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Henri Malahicade   862268736

Homework 1 - Math Review

① $\frac{u \cdot v}{|u||v|} = \cos\theta$ $\frac{2 \cdot 4 + 4 \cdot 3 + 0}{\sqrt{38} \cdot \sqrt{25}} = \frac{8+12}{5\sqrt{38}}$

A $\frac{4}{\sqrt{38}}$

② With $P = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and normal $n = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$ and $Q = (5, 6, 7)$
& using p as "origin"
Q is now $= (4, 3, 8)$
and Q is on the $\frac{4}{3} + \frac{6}{3} - \frac{16}{3} = -\frac{6}{3}$ so negative side of the product

a) $(0, 0, 0)$ transforms into $(-1, -3, 1)$
$-\frac{1}{3} - \frac{6}{3} - \frac{2}{3}$ so negative side, so same side as Q

b) $(-1, 1, 2)$ transforms into $(-2, -2, 3)$
$-\frac{2}{3} - \frac{4}{3} - \frac{6}{3}$ so negative side, so same side as Q

c) $(1, 4, 0)$ transforms into $(0, 1, 1)$
$0 + \frac{2}{3} - \frac{2}{3} = 0$ so on the plane, so not on side of Q

d) $(1, 5, 1)$ transforms into $(0, 2, 2)$
$0 + \frac{4}{3} - \frac{4}{3} = 0$ so on the plane, so not on the side of Q

e) $(-1, -1, -1)$ transforms into $(-2, -4, 0)$
$-\frac{2}{3} - \frac{8}{3} + 0 =$ negative, so same side as Q

③ $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = A$ $\det(A) = i(12-15) - j(6-12) + k(5-6)$
$= -3i + 6j - 1k$

Solution $= \langle -3, 6, -1 \rangle$

④ $\vec{w} = \frac{u}{|u|} \cdot |v|$

Math Review

5) a) $h^2+k^2=a^2$

b) $h^2+(b-k)^2=c^2$

c) $\cos\theta=\frac{k}{a}$

d) $a^2-k^2+(b-k)^2=c^2$ (from part a into part b)

$a^2-k^2+b^2-2bk+k^2=c^2$ (simplify)

$a^2+b^2-2bk=c^2$ (simplify)

$\underline{a^2+b^2-2ba\cos\theta=c^2}$ (from part c)

e) $|u|^2+|v|^2-2|u||v|\cos\theta=|u-v|^2$

f) $|u|^2+|v|^2-2\vec{u}\cdot\vec{v}=|u-v|^2$

g) $\vec{u}\cdot\vec{v}=\frac{|u|^2+|v|^2-|u-v|^2}{2}$

h) $$\begin{aligned}\|u\times v\|^2+(\vec{u}\cdot\vec{v})^2&=\|u\|^2\|v\|^2\sin^2\theta+\|u\|^2\|v\|^2\cos^2\theta\\&=\|u\|^2\|v\|^2(\sin^2\theta+\cos^2\theta)\\&=\|u\|^2\|v\|^2 \quad\square\end{aligned}$$

i)

6) Using $(1,0,0)$ as end point, our two vectors are $\langle -1,2,-1\rangle$ & $\langle 1,0,-1\rangle$

And to calculate the normal we do

$$\det\left(\begin{vmatrix} i & j & k\\ -1 & 2 & -1\\ 1 & 0 & -1\end{vmatrix}\right)=i(-2)-j(1+1)+k(-2)$$

normal $=\langle -2,-2,-2\rangle$

$$|u-v|^2 = (u_1-v_1)^2+(u_2-v_2)^2+(u_3-v_3)^2$$
$$u_1^2-2u_1v_1+v_1^2$$
$$u_1^2+v_1^2+u_2^2+v_2^2+u_3^2+v_3^2-2(u_1v_1+u_2v_2+u_3v_3)$$
$$|u|^2+|v|^2-2\vec{u}\cdot\vec{v}$$

Matrices

$$A=\begin{bmatrix}1&2&5\\3&7&19\end{bmatrix}\qquad B=\begin{bmatrix}5&2\\1&-3\\-1&1\end{bmatrix}\qquad v=\begin{bmatrix}1\\2\end{bmatrix}$$

7 a) $\begin{bmatrix}1&2&5\\3&7&19\end{bmatrix}+\begin{bmatrix}5&1&-1\\2&-3&1\end{bmatrix}=\begin{bmatrix}6&3&4\\5&4&20\end{bmatrix}$

b) $\begin{bmatrix}1&2&5\\3&7&19\end{bmatrix}\cdot\begin{bmatrix}5&2\\1&-3\\-1&1\end{bmatrix}=\begin{bmatrix}2&1\\3&4\end{bmatrix}$

c) $\det(AB)=8-3=5$

$\begin{bmatrix}2&1\\3&4\end{bmatrix}\cdot\begin{bmatrix}\frac{4}{5}&-\frac{1}{5}\\-\frac{3}{5}&\frac{2}{5}\end{bmatrix}=\begin{bmatrix}\frac{8}{5}-\frac{3}{5}&\frac{2}{5}-\frac{2}{5}\\\frac{12}{5}-\frac{12}{5}&\frac{8}{5}-\frac{3}{5}\end{bmatrix}$

$=\begin{bmatrix}1&0\\0&1\end{bmatrix}\qquad (AB)^{-1}=\begin{bmatrix}\frac{4}{5}&-\frac{1}{5}\\-\frac{3}{5}&\frac{2}{5}\end{bmatrix}$

8 $\begin{bmatrix}2&1\\3&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$

$\begin{bmatrix}\frac{4}{5}&-\frac{1}{5}\\-\frac{3}{5}&\frac{2}{5}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}\frac{4}{5}-\frac{2}{5}\\\frac{4}{5}-\frac{3}{5}\end{bmatrix}=\begin{bmatrix}\frac{2}{5}\\\frac{1}{5}\end{bmatrix}$

9 $AB=C$, A is 2x3, B is 3x2, multiplication is compatible and creates C which is 2x2

$BA=D$, but B is 3x2, and A is 2x3 so D is 3x3 and not equal to C

(10)
$$\begin{bmatrix} c_{11}d_{11} + c_{12}d_{21} & c_{11}d_{12} + c_{12}d_{22} \\ c_{21}d_{11} + c_{22}d_{21} & c_{21}d_{12} + c_{22}d_{22} \end{bmatrix} = A = CD$$

$$\begin{bmatrix} d_{11}v_1 + d_{12}v_2 \\ d_{21}v_1 + d_{22}v_2 \end{bmatrix} = w = Dv$$

$$\begin{bmatrix} (c_{11}d_{11} + c_{12}d_{21})v_1 + (c_{11}d_{12} + c_{12}d_{22})v_2 \\ (c_{22}d_{11} + c_{22}d_{21})v_1 + (c_{21}d_{12} + c_{22}d_{22})v_2 \end{bmatrix} = Av = (CD)v$$

$$\begin{bmatrix} c_{11}(d_{11}v_1 + d_{12}v_2) + c_{12}(d_{21}v_1 + d_{22}v_2) \\ c_{21}(d_{11}v_1 + d_{12}v_2) + c_{22}(d_{21}v_1 + d_{22}v_2) \end{bmatrix} = Cw = C(Dv)$$