Final Exercise

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Problem 1)

a)

Outputs of the system:	$I_1(t), q(t)$
Inputs of the system:	E(t)
Constants of the system:	R_1, R_2, R_3, L, C
Internal time-varying variables of the system:	$I_2(t), I_3$(t)

Where q(t) stands for the charge of the capacitor C

b)

Left loop:

$$U_1(t) + U_2(t) + U_L(t) = 0$$

$$R_1 I_1(t) + R_2 I_2(t) + L * \frac{d}{dt} I_1(t) = 0$$

$$R_1 I_1(t) + R_2\big(I_1(t) - I_3(t)\big) + L * \frac{dI_1(t)}{dt} = 0$$

$$R_1 I_1(t) + R_2 I_1(t) - R_2 I_3(t) + L * \frac{dI_1(t)}{dt} = 0$$

$$\frac{dI_1(t)}{dt} = -\frac{R_1 + R_2}{L} * I_1(t) + \frac{R_2}{L} * I_3(t)$$

$$\boldsymbol{\frac{dI_1(t)}{dt} = -\frac{R_1 + R_2}{L} * I_1(t) + \frac{R_2}{L} * \frac{dq(t)}{dt}}$$

Right loop:

$$U_3(t) + U_c(t) - U_2(t) = E(t)$$

$$R_3 I_3(t) + \frac{q(t)}{C} - R_2 I_2 = E(t)$$

$$R_3 \frac{dq(t)}{dt}(t) + \frac{q(t)}{C} - R_2(I_1 - I_3) = E(t)$$

$$R_3 \frac{dq(t)}{dt}(t) + \frac{q(t)}{C} - R_2 I_1 + R_2 \frac{dq(t)}{dt} = E(t)$$

$$R_3 \frac{dq(t)}{dt}(t) + \frac{q(t)}{C} - R_2 I_1 + R_2 \frac{dq(t)}{dt} = E(t)$$

$$(R_3 + R_2)\frac{dq(t)}{dt}(t) = E(t) - \frac{q(t)}{C} + R_2 I_1$$

$$\boldsymbol{\frac{dq(t)}{dt} = \frac{E(t)}{R_3 + R_2} - \frac{1}{(R_3 + R_2)C} q(t) + \frac{R_2}{R_3 + R_2} I_1(t)}$$

States:

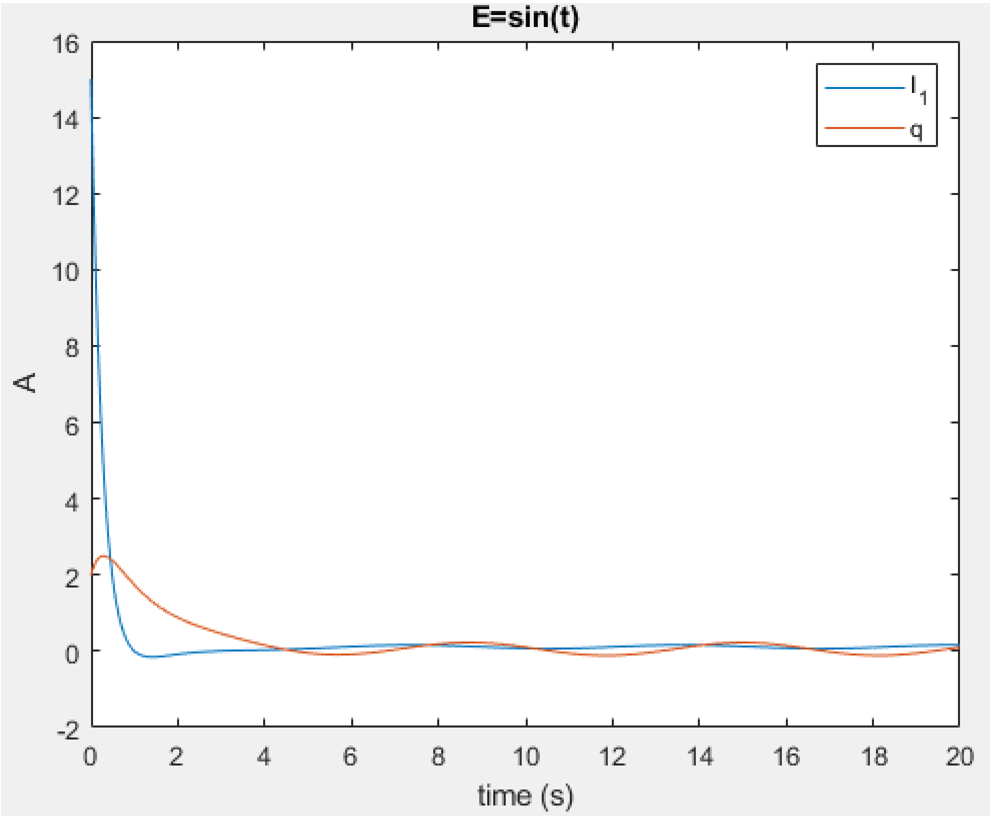
$\boldsymbol{x_1 = I_1}$

$\boldsymbol{x_2 = q}$

$$\dot{x}_1 = \dot{I}_1 = -\frac{R_1+R_2}{L}x_1 + \frac{R_2}{L}\dot{x}_2 = -\frac{R_1+R_2}{L}x_1 + \frac{R_2}{L}\left(\frac{E(t)}{R_3+R_2} - \frac{1}{(R_3+R_{2)C}}x_2 + \frac{R_2}{R_3+R_2}x_1\right)$$
$$= -\frac{R_1+R_2}{L}x_1 + \frac{R_2}{(R_3+R_2)L}E(t) - \frac{R_2}{(R_3+R_2)LC}x_2 + \frac{R_2R_2}{(R_3+R_2)L}x_1$$
$$= \boldsymbol{\left(\frac{R_2R_2}{(R_3+R_2)L} - \frac{R_1+R_2}{L}\right)x_1 - \frac{R_2}{(R_3+R_2)LC}x_2 + \frac{R_2}{(R_3+R_2)L}E(t)}$$

