

## [UFMG] Stenio Garcia (2016-17)

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## 1 InContests

## 1.1 Makefile

```
CXX=g++
CXXFLAGS=-std=c++11 -Wall

SRC=$(*.cpp)
OBJ=$(SRC:.cpp=%)
```

## 1.2 Vimrc

```
set ts=2 si ai sw=2 number mouse=a
syntax on
```

## 1.3 Template

```
#include <bits/stdc++.h>
using namespace std;
#define sc(a) scanf("%d", &a)
#define sc2(a, b) scanf("%d%d", &a, &b)
#define sc3(a, b, c) scanf("%d%d%d", &a, &b, &c)
#define pri(x) printf("%d\n", x)
#define prie(x) printf("%d ", x)
#define mp make_pair
#define pb push_back
#define BUFF ios::sync_with_stdio(false);
#define db(x) cerr << #x << " == " << x << endl
typedef long long int ll;
typedef long double ld;
typedef pair<int, int> ii;
typedef vector<int> vi;
const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const ld pi = acos(-1);
```

## 2 Graph Algorithms

## 2.1 2 SAT

```
/* Supondo que cada vertice u, o seu
 * positivo e 2*u, e negativo e 2*i+1
 * resposta[i]=0, significa que o positivo de i e resposta
 * resposta[i]=1, significa que o negativo de i e resposta
 * chamar Sat(n) , n e o numero de vertices do grafo
```

```

* contando com os negativos .. na maioria dos problemas
* chamar 2*n;
* testado em :http://codeforces.com/contest/781/problem/D
* */
int resposta[N];
vi graph[N], rev[N];
int us[N];
stack<int> pilha;
void dfs1(int u)
{
    us[u] = 1;
    for (int v : graph[u])
        if (!us[v]) dfs1(v);
    pilha.push(u);
}
void dfs2(int u, int color)
{
    us[u] = color;
    for (int v : rev[u])
        if (!us[v]) dfs2(v, color);
}
int Sat(int n)
{
    for (int i = 0; i < n; i++)
        if (!us[i]) dfs1(i);
    int color = 1;
    memset(us, 0, sizeof(us));
    while (!pilha.empty()) {
        int topo = pilha.top();
        pilha.pop();
        if (!us[topo]) dfs2(topo, color++);
    }
    for (int i = 0; i < n; i += 2) {
        if (us[i] == us[i + 1]) return 0;
        resposta[i / 2] = (us[i] < us[i + 1]);
    }
    return 1;
}
inline void add(int u, int v)
{
    graph[u].pb(v);
    rev[v].pb(u);
}
}

```

## 2.2 Kosaraju

```

//Retorna os componentes fortemente conectados
//Se o usados[i]=usados[j], temos que i e j
//pertencem ao mesmo componente, col-1= numero
//de componentes fortemente conectados do grafo
class kosaraju {
private:
    vi usados;
    vvi graph;
    vvi trans;
    vi pilha;

public:
    kosaraju(int N)
    {
        graph.resize(N);
        trans.resize(N);
    }
    void AddEdge(int u, int v)
    {
        graph[u].pb(v);
        trans[v].pb(u);
    }
    void dfs(int u, int pass, int color)
    {
        usados[u] = color;
        vi vizinhos;
        if (pass == 1)
            vizinhos = graph[u];
        else
            vizinhos = trans[u];
        for (int j = 0; j < vizinhos.size(); j++) {
            int v = vizinhos[j];
            if (usados[v] == 0) {
                dfs(v, pass, color);
            }
        }
        pilha.pb(u);
    }
    int SSC(int n)
    {
        pilha.clear();
    }
}

```

```

usados.assign(n, 0);
for (int i = 0; i < n; i++) {
    if (!usados[i]) dfs(i, 1, 1);
}
usados.assign(n, 0);
int color = 1;
for (int i = n - 1; i >= 0; i--) {
    if (usados[pilha[i]] == 0) {
        dfs(pilha[i], 2, color);
        color++;
    }
}
return color - 1;
}
};

```

## 2.3 Tree Isomorphism

```

vvi children, subtreeLabels, tree, L;
vi pred, map;
int n;

bool compare(int a, int b) { return subtreeLabels[a] < subtreeLabels[b]; }
bool equals(int a, int b) { return subtreeLabels[a] == subtreeLabels[b]; }
void generateMapping(int r1, int r2)
{
    map.resize(n);
    map[r1] = r2 - n;
    sort(children[r1].begin(), children[r1].end(), compare);
    sort(children[r2].begin(), children[r2].end(), compare);
    for (int i = 0; i < (int)children[r1].size(); i++) {
        int u = children[r1][i];
        int v = children[r2][i];
        generateMapping(u, v);
    }
}

vi findCenter(int offset = 0)
{
    int cnt = n;
    vi a;
    vi deg(n);
    for (int i = 0; i < n; i++) {
        deg[i] = tree[i + offset].size();
        if (deg[i] <= 1) {
            a.push_back(i + offset);
            --cnt;
        }
    }
    while (cnt > 0) {
        vi na;
        for (int i = 0; i < (int)a.size(); i++) {
            int u = a[i];
            for (int j = 0; j < (int)tree[u].size(); j++) {
                int v = tree[u][j];
                if (--deg[v - offset] == 1) {
                    na.push_back(v);
                    --cnt;
                }
            }
        }
        a = na;
    }
    return a;
}

int dfs(int u, int p = -1, int depth = 0)
{
    L[depth].push_back(u);
    int h = 0;
    for (int i = 0; i < (int)tree[u].size(); i++) {
        int v = tree[u][i];
        if (v == p) continue;
        pred[v] = u;
        children[u].push_back(v);
        h = max(h, dfs(v, u, depth + 1));
    }
    return h + 1;
}

bool rootedTreeIsomorphism(int r1, int r2)
{
    L.assign(n, vi());
    pred.assign(2 * n, -1);
    children.assign(2 * n, vi());

    int h1 = dfs(r1);
}

```

```

int h2 = dfs(r2);
if (h1 != h2) return false;

int h = h1 - 1;
vi label(2 * n);
subtreeLabels.assign(2 * n, vi());

for (int i = h - 1; i >= 0; i--) {
    for (int j = 0; j < (int)L[i + 1].size(); j++) {
        int v = L[i + 1][j];
        subtreeLabels[pred[v]].push_back(label[v]);
    }
    for (int j = 0; j < (int)L[i].size(); j++) {
        int v = L[i][j];
        sort(subtreeLabels[v].begin(), subtreeLabels[v].end());
    }

    sort(L[i].begin(), L[i].end(), compare);

    for (int j = 0, cnt = 0; j < (int)L[i].size(); j++) {
        if (j %& % != equals(L[i][j], L[i][j - 1])) ++cnt;
        label[L[i][j]] = cnt;
    }
}

if (!equals(r1, r2)) return false;

generateMapping(r1, r2);
return true;
}

bool treeIsomorphism()
{
    vi c1 = findCenter();
    vi c2 = findCenter(n);
    if (c1.size() == c2.size()) {
        if (rootedTreeIsomorphism(c1[0], c2[0]))
            return true;
        else if (c1.size() > 1)
            return rootedTreeIsomorphism(c1[1], c2[0]);
    }
    return false;
}

int main()
{
    n = 5;
    vvi t1(n);
    t1[0].pb(1); t1[1].pb(0);
    t1[1].pb(2); t1[2].pb(1);
    t1[1].pb(3); t1[3].pb(1);
    t1[0].pb(4); t1[4].pb(0);
    vvi t2(n);
    t2[0].pb(1); t2[1].pb(0);
    t2[0].pb(4); t2[4].pb(0);
    t2[4].pb(3); t2[3].pb(4);
    t2[4].pb(2); t2[2].pb(4);
    tree.assign(2 * n, vi());
    for (int u = 0; u < n; u++) {
        for (int i = 0; i < t1[u].size(); i++) {
            int v = t1[u][i];
            tree[u].push_back(v);
        }
        for (int i = 0; i < t2[u].size(); i++) {
            int v = t2[u][i];
            tree[u + n].push_back(v + n);
        }
    }
    bool res = treeIsomorphism();
    cout << res << endl;
    if (res)
        for (int i = 0; i < n; i++)
            cout << map[i] << endl;
}

```

## 2.4 LCA

```

//antes de usar as queries de lca, e etc..
//certifique-se de chamar a dfs da arvore e
//process()
const int N = 100000;
const int M = 22;
int P[N][M];
int big[N][M], low[N][M], level[N];
vii graph[N];
int n;

void dfs(int u, int last, int l)

```

```

{
    level[u] = l;
    P[u][0] = last;
    for (ii v : graph[u])
        if (v.first != last) {
            big[v.first][0] = low[v.first][0] = v.second;
            dfs(v.first, u, l + 1);
        }
}

void process()
{
    for (int j = 1; j < M; j++)
        for (int i = 1; i <= n; i++) {
            P[i][j] = P[P[i][j - 1]][j - 1];
            big[i][j] = max(big[i][j - 1], big[P[i][j - 1]][j - 1]);
            low[i][j] = min(low[i][j - 1], low[P[i][j - 1]][j - 1]);
        }
}

int lca(int u, int v)
{
    if (level[u] < level[v]) swap(u, v);
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[v]) u = P[u][i];
    if (u == v) return u;
    for (int i = M - 1; i >= 0; i--) {
        if (P[u][i] != P[v][i]) u = P[u][i], v = P[v][i];
    }
    return P[u][0];
}

int maximum(int u, int v, int x)
{
    int resp = 0;
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[x]) {
            resp = max(resp, big[u][i]);
            u = P[u][i];
        }
    for (int i = M - 1; i >= 0; i--)
        if (level[v] - (1 << i) >= level[x]) {
            resp = max(resp, big[v][i]);
            v = P[v][i];
        }
    return resp;
}

int minimum(int u, int v, int x)
{
    int resp = INF;
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[x]) {
            resp = min(resp, low[u][i]);
            u = P[u][i];
        }
    for (int i = M - 1; i >= 0; i--)
        if (level[v] - (1 << i) >= level[x]) {
            resp = min(resp, low[v][i]);
            v = P[v][i];
        }
    return resp;
}

```

## 2.5 Bridges and Articulation Points

```

class ponte {
private:
    vvi graph;
    vi usados;
    vi e_articulacao;
    vi dfs_low;
    vi dfs_prof;
    vector<ii> pontes;
    int tempo;

public:
    ponte(int N)
    {
        graph.clear();
        graph.resize(N);
        usados.assign(N, 0);
        dfs_low.assign(N, 0);
        dfs_prof.assign(N, 0);
        e_articulacao.assign(N, 0);
        tempo = 0;
    }
}

```

```

void AddEdge(int u, int v)
{
    graph[u].pb(v);
    graph[v].pb(u);
}

void dfs(int u, int pai)
{
    usados[u] = 1;
    int nf = 0;
    dfs_low[u] = dfs_prof[u] = tempo++;
    for (int v : graph[u]) {
        if (!usados[v]) {
            dfs(v, u);
            nf++;
            if (dfs_low[v] >= dfs_prof[u] and pai != -1) e_articulacao[u] = true;
            if (pai == -1 and nf > 1) e_articulacao[u] = true;
            if (dfs_low[v] > dfs_prof[u]) pontes.pb(mp(u, v));
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        }
        else if (v != pai)
            dfs_low[u] = min(dfs_low[u], dfs_prof[v]);
    }
}

void olha_as_pontes()
{
    for (int i = 0; i < graph.size(); i++)
        if (!usados[i]) dfs(i, -1);
    if (pontes.size() == 0)
        cout << " Que merda! nao tem ponte!" << endl;
    else {
        for (ii i : pontes) cout << i.first << " " << i.second << endl;
    }
}

void olha_as_art()
{
    for (int i = 0; i < graph.size(); i++)
        if (!usados[i]) dfs(i, -1);
    for (int i = 0; i < e_articulacao.size(); i++)
        if (e_articulacao[i]) cout << " OIAAA A PONTE " << i << endl;
}
}

