Chapter 1

Functions and Graphs

Exercise Solution

Exercise 1.1.15

Find the domain, range, and all zeros/intercepts, if any, of the function $g(x) = \sqrt{8x-1}$.

Solution

i The domain of the square root function is $[0, \infty)$, which implies $8x - 1 \geq 0$. Solving for x gives $x \geq \frac{1}{8}$.

ii To find the range of g, we note that $\sqrt{8x-1} \geq 0$. Therefore, the range of g must be a subset of the set $\{y \mid y \geq 0\}$. To show that every element in this set is in the range of g, we need to show that for a given y in this set, there exists a real number x in the domain such that $g(x) = y$.

Let $y \geq 0$. Then $g(x) = y$ if and only if

$$\sqrt{8x-1} = y.$$

We are interested in x, and will solve this equation for x. Since $y \geq 0$ such an x could exist. Squaring both sides of this equation, we have

$$8x - 1 = y^2.$$

Therefore, we need

$$8x = y^2 + 1,$$

which implies

$$x = \frac{y^2+1}{8}.$$

We just need to verify that x is in the domain of g. Since the domain of g consists of all real numbers greater than or equal to $1/8$, and

$$\frac{y^2+1}{8} \geq \frac{1}{8},$$

there does exist an x in the domain of g. We conclude that the range of g is $\{y \mid y \geq 0\}$.

iii To find the zeroes, solve $g(x) = \sqrt{8x - 1} = 0$. We discover that g have one zero at $x = -1/8$.

iv The y-intercept is given by $(0, g(0))$. Since $x = 0$ isn't in the domain of g, it follows that that there aren't any intercepts.

Answer

Domain = $x \geq \frac{1}{8}$, range = $\{y \mid y \geq 0\}$, zeroes $x = -1/8$, no intercepts.