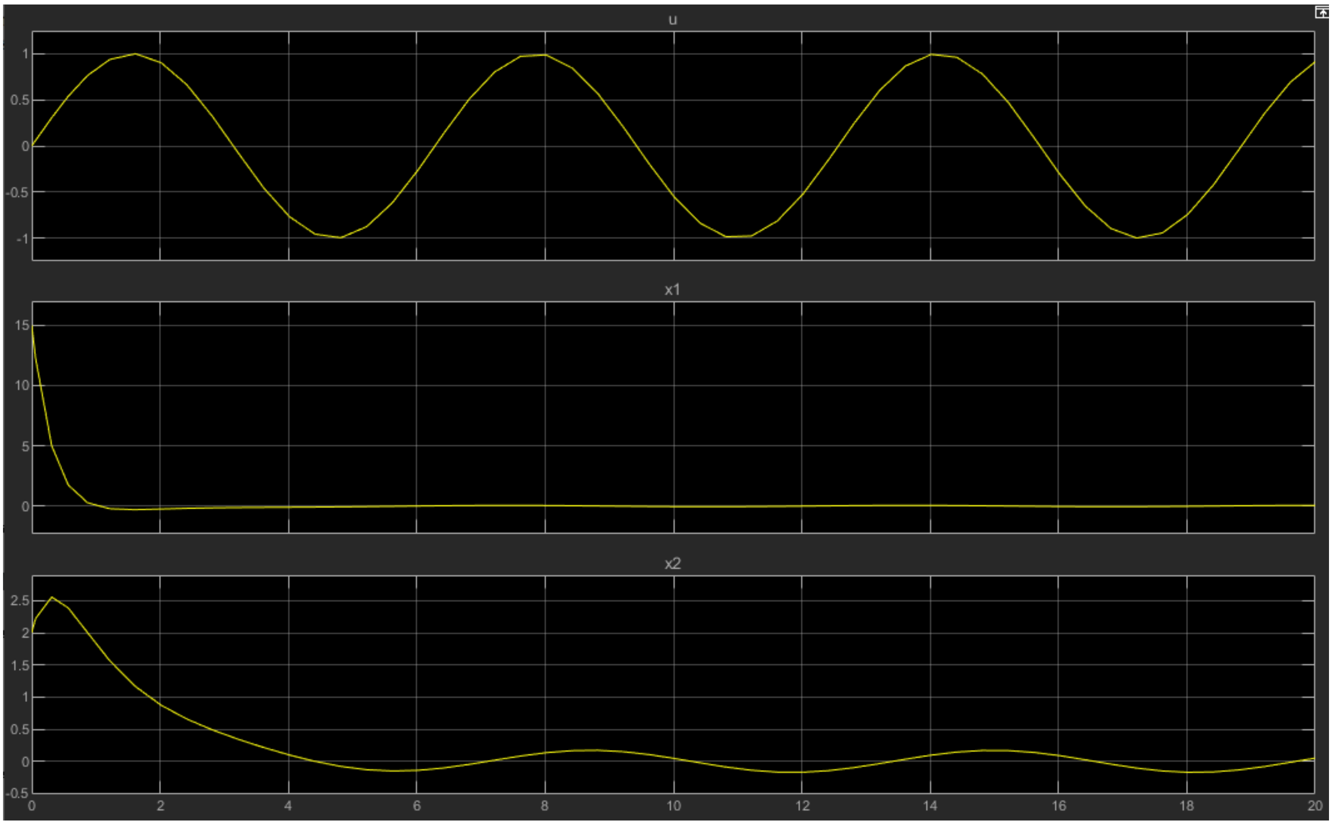
$$\boldsymbol{\dot{x}_2 = \dot{q} = \frac{R_2}{R_3+R_2}x_1 - \frac{1}{(R_3+R_2)C}x_2 + \frac{1}{R_3+R_2}E(t)}$$

c)

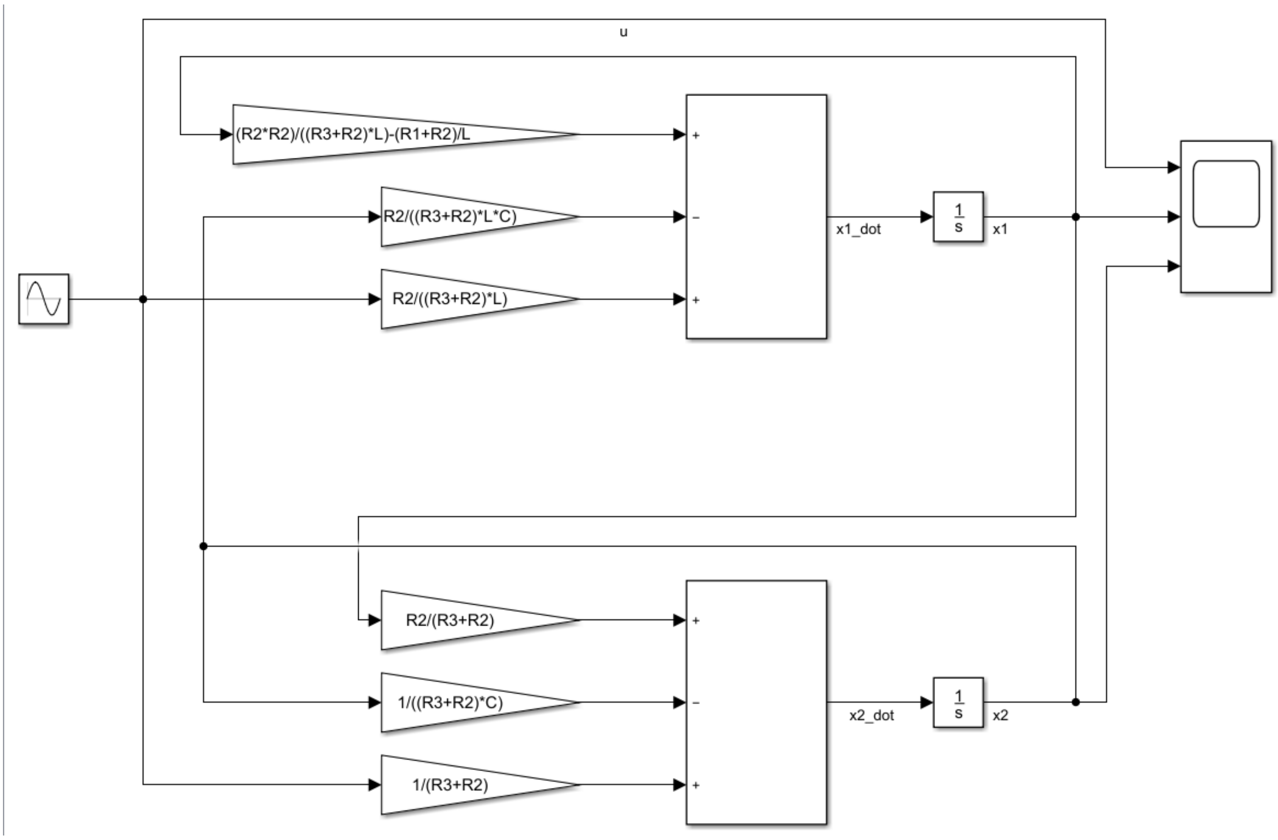


Differential equations solved with ode45 and plotted.

d)



The scope of the Simulink.