```

## 2.6 Eulerian Tour

```

multiset<int> graph[N];
stack<int> path;
// -> It suffices to call dfs1 just
// one time leaving from node 0.
// -> To calculate the path,
// call the dfs from the odd degree node.
// -> O(n + log(n))
void dfs1(int u)
{
    while (graph[u].size())
    {
        int v = *graph[u].begin();
        graph[u].erase(graph[u].begin());
        graph[v].erase(graph[v].find(u));
        dfs1(v);
    }
    path.push(u);
}

```

## 2.7 Floyd Warshall

```

//menor caminho para todos os vertices
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        if (graph[i][j] != INF) pai[i][j] = i;

for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (graph[i][j] > graph[i][k] + graph[k][j]) {
                graph[i][j] = graph[i][k] + graph[k][j];
                pai[i][j] = pai[k][j];
            }
        }
    }
}

```

## 3 Strings

### 3.1 Aho Corasick

```

//N= tamanho da trie, M tamanho do alfabeto
int to[N][M], Link[N], fim[N];
int idx = 1;
void add_str(string &s)
{
    int v = 0;
    for (int i = 0; i < s.size(); i++) {
        if (!to[v][s[i]]) to[v][s[i]] = idx++;
        v = to[v][s[i]];
    }
    fim[v] = 1;
}

void process()
{
    queue<int> fila;
    fila.push(0);
    while (!fila.empty()) {
        int cur = fila.front();
        fila.pop();
        int l = Link[cur];
        fim[cur] |= fim[l];
        for (int i = 0; i < 200; i++) {
            if (to[cur][i]) {
                if (cur != 0) {
                    Link[to[cur][i]] = to[l][i];
                }
                else
                    Link[to[cur][i]] = 0;
                fila.push(to[cur][i]);
            }
            else {
                to[cur][i] = to[l][i];
            }
        }
    }
}

int resolve(string &s)
{
    int v = 0, r = 0;
    for (int i = 0; i < s.size(); i++) {
        v = to[v][s[i]];
        if (fim[v]) r++;
    }
    return r;
}

```

### 3.2 KMP

```

int b[100000];
int szet, sizep;
void kmpPreprocess(string &text, string &pattern)
{
    int i = 0, j = -1;
    b[0] = -1;
    while (i < sizep) {
        while (j >= 0 and pattern[i] != pattern[j]) j = b[j];
        i++; j++;
        b[i] = j;
    }
}

void kmpSearch(string &text, string &pattern)
{
    kmpPreprocess(text, pattern);
    int i = 0, j = 0;
    while (i < szet) {
        while (j >= 0 and text[i] != pattern[j]) j = b[j];
        i++; j++;
        if (j == sizep) {
            cout << "Olha a substring do texto " << i - j << endl;
            j = b[j];
        }
    }
}

```

### 3.3 Suffix Array

```

/*
 * O(nlog^2(n)) para o sufux array
 * O(logn) para o LCP(i,j)
 * LCP de i para j;
 */
struct SA {
    const int L;
    string s;
    vvi P;
    vector<pair< ii,int> > M;

    SA(const string &s) : L(s.size()), s(s), P(1, vi(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = s[i] - 'a';
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.pb(vi(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = mp(mp(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
        }

        vi GetSA() {
            vi v=P.back();
            vi ret(v.size());
            for(int i=0;i<v.size();i++){
                ret[v[i]]=i;
            }
            return ret;
        }

        int LCP(int i, int j) {
            int len = 0;
            if (i == j) return L - i;
            for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
                if (P[k][i] == P[k][j]) {
                    i += 1 << k;
                    j += 1 << k;
                    len += 1 << k;
                }
            }
            return len;
        }

        vi GetLCP(vi &sa)
        {
            vi lcp(sa.size()-1);
            for(int i=0;i<sa.size()-1;i++){
                lcp[i]=LCP(sa[i],sa[i+1]);
            }
            return lcp;
        }
    };
};

```

### 3.4 Suffix Array 2

```

/*****
 * Suffix Array. Building works in O(NlogN).
 * Also LCP array is calculated in O(NlogN).
 * This code counts number of different substrings in the string.
 * Based on problem 1 from here: http://codeforces.ru/gym/100133
 *****/

const int MAXN = 205000;
const int ALPH = 256;
const int MAXLOG = 20;

int n;
char s[MAXN];
int p[MAXN]; // suffix array itself
int pcur[MAXN];
int c[MAXN][MAXLOG];
int num[MAXN];
int classesNum;
int lcp[MAXN];

void buildSuffixArray() {
    n++;

    for (int i = 0; i < n; i++)
        num[s[i]]++;

    for (int i = 1; i < ALPH; i++)

```

```

        num[i] += num[i - 1];

    for (int i = 0; i < n; i++) {
        p[num[s[i]] - 1] = i;
        num[s[i]]--;
    }

    c[p[0]][0] = 1;
    classesNum = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i - 1]])
            classesNum++;
        c[p[i]][0] = classesNum;
    }

    for (int i = 1; ; i++) {
        int half = (1 << (i - 1));

        for (int j = 0; j < n; j++) {
            pcur[j] = p[j] - half;
            if (pcur[j] < 0)
                pcur[j] += n;
        }

        for (int j = 1; j <= classesNum; j++)
            num[j] = 0;

        for (int j = 0; j < n; j++)
            num[c[pcur[j]][i - 1]]++;
        for (int j = 2; j <= classesNum; j++)
            num[j] += num[j - 1];

        for (int j = n - 1; j >= 0; j--) {
            p[num[c[pcur[j]][i - 1]] - 1] = pcur[j];
            num[c[pcur[j]][i - 1]]--;
        }

        c[p[0]][i] = 1;
        classesNum = 1;

        for (int j = 1; j < n; j++) {
            int p1 = (p[j] + half) % n, p2 = (p[j - 1] + half) % n;
            if (c[p[j]][i - 1] != c[p[j - 1]][i - 1] || c[p1][i - 1] != c[p2][i - 1])
                classesNum++;
            c[p[j]][i] = classesNum;
        }

        if ((1 << i) >= n)
            break;
    }

    for (int i = 0; i < n; i++)
        p[i] = p[i + 1];
    n--;
}

int getLcp(int a, int b) {
    int res = 0;
    for (int i = MAXLOG - 1; i >= 0; i--) {
        int curlen = (1 << i);
        if (curlen > n)
            continue;
        if (c[a][i] == c[b][i]) {
            res += curlen;
            a += curlen;
            b += curlen;
        }
    }
    return res;
}

void calcLcpArray() {
    for (int i = 0; i < n - 1; i++)
        lcp[i] = getLcp(p[i], p[i + 1]);
}

int main() {
    assert(freopen("substr.in", "r", stdin));
    assert(freopen("substr.out", "w", stdout));

    gets(s);
    n = strlen(s);

    buildSuffixArray();

    // Now p from 0 to n - 1 contains suffix array of original string

    /*for (int i = 0; i < n; i++) {
        printf("%d ", p[i + 1]);
    }*/
}

```

```

    calcLcpArray();

    long long ans = 0;
    for (int i = 0; i < n; i++)
        ans += n - p[i];
    for (int i = 1; i < n; i++)
        ans -= lcp[i - 1];

    cout << ans << endl;

    return 0;
}

```

## 3.5 Suffix Array Dilsunginha

```

struct SuffixArray{
    const string& s;
    int n;
    vector<int> order, rank, lcp;
    vector<int> count, x, y;

    SuffixArray(const string& s) : s(s), n(s.size()), order(n), rank(n),
        count(n + 1), x(n), y(n), lcp(n){

        build();
        buildLCP();
    }

    void build(){
        //sort suffixes by the first character
        for(int i = 0; i < n; i++) order[i] = i;
        sort(order.begin(), order.end(), [&](int a, int b){return s[a] < s[b];});
        rank[order[0]] = 0;
        for(int i = 1; i < n; i++){
            rank[order[i]] = rank[order[i - 1]];
            if(s[order[i]] != s[order[i - 1]]) rank[order[i]]++;
        }

        //sort suffixes by the first 2*p characters, for p in 1, 2, 4, 8,...
        for(int p = 1; p < n, rank[order[n - 1]] < n - 1; p <= 1){
            for(int i = 0; i < n; i++){
                x[i] = rank[i];
                y[i] = i + p < n ? rank[i + p] + 1 : 0;

                radixPass(y);
                radixPass(x);

                rank[order[0]] = 0;
                for(int i = 1; i < n; i++){
                    rank[order[i]] = rank[order[i - 1]];
                    if(x[order[i]] != x[order[i - 1]] or y[order[i]] != y[order[i - 1]]) rank[order[i]]++;
                }
            }

            //Stable counting sort
            void radixPass(vector<int>& key){
                fill(count.begin(), count.end(), 0);
                for(auto index : order) count[key[index]]++;
                for(int i = 1; i <= n; i++) count[i] += count[i - 1];
                for(int i = n - 1; i >= 0; i--) lcp[--count[key[order[i]]]] = order[i];
                order.swap(lcp);
            }

            //Kasai's algorithm to build the LCP array from order, rank and s
            //For i >= 1, lcp[i] refers to the suffixes starting at order[i] and order[i - 1]
            void buildLCP(){
                lcp[0] = 0;
                int k = 0;
                for(int i = 0; i < n; i++){
                    if(rank[i] == n - 1){
                        k = 0;
                    } else {
                        int j = order[rank[i] + 1];
                        while(i + k < n and j + k < n and s[i + k] == s[j + k]) k++;
                        lcp[rank[i]] = k;
                        if(k) k--;
                    }
                }
            }
        }
    };

    int main(){
        ios::sync_with_stdio(false);

```

```

string s;
cin >> s;
SuffixArray sa(s);
for(int i = 0; i < s.size(); i++) cout << sa.order[i] << '\n';
}

```

## 3.6 Manacher Algorithm

```

/*****
Manacher's algorithm for finding all subpalindromes in the string.
Based on problem L from here: http://codeforces.ru/gym/100133
*****/

const int MAXN = 105000;

string s;
int n;
int odd[MAXN], even[MAXN];
int l, r;
long long ans;

int main() {
    assert(freopen("palindrome.in", "r", stdin));
    assert(freopen("palindrome.out", "w", stdout));

    getline(cin, s);
    n = (int) s.length();

    // Odd case
    l = r = -1;
    for (int i = 0; i < n; i++) {
        int cur = 1;
        if (i < r)
            cur = min(r - i + 1, odd[l + r - i]);
        while (i + cur < n && i - cur >= 0 && s[i - cur] == s[i + cur])
            cur++;
        odd[i] = cur;
        if (i + cur - 1 > r) {
            l = i - cur + 1;
            r = i + cur - 1;
        }
    }

    // Even case
    l = r = -1;
    for (int i = 0; i < n; i++) {
        int cur = 0;
        if (i < r)
            cur = min(r - i + 1, even[l + r - i + 1]);
        while (i + cur < n && i - 1 - cur >= 0 && s[i - 1 - cur] == s[i + cur])
            cur++;
        even[i] = cur;
        if (i + cur - 1 > r) {
            l = i - cur;
            r = i + cur - 1;
        }
    }

    for (int i = 0; i < n; i++) {
        if (odd[i] > 1) {
            ans += odd[i] - 1;
        }
        if (even[i])
            ans += even[i];
    }

    cout << ans << endl;

    return 0;
}

```

## 3.7 Splitting String

```

/* String s to be splitted and the delimiter used to split it. */
vector<string> splitstr(string s, string delimiter)
{
    vector<string> result;
    string str = s, token;
    size_t pos=0;
    while((pos=str.find(delimiter)) != std::string::npos)
    {
        token = str.substr(0, pos);
        result.push_back(token);

