Chapter 1

Functions and Graphs

Checkpoint Solutions

Checkpoint 1.1: Evaluating Functions

Instruction

For the function $f(x) = x^2 - 3x + 5$ evaluate

(a) $f(1)$

(b) $f(a+h)$

Solution

(a) $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3$

(b) $f(a+h) = (a+h)^2 - 3(a+h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5$

Answer

(a) $f(1) = 3$

(b) $f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 5$

Checkpoint 1.2: Finding Domain and Range

Instruction

Find the domain and range for $f(x) = \sqrt{4-2x} + 5$.

Solution

i To find the domain of f, we need the expression $4 - 2x \geq 0$, due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is $\{x \mid x \leq 2\}$.

ii To find the range of f, we note that since $\sqrt{4-2x} \geq 0$, it follows that $f(x) = \sqrt{4-2x}+5 \geq 5$. Therefore, the range of f must be a subset of the set $\{y \mid y \geq 5\}$.

To show that every element in this set is in the range of f, we need to show that for all y in this set, there exists a real number x in the domain such that $f(x) = y$. Let $y \geq 5$. Then, $f(x) = y$ if and only if

$$\sqrt{4-2x}+5 = y.$$

Solving this equation for x, we see that x must solve the equation

$$\sqrt{4-2x} = y-5.$$

Since $y \geq 5$, such an x could exist. Squaring both sides of the above equation we have

$$4-2x = (y-5)^2.$$

Therefore we need

$$-2x = (y-5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y-5)^2}{2}.$$

We just need to verify that x is in the domain of f. Since the domain of f consists of all real numbers less or equal to 2, and

$$2 - \frac{(y-5)^2}{2} \leq 2,$$

there does exist an x in the domain of f. We conclude that the range of f is $\{y \mid y \geq 5\}$.

Answer

Domain $= \{x \mid x \leq 2\}$, range $= \{y \mid y \geq -4\}$

Checkpoint 1.3: Finding Zeroes

Instruction

Find the zeroes of $f(x) = x^3 - 5x^2 + 6x$.

Solution

The zeroes of a function are the values of x where $f(x) = 0$. To find the zeroes, we need to solve

$$f(x) = x^3 - 5x^2 + 6x = 0.$$

Factor out x

$$f(x) = x(x^2 - 5x + 6) = 0.$$

We can continue factoring by pure inspection, with the goal of finding a pair of numbers that add up to -5 and whose product is 6. This pair of numbers turns out to be -2 and -3, leading to the factoring

$$f(x) = x(x-2)(x-3) = 0.$$

From the above complete factoring of f, we conclude that there are three zeroes when x is 0, 2, and 3.

Answer

$x = 0, 2, 3$

Checkpoint 1.4: Combining Functions Using Mathematical Operations

Instruction

For $f(x) = x^2 + 3$ and $g(x) = 2x - 5$, find $(f/g)(x)$ and state its domain.

Solution

To find $(f/g)(x)$ we write the function with the quotient operator

$$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}.$$

The domain of this function is $\{x \mid x \neq \frac{5}{2}\}$.

Answer

$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}$. The domain is $\{x \mid x \neq \frac{5}{2}\}$.

Checkpoint 1.5: Compositions of Functions

Instruction

Let $f(x) = 2 - 5x$. Let $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$.

Solution

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2 - 5\sqrt{x}$

Answer

$(f \circ g)(x) = 2 - 5\sqrt{x}$

Checkpoint 1.6: Application Involving a Composite Function

Instruction

If items are on sale for 10% off their original price, and a customer has a coupon for an additional 30% off, what will be the final price for an item that is originally x dollars, after applying the coupon to the sale price?

Solution

Since the sale price 10% off the original price, if an item is x dollars, its sale price is given by

$$f(x) = 0.90x.$$

Since the coupon entitles an individual to 30% off the price of any item, if an item is y dollars, the price after applying the coupon, is given by

$$g(y) = 0.70y.$$

Therefore, if the price is originally x dollars, its price after applying the coupon to the sale price will be

$$(g \circ f)(x) = g(f(x)) = (0.70)0.90x = 0.63x..$$

Answer

$(g \circ f)(x) = 0.63x$

Exercise Solutions

For the following exercises, (a) determine the domain and the range of each relation, and (b) state whether the relation is a function.

Exercise 1.1.1

Instruction

Relation:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Solution

(a) The domain of the relation is the set of unique x values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

The range of the relation is the set of unique y values,

$$\{0, 1, 4, 9\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

Answer

(a) Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$, range = $\{0, 1, 4, 9\}$

(b) Yes, a function

Exercise 1.1.2

Instruction

Relation:

x	y
-3	-2
-2	-8
-1	-1
0	0
1	1
2	8
3	-2

Solution

(a) The domain of the relation is the set of unique x values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

The range of the relation is the set of unique y values,

$$\{-8, -2, -1, 0, 1, 8\}.$$

(b) This relation is a function, each input is a assigned to exactly one output.

Answer

(a) Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$, range = $\{-8, -2, -1, 0, 1, 8\}$

(b) Yes, a function

Exercise 1.1.3

Instruction

Relation:

x	y
1	-3
2	-2
3	-1
0	0
1	1
2	2
3	3

Solution

(a) The domain of the relation is the set of unique x values,

$$\{0, 1, 2, 3\}.$$

The range of the relation is the set of unique y values,

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

(b) This relation is not function, due to that each input is not assigned to exactly one output. Take for example $x = 1$ that can cause both $y = -3$ and $y = 1$.

Answer

(a) Domain = $\{0, 1, 2, 3\}$, range = { -3, -2, -1, 0, 1, 2, 3 }

(b) No, not a function