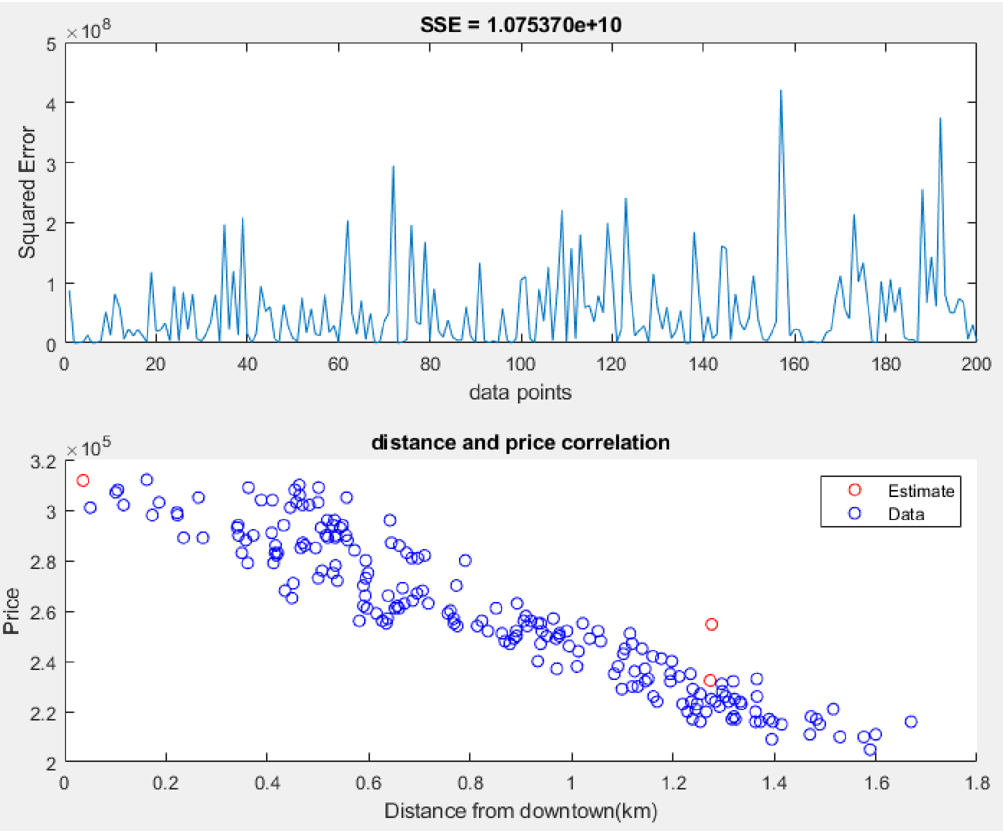
The Simulink used.

Problem 2)

Prices of the properties in table 1.

	Price
House No. 1	232335
House No. 2	254638
House No. 3	311722

SSE	10753704297
SST	168839120000
R^2	0.936
Variance	55431465



I plotted with distance from downtown as x and price as y, as the most important parameter regarding the price of the house is the distance from downtown, they correlate very strongly together. The variance is explained mostly by differences in area and year when the apartment is built. Also, the floor on which the apartment is and the number of rooms have a small impact on the price of the apartment.

We can see from the plot that its linear and we can see that our estimations for the houses 1-3 are quite well in line with the rest of the data. As $R^2 = 0.936$ our estimation is quite good. In other words, the sum of the square errors (SSE) is less than the sum of squares total (SST).

Problem 3)

Dataset 1)

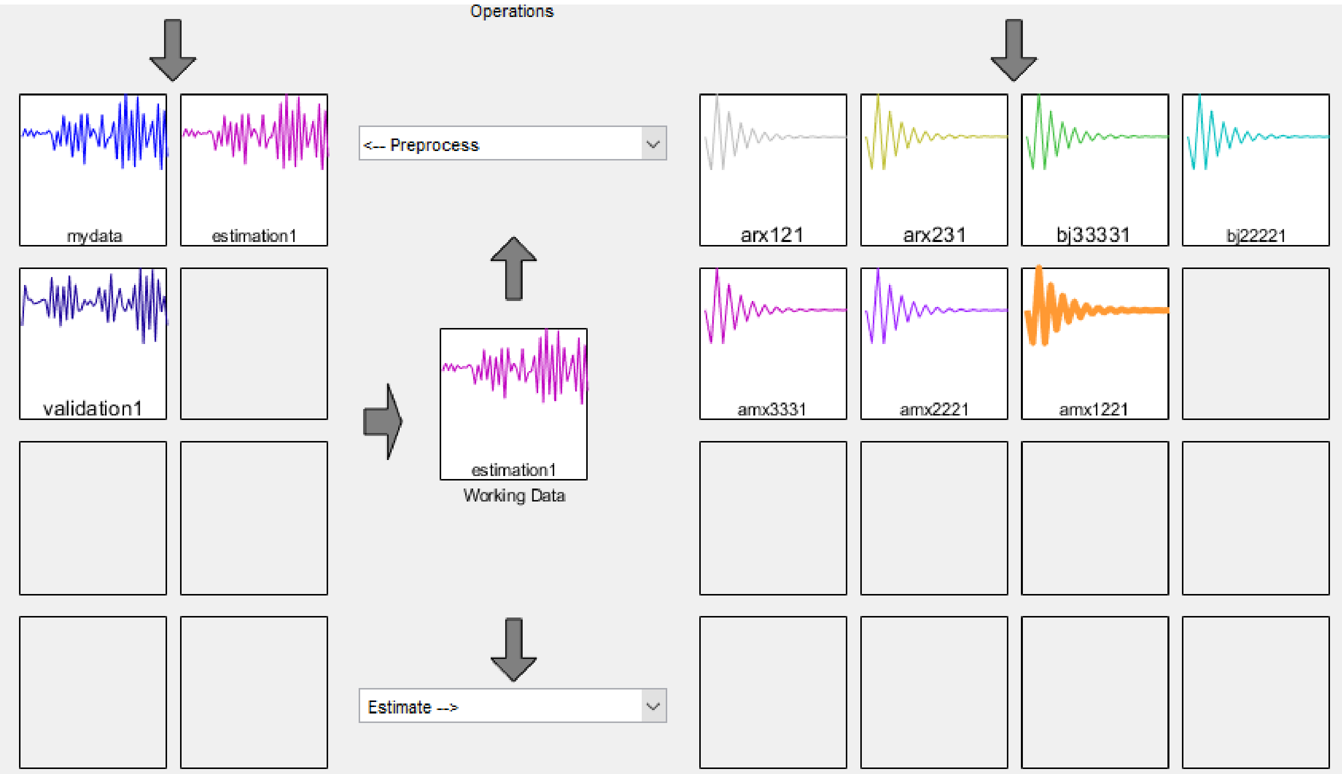
a)