```

```

    str.erase(0, pos + delimiter.length());
}
result.push_back(str);
return result;
}

```

## 4 Numerical Algorithms

### 4.1 Fast Fourier Transform

```

// FFT - The Iterative Version
//
// Running Time:
//   O(n*log n)
//
// How To Use:
//   fft(a,1)
//   fft(b,1)
//   mul(a,b)
//   fft(a,-1)
//
// INPUT:
//   - fft method:
//     * The vector representing the polynomial
//     * 1 to normal transform
//     * -1 to inverse transform
//   - mul method:
//     * The two polynomials to be multiplied
//
// OUTPUT:
//   - fft method: Transforms the vector sent.
//   - mul method: The result is kept in the first vector.
//
// NOTES:
//   - You can either use the struct defined or define difencil as complex<double>
//
// SOLVED:
//   * Codeforces Round #296 (Div. 1) D. Fuzzy Search
//

```

```

struct difencil {
    double real;
    double im;
    difencil() {
        real=0.0;
        im=0.0;
    }

    difencil(double real, double im):real(real),im(im){}

    difencil operator+(const difencil &o) const {
        return difencil(o.real+real, im+o.im);
    }

    difencil operator/(double v) const {
        return difencil(real/v, im/v);
    }

    difencil operator*(const difencil &o) const {
        return difencil(real*o.real-im*o.im, real*o.im+im*o.real);
    }

    difencil operator-(const difencil &o) const {
        return difencil(real-o.real, im-o.im);
    }
};

difencil tmp[MAXN*2];
int coco,maiorpot2[MAXN];

void fft(vector<difencil> &A, int s)
{
    int n = A.size(), p = 0;

    while(n>1) {
        p++;
        n >>= 1;
    }

    n = (1<<p);
    vector<difencil> a=A;

    for(int i = 0; i < n; ++i){
        int rev = 0;

```

```

        for(int j = 0; j < p; ++j){
            rev <= 1;
            rev |= (i >> j) & 1;
        }
        A[i] = a[rev];
    }

    difencil w, wn;

    for(int i = 1; i <= p; ++i){
        int M = 1 << i;
        int K = M >> 1;
        wn = difencil(cos(s*2.0*pi/(double)M), sin(s*2.0*pi/(double)M));
        for(int j = 0; j < n; j += M){
            w = difencil(1.0, 0.0);
            for(int l = j; l < K + j; ++l){
                difencil t = w;
                t = t*A[l + K];
                difencil u = A[l];
                A[l] = A[l] + t;
                u = u-t;
                A[l + K] = u;
                w = wn*w;
            }
        }

        if(s==1){
            for(int i = 0; i<n; ++i)
                A[i] = A[i]/(double)n;
        }
    }

    void mul(vector<difencil> &a, vector<difencil> &b)
    {
        for(int i=0; i<a.size(); i++)
        {
            a[i]=a[i]*b[i];
        }
    }
}

```

### 4.2 Fast Fourier Transform 2

```

// FFT - The Recursive Version
//
// Running Time:
//   O(n*log n)
//
// How To Use:
//   fft(&a[0], tam, 0)
//   fft(&b[0], tam, 0)
//   mul(a,b)
//   fft(&a[0], tam, 1)
//
// INPUT:
//   - fft method:
//     * The vector representing the polynomial
//     * 0 to normal transform
//     * 1 to inverse transform
//   - mul method:
//     * The two polynomials to be multiplied
//
// OUTPUT:
//   - fft method: Transforms the vector sent.
//   - mul method: The result is kept in the first vector.
//
// NOTES:
//   - Tam has to be a power of 2.
//   - You can either use the struct defined or define difencil as complex<double>
//
// SOLVED:
//   * Codeforces Round #296 (Div. 1) D. Fuzzy Search
//

```

```

difencil tmp[MAXN*2];
int coco, maiorpot2[MAXN];

void fft(difencil *v, int N, bool inv)
{
    if(N<=1) return;
    difencil *vodd = v;
    difencil *veven = v+N/2;
    for(int i=0; i<N; i++) tmp[i] = v[i];
    coco = 0;
    for(int i=0; i<N; i+=2)
    {
        veven[coco] = tmp[i];
        vodd[coco] = tmp[i+1];
    }
}

```

```

    coco++;
}
fft(&vodd[0], N/2, inv);
fft(&veven[0], N/2, inv);

difícil w(1);
double angucomleite = 2.0*PI/(double)N;
if(inv) angucomleite = -angucomleite;

difícil wn(cos(angucomleite), sin(angucomleite));
for(int i=0; i<N/2; i++)
{
    tmp[i] = veven[i]+w*vodd[i];
    tmp[i+N/2] = veven[i]-w*vodd[i];
    w *= wn;
    if(inv)
    {
        tmp[i] /= 2;
        tmp[i+N/2] /= 2;
    }
}
for(int i=0; i<N; i++) v[i] = tmp[i];
}

void mul(vector<difícil> &a, vector<difícil> &b)
{
    for(int i=0; i<a.size(); i++)
    {
        a[i] = a[i]*b[i];
    }
}

void precomp()
{
    int pot=0;
    for(int i=1; i<MAXN; i++)
    {
        if((1<<pot)<i) pot++;
        maiorpot2[i] = (1<<pot);
    }
}

```

## 4.3 Simpson Algorithm

```

const int NPASSOS = 100000;
const int W=1000000;
//W= tamanho do intervalo que eu estou integrando
double integrall()
{
    double h = W / (NPASSOS);
    double a = 0;
    double b = W;
    double s = f(a) + f(b);
    for (double i = 1; i <= NPASSOS; i += 2) s += f(a + i * h) * 4.0;
    for (double i = 2; i <= (NPASSOS - 1); i += 2) s += f(a + i * h) * 2.0;
    return s * h / 3.0;
}

```

## 4.4 Matrix Exponentiation

```

//matmul multiplica m1 por m2
//matpow exponencia a matrix m1 por p
//mul vet multiplica a matrix m1 pelo vetor vet
vvi matmul(vvi &m1, vvi &m2)
{
    vvi ans;
    ans.resize(m1.size(), vi(m2.size(), 0));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++) {
                ans[i][j] += m1[i][k] * m2[k][j];
                ans[i][j] %= MOD;
            }
    return ans;
}

vvi matpow(vvi &m1, ll p)
{
    vvi ans;
    ans.resize(m1.size(), vi(m1.size(), 0));
    for (int i = 0; i < n; i++) ans[i][i] = 1;
    while (p) {
        if (p & 1) ans = matmul(ans, m1);
    }
}

```

```

    m1 = matmul(m1, m1);
    p >>= 1;
}
return ans;
}

// VETOR TEM N LINHAS E A MATRIZ E QUADRADA
vvi mulvet(vvi &m1, vi &vet)
{
    vi ans;
    ans.resize(vet.size(), 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            ans[i] += (m1[i][j] * vet[j]);
            ans[i] %= MOD;
        }
    return ans;
}

```

# 5 Mathematics

## 5.1 Big Number

```

void zero_esq(string &resp)
{
    string retorno = resp;
    reverse(retorno.begin(), retorno.end());
    int i = resp.size() - 1;
    while (retorno[i] == '0' and i > 0) {
        retorno.erase(i);
        i--;
    }
    reverse(retorno.begin(), retorno.end());
    resp = retorno;
}

string sum_big(string a, string b)
{
    string resp;
    reverse(a.begin(), a.end());
    reverse(b.begin(), b.end());
    if (a.size() <= b.size()) {
        int carry = 0;
        for (int i = 0; i < a.size(); i++) {
            int x = b[i] - '0' + a[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        for (int i = a.size(); i < b.size(); i++) {
            int x = b[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        if (carry > 0) resp.push_back((char)(carry + '0'));
    }
    else {
        int carry = 0;
        for (int i = 0; i < b.size(); i++) {
            int x = a[i] - '0' + b[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        for (int i = b.size(); i < a.size(); i++) {
            int x = a[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        if (carry > 0) resp.push_back((char)(carry + '0'));
    }
    reverse(resp.begin(), resp.end());
    zero_esq(resp);
    return resp;
}

string mul_big(string a, string b)
{
    string resp;
    resp.push_back('0');
    string temp;
    int carry = 0;
    reverse(a.begin(), a.end());
    reverse(b.begin(), b.end());
    for (int i = 0; i < a.size(); i++) {
        temp.clear();
        for (int k = 0; k < i; k++) temp.push_back('0');
        int x = a[i] - '0';
        for (int j = 0; j < b.size(); j++) {
            int y = b[j] - '0';

```



```

    int novo = (x * y + carry);
    temp.push_back((novo % 10) + '0');
    carry = novo / 10;
}
if (carry > 0) temp.push_back(carry + '0');
reverse(temp.begin(), temp.end());
carry = 0;
resp = sum_big(temp, resp);
}
zero_esq(resp);
return resp;
}

```

## 5.2 Big Number 2

```

/*****
Structure implementing long arithmetic in C++
Analogue to BigInteger in Java.
Tested on many problems.
TODO: list some problems
*****/

struct BigInt {
    vector<int> num;

    static const int base = 1000 * 1000 * 1000;
    static const int baseDigits = 9;
    string leadingZerosModifier;

    /*****
    * CONSTRUCTORS & SETTERS
    *****/

    void setLeadingZerosModifier() {
        leadingZerosModifier = "%0xd";
        leadingZerosModifier[2] = '0' + baseDigits;
    }

    void set(int value) {
        num.clear();
        if (value == 0)
            num.push_back(0);
        while (value) {
            num.push_back(value % base);
            value /= base;
        }
    }

    void set(long long value) {
        num.clear();
        if (value == 0)
            num.push_back(0);
        while (value) {
            num.push_back(value % base);
            value /= base;
        }
    }

    void set(string &value) {
        num.clear();
        for (int i = (int)value.length() - 1; i >= 0; i -= baseDigits) {
            int add = 0;
            for (int j = max(0, i - baseDigits + 1); j <= i; j++)
                add = add * 10 + value[j] - '0';
            num.push_back(add);
        }
    }

    void operator = (int value) {
        set(value);
    }

    void operator = (long long value) {
        set(value);
    }

    void operator = (string &value) {
        set(value);
    }

    BigInt() {
        setLeadingZerosModifier();
        set(0);
    }

    BigInt(int value) {

```

```

        setLeadingZerosModifier();
        set(value);
    }

    BigInt(string value) {
        setLeadingZerosModifier();
        set(value);
    }

    /*****
    * SIZE METHODS
    *****/

    //returns size of vector
    int size() {
        return (int)num.size();
    }

    //returns length of the number
    int digitNum() {
        int result = 0;
        for (int i = 0; i < (int)num.size() - 1; i++)
            result += baseDigits;
        int lastNum = num.back();
        while (lastNum) {
            result++;
            lastNum /= 10;
        }
        return result;
    }

    /*****
    * I/O
    *****/

    void read() {
        string s;
        cin >> s;
        num.clear();
        for (int i = (int)s.length() - 1; i >= 0; i -= baseDigits) {
            int add = 0;
            for (int j = max(0, i - baseDigits + 1); j <= i; j++)
                add = add * 10 + s[j] - '0';
            num.push_back(add);
        }
    }

    void print() {
        printf("%d", num.back());
        for (int i = (int)num.size() - 2; i >= 0; i--)
            printf(leadingZerosModifier.c_str(), num[i]);
    }

    void println() {
        print();
        printf("\n");
    }

    string toString() {
        string result = "";
        for (int i = 0; i < (int)num.size(); i++) {
            int cur = num[i];
            for (int j = 1; j <= baseDigits; j++) {
                if (cur == 0 && i == (int) num.size() - 1)
                    break;
                result.append(1, (char) '0' + cur % 10);
                cur /= 10;
            }
        }
        reverse(result.begin(), result.end());
        return result;
    }

    /*****
    * ADDITION
    *****/

    void sumThis(BigInt number) {
        int carry = 0;
        for (int i = 0; i < max((int)num.size(), number.size()) || carry; i++) {
            if (i == num.size())
                num.push_back(0);
            if (i >= number.size())
                carry = num[i] + carry;
            else
                carry = num[i] + carry + number.num[i];
            num[i] = carry % base;
            carry /= base;
        }
    }

    void sumThis(int number) {

