Chapter 1

Basic Classes of Functions

Checkpoint Solution

Checkpoint 1.10: Graphing Polynomial Functions

Instruction

Consider the quadratic function $f(x) = 3x^2 - 6x + 2$.

(a) Find the zeroes of f.

(b) Doest the parabola open upward or downward?

(c) Sketch a graph of f.

Solution

(a) We find the zeroes of f using the quadratic function. In this case we have $a = 3$, $b = -6$, $c = 2$. The two zeroes are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3} = 1 \pm \frac{\sqrt{3}}{3}.$$

Using an calculator we can find the alternate form $x_1 \approx 1.58$, $x_2 \approx 0.423$.

(b) We have an quadratic function on the form $f(x) = ax^2 + bx + c$. The plot for this type of function will be a parabola. If $a > 0$, then $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$. This is due to that x^2 will eventually start to dominate as x grows, the other part of the function will not matter. This leads to that the parabola will open upwards for $a > 0$. In this case we have $a = 3$, which is greater than zero. We conclude that the parabola will open upward.

(c) We can manually sketch the parabola by first calculating points $(x, f(x))$ for some different x values around to the zeroes calculated in part a. Then plot these points.

Finally connect the points with a parabola shape, remembering from part b that the parabola will open upward.

Another way is to use graphing tool, can be a calculator or a computer software, see figure 1.1 for an example of the result.

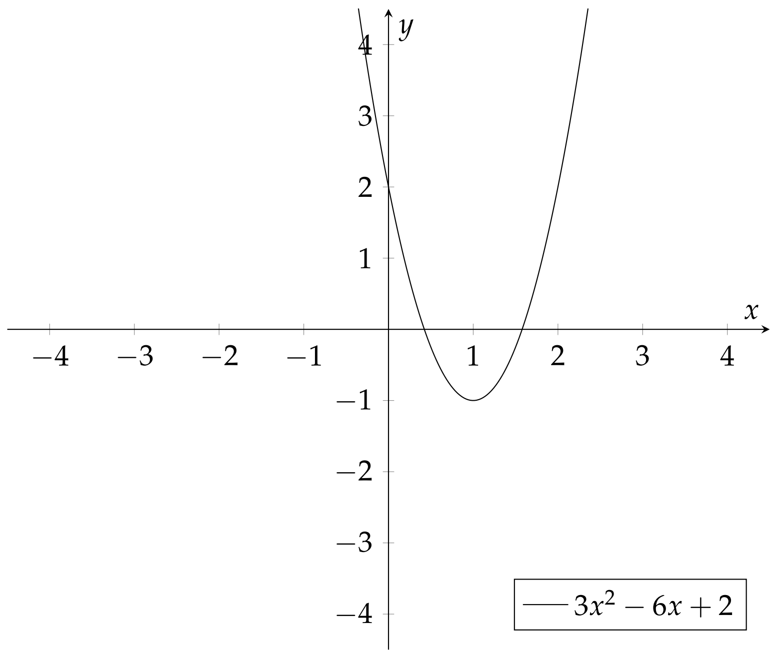

Figure 1.1: Graph of the quadratic functino in checkpoint 1.10

Answer

(a) The zeroes are $1 \pm \sqrt{3}/3$.

(b) The parabola opens upward.

(c) See the graph in figure 1.1.