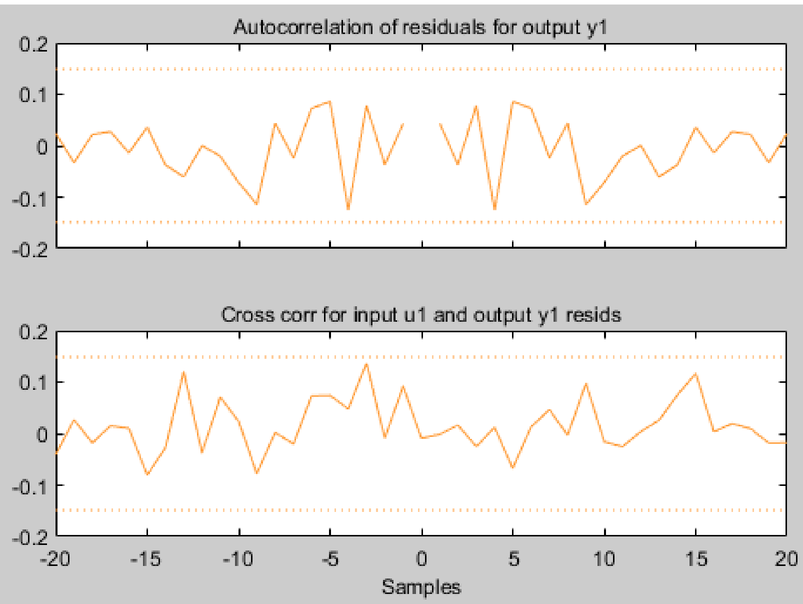
chosen model: amx1221

b)

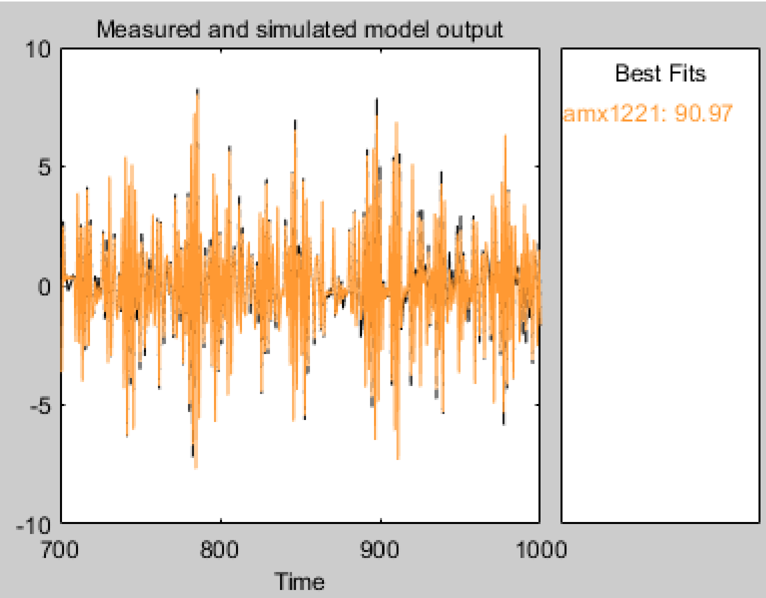
When determining the delay, all the arx models had the same delay. Therefore, delay of 1 was chosen.

$n_k = 1$

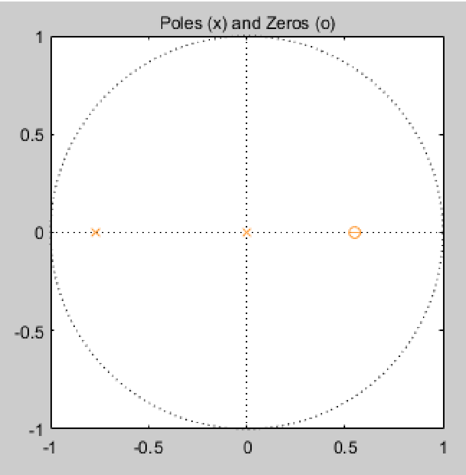




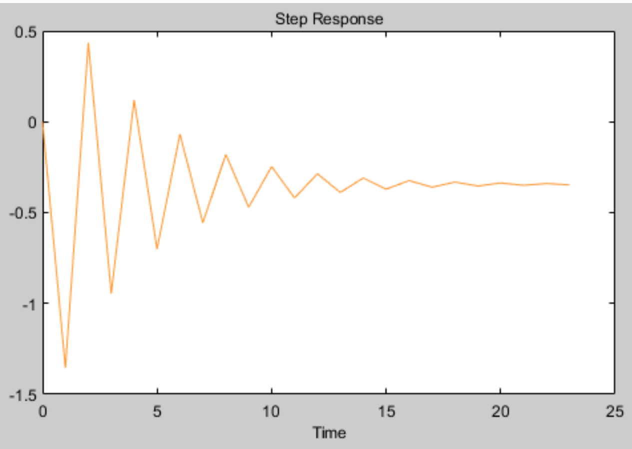
Inside confidence levels. Does not go over the bands.