```

```

    int carry = number;
    for (int i = 0; i < (int)num.size() || carry; i++) {
        if (i == num.size())
            num.push_back(0);
        carry = num[i] + carry;
        num[i] = carry % base;
        carry /= base;
    }
}

BigInt sum(BigInt number) {
    BigInt result = *this;
    result.sumThis(number);
    return result;
}

BigInt sum(int number) {
    BigInt result = *this;
    result.sumThis(number);
    return result;
}

void operator += (BigInt number) {
    sumThis(number);
}

void operator += (int number) {
    sumThis(number);
}

BigInt operator + (BigInt number) {
    return sum(number);
}

BigInt operator + (int number) {
    return sum(number);
}

/*=====
 * SUBTRACTION
=====*/

void subThis(BigInt number) {
    int carry = 0;
    for (int i = 0; i < (int)number.size() || carry; i++) {
        if (i < (int)number.size())
            num[i] -= carry + number.num[i];
        else
            num[i] -= carry;
        if (num[i] < 0) {
            carry = 1;
            num[i] += base;
        }
        else
            carry = 0;
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

void subThis(int number) {
    int carry = -number;
    for (int i = 0; carry > 0; i++) {
        num[i] -= carry;
        if (num[i] < 0) {
            carry = 1;
            num[i] += base;
        }
        else
            carry = 0;
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

BigInt sub(BigInt number) {
    BigInt result = *this;
    result.subThis(number);
    return result;
}

BigInt sub(int number) {
    BigInt result = *this;
    result.subThis(number);
    return result;
}

void operator -= (BigInt number) {
    subThis(number);
}

void operator -= (int number) {
    subThis(number);
}

    subThis(number);
}

BigInt operator - (BigInt number) {
    return sub(number);
}

BigInt operator - (int number) {
    return sub(number);
}

/*=====
 * MULTIPLICATION
=====*/

BigInt mult(BigInt number) {
    BigInt product;
    product.num.resize(num.size() + number.size());
    for (int i = 0; i < (int)num.size(); i++) {
        for (int j = 0, carry = 0; j < (int)number.size() || carry; j++) {
            long long cur = product.num[i + j] + num[i] * 11l * (j < (int)number.size() ? number.num[j] :
                0) + carry;
            product.num[i + j] = int (cur % base);
            carry = int (cur / base);
        }
    }
    while (product.size() > 1 && product.num.back() == 0)
        product.num.pop_back();
    return product;
}

void multThis(BigInt number) {
    *this = mult(number);
}

void multThis(int number) {
    int carry = 0;
    for (int i = 0; i < (int)num.size() || carry; ++i) {
        if (i == num.size())
            num.push_back(0);
        long long cur = carry + num[i] * 11l * number;
        num[i] = int (cur % base);
        carry = int (cur / base);
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

BigInt mult(int number) {
    BigInt result = *this;
    result.multThis(number);
    return result;
}

void operator *= (BigInt number) {
    multThis(number);
}

void operator *= (int number) {
    multThis(number);
}

BigInt operator * (BigInt number) {
    return mult(number);
}

BigInt operator * (int number) {
    return mult(number);
}

void multThisByPowerOfTen(int power) {
    int baseNums = power / baseDigits;
    int curLen = (int)num.size();
    num.resize(curLen + baseNums);
    for (int i = num.size() - 1; i >= baseNums; i--) {
        num[i] = num[i - baseNums];
    }
    for (int i = baseNums - 1; i >= 0; i--)
        num[i] = 0;
    power %= baseDigits;
    int multBy = (int)pow(10.0, power);
    multThis(multBy);
}

/*=====
 * DIVISION
=====*/

void divThis(int number) {
    int carry = 0;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        long long cur = num[i] + carry * 11l * base;
    }
}

```

```

    num[i] = int (cur / number);
    carry = int (cur % number);
}
while (num.size() > 1 && num.back() == 0)
    num.pop_back();
}

BigInt div(int number) {
    BigInt result = *this;
    result.divThis(number);
    return result;
}

void operator /= (int number) {
    divThis(number);
}

BigInt operator / (int number) {
    return div(number);
}

void divThisByPowerOfTen(int power) {
    int baseNums = power / baseDigits;
    int curLen = (int)num.size();
    for (int i = 0; i < (int)num.size() - baseNums; i++) {
        num[i] = num[i + baseNums];
    }
    for (int i = 1; i <= baseNums; i++)
        num.pop_back();
    power %= baseDigits;
    int divBy = (int)pow(10.0, power);
    divThis(divBy);
}

/*=====
 * MODULO
=====*/

void modThis(int number) {
    int carry = 0;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        long long cur = num[i] + carry * 111 * base;
        num[i] = int (cur / number);
        carry = int (cur % number);
    }
    set(carry);
}

BigInt mod(int number) {
    BigInt result = *this;
    result.modThis(number);
    return result;
}

void operator %= (int number) {
    modThis(number);
}

BigInt operator % (int number) {
    return mod(number);
}

/*=====
 * COMPARISON
=====*/

//Returns: -1 - this number is less than argument, 0 - equal, 1 - this number is greater
int compareTo(BigInt number) {
    if ((int)num.size() < number.size())
        return -1;
    if ((int)num.size() > number.size())
        return 1;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        if (num[i] > number.num[i])
            return 1;
        if (num[i] < number.num[i])
            return -1;
    }
    return 0;
}

//Returns: -1 - this number is less than argument, 0 - equal, 1 - this number is greater
int compareTo(int number) {
    if (num.size() > 1 || num[0] > number)
        return 1;
    if (num[0] < number)
        return -1;
    return 0;
}

bool operator < (BigInt number) {
    return compareTo(number) == -1;
}

```

```

}

bool operator < (int number) {
    return compareTo(number) == -1;
}

bool operator <= (BigInt number) {
    return compareTo(number) != 1;
}

bool operator <= (int number) {
    return compareTo(number) != 1;
}

bool operator == (BigInt number) {
    return compareTo(number) == 0;
}

bool operator == (int number) {
    return compareTo(number) == 0;
}

bool operator > (BigInt number) {
    return compareTo(number) == 1;
}

bool operator > (int number) {
    return compareTo(number) == 1;
}

bool operator >= (BigInt number) {
    return compareTo(number) != -1;
}

bool operator >= (int number) {
    return compareTo(number) != -1;
}

bool operator != (BigInt number) {
    return compareTo(number) != 0;
}

bool operator != (int number) {
    return compareTo(number) != 0;
}
};

```

## 5.3 Chinese Remainder

```

11 mulmod(11 a, 11 b, 11 m)
{
    11 ret = 0;
    while (b > 0) {
        if (b % 2 != 0) ret = (ret + a) % m;
        a = (a + a) % m;
        b >>= 1;
    }
    return ret;
}

11 expmod(11 a, 11 e, 11 m)
{
    11 ret = 1;
    while (e > 0) {
        if (e % 2 != 0) ret = mulmod(ret, a, m);
        a = mulmod(a, a, m);
        e >>= 1;
    }
    return ret;
}

11 invmul(11 a, 11 m) { return expmod(a, m - 2, m); }
11 chinese(vector<11> r, vector<11> m)
{
    int sz = m.size();
    11 M = 1;
    for (int i = 0; i < sz; i++) {
        M *= m[i];
    }
    11 ret = 0;
    for (int i = 0; i < sz; i++) {
        ret += mulmod(mulmod(M / m[i], r[i], M), invmul(M / m[i], M), M);
        ret = ret % M;
    }
    return ret;
}

```

## 5.4 Chinese Remainder 2

```
// Chinese remainder theorem (special case): find z such that // z % m1 = r1, z
// % m2 = r2. Here, z is unique modulo M = lcm(m1, m2). // Return (z, M). On
// failure, M = -1;
ii chinese_remainder_theorem(int m1, int r1, int m2, int r2)
{
    int s, t;
    int g = extended_euclid(m1, m2, s, t);
    if (r1 % g != r2 % g) return mp(0, -1);
    return mp(mod(s * r2 * m1 + t * r1 * m2, m1 * m2) / g, m1 * m2 / g);
}
// Chinese remainder theorem: find z such that // z % m[i] =
// r[i] for all i
// .Note that the solution is unique modulo M = lcm_i (m[i]).
// Return(z, M)
// .On // failure, M = -1. Note that we do not require the a[i] s
// to be relatively prime.
ii chinese_remainder_theorem(const vi &m, const vi &r)
{
    ii ret = make_pair(r[0], m[0]);
    for (int i = 1; i < m.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
        if (ret.second == -1) break;
    }
    return ret;
}
```

## 5.5 Matrix Exponentiation

```
//matmul multiplica m1 por m2
//matpow exponencia a matrix m1 por p
//mul vet multiplica a matrix m1 pelo vetor vet
vvi matmul(vvi &m1, vvi &m2)
{
    vvi ans;
    ans.resize(m1.size(), vi(m2.size(), 0));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++) {
                ans[i][j] += m1[i][k] * m2[k][j];
                ans[i][j] %= MOD;
            }
    return ans;
}
vvi matpow(vvi &m1, ll p)
{
    vvi ans;
    ans.resize(m1.size(), vi(m1.size(), 0));
    for (int i = 0; i < n; i++) ans[i][i] = 1;
    while (p) {
        if (p & 1) ans = matmul(ans, m1);
        m1 = matmul(m1, m1);
        p >>= 1;
    }
    return ans;
}
// VETOR TEM N LINHAS E A MATRIZ E QUADRADA
vi mulvet(vvi &m1, vi &vet)
{
    vi ans;
    ans.resize(vet.size(), 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            ans[i] += (m1[i][j] * vet[j]);
            ans[i] %= MOD;
        }
    return ans;
}
```

## 5.6 Pascal Triangle

```
//Fazer combinacao de N escolhe M
//por meio do triangulo de pascal
//Complexidade: O(m*n)
unsigned long long comb[61][61];
for (int i = 0; i < 61; i++) {
    comb[i][i] = 1;
    comb[i][0] = 1;
}
```

```
}
for (int i = 2; i < 61; i++)
    for (int j = 1; j < i; j++)
        comb[i][j] = comb[i - 1][j] + comb[i - 1][j - 1];
```

## 5.7 Euler's Totient Function

```
//retorna quantos elementos coprimos
//a N e menores que n existem
int phi(int n)
{
    int result = n;
    for (int i = 2; i * i <= n; ++i)
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    if (n > 1) result -= result / n;
    return result;
}
```

