

[UFMG] Stenio Garcia (2016-17)

Contents

1 InContests

1.1	Makefile	1
1.2	Vimrc	1
1.3	Template	1

2 Graph Algorithms

2.1	2 SAT	1
2.2	Kosaraju	2
2.3	Tree Isomorphism	2
2.4	LCA	3
2.5	Bridges and Articulation Points	3
2.6	Eulerian Tour	4
2.7	Floyd Warshall	4

3 Strings

3.1	Aho Corasick	4
3.2	KMP	4
3.3	Suffix Array	5
3.4	Suffix Array 2	5
3.5	Suffix Array Dilsunginha	6
3.6	Manacher Algorithm	6
3.7	Splitting String	6

4 Numerical Algorithms

4.1	Fast Fourier Transform	7
4.2	Fast Fourier Transform 2	7
4.3	Simpson Algorithm	8
4.4	Matrix Exponentiation	8

5 Mathematics

5.1	Big Number	8
5.2	Big Number 2	9
5.3	Chinese Remainder	11
5.4	Chinese Remainder 2	12
5.5	Matrix Exponentiation	12
5.6	Pascal Triangle	12
5.7	Euler's Totient Function	12
5.8	Pollard Rho	12
5.9	Sieve of Eratosthenes	13
5.10	Extended Euclidean Algorithm	13
5.11	Multiplicative Inverse	13
5.12	Multiplicative Inverse 2	13

6 Combinatorial Optimization

6.1	Dinic	13
6.2	Hopcroft-Karp Bipartite Matching	13
6.3	Max Bipartite Matching 2	14
6.4	Maximum Matching in General Graphs (Blossom)	14
6.5	Min Cost Matching	15
6.6	Min Cost Max Flow	16
6.7	Edmonds Karp	17

7 Dynamic Programming

7.1	Convex Hull Trick	17
7.2	Convex Hull Trick 2	17
7.3	Divide and Conquer	18
7.4	Longest Increasing Subsequence	18

8 Geometry

8.1	Convex Hull Monotone Chain	18
8.2	Minimum Enclosing Circle	19
8.3	Minimum Enclosing Circle 2	19

8.4	Fast Geometry in Cpp	20
8.5	Point Inside Polygon $O(\lg n)$	22

9 Data Structures

9.1	Disjoint Set Union	22
9.2	Persistent Segment Tree	23
9.3	RMQ of Indices	23
9.4	RSQ with Lazy Propagation	23
9.5	Segment Tree	24
9.6	Sparse Table	24

10 Miscellaneous

10.1	Hashing	24
10.2	Inversion Count	25
10.3	Distinct Elements in ranges	25
10.4	Maximum Rectangular Area in Histogram	25
10.5	Multiplying Two $LL \bmod n$	25
10.6	Josephus Problem	25
10.7	Josephus Problem 2	26
10.8	Dynamic MST	26
10.9	Ordered Static Set (Examples)	26

1 InContests

1.1 Makefile

```
CXX=g++
CXXFLAGS=-std=c++11 -Wall

SRC=$(*.cpp)
OBJ=$(SRC:%.cpp=%)
```

1.2 Vimrc

```
set ts=2 si ai sw=2 number mouse=a
syntax on
```

1.3 Template

```
#include <bits/stdc++.h>
using namespace std;
#define sc(a) scanf("%d", &a)
#define sc2(a, b) scanf("%d%d", &a, &b)
#define sc3(a, b, c) scanf("%d%d%d", &a, &b, &c)
#define pri(x) printf("%d\n", x)
#define prie(x) printf("%d ", x)
#define mp make_pair
#define pb push_back
#define BUFF ios::sync_with_stdio(false);
#define LER freopen("x.in", "r", stdin); freopen("x.out", "w", stdout);
#define db(x) cerr << x << " == " << x << endl
typedef long long int ll;
typedef long double ld;
typedef pair<int, int> ii;
typedef vector<int> vi;
const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const ld pi = acos(-1);
```

2 Graph Algorithms

2.1 2 SAT

```

/* Supondo que cada vertice u, o seu
 * positivo e 2*u, e negativo e 2*i+1
 * resposta[i]=0, significa que o positivo de i e resposta
 * resposta[i]=1, significa que o negativo de i e resposta
 * chamar Sat(n) , n e o numero de vertices do grafo
 * contando com os negativos .. na maioria dos problemas
 * chamar 2*n;
 * testado em :http://codeforces.com/contest/781/problem/D
 * */
int resposta[N];
vi graph[N], rev[N];
int us[N];
stack<int> pilha;
void dfs1(int u)
{
    us[u] = 1;
    for (int v : graph[u])
        if (!us[v]) dfs1(v);
    pilha.push(u);
}
void dfs2(int u, int color)
{
    us[u] = color;
    for (int v : rev[u])
        if (!us[v]) dfs2(v, color);
}
int Sat(int n)
{
    for (int i = 0; i < n; i++)
        if (!us[i]) dfs1(i);
    int color = 1;
    memset(us, 0, sizeof(us));
    while (!pilha.empty()) {
        int topo = pilha.top();
        pilha.pop();
        if (!us[topo]) dfs2(topo, color++);
    }
    for (int i = 0; i < n; i += 2) {
        if (us[i] == us[i + 1]) return 0;
        resposta[i / 2] = (us[i] < us[i + 1]);
    }
    return 1;
}
inline void add(int u, int v)
{
    graph[u].pb(v);
    rev[v].pb(u);
}
}

```

2.2 Kosaraju

```

//Retorna os componentes fortemente conectados
//Se o usados[i]=usados[j], temos que i e j
//pertencem ao mesmo componente, col=i= numero
//de componentes fortemente conectados do grafo
class Kosaraju {
private:
    vi usados;
    vvi graph;
    vvi trans;
    vi pilha;

public:
    Kosaraju(int N)
    {
        graph.resize(N);
        trans.resize(N);
    }
    void AddEdge(int u, int v)
    {
        graph[u].pb(v);
        trans[v].pb(u);
    }
    void dfs(int u, int pass, int color)
    {
        usados[u] = color;
        vi vizinhos;
        if (pass == 1)
            vizinhos = graph[u];
        else
            vizinhos = trans[u];
        for (int j = 0; j < vizinhos.size(); j++) {
            int v = vizinhos[j];
            if (usados[v] == 0) {
                dfs(v, pass, color);
            }
        }
    }
}

```

```

    pilha.pb(u);
}
int SSC(int n)
{
    pilha.clear();
    usados.assign(n, 0);
    for (int i = 0; i < n; i++) {
        if (!usados[i]) dfs(i, 1, 1);
    }
    usados.assign(n, 0);
    int color = 1;
    for (int i = n - 1; i >= 0; i--) {
        if (usados[pilha[i]] == 0) {
            dfs(pilha[i], 2, color);
            color++;
        }
    }
    return color - 1;
}
};

```

2.3 Tree Isomorphism

```

vvi children, subtreeLabels, tree, L;
vi pred, map;
int n;

bool compare(int a, int b) { return subtreeLabels[a] < subtreeLabels[b]; }
bool equals(int a, int b) { return subtreeLabels[a] == subtreeLabels[b]; }
void generateMapping(int r1, int r2)
{
    map.resize(n);
    map[r1] = r2 - n;
    sort(children[r1].begin(), children[r1].end(), compare);
    sort(children[r2].begin(), children[r2].end(), compare);
    for (int i = 0; i < (int)children[r1].size(); i++) {
        int u = children[r1][i];
        int v = children[r2][i];
        generateMapping(u, v);
    }
}

vi findCenter(int offset = 0)
{
    int cnt = n;
    vi a;
    vi deg(n);
    for (int i = 0; i < n; i++) {
        deg[i] = tree[i + offset].size();
        if (deg[i] <= 1) {
            a.push_back(i + offset);
            --cnt;
        }
    }
    while (cnt > 0) {
        vi na;
        for (int i = 0; i < (int)a.size(); i++) {
            int u = a[i];
            for (int j = 0; j < (int)tree[u].size(); j++) {
                int v = tree[u][j];
                if (--deg[v - offset] == 1) {
                    na.push_back(v);
                    --cnt;
                }
            }
        }
        a = na;
    }
    return a;
}

int dfs(int u, int p = -1, int depth = 0)
{
    L[depth].push_back(u);
    int h = 0;
    for (int i = 0; i < (int)tree[u].size(); i++) {
        int v = tree[u][i];
        if (v == p) continue;
        pred[v] = u;
        children[u].push_back(v);
        h = max(h, dfs(v, u, depth + 1));
    }
    return h + 1;
}

bool rootedTreeIsomorphism(int r1, int r2)
{
}

```

```

L.assign(n, vi());
pred.assign(2 * n, -1);
children.assign(2 * n, vi());

int h1 = dfs(r1);
int h2 = dfs(r2);
if (h1 != h2) return false;

int h = h1 - 1;
vi label(2 * n);
subtreeLabels.assign(2 * n, vi());

for (int i = h - 1; i >= 0; i--) {
    for (int j = 0; j < (int)L[i + 1].size(); j++) {
        int v = L[i + 1][j];
        subtreeLabels[pred[v]].push_back(label[v]);
    }
    for (int j = 0; j < (int)L[i].size(); j++) {
        int v = L[i][j];
        sort(subtreeLabels[v].begin(), subtreeLabels[v].end());
    }

    sort(L[i].begin(), L[i].end(), compare);

    for (int j = 0, cnt = 0; j < (int)L[i].size(); j++) {
        if (j % equals(L[i][j], L[i][j - 1])) ++cnt;
        label[L[i][j]] = cnt;
    }
}

if (!equals(r1, r2)) return false;

generateMapping(r1, r2);
return true;
}

bool treeIsomorphism()
{
    vi c1 = findCenter();
    vi c2 = findCenter(n);
    if (c1.size() == c2.size()) {
        if (rootedTreeIsomorphism(c1[0], c2[0]))
            return true;
        else if (c1.size() > 1)
            return rootedTreeIsomorphism(c1[1], c2[0]);
    }
    return false;
}

int main()
{
    n = 5;
    vvi t1(n);
    t1[0].pb(1); t1[1].pb(0);
    t1[1].pb(2); t1[2].pb(1);
    t1[1].pb(3); t1[3].pb(1);
    t1[0].pb(4); t1[4].pb(0);
    vvi t2(n);
    t2[0].pb(1); t2[1].pb(0);
    t2[0].pb(4); t2[4].pb(0);
    t2[4].pb(3); t2[3].pb(4);
    t2[4].pb(2); t2[2].pb(4);
    tree.assign(2 * n, vi());
    for (int u = 0; u < n; u++) {
        for (int i = 0; i < t1[u].size(); i++) {
            int v = t1[u][i];
            tree[u].push_back(v);
        }
        for (int i = 0; i < t2[u].size(); i++) {
            int v = t2[u][i];
            tree[u + n].push_back(v + n);
        }
    }

    bool res = treeIsomorphism();
    cout << res << endl;
    if (res)
        for (int i = 0; i < n; i++)
            cout << map[i] << endl;
}

```

2.4 LCA

```

//antes de usar as queries de lca, e etc..
//certifique-se de chamar a dfs da arvore e
//process()
const int N = 100000;
const int M = 22;
int P[N][M];

```

```

int big[N][M], low[N][M], level[N];
vii graph[N];
int n;

void dfs(int u, int last, int l)
{
    level[u] = l;
    P[u][0] = last;
    for (ii v : graph[u])
        if (v.first != last) {
            big[v.first][0] = low[v.first][0] = v.second;
            dfs(v.first, u, l + 1);
        }
}

void process()
{
    for (int j = 1; j < M; j++)
        for (int i = 1; i <= n; i++) {
            P[i][j] = P[P[i][j - 1]][j - 1];
            big[i][j] = max(big[i][j - 1], big[P[i][j - 1]][j - 1]);
            low[i][j] = min(low[i][j - 1], low[P[i][j - 1]][j - 1]);
        }
}

int lca(int u, int v)
{
    if (level[u] < level[v]) swap(u, v);
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[v]) u = P[u][i];
    if (u == v) return u;
    for (int i = M - 1; i >= 0; i--) {
        if (P[u][i] != P[v][i]) u = P[u][i], v = P[v][i];
    }
    return P[u][0];
}

int maximum(int u, int v, int x)
{
    int resp = 0;
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[x]) {
            resp = max(resp, big[u][i]);
            u = P[u][i];
        }
    for (int i = M - 1; i >= 0; i--)
        if (level[v] - (1 << i) >= level[x]) {
            resp = max(resp, big[v][i]);
            v = P[v][i];
        }
    return resp;
}

int minimum(int u, int v, int x)
{
    int resp = INF;
    for (int i = M - 1; i >= 0; i--)
        if (level[u] - (1 << i) >= level[x]) {
            resp = min(resp, low[u][i]);
            u = P[u][i];
        }
    for (int i = M - 1; i >= 0; i--)
        if (level[v] - (1 << i) >= level[x]) {
            resp = min(resp, low[v][i]);
            v = P[v][i];
        }
    return resp;
}

```

2.5 Bridges and Articulation Points

```

class ponte {
private:
    vvi graph;
    vi usados;
    vi e_articulacao;
    vi dfs_low;
    vi dfs_prof;
    vector<ii> pontes;
    int tempo;

public:
    ponte(int N)
    {
        graph.clear();
        graph.resize(N);
        usados.assign(N, 0);
    }

