

# Chapter 1

## Functions and Graphs

### Checkpoint Solutions

#### 1.1 Evaluating Functions

##### Instruction

For the function  $f(x) = x^2 - 3x + 5$  evaluate

(a)  $f(1)$

(b)  $f(a + h)$

##### Solution

(a)  $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3$

(b)  $f(a + h) = (a + h)^2 - 3(a + h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5$

##### Answer

(a)  $f(1) = 3$

(b)  $f(a + h) = a^2 + 2ah + h^2 - 3a - 3h + 5$

#### 1.2 Finding Domain and Range

##### Instruction

Find the domain and range for  $f(x) = \sqrt{4 - 2x} + 5$ .

### Solution

- i To find the domain of  $f$ , we need the expression  $4 - 2x \geq 0$ , due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is  $\{x \mid x \leq 2\}$ .
- ii To find the range of  $f$ , we note that since  $\sqrt{4 - 2x} \geq 0$ , it follows that  $f(x) = \sqrt{4 - 2x} + 5 \geq 5$ . Therefore, the range of  $f$  must be a subset of the set  $\{y \mid y \geq 5\}$ .

To show that every element in this set is in the range of  $f$ , we need to show that for all  $y$  in this set, there exists a real number  $x$  in the domain such that  $f(x) = y$ . Let  $y \geq 5$ . Then,  $f(x) = y$  if and only if

$$\sqrt{4 - 2x} + 5 = y.$$

Solving this equation for  $x$ , we see that  $x$  must solve the equation

$$\sqrt{4 - 2x} = y - 5.$$

Since  $y \geq 5$ , such an  $x$  could exist. Squaring both sides of the above equation we have

$$4 - 2x = (y - 5)^2.$$

Therefore we need

$$-2x = (y - 5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y - 5)^2}{2}.$$

We just need to verify that  $x$  is in the domain of  $f$ . Since the domain of  $f$  consists of all real numbers less or equal to 2, and

$$2 - \frac{(y - 5)^2}{2} \leq 2,$$

there does exist an  $x$  in the domain of  $f$ . We conclude that the range of  $f$  is  $\{y \mid y \geq 5\}$ .

### Answer

Domain =  $\{x \mid x \leq 2\}$ , range =  $\{y \mid y \geq -4\}$

## 1.3 Finding Zeroes

### Instruction

Find the zeroes of  $f(x) = x^3 - 5x^2 + 6x$ .

### Solution

The zeroes of a function are the values of  $x$  where  $f(x) = 0$ . To find the zeroes, we need to solve

$$f(x) = x^3 - 5x^2 + 6x = 0.$$

Factor out  $x$

$$f(x) = x(x^2 - 5x + 6) = 0.$$

We can continue factoring by pure inspection, with the goal of finding a pair of numbers that add up to  $-5$  and whose product is  $6$ . This pair of numbers turns out to be  $-2$  and  $-3$ , leading to the factoring

$$f(x) = x(x - 2)(x - 3) = 0.$$

From the above complete factoring of  $f$ , we conclude that there are three zeroes when  $x$  is  $0$ ,  $2$ , and  $3$ .

### Answer

$$x = 0, 2, 3$$

## 1.4 Combining Functions Using Mathematical Operations

### Instruction

For  $f(x) = x^2 + 3$  and  $g(x) = 2x - 5$ , find  $(f/g)(x)$  and state its domain.

### Solution

$$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}. \text{ The domain of this function is } \{x \mid x \neq \frac{5}{2}\}.$$

### Answer

$$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}. \text{ The domain is } \{x \mid x \neq \frac{5}{2}\}.$$

## 1.5 Compositions of Functions

### Instruction

Let  $f(x) = 2 - 5x$ . Let  $g(x) = \sqrt{x}$ . Find  $(f \circ g)(x)$ .

### Solution

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2 - 5\sqrt{x}$$

**Answer**

$$(f \circ g)(x) = 2 - 5\sqrt{x}$$

**1.6 Application Involving a Composite Function****Instruction**

If items are on sale for 10% off their original price, and a customer has a coupon for an additional 30% off, what will be the final price for an item that is originally  $x$  dollars, after applying the coupon to the sale price?

**Solution**

Since the sale price 10% off the original price, if an item is  $x$  dollars, its sale price is given by

$$f(x) = 0.90x.$$

Since the coupon entitles an individual to 30% off the price of any item, if an item is  $y$  dollars, the price after applying the coupon, is given by

$$g(y) = 0.70y.$$

Therefore, if the price is originally  $x$  dollars, its price after applying the coupon to the sale price will be

$$(g \circ f)(x) = g(f(x)) = (0.70)0.90x = 0.63x..$$

**Answer**

$$(g \circ f)(x) = 0.63x$$

**Exercise Solutions**

For the following exercises, (a) determine the domain and the range of each relation, and (b) state whether the relation is a function.

**1.1.1****Instruction**

Relation:

$x$	$y$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

**Solution**

(a) The domain of the relation is the set of different  $x$  values

$$\{-3, -2, -1, 0, 1, 2, 3\}.$$

The range of the relation is the set of different  $y$  values

$$\{0, 1, 4, 9\}.$$

(b) TODO