## 5.8 Pollard Rho

```
ll u;
ll t;
const int tamteste=5;
ll abss(ll v){ return v>=0 ? v : -v;}
ll randerson()
{
    ld pseudo=(ld)rand()/(ld)RAND_MAX;
    return (ll)(round((ld)range+pseudo))+1LL;
}

ll mulmod(ll a, ll b, ll mod)
{
    ll ret=0;
    while(b>0)
    {
        if(b%2!=0) ret=(ret+a)%mod;
        a=(a+a)%mod;
        b=b/2LL;
    }
    return ret;
}

ll expmod(ll a, ll e, ll mod)
{
    ll ret=1;
    while(e>0)
    {
        if(e%2!=0) ret=mulmod(ret,a,mod);
        a=mulmod(a,a,mod);
        e=e/2LL;
    }
    return ret;
}

bool jeova(ll a, ll n)
{
    ll x = expmod(a,u,n);
    ll last=x;
    for(int i=0;i<t;i++)
    {
        x=mulmod(x,x,n);
        if(x==1 and last!=1 and last!=(n-1)) return true;
        last=x;
    }
    if(x==1) return false;
    return true;
}

bool isprime(ll n)
{
    u=n-1;
    t=0;
    while(u%2==0)
    {
        t++;
        u/=2LL;
    }
    if(n==2) return true;
    if(n==3) return true;
    if(n%2==0) return false;
```

```

if(n<2) return false;
for(int i=0;i<tamteste;i++)
{
    ll v = randerson()% (n-2)+1;
    //cout<<"jeova "<<v<<" "<<n<<endl;
    if(jeova(v,n)) return false;
}
return true;
}

ll gcd(ll a, ll b){ return !b ? a : gcd(b,a%b);}

ll calc(ll x, ll n, ll c)
{
    return (mulmod(x,x,n)+c)%n;
}

ll pollard(ll n)
{
    ll d=1;
    ll i=1;
    ll k=1;
    ll x=2;
    ll y=x;
    ll c;
    do
    {
        c=randerson()%n;
    }while(c==0 or (c+2)%n==0);
    while(d!=n)
    {
        if(i==k)
        {
            k*=2LL;
            y=x;
            i=0;
        }
        x=calc(x,n,c);
        i++;
        d=gcd(abss(y-x),n);
        if(d!=1) return d;
    }
}

vector<ll> getdiv(ll n)
{
    vector<ll> ret;
    if(n==1) return ret;
    if(isprime(n))
    {
        ret.pb(n);
        return ret;
    }
    ll d = pollard(n);
    ret=getdiv(d);
    vector<ll> ret2=getdiv(n/d);
    for(int i=0;i<ret2.size();i++) ret.pb(ret2[i]);
    return ret;
}

```

## 5.9 Sieve of Eratosthenes

```

//esse crivo gera MAXN primos
const int MAX = 1e6;
int primes[MAXN];
void gen_primes()
{
    int i, j;
    for (i = 2; i*i <= MAX; i++)
        if (primes[i])
            for (j = i; j * i < MAX; j++) primes[i * j] = 0;
}

```

## 5.10 Extended Euclidean Algorithm

```

//returns g = gcd(a, b);
//finds x,y such that d= ax+by;
int extended_euclid(int a, int b, int &x, int &y)
{
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a / b;
        int t = b;

```

```

        b = a % b;
        a = t;
        t = xx;
        xx = x - q * xx;
        x = t;
        t = yy;
        yy = y - q * yy;
        y = t;
    }
    return a;
}

```

## 5.11 Multiplicative Inverse

```

//computes b such that ab = 1(mod n), returns -1 on failure
int mod_inverse(int a, int n)
{
    int x, y;
    int g = extended_euclid(a, n, x, y);
    if (g > 1) return -1;
    return (x+n)%n;
}

```

## 5.12 Multiplicative Inverse 2

```

//inverso multiplicativo de A % MOD
//certifique de MOD estar definido antes bonito!
//complexidade: O(log(a))
ll mul_inv(ll a)
{
    ll pin0 = MOD, pin = MOD, t, q;
    ll x0 = 0, x1 = 1;
    if (pin == 1) return 1;
    while (a > 1) {
        q = a / pin;
        t = pin, pin = a % pin, a = t;
        t = x0, x0 = x1 - q * x0, x1 = t;
    }
    if (x1 < 0) x1 += pin0;
    return x1;
}

```

# 6 Combinatorial Optimization

## 6.1 Dinic

```

//grafo bipartido O(Esqrt(v))
//Para recuperar a resposta, e so colocar um bool
//de false na aresta de retorno e fazer uma bfs/dfs
//andando pelos vertices de capacidade =0 e arestas
//que nao sao de retorno
struct Edge {
    int v, rev;
    int cap;
    Edge(int v_, int cap_, int rev_) : v(v_), rev(rev_), cap(cap_) {}
};

struct MaxFlow {
    vector<vector<Edge>> > g;
    vector<int> level;
    queue<int> q;
    int flow, n;

    MaxFlow(int n_) : g(n_), level(n_), n(n_) {}
    void addEdge(int u, int v, int cap)
    {
        if (u == v) return;
        Edge e(v, cap, int(g[v].size()));
        Edge r(u, 0, int(g[u].size()));
        g[u].push_back(e);
        g[v].push_back(r);
    }

    bool buildLevelGraph(int src, int sink)
    {
        fill(level.begin(), level.end(), -1);

```

```

while (not q.empty()) q.pop();
level[src] = 0;
q.push(src);
while (not q.empty()) {
    int u = q.front();
    q.pop();
    for (auto e = g[u].begin(); e != g[u].end(); ++e) {
        if (not e->cap or level[e->v] != -1) continue;
        level[e->v] = level[u] + 1;
        if (e->v == sink) return true;
        q.push(e->v);
    }
}
return false;
}

int blockingFlow(int u, int sink, int f)
{
    if (u == sink or not f) return f;
    int fu = f;
    for (auto e = g[u].begin(); e != g[u].end(); ++e) {
        if (not e->cap or level[e->v] != level[u] + 1) continue;
        int mincap = blockingFlow(e->v, sink, min(fu, e->cap));
        if (mincap) {
            g[e->v][e->rev].cap += mincap;
            e->cap -= mincap;
            fu -= mincap;
        }
    }
    if (f == fu) level[u] = -1;
    return f - fu;
}

int maxFlow(int src, int sink)
{
    flow = 0;
    while (buildLevelGraph(src, sink))
        flow += blockingFlow(src, sink, numeric_limits<int>::max());
    return flow;
}
};

```

## 6.2 Hopcroft-Karp Bipartite Matching

```

/* O(V^3)
 * Matching maximo de grafo bipartido de peso 1 nas arestas
 * supondo que o grafo bipartido seja enumerado de 0-n-1
 * chamamos maxMatch(n)
 */
class MaxMatch {
    vi graph[N];
    int match[N], us[N];

public:
    MaxMatch(){};
    void addEdge(int u, int v) { graph[u].pb(v); }
    int dfs(int u)
    {
        if (us[u]) return 0;
        us[u] = 1;
        for (int v : graph[u]) {
            if (match[v] == -1 or (dfs(match[v]))) {
                match[v] = u;
                return 1;
            }
        }
        return 0;
    }
    int maxMatch(int n)
    {
        memset(match, -1, sizeof(match));
        int ret = 0;
        for (int i = 0; i < n; i++) {
            memset(us, 0, sizeof(us));
            ret += dfs(i);
        }
        return ret;
    }
};

```

## 6.3 Max Bipartite Matching 2

```
// This code performs maximum bipartite matching.
```

```

//
// Running time: O(|E| |V|) -- often much faster in practice
//
// INPUT: w[i][j] = edge between row node i and column node j
// OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//         mc[j] = assignment for column node j, -1 if unassigned
//         function returns number of matches made
#include <vector>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {
        if (w[i][j] && !seen[j]) {
            seen[j] = true;
            if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
                mr[i] = j;
                mc[j] = i;
                return true;
            }
        }
    }
    return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);
    int ct = 0;
    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}

```

## 6.4 Min Cost Matching

```

////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[i][j] matrix.
////////////////////////////////////
#include <algorithm>
#include <cmath>
#include <cstdio>
#include <vector>

using namespace std;

typedef vector<double> VVD;
typedef vector<VVD> VVVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate)
{
    int n = int(cost.size());

    // construct dual feasible solution
    VVD u(n);
    VVD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;

```

```

    if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
    }
}

VD dist(n);
VI dad(n);
VI seen(n);

// repeat until primal solution is feasible
while (mated < n) {
    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++) dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {
        // find closest
        j = -1;
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            if (j == -1 || dist[k] < dist[j]) j = k;
        }
        seen[j] = 1;

        // termination condition
        if (Rmate[j] == -1) break;

        // relax neighbors
        const int i = Rmate[j];
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
            if (dist[k] > new_dist) {
                dist[k] = new_dist;
                dad[k] = j;
            }
        }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
        if (k == j || !seen[k]) continue;
        const int i = Rmate[k];
        v[k] += dist[k] - dist[j];
        u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
        const int d = dad[j];
        Rmate[j] = Rmate[d];
        Lmate[Rmate[j]] = j;
        j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
}

double value = 0;
for (int i = 0; i < n; i++) value += cost[i][Lmate[i]];

return value;
}

```

## 6.5 Min Cost Max Flow

```

int flow[N][N];
vector<pair<int, int>> g[N];

int n, m, k;

inline int ent(int a) { return a + 2; }
inline int out(int a) { return a + 2 + 1; }
inline void addEdge(int a, int b, int custo, int fluxo)

```

```

{
    flow[a][b] += fluxo;
    g[a].push_back(make_pair(b, custo));
    g[b].push_back(make_pair(a, -custo));
}

int src = N - 1, tgt = N - 2;
int dis[N], pai[N];

inline int dij()
{
    memset(dis, INF, sizeof dis);
    memset(pai, -1, sizeof pai);
    priority_queue<pair<int, int>> q;
    dis[src] = 0;
    q.push(make_pair(0, src));
    while (!q.empty()) {
        pair<int, int> foo = q.top();
        q.pop();
        int x = foo.second, cost = -foo.first;
        if (dis[x] != cost) continue;
        for (int i = 0; i < g[x].size(); ++i) {
            int y = g[x][i].first, w = g[x][i].second;
            if (flow[x][y] <= 0) continue;
            if (dis[y] > dis[x] + w) {
                dis[y] = dis[x] + w;
                pai[y] = x;
                q.push(make_pair(-dis[y], y));
            }
        }
    }
    return dis[tgt] != INF;
}

int minCost()
{
    int maxFlow = 0;
    int minC = 0;
    while (dij()) {
        int u = tgt;
        int minFlow = INF;
        while (pai[u] != -1) {
            minFlow = min(minFlow, flow[pai[u]][u]);
            u = pai[u];
        }
        maxFlow += minFlow;
        minC += minFlow * dis[tgt];
        u = tgt;
        while (pai[u] != -1) {
            flow[pai[u]][u] -= minFlow;
            flow[u][pai[u]] += minFlow;
            u = pai[u];
        }
    }
    if (maxFlow != n * k) minC = -1;
    return minC;
}

inline void init()
{
    memset(flow, 0, sizeof flow);
    for (int i = 0; i < N; ++i) {
        g[i].clear();
    }
}

```

## 6.6 Min Cost Max Flow 2

```

// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.

// Running time: O(|V|^2) cost per augmentation
// max flow: O(|V|^3) augmentations
// min cost max flow: O(|V|^4 * MAX_EDGE_COST) augmentations

// INPUT:
// - graph, constructed using AddEdge()
// - source
// - sink

// OUTPUT:
// - (maximum flow value, minimum cost value)
// - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <iostream>

```

```

#include <vector>

using namespace std;

typedef vector<VI> VVI;
typedef long long LL;
typedef vector<LL> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const LL INF = numeric_limits<LL>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;

    MinCostMaxFlow(int N): N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, LL cap, LL cost)
    {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, LL cap, LL cost, int dir)
    {
        LL val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    LL Dijkstra(int s, int t)
    {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
            s = best;
        }

        for (int k = 0; k < N; k++) pi[k] = min(pi[k] + dist[k], INF);
        return width[t];
    }

    pair<LL, LL> GetMaxFlow(int s, int t)
    {
        LL totflow = 0, totcost = 0;
        while (LL amt = Dijkstra(s, t)) {
            totflow += amt;
            for (int x = t; x != s; x = dad[x].first) {
                if (dad[x].second == 1) {
                    flow[dad[x].first][x] += amt;
                    totcost += amt * cost[dad[x].first][x];
                }
                else {
                    flow[x][dad[x].first] -= amt;
                    totcost -= amt * cost[x][dad[x].first];
                }
            }
        }
        return make_pair(totflow, totcost);
    }
};