```

```

dfs_low.assign(N, 0);
dfs_prof.assign(N, 0);
e_articulacao.assign(N, 0);
tempo = 0;
}
void AddEdge(int u, int v)
{
    graph[u].pb(v);
    graph[v].pb(u);
}
void dfs(int u, int pai)
{
    usados[u] = 1;
    int nf = 0;
    dfs_low[u] = dfs_prof[u] = tempo++;
    for (int v : graph[u]) {
        if (!usados[v]) {
            dfs(v, u);
            nf++;
            if (dfs_low[v] >= dfs_prof[u] and pai != -1) e_articulacao[u] = true;
            if (pai == -1 and nf > 1) e_articulacao[u] = true;
            if (dfs_low[v] > dfs_prof[u]) pontes.pb(mp(u, v));
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        }
        else if (v != pai)
            dfs_low[u] = min(dfs_low[u], dfs_prof[v]);
    }
}
void olha_as_pontes()
{
    for (int i = 0; i < graph.size(); i++)
        if (!usados[i]) dfs(i, -1);
    if (pontes.size() == 0)
        cout << " Que merda! nao tem ponte!" << endl;
    else {
        for (ii i : pontes) cout << i.first << " " << i.second << endl;
    }
}
void olha_as_art()
{
    for (int i = 0; i < graph.size(); i++)
        if (!usados[i]) dfs(i, -1);
    for (int i = 0; i < e_articulacao.size(); i++)
        if (e_articulacao[i]) cout << " OIAAA A PONTE " << i << endl;
}
}

```

2.6 Eulerian Tour

```

multiset<int> graph[N];
stack<int> path;
// -> It suffices to call dfs1 just
// one time leaving from node 0.
// -> To calculate the path,
// call the dfs from the odd degree node.
// -> O(n * log(n))
void dfs1(int u)
{
    while(graph[u].size())
    {
        int v = *graph[u].begin();
        graph[u].erase(graph[u].begin());
        graph[v].erase(graph[v].find(u));
        dfs1(v);
    }
    path.push(u);
}

```

2.7 Floyd Warshall

```

//menor caminho para todos os vertices
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        if (graph[i][j] != INF) pai[i][j] = i;

for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (graph[i][j] > graph[i][k] + graph[k][j]) {
                graph[i][j] = graph[i][k] + graph[k][j];
                pai[i][j] = pai[k][j];
            }
        }
    }
}

```

```

}
}

```

3 Strings

3.1 Aho Corasick

```

//N= tamanho da trie, M tamanho do alfabeto
int to[N][M], Link[N], fim[N];
int idx = 1;
void add_str(string &s)
{
    int v = 0;
    for (int i = 0; i < s.size(); i++) {
        if (!to[v][s[i]]) to[v][s[i]] = idx++;
        v = to[v][s[i]];
    }
    fim[v] = 1;
}

void process()
{
    queue<int> fila;
    fila.push(0);
    while (!fila.empty()) {
        int cur = fila.front();
        fila.pop();
        int l = Link[cur];
        fim[cur] |= fim[l];
        for (int i = 0; i < 200; i++) {
            if (to[cur][i]) {
                if (cur != 0) {
                    Link[to[cur][i]] = to[l][i];
                }
                else
                    Link[to[cur][i]] = 0;
                fila.push(to[cur][i]);
            }
            else {
                to[cur][i] = to[l][i];
            }
        }
    }
}

int resolve(string &s)
{
    int v = 0, r = 0;
    for (int i = 0; i < s.size(); i++) {
        v = to[v][s[i]];
        if (fim[v]) r++, v = 0;
    }
    return r;
}

```

3.2 KMP

```

int p[N];
int n;
void process(vi &s)
{
    int i = 0, j = -1;
    p[0] = -1;
    while (i < s.size()) {
        while (j >= 0 and s[i] != s[j]) j = p[j];
        i++, j++;
        p[i] = j;
    }
}

// s=texto, t=padrao
int match(string &s, string &t)
{
    int ret = 0;
    process(t);
    int i = 0, j = 0;
    while (i < s.size()) {
        while (j >= 0 and (s[i] != t[j])) j = p[j];
        i++, j++;
        if (j == t.size()) {
            j = p[j];
        }
    }
}

```

```

    }
    ret++;
}
return ret;
}

```

3.3 Suffix Array

```

/*
 * O(nlog^2(n)) para o sufix array
 * O(logn) para o LCP(i,j)
 * LCP de i para j;
 */
struct SA {
    const int L;
    string s;
    vvi P;
    vector<pair< ii,int> > M;

    SA(const string &s) : L(s.size()), s(s), P(1, vi(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = s[i] - 'a';
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.pb(vi(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = mp(mp(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
        }
    }

    vi GetSA() {
        vi v=P.back();
        vi ret(v.size());
        for(int i=0;i<v.size();i++){
            ret[v[i]]=i;
        }
        return ret;
    }

    int LCP(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }

    vi GetLCP(vi &sa)
    {
        vi lcp(sa.size()-1);
        for(int i=0;i<sa.size()-1;i++){
            lcp[i]=LCP(sa[i],sa[i+1]);
        }
        return lcp;
    }
};

```

3.4 Suffix Array 2

```

/*****
 Suffix Array. Building works in O(NlogN).
 Also LCP array is calculated in O(NlogN).
 This code counts number of different substrings in the string.
 Based on problem 1 from here: http://codeforces.ru/gym/100133
 *****/

const int MAXN = 205000;
const int ALPH = 256;
const int MAXLOG = 20;

int n;
char s[MAXN];
int p[MAXN]; // suffix array itself
int pcur[MAXN];
int c[MAXN][MAXLOG];
int num[MAXN];
int classesNum;
int lcp[MAXN];

```

```

void buildSuffixArray() {
    n++;

    for (int i = 0; i < n; i++)
        num[s[i]]++;

    for (int i = 1; i < ALPH; i++)
        num[i] += num[i - 1];

    for (int i = 0; i < n; i++) {
        p[num[s[i]] - 1] = i;
        num[s[i]]--;
    }

    c[p[0]][0] = 1;
    classesNum = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i - 1]])
            classesNum++;
        c[p[i]][0] = classesNum;
    }

    for (int i = 1; ; i++) {
        int half = (1 << (i - 1));

        for (int j = 0; j < n; j++) {
            pcur[j] = p[j] - half;
            if (pcur[j] < 0)
                pcur[j] += n;
        }

        for (int j = 1; j <= classesNum; j++)
            num[j] = 0;

        for (int j = 0; j < n; j++)
            num[c[pcur[j]][i - 1]]++;
        for (int j = 2; j <= classesNum; j++)
            num[j] += num[j - 1];

        for (int j = n - 1; j >= 0; j--) {
            p[num[c[pcur[j]][i - 1]] - 1] = pcur[j];
            num[c[pcur[j]][i - 1]]--;
        }

        c[p[0]][i] = 1;
        classesNum = 1;

        for (int j = 1; j < n; j++) {
            int p1 = (p[j] + half) % n, p2 = (p[j - 1] + half) % n;
            if (c[p[j]][i - 1] != c[p[j - 1]][i - 1] || c[p1][i - 1] != c[p2][i - 1])
                classesNum++;
            c[p[j]][i] = classesNum;
        }

        if ((1 << i) >= n)
            break;
    }

    for (int i = 0; i < n; i++)
        p[i] = p[i + 1];
    n--;
}

int getLcp(int a, int b) {
    int res = 0;
    for (int i = MAXLOG - 1; i >= 0; i--) {
        int curlen = (1 << i);
        if (curlen > n)
            continue;
        if (c[a][i] == c[b][i]) {
            res += curlen;
            a += curlen;
            b += curlen;
        }
    }
    return res;
}

void calcLcpArray() {
    for (int i = 0; i < n - 1; i++)
        lcp[i] = getLcp(p[i], p[i + 1]);
}

int main() {
    assert(freopen("substr.in", "r", stdin));
    assert(freopen("substr.out", "w", stdout));

    gets(s);
    n = strlen(s);

    buildSuffixArray();
}

```

```
// Now p from 0 to n - 1 contains suffix array of original string

/*for (int i = 0; i < n; i++) {
    printf("%d ", p[i] + 1);
}*/

    calcLcpArray();

    long long ans = 0;
    for (int i = 0; i < n; i++)
        ans += n - p[i];
    for (int i = 1; i < n; i++)
        ans -= lcp[i - 1];

    cout << ans << endl;

    return 0;
}
```

3.5 Suffix Array Dilsunginha

```
struct SuffixArray{
    const string& s;
    int n;
    vector<int> order, rank, lcp;
    vector<int> count, x, y;

    SuffixArray(const string& s) : s(s), n(s.size()), order(n), rank(n),
        count(n + 1), x(n), y(n), lcp(n){

        build();
        buildLCP();
    }

    void build(){
        //sort suffixes by the first character
        for(int i = 0; i < n; i++) order[i] = i;
        sort(order.begin(), order.end(), [&](int a, int b){return s[a] < s[b];});
        rank[order[0]] = 0;
        for(int i = 1; i < n; i++){
            rank[order[i]] = rank[order[i - 1]];
            if(s[order[i]] != s[order[i - 1]]) rank[order[i]]++;
        }

        //sort suffixes by the the first 2*p characters, for p in 1, 2, 4, 8,...
        for(int p = 1; p < n, rank[order[n - 1]] < n - 1; p <= 1){
            for(int i = 0; i < n; i++){
                x[i] = rank[i];
                y[i] = i + p < n ? rank[i + p] + 1 : 0;
            }

            radixPass(y);
            radixPass(x);

            rank[order[0]] = 0;
            for(int i = 1; i < n; i++){
                rank[order[i]] = rank[order[i - 1]];
                if(x[order[i]] != x[order[i - 1]] or y[order[i]] != y[order[i - 1]]) rank[order[i]]++;
            }
        }

        //Stable counting sort
        void radixPass(vector<int>& key){
            fill(count.begin(), count.end(), 0);
            for(auto index : order) count[key[index]]++;
            for(int i = 1; i <= n; i++) count[i] += count[i - 1];
            for(int i = n - 1; i >= 0; i--) lcp[--count[key[order[i]]]] = order[i];
            order.swap(lcp);
        }

        //Kasai's algorithm to build the LCP array from order, rank and s
        //For i >= 1, lcp[i] refers to the suffixes starting at order[i] and order[i - 1]
        void buildLCP(){
            lcp[0] = 0;
            int k = 0;
            for(int i = 0; i < n; i++){
                if(rank[i] == n - 1){
                    k = 0;
                }
                else{
                    int j = order[rank[i] + 1];
                    while(i + k < n and j + k < n and s[i + k] == s[j + k]) k++;
                    lcp[rank[i]] = k;
                    if(k) k--;
                }
            }
        }
    }
}
```

```
}
}

int main(){
    ios::sync_with_stdio(false);
    string s;
    cin >> s;
    SuffixArray sa(s);
    for(int i = 0; i < s.size(); i++) cout << sa.order[i] << '\n';
}
```

3.6 Manacher Algorithm

```
/******
Manacher's algorithm for finding all subpalindromes in the string.
Based on problem L from here: http://codeforces.ru/gym/100133
******/

const int MAXN = 105000;

string s;
int n;
int odd[MAXN], even[MAXN];
int l, r;
long long ans;

int main() {
    assert(freopen("palindrome.in", "r", stdin));
    assert(freopen("palindrome.out", "w", stdout));

    getline(cin, s);
    n = (int) s.length();

    // Odd case
    l = r = -1;
    for (int i = 0; i < n; i++) {
        int cur = 1;
        if (i < r)
            cur = min(r - i + 1, odd[l + r - i]);
        while (i + cur < n && i - cur >= 0 && s[i - cur] == s[i + cur])
            cur++;
        odd[i] = cur;
        if (i + cur - 1 > r) {
            l = i - cur + 1;
            r = i + cur - 1;
        }
    }

    // Even case
    l = r = -1;
    for (int i = 0; i < n; i++) {
        int cur = 0;
        if (i < r)
            cur = min(r - i + 1, even[l + r - i + 1]);
        while (i + cur < n && i - 1 - cur >= 0 && s[i - 1 - cur] == s[i + cur])
            cur++;
        even[i] = cur;
        if (i + cur - 1 > r) {
            l = i - cur;
            r = i + cur - 1;
        }
    }

    for (int i = 0; i < n; i++) {
        if (odd[i] > 1) {
            ans += odd[i] - 1;
        }
        if (even[i])
            ans += even[i];
    }

    cout << ans << endl;

    return 0;
}
```

3.7 Splitting String

```
/* String s to be splitted and the delimiter used to split it. */
vector<string> splitstr(string s, string delimiter)
{
```

```

vector<string> result;
string str = s, token;
size_t pos=0;
while((pos=str.find(delimiter)) != std::string::npos)
{
    token = str.substr(0, pos);
    result.push_back(token);
    str.erase(0, pos + delimiter.length());
}
result.push_back(str);
return result;
}

```

4 Numerical Algorithms

4.1 Fast Fourier Transform

```

// FFT - The Iterative Version
//
// Running Time:
//   O(n*log n)
//
// How To Use:
//   fft(a,1)
//   fft(b,1)
//   mul(a,b)
//   fft(a,-1)
//
// INPUT:
//   - fft method:
//     * The vector representing the polynomial
//     * 1 to normal transform
//     * -1 to inverse transform
//   - mul method:
//     * The two polynomials to be multiplied
//
// OUTPUT:
//   - fft method: Transforms the vector sent.
//   - mul method: The result is kept in the first vector.
//
// NOTES:
//   - You can either use the struct defined or define difcil as complex<double>
//
// SOLVED:
//   * Codeforces Round #296 (Div. 1) D. Fuzzy Search
//
struct difcil {
    double real;
    double im;
    difcil() {
        real=0.0;
        im=0.0;
    }

    difcil(double real, double im):real(real),im(im){}

    difcil operator+(const difcil &o) const {
        return difcil(o.real+real, im+o.im);
    }

    difcil operator/(double v) const {
        return difcil(real/v, im/v);
    }

    difcil operator*(const difcil &o) const {
        return difcil(real*o.real-im*o.im, real*o.im+im*o.real);
    }

    difcil operator-(const difcil &o) const {
        return difcil(real-o.real, im-o.im);
    }
};

difcil tmp[MAXN*2];
int coco, maiorpot2[MAXN];

void fft(vector<difcil> &A, int s)
{
    int n = A.size(), p = 0;

    while(n>1) {
        p++;
        n >>= 1;
    }
}

