Example document in LaTeX about the square root function

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March 14, 2020

This is a quick document about the square root function. The square root function is the solution to the simple differential equation

$$y' = \frac{1}{2y}. \tag{1}$$

This differential equation can for example be solved by a numerical ordinary differential equation solver, like the *Runge-Kutta method*. By doing that, one can obtain data points that can be used to create a plot of the function, like the one seen in figure 1.
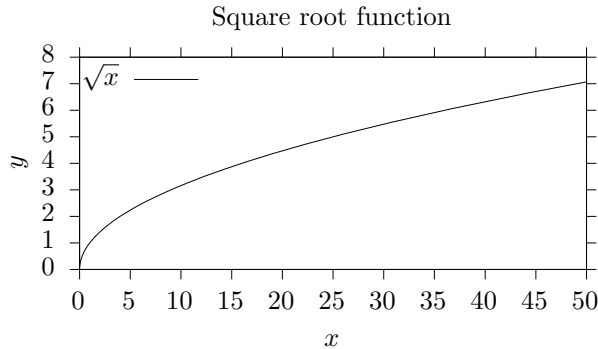

Figure 1: Plot of the function $\sqrt{x}$. The plot has been created using Gnuplot.

1 Square roots of positive integers

The following paragraphs are taken directly from Wikipedia[1]. A positive number has two square roots, one positive, and one negative, which are opposite to each other. So, when talking of the square root of a positive integer, this is the positive square root that is meant.

The square roots of an integer are algebraic integers and, more specifically, quadratic integers.

The square root of a positive integer is the product of the roots of its prime factors, because the square root of a product is the product of the square roots of the factors. Since $\sqrt{p^{2k}} = p^k$, only roots of those primes having an odd power in the factorization are necessary. More precisely, the square root of a prime factorization is

$$\sqrt{p_1^{2e_1+1} \ldots p_k^{2e_k+1} p_{k+1}^{2e_k+1} \ldots p_n^{2e_n}}$$
$$= p_1^{e_1} \ldots p_n^{e_n} \sqrt{p_1 \ldots p_k}. \tag{2}$$

2 Practical application of the square root

The square root is a very useful function. The function makes it easy to calculate for example the distance between two points in n-dimensional Euclidean space. This can be done as

$$l = \sqrt{\Delta x_1^2 + \Delta x_2^2 \ldots \Delta x_n^2}. \tag{3}$$

References

[1] Wikipedia,
    https://en.wikipedia.org/wiki/Square_root