CS130 - Parametric functions

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Given a parametric surface parameterized as $f(u,v) = \begin{pmatrix} u^2 - v^2 \\ 2uv \\ u^2 + v^2 \end{pmatrix}$ and a ray with endpoint $(-5, 1, 7)$ and direction $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

1. Normalize the ray's direction.

$$\sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{9} = 3$$

$$\hat{d} =< \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} >$$

2. Compute the intersection location and distance along the ray. *Hint: note that* $u \to -u$ *and* $v \to -v$ *results in the same point, so we may assume that* $u > 0$*. Solve for* u^2 *to find* u*. Then, eliminate* v*.*

$$R(t) = (-5, 1, 7) + t \cdot < \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} >$$

Turned into parametric components, we have:

$$R_x(t) = -5 + t \cdot \frac{2}{3}$$

$$R_y(t) = 1 + t \cdot \frac{1}{3}$$

$$R_z(t) = 7 + t \cdot \frac{-2}{3}$$

Then comparing them in the components of f(u,v):

$$u^2 - v^2 = -5 + t \cdot \frac{2}{3}$$

$$2uv = 1 + t \cdot \frac{1}{3}$$

$$u^2 + v^2 = 7 + t \cdot \frac{-2}{3}$$

Let's add the x component with the z component:

$$2u^2 - v^2 + v^2 = -5 + 7 + t \cdot \frac{2}{3} + t\frac{-2}{3}$$

$$2u^2 = 2$$

$$u^2 = 1$$

$$u = \pm 1$$

Now we can compare the v components for +1 (since -1 = u will just result in +1):

$$v_x = \sqrt{6 - t \cdot \frac{2}{3}}$$

$$v_y = \frac{1 + t \cdot \frac{1}{3}}{2}$$

$$v_z = \sqrt{6 + t \cdot \frac{-2}{3}} = \sqrt{6 - t \cdot \frac{2}{3}}$$

Compare x and y components:

$$\sqrt{6 - \frac{2}{3}t} = \frac{1 + \frac{1}{3}t}{2}$$

$$6 - \frac{2}{3}t = \frac{1 + \frac{1}{9}t^2}{4}$$

$$24 - \frac{8}{3}t = 1 + \frac{1}{9}t^2$$

$$0 = \frac{1}{9}t^2 + \frac{8}{3}t - 23$$

Using the quadratic formula we get two different intersection points:

$$t = -30.734, 6.734$$

Therefore our solution is at t = 6.734, and our coordinates are:

$$R(6.734) = (-1.489, 3.245, 2.511)$$

And our u,v is:

$$(1, 1.622)$$

3. Compute the normal direction for the surface at the intersection location.

Our gradient function is:

$$\triangledown f(u,v) = [\begin{pmatrix} 2u \\ 2v \\ 2u \end{pmatrix}, \begin{pmatrix} -2v \\ 2u \\ 2v \end{pmatrix}]$$

Then we calculate the gradients in each direction:

$$\triangledown f(1, 1.622) = [\begin{pmatrix} 2 \\ 3.244 \\ 2 \end{pmatrix}, \begin{pmatrix} -3.244 \\ 2 \\ 3.244 \end{pmatrix}]$$

And then we cross them:

$$< 2, 3.244, 2 > \times < -3.244, 2, 3.244 >$$

We can manually do this by getting the determinant of the following matrix:

$$\begin{pmatrix} i & j & k \\ 2 & 3.244 & 2 \\ -3.244 & 2 & 3.244 \end{pmatrix}$$

$$n = (10.527 - 4)i - (6.489 + 6.489)j + (4 + 10.527)k$$

$$n =< 6.527, 12.979, 14.527 >$$

$$\hat{n} =< 0.318, 0.632, 0.707 >$$