# Homework #2

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## Problem 1)

### a)

Forces applied to the car:

and if we can rewrite it as:

### b)

Design specification: 0 km/h to 100 km/h in 8 seconds with an overshoot less than 20%

#### i)

As stated in Problem 1 part a), the system can be represented by:

We take the Laplace transform:

We get:

The PI controller:

Closed loop:

We need an overshoot less than 20%:

We can find and from:

I assume that the car "reaches 100km/h" when at the settling time ,and not at the time it raises above 100km/h for the first time:

Therefore, we want the following closed loop transfer function H(s):

We get the following gains:

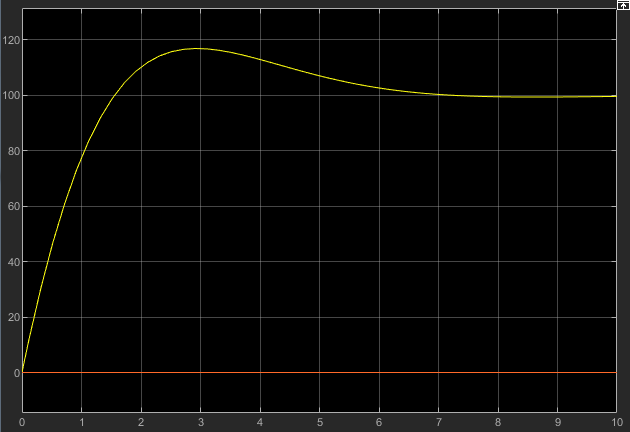
When simulating with these gain values, the overshoot is around 28% which is not acceptable. We increase the damping factor by half:

When simulating with these new gain values, the overshoot is 16,8% and a settling time of 7 seconds, which is acceptable.

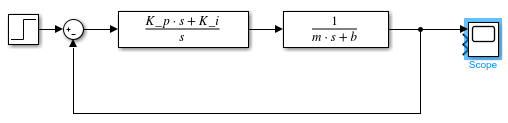
Therefore, our PI controller is:

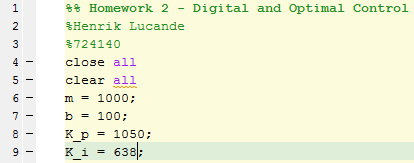
#### ii)

Simulink:



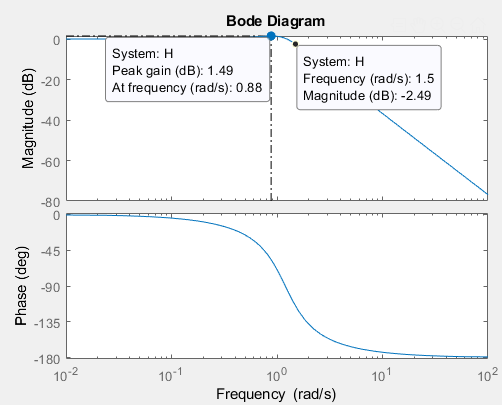
We get an overshoot of 16.8% and a settling time of 7 seconds.



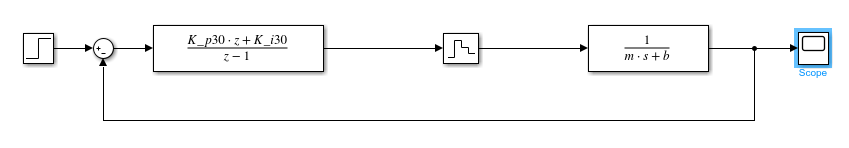


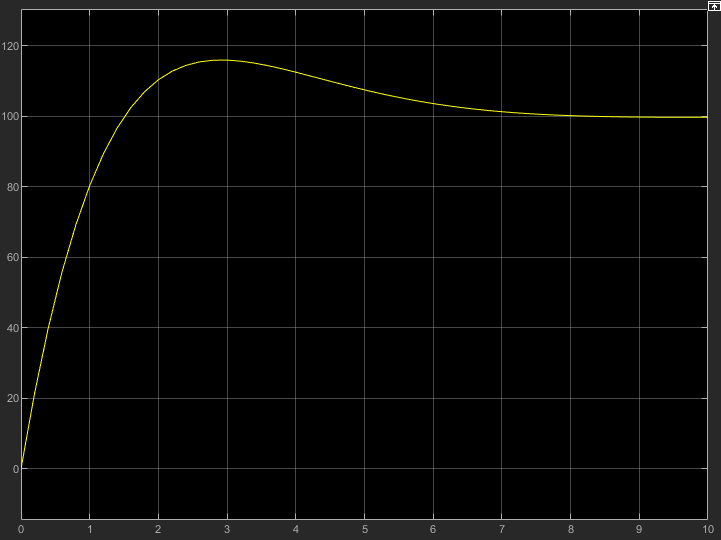
### c)

