# Chapter 1

# Functions and Graphs

## Checkpoint Solutions

### Checkpoint 1.1: Evaluating Functions

#### Instruction

For the function $f(x) = x^2 - 3x + 5$ evaluate

(a) $f(1)$

(b) $f(a+h)$

#### Solution

(a) $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3.$

(b) $f(a+h) = (a+h)^2 - 3(a+h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5.$

#### Answer

(a) $f(1) = 3.$

(b) $f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 5.$

### Checkpoint 1.2: Finding Domain and Range

#### Instruction

Find the domain and range for $f(x) = \sqrt{4-2x} + 5.$

#### Solution

i To find the domain of $f$, we need the expression $4 - 2x \geq 0$, due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is $\{x \mid x \leq 2\}$.

ii To find the range of $f$, we note that since $\sqrt{4-2x} \geq 0$, it follows that $f(x) = \sqrt{4-2x} + 5 \geq 5$. Therefore, the range of $f$ must be a subset of the set $\{y \mid y \geq 5\}$.

To show that every element in this set is in the range of $f$, we need to show that for all $y$ in this set, there exists a real number $x$ in the domain such that $f(x) = y$. Let $y \geq 5$. Then, $f(x) = y$ if and only if

$$\sqrt{4-2x} + 5 = y.$$

Solving this equation for $x$, we see that $x$ must solve the equation

$$\sqrt{4-2x} = y - 5.$$

Since $y \geq 5$, such an $x$ could exist. Squaring both sides of the above equation we have

$$4 - 2x = (y-5)^2.$$

Therefore we need

$$-2x = (y-5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y-5)^2}{2}.$$

We just need to verify that $x$ is in the domain of $f$. Since the domain of $f$ consists of all real numbers less or equal to 2, and

$$2 - \frac{(y-5)^2}{2} \leq 2,$$

there does exist an $x$ in the domain of $f$. We conclude that the range of $f$ is $\{y \mid y \geq 5\}$.

### Answer

Domain $= \{x \mid x \leq 2\}$, range $= \{y \mid y \geq 5\}$.

## Checkpoint 1.3: Finding Zeroes

### Instruction

Find the zeroes of $f(x) = x^3 - 5x^2 + 6x$.

### Solution

The zeroes of a function are the values of $x$ where $f(x) = 0$. To find the zeroes, we need to solve

$$f(x) = x^3 - 5x^2 + 6x = 0.$$

Factor out $x$

$$f(x) = x(x^2 - 5x + 6) = 0.$$

We can continue factoring by pure inspection, with the goal of finding a pair of numbers that add up to $-5$ and whose product is 6. This pair of numbers turns out to be $-2$ and $-3$, leading to the factoring

$$f(x) = x(x-2)(x-3) = 0.$$

From the above complete factoring of $f$, we conclude that there are three zeroes when $x$ is 0, 2, and 3.

**Answer**

$x = 0, 2, 3.$

## Checkpoint 1.4: Combining Functions Using Mathematical Operations

**Instruction**

For $f(x) = x^2 + 3$ and $g(x) = 2x - 5$, find $(f/g)(x)$ and state its domain.

**Solution**

To find $(f/g)(x)$ we write the function with the quotient operator

$$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}.$$

The domain of this function is $\{x \mid x \neq \frac{5}{2}\}$.

**Answer**

$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}$. The domain is $\{x \mid x \neq \frac{5}{2}\}$.

## Checkpoint 1.5: Compositions of Functions

**Instruction**

Let $f(x) = 2 - 5x$. Let $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$.

**Solution**

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2 - 5\sqrt{x}.$

**Answer**

$(f \circ g)(x) = 2 - 5\sqrt{x}.$

### Checkpoint 1.6: Application Involving a Composite Function

#### Instruction

If items are on sale for 10% off their original price, and a customer has a coupon for an additional 30% off, what will be the final price for an item that is originally x dollars, after applying the coupon to the sale price?

