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### Contents

$\mathbf{Gra}$	ph Algorithms	l
1.1	2-SAT	1
1.2	Kosaraju	1
1.3	Tree Isomorphism	2
1.4	LCA	2
1.5	Bridges and Articulation Points	3
1.6	Eulerian Tour	3
1.7	Floyd Warshall	3
Stri	ngs 3	3
2.1	Aho-Corasick	3
2.2	KMP	1
2.3	Suffix Array	1
2.4	Rabin Karp	1
Mat	chematics	ı
3.1	Big Number	1
3.2	Chinese Remainder	5
3.3	Matrix Exponentiation	5
3.4	Pascal Triangle	5
3.5	Eulers Totient Function	5
3.6	Pollard Rho	3
3.7	Sieve of Eratosthenes	3
3.8	Extended Euclidean Algorithm	3
3.9	Multiplicative Inverse	7
Con	nbinatorial Optimization	7
4.1	•	7
4.2		7
4.3		7
4.4	Edmonds Karp	3
	1.1 1.2 1.3 1.4 1.5 1.6 1.7  Stri 2.1 2.2 2.3 2.4  Mat 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9  Con 4.1 4.2 4.3	1.1    2-SAT

# 1 Graph Algorithms

#### 1.1 2-SAT

```
const int N = 510;
vi graph[N], rev[N];
int us[N];
stack<int> pilha;
int resposta[N];
void dfs1(int u)
  for (int v : graph[u])
  if (!us[v]) dfs1(v);
  pilha.push(u);
void dfs2(int u, int color)
  us[u] = color;
  for (int v : rev[u])
    if (!us[v]) dfs2(v, color);
int Sat(int n)
  for (int i = 0; i < n; i++)
    if (!us[i]) dfs1(i);
  int color = 1;
  memset(us, 0, sizeof(us));
while (!pilha.empty()) {
  int topo = pilha.top();
    r.pb(topo);
    pilha.pop();
    if (!us[topo]) dfs2(topo, color++);
```

```
for (int i = 0; i < n; i += 2) {
    if (us[i] == us[i + 1]) return 0;
}
memset(resposta, -1, sizeof(resposta));
for (int i = r.size() - 1; i >= 0; i--) {
    int vert = r[i] / 2;
    int ok = r[i] % 2;
    if (resposta[vert] == -1) resposta[vert] = !ok;
}
return 1;
}
inline void add(int u, int v)
{
    graph[u].pb(v);
    rev[v].pb(u);
}
inline int pos(int u) { return 2 * u; }
inline int neg(int u) { return 2 * u + 1; }
```

## 1.2 Kosaraju

```
class kosaraju {
 private:
   vi usados;
  vvi graph;
  vvi trans;
  vi pilha;
 public:
   kosaraju(int N)
     graph.resize(N);
     trans.resize(N);
   void AddEdge(int u, int v)
        graph[u]
  .pb(v);
trans[v].pb(u);
} void dfs(int u, int pass, int color)
  usados[u] = color;
   vi vizinhos;
   if (pass == 1)
     vizinhos = graph[u];
     vizinhos = trans[u];
  for (int j = 0; j < vizinhos.size(); j++) {
  int v = vizinhos[j];</pre>
     if (usados[v] == 0) {
       dfs(v, pass, color);
  pilha.pb(u);
int SSC(int n)
  pilha.clear();
  usados.assign(n, 0);
for (int i = 0; i < n; i++) {
   if (!usados[i]) dfs(i, 1, 1);</pre>
   usados.assign(n, 0);
  dfs(pilha[i], 2, color);
       color++:
  return color - 1;
vvi compression(int n)
  int tam = SSC(n);
   vvi Trans;
   resp.resize(tam);
   Trans.resize(tam);
  frans.resize(tam);
for (int u = 0; u < graph.size(); u++) {
  for (int j = 0; j < graph[u].size(); j++) {
    int v = graph[u][j];
    if (usados[u] != usados[v]) {
      resp[usados[u] - 1].pb(usados[v] - 1);
      Trans[usados[v] - 1].pb(usados[u] - 1);
}</pre>
```

```
}
return Trans;
}
```