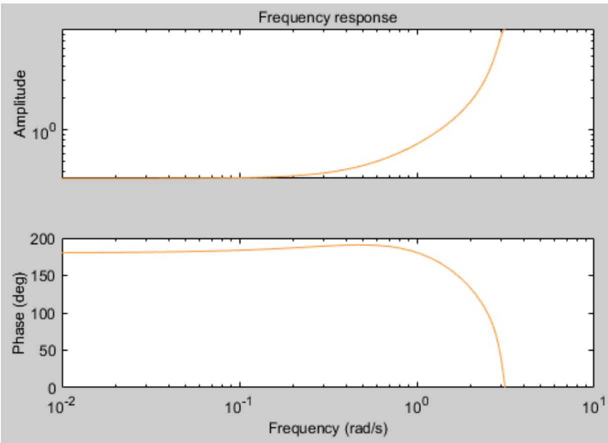


The model is a good fit: 90.97



Poles and zeros are within the unit circle and they are not close together.





```
amx1221 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 + 0.7712 (+/- 0.0004324) z^-1

  B(z) = -1.355 (+/- 0.007083) z^-1 + 0.7456 (+/- 0.008892) z^-2

  C(z) = 1 + 1.513 (+/- 0.03178) z^-1 + 0.5476 (+/- 0.03178) z^-2
```

The standard deviation analysis is good.

c)

The best model was quite obvious and a very good fit, so no alternative models were considered.

Dataset 2)

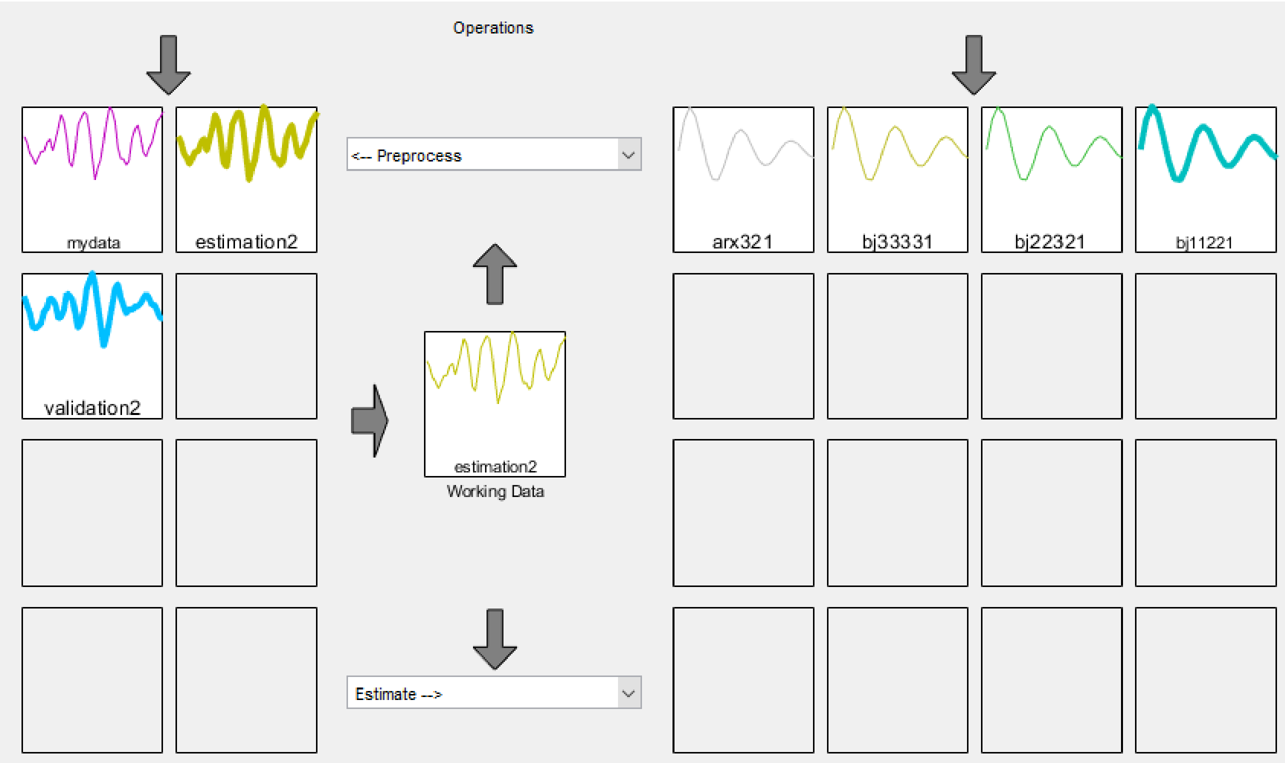
a)

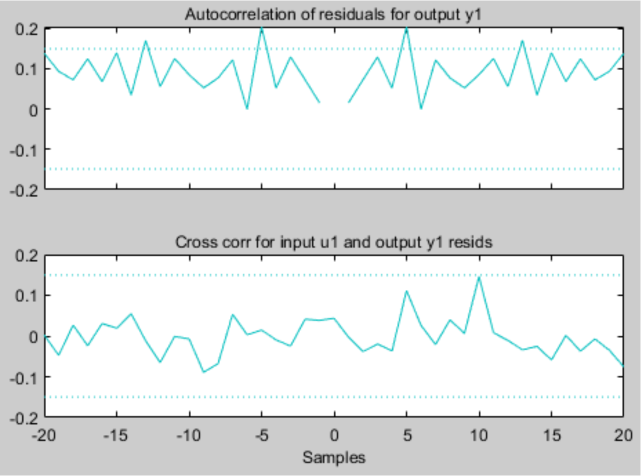
chosen model: bj11221

b)

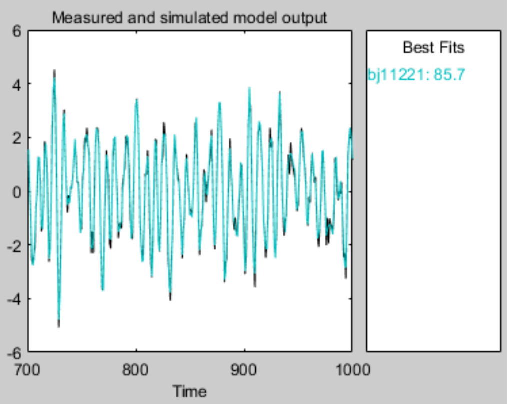
When determining the delay, all the arx models had the same delay. Therefore, delay of 1 was chosen.

