

# Chapter 1: Functions and Graphs

## Checkpoint Solutions

### 1.1 Evaluating Functions

For the function  $f(x) = x^2 - 3x + 5$  evaluate

- (a)  $f(1)$
- (b)  $f(a + h)$

#### Solution

- (a)  $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3$
- (b)  $f(a + h) = (a + h)^2 - 3(a + h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5$

### 1.2 Finding Domain and Range

Find the domain and range for  $f(x) = \sqrt{4 - 2x} + 5$ .

#### Solution

- i To find the domain of  $f$ , we need the expression  $4 - 2x \geq 0$ , due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is  $\{x \mid x \leq 2\}$ .
- ii To find the range of  $f$ , we note that since  $\sqrt{4 - 2x} \geq 0$ , it follows that  $f(x) = \sqrt{4 - 2x} + 5 \geq 5$ . Therefore, the range of  $f$  must be a subset of the set  $\{y \mid y \geq 5\}$ .

To show that every element in this set is in the range of  $f$ , we need to show that for all  $y$  in this set, there exists a real number  $x$  in the domain such that  $f(x) = y$ . Let  $y \geq 5$ . Then,  $f(x) = y$  if and only if

$$\sqrt{4 - 2x} + 5 = y.$$

Solving this equation for  $x$ , we see that  $x$  must solve the equation

$$\sqrt{4 - 2x} = y - 5.$$

Since  $y \geq 5$ , such an  $x$  could exist. Squaring both sides of the above equation we have

$$4 - 2x = (y - 5)^2.$$

Therefore we need

$$-2x = (y - 5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y - 5)^2}{2}.$$

We just need to verify that  $x$  is in the domain of  $f$ . Since the domain of  $f$  consists of all real numbers less or equal to 2, and

$$2 - \frac{(y - 5)^2}{2} \leq 2,$$

there does exist an  $x$  in the domain of  $f$ . We conclude that the range of  $f$  is  $\{y \mid y \geq 5\}$ .