Solutions Programme F.1 Arithmetic

Solutions to exercises from the book Engineering Mathematics 7th edition. The book is divided into frames and the numbering of the following exercises refers to the numbers of these frames.

2. The numbers -10, 4, 0, -13 are of a type called integers.

3. (a) $-3 > -6$

(b) $2 > -4$

(c) $-7 < 12$

5. (a) $8 + (-3) = 8 - 3 = 5$

(b) $9 - (-6) = 9 + 6 = 15$

(c) $(-14) - (-7) = -14 + 7 = -7$

7. (a) $(-5) \times 3 = -15$

(b) $12 \div (-6) = -2$

(c) $(-2) \times (-8) = 16$

(d) $(-14) \div (-7) = 2$

9. $34 + 10 \div (2 - 3) \times 5 = 34 + 10 \div (-1) \times 5 = 34 - 10 \times 5 = 34 - 50 = -16$

Some numbers and the rounding of these numbers to the nearest 10, 100 and 1000.

(a) $1846, 1850, 1800, 2000$

(b) $-638, -640, -600, -1000$

(c) $445, 450, 400, 0$

14. (a) $18 \times 21 - 19 \div 11 \approx 20 \times 20 - 20 \div 10 = 398$

(b) $99 \div 101 - 49 \times 8 \approx 100 \div 100 - 50 \times 10 = -499$

17. This frame holds multiple review exercises that follow below.

1. (a) $-1 > -6$

(b) $5 > -29$

(c) $-14 < 7$

2. (a) $16 - 12 \times 4 + 8 \div 2 = 16 - 48 + 4 = -28$

(b) $(16 - 12) \times (4 + 8) \div 2 = 4 \times 12 \div 2 = 24$

(c) $9 - 3(17 + 5[5 - 7]) = 9 - 3(17 - 10) = 9 - 21 = -12$

(d) $8(3[2+4]-2[5+7]) = 8(3\times6-2\times12) = 8(18-24) = 8(-6) = -48$

3. (a) Show that:
$6 - (3 - 2) \neq (6 - 3) - 2$
Proof:
$LHS = 6 - (3 - 2) = 6 - 1 = 5$
$RHS = (6 - 3) - 2 = 3 - 2 = 1$
$LHS \neq RHS$

(b) Show that:
$100 \div (10 \div 5) \neq (100 \div 10) \div 5$
Proof:
$LHS = 100 \div (10 \div 5)) = 100 \div 2 = 50$
$RHS = (100 \div 10) \div 5 = 10 \div 5 = 2$
$LHS \neq RHS$

(c) Show that:
$24 \div (2 - 6) \neq 24 \div 2 - 24 \div 6$
Proof:
$LHS = 24 \div (2 - 6) = 24 \div (-4) = -6$
$RHS = 24 \div 2 - 24 \div 6 = 12 - 4 = 8$
$LHS \neq RHS$

4. Some numbers and the rounding of these numbers to the nearest 10, 100 and 1000.

(a) $2562, 2560, 2600, 3000$
(b) $1500, 1500, 1500, 2000$
(c) $-3451, -3450, -3500, -3000$
(d) $-14525, -14530, -14500, -15000$

19. (a) $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$
So the factors of 12 are 1, 2, 3, 4, 6, 12.

(b) $25 = 1 \times 25 = 5 \times 5$
So the factors of 25 are 1, 5, 25.

(c) $17 = 1 \times 17$
So the factors of 17 are 1, 17.

