Benders Version

Nomenclature

Indices and Sets

$g \in \mathcal{G}$	Set of thermal generators.
$g \in \mathcal{G}^0_{on}$	Set of thermal generators which are initially committed (on).
$g \in \mathcal{G}^0_{off}$	Set of thermal generators which are not initially committed (off).
$w \in \mathcal{W}$	Set of renewable generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$, $T =$ `time_periods`
$l \in \mathcal{L}_g$	Piecewise production cost intervals for thermal generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}_g$	Startup categories for thermal generator g, from hottest (1) to coldest (S_g): $1, \ldots, S_g$.

System Parameters

$D(t)$	Load (demand) at time t (MW), `demand`.
$R(t)$	Spinning reserve at time t (MW), `reserves`.

Thermal Generator Parameters

CS^s_g	Startup cost in category s for generator g (\$), `startup['cost']`.
CP^l_g	Cost of operating at piecewise generation point l for generator g (MW), `piecewise_production['cost']`.
DT_g	Minimum down time for generator g (h), `time_down_minimum`.
DT^0_g	Number of time periods the unit has been off prior to the first time period for generator g, `time_down_t0`.
$\overline{P}_g$	Maximum power output for generator g (MW), `power_output_maximum`.
$\underline{P}_g$	Minimum power output for generator g (MW), `power_output_minimum`.
P^0_g	Power output for generator g (MW) in the time period prior to t=1, `power_output_t0`.
P^l_g	Power level for piecewise generation point l for generator g (MW); $P^1_g = \underline{P}_g$ and $P^{L_g}_g = \overline{P}_g$, `piecewise_production['mw']`.
RD_g	Ramp-down rate for generator g (MW/h), `ramp_down_limit`.
RU_g	Ramp-up rate for generator g (MW/h), `ramp_up_limit`.
SD_g	Shutdown capability for generator g (MW), `ramp_shutdown_limit`.
SU_g	Startup capability for generator g (MW), `ramp_startup_limit`
TS^s_g	Time offline after which the startup category s becomes active (h), `startup['lag']`.
UT_g	Minimum up time for generator g (h), `time_up_minimum`.
UT^0_g	Number of time periods the unit has been on prior to the first time period for generator g, `time_up_t0`.
U^0_g	Initial on/off status for generator g, $U^0_g = 1$ for $g \in \mathcal{G}^0_{on}$, $U^0_g = 0$ for $g \in \mathcal{G}^0_{off}$, `unit_on_t0`.
U_g	Must-run status for generator g, `must_run`.

Renewable Generator Parameters

$\overline{P}_w(t)$	Maximum renewable generation available from renewable generator w at time t (MW), `power_output_maximum`.
$\underline{P}_w(t)$	Minimum renewable generation available from renewable generator w at time t (MW), `power_output_minimum`.

Variables

$c_g(t)$	Cost of power produced above minimum for thermal generator g at time t (MW), $\in \mathbb{R}$.

$p_g(t)$	Power above minimum for thermal generator g at time t (MW), ≥ 0.
$p_w(t)$	Renewable generation used from renewable generator w at time t (MW), ≥ 0.
$r_g(t)$	Spinning reserves provided by thermal generator g at time t (MW), ≥ 0.
$u_g(t)$	Commitment status of thermal generator g at time t, $\in \{0,1\}$.
$v_g(t)$	Startup status of thermal generator g at time t, $\in \{0,1\}$.
$w_g(t)$	Shutdown status of thermal generator g at time t, $\in \{0,1\}$.
$\delta^s_g(t)$	Startup in category s for thermal generator g at time t, $\in \{0,1\}$.
$\lambda^l_g(t)$	Fraction of power from piecewise generation point l for generator g at time t (MW), $\in [0,1]$.
β	Estimated cost of sub problem.

Master Problem

$$\min \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left(CP^1_g\, u_g(t) + \sum_{s=1}^{S_g} \left(CS^s_g \delta^s(t) \right) \right) + \beta \tag{1}$$

subject to:

$$\sum_{t=1}^{\min\{UT_g - UT^0_g, T\}} (u_g(t) - 1) = 0 \qquad \forall g \in \mathcal{G}^0_{on} \tag{2}$$

$$\sum_{t=1}^{\min\{DT_g - DT^0_g, T\}} u_g(t) = 0 \qquad \forall g \in \mathcal{G}^0_{\mathit{off}} \tag{3}$$

$$u_g(1) - U^0_g = v_g(1) - w_g(1) \qquad \forall g \in \mathcal{G} \tag{4}$$

$$\sum_{s=1}^{S_g - 1} \sum_{t=\max\{1, TS^{s+1}_g - DT^0_g + 1\}}^{\min\{TS^{s+1}_g - 1, T\}} \delta^s_g(t) = 0 \qquad \forall g \in \mathcal{G} \tag{5}$$

$$U^0_g(P^0_g - \underline{P}_g) \leq (\overline{P}_g - \underline{P}_g)U^0_g - \max\{(\overline{P}_g - SD_g), 0\} w_g(1) \qquad \forall g \in \mathcal{G} \tag{6}$$

