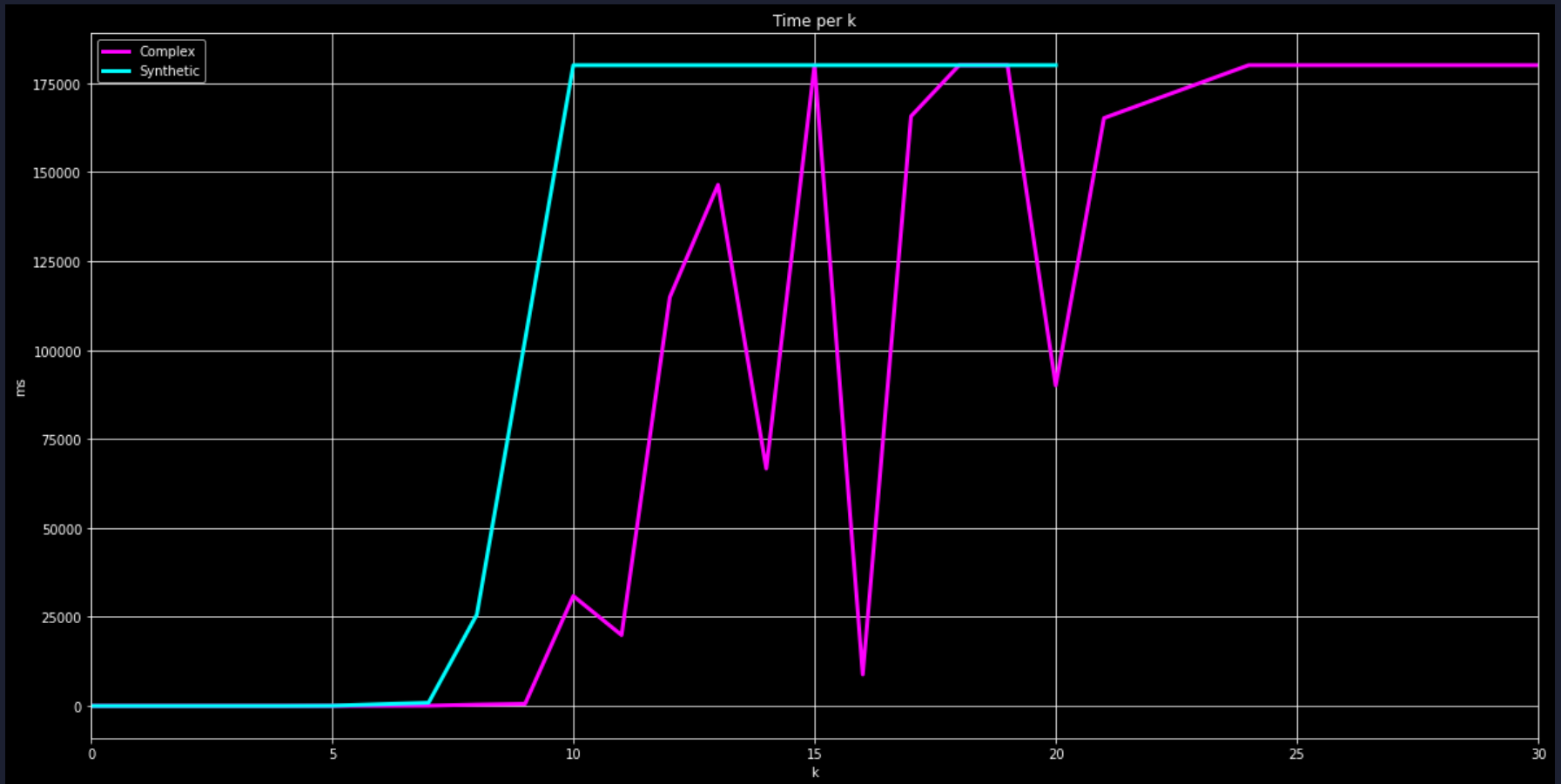


# *Performance*

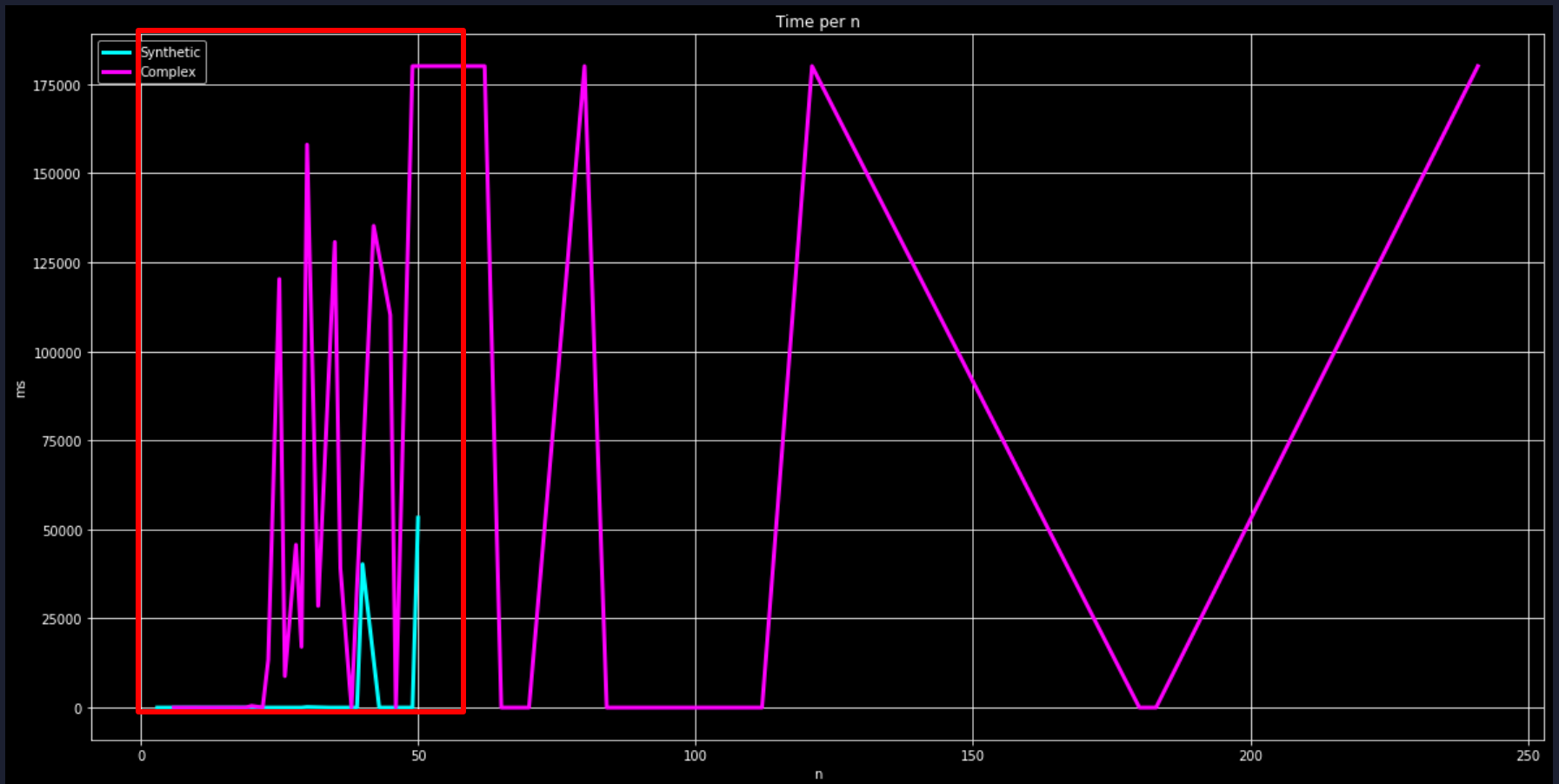
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- We set a timeout after 3 minutes for the plots
- The best we could solve was  $k = 21$
- The algorithm was executed on an i9-9900K
- Next time we will show the server performance too

