Chapter 1

Functions and Graphs

Exercise Solution

Exercise 1.1.53

Instruction

A vehicle has a 20-gal tank and gets 15 mpg. The number of miles N that can be driven depends on the amount of gas x in the tank.

(a) Write a formula that models the situation.

(b) Determine the number of miles the vehicle can travel on (i) a full tank of gas and (ii) 3/4 of a tank of gas.

(c) Determine the (i) domain and (ii) range of the function.

(d) Determine how many times the driver had to stop for gas if she has driven a total 578 miles.

Solution

(a) A gallon makes the vehicle go 15 miles. This information leads to that the number of miles N that can be driven on x gallons of gas is $N(x) = 15x$. This is formula is a function because it maps each input to exactly on output.

(i) A full tank holds 20 gallons. This is our known x, that we can use in the formula from above. The number of miles that can be traveled is $N(20) = 15 \cdot x = 15 \cdot 20 = 300$ miles.

(ii) 3/4 of the tank is 15 gallons. Again, this is our known x, that we can use in the formula from above. The number of miles that can be traveled is $N(15) = 15 \cdot x = 15 \cdot 15 = 225$ miles.

(b) (i) The domain of the function is all the different amount of gas that is possible to put in the tank, from empty to full, $0 \leq x \leq 20$.

(ii) The function between the amount of gas and miles traveled is a linear relation. The more gas we have the further we can travel. With an empty tank we can travel $N(0) = 15 \cdot x = 15 \cdot 0 = 0$ miles. With an full tank we can travel $N(20) = 15 \cdot x = 15 \cdot 20 = 300$ miles. When we have something in between the extreme values in the tank we will be able to travel in between 0 and 300 miles. The range of the function is hence $[0, 300]$.

(c) We start bay calculating the number of gallons of gas required for the trip. We solve $N = 15x$ for x by dividing both sides by 15, $x = N/15$. We now have a relation between number of gallons and distance. Plug in the known distance to calculate number of gallons, $x = 578/15 \approx 39$ gallons. If assuming that the trip was started with a full tank, holding 20 gallons, we conclude that the driver was $39 - 20 = 19$ gallons short. 19 being less than one full tank means that driver had to stop at least one time during the trip to fill up the tank.

Answer

(a) $N(x) = 15x$.

(b) 300 miles can be traveled on a full tank of gas. 225 miles can be traveled on 3/4 of a full tank of gas.

(c) Domain: $0 \geq x \geq 20$, range: $[0, 300]$.

(d) The driver had to stop for gas refill at least once.