$$u_g(t) \geq U_g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{7}$$

$$u_g(t) - u_g(t-1) = v_g(t) - w_g(t) \qquad \forall t \in \mathcal{T} \setminus \{1\},\ \forall g \in \mathcal{G} \tag{8}$$

$$\sum_{i=t-\min\{UT_g, T\}+1}^{t} v_g(i) \leq u_g(t) \qquad \forall t \in \{\min\{UT_g, T\} \ldots, T\},\ \forall g \in \mathcal{G} \tag{9}$$

$$\sum_{i=t-\min\{DT_g, T\}+1}^{t} w_g(i) \leq 1 - u_g(t) \qquad \forall t \in \{\min\{DT_g, T\}, \ldots, T\},\ \forall g \in \mathcal{G} \tag{10}$$

$$\delta^s_g(t) \leq \sum_{i=TS^s_g}^{TS^{s+1}_g - 1} w_g(t-i) \qquad \forall t \in \{TS^{s+1}_g, \ldots, T\},\ \forall s \in \mathcal{S}_g \backslash \{S_g\},\ \forall g \in \mathcal{G} \tag{11}$$

$$v_g(t) = \sum_{s=1}^{S_g} \delta^s_g(t) \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{12}$$

$$\beta \geq \theta_j + \phi_j^\top (x - x_j) \tag{13}$$

$$u_g(t),\ v_g(t),\ w_g(t),\ \delta^s_g(t) \in \{0,1\} \qquad \forall t \in \mathcal{T},\ \forall s \in \mathcal{S}_g,\ \forall g \in \mathcal{G} \tag{14}$$

Sub Problem

$$\min \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} c_g(t) \tag{15}$$

subject to:

$$\sum_{g\in\mathcal{G}} \big(p_g(t) + \underline{P}_g u_g(t)\big) + \sum_{w\in\mathcal{W}} p_w(t) = D(t) \qquad \forall t \in \mathcal{T} \tag{16}$$

$$\sum_{g\in\mathcal{G}} r_g(t) \geq R(t) \qquad \forall t \in \mathcal{T} \tag{17}$$

$$p_g(1) + r_g(1) - U_g^0(P_g^0 - \underline{P}_g) \leq RU_g \qquad \forall g \in \mathcal{G} \tag{18}$$

$$U_g^0(P_g^0 - \underline{P}_g) - p_g(1) \leq RD_g \qquad \forall g \in \mathcal{G} \tag{19}$$

$$p_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - \max\{(\overline{P}_g - SU_g), 0\}v_g(t) \qquad \forall t \in \mathcal{T},\, \forall g \in \mathcal{G} \tag{20}$$

$$p_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - \max\{(\overline{P}_g - SD_g), 0\}w_g(t+1) \qquad \forall t \in \mathcal{T} \setminus \{T\},\, \forall g \in \mathcal{G} \tag{21}$$

$$p_g(t) + r_g(t) - p_g(t-1) \leq RU_g \qquad \forall t \in \mathcal{T} \setminus \{1\},\, \forall g \in \mathcal{G} \tag{22}$$

$$p_g(t-1) - p_g(t) \leq RD_g \qquad \forall t \in \mathcal{T} \setminus \{1\},\, \forall g \in \mathcal{G} \tag{23}$$

$$p_g(t) = \sum_{l\in\mathcal{L}_g} (P_g^l - P_g^1)\lambda_g^l(t) \qquad \forall t \in \mathcal{T},\, \forall g \in \mathcal{G} \tag{24}$$

$$c_g(t) = \sum_{l\in\mathcal{L}_g} (CP_g^l - CP_g^1)\lambda_g^l(t) \qquad \forall t \in \mathcal{T},\, \forall g \in \mathcal{G} \tag{25}$$

$$u_g(t) = \sum_{l\in\mathcal{L}_g} \lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{26}$$

$$\underline{P}_w(t) \leq p_w(t) \leq \overline{P}_w(t) \qquad \forall t \in \mathcal{T},\, \forall w \in \mathcal{W} \tag{27}$$

$$p_g(t),\ r_g(t),\ p_w(t) \geq 0 \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall w \in \mathcal{W} \tag{28}$$

$$\lambda_g^l(t) \in [0,1] \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall l \in \mathcal{L}_g \tag{29}$$

$$c_g(t) \in \mathbb{R} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{30}$$

The algorithm for solving this problem is as follows:

Algorithm 1 Benders decomposition

choose ϵ (convergence tolerance), $U_0^* := +\infty$ (initial upper bound), $j := 0$, set quantum computer to solve the MP;
repeat
    $j := j + 1$;
    solve the MP and obtain β_j and x_j^{RMP}; $L_j^* := c^\top x_j^{RMP} + \beta_j$;
    solve subproblem at x_j^{RMP} and obtain θ_j and ϕ_j;
    $U_j^* := \min(U_{j-1}^*, c^\top x_j^{RMP} + \theta_j)$;
    add cut $\beta \geq \theta_j + \phi_j^\top (x - x_j)$ to the MP;
until $U_j^* - L_j^* \leq \epsilon$.