```

```

ll cap;
void init(int _at, ll _cap, int _where)
{
    at = _at, cap = _cap, where = _where;
}

struct dad {
    int at, up, down;
    dad() { at = -1; }
    dad(int _at, int _up, int _down) { at = _at, up = _up, down = _down; }
};

class MaxFlow {
private:
    vector<vector<Edge>> > g;
    ll mf, f;
    int s, t;
    vector<dad> p;

public:
    void augment(int v, ll minEdge)
    {
        if (v == s) {
            f = minEdge;
            return;
        }
        else if (p[v].at != -1) {
            augment(p[v].at, min(minEdge, g[p[v].at][p[v].up].cap));
            g[p[v].at][p[v].up].cap -= f;
            g[v][p[v].down].cap += f;
        }
    }

    void init(int N)
    {
        for (int i = 0; i < g.size(); i++) g[i].clear();
        mf = 0, f = 0;
        g.resize(N);
    }

    void addEdge(int u, int v, ll cap)
    {
        Edge A;
        A.init(v, cap, g[v].size());
        Edge B;
        B.init(u, 0, g[u].size());
        g[u].pb(A);
        g[v].pb(B);
    }

    int maxFlow(int source, int sink)
    {
        s = source;
        t = sink;
        mf = 0;
        while (true) {
            f = 0;
            vector<int> dist(g.size(), INF);
            dist[s] = 0;
            queue<int> q;
            q.push(s);
            q.clear();
            p.resize(g.size());
            while (!q.empty()) {
                int u = q.front();
                q.pop();
                if (u == t) break;
                for (int i = 0; i < g[u].size(); i++) {
                    Edge prox = g[u][i];
                    if (dist[prox.at] == INF and prox.cap > 0) {
                        dist[prox.at] = dist[u] + 1;
                        q.push(prox.at);
                        dad paizao(u, i, prox.where);
                        p[prox.at] = paizao;
                    }
                }
            }
            augment(t, INF);
            if (f == 0) break;
            mf += f;
        }
        return mf;
    }
};

```

## 6.7 Edmonds Karp

```

struct Edge {
    int at, where;
};

```

## 7 Dynamic Programming

### 7.1 Convex Hull Trick



```

/* Esse convex hull trick e para achar a reta minima!
 * Para maximizar a reta dada , basta trocar o '>' para
 * para '<' na funcao query;
 * Nao chamar query com B ou A vazios! Atualizar dp para
 * depois fazer a query =)
 * ATENCAO COM O DOUBLE!! ESTA EM LONG LONG :)
 */
vi A[N], B[N];
int pont[N];
bool odomeioehlixo(int r1, int r2, int r3, int j)
{
    return (B[j][r1] - B[j][r3]) * (A[j][r2] - A[j][r1]) <
           (B[j][r1] - B[j][r2]) * (A[j][r3] - A[j][r1]);
}

void add(ll a, ll b, int j)
{
    B[j].pb(b);
    A[j].pb(a);
    while (B[j].size() >= 3 and
           odomeioehlixo(B[j].size() - 3, B[j].size() - 2, B[j].size() - 1, j)) {
        B[j].erase(B[j].end() - 2);
        A[j].erase(A[j].end() - 2);
    }
}

ll query(ll x, int j)
{
    if (pont[j] >= B[j].size()) pont[j] = B[j].size() - 1;
    while (pont[j] < B[j].size() - 1 and
           (A[j][pont[j] + 1] * x + B[j][pont[j] + 1] >
            A[j][pont[j]] * x + B[j][pont[j]]))
        pont[j]++;
    return A[j][pont[j]] * x + B[j][pont[j]];
}

/* Testado em :
 * http://www.spoj.com/problems/APIO10A/
 * http://www.spoj.com/problems/ACQUIRE/
 */

```

## 7.2 Convex Hull Trick 2

```

/*
 * Given a set of pairs (m, b) specifying lines of the form y = m*x + b, process
 * a
 * set of x-coordinate queries each asking to find the minimum y-value when any
 * of
 * the given lines are evaluated at the specified x. To instead have the queries
 * optimize for maximum y-value, set the QUERY_MAX flag to true.
 * The following implementation is a fully dynamic variant of the convex hull
 * optimization technique, using a self-balancing binary search tree (std::set)
 * to
 * support the ability to call add_line() and get_best() in any desired order.
 * Explanation: http://wcipeg.com/wiki/Convex_hull_trick#Fully_dynamic_variant
 * Time Complexity: O(n log n) on the total number of calls made to add_line(),
 * for
 * any length n sequence of arbitrarily interlaced add_line() and get_min()
 * calls.
 * Each individual call to add_line() is O(log n) amortized and each individual
 * call to get_best() is O(log n), where n is the number of lines added so far.
 * Space Complexity: O(n) auxiliary on the number of calls made to add_line().
 */

#include <limits> // std::numeric_limits
#include <set>

class hull_optimizer {
    struct line {
        long long m, b, val;
        double xlo;
        bool is_query;
        bool query_max;
    };

    line(long long m, long long b, long long val, bool is_query, bool query_max)
    {
        this->m = m;
        this->b = b;
        this->val = val;
        this->xlo = -std::numeric_limits<double>::max();
        this->is_query = is_query;
        this->query_max = query_max;
    }

    bool parallel(const line &l) const { return m == l.m; }
    double intersect(const line &l) const
    {
        if (parallel(l)) return std::numeric_limits<double>::max();
        return (double) (l.b - b) / (m - l.m);
    }
}

```

```

}

bool operator<(const line &l) const
{
    if (l.is_query) return query_max ? (xlo < l.val) : (l.val < xlo);
    return m < l.m;
};

std::set<line> hull;
bool _query_max;

typedef std::set<line>::iterator hulliter;

bool has_prev(hulliter it) const { return it != hull.begin(); }
bool has_next(hulliter it) const
{
    return (it != hull.end()) && (++it != hull.end());
}

bool irrelevant(hulliter it) const
{
    if (!has_prev(it) || !has_next(it)) return false;
    hulliter prev = it, next = it;
    --prev;
    ++next;
    return _query_max ? prev->intersect(*next) <= prev->intersect(*it)
                      : next->intersect(*prev) <= next->intersect(*it);
}

hulliter update_left_border(hulliter it)
{
    if ((_query_max && !has_prev(it)) || (!_query_max && !has_next(it)))
        return it;
    hulliter it2 = it;
    double val = it->intersect(_query_max ? --it2 : ++it2);
    line l(*it);
    l.xlo = val;
    hull.erase(it++);
    return hull.insert(it, l);
}

public:
hull_optimizer(bool query_max = false) { this->_query_max = query_max; }
void add_line(long long m, long long b)
{
    line l(m, b, 0, false, _query_max);
    hulliter it = hull.lower_bound(l);
    if (it != hull.end() && it->parallel(l)) {
        if (_query_max && it->b < b) || (!_query_max && b < it->b)
            hull.erase(it++);
        else
            return;
    }
    it = hull.insert(it, l);
    if (irrelevant(it)) {
        hull.erase(it);
        return;
    }
    while (has_prev(it) && irrelevant(--it)) hull.erase(it++);
    while (has_next(it) && irrelevant(++it)) hull.erase(it--);
    it = update_left_border(it);
    if (has_prev(it)) update_left_border(--it);
    if (has_next(++it)) update_left_border(++it);
}

long long get_best(long long x) const
{
    line q(0, 0, x, true, _query_max);
    hulliter it = hull.lower_bound(q);
    if (_query_max) --it;
    return it->m * x + it->b;
}

}

/** Example Usage **/

#include <cassert>

int main()
{
    hull_optimizer h;
    h.add_line(3, 0);
    h.add_line(0, 6);
    h.add_line(1, 2);
    h.add_line(2, 1);
    assert(h.get_best(0) == 0);
    assert(h.get_best(2) == 4);
    assert(h.get_best(1) == 3);
    assert(h.get_best(3) == 5);
    return 0;
}

```

## 7.3 Divide and Conquer

```
//Um exemplo de Divide and conquer:
int MOD = 1e9 + 7;
const int N = 1010;
int dp[N][N], cost[N][N], v[N], pref[N], n, m;
void compDP(int j, int L, int R, int b, int e)
{
    if (L > R) return;
    int mid = (L + R) / 2;
    int idx = -1;
    for (int i = b; i <= min(mid, e); i++)
        if (dp[mid][j] > dp[i][j - 1] + cost[i + 1][mid]) {
            idx = i;
            dp[mid][j] = dp[i][j - 1] + cost[i + 1][mid];
        }
    compDP(j, L, mid - 1, b, idx);
    compDP(j, mid + 1, R, idx, e);
}
//chamada!
for(int i=1;i<=n;i++) dp[i][0]=cost[1][i];
for(int i=1;i<=m;i++) compDP(i,1,n,1,n);
```

## 7.4 Longest Increasing Subsequence

```
//asw -> vetor com resposta!!
//asw.size() o tamanho da maior lis
void lis( const vector< int > & v, vector< int > & asw )
{
    vector<int> pd(v.size(),0), pd_index(v.size()), pred(v.size());
    int maxi = 0, x=0, j=0, ind=0;
    for(int i=0;i<v.size();i++)
    {
        x = v[i];
        j=lower_bound(pd.begin(),pd.begin()+maxi,x) -pd.begin();
        pd[j] = x;
        pd_index[j] = i;
        if(j==maxi)
        {
            maxi++;
            ind = i;
        }
        if(pred[i] == j) pd_index[j-1] = -1;
    }
    int pos=maxi-1,k=v[ind];
    asw.resize( maxi );
    while ( pos >= 0 )
    {
        asw[pos--] = k;
        ind = pred[ind];
        k = v[ind];
    }
}
```

# 8 Geometry

## 8.1 Convex Hull Monotone Chain

```
typedef struct sPoint {
    int x, y;
    sPoint(int _x, int _y)
    {
        x = _x;
        y = _y;
    }
} point;
bool comp(point a, point b)
{
    if (a.x == b.x) return a.y < b.y;
    return a.x < b.x;
}
int cross(point a, point b, point c) // AB x BC
{
    a.x -= b.x;
    a.y -= b.y;
    b.x -= c.x;
    b.y -= c.y;
```

```
        return a.x * b.y - a.y * b.x;
    }

    bool isCw(point a, point b, point c) // Clockwise
    {
        return cross(a, b, c) < 0;
    }

    // >= if you want to put collinear points on the convex hull
    bool isCcw(point a, point b, point c) // Counter Clockwise
    {
        return cross(a, b, c) > 0;
    }

    vector<point> convexHull(vector<point> p)
    {
        vector<point> u, l; // Upper and Lower hulls

        sort(p.begin(), p.end(), comp);
        for (unsigned int i = 0; i < p.size(); i++) {
            while (l.size() > 1 && !isCcw(l[l.size() - 1], l[l.size() - 2], p[i]))
                l.pop_back();
            l.push_back(p[i]);
        }

        for (int i = p.size() - 1; i >= 0; i--) {
            while (u.size() > 1 && !isCcw(u[u.size() - 1], u[u.size() - 2], p[i]))
                u.pop_back();
            u.push_back(p[i]);
        }

        u.pop_back();
        l.pop_back();
        l.insert(l.end(), u.begin(), u.end());
        return l;
    }
}
```

