Some solutions to exercises from the book Numerical Mathematics and Computing, Seventh Edition, written by Cheney W., and Kincaid D., published 2013 by Cengage Learning.

1 Mathematical Preliminaries and Floating-Point Representation

1.1 Introduction - Exercise Solutions

1. The number π can be approximated by fractions with varying degree of error. A first approximation is 22/7. To measure how good this approximation is we can calculate the absolute and relative error:

$$AbsoluteError = |\pi - 22/7| = -0.00126448926$$

$$RelativeError = \frac{|\pi - 22/7|}{|\pi|} = -0.00040249943$$

There are other fractions (Cook 2018) that gives better approximations than 22/7 A selection is presented in the below table, including absolute and relative errors. The actual error calculations are omitted because are not very interesting, having been done in the same way as presented above.

Fraction	Absolute Error	Relative Error
22/7	-0.00126448926	-0.00040249943
355/113	-2.66764189e-7	-8.49136786e-8
208341/66317	1.2235635e-10	3.8947235e-11
1146408/364913	1.6107116e-12	5.1270541e-13

The last two approximations from the above table both have an absolute error that is less than 10^{-9}.

2. This solution shows how to calculate the real number x given that if 0.6032 is used to approximate x so will the relative error be at most 0.1%.

The relative error is defined as

$$RelativeError = \frac{|Exact\,Value - Approximate\,Value|}{|Exact\,Value|}$$

We know that the relative error is $0.1\% = 0.001$, and the approximate value is 0.6032 and it follows that

$$0.001 = \frac{|x - 0.6032|}{|x|}$$

$$0.001\,|x| = |x - 0.6032|$$

2 References

Cook, J. D. (2018, May, 22). 10 best rational approximations for pi. *John D. Cock Consulting.* https://www.johndcook.com/blog/2018/05/22/best-approximations-for-pi/