Some solutions to exercises from the book Numerical Mathematics and Computing, Seventh Edition, written by Cheney W., and Kincaid D., published 2013 by Cengage Learning.

1 Mathematical Preliminaries and Floating-Point Representation

1.1 Introduction - Exercise Solutions

1. The number π can be approximated by fractions with varying degree of error. A first approximation is 22/7. To measure how good this approximation is we can calculate the absolute and relative error:

$$AbsoluteError = |\pi - 22/7| = -0.00126448926$$

$$RelativeError = \frac{|\pi - 22/7|}{|\pi|} = -0.00040249943$$

There are other fractions (Cook 2018) that gives better approximations than 22/7 A selection is presented in the below table, including absolute and relative errors. The actual error calculations are omitted because are not very interesting, having been done in the same way as presented above.

Fraction	Absolute Error	Relative Error
22/7	-0.00126448926	-0.00040249943
355/113	-2.66764189e-7	-8.49136786e-8
208341/66317	1.2235635e-10	3.8947235e-11
1146408/364913	1.6107116e-12	5.1270541e-13

The last two approximations from the above table both have an absolute error that is less than 10^{-9}.

2. This solution shows how to calculate the real number x given that if 0.6032 is used to approximate x so will the relative error be at most 0.1%. Note that there will be two answers.

The relative error is defined as

$$RelativeError = \frac{|Exact\,Value - Approximate\,Value|}{|Exact\,Value|}$$

We know that the relative error is $0.1\% = 0.001$, and the approximate value is 0.6032, plugging in these values gives

$$0.001 = \frac{|x - 0.6032|}{|x|}$$

The calculation now splits into two cases, first case being that $x \geq 0.6032$. Then

$$\begin{aligned}
0.001 &= \frac{x - 0.6032}{x} \\
0.001x &= x - 0.6032 \\
10x &= 10000x - 6032 \\
9990x &= 6032 \\
x &= \frac{6032}{9990}
\end{aligned}$$

We have our first answer now, and can continue with the second case being that $x < 0.6032$ in this case, if we for now assume that $x > 0$, we get

$$\begin{aligned}
0.001 &= \frac{-(x - 0.6032)}{x} \\
0.001x &= 0.6032 - x \\
10x &= 6032 - 10000x \\
10010x &= 6032 \\
x &= \frac{6032}{10010}
\end{aligned}$$

Our assumption from above, about x being greater than zero held up, meaning that we now also have found the second answer.

3. The relative error when rounding 4.99997 to 5.000 is

$$\frac{|5.000 - 4.99997|}{|5.000|} = \frac{|0.00003|}{|5.000|} = \frac{0.00003}{5.000} = 0.000006 = 6 \times 10^{-5}$$

2 References

Cook, J. D. (2018, May, 22). 10 best rational approximations for pi. *John D. Cock Consulting.* https://www.johndcook.com/blog/2018/05/22/best-approximations-for-pi/