Chapter 1

Functions and Graphs

Exercise Solution

Exercise 1.1.29

Use the vertical line test to determine whether the graph in figure 1.1 represent a function. Assume that the graph continues at both ends beyond the given grid. If the graph represents a function, then determine the following for the graph:

(a) Domain and range

(b) x-intercept, if any (estimate where necessary)

(c) y-intercept, if any (estimate where necessary)

(d) The intervals for which the function is increasing

(e) The intervals for which the function is decreasing

(f) The intervals for which the function is constant

(g) Symmetry about any axis and/or the origin

(h) Whether the function is even, odd, or neither

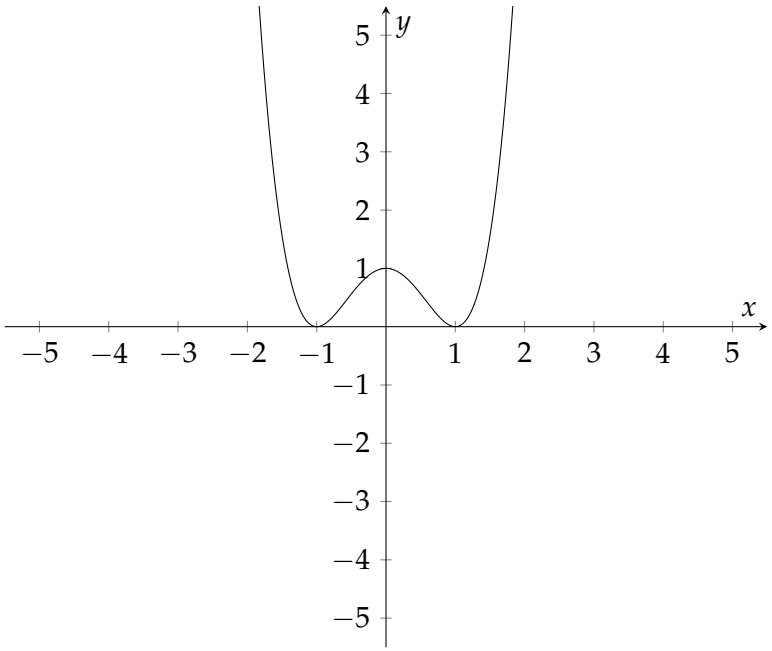

Figure 1.1: Graph for exercise 1.1.29

Solution

The graph in figure 1.1 do represent a function because every vertical line that may be drawn intersects the graph no more than once. See figure 1.2 for an example of a vertical line with one intersection of the graph. We could slide this line over the entire graph and there would always only be at most one intersection.

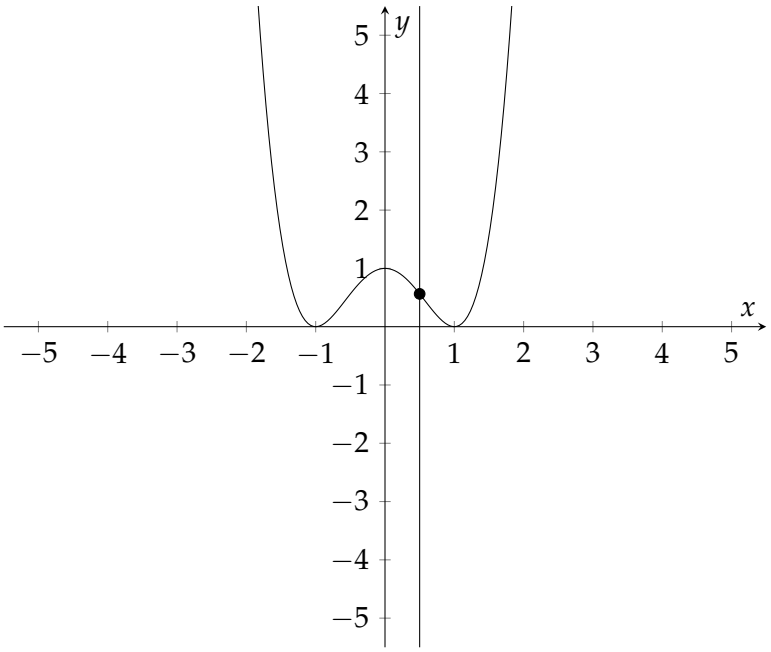

Figure 1.2: Vertical line test illustration

(a) i The function seems to grow rapidly as x goes towards $\pm\infty$, but there will still always be a y value. We conclude that the domain is all real numbers.

ii y is always greater or equal to 0, this is the range.

(b) y is zero for $x = -1$, and $x = 1$, these are the x-intercepts.

(c) The y-intercept is $y = 1$.

(d) The function is increasing for the intervals $-1 < x < 0$ and $1 < x < \infty$.

(e) The function is decreasing for the intervals $-\infty < x < -1$ and $0 < x < 1$.

(f) The function changes from decreasing/increasing when x is -1, 0, and 1, but there are no intervals for which the function is constant.

(g) $(-x, y)$ is on the graph whenever (x, y) is on the graph, in other words the function is symmetric around the y-axis.

(h) The function is not odd because $f(-x) \neq -f(x)$ for all x in the domain. Take for example $x = 0.5$ for which $f(-x) \approx 0.6$ and $-f(x) \approx -0.6$.

The function is even because $f(-x) = f(x)$ for all x. Take for example $x = 0.5$ for which $f(-x) \approx 0.6$ and $f(x) \approx 0.6$.

Answer

Graph represents a function.

(a) Domain: all real numbers, range: $y \geq 0$.

(b) $x = -1$ and $x = 1$.

(c) $y = 1$.

(d) $-1 < x < 0$ and $1 < x < \infty$.

(e) $-\infty < x < -1$ and $0 < x < 1$.

(f) Not constant.

(g) y-axis.

(h) Even.