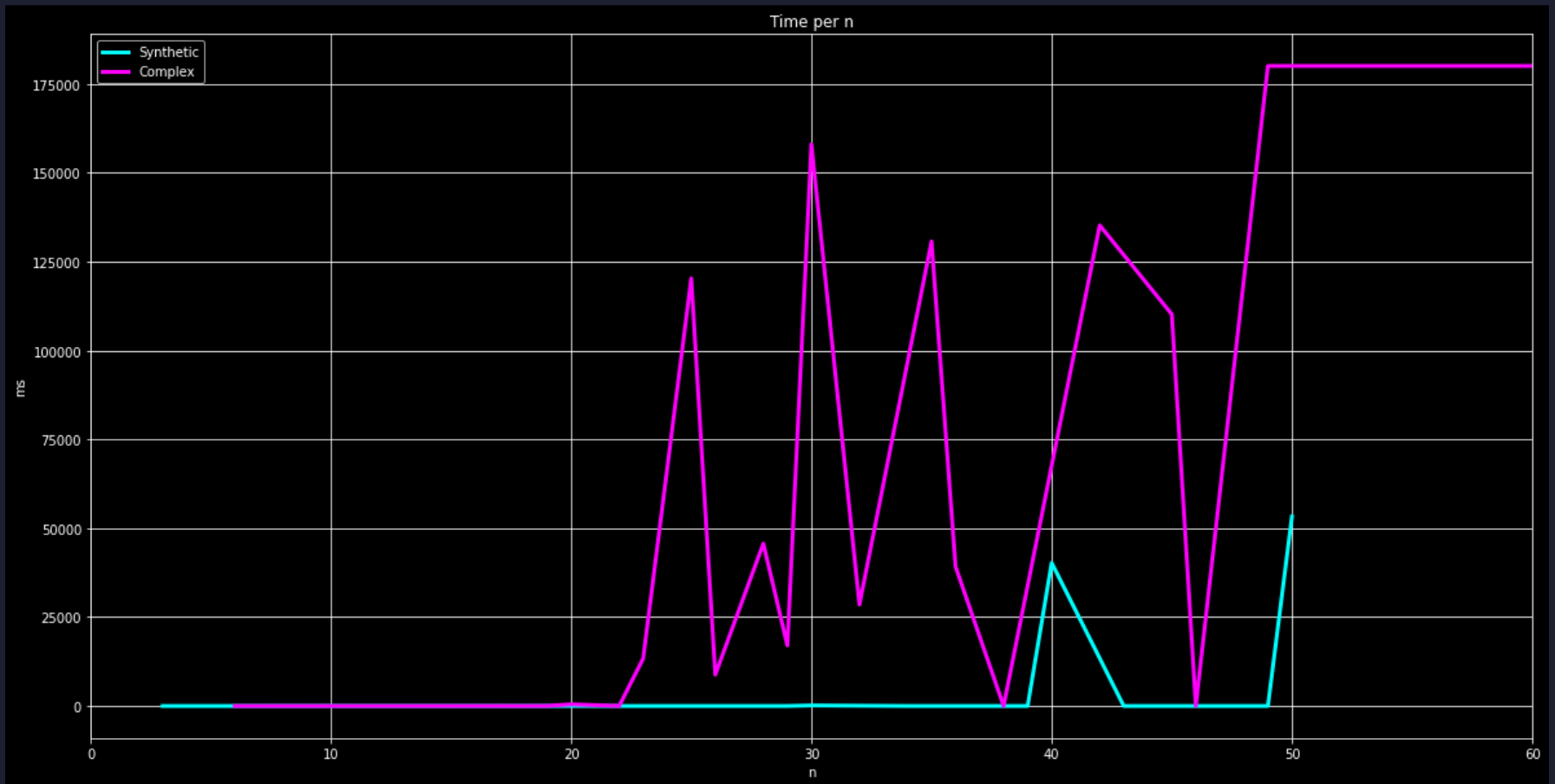
# *Performance – per k*



# *Performance – per $n$*



# *Performance – per $n$*



*Do you have any questions?*

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