Chapter 1

Basic Classes of Functions

Checkpoint Solution

Checkpoint 1.11: Finding Domain and Range for Algebraic Functions

Instruction

Consider the function $f(x) = (5x + 2)/(2x - 1)$.

(a) Find the domain of the function.

(b) Find the range of the function.

Solution

(a) It is not possible to divide by zero, so the domain is the set of real numbers x such that $x \neq 1/2$.

(b) To find the range, we need to find the values y for which there exists a real number x such that

$$y = \frac{5x + 2}{2x - 1}.$$

We solve this equation for x,

$$y(2x - 1) = 5x + 2,$$

$$2xy - y = 5x + 2,$$

$$2xy - 5x = y + 2,$$

$$x(2y - 5) = y + 2,$$

$$x = \frac{y + 2}{2y - 5}.$$

If $y = 5/2$, the above equation have no solution. On the other hand as long as $y \neq 5/2$, there will be a real number x that satisfies the equation. We conclude that the range of f is $\{y \mid y \neq 5/2\}$.

Answer

(a) The domain is the set of real numbers such x such that $x \neq 1/2$.

(b) The range is the set $\{y \mid y \neq 5/2\}$.