#### 1.3 Tree Isomorphism

```
vvichildren.subtreeLabels.tree. L:
vipred, map;
int n:
boolcompare(int a, int b) {
  return subtreeLabels[a] <subtreeLabels[b];
boolequals(inta, int b) {
  return subtreeLabels[a] == subtreeLabels[b];
voidgenerateMapping(int r1, int r2) {
     map resize(n);
     m a p [r1] = r2 - n;
     s o r t (children[r1].begin(), children[r1].end(), compare);
     s o r t (children[r2].begin(), children[r2].end(), compare);
     for (int i = 0; i < (int) children[rl].size(); i++) {
  int u = children[rl][i];
  int v = children[r2][i];</pre>
        generateMapping(u, v);
vifindCenter(int offset = 0) {
     int cnt = n;
     vi a;
     vi deg(n);
     for (int i = 0; i < n; i++) {
    deg[i] = tree[i + offset].size();</pre>
             i f (deg[i] <= 1) {
                      a push_back(i + offset);
                       --cnt:
     while (cnt > 0) {
            vi na;
             for (int i = 0; i < (int) a.size(); i++) {
                    int u = a[i];
                    for (int j = 0; j < (int) tree[u].size(); j++) {
    int v = tree[u][j];
                             if (--deg[v - offset] == 1) {
                                    n a .push_back(v);
                                      --cnt;
              a = na;
    return a:
int dfs(int u, int p = -1, int depth = 0) {
      L [depth] push_back(u);
      int h = 0;
      for (int i = 0; i < (int) tree[u].size(); i++) {
            int v = tree[u][i];
             i f (v == p)
                continue
            pred[v] = u;
          children [u].push_back(v);
h = max(h, dfs(v, u, depth + 1));
     return h + 1:
boolrootedTreeIsomorphism(int r1, int r2) {
      L .assign(n, vi());
     pred.assign(2 * n, -1);
   children.assign(2 * n, vi());
    int h1 = dfs(r1);
int h2 = dfs(r2);
      if (h1 != h2)
         return false;
      int h = h1 - 1;
    vi label(2 * n);
  vi label(z * n);

subtreeLabels.assign(2 * n, vi());

for (int i = h - 1; i >= 0; i --) {

    for (int j = 0; j < (int) L[i + 1].size(); j++) {

        int v = L[i + 1][j];
               s u b t r e e L a b e l s [pred[v]].push_back(label[v]);
             f o r (int j = 0; j < (int) L[i].size(); j++) {</pre>
                    int v = L[i][j];
```

```
s o r t (subtreeLabels[v].begin(), subtreeLabels[v].end());
            sort(L[i].begin(), L[i].end(), compare);
             for (int j = 0, cnt = 0; j < (int) L[i].size(); j++) {
                     if (j && !equals(L[i][j], L[i][j - 1]))
                               ++cnt;
                   label[L[i][j]] = cnt;
      i f (!equals(r1, r2))
         return false;
  generateMapping(r1, r2);
   return true;
booltreeIsomorphism() {
     vi c1 = findCenter();
vi c2 = findCenter(n);
      i f (c1.size() == c2.size()) {
             i f (rootedTreeIsomorphism(c1[0], c2[0]))
                return true;
            else if (c1.size() > 1)
           return rootedTreeIsomorphism(c1[1], c2[0]);
   return false:
int main() {
     n = 5;
vvi t1(n);
     t 1 [0].push_back(1);
      t 1 [1].push_back(0);
      t 1 [1].push_back(2);
      t 1 [2].push_back(1);
      t 1 [1].push_back(3);
      t 1 [3].push_back(1);
      t 1 [0].push_back(4);
     t 1 [4].push_back(0);
vvi t2(n);
     t 2 [0] .push_back(1);
      t 2 [1].push_back(0);
      t 2 [0].push_back(4);
      t 2 [4].push_back(0);
      t 2 [4].push_back(3);
      t 2 [3].push back(4);
      t 2 [4].push_back(2);
      t 2 [2].push_back(4);
     tz [2].pusn_back(*);
tree.assign(2 * n, vi());
for (int u = 0; u < n; u++) {
    for (int i = 0; i < t1[u].size(); i++) {
        int v = t1[u][i];
}</pre>
                    tree[u].push_back(v);
             for (int i = 0; i < t2[u].size(); i++) {
   int v = t2[u][i];</pre>
                    tree[u + n].push_back(v + n);
     bool res =treeIsomorphism();
     cout << res << endl;
      i f (res)
             for (int i = 0; i < n; i++)
                    cout << map[i] << endl;
```