```

```

n = 1<<p;

vector<difcil> a=A;

for(int i = 0; i < n; ++i){
    int rev = 0;
    for(int j = 0; j < p; ++j){
        rev <<= 1;
        rev |= (i >> j) & 1;
    }
    A[i] = a[rev];
}

difcil w, wn;

for(int i = 1; i <= p; ++i){
    int M = 1 << i;
    int K = M >> 1;
    wn = difcil(cos(s*2.0*pi/(double)M), sin(s*2.0*pi/(double)M));
    for(int j = 0; j < n; j += M){
        w = difcil(1.0, 0.0);
        for(int l = j; l < K + j; ++l){
            difcil t = w;
            t = t*A[l + K];
            difcil u = A[l];
            A[l] = A[l] + t;
            u = u - t;
            A[l + K] = u;
            w = wn*w;
        }
    }

    if(s==1){
        for(int i = 0; i<n; ++i)
            A[i] = A[i]/(double)n;
    }
}

void mul(vector<difcil> &a, vector<difcil> &b)
{
    for(int i=0; i<a.size(); i++)
    {
        a[i]=a[i]*b[i];
    }
}

```

4.2 Fast Fourier Transform 2

```

// FFT - The Recursive Version
//
// Running Time:
//   O(n*log n)
//
// How To Use:
//   fft(&a[0], tam, 0)
//   fft(&b[0], tam, 0)
//   mul(a,b)
//   fft(&a[0], tam, 1)
//
// INPUT:
//   - fft method:
//     * The vector representing the polynomial
//     * 0 to normal transform
//     * 1 to inverse transform
//   - mul method:
//     * The two polynomials to be multiplied
//
// OUTPUT:
//   - fft method: Transforms the vector sent.
//   - mul method: The result is kept in the first vector.
//
// NOTES:
//   - Tam has to be a power of 2.
//   - You can either use the struct defined or define difcil as complex<double>
//
// SOLVED:
//   * Codeforces Round #296 (Div. 1) D. Fuzzy Search
//
difcil tmp[MAXN*2];
int coco, maiorpot2[MAXN];

void fft(difcil *v, int N, bool inv)
{
    if(N<=1) return;
    difcil *vodd = v;
}

```

```

difícil +veven = v+N/2;
for(int i=0; i<N; i++) tmp[i] = v[i];
coco = 0;
for(int i=0; i<N; i+=2)
{
    veven[coco] = tmp[i];
    vodd[coco] = tmp[i+1];
    coco++;
}
fft(&vodd[0], N/2, inv);
fft(&veven[0], N/2, inv);

difícil w(1);
double angucomleite = 2.0*PI/(double)N;
if(inv) angucomleite = -angucomleite;

difícil wn(cos(angucomleite), sin(angucomleite));
for(int i=0; i<N/2; i++)
{
    tmp[i] = veven[i]+w*vodd[i];
    tmp[i+N/2] = veven[i]-w*vodd[i];
    w *= wn;
    if(inv)
    {
        tmp[i] /= 2;
        tmp[i+N/2] /= 2;
    }
}
for(int i=0; i<N; i++) v[i] = tmp[i];
}

void mul(vector<difícil> &a, vector<difícil> &b)
{
    for(int i=0; i<a.size(); i++)
    {
        a[i] = a[i]+b[i];
    }
}

void precomp()
{
    int pot=0;
    for(int i=1; i<MAXN; i++)
    {
        if((1<<pot)<i) pot++;
        maiorpot2[i] = (1<<pot);
    }
}

```

4.3 Simpson Algorithm

```

const int NPASSOS = 100000;
const int W=1000000;
//W= tamanho do intervalo que eu estou integrando
double integrall()
{
    double h = W / (NPASSOS);
    double a = 0;
    double b = W;
    double s = f(a) + f(b);
    for (double i = 1; i <= NPASSOS; i += 2) s += f(a + i * h) * 4.0;
    for (double i = 2; i <= (NPASSOS - 1); i += 2) s += f(a + i * h) * 2.0;
    return s * h / 3.0;
}

```

4.4 Matrix Exponentiation

```

//matmul multiplica m1 por m2
//matpow exponencia a matrix m1 por p
//mul vet multiplica a matrix m1 pelo vetor vet
vvi matmul(vvi &m1, vvi &m2)
{
    vvi ans;
    ans.resize(m1.size(), vi(m2.size(), 0));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++) {
                ans[i][j] += m1[i][k] * m2[k][j];
                ans[i][j] %= MOD;
            }
    return ans;
}

```

```

vvi matpow(vvi &m1, ll p)
{
    vvi ans;
    ans.resize(m1.size(), vi(m1.size(), 0));
    for (int i = 0; i < n; i++) ans[i][i] = 1;
    while (p) {
        if (p & 1) ans = matmul(ans, m1);
        m1 = matmul(m1, m1);
        p >>= 1;
    }
    return ans;
}
// VETOR TEM N LINHAS E A MATRIZ E QUADRADA
vi mulvet(vvi &m1, vi &vet)
{
    vi ans;
    ans.resize(vet.size(), 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            ans[i] += (m1[i][j] * vet[j]);
            ans[i] %= MOD;
        }
    return ans;
}

```

5 Mathematics

5.1 Big Number

```

void zero_esq(string &resp)
{
    string retorno = resp;
    reverse(retorno.begin(), retorno.end());
    int i = retorno.size() - 1;
    while (retorno[i] == '0' and i > 0) {
        retorno.erase(i);
        i--;
    }
    reverse(retorno.begin(), retorno.end());
    resp = retorno;
}

string sum_big(string a, string b)
{
    string resp;
    reverse(a.begin(), a.end());
    reverse(b.begin(), b.end());
    if (a.size() <= b.size()) {
        int carry = 0;
        for (int i = 0; i < a.size(); i++) {
            int x = b[i] - '0' + a[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        for (int i = a.size(); i < b.size(); i++) {
            int x = b[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        if (carry > 0) resp.push_back((char)(carry + '0'));
    }
    else {
        int carry = 0;
        for (int i = 0; i < b.size(); i++) {
            int x = a[i] - '0' + b[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        for (int i = b.size(); i < a.size(); i++) {
            int x = a[i] - '0' + carry;
            resp.push_back((char)(x % 10 + '0'));
            carry = x / 10;
        }
        if (carry > 0) resp.push_back((char)(carry + '0'));
    }
    reverse(resp.begin(), resp.end());
    zero_esq(resp);
    return resp;
}

string mul_big(string a, string b)
{
    string resp;
    resp.push_back('0');
    string temp;
    int carry = 0;
    reverse(a.begin(), a.end());

```



```

reverse(b.begin(), b.end());
for (int i = 0; i < a.size(); i++) {
    temp.clear();
    for (int k = 0; k < i; k++) temp.push_back('0');
    int x = a[i] - '0';
    for (int j = 0; j < b.size(); j++) {
        int y = b[j] - '0';
        int novo = (x * y + carry);
        temp.push_back((novo % 10) + '0');
        carry = novo / 10;
    }
    if (carry > 0) temp.push_back(carry + '0');
    reverse(temp.begin(), temp.end());
    carry = 0;
    resp = sum_big(temp, resp);
}
zero_esq(resp);
return resp;
}

```

5.2 Big Number 2

```

/*****
Structure implementing long arithmetic in C++
Analogue to BigInteger in Java.
Tested on many problems.
TODO: list some problems
*****/

struct BigInt {
    vector<int> num;

    static const int base = 1000 * 1000 * 1000;
    static const int baseDigits = 9;
    string leadingZerosModifier;

    /*****
    * CONSTRUCTORS & SETTERS
    *****/

    void setLeadingZerosModifier() {
        leadingZerosModifier = "%0xd";
        leadingZerosModifier[2] = '0' + baseDigits;
    }

    void set(int value) {
        num.clear();
        if (value == 0)
            num.push_back(0);
        while (value) {
            num.push_back(value % base);
            value /= base;
        }
    }

    void set(long long value) {
        num.clear();
        if (value == 0)
            num.push_back(0);
        while (value) {
            num.push_back(value % base);
            value /= base;
        }
    }

    void set(string &value) {
        num.clear();
        for (int i = (int)value.length() - 1; i >= 0; i -= baseDigits) {
            int add = 0;
            for (int j = max(0, i - baseDigits + 1); j <= i; j++)
                add = add * 10 + value[j] - '0';
            num.push_back(add);
        }
    }

    void operator = (int value) {
        set(value);
    }

    void operator = (long long value) {
        set(value);
    }

    void operator = (string &value) {
        set(value);
    }
}

```

```

BigInt() {
    setLeadingZerosModifier();
    set(0);
}

BigInt(int value) {
    setLeadingZerosModifier();
    set(value);
}

BigInt(string value) {
    setLeadingZerosModifier();
    set(value);
}

/*****
* SIZE METHODS
*****/

//returns size of vector
int size() {
    return (int)num.size();
}

//returns length of the number
int digitNum() {
    int result = 0;
    for (int i = 0; i < (int)num.size() - 1; i++)
        result += baseDigits;
    int lastNum = num.back();
    while (lastNum) {
        result++;
        lastNum /= 10;
    }
    return result;
}

/*****
* I/O
*****/

void read() {
    string s;
    cin >> s;
    num.clear();
    for (int i = (int)s.length() - 1; i >= 0; i -= baseDigits) {
        int add = 0;
        for (int j = max(0, i - baseDigits + 1); j <= i; j++)
            add = add * 10 + s[j] - '0';
        num.push_back(add);
    }
}

void print() {
    printf("%d", num.back());
    for (int i = (int)num.size() - 2; i >= 0; i--)
        printf(leadingZerosModifier.c_str(), num[i]);
}

void println() {
    print();
    printf("\n");
}

string toString() {
    string result = "";
    for (int i = 0; i < (int)num.size(); i++) {
        int cur = num[i];
        for (int j = 1; j <= baseDigits; j++) {
            if (cur == 0 && i == (int) num.size() - 1)
                break;
            result.append(1, (char) '0' + cur % 10);
            cur /= 10;
        }
        reverse(result.begin(), result.end());
        return result;
    }
}

/*****
* ADDITION
*****/

void sumThis(BigInt number) {
    int carry = 0;
    for (int i = 0; i < max((int)num.size(), number.size()) || carry; i++) {
        if (i == num.size())
            num.push_back(0);
        if (i >= number.size())
            carry = num[i] + carry;
        else

```

```

        carry = num[i] + carry + number.num[i];
        num[i] = carry % base;
        carry /= base;
    }
}

void sumThis(int number) {
    int carry = number;
    for (int i = 0; i < (int)num.size() || carry; i++) {
        if (i == num.size())
            num.push_back(0);
        carry = num[i] + carry;
        num[i] = carry % base;
        carry /= base;
    }
}

BigInt sum(BigInt number) {
    BigInt result = *this;
    result.sumThis(number);
    return result;
}

BigInt sum(int number) {
    BigInt result = *this;
    result.sumThis(number);
    return result;
}

void operator += (BigInt number) {
    sumThis(number);
}

void operator += (int number) {
    sumThis(number);
}

BigInt operator + (BigInt number) {
    return sum(number);
}

BigInt operator + (int number) {
    return sum(number);
}

/*=====
* SUBTRACTION
=====*/

void subThis(BigInt number) {
    int carry = 0;
    for (int i = 0; i < (int)number.size() || carry; i++) {
        if (i < (int)number.size())
            num[i] -= carry + number.num[i];
        else
            num[i] -= carry;
        if (num[i] < 0) {
            carry = 1;
            num[i] += base;
        }
        else
            carry = 0;
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

void subThis(int number) {
    int carry = -number;
    for (int i = 0; carry > 0; i++) {
        num[i] -= carry;
        if (num[i] < 0) {
            carry = 1;
            num[i] += base;
        }
        else
            carry = 0;
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

BigInt sub(BigInt number) {
    BigInt result = *this;
    result.subThis(number);
    return result;
}

BigInt sub(int number) {
    BigInt result = *this;
    result.subThis(number);
    return result;
}

}

void operator -= (BigInt number) {
    subThis(number);
}

void operator -= (int number) {
    subThis(number);
}

BigInt operator - (BigInt number) {
    return sub(number);
}

BigInt operator - (int number) {
    return sub(number);
}

/*=====
* MULTIPLICATION
=====*/

BigInt mult(BigInt number) {
    BigInt product;
    product.num.resize(num.size() + number.size());
    for (int i = 0; i < (int)num.size(); i++) {
        for (int j = 0, carry = 0; j < (int)number.size() || carry; j++) {
            long long cur = product.num[i + j] + num[i] * 11l * (j < (int)number.size() ? number.num[j] : 0) + carry;
            product.num[i + j] = int (cur % base);
            carry = int (cur / base);
        }
        while (product.size() > 1 && product.num.back() == 0)
            product.num.pop_back();
        return product;
    }
}

void multThis(BigInt number) {
    *this = mult(number);
}

void multThis(int number) {
    int carry = 0;
    for (int i = 0; i < (int)num.size() || carry; ++i) {
        if (i == num.size())
            num.push_back(0);
        long long cur = carry + num[i] * 11l * number;
        num[i] = int (cur % base);
        carry = int (cur / base);
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

BigInt mult(int number) {
    BigInt result = *this;
    result.multThis(number);
    return result;
}

void operator *= (BigInt number) {
    multThis(number);
}

void operator *= (int number) {
    multThis(number);
}

BigInt operator * (BigInt number) {
    return mult(number);
}

BigInt operator * (int number) {
    return mult(number);
}

void multThisByPowerOfTen(int power) {
    int baseNums = power / baseDigits;
    int curLen = (int)num.size();
    num.resize(curLen + baseNums);
    for (int i = num.size() - 1; i >= baseNums; i--) {
        num[i] = num[i - baseNums];
    }
    for (int i = baseNums - 1; i >= 0; i--)
        num[i] = 0;
    power %= baseDigits;
    int multBy = (int)pow(10.0, power);
    multThis(multBy);
}

/*=====

