

Chapter 1

Functions and Graphs

Exercise Solution

Exercise 1.1.29

Use the vertical line test to determine whether the graph in figure 1.1 represent a function. Assume that the graph continues at both ends beyond the given grid. If the graph represents a function, then determine the following for the graph:

- (a) Domain and range
- (b) x -intercept, if any (estimate where necessary)
- (c) y -intercept, if any (estimate where necessary)
- (d) The intervals for which the function is increasing
- (e) The intervals for which the function is decreasing
- (f) The intervals for which the function is constant
- (g) Symmetry about any axis and/or the origin
- (h) Whether the function is even, odd, or neither

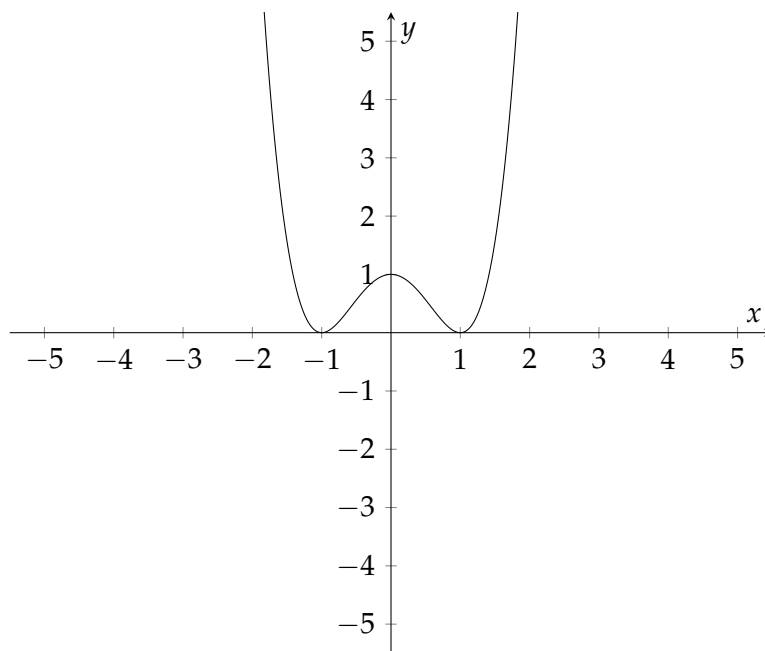


Figure 1.1: Graph for exercise 1.1.29

Solution

The graph in figure 1.1 do represent a function because every vertical line that may be drawn intersects the graph no more than once. See figure 1.2 for an example of a vertical line with one intersection of the graph. We could slide this line over the entire graph and there would always only be at most one intersection.

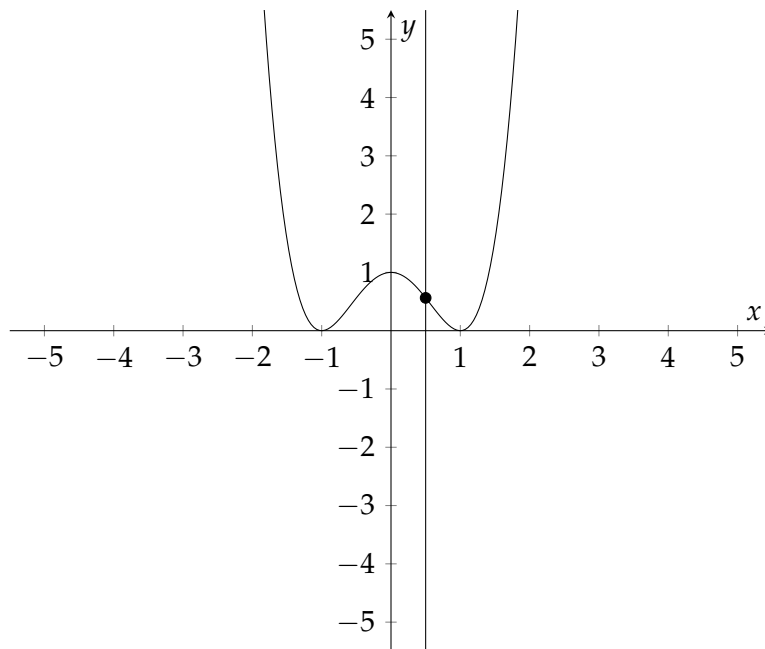


Figure 1.2: Vertical line test illustration

- (a)
 - i The function seems to grow rapidly as x goes towards $\pm\infty$, but there will still always be a y value. We conclude that the domain is all real numbers.
 - ii y is always greater or equal to 0, this is the range.
- (b) y is zero for $x = -1$, and $x = 1$, these are the x -intercepts.
- (c) The y -intercept is $y = 1$.
- (d) The function is increasing for the intervals $-1 < x < 0$ and $1 < x < \infty$.
- (e) The function is decreasing for the intervals $-\infty < x < -1$ and $0 < x < 1$.
- (f) The function changes from decreasing/increasing when x is -1 , 0 , and 1 , but there are no intervals for which the function is constant.
- (g) $(-x, y)$ is on the graph whenever (x, y) is on the graph, in other words the function is symmetric around the y -axis.
- (h) The function is not odd because $f(-x) \neq -f(x)$ for all x in the domain. Take for example $x = 0.5$ for which $f(-x) \approx 0.6$ and $-f(x) \approx -0.6$.
 The function is even because $f(-x) = f(x)$ for all x . Take for example $x = 0.5$ for which $f(-x) \approx 0.6$ and $f(x) \approx 0.6$.

Answer

Graph represents a function.

- (a) Domain: all real numbers, range: $y \geq 0$.
- (b) $x = -1$ and $x = 1$.
- (c) $y = 1$.
- (d) $-1 < x < 0$ and $1 < x < \infty$.
- (e) $-\infty < x < -1$ and $0 < x < 1$.
- (f) Not constant.
- (g) y -axis.
- (h) Even.