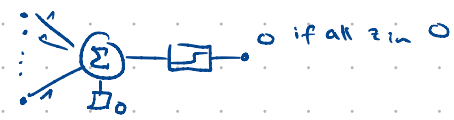


# Ex 2 - Task 1

Charlotte Doll and Johannes Bahrenberg

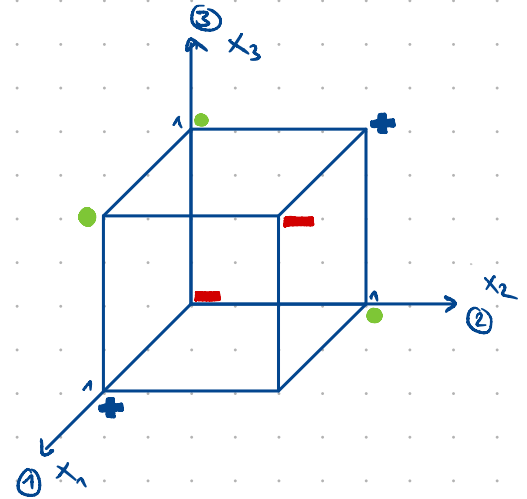
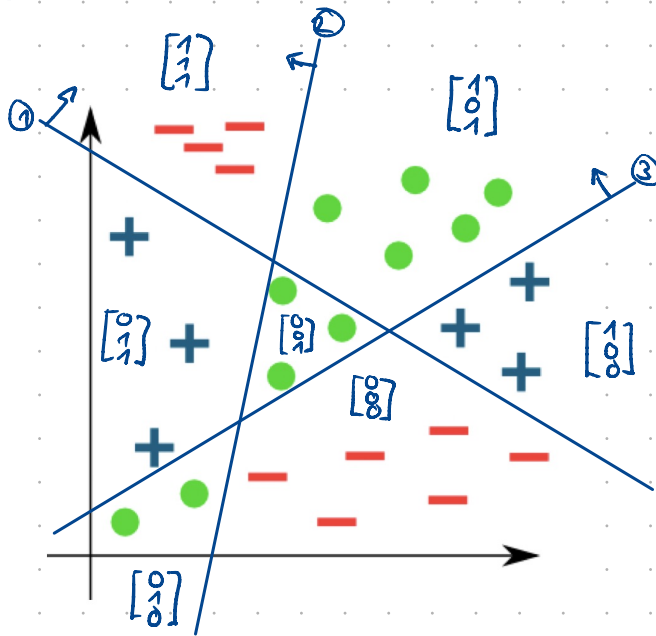


logical or :  $z_{out} = f(z) = \Psi(z \cdot \beta + b)$  with  $\Psi = \sigma$  sigmoid fct.,  $\beta = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$ ,  $b = -50$

masked logical or :  $z_{out} = g(z; c) = \Psi(z \cdot \beta + b; c)$  with  $\Psi = \sigma$ ,  $\beta = c \cdot 100$ ,  $b = -50$

perfect match :  $z_{out} = h(z; c) = \Psi(\beta \cdot \|z - c\| + b)$   $\beta = -100$  scalar now,  $b = 50$ ,  $\Psi = \sigma$

layer 1:



layer 2:

green:  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

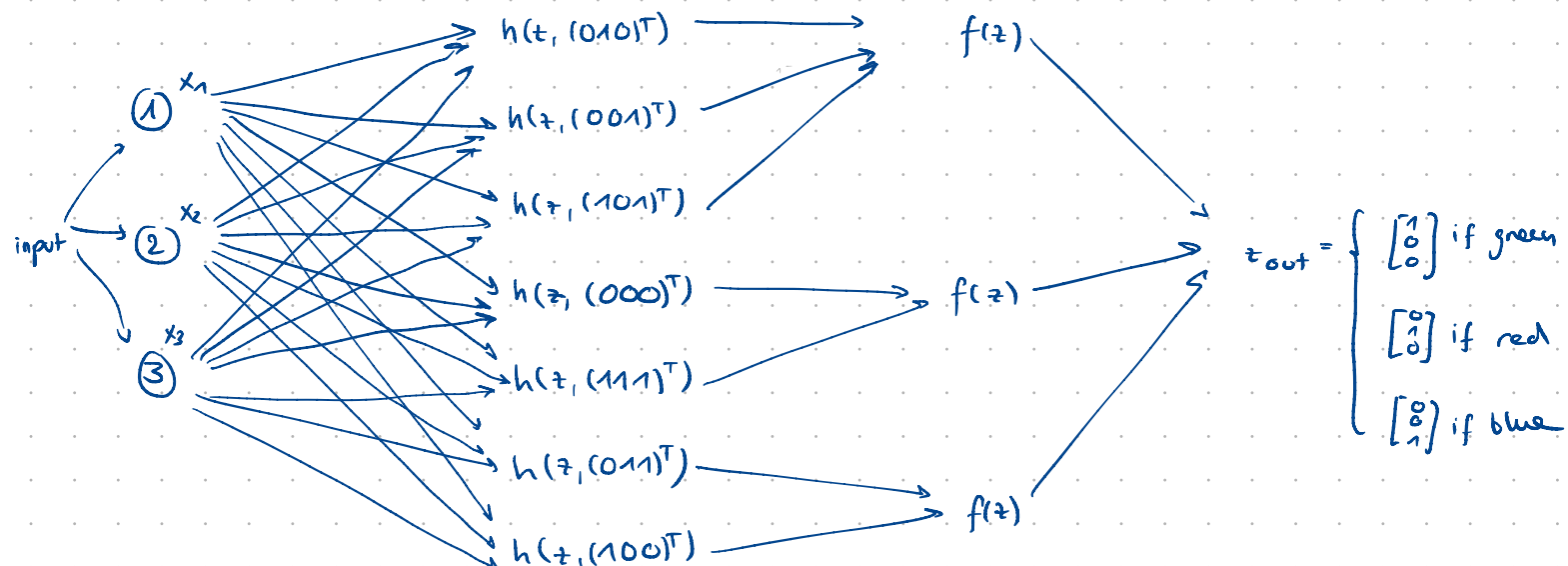
red:  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

blue:  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

layer 1

layer 2

layer 3



## Ex 2 - Task 2

$$z_0 = X, \quad \tilde{z}_L = z_{L-1} \cdot B_L + b_L, \quad z_L = \phi_L(\tilde{z}_L) \quad X, z_L, b_L : \text{row vectors}$$

assume  $\phi_L$  is identity fct  $\rightarrow z_L = \tilde{z}_L$

$$\text{for } L > 1 \text{ layers} \rightarrow z_L = (\dots ((z_0 B_1 + b_1) B_2 + b_2) B_3 + b_3) \dots B_L + b_L$$

$$= (\dots (z_0 B_1 B_2 + b_1 B_2 + b_2) B_3 + b_3) \dots B_L + b_L$$

$$= (\dots (z_0 B_1 B_2 B_3 + b_1 B_2 B_3 + b_2 B_3 + b_3) \dots) B_L + b_L$$

$$= z_0 \cdot \underbrace{\prod_{i=1}^{L-1} B_i}_{\text{new 1-layer weights}} + \sum_{i=1}^L \underbrace{b_i \prod_{j=i+1}^L B_j}_{\text{new 1-layer bias}}$$

new 1-layer weights      new 1-layer bias