Mathematics Behind Support Vector Machine

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upport Vector Machine (SVM) Mathematical Formulas

1. **Equation of the Hyperplane**

$$w \cdot x + b = 0$$

This equation represents the decision boundary that separates two classes.

2. **Classifier Function**

$$h(x) = \{+1, if w \cdot x + b \geq 0 - 1, if w \cdot x + b < 0$$

This function determines the class of a given data point.

3. **Margin Optimization (Hard Margin SVM)**

$$\min_{w,b} \frac{1}{2}||w||^2$$

Subject to:

$$y_i(w \cdot x_i + b) \geq 1, \quad \forall i$$

This ensures that all points are correctly classified and lie outside the margin.

4. **Soft Margin SVM (Handling Overlapping Classes)**

$$\min_{w,b,\xi} \frac{1}{2}||w||^2 + C\sum_i \xi_i$$

Subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \xi_i, \quad \xi_i \geq 0$$

The slack variable ξ_i allows some misclassification, controlled by the regularization parameter C.

5. **Lagrangian Dual Formulation**

$$\mathcal{L}(w, b, \alpha) = \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j (x_i \cdot x_j)$$

Subject to:

$$\sum_i \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C$$

This formulation allows solving SVM efficiently using quadratic programming.

6. **Kernel Trick (Handling Non-Linearity)**

$$K(x_i, x_j) = \phi(x_i) \cdot \phi(x_j)$$

This transformation maps data to a higher-dimensional space for better separation.

7. **Common Kernel Functions** - Linear Kernel:

$$K(x_i, x_j) = x_i \cdot x_j$$

- Polynomial Kernel:

$$K(x_i, x_j) = (x_i \cdot x_j + c)^d$$

- **Radial Basis Function (RBF) Kernel**:

$$K(x_i, x_j) = \exp(-\gamma ||x_i - x_j||^2)$$

These formulas define how SVMs work mathematically, including margin maximization, dual optimization, and kernel transformations.