## 8.2 Minimum Enclosing Circle

```
//6.5- Minimum Enclosing Circle
const double eps = 1e-6;
#define CIRCLE circ
#define PT Ponto
#define MP 101
#define eps 1e-9
#define x first
#define y second
typedef double cood;
typedef int num;
typedef int point;
double resp;
cood x[MP], y[MP], ar, ax, ay;
int p[MP];
typedef pair<double, double> ponto;
typedef pair<double, double> Ponto;
double dista(ponto a, ponto b)
{
    return sqrt((a.first - b.first) * (a.first - b.first) +
        (a.second - b.second) * (a.second - b.second));
}

bool in(ponto a, pair<double, ponto> c)
{
    if (dista(a, c.second) - eps < c.first) return true;
    return false;
}

bool same(point a, point b)
{
    return (fabs(x[a] - x[b]) < eps && fabs(y[a] - y[b]) < eps);
}

bool lexLess(point a, point b)
{
    if (fabs(x[a] - x[b]) < eps) return y[a] < y[b];
    return x[a] < x[b];
}

inline cood dist(cood xx, cood yy, point a)
{
    return sqrt((xx - x[a]) * (xx - x[a]) + (yy - y[a]) * (yy - y[a]));
}

inline cood cP(point a, point b, point c)
{
    return (x[a] - x[b]) * (y[c] - y[b]) - (x[c] - x[b]) * (y[a] - y[b]);
}

void findCircle(point a, point b, point c, cood& cx, cood& cy)
{
}
```

```

    cx = 0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (y[b] - y[c]) -
        0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (y[a] - y[b]),
    cy = 0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (x[a] - x[b]) -
        0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (x[b] - x[c]);
    cx /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
    cy /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
}

void spanCircle2(int k, point p0, point p1, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p1]);
    cy = 0.5 * (y[p0] + y[p1]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) {
            findCircle(p0, p1, p[i], cx, cy);
            r = dist(cx, cy, p[i]);
        }
}

void spanCircle1(int k, point p0, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p[0]]);
    cy = 0.5 * (y[p0] + y[p[0]]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) spanCircle2(i, p0, p[i], cx, cy, r);
}

void spanCircle(int n, cood& cx, cood& cy, cood& r)
{
    // Bem importante, retirar repetidos
    sort(p, p + 1, lexLess);
    n = unique(p, p + n) - p;
    random_shuffle(p, p + n);
    if (n > 1) {
        cx = 0.5 * (x[p[0]] + x[p[1]]);
        cy = 0.5 * (y[p[0]] + y[p[1]]);
        r = dist(cx, cy, p[1]);
        for (int i = 2; i < n; i++)
            if (dist(cx, cy, p[i]) > r) spanCircle1(i, p[i], cx, cy, r);
    }
    else {
        cx = x[0];
        cy = y[0];
        r = 0.0;
    }
}

void solve(vector<pair<double, double>> &v)
{
    int N = v.size();
    for (int i = 0; i < N; i++) {
        x[i] = v[i].first;
        y[i] = v[i].second;
        p[i] = i;
    }
    spanCircle(N, ax, ay, ar);
}

```

## 8.3 Minimum Enclosing Circle 2

```

const double eps = 1e-6;
#define CIRCLE circ
#define PT Ponto
#define MP 101
#define eps 1e-9
#define x first
#define y second
typedef double cood;
typedef int num;
typedef int point;
double resp;
cood x[MP], y[MP], ar, ax, ay;
int p[MP];
typedef pair<double, double> ponto;
typedef pair<double, double> Ponto;
double dista(ponto a, ponto b)
{
    return sqrt((a.first - b.first) * (a.first - b.first) +
        (a.second - b.second) * (a.second - b.second));
}

bool in(ponto a, pair<double, double> c)
{

```

```

    if (dista(a, c.second) - eps < c.first) return true;
    return false;
}

bool same(point a, point b)
{
    return (fabs(x[a] - x[b]) < eps && fabs(y[a] - y[b]) < eps);
}

bool lexLess(point a, point b)
{
    if (fabs(x[a] - x[b]) < eps) return y[a] < y[b];
    return x[a] < x[b];
}

inline cood dist(cood xx, cood yy, point a)
{
    return sqrt((xx - x[a]) * (xx - x[a]) + (yy - y[a]) * (yy - y[a]));
}

inline cood cP(point a, point b, point c)
{
    return (x[a] - x[b]) * (y[c] - y[b]) - (x[c] - x[b]) * (y[a] - y[b]);
}

void findCircle(point a, point b, point c, cood& cx, cood& cy)
{
    cx = 0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (y[b] - y[c]) -
        0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (y[a] - y[b]),
    cy = 0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (x[a] - x[b]) -
        0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (x[b] - x[c]);
    cx /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
    cy /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
}

void spanCircle2(int k, point p0, point p1, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p1]);
    cy = 0.5 * (y[p0] + y[p1]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) {
            findCircle(p0, p1, p[i], cx, cy);
            r = dist(cx, cy, p[i]);
        }
}

void spanCircle1(int k, point p0, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p[0]]);
    cy = 0.5 * (y[p0] + y[p[0]]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) spanCircle2(i, p0, p[i], cx, cy, r);
}

void spanCircle(int n, cood& cx, cood& cy, cood& r)
{
    // Bem importante, retirar repetidos
    sort(p, p + 1, lexLess);
    n = unique(p, p + n) - p;
    random_shuffle(p, p + n);
    if (n > 1) {
        cx = 0.5 * (x[p[0]] + x[p[1]]);
        cy = 0.5 * (y[p[0]] + y[p[1]]);
        r = dist(cx, cy, p[1]);
        for (int i = 2; i < n; i++)
            if (dist(cx, cy, p[i]) > r) spanCircle1(i, p[i], cx, cy, r);
    }
    else {
        cx = x[0];
        cy = y[0];
        r = 0.0;
    }
}

void solve(vector<pair<double, double>> &v)
{
    int N = v.size();
    for (int i = 0; i < N; i++) {
        x[i] = v[i].first;
        y[i] = v[i].second;
        p[i] = i;
    }
    spanCircle(N, ax, ay, ar);
}

```

## 8.4 Fast Geometry in Cpp

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
    double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }
}

if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=d-c; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
            return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterClockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}
```

```

}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (l == 1 || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

int main() {
    // expected: (-5,2)
    cerr << RotateCCW90(PT(2,5)) << endl;

    // expected: (5,-2)
    cerr << RotateCW90(PT(2,5)) << endl;

    // expected: (-5,2)
    cerr << RotateCCW(PT(2,5), M_PI/2) << endl;

    // expected: (5,2)
    cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

    // expected: (5,2) (7.5,3) (2.5,1)
    cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

    // expected: 6.78903
    cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

    // expected: 1 0 1
    cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 0 0 1
    cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 1 1 1 0
    cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

    // expected: (1,2)
    cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

    // expected: (1,1)
    cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

    vector<PT> v;
    v.push_back(PT(0,0));
    v.push_back(PT(5,0));
    v.push_back(PT(5,5));
    v.push_back(PT(0,5));

    // expected: 1 1 1 0 0
    cerr << PointInPolygon(v, PT(2,2)) << " "
        << PointInPolygon(v, PT(2,0)) << " "
        << PointInPolygon(v, PT(0,2)) << " "
        << PointInPolygon(v, PT(5,2)) << " "
        << PointInPolygon(v, PT(2,5)) << endl;

    // expected: 0 1 1 1 1
    cerr << PointOnPolygon(v, PT(2,2)) << " "
        << PointOnPolygon(v, PT(2,0)) << " "
        << PointOnPolygon(v, PT(0,2)) << " "
        << PointOnPolygon(v, PT(5,2)) << " "
        << PointOnPolygon(v, PT(2,5)) << " "

```

```

    << PointOnPolygon(v, PT(2,5)) << endl;

    // expected: (1,6)
    // (5,4) (4,5)
    // blank line
    // (4,5) (5,4)
    // blank line
    // (4,5) (5,4)
    vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

    // area should be 5.0
    // centroid should be (1.1666666, 1.1666666)
    PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
    vector<PT> p(pa, pa+4);
    PT c = ComputeCentroid(p);
    cerr << "Area: " << ComputeArea(p) << endl;
    cerr << "Centroid: " << c << endl;

    return 0;
}

```

## 9 Data Structures

### 9.1 Disjoint Set Union

```

const int N=500010;
int p[N],Rank[N];
void init()
{
    memset(Rank,0,sizeof(Rank));
    for(int i=0;i<N;i++) p[i]=i;
}
int findset(int i)
{
    if(p[i]==i) return i;
    return p[i]=findset(p[i]);
}
bool same(int i, int j)
{
    return (findset(i) == findset(j));
}
void unionSet(int i, int j)
{
    if (!same(i, j)) {
        int x = findset(i), y=findset(j);
        if (Rank[x] > Rank[y])
            p[y] = x;
        else {
            p[x] = y;
            if (Rank[x] == Rank[y]) Rank[y]++;
        }
    }
}

```

### 9.2 Persistent Segment Tree

```

//PRINTAR O NUMERO DE ELEMENTOS DISTINTOS
//EM UM INTERVALO DO ARRAY
const int N = 30010;
int tr[100 * N], L[100 * N], R[100 * N], root[100 * N];
int v[N], mapa[100 * N];
int cont = 0;
void build(int node, int b, int e)
{
    if (b == e) {
        tr[node] = 0;
    }
    else {
        L[node] = cont++;

```

```

    R[node] = cont++;
    build(L[node], b, (b + e) / 2);
    build(R[node], (b + e) / 2 + 1, e);
    tr[node] = tr[L[node]] + tr[R[node]];
}

int update(int node, int b, int e, int i, int val)
{
    int idx = cont++;
    tr[idx] = tr[node] + val;
    L[idx] = L[node];
    R[idx] = R[node];
    if (b == e) return idx;
    int mid = (b + e) / 2;
    if (i <= mid)
        L[idx] = update(L[node], b, mid, i, val);
    else
        R[idx] = update(R[node], mid + 1, e, i, val);
    return idx;
}

int query(int nodeL, int nodeR, int b, int e, int i, int j)
{
    if (b > j or i > e) return 0;
    if (i <= b and j >= e) {
        int p1 = tr[nodeR];
        int p2 = tr[nodeL];
        return p1 - p2;
    }
    int mid = (b + e) / 2;
    return query(L[nodeL], L[nodeR], b, mid, i, j) +
           query(R[nodeL], R[nodeR], mid + 1, e, i, j);
}

int main()
{
    int n;
    sc(n);
    memset(mapa, -1, sizeof(mapa));
    for (int i = 0; i < n; i++) sc(v[i]);
    build(l, 0, n - 1);
    for (int i = 0; i < n; i++) {
        if (mapa[v[i]] == -1) {
            root[i + 1] = update(root[i], 0, n - 1, i, 1);
            mapa[v[i]] = i;
        }
        else {
            root[i + 1] = update(root[i], 0, n - 1, mapa[v[i]], -1);
            mapa[v[i]] = i;
            root[i + 1] = update(root[i + 1], 0, n - 1, i, 1);
        }
    }
    int q;
    sc(q);
    for (int i = 0; i < q; i++) {
        int l, r;
        sc2(l, r);
        int resp = query(root[l - 1], root[r], 0, n - 1, l - 1, r - 1);
        pri(resp);
    }
    return 0;
}

```

## 9.3 RMQ of Indices

```

//RMQ DE INDICE
class RMQ {
private:
    vi A;
    vi M;
public:
    RMQ(vi &v)
    {
        A = v;
        M.resize(4 * v.size());
        build(1, 0, v.size() - 1);
    }
    void build(int node, int b, int e)
    {
        if (b == e)
            M[node] = b;
        else {
            build(2 * node, b, (b + e) / 2);
            build(2 * node + 1, (b + e) / 2 + 1, e);
            if (A[M[2 * node]] <= A[M[2 * node + 1]])
                M[node] = M[2 * node];
            else
                M[node] = M[2 * node + 1];
        }
    }
}

```