$n_k = 1$

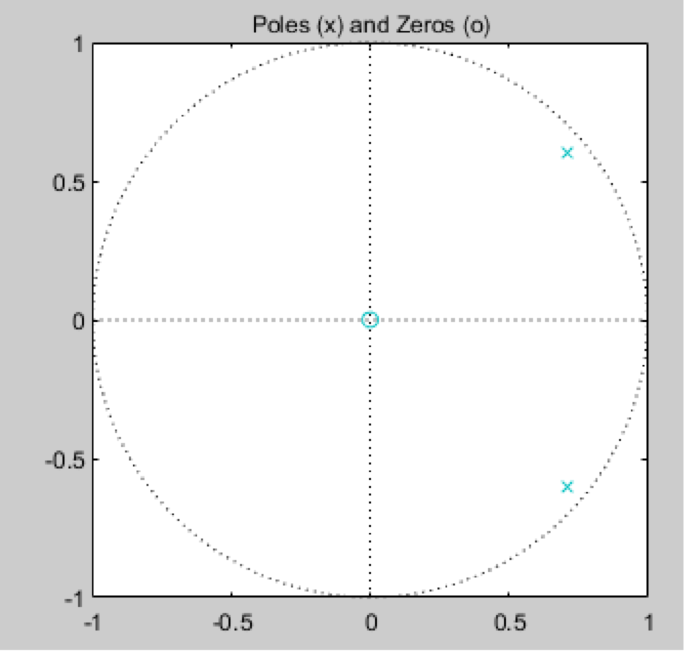




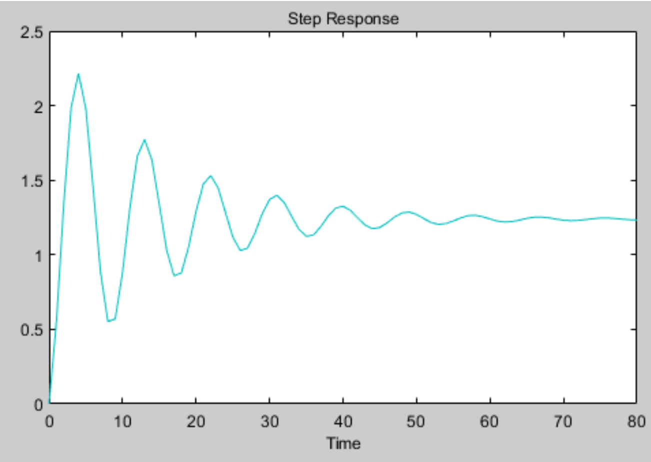
Inside confidence levels. Does not go over the bands (autocorrelation is shifted but it is okay as the variance is smaller than the bands).

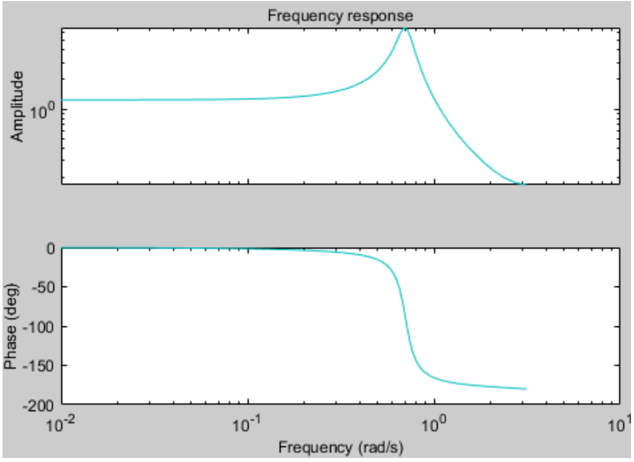


The model is a good fit: 85.7



Poles and zeros are within the unit circle and not too close together.






```
bjl1221 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.5527 (+/- 0.00247) z^-1

  C(z) = 1 - 0.9827 (+/- 0.02024) z^-1

  D(z) = 1 - 0.4525 (+/- 0.04099) z^-1 - 0.4888 (+/- 0.03784) z^-2

  F(z) = 1 - 1.428 (+/- 0.0007057) z^-1 + 0.8741 (+/- 0.0007582) z^-2

Name: bjl1221
Sample time: 1 seconds
```

The standard deviation analysis is good.

c)

The best model was quite obvious and a very good fit, so no alternative models were considered.

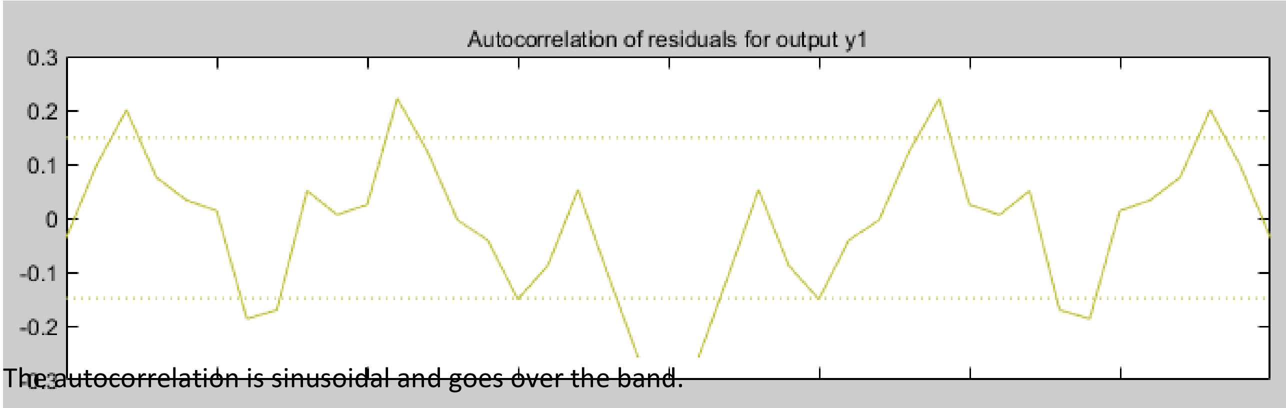
Dataset 3)

a)