```

```

* DIVISION
=====*/

void divThis(int number) {
    int carry = 0;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        long long cur = num[i] + carry * 11l * base;
        num[i] = int (cur / number);
        carry = int (cur % number);
    }
    while (num.size() > 1 && num.back() == 0)
        num.pop_back();
}

BigInt div(int number) {
    BigInt result = *this;
    result.divThis(number);
    return result;
}

void operator /= (int number) {
    divThis(number);
}

BigInt operator / (int number) {
    return div(number);
}

void divThisByPowerOfTen(int power) {
    int baseNums = power / baseDigits;
    int curLen = (int)num.size();
    for (int i = 0; i < (int)num.size() - baseNums; i++) {
        num[i] = num[i + baseNums];
    }
    for (int i = 1; i <= baseNums; i++)
        num.pop_back();
    power %= baseDigits;
    int divBy = (int)pow(10.0, power);
    divThis(divBy);
}

/*=====
* MODULO
=====*/

void modThis(int number) {
    int carry = 0;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        long long cur = num[i] + carry * 11l * base;
        num[i] = int (cur / number);
        carry = int (cur % number);
    }
    set(carry);
}

BigInt mod(int number) {
    BigInt result = *this;
    result.modThis(number);
    return result;
}

void operator %= (int number) {
    modThis(number);
}

BigInt operator % (int number) {
    return mod(number);
}

/*=====
* COMPARISON
=====*/

//Returns: -1 - this number is less than argument, 0 - equal, 1 - this number is greater
int compareTo(BigInt number) {
    if ((int)num.size() < number.size())
        return -1;
    if ((int)num.size() > number.size())
        return 1;
    for (int i = (int)num.size() - 1; i >= 0; i--) {
        if (num[i] > number.num[i])
            return 1;
        if (num[i] < number.num[i])
            return -1;
    }
    return 0;
}

//Returns: -1 - this number is less than argument, 0 - equal, 1 - this number is greater
int compareTo(int number) {
    if (num.size() > 1 || num[0] > number)
        return 1;

```

```

    if (num[0] < number)
        return -1;
    return 0;
}

bool operator < (BigInt number) {
    return compareTo(number) == -1;
}

bool operator < (int number) {
    return compareTo(number) == -1;
}

bool operator <= (BigInt number) {
    return compareTo(number) != 1;
}

bool operator <= (int number) {
    return compareTo(number) != 1;
}

bool operator == (BigInt number) {
    return compareTo(number) == 0;
}

bool operator == (int number) {
    return compareTo(number) == 0;
}

bool operator > (BigInt number) {
    return compareTo(number) == 1;
}

bool operator > (int number) {
    return compareTo(number) == 1;
}

bool operator >= (BigInt number) {
    return compareTo(number) != -1;
}

bool operator >= (int number) {
    return compareTo(number) != -1;
}

bool operator != (BigInt number) {
    return compareTo(number) != 0;
}

bool operator != (int number) {
    return compareTo(number) != 0;
}
};

```

5.3 Chinese Remainder

```

11 mulmod(11 a, 11 b, 11 m)
{
    11 ret = 0;
    while (b > 0) {
        if (b % 2 != 0) ret = (ret + a) % m;
        a = (a + a) % m;
        b >>= 1;
    }
    return ret;
}

11 expmod(11 a, 11 e, 11 m)
{
    11 ret = 1;
    while (e > 0) {
        if (e % 2 != 0) ret = mulmod(ret, a, m);
        a = mulmod(a, a, m);
        e >>= 1;
    }
    return ret;
}

11 invmul(11 a, 11 m) { return expmod(a, m - 2, m); }
11 chinese(vector<11> r, vector<11> m)
{
    int sz = m.size();
    11 M = 1;
    for (int i = 0; i < sz; i++) {
        M *= m[i];
    }
    11 ret = 0;
    for (int i = 0; i < sz; i++) {

```

```

    ret += mulmod(mulmod(M / m[i], r[i], M), invmul(M / m[i], M), M);
    ret = ret % M;
}
return ret;
}

```

5.4 Chinese Remainder 2

```

// Chinese remainder theorem (special case): find z such that // z % m1 = r1, z
// % m2 = r2. Here, z is unique modulo M = lcm(m1, m2). // Return (z, M). On
// failure, M = -1;
ii chinese_remainder_theorem(int m1, int r1, int m2, int r2)
{
    int s, t;
    int g = extended_euclid(m1, m2, s, t);
    if (r1 % g != r2 % g) return mp(0, -1);
    return mp(mod(s * r2 * m1 + t * r1 * m2, m1 * m2) / g, m1 * m2 / g);
}
// Chinese remainder theorem: find z such that // z % m[i] =
// r[i] for all i
// .Note that the solution is unique modulo M = lcm_i (m[i]).
// Return(z, M)
// .On // failure, M = -1. Note that we do not require the a[i] s
// to be relatively prime.
ii chinese_remainder_theorem(const vi &m, const vi &r)
{
    ii ret = make_pair(r[0], m[0]);
    for (int i = 1; i < m.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
        if (ret.second == -1) break;
    }
    return ret;
}

```

5.5 Matrix Exponentiation

```

//matmul multiplica m1 por m2
//matpow exponencia a matrix m1 por p
//mul vet multiplica a matrix m1 pelo vetor vet
vvi matmul(vvi &m1, vvi &m2)
{
    vvi ans;
    ans.resize(m1.size(), vi(m2.size(), 0));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++) {
                ans[i][j] += m1[i][k] * m2[k][j];
                ans[i][j] %= MOD;
            }
    return ans;
}
vvi matpow(vvi &m1, ll p)
{
    vvi ans;
    ans.resize(m1.size(), vi(m1.size(), 0));
    for (int i = 0; i < n; i++) ans[i][i] = 1;
    while (p) {
        if (p & 1) ans = matmul(ans, m1);
        m1 = matmul(m1, m1);
        p >>= 1;
    }
    return ans;
}
// VETOR TEM N LINHAS E A MATRIZ E QUADRADA
vi mulvet(vvi &m1, vi &vet)
{
    vi ans;
    ans.resize(vet.size(), 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            ans[i] += (m1[i][j] * vet[j]);
            ans[i] %= MOD;
        }
    return ans;
}

```

5.6 Pascal Triangle

```

//Fazer combinacao de N escolhe M
//por meio do triangulo de pascal
//Complexidade: O(m*n)
unsigned long long comb[61][61];
for (int i = 0; i < 61; i++) {
    comb[i][i] = 1;
    comb[i][0] = 1;
}
for (int i = 2; i < 61; i++)
    for (int j = 1; j < i; j++)
        comb[i][j] = comb[i - 1][j] + comb[i - 1][j - 1];

```

5.7 Euler's Totient Function

```

//retorna quantos elementos coprimos
//a N e menores que n existem
int phi(int n)
{
    int result = n;
    for (int i = 2; i * i <= n; ++i)
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    if (n > 1) result -= result / n;
    return result;
}

```

5.8 Pollard Rho

```

ll u;
ll t;
const int tamteste=5;
ll abss(ll v){ return v>=0 ? v : -v;}
ll randerson()
{
    ld pseudo=(ld)rand()/(ld)RAND_MAX;
    return (ll)(round((ld)range+pseudo))+1LL;
}

ll mulmod(ll a, ll b, ll mod)
{
    ll ret=0;
    while(b>0)
    {
        if(b%2!=0) ret=(ret+a)%mod;
        a=(a+a)%mod;
        b=b/2LL;
    }
    return ret;
}

ll expmod(ll a, ll e, ll mod)
{
    ll ret=1;
    while(e>0)
    {
        if(e%2!=0) ret=mulmod(ret,a,mod);
        a=mulmod(a,a,mod);
        e=e/2LL;
    }
    return ret;
}

bool jeova(ll a, ll n)
{
    ll x = expmod(a,u,n);
    ll last=x;
    for(int i=0;i<t;i++)
    {
        x=mulmod(x,x,n);
        if(x==1 and last!=1 and last!=(n-1)) return true;
        last=x;
    }
    if(x==1) return false;
    return true;
}

bool isprime(ll n)
{
    u=n-1;
    t=0;
    while(u%2==0)

```

```

{
    t++;
    u/=2LL;
}
if (n==2) return true;
if (n==3) return true;
if (n%2==0) return false;
if (n<2) return false;
for(int i=0; i<tamteste; i++)
{
    ll v = randerson()%(n-2)+1;
    //cout<<"jeova "<<v<<" "<<n<<endl;
    if(jeova(v,n)) return false;
}
return true;
}

ll gcd(ll a, ll b){ return !b ? a : gcd(b,a%b);}

ll calc(ll x, ll n, ll c)
{
    return (mulmod(x,x,n)+c)%n;
}

ll pollard(ll n)
{
    ll d=1;
    ll i=1;
    ll k=1;
    ll x=2;
    ll y=x;
    ll c;
    do
    {
        c=randerson()%n;
    }while(c==0 or (c+2)%n==0);
    while(d!=n)
    {
        if(i==k)
        {
            k+=2LL;
            y=x;
            i=0;
        }
        x=calc(x,n,c);
        i++;
        d=gcd(abss(y-x),n);
        if(d!=1) return d;
    }
}

vector<ll> getdiv(ll n)
{
    vector<ll> ret;
    if (n==1) return ret;
    if (isprime(n))
    {
        ret.pb(n);
        return ret;
    }
    ll d = pollard(n);
    ret=getdiv(d);
    vector<ll> ret2=getdiv(n/d);
    for(int i=0; i<ret2.size(); i++) ret.pb(ret2[i]);
    return ret;
}

```

5.9 Sieve of Eratosthenes

```

//esse crivo gera MAXN primos
const int MAX = 1e6;
int primes[MAX];
void gen_primes()
{
    int i, j;
    for (i = 2; i+i <= MAX; i++)
        if (primes[i])
            for (j = i; j * i < MAX; j++) primes[i * j] = 0;
}

```

5.10 Extended Euclidean Algorithm

```

//returns g = gcd(a, b);
//finds x,y such that d= ax+by;

```

```

int extended_euclid(int a, int b, int &x, int &y)
{
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a / b;
        int t = b;
        b = a % b;
        a = t;
        t = xx;
        xx = x - q * xx;
        x = t;
        t = yy;
        yy = y - q * yy;
        y = t;
    }
    return a;
}

```

5.11 Multiplicative Inverse

```

//computes b such that ab = 1(mod n), returns -1 on failure
int mod_inverse(int a, int n)
{
    int x, y;
    int g = extended_euclid(a, n, x, y);
    if (g > 1) return -1;
    return (x+n)%n;
}

```

5.12 Multiplicative Inverse 2

```

//inverso multiplicativo de A % MOD
//certifique de MOD estar definido antes bonito!
//complexidade: O(log(a))
ll mul_inv(ll a)
{
    ll pin0 = MOD, pin = MOD, t, q;
    ll x0 = 0, x1 = 1;
    if (pin == 1) return 1;
    while (a > 1) {
        q = a / pin;
        t = pin, pin = a % pin, a = t;
        t = x0, x0 = x1 - q * x0, x1 = t;
    }
    if (x1 < 0) x1 += pin0;
    return x1;
}

```

6 Combinatorial Optimization

6.1 Dinic

```

//grafo bipartido O(Esqrt(v))
//Para recuperar a resposta, e so colocar um bool
//de false na aresta de retorno e fazer uma bfs/dfs
//andando pelos vertices de capacidade =0 e arestas
//que nao sao de retorno
struct Edge {
    int v, rev;
    int cap;
    Edge(int v_, int cap_, int rev_) : v(v_), rev(rev_), cap(cap_) {}
};

struct MaxFlow {
    vector<vector<Edge>> > g;
    vector<int> level;
    queue<int> q;
    int flow, n;

    MaxFlow(int n_) : g(n_), level(n_), n(n_) {}
    void addEdge(int u, int v, int cap)
    {
        if (u == v) return;
        Edge e(v, cap, int(g[v].size()));
        Edge r(u, 0, int(g[u].size()));
    }
}

