Chapter 1

Functions and Graphs

Checkpoint Solutions

1.1 Evaluating Functions

Instruction

For the function $f(x) = x^2 - 3x + 5$ evaluate

(a) $f(1)$

(b) $f(a + h)$

Solution

(a) $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3$

(b) $f(a + h) = (a + h)^2 - 3(a + h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5$

1.2 Finding Domain and Range

Instruction

Find the domain and range for $f(x) = \sqrt{4 - 2x} + 5$.

Solution

i To find the domain of f, we need the expression $4 - 2x \geq 0$, due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is $\{x \mid x \leq 2\}$.

ii To find the range of f, we note that since $\sqrt{4 - 2x} \geq 0$, it follows that $f(x) = \sqrt{4 - 2x} + 5 \geq 5$. Therefore, the range of f must be a subset of the set $\{y \mid y \geq 5\}$.

To show that every element in this set is in the range of f, we need to show that for all y in this set, there exists a real number x in the domain such that $f(x) = y$. Let $y \geq 5$. Then, $f(x) = y$ if and only if

$$\sqrt{4 - 2x} + 5 = y.$$

Solving this equation for x, we see that x must solve the equation

$$\sqrt{4 - 2x} = y - 5.$$

Since $y \geq 5$, such an x could exist. Squaring both sides of the above equation we have

$$4 - 2x = (y - 5)^2.$$

Therefore we need

$$-2x = (y - 5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y - 5)^2}{2}.$$

We just need to verify that x is in the domain of f. Since the domain of f consists of all real numbers less or equal to 2, and

$$2 - \frac{(y - 5)^2}{2} \leq 2,$$

there does exist an x in the domain of f. We conclude that the range of f is $\{y \mid y \geq 5\}$.