Homework 1.3

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1 Problem a

For three masses and four springs:

$$m\ddot{x}_1 = -kx_1 - k(x_1 - x_2) \rightarrow m\ddot{x}_1 = -k(2x_1 - x_2)$$

$$m\ddot{x}_2 = -k(x_2 - x_1) - k(x_2 - x_3) \rightarrow m\ddot{x}_2 = -k(x_1 + 2x_2 - x_3)$$

$$m\ddot{x}_3 = -k(-x_2 + 2x_3) \rightarrow m\ddot{x}_3 = -k(-x_2 + 2x_3)$$

The three equations above are broken down and put into two separate matrices.

$$m = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, k = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix}$$

So then,

$$(k - \omega^2 m) = \begin{bmatrix} 2k - \omega^2 m & -k & 0 \\ -k & 2k - \omega^2 m & -k \\ 0 & -k & 2k - \omega^2 m \end{bmatrix}$$

After the matrices are combined, the eigenvalues are needed to be found. To find the eigenvalues the determinate of the equation $(k - \omega^2 m)$ is found,

$$det(k - \omega^2 m) = (2k - \omega^2 m)(4k\omega^2 m + 2k^2 + m^2\omega^4) = 0$$

This 3 x 3 matrix means that there will be three unique eigenvalues calculated.

$$2k - m\omega^2 = 0 \rightarrow 2k = m\omega^2 \rightarrow \omega^2 = \frac{2k}{m} \rightarrow \omega_1 = \sqrt{\frac{2k}{m}}$$

$$-\sqrt{2}k + 2k - m\omega^2 = 0 \rightarrow -\sqrt{2}k + 2k = m\omega^2 \rightarrow \omega^2 = \frac{-\sqrt{2}k + 2k}{m} \rightarrow \omega_2 = \sqrt{\frac{-\sqrt{2}k + 2k}{m}}$$

$$\sqrt{2}k + 2k - m\omega^2 = 0 \rightarrow \sqrt{2}k + 2k = m\omega^2 \rightarrow \omega^2 = \frac{\sqrt{2}k + 2k}{m} \rightarrow \omega_3 = \sqrt{\frac{\sqrt{2}k + 2k}{m}}$$

Thus the three eigenvalues are

$$\omega_1 = \sqrt{\frac{2k}{m}}, \qquad \omega_2 = \sqrt{\frac{-\sqrt{2}k + 2k}{m}}, \qquad \omega_3 = \sqrt{\frac{\sqrt{2}k + 2k}{m}}$$

The mass of the cart is 210.2 g and measure the spring constant is -3.37 $\frac{N}{m}$. Where the spring constant is k and the mass is m. The eigenvectors for the identical springs and masses are as follows,

$$v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, v_2 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}, v_3 \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}$$

2 Problem b

First things first, the values for ω_1, ω_2, and ω_3 must be calculated with the k and m coefficients. For ω_1,

$$\omega_1 = \sqrt{\frac{2(3.374)\frac{N}{m}}{0.2102}g} = \sqrt{33.7} = \frac{5.8}{2\pi} = 0.923Hz$$

$$\omega_2 = \sqrt{\frac{-\sqrt{2}k + 2k}{m}} = \sqrt{\frac{-\sqrt{2}(3.37)\frac{N}{m} + 2(3.37)\frac{N}{m}}{0.2102g}} = \sqrt{\frac{1.97\frac{N}{m}}{0.2102g}} = \frac{3.06}{2\pi} = 0.49Hz$$

$$\omega_3 = \sqrt{\frac{\sqrt{2}k + 2k}{m}} = \sqrt{\frac{\sqrt{2}(3.37)\frac{N}{m} + 2(3.37)\frac{N}{m}}{0.2102g}} = \sqrt{\frac{11.5059\frac{N}{m}}{0.2102g}} = \frac{7.6}{2\pi} = 1.21Hz$$

Now that the official eigenvalues have been calculated with data from the lab, the uncertainties for each of the eigenvalues can be calculated. The uncertainty for mass is $210.2 \pm 0.1g$ and the uncertainty for the spring constant is $3.37 \pm 0.09\frac{N}{m}$. Since the uncertainties for out coefficients are found, the uncertainty for $\omega_1, \omega_2, and \omega_3$ can be calculated.

$$\frac{\delta(2k)}{|2k|} = \frac{\delta(k)}{|k|} \rightarrow \frac{\delta(2k)}{|6.74|} = \frac{0.09}{3.37} = 0.18$$

$$\frac{\frac{\delta(2k)}{m}}{|\frac{(2k)}{m}|} = \sqrt{\left(\frac{\delta(2k)}{2k}\right)^2 + \left(\frac{\delta m}{m}\right)^2} = \sqrt{0.0324 + 0.0005} = 0.181$$

$$\frac{\delta\left(\sqrt{\frac{2k}{m}}\right)}{\sqrt{\frac{2k}{m}}} = \left(\frac{1}{2}\right)\frac{\delta\left(\sqrt{\frac{2k}{m}}\right)}{\sqrt{\frac{2k}{m}}} = \left(\frac{1}{2}\right)\left(\frac{0.035}{33.7}\right) = 0.00269$$