```

}
int query(int node, int b, int e, int i, int j)
{
    int p1, p2;
    if (i > e || j < b) return -1;
    if (b >= i and e <= j) return M[node];
    p1 = query(2 * node, b, (b + e) / 2, i, j);
    p2 = query(2 * node + 1, (b + e) / 2 + 1, e, i, j);
    if (p1 == -1) return p2;
    if (p2 == -1) return p1;
    if (A[p1] <= A[p2]) return p1;
    return p2;
}

void atualiza(int node, int b, int e, int i, int val)
{
    if (i > e || i < b) return;
    if (e == b) {
        A[i] = val;
    }
    else {
        atualiza(2 * node, b, (b + e) / 2, i, val);
        atualiza(2 * node + 1, (b + e) / 2 + 1, e, i, val);
        if (A[M[2 * node]] <= A[M[2 * node + 1]])
            M[node] = M[2 * node];
        else
            M[node] = M[2 * node + 1];
    }
}
};

```

## 9.4 RSQ with Lazy Propagation

```

//RSQ COM LAZY PROPAGATION!
class RSQ {
private:
    vii A;
    vii M;
    vii lazy;
public:
    RSQ(vii &v)
    {
        A = v;
        M.resize(v.size() * 4);
        lazy.assign(v.size() * 4, 0);
        build(1, 0, v.size() - 1);
    }
    void build(int node, int b, int e)
    {
        if (b == e) {
            M[node] = A[b];
            return;
        }
        build(2 * node, b, (b + e) / 2);
        build(2 * node + 1, (b + e) / 2 + 1, e);
        M[node] = M[2 * node] + M[2 * node + 1];
    }
    void atualiza(int node, int b, int e, int i, int j, ll val)
    {
        if (lazy[node] != 0) {
            M[node] += lazy[node];
            if (b != e) {
                ll inter = (e - b + 1);
                ll i1 = (b + e) / 2 - b + 1;
                ll i2 = e - (b + e) / 2;
                ll un = lazy[node] / inter;
                lazy[2 * node] += un * i1;
                lazy[2 * node + 1] += un * i2;
            }
            lazy[node] = 0;
        }
        if (i > e or j < b) return;
        if (i <= b and j >= e) {
            ll inter = (e - b + 1);
            M[node] += val * inter;
            if (b != e) {
                ll i1 = (b + e) / 2 - b + 1;
                ll i2 = e - (b + e) / 2;
                lazy[2 * node] += i1 * (ll)val;
                lazy[2 * node + 1] += i2 * (ll)val;
            }
            return;
        }
        atualiza(2 * node, b, (b + e) / 2, i, j, val);
        atualiza(2 * node + 1, (b + e) / 2 + 1, e, i, j, val);
        M[node] = M[2 * node] + M[2 * node + 1];
    }
}

```

```

}
ll query(int node, int b, int e, int i, int j)
{
    if (i > e or j < b) return 0;
    ll p1, p2;
    if (lazy[node] != 0) {
        M[node] += lazy[node];
        if (b != e) {
            ll inter = (e - b + 1);
            ll i1 = (b + e) / 2 - b + 1;
            ll i2 = e - (b + e) / 2;
            ll un = lazy[node] / inter;
            lazy[2 * node] += un * i1;
            lazy[2 * node + 1] += un * i2;
        }
        lazy[node] = 0;
    }
    if (i <= b and j >= e) return M[node];
    p1 = query(2 * node, b, (b + e) / 2, i, j);
    p2 = query(2 * node + 1, (b + e) / 2 + 1, e, i, j);
    return p1 + p2;
}
};

```

## 9.5 Segment Tree

```

//compilar em C++11, essa segment tree
//computa qual e o k's elemento compreendido
//no intervalo entre i,j
//presntes no array
vi tr[5 * N];
void build(int node, int b, int e)
{
    if (b == e)
        tr[node].pb(v[b]);
    else {
        build(2 * node, b, (b + e) / 2);
        build(2 * node + 1, (b + e) / 2 + 1, e);
        merget(tr[2 * node], tr[2 * node + 1], tr[node]);
        merge(tr[2 * node].begin(), tr[2 * node].end(), tr[2 * node + 1].begin(),
              tr[2 * node + 1].end(), back_inserter(tr[node]));
    }
}
int query(int node, int b, int e, int i, int j, int k)
{
    if (i > e or b > j) return 0;
    if (i <= b and j >= e) {
        int resp =
            upper_bound(tr[node].begin(), tr[node].end(), k) - tr[node].begin();
        return tr[node].size() - resp;
    }
    return query(2 * node, b, (b + e) / 2, i, j, k) +
           query(2 * node + 1, (b + e) / 2 + 1, e, i, j, k);
}
}

```

## 9.6 Sparse Table

```

//comutar RMQ , favor inicializar: dp[i][0]=v[0]
//sendo v[0] o vetor do rmq
//chamar o build!
int dp[200100][22];
int n;
int d[200100];
void build()
{
    d[0] = d[1] = 0;
    for (int i = 2; i < n; i++) d[i] = d[i >> 1] + 1;
    for (int j = 1; j < 22; j++) {
        for (int i = 0; i + (1 << (j - 1)) < n; i++) {
            dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
        }
    }
}
int query(int i, int j)
{
    int k = d[j - i];
    int x = min(dp[i][k], dp[j - (1 << k) + 1][k]);
    return x;
}

```

## 10 Miscellaneous

### 10.1 Hashing

```

//certificar que gethash() foi chamado
//antes de getHash(i,j);
struct Hashing {
    const string &s;
    int n, idx;
    vector<ll> hashes, M, B;
    Hashing(const string &s) : s(s), hashes(s.size()) {
        M={1000000409, 2000003273, 2000003281, 2000003293};
        B={31, 53, 61, 41};
        srand(time(NULL));
        idx=rand() % 4;
        getHash();
    }
    void otherprime() {
        idx=(idx+1) % 4;
    }
    ll int_mod(ll a) { return (a % M[idx] + M[idx]) % M[idx]; }
    ll eleva(ll a, ll b)
    {
        if (b == 0)
            return 1;
        else if (b == 1)
            return a;
        ll x = eleva(a, b / 2);
        if (b % 2 == 0)
            return (x * x) % M[idx];
        else
            return (a * ((x * x) % M[idx])) % M[idx];
    }
    /*hash da string de 0 ate i*/
    void getHash()
    {
        int n = s.size();
        ll hp = 0;
        for (int i = 0; i < s.size(); i++) {
            hp = int_mod(hp * B[idx] + s[i]);
            hashes[i] = hp;
        }
    }
    /*Hash da string compreendida entre i e j*/
    ll getHash(int i, int j)
    {
        if (i == 0) return hashes[j];
        ll h1 = hashes[j];
        ll h2 = (hashes[i - 1] * eleva(B[idx], j - i + 1)) % M[idx];
        ll ret = (h1 - h2) % M[idx] + M[idx];
        return ret % M[idx];
    }
};

```

### 10.2 Inversion Count

```

//conta o numero de inversoes de um array
//x e o tamanho do array, v e o array que quero contar
ll inversoes = 0;
void merge_sort(vi &v, int x)
{
    if (x == 1) return;
    int tam_esq = (x + 1) / 2, tam_dir = x / 2;
    int esq[tam_esq], dir[tam_dir];
    for (int i = 0; i < tam_esq; i++) esq[i] = v[i];
    for (int i = 0; i < tam_dir; i++) dir[i] = v[i + tam_esq];
    merge_sort(esq, tam_esq);
    merge_sort(dir, tam_dir);
    int i_esq = 0, i_dir = 0, i = 0;
    while (i_esq < tam_esq or i_dir < tam_dir) {
        if (i_esq == tam_esq) {
            while (i_dir != tam_dir) {
                v[i] = dir[i_dir];
                i_dir++; i++;
            }
        }
        else if (i_dir == tam_dir) {
            while (i_esq != tam_esq) {
                v[i] = esq[i_esq];
                i_esq++; i++;
                inversoes += i_dir;
            }
        }
    }
}

```

```

    }
} else {
    if (esq[i_esq] <= dir[i_dir]) {
        v[i] = esq[i_esq];
        i++, i_esq++;
        inversoes += i_dir;
    }
    else {
        v[i] = dir[i_dir];
        i++, i_dir++;
    }
}
}
}
}

```

## 10.3 Distinct Elements in ranges

```

const int MOD = 1e9 + 7;
const int N = 1e6 + 10;
int bit[N], v[N], id[N], r[N];
ii query[N];
int mapa[N];
bool compare(int x, int y) { return query[x] < query[y]; }
void add(int idx, int val)
{
    while (idx < N) {
        bit[idx] += val;
        idx += idx & -idx;
    }
}
int sum(int idx)
{
    int ret = 0;
    while (idx > 0) {
        ret += bit[idx];
        idx -= idx & -idx;
    }
    return ret;
}
int main()
{
    memset(bit, 0, sizeof(bit));
    memset(mapa, 0, sizeof(mapa));
    int n;
    sc(n);
    for (int i = 1; i <= n; i++) sc(v[i]);
    int q;
    sc(q);
    for (int i = 0; i < q; i++) {
        sc2(query[i].second, query[i].first);
        id[i] = i;
    }
    sort(id, id + q, compare);
    sort(query, query + q);
    int j = 1;
    for (int i = 0; i < q; i++) {
        int L = query[i].second;
        int R = query[i].first;
        while (j <= R) {
            if (mapa[v[j]] > 0) {
                add(mapa[v[j]], -1);
                mapa[v[j]] = j;
                add(mapa[v[j]], 1);
            }
            else {
                mapa[v[j]] = j;
                add(mapa[v[j]], 1);
            }
            j++;
        }
        r[id[i]] = sum(R);
        if (L > 1) r[id[i]] -= sum(L - 1);
    }
    for (int i = 0; i < q; i++) pri(r[i]);
    return 0;
}

```

## 10.4 Maximum Rectangular Area in Histogram

```

/*
 * Complexidade : O(N)
 */

ll solve(vi &h)
{
    int n = h.size();
    ll resp = 0;
    stack<int> pilha;
    ll i = 0;
    while (i < n) {
        if (pilha.empty() || h[pilha.top()] <= h[i]) {
            pilha.push(i++);
        }
        else {
            int aux = pilha.top();
            pilha.pop();
            resp =
                max(resp, (ll)h[aux] * ((pilha.empty() ? i + 1 : i - pilha.top())));
        }
    }
    while (!pilha.empty()) {
        int aux = pilha.top();
        pilha.pop();
        resp =
            max(resp, (ll)h[aux] * ((pilha.empty() ? n + 1 : n - pilha.top())));
    }
    return resp;
}

```

## 10.5 Multiplying Two LL mod n

```

/* Metodo para calcular (a*b) mod m onde a e b s o inteiros com 64-bits cada.
Fonte: http://bit.ly/1pp7xEZ */
ll mulmod(ll a, ll b, ll m) {
    ll y = (ll) ( (ld)a*(ld)b/m + (ld)1/2 );
    y = y * m;
    ll x = a * b, r = x - y;
    if ((ll) r < 0) { r = r + m; y = y - 1; }
    return r;
}

```

## 10.6 Josephus Problem

```

/* Josephus Problem - It returns the position to be, in order to not die. O(n)*/
/* With k=2, for instance, the game begins with 2 being killed and then n+2, n+4, ... */
ll josephus(ll n, ll k) {
    if (n==1) return 1;
    else return (josephus(n-1, k)+k-1)%n+1;
}

```

## 10.7 Josephus Problem 2

```

/* Another Way to compute the last position to be killed O ( d * log n ) */
ll josephus(ll n, ll d) {
    ll K = 1;
    while (K <= (d - 1)*n) K = (d * K + d - 2) / (d - 1);
    return d * n + 1 - K;
}

```