```

```

    g[u].push_back(e);
    g[v].push_back(r);
}

bool buildLevelGraph(int src, int sink)
{
    fill(level.begin(), level.end(), -1);
    while (not q.empty()) q.pop();
    level[src] = 0;
    q.push(src);
    while (not q.empty()) {
        int u = q.front();
        q.pop();
        for (auto e = g[u].begin(); e != g[u].end(); ++e) {
            if (not e->cap or level[e->v] != -1) continue;
            level[e->v] = level[u] + 1;
            if (e->v == sink) return true;
            q.push(e->v);
        }
    }
    return false;
}

int blockingFlow(int u, int sink, int f)
{
    if (u == sink or not f) return f;
    int fu = f;
    for (auto e = g[u].begin(); e != g[u].end(); ++e) {
        if (not e->cap or level[e->v] != level[u] + 1) continue;
        int mincap = blockingFlow(e->v, sink, min(fu, e->cap));
        if (mincap) {
            g[e->v][e->rev].cap += mincap;
            e->cap -= mincap;
            fu -= mincap;
        }
    }
    if (f == fu) level[u] = -1;
    return f - fu;
}

int maxFlow(int src, int sink)
{
    flow = 0;
    while (buildLevelGraph(src, sink))
        flow += blockingFlow(src, sink, numeric_limits<int>::max());
    return flow;
}
};

```

6.2 Hopcroft-Karp Bipartite Matching

```

/* O(v^3)
 * Matching maximo de grafo bipartido de peso 1 nas arestas
 * supondo que o grafo bipartido seja enumerado de 0-n-1
 * chamamos maxMatch(n)
 */
class MaxMatch {
    vi graph[N];
    int match[N], us[N];

public:
    MaxMatch(){};
    void addEdge(int u, int v) { graph[u].pb(v); }
    int dfs(int u)
    {
        if (us[u]) return 0;
        us[u] = 1;
        for (int v : graph[u]) {
            if (match[v] == -1 or (dfs(match[v]))) {
                match[v] = u;
                return 1;
            }
        }
        return 0;
    }
    int maxMatch(int n)
    {
        memset(match, -1, sizeof(match));
        int ret = 0;
        for (int i = 0; i < n; i++) {
            memset(us, 0, sizeof(us));
            ret += dfs(i);
        }
        return ret;
    }
};

```

6.3 Max Bipartite Matching 2

```

// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
// INPUT: w[i][j] = edge between row node i and column node j
// OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//         mc[j] = assignment for column node j, -1 if unassigned
//         function returns number of matches made
#include <vector>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {
        if (w[i][j] && !seen[j]) {
            seen[j] = true;
            if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
                mr[i] = j;
                mc[j] = i;
                return true;
            }
        }
    }
    return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);
    int ct = 0;
    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}

```

6.4 Maximum Matching in General Graphs (Blossom)

```

/*
GETS:
V->number of vertices
E->number of edges
pair of vertices as edges (vertices are 1..V)

GIVES:
output of edmonds() is the maximum matching
match[i] is matched pair of i (-1 if there isn't a matched pair)

Code for the SEAGRP problem at CodeChef.
SEAGRP's limits are: 1 <= V, E <= 100.
The problem asked if there is a perfect matching.
*/

#include <bits/stdc++.h>
using namespace std;
const int M=500;
struct struct_edge { int v; struct_edge* n; };
typedef struct_edge* edge;
struct_edge pool[M*M*2];
int topindex;
edge adj[M];
int V,E,match[M],qh,qt,q[M],father[M],base[M];
bool inq[M],inb[M],ed[M][M];

void clean()
{
    memset(ed, false, sizeof(ed));
    topindex=0;

    for(int i = 0; i < M; i++)
        adj[i] = NULL;
}

void add_edge(int u,int v)
{
    edge top = &pool[topindex++];
    top->v=v,top->n=adj[u],adj[u]=top;
    top = &pool[topindex++];
    top->v=u,top->n=adj[v],adj[v]=top;
}

int LCA(int root,int u,int v)
{

```

```

static bool inp[M];
memset(inp,0,sizeof(inp));
while(1)
{
    inp[u=base[u]]=true;
    if (u==root) break;
    u=father[match[u]];
}
while(1)
{
    if (inp[v=base[v]]) return v;
    else v=father[match[v]];
}
}
void mark_blossom(int lca,int u)
{
    while (base[u]!=lca)
    {
        int v=match[u];
        inb[base[u]]=inb[base[v]]=true;
        u=father[v];
        if (base[u]!=lca) father[u]=v;
    }
}
void blossom_contraction(int s,int u,int v)
{
    int lca=LCA(s,u,v);
    memset(inb,0,sizeof(inb));
    mark_blossom(lca,u);
    mark_blossom(lca,v);
    if (base[u]!=lca)
        father[u]=v;
    if (base[v]!=lca)
        father[v]=u;
    for (int u=0;u<V;u++)
        if (inb[base[u]])
        {
            base[u]=lca;
            if (!inq[u])
                inq[q[++qt]]=u;
        }
}
int find_augmenting_path(int s)
{
    memset(inq,0,sizeof(inq));
    memset(father,-1,sizeof(father));
    for (int i=0;i<V;i++) base[i]=i;
    inq[q[qh=qt=0]=s]=true;
    while (qh<=qt)
    {
        int u=q[qh++];
        for (edge e=adj[u];e!=NULL;e=e->n)
        {
            int v=e->v;
            if (base[u]!=base[v] && match[u]!=v)
            {
                if ((v==s) || (match[v]!=-1 && father[match[v]]!=-1))
                    blossom_contraction(s,u,v);
                else if (father[v]==-1)
                {
                    father[v]=u;
                    if (match[v]==-1)
                        return v;
                    else if (!inq[match[v]])
                        inq[q[++qt]=match[v]]=true;
                }
            }
        }
    }
    return -1;
}
int augment_path(int s,int t)
{
    int u=t,v,w;
    while (u!=-1)
    {
        v=father[u];
        w=match[v];
        match[v]=u;
        match[u]=v;
        u=w;
    }
    return t!=-1;
}
int edmonds()
{
    int matchc=0;
    memset(match,-1,sizeof(match));
    for (int u=0;u<V;u++)
        if (match[u]==-1)
            matchc+=augment_path(u,find_augmenting_path(u));
    return matchc;
}

```

```

}
int main()
{
    int u, v, t;

    cin >> t;
    while(t-->0)
    {
        cin >> V >> E;
        clean();
        while(E-->0)
        {
            cin >> u >> v;
            if (!ed[u-1][v-1])
            {
                add_edge(u-1,v-1);
                ed[u-1][v-1]=ed[v-1][u-1]=true;
            }
        }

        //cout << "UE\n";
        //cout << V << " " << edmonds() << endl;
        //for (int i=0;i<V;i++)
        //    if (!match[i])
        //        cout<<i+1<<" "<<match[i]+1<<endl;
        //cout << endl;
        if(2*edmonds() == V) cout << "YES\n";
        else cout << "NO\n";
    }
    return 0;
}

```

6.5 Min Cost Matching

```

////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[i][j] matrix.
////////////////////////////////////

#include <algorithm>
#include <cmath>
#include <cstdio>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate)
{
    int n = int(cost.size());

    // construct dual feasible solution
    VD u(n);
    VD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
                Lmate[i] = j;
                Rmate[j] = i;
                mated++;
            }
        }
    }
}

```

```

        mated++;
        break;
    }
}

VD dist(n);
VI dad(n);
VI seen(n);

// repeat until primal solution is feasible
while (mated < n) {
    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++) dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {
        // find closest
        j = -1;
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            if (j == -1 || dist[k] < dist[j]) j = k;
        }
        seen[j] = 1;

        // termination condition
        if (Rmate[j] == -1) break;

        // relax neighbors
        const int i = Rmate[j];
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
            if (dist[k] > new_dist) {
                dist[k] = new_dist;
                dad[k] = j;
            }
        }

        // update dual variables
        for (int k = 0; k < n; k++) {
            if (k == j || !seen[k]) continue;
            const int i = Rmate[k];
            v[k] += dist[k] - dist[j];
            u[i] -= dist[k] - dist[j];
        }
        u[s] += dist[j];

        // augment along path
        while (dad[j] >= 0) {
            const int d = dad[j];
            Rmate[j] = Rmate[d];
            Lmate[Rmate[j]] = j;
            j = d;
        }
        Rmate[j] = s;
        Lmate[s] = j;

        mated++;
    }

    double value = 0;
    for (int i = 0; i < n; i++) value += cost[i][Lmate[i]];

    return value;
}

```

6.6 Min Cost Max Flow

```

// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
//
// Running time,  $O(|V|^2)$  cost per augmentation
// max flow:  $O(|V|^3)$  augmentations
// min cost max flow:  $O(|V|^4 * \text{MAX\_EDGE\_COST})$  augmentations
//
// INPUT:
// - graph, constructed using AddEdge()
// - source

```

```

// - sink
//
// OUTPUT:
// - (maximum flow value, minimum cost value)
// - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <iostream>
#include <vector>

using namespace std;

typedef vector<VI> VVI;
typedef long long LL;
typedef vector<LL> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPPI;

const LL INF = numeric_limits<LL>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPPI dad;

    MinCostMaxFlow(int N): N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, LL cap, LL cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, LL cap, LL cost, int dir) {
        LL val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    LL Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
            s = best;
        }

        for (int k = 0; k < N; k++) pi[k] = min(pi[k] + dist[k], INF);
        return width[t];
    }

    pair<LL, LL> GetMaxFlow(int s, int t) {
        LL totflow = 0, totcost = 0;
        while (LL amt = Dijkstra(s, t)) {
            totflow += amt;
            for (int x = t; x != s; x = dad[x].first) {
                if (dad[x].second == 1) {
                    flow[dad[x].first][x] += amt;
                    totcost += amt * cost[dad[x].first][x];
                }
                else {
                    flow[x][dad[x].first] -= amt;
                    totcost -= amt * cost[x][dad[x].first];
                }
            }
        }
        return make_pair(totflow, totcost);
    }
};

```


6.7 Edmonds Karp

```

struct Edge {
    int at, where;
    ll cap;
    void init(int _at, ll _cap, int _where)
    {
        at = _at, cap = _cap, where = _where;
    }
};

struct dad {
    int at, up, down;
    dad() { at = -1; }
    dad(int _at, int _up, int _down) { at = _at, up = _up, down = _down; }
};

class MaxFlow {
private:
    vector<vector<Edge>> > g;
    ll mf, f;
    int s, t;
    vector<dad> p;

public:
    void augment(int v, ll minEdge)
    {
        if (v == s) {
            f = minEdge;
            return;
        }
        else if (p[v].at != -1) {
            augment(p[v].at, min(minEdge, g[p[v].at][p[v].up].cap));
            g[p[v].at][p[v].up].cap -= f;
            g[p[v].down].cap += f;
        }
    }

    void init(int N)
    {
        for (int i = 0; i < g.size(); i++) g[i].clear();
        mf = 0, f = 0;
        g.resize(N);
    }

    void addEdge(int u, int v, ll cap)
    {
        Edge A;
        A.init(v, cap, g[v].size());
        Edge B;
        B.init(u, 0, g[u].size());
        g[u].pb(A);
        g[v].pb(B);
    }

    int maxFlow(int source, int sink)
    {
        s = source;
        t = sink;
        mf = 0;
        while (true) {
            f = 0;
            vector<int> dist(g.size(), INF);
            dist[s] = 0;
            queue<int> q;
            q.push(s);
            p.clear();
            p.resize(g.size());
            while (!q.empty()) {
                int u = q.front();
                q.pop();
                if (u == t) break;
                for (int i = 0; i < g[u].size(); i++) {
                    Edge prox = g[u][i];
                    if (dist[prox.at] == INF and prox.cap > 0) {
                        dist[prox.at] = dist[u] + 1;
                        q.push(prox.at);
                        dad paizao(u, i, prox.where);
                        p[prox.at] = paizao;
                    }
                }
            }
            augment(t, INF);
            if (f == 0) break;
            mf += f;
        }
        return mf;
    }
};

```

7 Dynamic Programming

7.1 Convex Hull Trick

```

/* Esse convex hull trick e para achar a reta minima!
 * Para maximizar a reta dada , basta trocar o '>' para
 * para '<' na funcao query;
 * Nao chamar query com B ou A vazios! Atualizar dp para
 * depois fazer a query =)
 * ATENCAO COM O DOUBLE!! ESTA EM LONG LONG :)
 */
vi A[N], B[N];
int pont[N];
bool odomeioehlixo(int r1, int r2, int r3, int j)
{
    return (B[j][r1] - B[j][r3]) * (A[j][r2] - A[j][r1]) <
           (B[j][r1] - B[j][r2]) * (A[j][r3] - A[j][r1]);
}

void add(ll a, ll b, int j)
{
    B[j].pb(b);
    A[j].pb(a);
    while (B[j].size() >= 3 and
           odomeioehlixo(B[j].size() - 3, B[j].size() - 2, B[j].size() - 1, j)) {
        B[j].erase(B[j].end() - 2);
        A[j].erase(A[j].end() - 2);
    }
}

ll query(ll x, int j)
{
    if (pont[j] >= B[j].size() pont[j] = B[j].size() - 1;
    while (pont[j] < B[j].size() - 1 and
           (A[j][pont[j] + 1] * x + B[j][pont[j] + 1] >
            A[j][pont[j]] * x + B[j][pont[j]]))
        pont[j]++;
    return A[j][pont[j]] * x + B[j][pont[j]];
}

/* Testado em :
 * http://www.spoj.com/problems/APIO10A/
 * http://www.spoj.com/problems/ACQUIRE/
 */