#### Solution

Since the sale price 10% off the original price, if an item is $x$ dollars, its sale price is given by

$$f(x) = 0.90x.$$

Since the coupon entitles an individual to 30% off the price of any item, if an item is $y$ dollars, the price after applying the coupon, is given by

$$g(y) = 0.70y.$$

Therefore, if the price is originally $x$ dollars, its price after applying the coupon to the sale price will be

$$(g \circ f)(x) = g(f(x)) = (0.70)0.90x = 0.63x..$$

#### Answer

$(g \circ f)(x) = 0.63x.$

## Exercise Solutions

### Exercise 1.1.1

#### Instruction

Assuming the relation in table 1.1.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |

Table 1.1: Relation between $x$ and $y$ in exercise 1.1.1

### Solution

(a) The domain of the relation is the set of unique $x$ values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

The range of the relation is the set of unique $y$ values,

$$\{0, 1, 4, 9\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

### Answer

(a) Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$, range = $\{0, 1, 4, 9\}$.

(b) Yes, a function.

## Exercise 1.1.2

### Instruction

Assuming the relation in table 1.2.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-2$ | $-8$ | $-1$ | $0$ | $1$ | $8$ | $-2$ |

Table 1.2: Relation between $x$ and $y$ in exercise 1.1.2

### Solution

(a) The domain of the relation is the set of unique $x$ values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

The range of the relation is the set of unique $y$ values,

$$\{-8, -2, -1, 0, 1, 8\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

**Answer**

(a) Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$, range = $\{-8, -2, -1, 0, 1, 8\}$.

(b) Yes, a function.

## Exercise 1.1.3

**Instruction**

Assuming the relation in table 1.3.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |

Table 1.3: Relation between $x$ and $y$ in exercise 1.1.3

**Solution**

(a) The domain of the relation is the set of unique $x$ values,

$$\{0, 1, 2, 3\}.$$

The range of the relation is the set of unique $y$ values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

(b) This relation is not a function, each input is not assigned to exactly one output. Take for example $x = 1$ that can cause both $y = -3$ and $y = 1$.

**Answer**

(a) Domain = $\{0, 1, 2, 3\}$, range = $\{-3, -2, -1, 0, 1, 2, 3\}$.

(b) No, not a function.

## Exercise 1.1.4

**Instruction**

Assuming the relation in table 1.4.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $y$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Table 1.4: Relation between $x$ and $y$ in exercise 1.1.4

**Solution**

(a) The domain of the relation is the set of unique $x$ values,

$$\{1, 2, 3, 4, 5, 6, 7\}.$$

The range of the relation is the set of unique $y$ values,

$$\{1\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

**Answer**

(a) Domain = $\{1, 2, 3, 4, 5, 6, 7\}$, range = $\{1\}$.

(b) Yes, a function.

### Exercise 1.1.5

**Instruction**

Assuming the relation in table 1.5.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | 3 | 5 | 8 | 10 | 15 | 21 | 33 |
|---|---|---|---|---|---|---|---|
| $y$ | 3 | 2 | 1 | 0 | 1 | 2 | 3 |

Table 1.5: Relation between $x$ and $y$ in exercise 1.1.5

**Solution**

(a) The domain of the relation is the set of unique $x$ values,

$$\{3, 5, 8, 10, 15, 21, 33\}.$$

The range of the relation is the set of unique $y$ values,

$$\{0, 1, 2, 3\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

**Answer**

(a) Domain = $\{3, 5, 8, 10, 15, 21, 33\}$, range = $\{0, 1, 2, 3\}$.

(b) Yes, a function.

## Exercise 1.1.6

**Instruction**

Assuming the relation in table 1.6.

(a) Determine the domain and the range of the relation.

(b) State whether the relation is a function.

| $x$ | $-7$ | $-2$ | $-2$ | $0$ | $1$ | $3$ | $6$ |
|---|---|---|---|---|---|---|---|
| $y$ | $11$ | $5$ | $1$ | $-1$ | $-2$ | $4$ | $11$ |

Table 1.6: Relation between $x$ and $y$ in exercise 1.1.6

**Solution**

(a) The domain of the relation is the set of unique $x$ values,

$$\{-7, -2, 0, 1, 3, 6\}.$$

The range of the relation is the set of unique $y$ values,

$$\{-2, -1, 1, 4, 5, 11\}.$$

(b) This relation is not a function, each input is not assigned to exactly one output. See $x = -2$ that can cause both $y = 1$ and $y = 5$.

**Answer**

(a) Domain = $\{-7, -2, 0, 1, 3, 6\}$, range = $\{-2, -1, 1, 4, 5, 11\}$.

(b) No, not a function.