We get the bandwidth from the bode diagram. Bandwidth is the frequency where magnitude drops -3dB from peak value. Therefore, bandwidth = 1.5rad/s = 0.2387 Hz



Discretization with Tustin method where

With the discretized controller with a sampling rate 30 times the bandwidth, we got the following results:

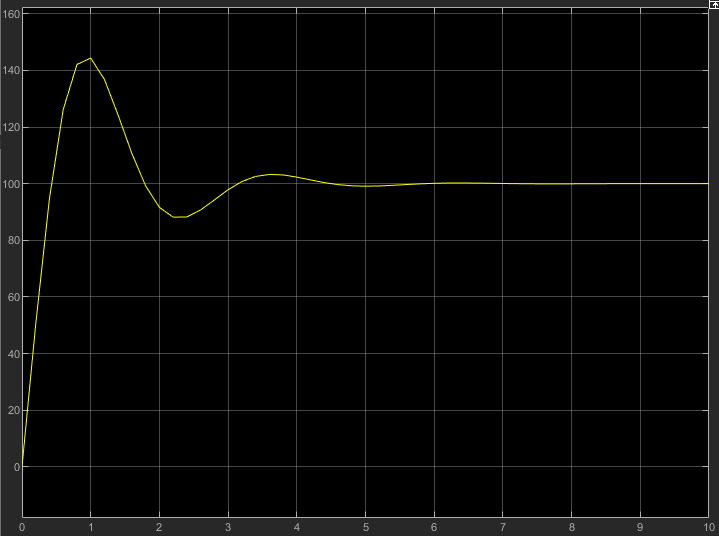


As we can see from the plot above, performance is almost the same as with the continuous controller. With an 16% which is slightly less than the continuous one, but with a slightly slower settling time.

### d)

#### i)

We discretize the controller with a sampling rate 6 times the bandwidth using the Tustin transformation.

With this controller we get the following response:

As we can see from the plot above, the controller doesn’t perform nearly as good as the continuous one. We get an overshoot of 44.3%, which is far from the system specifications.

#### ii)

We discretize the plant with :

Our digital controller:

Our closed loop transfer function therefore is:

We need to find the desired H(z) by using the zeta and omega from the continuous case:

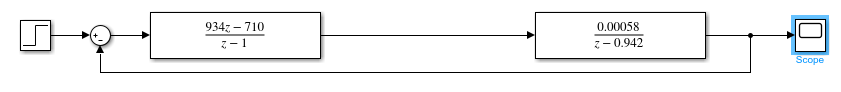
From the “s-plane to z-plane”-map we get:

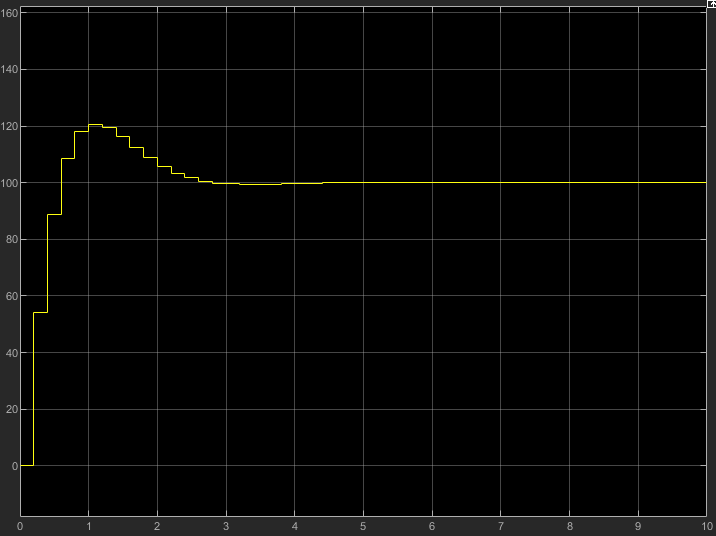
From which we get the desired H(z):

Now we can get the following gains:

We get our final controller:

Now we simulate:





As we can see from the plot above, the discretized plant with a sampling rate of 6 times the bandwidth with a discrete PI controller, does not meet the design specifications.

We get an overshoot of 20.7%

The m-code used in this homework is short as most was calculated for hand in this document (in the last simulation I just added the gains straight to Simulink):