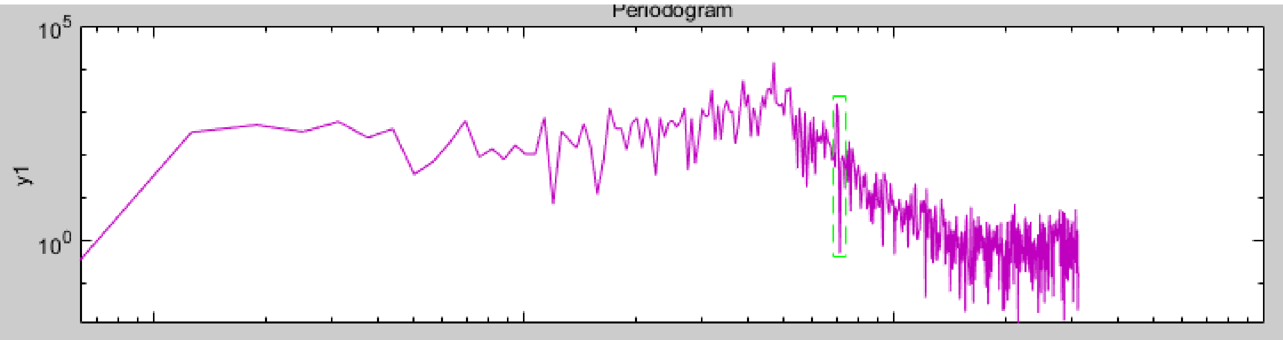
model chosen: oe322

b)

When determining the delay from arx models, we can see that the autocorrelation of arx332 is sinusoidal and goes over the band.



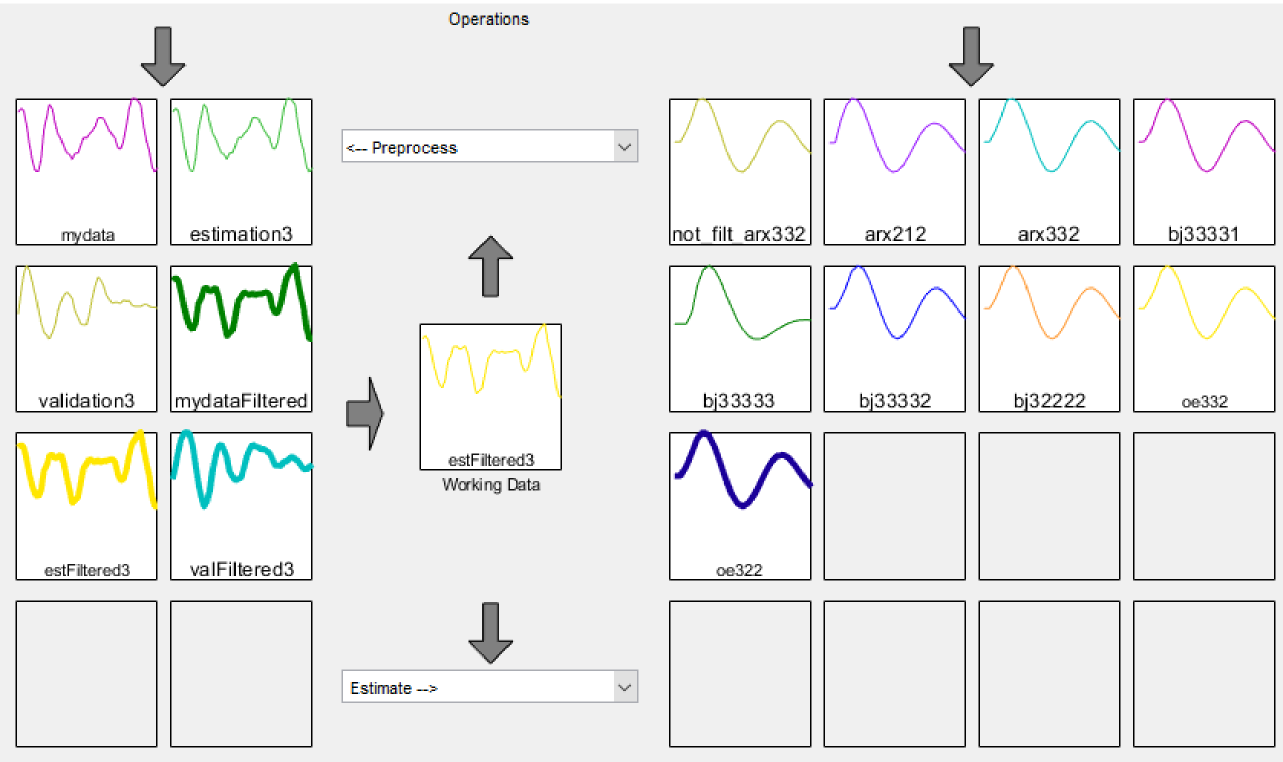
~~The autocorrelation is sinusoidal and goes over the band.~~

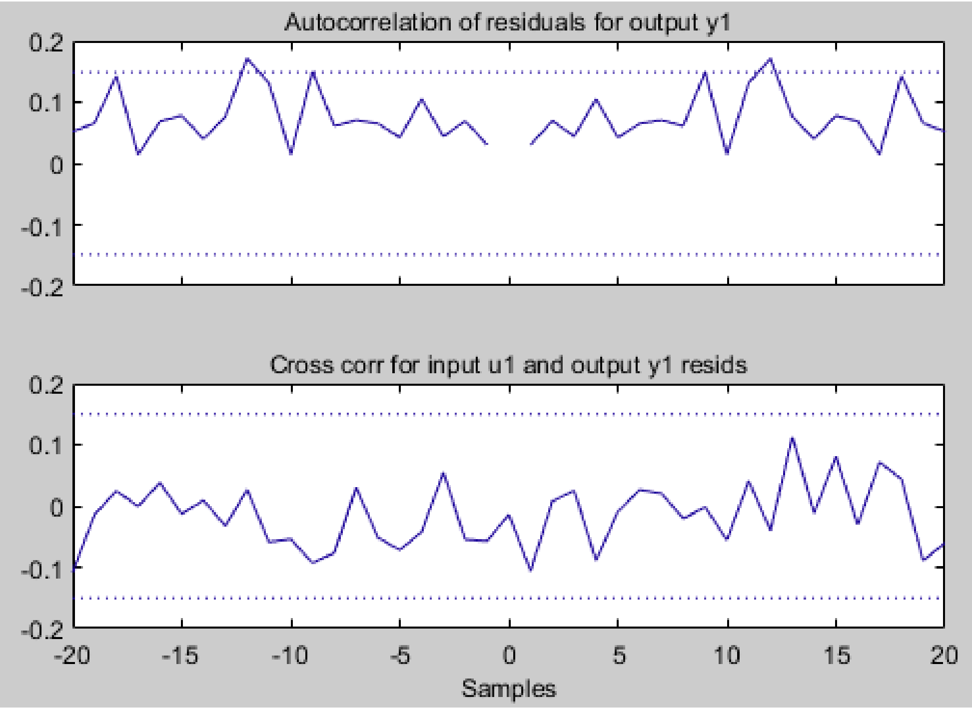


From filtering the data, we can see that the noise peaks around 0.7 Rad/s. Also, the bj33332 has a huge spike in its noise spectrum at 0.7 Rad/s. Therefore, we filter it out from the range: [0.68 0.72]

