

Chapter 1

Basic Classes of Functions

Checkpoint Solution

Checkpoint 1.10: Graphing Polynomial Functions

Instruction

Consider the quadratic function $f(x) = 3x^2 - 6x + 2$.

- (a) Find the zeroes of f .
- (b) Does the parabola open upward or downward?
- (c) Sketch a graph of f .

Solution

- (a) We find the zeroes of f using the quadratic function. In this case we have $a = 3$, $b = -6$, $c = 2$. The two zeroes are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3} = 1 \pm \frac{\sqrt{3}}{3}.$$

Using an calculator we can find the alternate form $x_1 \approx 1.58$, $x_2 \approx 0.423$.

- (b) We have an quadratic function on the form $f(x) = ax^2 + bx + c$. The plot for this type of function will be a parabola. If $a > 0$, then $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$. This is due to that x^2 will eventually start to dominate as x grows, the other part of the function will not matter. This leads to that the parabola will open upwards for $a > 0$. In this case we have $a = 3$, which is greater than zero. We conclude that the parabola will open upward.
- (c) We can manually sketch the parabola by first calculating points $(x, f(x))$ for some different x values close to the zeroes calculated in part a. Then plot this points

and finally connect the points with a parabola shape, remembering from part b that the parabola will open upward.

Another way is to use graphing tool, can be a calculator or a computer software. Below is an example based on using TODO

Answer

- (a) The zeroes are $1 \pm \sqrt{3}/3$.
- (b) The parabola opens upward.
- (c)