

Example document in L^AT_EX about the square root function

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This is a quick document about the square root function. The square root function is the solution to the simple differential equation

$$y' = \frac{1}{2y}. \quad (1)$$

This differential equation can for example be solved by a numerical ordinary differential equation solver, like the *Runge-Kutta method*. By doing that, one can obtain data points that can be used to create a plot of the function, like the one seen in figure 1.

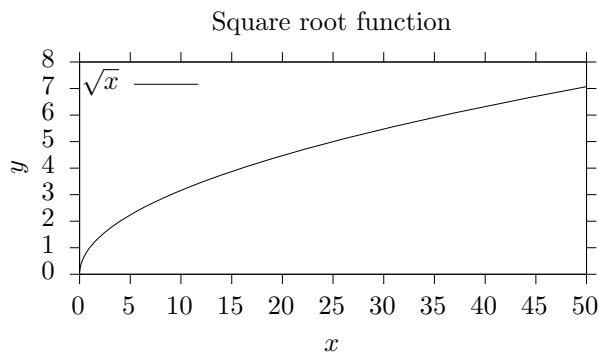


Figure 1: Plot of the function \sqrt{x} . The plot has been created using Gnuplot.

1 Square roots of positive integers

The following paragraphs are taken directly from Wikipedia[1]. A positive number has two square

roots, one positive, and one negative, which are opposite to each other. So, when talking of the square root of a positive integer, this is the positive square root that is meant.

The square roots of an integer are algebraic integers and, more specifically, quadratic integers.

The square root of a positive integer is the product of the roots of its prime factors, because the square root of a product is the product of the square roots of the factors. Since $\sqrt{p^{2k}} = p^k$, only roots of those primes having an odd power in the factorization are necessary. More precisely, the square root of a prime factorization is

$$\begin{aligned} \sqrt{p_1^{2e_1+1} \cdots p_k^{2e_k+1} p_{k+1}^{2e_{k+1}} \cdots p_n^{2e_n}} \\ = p_1^{e_1} \cdots p_n^{e_n} \sqrt{p_1 \cdots p_k}. \end{aligned} \quad (2)$$

2 Practical application of the square root

The square root is a very useful function. The function makes it easy to calculate for example the distance between two points in n-dimensional Euclidean space. This can be done as

$$l = \sqrt{\Delta x_1^2 + \Delta x_2^2 \cdots \Delta x_n^2}. \quad (3)$$

References

- [1] Wikipedia,
https://en.wikipedia.org/wiki/Square_root