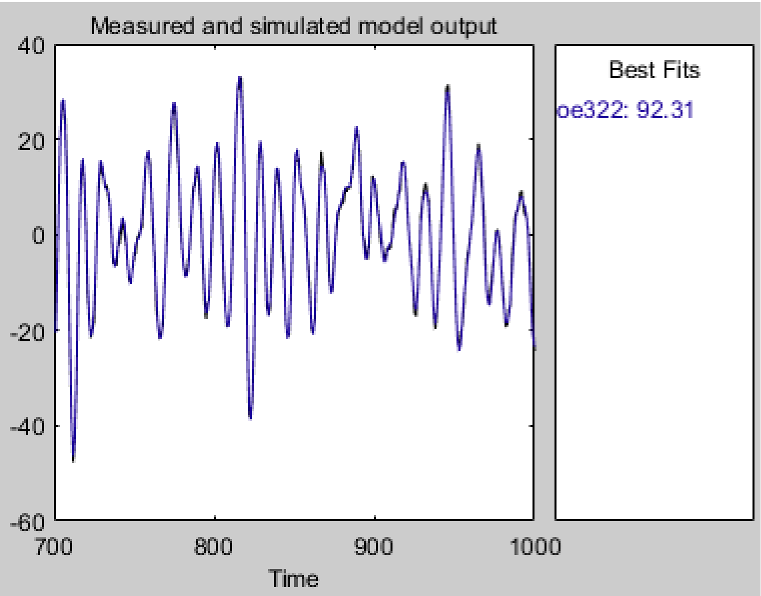
With the new filtered data, we determine the delay from arx models. All but the 2nd has a delay of 2. Also, the bj33331 and bj33333 both have a peak in cross correlation at 2. Therefore, delay of 2 is chosen.

$n_k = 2$

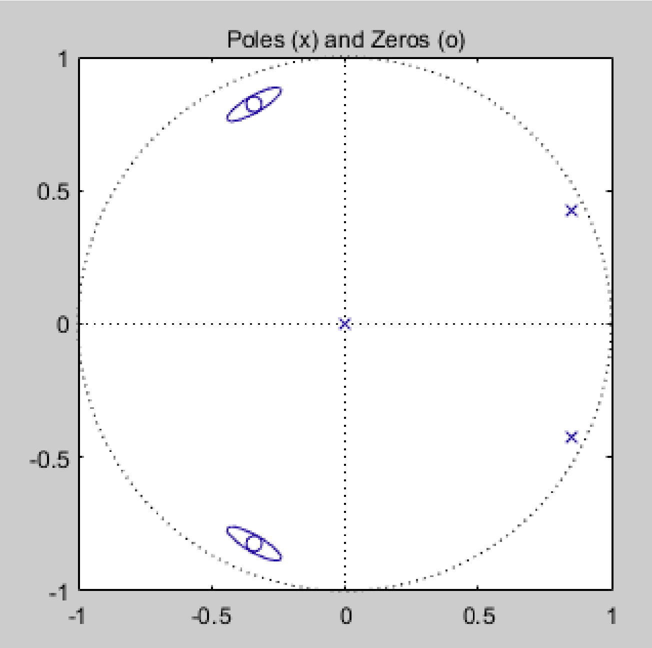




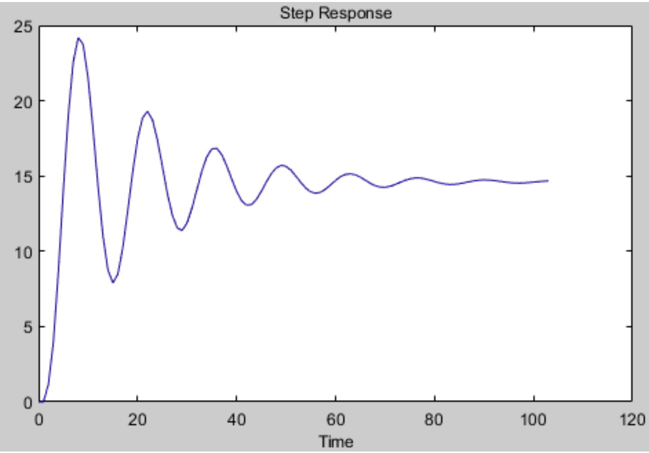
Inside confidence levels. Does not go over the bands (autocorrelation is shifted but it is okay as the variance is smaller than the bands).

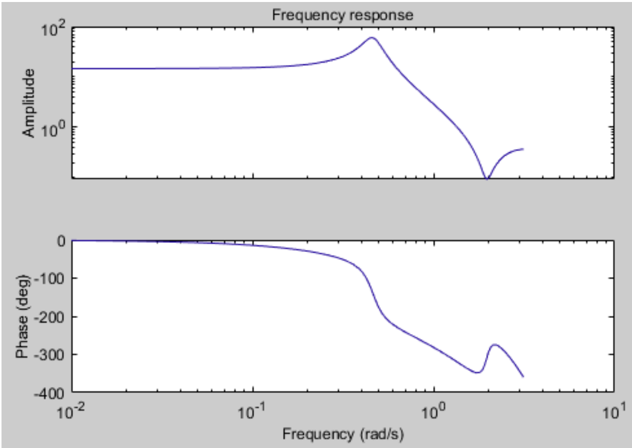


The model is a good fit: 92.31



Poles and zeros are within the unit circle and not too close together






```
oe322 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

 B(z) = 1.183 (+/- 0.03788) z^-2 + 0.8053 (+/- 0.06781) z^-3 + 0.9386 (+/- 0.04253) z^-4

  F(z) = 1 - 1.699 (+/- 0.000748) z^-1 + 0.8992 (+/- 0.0006707) z^-2

Name: oe322
Sample time: 1 seconds
```

The standard deviation analysis is good.

c)

Basically, bj32222 was an okay model, but as C&D were similar, we were able move to Output Error (OE) models.

The oe322 was a very good model and therefore, other models were not very seriously considered.