$\sum_{i=1}^{N} X_i = 0 \qquad N_1 = N_{-1} = N/2$

$$\mu_{-1} = \frac{1}{N_{-1}} \sum_{i:\, y_i^* = -1} X_i \qquad \mu_1 = \frac{1}{N_1} \sum_{i:\, y_i^* = 1} X_i \qquad \Sigma = \frac{1}{N} \left[\sum_{i:\, y_i^* = -1} (X_i - \mu_{-1})^T \cdot (X_i - \mu_{-1}) + \sum_{i:\, y_i^* = 1} (X_i - \mu_1)^T \cdot (X_i - \mu_1) \right]$$

$$\frac{\partial}{\partial \beta} \sum_{i=1}^{N} (y_i^* - X_i \cdot \beta)^2 \stackrel{!}{=} 0$$

$\mu_1 + \mu_{-1} = 0$

$\mu_1 = -\mu_{-1}$

$$\Sigma \cdot \beta + \frac{1}{4}(\mu_1 - \mu_{-1})^T \cdot (\mu_1 - \mu_{-1}) \cdot \beta = \frac{1}{2}(\mu_1 - \mu_{-1})^T$$

$= \mu_1^T \mu_1 - \mu_1^T \mu_{-1} - \mu_{-1}^T \mu_1 + \mu_{-1}^T \mu_{-1}$

$$0 = \sum_{i=1}^{N} -2 x_i^T (y_i^* - X_i \beta) = -2 \sum_{i=1}^{N} (y_i^* x_i^T - x_i^T (X_i \beta)) \quad | : 2 \quad | \cdot \frac{1}{N}$$

$$0 = -\left[\frac{1}{N} \sum_{y_i^* = -1} x_i^T + \frac{1}{N} \sum_{y_i^* = 1} x_i^T - \frac{2}{N} \sum_{i=1}^{N} x_i^T (x_i \beta) \right]$$

$$= -\left(-\mu_{-1}^T + \mu_1^T \right) + \frac{2}{N} \left(\sum_{i:\, y_i = 1} x_i^T (x_i \beta) + \sum_{i:\, y_i = -1} x_i^T (x_i \beta) \right) \quad | + (\mu_1 - \mu_{-1})^T \quad | : 2$$

$$\frac{1}{2}(\mu_1 - \mu_{-1})^T = \frac{1}{N} \left(\sum_{y_i = -1} x_i^T x_i + \sum_{y_i = 1} x_i^T x_i \right) \beta$$

$$= \frac{1}{N} \left[\sum_{y_i = 1} (x_i^T - \mu_1^T + \mu_1^T)((x_i - \mu_{-1}) - \mu_1) + \sum_{y_i = -1} (x_i^T - \mu_{-1}^T + \mu_{-1}^T)((x_i - \mu_1) - \mu_{-1}) \right] \beta$$

$$= \frac{1}{N} \left[\sum_{y_i = 1} \left((x_i - \mu_1)^T (x_i - \mu_1) - (x_i - \mu_1)^T \mu_1 - \mu_1^T (x_i - \mu_1) + \mu_1^T \mu_1 \right) \right.$$

$$\left. + \sum_{y_i = -1} \left((x_i - \mu_{-1})^T (x_i - \mu_{-1}) - (x_i - \mu_{-1})^T \mu_{-1} - \mu_{-1}^T (x_i - \mu_{-1}) + \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \frac{1}{N} \left[\underbrace{\sum_{y_i = 1} (x_i - \mu_1)^T (x_i - \mu_1) + \sum_{y_i = -1} (x_i - \mu_{-1})^T (x_i - \mu_{-1})}_{= N \cdot \Sigma} + \sum_{y_i = 1} \left[-x_i^T \mu_1 - \mu_1^T x_i \right] + \sum_{y_i = -1} \left[-x_i^T \mu_{-1} - \mu_{-1}^T x_i \right] \right.$$

$$\left. + \mu_{-1} \cdot \left(\mu_{-1}^T \mu_1 + \mu_1^T \mu_{-1} + \mu_1^T \mu_1 \right) + \mu_1 \left(\mu_1^T \mu_{-1} + \mu_{-1}^T \mu_1 + \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \frac{1}{N} \left[N \Sigma + \frac{N}{2} \left(-\mu_{-1}^T \mu_1 - \mu_1^T \mu_{-1} - \mu_1^T \mu_{-1} - \mu_{-1}^T \mu_1 \right) + \frac{N}{2} \left[\mu_{-1}^T \mu_1 + \mu_1^T \mu_{-1} \right] \right] \beta$$

$$= \left[\Sigma + \frac{1}{2} \left(\mu_1^T \mu_1 + \underbrace{2 \mu_{-1}^T \mu_1 - 2 \mu_{-1}^T \mu_{-1} - 2 \mu_1^T \mu_{-1}}_{= 0} + 2 \mu_1^T \mu_{-1} + \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \left[\Sigma + \frac{1}{2} \left(\mu_1^T \mu_1 + \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \left[\Sigma + \frac{1}{4} \left(2 \mu_1^T \mu_1 + 2 \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \left[\Sigma + \frac{1}{4} \left(\mu_1^T \mu_1 - \mu_1^T \mu_{-1} - \mu_{-1}^T \mu_1 + \mu_{-1}^T \mu_{-1} \right) \right] \beta$$

$$= \Sigma \beta + \frac{1}{4} \left((\mu_1 - \mu_{-1})^T \cdot (\mu_1 - \mu_{-1}) \right) \beta \qquad = \frac{1}{2}(\mu_1 - \mu_{-1})^T$$