Uncertainty in ω_1±0.003 Hz, thus $\omega_2 = 0.923 \pm 0.003Hz$.

$$\frac{\delta(2k)}{|2k|} = \frac{\delta(k)}{|k|} \rightarrow \frac{\delta(2k)}{|6.74|} = \frac{0.09}{3.37} = 0.18$$

$$\frac{\delta\sqrt{2}k}{|\sqrt{2}k|} \to \frac{\delta\sqrt{2}k}{4.7659} = \frac{0.09}{3.37} \to \delta\sqrt{2}k = 0.127$$

$$\delta(\sqrt{2}k + 2k) = \delta(\sqrt{2k}) + \delta(2k) = 0.18 + 0.127 = 0.307$$

$$\frac{\delta m}{|m|} = \frac{0.001}{0.2102} = 0.000476$$

$$\frac{\delta\left(\frac{\sqrt{2}k+2k}{m}\right)}{\left|\left(\frac{\sqrt{2}k+2k}{m}\right)\right|} = \sqrt{\left(\frac{\delta\sqrt{2}k + 2k}{\sqrt{2}k + 2k}\right)^2 + \left(\frac{\delta m}{m}\right)^2} = \sqrt{0.000712} = 0.0267$$

$$\frac{\delta\left(\sqrt{\frac{\sqrt{2}k+2k}{m}}\right)}{\left|\left(\sqrt{\frac{\sqrt{2}k+2k}{m}}\right)\right|} = \frac{1}{2}\frac{0.0267}{\frac{11.5059}{0.2}} = \pm 0.004$$

Uncertainty in $\omega_2 \pm 0.004$ Hz, thus $\omega_1 = 1.207 \pm 0.004 Hz$.

$$\frac{\delta(2k)}{|2k|} = \frac{\delta(k)}{|k|} \to \frac{\delta(2k)}{|6.74|} = \frac{0.09}{3.37} = 0.18$$

$$\frac{\delta - \sqrt{2}k}{|\sqrt{2}k|} \to \frac{\delta - \sqrt{2}k}{4.7659} = \frac{0.09}{3.37} \to \delta - \sqrt{2}k = 0.127$$

$$\delta(\sqrt{2}k + 2k) = \delta(\sqrt{2k}) + \delta(2k) = 0.18 - 0.127 = 0.053$$

$$\frac{\delta m}{|m|} = \frac{0.001}{0.2102} = 0.000476$$

$$\frac{\delta\left(\frac{-\sqrt{2}k+2k}{m}\right)}{\left|\left(\frac{-\sqrt{2}k+2k}{m}\right)\right|} = \sqrt{\left(\frac{\delta - \sqrt{2}k + 2k}{-\sqrt{2}k + 2k}\right)^2 + \left(\frac{\delta m}{m}\right)^2} = \sqrt{0.000002} = 0.0015$$

$$\frac{\delta\left(\sqrt{\frac{-\sqrt{2}k+2k}{m}}\right)}{\left|\left(\sqrt{\frac{-\sqrt{2}k+2k}{m}}\right)\right|} = \frac{1}{2}\frac{0.0005}{\frac{1.9741}{0.2}} = \pm 0.007$$

Uncertainty in $\omega_3 \pm 0.007$ Hz, thus $\omega_3 = 0.5000 \pm 0.007 Hz$.

3 Problem c

The frequencies that were calculated and the frequencies that were measured are roughly the same with them having very little different values between them. Mode 1 frequency is around 0.5, mode 2 is around 0.9, mode 3 is around 1.15. Energy is not evenly distributed between masses and modes. We can look at examples of modes 1 and 2. Mode 1 is when are the masses are moving with the same motion and create a sinusoidal plot. While mode 2 has the outside masses move opposite of each other while the middle mass remains still with no movement, or energy. Thus, this is a demonstration to say than no energy is not distributed evenly between masses and nor is is evenly distributed between modes. Looking at the difference in all the amplitudes in the frequencies can tell us this too.

4 Problem d

Yes, it is true that the motions are sinusoidal when this occurs. However, they are not in sink with each other. Thus, for each mode the waves are smooth for all of the masses. Simply speaking, looking at the data found in Appendix A will tell us a lot about how these modes functions. As for mode 3 when the two out side masses move in the same direction and the outside mass move is the opposite direction the plot of the graph looks chaotic. However a closer observation shows that each one of these masses has a sinusoidal graph to it, as does modes 1 and 2.