21. (a) $84 = 2 \times 42 = 2 \times 2 \times 21 = 2 \times 2 \times 3 \times 7$
(b) $512 = 2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

23. The prime factorizations of 84 and 512 are

$84 = 2 \times 2 \times 3 \times 7$

$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

The highest common factor of 84 and 512 is hence

$HCF = 2 \times 2 = 4$

And the lowest common multiple is

$LCF = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 10752$

26. This frame holds multiple review exercises that follow below.

1. Repeated integer division by increasingly bigger numbers gives the following products of prime factors.

(a) $429 = 3 \times 11 \times 13$

(b) $1820 = 2 \times 2 \times 5 \times 7 \times 13$

(c) $2992 = 2 \times 2 \times 2 \times 2 \times 11 \times 17$

(d) $3185 = 5 \times 7 \times 7 \times 13$

2. (a) The prime factorizations of 63 and 42 are
$63 = 3 \times 3 \times 7$
$42 = 2 \times 3 \times 7$
The highest common factor of 63 and 42 is hence
$HCF = 3 \times 7 = 21$
And the lowest common multiple is
$LCF = 2 \times 3 \times 3 \times 7 = 126$

(b) The prime factorization of 34 and 92 are
$34 = 2 \times 17$
$92 = 2 \times 2 \times 23$
The highest common factor of 34 and 92 is hence
$HCF = 2$
And the lowest common multiple is
$LCF = 2 \times 2 \times 17 \times 23 = 1564$

28. An example of a proper fraction is $\dfrac{-8}{11}$

30. $\dfrac{5}{9} \times \dfrac{2}{7} = \dfrac{5 \times 2}{9 \times 7} = \dfrac{10}{63}$

33. $\dfrac{3}{8}\ of\ \dfrac{5}{7} = \dfrac{3}{8} \times \dfrac{5}{7} = \dfrac{3 \times 5}{8 \times 7} = \dfrac{15}{56}$

34. $\dfrac{7}{5}$ and $\dfrac{28}{20}$ are equivalent fractions because $\dfrac{7}{5} \times \dfrac{4}{4} = \dfrac{7 \times 4}{5 \times 4} = \dfrac{28}{20}$

35. $\dfrac{84}{108} = \dfrac{42 \times 2}{54 \times 2} = \dfrac{42}{54} = \dfrac{21 \times 2}{27 \times 2} = \dfrac{21}{27} = \dfrac{7 \times 3}{9 \times 3} = \dfrac{7}{9}$

37. $\dfrac{7}{13} \div \dfrac{3}{4} = \dfrac{7}{13} \times \dfrac{4}{3} = \dfrac{28}{39}$

38. The reciprocal of $\dfrac{17}{4}$ is $\dfrac{4}{17}$

39. The reciprocal of -5 is $-\dfrac{1}{5}$

41. $\dfrac{5}{9} + \dfrac{1}{6} = \dfrac{10}{18} + \dfrac{3}{18} = \dfrac{13}{18}$

43. $\frac{11}{15} - \frac{2}{3} = \frac{11}{15} - \frac{10}{15} = \frac{1}{15}$

47. We have $\frac{3}{4} = \frac{9}{12}$ of A, $\frac{1}{6} = \frac{2}{12}$ of B and $\frac{1}{12}$ of C in the compound.
This means that the ratio in the compound is
$9 : 2 : 1$

49. The percentage of cars that are red is
$\frac{13}{100} = 13\%$

50. $\frac{12}{25} = \left(\frac{12}{25} \times 100\right)\% = \left(\frac{12}{25} \times 25 \times 4\right)\% = (12 \times 4)\% = 48\%$

52. $\frac{8}{100} \times 25 = \frac{2 \times 4}{4 \times 25} \times 25 = 2$

54. This frame holds multiple review exercises that follow below.

1. Reduction of fractions to their lowest terms.

(a) $\frac{24}{30} = \frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 5} = \frac{2 \times 2}{5} = \frac{4}{5}$

(b) $\frac{72}{15} = \frac{2 \times 2 \times 2 \times 3 \times 3}{3 \times 5} = \frac{2 \times 2 \times 2 \times 3}{5} = \frac{24}{5}$

(c) $-\frac{52}{65} = -\frac{2 \times 2 \times 13}{5 \times 13} = -\frac{2 \times 2}{5} = -\frac{4}{5}$