```

7.2 Convex Hull Trick 2

```

/*
 * Given a set of pairs (m, b) specifying lines of the form y = m*x + b, process
 * a
 * set of x-coordinate queries each asking to find the minimum y-value when any
 * of
 * the given lines are evaluated at the specified x. To instead have the queries
 * optimize for maximum y-value, set the QUERY_MAX flag to true.
 * The following implementation is a fully dynamic variant of the convex hull
 * optimization technique, using a self-balancing binary search tree (std::set)
 * to
 * support the ability to call add_line() and get_best() in any desired order.
 * Explanation: http://wcipeg.com/wiki/Convex_hull_trick#Fully_dynamic_variant
 * Time Complexity: O(n log n) on the total number of calls made to add_line(),
 * for
 * any length n sequence of arbitrarily interlaced add_line() and get_min()
 * calls.
 * Each individual call to add_line() is O(log n) amortized and each individual
 * call to get_best() is O(log n), where n is the number of lines added so far.
 * Space Complexity: O(n) auxiliary on the number of calls made to add_line().
 */

#include <limits> // std::numeric_limits
#include <set>

class hull_optimizer {
    struct line {
        long long m, b, val;
        double xlo;
        bool is_query;
        bool query_max;

        line(long long m, long long b, long long val, bool is_query, bool query_max)
        {
            this->m = m;
            this->b = b;
        }
    };
};

```

```

    this->val = val;
    this->xlo = -std::numeric_limits<double>::max();
    this->is_query = is_query;
    this->query_max = query_max;
}

bool parallel(const line &l) const { return m == 1.m; }
double intersect(const line &l) const
{
    if (parallel(l)) return std::numeric_limits<double>::max();
    return (double) (l.b - b) / (m - 1.m);
}

bool operator<(const line &l) const
{
    if (l.is_query) return query_max ? (xlo < l.val) : (l.val < xlo);
    return m < 1.m;
}
};

std::set<line> hull;
bool _query_max;

typedef std::set<line>::iterator hulliter;

bool has_prev(hulliter it) const { return it != hull.begin(); }
bool has_next(hulliter it) const
{
    return (it != hull.end()) && (++it != hull.end());
}

bool irrelevant(hulliter it) const
{
    if (!has_prev(it) || !has_next(it)) return false;
    hulliter prev = it, next = it;
    --prev;
    ++next;
    return _query_max ? prev->intersect(*next) <= prev->intersect(*it)
        : next->intersect(*prev) <= next->intersect(*it);
}

hulliter update_left_border(hulliter it)
{
    if ((_query_max && !has_prev(it)) || (!_query_max && !has_next(it)))
        return it;
    hulliter it2 = it;
    double val = it->intersect(_query_max ? *--it2 : *++it2);
    line l(*it);
    l.xlo = val;
    hull.erase(it++);
    return hull.insert(it, l);
}

public:
hull_optimizer(bool query_max = false) { this->_query_max = query_max; }
void add_line(long long m, long long b)
{
    line l(m, b, 0, false, _query_max);
    hulliter it = hull.lower_bound(l);
    if (it != hull.end() && it->parallel(l)) {
        if (_query_max && it->b < b || (!_query_max && b < it->b))
            hull.erase(it++);
        else
            return;
    }
    it = hull.insert(it, l);
    if (irrelevant(it)) {
        hull.erase(it);
        return;
    }
    while (has_prev(it) && irrelevant(--it)) hull.erase(it++);
    while (has_next(it) && irrelevant(++it)) hull.erase(it--);
    it = update_left_border(it);
    if (has_prev(it)) update_left_border(--it);
    if (has_next(++it)) update_left_border(++it);
}

long long get_best(long long x) const
{
    line q(0, 0, x, true, _query_max);
    hulliter it = hull.lower_bound(q);
    if (_query_max) --it;
    return it->m * x + it->b;
}
};

/** Example Usage ***/

#include <cassert>

int main()
{

```

```

hull_optimizer h;
h.add_line(3, 0);
h.add_line(0, 6);
h.add_line(1, 2);
h.add_line(2, 1);
assert(h.get_best(0) == 0);
assert(h.get_best(2) == 4);
assert(h.get_best(1) == 3);
assert(h.get_best(3) == 5);
return 0;
}

```

7.3 Divide and Conquer

```

//Um exemplo de Divide and conquer:
int MOD = 1e9 + 7;
const int N = 1010;
int dp[N][N], cost[N][N], v[N], pref[N], n, m;
void compDP(int j, int L, int R, int b, int e)
{
    if (L > R) return;
    int mid = (L + R) / 2;
    int idx = -1;
    for (int i = b; i <= min(mid, e); i++)
        if (dp[mid][j] > dp[i][j - 1] + cost[i + 1][mid]) {
            idx = i;
            dp[mid][j] = dp[i][j - 1] + cost[i + 1][mid];
        }
    compDP(j, L, mid - 1, b, idx);
    compDP(j, mid + 1, R, idx, e);
}
//chamada!
for (int i = 1; i <= n; i++) dp[i][0] = cost[1][i];
for (int i = 1; i <= m; i++) compDP(i, 1, n, 1, n);

```

7.4 Longest Increasing Subsequence

```

//asw -> vetor com resposta!!
//asw.size() o tamanho da maior lis
void lis(const vector<int> &v, vector<int> &asw)
{
    vector<int> pd(v.size(), 0), pd_index(v.size()), pred(v.size());
    int maxi = 0, x = 0, j = 0, ind = 0;
    for (int i = 0; i < v.size(); i++)
    {
        x = v[i];
        j = lower_bound(pd.begin(), pd.begin() + maxi, x) - pd.begin();
        pd[j] = x;
        pd_index[j] = i;
        if (j == maxi)
        {
            maxi++;
            ind = i;
        }
        if (pred[i] == j) pd_index[j - 1] = -1;
    }
    int pos = maxi - 1, k = v[ind];
    asw.resize(maxi);
    while (pos >= 0)
    {
        asw[pos--] = k;
        ind = pred[ind];
        k = v[ind];
    }
}

```

8 Geometry

8.1 Convex Hull Monotone Chain

```

typedef struct sPoint {
    int x, y;
    sPoint(int _x, int _y)
    {
        x = _x;
        y = _y;
    }
}

```

```

    }
} point;
bool comp(point a, point b)
{
    if (a.x == b.x) return a.y < b.y;
    return a.x < b.x;
}
int cross(point a, point b, point c) // AB x BC
{
    a.x -= b.x;
    a.y -= b.y;
    b.x -= c.x;
    b.y -= c.y;
    return a.x * b.y - a.y * b.x;
}

bool isCw(point a, point b, point c) // Clockwise
{
    return cross(a, b, c) < 0;
}

// >= if you want to put collinear points on the convex hull
bool isCcw(point a, point b, point c) // Counter Clockwise
{
    return cross(a, b, c) > 0;
}

vector<point> convexHull(vector<point> p)
{
    vector<point> u, l; // Upper and Lower hulls

    sort(p.begin(), p.end(), comp);
    for (unsigned int i = 0; i < p.size(); i++) {
        while (l.size() > 1 && !isCcw(l[l.size() - 1], l[l.size() - 2], p[i]))
            l.pop_back();
        l.push_back(p[i]);
    }

    for (int i = p.size() - 1; i >= 0; i--) {
        while (u.size() > 1 && !isCcw(u[u.size() - 1], u[u.size() - 2], p[i]))
            u.pop_back();
        u.push_back(p[i]);
    }

    u.pop_back();
    l.pop_back();
    l.insert(l.end(), u.begin(), u.end());
    return l;
}

```

8.2 Minimum Enclosing Circle

```

//6.5- Minimum Enclosing Circle
const double eps = 1e-6;
#define CIRCLE circ
#define PT Ponto
#define MP 101
#define eps 1e-9
#define x first
#define y second
typedef double cood;
typedef int num;
typedef int point;
double resp;
cood x[MP], y[MP], ar, ax, ay;
int p[MP];
typedef pair<double, double> ponto;
typedef pair<double, double> Ponto;
double dista(ponto a, ponto b)
{
    return sqrt((a.first - b.first) * (a.first - b.first) +
                (a.second - b.second) * (a.second - b.second));
}

bool in(ponto a, pair<double, ponto> c)
{
    if (dista(a, c.second) - eps < c.first) return true;
    return false;
}

bool same(point a, point b)
{
    return (fabs(x[a] - x[b]) < eps && fabs(y[a] - y[b]) < eps);
}

bool lexLess(point a, point b)
{
    if (fabs(x[a] - x[b]) < eps) return y[a] < y[b];
    return x[a] < x[b];
}

```

```

inline cood dist(cood xx, cood yy, point a)
{
    return sqrt((xx - x[a]) * (xx - x[a]) + (yy - y[a]) * (yy - y[a]));
}

inline cood cP(point a, point b, point c)
{
    return (x[a] - x[b]) * (y[c] - y[b]) - (x[c] - x[b]) * (y[a] - y[b]);
}

void findCircle(point a, point b, point c, cood& cx, cood& cy)
{
    cx = 0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (y[b] - y[c]) -
        0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (y[a] - y[b]),
    cy = 0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (x[a] - x[b]) -
        0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (x[b] - x[c]);
    cx /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
    cy /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
}

void spanCircle2(int k, point p0, point p1, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p1]);
    cy = 0.5 * (y[p0] + y[p1]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) {
            findCircle(p0, p1, p[i], cx, cy);
            r = dist(cx, cy, p[i]);
        }
}

void spanCircle1(int k, point p0, cood& cx, cood& cy, cood& r)
{
    cx = 0.5 * (x[p0] + x[p[0]]);
    cy = 0.5 * (y[p0] + y[p[0]]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) spanCircle2(i, p0, p[i], cx, cy, r);
}

void spanCircle(int n, cood& cx, cood& cy, cood& r)
{
    // Bem importante, retirar repetidos
    sort(p, p + 1, lexLess);
    n = unique(p, p + n) - p;
    random_shuffle(p, p + n);
    if (n > 1) {
        cx = 0.5 * (x[p[0]] + x[p[1]]);
        cy = 0.5 * (y[p[0]] + y[p[1]]);
        r = dist(cx, cy, p[1]);
        for (int i = 2; i < n; i++)
            if (dist(cx, cy, p[i]) > r) spanCircle1(i, p[i], cx, cy, r);
    }
    else {
        cx = x[0];
        cy = y[0];
        r = 0.0;
    }
}

void solve(vector<pair<double, double> >& v)
{
    int N = v.size();
    for (int i = 0; i < N; i++) {
        x[i] = v[i].first;
        y[i] = v[i].second;
        p[i] = i;
    }
    spanCircle(N, ax, ay, ar);
}

```

8.3 Minimum Enclosing Circle 2

```

const double eps = 1e-6;
#define CIRCLE circ
#define PT Ponto
#define MP 101
#define eps 1e-9
#define x first
#define y second
typedef double cood;
typedef int num;

```

```

typedef int point;
double resp;
coord x[MP], y[MP], ar, ax, ay;
int p[MP];
typedef pair<double, double> ponto;
typedef pair<double, double> Ponto;
double dista(ponto a, ponto b)
{
    return sqrt((a.first - b.first) * (a.first - b.first) +
                (a.second - b.second) * (a.second - b.second));
}
bool in(ponto a, pair<double, ponto> c)
{
    if (dista(a, c.second) - eps < c.first) return true;
    return false;
}
bool same(point a, point b)
{
    return (fabs(x[a] - x[b]) < eps && fabs(y[a] - y[b]) < eps);
}
bool lexLess(point a, point b)
{
    if (fabs(x[a] - x[b]) < eps) return y[a] < y[b];
    return x[a] < x[b];
}
inline coord dist(coord xx, coord yy, point a)
{
    return sqrt((xx - x[a]) * (xx - x[a]) + (yy - y[a]) * (yy - y[a]));
}
inline coord cP(point a, point b, point c)
{
    return (x[a] - x[b]) * (y[c] - y[b]) - (x[c] - x[b]) * (y[a] - y[b]);
}
void findCircle(point a, point b, point c, coord& cx, coord& cy)
{
    cx = 0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (y[b] - y[c]) -
        0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (y[a] - y[b]),
    cy = 0.5 * (x[b] * x[b] + y[b] * y[b] - x[c] * x[c] - y[c] * y[c]) *
        (x[a] - x[b]) -
        0.5 * (x[a] * x[a] + y[a] * y[a] - x[b] * x[b] - y[b] * y[b]) *
        (x[b] - x[c]);
    cx /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
    cy /= (x[a] - x[b]) * (y[b] - y[c]) - (x[b] - x[c]) * (y[a] - y[b]);
}
void spanCircle2(int k, point p0, point p1, coord& cx, coord& cy, coord& r)
{
    cx = 0.5 * (x[p0] + x[p1]);
    cy = 0.5 * (y[p0] + y[p1]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) {
            findCircle(p0, p1, p[i], cx, cy);
            r = dist(cx, cy, p[i]);
        }
}
void spanCircle1(int k, point p0, coord& cx, coord& cy, coord& r)
{
    cx = 0.5 * (x[p0] + x[p[0]]);
    cy = 0.5 * (y[p0] + y[p[0]]);
    r = dist(cx, cy, p0);
    for (int i = 0; i < k; i++)
        if (dist(cx, cy, p[i]) > r) spanCircle2(i, p0, p[i], cx, cy, r);
}
void spanCircle(int n, coord& cx, coord& cy, coord& r)
{
    // Bem importante, retirar repetidos
    sort(p, p + 1, lexLess);
    n = unique(p, p + n) - p;
    random_shuffle(p, p + n);
    if (n > 1) {
        cx = 0.5 * (x[p[0]] + x[p[1]]);
        cy = 0.5 * (y[p[0]] + y[p[1]]);
        r = dist(cx, cy, p[1]);
        for (int i = 2; i < n; i++)
            if (dist(cx, cy, p[i]) > r) spanCircle1(i, p[i], cx, cy, r);
    }
    else {
        cx = x[0];
        cy = y[0];
        r = 0.0;
    }
}
void solve(vector<pair<double, double> >& v)

