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1

$$\begin{cases} \dot{q} + 18,2q = 435,5u - 5,1v \\ 0,3q - \dot{v} = 36,7u \\ y = -0,2q \end{cases}$$

$$\begin{cases} \dot{q} = -18,2q - 5,1v + 435,5u \\ \dot{v} = 0,3q - 36,7u \\ y = -0,2q \end{cases}$$

State space:

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -18,2 & -5,1 \\ 0,3 & 0 \end{bmatrix} \begin{bmatrix} q \\ v \end{bmatrix} + \begin{bmatrix} 435,5 \\ -36,7 \end{bmatrix} u$$

$$y = \begin{bmatrix} -0,2 & 0 \end{bmatrix} \begin{bmatrix} q \\ v \end{bmatrix} + 0u$$

b/

$$TF = G(s) = C(sI - A)^{-1}B + D$$

from matlab: $G(s) = \dfrac{-87,1s - 37,43}{s^2 + 18,2s + 1,53}$

Answer in file TF.m

(2) TANK:

constant volume V c

homogeneous temp T c

reactant concentration $A(t)$ (mol/L) o

product concentration $B(t)$ (mol/L) o

inflow:

reactant concentration A_i (mol/L) i

volumetric flow rate q_i (L/s) c

outflow:

volumetric flow rate q_o (L/s) c

the tank reactant $A(t)$ is transformed to product $B(t)$ at rate: $r = kA(t)$

- reaction rate r (mol/(s·L))
- rate constant (Arrhenius) $k = k_0 e^{-\frac{E_a}{RT}}$

A & B ? controlled by A_i

a/ Outputs: $A(t)$ and $B(t)$ in the outflow

Inputs: concentration A_i in the inflow

Constant: temp T, volume V, flow rates q_i & q_o

Internal time varying: none

b/

$$\begin{Bmatrix}\text{rate of accumulation} \\ \text{or depletion of} \\ \text{component } j\end{Bmatrix} = \begin{Bmatrix}\text{rate of} \\ \text{inflow } j\end{Bmatrix} - \begin{Bmatrix}\text{rate of} \\ \text{outflow } j\end{Bmatrix} +/- \begin{Bmatrix}\text{rate of} \\ \text{transforma} \\ \text{of } j \text{ by} \\ \text{chem. reac.}\end{Bmatrix}$$

$$V \cdot \dot{A}(t) = q_i A_i - q_o A(t) - V k_o e^{-\frac{E_a}{RT}} A(t)$$

$$V \cdot \dot{B}(t) = 0 - q_o B(t) + V k_o e^{-\frac{E_a}{RT}} A(t)$$

$$\Rightarrow \begin{aligned} x_1 &= A(t) \\ x_2 &= B(t) \\ u &= A_i \end{aligned}$$

c/

$$\begin{bmatrix}\dot{A} \\ \dot{B}\end{bmatrix} = \begin{bmatrix} -\frac{q_o + V k_o e^{-\frac{E_a}{RT}}}{V} & 0 \\ k_o e^{-\frac{E_a}{RT}} & -\frac{q}{V} \end{bmatrix} \begin{bmatrix}A \\ B\end{bmatrix} + \begin{bmatrix}\frac{q}{V} \\ 0\end{bmatrix} A_i$$

$$\begin{bmatrix}A \\ B\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} \begin{bmatrix}A \\ B\end{bmatrix} + 0 A_i$$

d/ File chemReactor.m

3.

a/ Outputs: distance $z(t)$

Inputs: current $i(t)$

Constant: mass m, acceleration g, inductance L

Internal time varying: magnetic force F_m

b/ $\sum F = ma$

$$\Rightarrow \quad m\ddot{x} = F_m - mg$$

$$m\ddot{x} = -\frac{L}{2a} i^2 e^{-\frac{z}{a}} + mg$$

$$\ddot{x} = -\frac{L}{2am} i^2 e^{-\frac{z}{a}} + g$$

c/ Nonlinear because of the exponential function.
State variables are z & $\dot{z}$

d/ Equilibrium when $\ddot{x} = 0$

$$g = \frac{L}{2am} i^2 e^{-\frac{z}{a}}$$

$$i = \sqrt{\frac{2amg}{L e^{-\frac{z}{a}}}} = 1{,}4314$$

e/ $\dot{x} = f(x^{i}, u^{z}) \quad f(x_0, u_0) +$

$$\left(\frac{df}{dx} \cdot (x - x_0) + \frac{df}{du}(u - u_0)\right) = \frac{df}{dx} x + \frac{df}{du} u$$

$$\frac{df}{di} \; -\frac{L}{2a} i^2 e^{-\frac{z}{a}} + \frac{df}{dz} \; -\frac{L}{2a} i^2 e^{-\frac{z}{a}}$$

$\left(e^{u(x)}\right)' = e^{u(x)} \cdot u'(x)$

$$-2\frac{L}{2a} i e^{-\frac{z}{a}} + -\frac{L}{2a} i^2 e^{-\frac{z}{a}} \cdot \frac{d}{dz} - \frac{z}{a}$$

$$-2\frac{L}{2a} i e^{-\frac{z}{a}} + -\frac{L}{2a} i^2 e^{-\frac{z}{a}} \cdot \left(-\frac{1}{a}\right)$$

$$-10{,}9655\,\Delta i + 1178{,}38\,\Delta z$$

$$f(1{,}4314, 0{,}01) - 10{,}9655\, i + 1178{,}38\, z$$

f/ because $f(1{,}4314, 0{,}01)$ is equal to mg, they cancel eachother

$$m\ddot{z} = -10{,}9655\, i + 1178{,}38\, z$$

state variables:

$x_1 = z$

$x_2 = \dot{z}$

$$\dot{x}_1 = \dot{z} = x_2$$

$$\dot{x}_2 = \ddot{z} = \frac{-10{,}9655\, i + 1178{,}38\, z}{0{,}8} = 1472{,}975\, z - 13{,}706875\, i$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1472{,}975 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -13{,}706875 \end{bmatrix} i$$

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

f/ File MagLevit.m

$$G(s) = \frac{-13,71}{s^2 - 7,105e^{-15}s - 1473}$$

h/ File MagLevit.m

The system is not stable.