Chapter 1

Functions and Graphs

Checkpoint Solutions

1.1 Evaluating Functions

Instruction

For the function $f(x) = x^2 - 3x + 5$ evaluate

(a) $f(1)$

(b) $f(a+h)$

Solution

(a) $f(1) = 1^2 - 3 \cdot 1 + 5 = 1 - 3 + 5 = 3$

(b) $f(a+h) = (a+h)^2 - 3(a+h) + 5 = a^2 + 2ah + h^2 - 3a - 3h + 5$

Answer

(a) $f(1) = 3$

(b) $f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 5$

1.2 Finding Domain and Range

Instruction

Find the domain and range for $f(x) = \sqrt{4 - 2x} + 5$.

Solution

i To find the domain of f, we need the expression $4 - 2x \geq 0$, due to that real negative numbers do not have a square root. Solving this inequality, we conclude that the domain is $\{x \mid x \leq 2\}$.

ii To find the range of f, we note that since $\sqrt{4-2x} \geq 0$, it follows that $f(x) = \sqrt{4-2x} + 5 \geq 5$. Therefore, the range of f must be a subset of the set $\{y \mid y \geq 5\}$.

To show that every element in this set is in the range of f, we need to show that for all y in this set, there exists a real number x in the domain such that $f(x) = y$. Let $y \geq 5$. Then, $f(x) = y$ if and only if

$$\sqrt{4-2x} + 5 = y.$$

Solving this equation for x, we see that x must solve the equation

$$\sqrt{4-2x} = y - 5.$$

Since $y \geq 5$, such an x could exist. Squaring both sides of the above equation we have

$$4 - 2x = (y-5)^2.$$

Therefore we need

$$-2x = (y-5)^2 - 4,$$

which implies

$$x = 2 - \frac{(y-5)^2}{2}.$$

We just need to verify that x is in the domain of f. Since the domain of f consists of all real numbers less or equal to 2, and

$$2 - \frac{(y-5)^2}{2} \leq 2,$$

there does exist an x in the domain of f. We conclude that the range of f is $\{y \mid y \geq 5\}$.

Answer

Domain $= \{x \mid x \leq 2\}$, range $= \{y \mid y \geq -4\}$

1.3 Finding Zeroes

Instruction

Find the zeroes of $f(x) = x^3 - 5x^2 + 6x$.

Solution

The zeroes of a function are the values of x where $f(x) = 0$. To find the zeroes, we need to solve

$$f(x) = x^3 - 5x^2 + 6x = 0.$$

Factor out x

$$f(x) = x(x^2 - 5x + 6) = 0.$$

We can continue factoring by pure inspection, with the goal of finding a pair of numbers that add up to -5 and whose product is 6. This pair of numbers turns out to be -2 and -3, leading to the factoring

$$f(x) = x(x-2)(x-3) = 0.$$

From the above complete factoring of f, we conclude that there are three zeroes when x is 0, 2, and 3.

Answer

$x = 0, 2, 3$

1.4 Combining Functions Using Mathematical Operations

Instruction

For $f(x) = x^2 + 3$ and $g(x) = 2x - 5$, find $(f/g)(x)$ and state its domain.

Solution

$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}$. The domain of this function is $\{x \mid x \neq \frac{5}{2}\}$.

Answer

$\frac{f}{g}(x) = \frac{x^2+3}{2x-5}$. The domain is $\{x \mid x \neq \frac{5}{2}\}$.

1.5 Compositions of Functions

Instruction

Let $f(x) = 2 - 5x$. Let $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$.

Solution

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2 - 5\sqrt{x}$

Answer

$(f \circ g)(x) = 2 - 5\sqrt{x}$

1.6 Application Involving a Composite Function

Instruction

If items are on sale for 10% off their original price, and a customer has a coupon for an additional 30% off, what will be the final price for an item that is originally x dollars, after applying the coupon to the sale price?

Solution

Since the sale price 10% off the original price, if an item is x dollars, its sale price is given by $f(x) = 0.90x$. Since the coupon entitles an individual to 30% off the price of any item, if an item is y dollars, the price after applying the coupon, is given by $g(y) = 0.70y$. Therefore, if the price is originally x dollars, its price after applying the coupon to the sale price will be $(g \circ f)(x) = g(f(x)) = (0.70)0.90x = 0.63x$.

Answer

$(g \circ f)(x) = 0.63x$