```

```

{
    int N = v.size();
    for (int i = 0; i < N; i++) {
        x[i] = v[i].first;
        y[i] = v[i].second;
        p[i] = i;
    }
    spanCircle(N, ax, ay, ar);
}

```

8.4 Fast Geometry in Cpp

// C++ routines for computational geometry.

```

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                           double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

```

```

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }
    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
    return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=d-c; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
            return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d+d-R+r+r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

```

```

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == 1 || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

int main() {
    // expected: (-5,2)
    cerr << RotateCCW90(PT(2,5)) << endl;

    // expected: (5,-2)
    cerr << RotateCW90(PT(2,5)) << endl;

    // expected: (-5,2)
    cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

    // expected: (5,2)
    cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

    // expected: (5,2) (7.5,3) (2.5,1)
    cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

    // expected: 6.78903
    cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

    // expected: 1 0 1
    cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 0 0 1
    cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 1 1 1 0
    cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

    // expected: (1,2)
    cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

    // expected: (1,1)
    cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

    vector<PT> v;
    v.push_back(PT(0,0));
    v.push_back(PT(5,0));
    v.push_back(PT(5,5));
    v.push_back(PT(0,5));

    // expected: 1 1 1 0 0

```

```

cerr << PointInPolygon(v, PT(2,2)) << " "
<< PointInPolygon(v, PT(2,0)) << " "
<< PointInPolygon(v, PT(0,2)) << " "
<< PointInPolygon(v, PT(5,2)) << " "
<< PointInPolygon(v, PT(2,5)) << endl;

// expected: 0 1 1 1 1
cerr << PointOnPolygon(v, PT(2,2)) << " "
<< PointOnPolygon(v, PT(2,0)) << " "
<< PointOnPolygon(v, PT(0,2)) << " "
<< PointOnPolygon(v, PT(5,2)) << " "
<< PointOnPolygon(v, PT(2,5)) << endl;

// expected: (1,6)
// (5,4) (4,5)
// blank line
// (4,5) (5,4)
// blank line
// (4,5) (5,4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

// area should be 5.0
// centroid should be (1.1666666, 1.1666666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;

return 0;
}

```

8.5 Point Inside Polygon $O(\lg n)$

```

/*
 * Solution for UVa 11072 - Points
 *
 * On this problem you must calculate the convex hull on the
 * first set of points.
 *
 * And for each point of the second set, answer if the point
 * is inside or outside the convex hull.
 */
typedef struct sPoint {
    ll x, y;

    sPoint() {}
    sPoint (ll _x, ll _y) : x(_x), y(_y) {}
    bool operator<(const sPoint& other) const
    {
        if(x == other.x) return y < other.y;
        return x < other.x;
    }
} point;

vector<point> vp, ch;

ll cross(point a, point b, point c) // AB x BC
{
    a.x -= b.x; a.y -= b.y;
    b.x -= c.x; b.y -= c.y;
    return a.x*b.y - a.y*b.x;
}

vector<point> convexhull()
{
    sort(vp.begin(), vp.end());

    vector<point> l, u;
    for(int i = 0; i < vp.size(); i++)
    {
        while(l.size() > 1 && cross(l[l.size()-2], l[l.size()-1], vp[i]) <= 0)
            l.pop_back();
        l.pb(vp[i]);
    }
}

```

```

for(int i = vp.size()-1; i >= 0; i--)
{
    while(u.size() > 1 && cross(u[u.size()-2], u[u.size()-1], vp[i]) <= 0)
        u.pop_back();
    u.pb(vp[i]);
}
l.pop_back(); u.pop_back();
l.insert(l.end(), u.begin(), u.end());
return l;
}

ll area(point a, point b, point c)
{ return llabs(cross(a, b, c)); }

bool insideTriangle(point a, point b, point c, point p)
{
    return area(a, b, c) == (area(a, b, p) +
        area(a, c, p) +
        area(b, c, p));
}

bool isInside(point p)
{
    if(ch.size() < 3) return false;

    int i = 2, j = ch.size()-1;

    while(i < j)
    {
        int mid = (i+j)/2;
        ll c = cross(ch[0], ch[mid], p);
        if(c > 0) i = mid+1;
        else j = mid;
    }
    return insideTriangle(ch[0], ch[i], ch[i-1], p);
}

int main()
{
    int n;

    while(true)
    {
        ch.clear();
        vp.clear();
        cin >> n;
        if(not cin) break;

        while(n--)
        {
            point p;
            cin >> p.x >> p.y;
            vp.pb(p);
        }

        ch = convexhull();

        cin >> n;
        while(n--)
        {
            point p;
            cin >> p.x >> p.y;
            if(isInside(p)) cout << "inside\n";
            else cout << "outside\n";
        }

        return 0;
    }
}

```

9 Data Structures

9.1 Disjoint Set Union

```

const int N=500010;
int p[N], Rank[N];
void init()
{
    memset(Rank, 0, sizeof(Rank));
    for(int i=0; i<N; i++) p[i]=i;
}

int findset(int i)
{
    if(p[i]==i) return i;
    return p[i]=findset(p[i]);
}

```

```

}
bool same(int i, int j)
{
    return (findset(i) == findset(j));
}
void unionSet(int i, int j)
{
    if (!same(i, j)) {
        int x = findset(i), y=findset(j);
        if (Rank[x] > Rank[y])
            p[y] = x;
        else {
            p[x] = y;
            if (Rank[x] == Rank[y]) Rank[y]++;
        }
    }
}
}

```

9.2 Persistent Segment Tree

```

//PRINTAR O NUMERO DE ELEMENTOS DISTINTOS
//EM UM INTERVALO DO ARRAY
const int N = 30010;
int tr[100 * N], L[100 * N], R[100 * N], root[100 * N];
int v[N], mapa[100 * N];
int cont = 0;
void build(int node, int b, int e)
{
    if (b == e) {
        tr[node] = 0;
    }
    else {
        L[node] = cont++;
        R[node] = cont++;
        build(L[node], b, (b + e) / 2);
        build(R[node], (b + e) / 2 + 1, e);
        tr[node] = tr[L[node]] + tr[R[node]];
    }
}
int update(int node, int b, int e, int i, int val)
{
    int idx = cont++;
    tr[idx] = tr[node] + val;
    L[idx] = L[node];
    R[idx] = R[node];
    if (b == e) return idx;
    int mid = (b + e) / 2;
    if (i <= mid)
        L[idx] = update(L[node], b, mid, i, val);
    else
        R[idx] = update(R[node], mid + 1, e, i, val);
    return idx;
}
int query(int nodeL, int nodeR, int b, int e, int i, int j)
{
    if (b > j or i > e) return 0;
    if (i <= b and j >= e) {
        int p1 = tr[nodeR];
        int p2 = tr[nodeL];
        return p1 - p2;
    }
    int mid = (b + e) / 2;
    return query(L[nodeL], L[nodeR], b, mid, i, j) +
           query(R[nodeL], R[nodeR], mid + 1, e, i, j);
}
int main()
{
    int n;
    sc(n);
    memset(mapa, -1, sizeof(mapa));
    for (int i = 0; i < n; i++) sc(v[i]);
    build(1, 0, n - 1);
    for (int i = 0; i < n; i++) {
        if (mapa[v[i]] == -1) {
            root[i + 1] = update(root[i], 0, n - 1, i, 1);
            mapa[v[i]] = i;
        }
        else {
            root[i + 1] = update(root[i], 0, n - 1, mapa[v[i]], -1);
            mapa[v[i]] = i;
            root[i + 1] = update(root[i + 1], 0, n - 1, i, 1);
        }
    }
    int q;
    sc(q);
    for (int i = 0; i < q; i++) {
        int l, r;

```

```

        sc2(l, r);
        int resp = query(root[l - 1], root[r], 0, n - 1, l - 1, r - 1);
        pri(resp);
    }
    return 0;
}

```

9.3 RMQ of Indices

```

//RMQ DE INDICE
class RMQ {
private:
    vi A;
    vi M;
public:
    RMQ(vi &v)
    {
        A = v;
        M.resize(4 * v.size());
        build(1, 0, v.size() - 1);
    }
    void build(int node, int b, int e)
    {
        if (b == e)
            M[node] = b;
        else {
            build(2 * node, b, (b + e) / 2);
            build(2 * node + 1, (b + e) / 2 + 1, e);
            if (A[M[2 * node]] <= A[M[2 * node + 1]])
                M[node] = M[2 * node];
            else
                M[node] = M[2 * node + 1];
        }
    }
    int query(int node, int b, int e, int i, int j)
    {
        int p1, p2;
        if (i > e || j < b) return -1;
        if (b >= i and e <= j) return M[node];
        p1 = query(2 * node, b, (b + e) / 2, i, j);
        p2 = query(2 * node + 1, (b + e) / 2 + 1, e, i, j);
        if (p1 == -1) return p2;
        if (p2 == -1) return p1;
        if (A[p1] <= A[p2]) return p1;
        return p2;
    }
    void atualiza(int node, int b, int e, int i, int val)
    {
        if (i > e || i < b) return;
        if (e == b) {
            A[i] = val;
        }
        else {
            atualiza(2 * node, b, (b + e) / 2, i, val);
            atualiza(2 * node + 1, (b + e) / 2 + 1, e, i, val);
            if (A[M[2 * node]] <= A[M[2 * node + 1]])
                M[node] = M[2 * node];
            else
                M[node] = M[2 * node + 1];
        }
    }
};

```

9.4 RSQ with Lazy Propagation

```

//RSQ COM LAZY PROPAGATION!
class RSQ {
private:
    vll A;
    vll M;
    vll lazy;
public:
    RSQ(vll &v)
    {
        A = v;
        M.resize(v.size() * 4);
        lazy.assign(v.size() * 4, 0);
        build(1, 0, v.size() - 1);
    }
    void build(int node, int b, int e)
    {

```

```

if (b == e) {
    M[node] = A[b];
    return;
}
build(2 * node, b, (b + e) / 2);
build(2 * node + 1, (b + e) / 2 + 1, e);
M[node] = M[2 * node] + M[2 * node + 1];
}
void atualiza(int node, int b, int e, int i, int j, ll val)
{
    if (lazy[node] != 0) {
        M[node] += lazy[node];
        if (b != e) {
            ll inter = (e - b + 1);
            ll i1 = (b + e) / 2 - b + 1;
            ll i2 = e - (b + e) / 2;
            ll un = lazy[node] / inter;
            lazy[2 * node] += un * i1;
            lazy[2 * node + 1] += un * i2;
        }
        lazy[node] = 0;
    }
    if (i > e or j < b) return;
    if (i <= b and j >= e) {
        ll inter = (e - b + 1);
        M[node] += val * inter;
        if (b != e) {
            ll i1 = (b + e) / 2 - b + 1;
            ll i2 = e - (b + e) / 2;
            lazy[2 * node] += i1 * (ll)val;
            lazy[2 * node + 1] += i2 * (ll)val;
        }
        return;
    }
    atualiza(2 * node, b, (b + e) / 2, i, j, val);
    atualiza(2 * node + 1, (b + e) / 2 + 1, e, i, j, val);
    M[node] = M[2 * node] + M[2 * node + 1];
}
ll query(int node, int b, int e, int i, int j)
{
    if (i > e or j < b) return 0;
    ll p1, p2;
    if (lazy[node] != 0) {
        M[node] += lazy[node];
        if (b != e) {
            ll inter = (e - b + 1);
            ll i1 = (b + e) / 2 - b + 1;
            ll i2 = e - (b + e) / 2;
            ll un = lazy[node] / inter;
            lazy[2 * node] += un * i1;
            lazy[2 * node + 1] += un * i2;
        }
        lazy[node] = 0;
    }
    if (i <= b and j >= e) return M[node];
    p1 = query(2 * node, b, (b + e) / 2, i, j);
    p2 = query(2 * node + 1, (b + e) / 2 + 1, e, i, j);
    return p1 + p2;
}
};

```

9.5 Segment Tree

```

//compilar em C++11, essa segment tree
//computa qual e o k's elemento compreendido
//no intervalo entre i,j
//presentes no array
vi tr[5 * N];
void build(int node, int b, int e)
{
    if (b == e)
        tr[node].pb(v[b]);
    else {
        build(2 * node, b, (b + e) / 2);
        build(2 * node + 1, (b + e) / 2 + 1, e);
        merget(tr[2 * node], tr[2 * node + 1], tr[node]);
        merge(tr[2 * node].begin(), tr[2 * node].end(), tr[2 * node + 1].begin(),
            tr[2 * node + 1].end(), back_inserter(tr[node]));
    }
}
int query(int node, int b, int e, int i, int j, int k)
{
    if (i > e or b > j) return 0;
    if (i <= b and j >= e) {
        int resp =
            upper_bound(tr[node].begin(), tr[node].end(), k) - tr[node].begin();
        return tr[node].size() - resp;
    }
}

```

```

}
return query(2 * node, b, (b + e) / 2, i, j, k) +
    query(2 * node + 1, (b + e) / 2 + 1, e, i, j, k);
}

```

9.6 Sparse Table

```

//computar RMQ , favor inicializar: dp[i][0]=v[0]
//sendo v[0] o vetor do rmq
//chamar o build!
int dp[200100][22];
int n;
int d[200100];
void build()
{
    d[0] = d[1] = 0;
    for (int i = 2; i < n; i++) d[i] = d[i >> 1] + 1;
    for (int j = 1; j < 22; j++) {
        for (int i = 0; i + (1 << (j - 1)) < n; i++) {
            dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
        }
    }
}
int query(int i, int j)
{
    int k = d[j - i];
    int x = min(dp[i][k], dp[j - (1 << k) + 1][k]);
    return x;
}