Appendix A

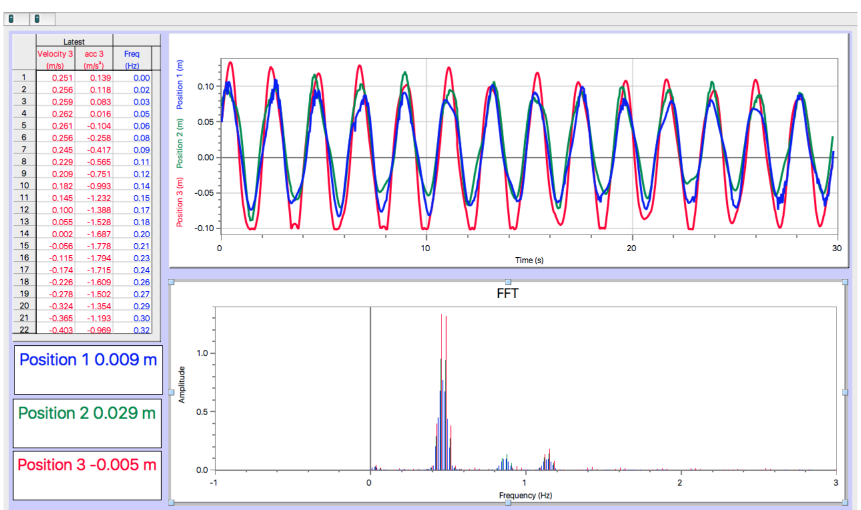

Figure 1: mode 1

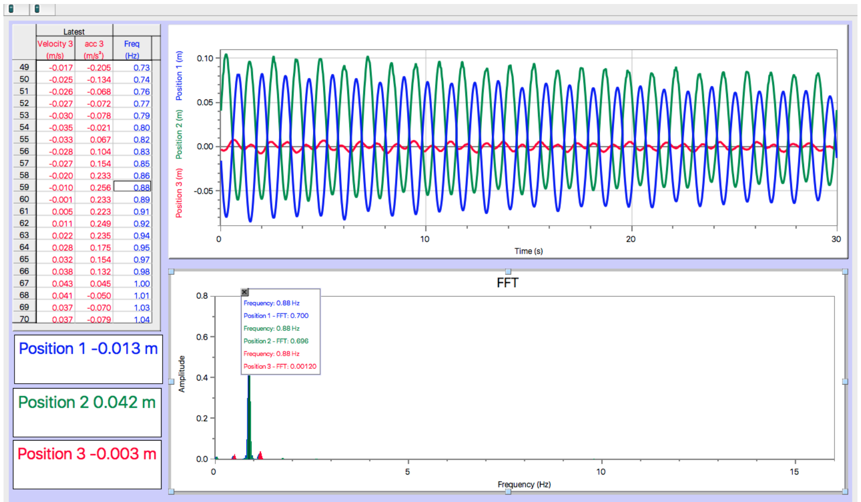

Figure 2: mode 2

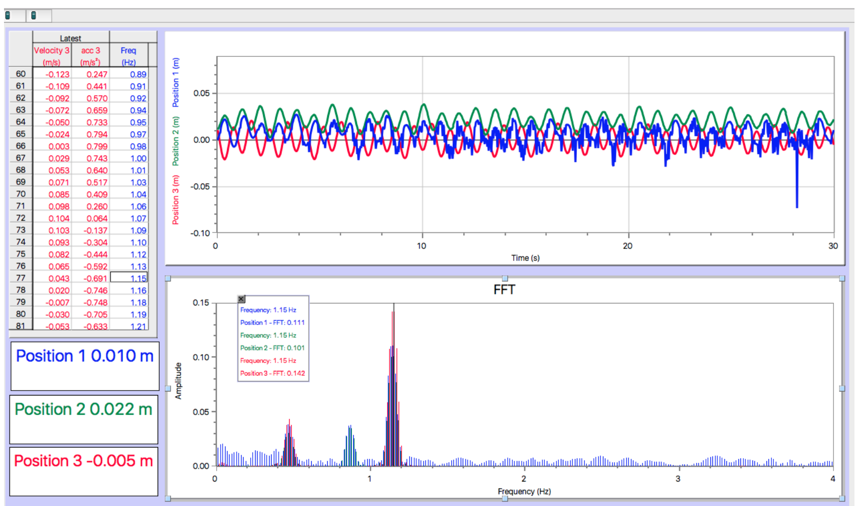

Figure 3: mode 3

Appendix B

Code for eigenvalues:

```
function homework1_3(m,k)
%my data k = 3.37 and 2*k = 6.74 for identical spring and masses
mass=[m 0 0;0 m 0;0 0 m];
kspring=[(2*k) k 0;k (2*k) k;0 k (2*k)];
```

```
B=(inv(mass)*kspring);
eigValue=eig(B);
w=sqrt(eigValue);
answer=w/(2*pi);
fspec='The eigenvalues are %g. \n';
fprintf(fspec,answer);
end
```

Output:

```
>> homework1_3(0.2, 3.37)
The eigenvalues are 0.500023.
The eigenvalues are 0.923922.
The eigenvalues are 1.20716.
```

Attached: additional hand work and data