The Minesweeper-π Conjecture

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1 Introduction

Minesweeper is an old single player game with no known origins. Its popularity grew as software companies typically included it as an out-of-the-box game with their operating systems starting in the early 90s.

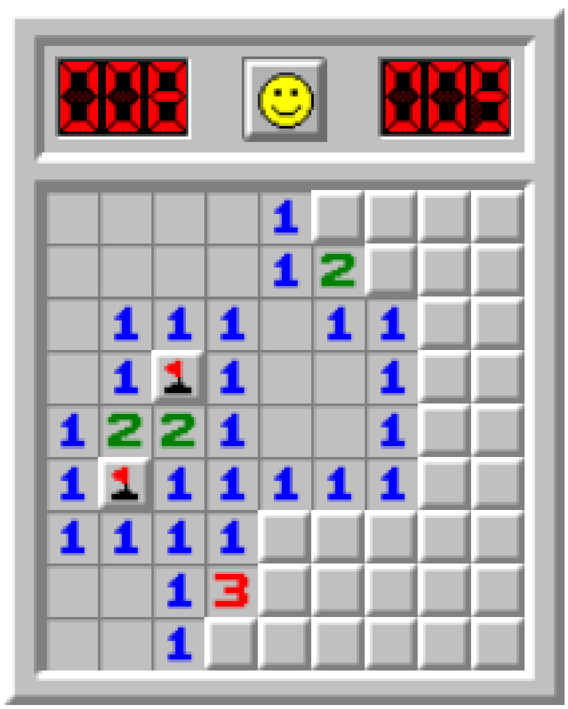

Figure 1: A game of Minesweeper

The rules of the game are simple; on an $n * m$ sized grid with l number of mines, the target for the player is to flag all fields on the grid that contain mines, without reveling the mine fields themselves. Numbers on fields adjacent to mine fields will tell the player how many mines are in its direct vicinity (vertically, horizontally or diagonally), and thus each field not containing a mine will have a number between 0 and 8.

2 Game restrictions

In this case, the grid, hereafter called the "board", must adhere to the following rules.

- The board must be quadratic, with side length of N.
- There must be N mines on a board with dimensions $N * N$.
- N must be greater than, or equal to, 2.

3 Observations

A Monte Carlo Simulation was carried out to check the average sums S for boards of size $N = 2$ up to $N = 200$ with $R = 100$ repetitions. The algorithm is given as follows.

Algorithm 1 Monte Carlo algorithm

$averages \leftarrow []$
for $i = 2...R$ **do**
  Set up B boards
  $avg \leftarrow$ avg sum for all B
  $averages \leftarrow avg$
end for
$avg\_diff \leftarrow []$
for $i = len(averages)...0$ **do**
  $avg\_diff \leftarrow (averages_i - averages_{i-1})$
end for
$total\_avg \leftarrow average(avg\_diff)$
$res \leftarrow (total\_avg/2.5)$ ▷ $res \approx \pi$ at this stage

The full code can be found at GitHub[1] From this algorithm we can deduce the following by doing some algebra.

$(S_1 - S_0) + (S_2 - S_1) + ... + (S_n - S_{(n-1)})/N$

will telescope and can be expressed as

$(S_n - S_0)/N$

[1] GitHub code for Minesweeper with Monte Carlo algorithm given under the `if __name__ == "__main__"` section:
`https://gist.github.com/henrik2706/1ab2bed48bab098f4a9000274d41c042`

It follows that

$$\lim_{N\to\infty} S_n - S_0/N = S_n/N$$

What we wish to prove is that

$$\lim_{N\to\infty} S_n/N \to 5\pi/2$$

This is plausible. Looking at edge effects and overlap, we get that $S_n = 8N$, which would have given $S_n/N \to 8$. Edge effects and overlap makes S_n a bit lower than $8N$, and we get

$$S_n/N \to 5\pi/2 \approx 7.854$$

4 Conjecture

The observations point to the following conjecture.

1. We define S_n as the average sum of all numbers on a quadratic Minesweeper board with dimensions $n * n$, and a number of mines equal to n.

2. We define S_{n-1} as the average sum of all numbers on a quadratic Minesweeper board with dimensions $(n-1) * (n-1)$, and a number of mines equal to $n-1$.

3. Subtracting S_{n-1} from S_n yields $\approx 7,85398 = 2.5\pi$.