```

10 Miscellaneous

10.1 Hashing

```

//certificar que gethash() foi chamado
//antes de getHash(i,j);
struct Hashing {
    const string &s;
    int n, idx;
    vector<ll> hashes, M, B;
    Hashing(const string &s) : s(s), hashes(s.size()) {
        M = {1000000409, 2000003273, 2000003281, 2000003293};
        B = {31, 53, 61, 41};
        srand(time(NULL));
        idx = rand() % 4;
        getHash();
    }
    void otherprime() {
        idx = (idx + 1) % 4;
    }
    ll int_mod(ll a) { return (a % M[idx] + M[idx]) % M[idx]; }
    ll eleva(ll a, ll b)
    {
        if (b == 0)
            return 1;
        else if (b == 1)
            return a;
        ll x = eleva(a, b / 2);
        if (b % 2 == 0)
            return (x * x) % M[idx];
        else
            return (a * ((x * x) % M[idx])) % M[idx];
    }
    /*hash da string de 0 ate i*/
    void getHash()
    {
        int n = s.size();
        ll hp = 0;
        for (int i = 0; i < s.size(); i++) {
            hp = int_mod(hp * B[idx] + s[i]);
            hashes[i] = hp;
        }
    }
    /*Hash da string compreendida entre i e j*/
    ll getHash(int i, int j)
    {
        if (i == 0) return hashes[j];
        ll h1 = hashes[j];
        ll h2 = (hashes[i - 1] * eleva(B[idx], j - i + 1)) % M[idx];
        ll ret = (h1 - h2) % M[idx] + M[idx];
    }
}

```



```

    return ret % M[idx];
}
};

```

10.2 Inversion Count

```

//conta o numero de inversoes de um array
//x e o tamanho do array, v e o array que quero contar
ll inversoes = 0;
void merge_sort(vi &v, int x)
{
    if (x == 1) return;
    int tam_esq = (x + 1) / 2, tam_dir = x / 2;
    int esq[tam_esq], dir[tam_dir];
    for (int i = 0; i < tam_esq; i++) esq[i] = v[i];
    for (int i = 0; i < tam_dir; i++) dir[i] = v[i + tam_esq];
    merge_sort(esq, tam_esq);
    merge_sort(dir, tam_dir);
    int i_esq = 0, i_dir = 0, i = 0;
    while (i_esq < tam_esq or i_dir < tam_dir) {
        if (i_esq == tam_esq) {
            while (i_dir != tam_dir) {
                v[i] = dir[i_dir];
                i_dir++; i++;
            }
        }
        else if (i_dir == tam_dir) {
            while (i_esq != tam_esq) {
                v[i] = esq[i_esq];
                i_esq++; i++;
                inversoes += i_dir;
            }
        }
        else {
            if (esq[i_esq] <= dir[i_dir]) {
                v[i] = esq[i_esq];
                i++, i_esq++;
                inversoes += i_dir;
            }
            else {
                v[i] = dir[i_dir];
                i++, i_dir++;
            }
        }
    }
}
}

```

10.3 Distinct Elements in ranges

```

const int MOD = 1e9 + 7;
const int N = 1e6 + 10;
int bit[N], v[N], id[N], r[N];
ii query[N];
int mapa[N];
bool compare(int x, int y) { return query[x] < query[y]; }
void add(int idx, int val)
{
    while (idx < N) {
        bit[idx] += val;
        idx += idx & -idx;
    }
}
int sum(int idx)
{
    int ret = 0;
    while (idx > 0) {
        ret += bit[idx];
        idx -= idx & -idx;
    }
    return ret;
}
int main()
{
    memset(bit, 0, sizeof(bit));
    memset(mapa, 0, sizeof(mapa));
    int n;
    sc(n);
    for (int i = 1; i <= n; i++) sc(v[i]);
    int q;
    sc(q);
    for (int i = 0; i < q; i++) {
        sc2(query[i].second, query[i].first);
        id[i] = i;
    }
}

```

```

}
sort(id, id + q, compare);
sort(query, query + q);
int j = 1;
for (int i = 0; i < q; i++) {
    int L = query[i].second;
    int R = query[i].first;
    while (j <= R) {
        if (mapa[v[j]] > 0) {
            add(mapa[v[j]], -1);
            mapa[v[j]] = j;
            add(mapa[v[j]], 1);
        }
        else {
            mapa[v[j]] = j;
            add(mapa[v[j]], 1);
        }
        j++;
    }
    r[id[i]] = sum(R);
    if (L > 1) r[id[i]] -= sum(L - 1);
}
for (int i = 0; i < q; i++) pri(r[i]);
return 0;
}

```

10.4 Maximum Rectangular Area in Histogram

```

/*
 * Complexidade : O(N)
 */
ll solve(vi &h)
{
    int n = h.size();
    ll resp = 0;
    stack<int> pilha;
    ll i = 0;
    while (i < n) {
        if (pilha.empty() or h[pilha.top()] <= h[i]) {
            pilha.push(i++);
        }
        else {
            int aux = pilha.top();
            pilha.pop();
            resp =
                max(resp, (ll)h[aux] * ((pilha.empty()) ? i : i - pilha.top() - 1));
        }
    }
    while (!pilha.empty()) {
        int aux = pilha.top();
        pilha.pop();
        resp = max(resp, (ll)h[aux] * ((pilha.empty()) ? n : n - pilha.top() - 1));
    }
    return resp;
}

```

10.5 Multiplying Two LL mod n

```

/* Metodo para calcular (a*b) mod m onde a e b s o inteiros com 64-bits cada.
Fonte: http://bit.ly/lpp7xEZ */
ll mulmod(ll a, ll b, ll m) {
    ll y = (ll) ( (ld)a*(ld)b/m + (ld)1/2 );
    y = y * m;
    ll x = a * b, r = x - y;
    if ((ll) r < 0) { r = r + m; y = y - 1; }
    return r;
}

```

10.6 Josephus Problem

```

/* Josephus Problem - It returns the position to be, in order to not die. O(n)*/
/* With k=2, for instance, the game begins with 2 being killed and then n+2, n+4, ... */
ll josephus(ll n, ll k) {
    if (n==1) return 1;
    else return (josephus(n-1, k)+k-1)%n+1;
}

```

10.7 Josephus Problem 2

```
/* Another Way to compute the last position to be killed O ( d * log n ) */
ll josephus(ll n, ll d) {
    ll K = 1;
    while (K <= (d - 1) * n) K = (d + K + d - 2) / (d - 1);
    return d * n + 1 - K;
}
```

10.8 Dynamic MST

```
/*
 * Code for URI 1887
 * It gives an tree and a bunch of queries to add
 * edges from a to b with cost c.
 */
const int MOD = 1e9 + 9;
struct ed{
    int u,v,w,t;
    ed(int _u, int _v, int _w, int _t){ u=_u,v=_v,w=_w,t=_t; }
    ed(){};
    bool operator < ( const ed &a) const {
        return w<a.w;
    }
};
const int N=50010;
int p[N],id[N];
void init(int n)
{
    for(int i=1;i<=n;i++) p[i]=i;
}
int findSet(int i)
{
    if(p[i]==i) return i;
    return p[i]=findSet(p[i]);
}
bool unionSet(int i, int j)
{
    int x=findSet(i),y=findSet(j);
    if(x==y) return false;
    p[x]=y;
    return true;
}
void reduction(int l,int r,int &n,vector<ed> &graph,int &res)
{
    vector<ed> g;
    init(n);
    sort(graph.begin(),graph.end());
    for(int i=0;i<(int)graph.size();i++)
        if(graph[i].t<=r and (graph[i].u>1 or unionSet(graph[i].u,graph[i].v))) {
            g.pb(graph[i]);
        }
    graph=g;
}
void contraction(int l,int r,int &n,vector<ed> &graph,int &res)
{
    vector<ed> g;
    init(n);
    sort(graph.begin(),graph.end());
    for(int i=0;i<(int)graph.size();i++)
        if(graph[i].t>=l unionSet(graph[i].u,graph[i].v);
    for(int i=0;i<(int)graph.size();i++) {
        if(graph[i].t<=l and unionSet(graph[i].u,graph[i].v)) {
            g.pb(graph[i]);
            res+=graph[i].w;
        }
    }
    init(n);
    for(int i=0;i<g.size();i++) {
        unionSet(g[i].u,g[i].v);
    }
    int tot=0;
    for(int i=1;i<=n;i++) id[i]=0;
    for(int i=1;i<=n;i++) {
        int f=findSet(i);
        if(!id[f]) id[f]=++tot;
        id[i]=id[f];
    }
    for(int i=0;i<graph.size();i++) {
```

```
        graph[i].u=id[graph[i].u],graph[i].v=id[graph[i].v];
        n=tot;
    }
}
void solve(int l,int r,int n,vector<ed> graph,int res)
{
    reduction(l,r,n,graph,res);
    contraction(l,r,n,graph,res);
    if(l==r)
    {
        init(n);
        sort(graph.begin(),graph.end());
        for(int i=0;i<(int)graph.size();i++)
            if(unionSet(graph[i].u,graph[i].v)) {
                res+=graph[i].w;
            }
        pri(res);
        return;
    }
    int mid=(l+r)/2;
    solve(l,mid,n,graph,res);
    solve(mid+1,r,n,graph,res);
}
int main()
{
    int T;
    sc(T);
    while(T--)
    {
        int n,m,q;
        sc3(n,m,q);
        vector<ed> graph;
        for(int i=1;i<=m;i++)
        {
            int u,v,w;
            sc3(u,v,w);
            int t=0;
            graph.pb(ed(u,v,w,t));
        }
        for(int i=1;i<=q;i++)
        {
            int u,v,w;
            sc3(u,v,w);
            int t=i;
            graph.pb(ed(u,v,w,t));
        }
        solve(1,q,n,graph,0);
    }
    return 0;
}
```

10.9 Ordered Static Set (Examples)

```
#include<bits/stdc++.h>
using namespace std;
#define sc(a) scanf("%d", &a)
#define sc2(a,b) scanf("%d%d", &a, &b)
#define sc3(a,b,c) scanf("%d%d%d", &a, &b, &c)
#define pri(x) printf("%d\n", x)
#define mp make_pair
#define pb push_back
#define BUFF ios::sync_with_stdio(false);
#define imprime(v) for(int X=0;X<v.size();X++) printf("%d ", v[X]); printf("\n");
#define endl "\n"
const int INF= 0x3f3f3f3f;
const long double pi= acos(-1);
typedef long long int ll;
typedef long double ld;
typedef pair<int,int> ii;
typedef vector<int> vi;
typedef vector< vector< int > > vvi;
const int MOD=1e9+7;
const ll LINF=0x3f3f3f3f3f3f3f3f;
///USANDO ORDERED STATIC SET PRA ESTRUTURA

#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update

using namespace __gnu_pbds;
typedef struct cu {
    int a;
    int b;
    bool operator < (const struct cu &other) const {
        if(a != other.a) return a < other.a;
        return b < other.b;
    }
}
```

```

        bool operator == (const struct cu &other) const {
            return(a == other.a and b == other.b);
        }
    }cuzao;

    bool cmp(const cuzao &a, const cuzao &b) {
        return true;
    }

    typedef tree<
        cuzao,
            null_type,
            less<cuzao>,
            rb_tree_tag,
            tree_order_statistics_node_update>
        ordered_set;

    int main()
    {
        ordered_set os;
        cuzao asd;
        asd.a = 1;
        asd.b = 2;
        os.insert(asd);
        asd.a = 4;
        os.insert(asd);
        cout<<(os.find(asd) == end(os))<<endl; //0
        cout<<os.order_of_key(asd)<<endl; //1
        asd.a = 1;
        cout<<os.order_of_key(asd)<<endl; //0
        cout<<os.find_by_order(0)->a<<" "<<os.find_by_order(0)->b<<endl; //1 2
        cout<<os.find_by_order(1)->a<<" "<<os.find_by_order(1)->b<<endl; //4 2
        return 0;
    }

//USANDO ORDERED STATIC SET PRA CONTAINER DO STL MESMO
#include<bits/stdc++.h>
using namespace std;
#define sc(a) scanf("%d", &a)
#define sc2(a,b) scanf("%d%d", &a, &b)
#define sc3(a,b,c) scanf("%d%d%d", &a, &b, &c)
#define pri(x) printf("%d\n", x)
#define mp make_pair

```

```

#define pb push_back
#define BUFF ios::sync_with_stdio(false);
#define imprime(v) for(int X=0;X<v.size();X++) printf("%d ", v[X]); printf("\n");
#define endl "\n"
const int INF= 0x3f3f3f3f;
const long double pi= acos(-1);
typedef long long int ll;
typedef long double ld;
typedef pair<int,int> ii;
typedef vector<int> vi;
typedef vector< vector< int > > vvi;
const int MOD=1e9+7;
const ll LINF=0x3f3f3f3f3f3f3f3f;

#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update

using namespace __gnu_pbds;

typedef tree<
    int,
        null_type,
        less<int>,
        rb_tree_tag,
        tree_order_statistics_node_update>
    ordered_set; //n multi

int main()
{
    ordered_set os;
    os.insert(1);
    os.insert(10);
    os.insert(1);
    os.insert(15);
    cout<<(os.find(10) == end(os))<<endl; //0 mesma coisa q !count
    cout<<os.order_of_key(10)<<endl; //1 qual o indice do valor 10, se n tem o indice, pega o proximo
    cout<<os.order_of_key(2)<<endl; //1
    cout<<*os.upper_bound(2)<<endl; //10
    cout<<*os.find_by_order(0)<<endl; //1
    cout<<*os.find_by_order(2)<<endl; //15
    return 0;
}

```