#### 1.4 LCA

```
const int N = 100000;
const int M = 22;
int P[N][M];
int big[N][M], low[N][M], level[N];
vii graph[N];
int n;
void dfs(int u, int last, int 1)
{
    level[u] = 1;
    P[u][0] = last;
    for (ii v : graph[u])
        if (v.first != last) {
            big[v.first][0] = low[v.first][0] = v.second;
            dfs(v.first, u, 1 + 1);
    }

void process()
{
    for (int j = 1; j < M; j++)
        for (int i = 1; i <= n; i++) {
            P[i][j] = P[P[i][j - 1]][j - 1];
    }
}</pre>
```

```
big[i][j] = max(big[i][j-1], big[P[i][j-1]][j-1]);
      low[i][j] = min(low[i][j-1], low[P[i][j-1]][j-1]);
int lca(int u, int v)
  if (level[u] < level[v]) swap(u, v);</pre>
  for (int i = M - 1; i >= 0; i--)
   if (level[u] - (1 << i) >= level[v]) u = P[u][i];
  if (u == v) return u;
  for (int i = M - 1; i >= 0; i--) {
  if (P[u][i] != P[v][i]) u = P[u][i], v = P[v][i];
  return P[u][0];
int maximum(int u, int v, int x)
  for (int i = M - 1; i >= 0; i--)
    if (level[u] - (1 << i) >= level[x]) {
     resp = max(resp, big[u][i]);
      u = P[u][i];
 for (int i = M - 1; i >= 0; i--)
if (level[v] - (1 << i) >= level[x]) {
     resp = max(resp, big[v][i]);
      v = P[v][i];
  return resp:
int minimum(int u, int v, int x)
  for (int i = M - 1; i >= 0; i--)
    if (level[u] - (1 << i) >= level[x]) {
      resp = min(resp, low[u][i]);
      u = P[u][i];
  for (int i = M - 1; i >= 0; i--)
  if (level[v] - (1 << i) >= level[x]) {
     resp = min(resp, low[v][i]);
      v = P[v][i];
  return resp;
```

## 1.5 Bridges and Articulation Points

```
class ponte {
private:
  vvi graph;
  vi usados;
 vi e_articulacao;
  vi dfs_low;
 vi dfs_prof;
vector<ii> pontes;
  int tempo;
public:
  ponte(int N)
    graph.clear();
    graph.resize(N);
    usados.assign(N, 0);
    dfs_low.assign(N, 0);
    dfs_prof.assign(N, 0);
    e_articulacao.assign(N, 0);
    tempo = 0;
  void AddEdge(int u, int v)
    graph[u].pb(v);
    graph[v].pb(u);
  void dfs(int u, int pai)
    usados[u] = 1;
    int nf = 0;
    dfs_low[u] = dfs_prof[u] = tempo++;
    for (int v : graph[u]) {
      if (!usados[v]) {
         dfs(v, u);
         nf++:
         iff (dfs_low[v] >= dfs_prof[u] and pai != -1) e_articulacao[u] = true;
if (pai == -1 and nf > 1) e_articulacao[u] = true;
if (dfs_low[v] > dfs_prof[u]) pontes.pb(mp(u, v));
         dfs_low[u] = min(dfs_low[u], dfs_low[v]);
```

#### 1.6 Eulerian Tour

```
multiset<int> graph[N];
stack<int> path;

// -> It suffices to call dfs1 just
// one time leaving from node 0.

// -> To calculate the path,
// call the dfs from the odd degree node.
// -> O(n * log(n))
void dfs1(int u)

{
   while(graph[u].size())
   {
      int v = *graph[u].begin();
        graph[u].erase(graph[u].begin());
        graph[v].erase(graph[v].find(u));
        dfs1(v);
   }
   path.push(u);
}
```

## 1.7 Floyd Warshall

## 2 Strings

#### 2.1 Aho-Corasick

```
int to[N][M], Link[N], fim[N];
int idx = 1;
void add_str(string &s)
{
   int v = 0;
   for (int i = 0; i < s.size(); i++) {
      if (!to[v][s[i]]) to[v][s[i]] = idx++;
      v = to[v][s[i]];
   }
fim[v] = 1;</pre>
```

```
void process()
   queue<int> fila;
   fila.push(0);
   while (!fila.empty()) {
     int cur = fila.front();
     fila.pop();
     int 1 = Link[cur];
fim[cur] |= fim[1];
for (int i = 0; i < 200; i++) {</pre>
       if (to[cur][i]) {
  if (cur != 0) {
             Link[to[cur][i]] = to[1][i];
            Link[to[cur][i]] = 0;
          fila.push(to[cur][i]);
          to[cur][i] = to[1][i];
int resolve (string &s)
  int v = 0, r = 0;
for (int i = 0; i < s.size(); i++) {
  v = to[v][s[i]];</pre>
     if (fim[v]) r++, v = 0;
   return r;
```

#### 2.2 KMP

```
int b[100000];
int sizet, sizep;
void kmpPreprocess(string &text, string &pattern)
 b[0] = -1;
  while (i < sizep)
    while (j >= 0 and pattern[i] != pattern[j]) j = b[j];
    i++, j++;
    b[i] = j;
void kmpSearch(string &text, string &pattern)
  kmpPreprocess(text, pattern);
  int i = 0, j = 0;
while (i < sizet)</pre>
    while (j >= 0 and text[i] != pattern[j]) j = b[j];
    i++. i++:
    if (j == sizep) {
     cout << "Olha a substring do texto " << i - j << endl;
      j = b[j];
```

### 2.3 Suffix Array

```
sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
         P[level][M[i].second] = (i > 0 \&\& M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i; 
  vi GetSA() {
    vi v=P.back();
    vi ret(v.size());
    for (int i=0; i < v.size(); i++) {</pre>
      ret[v[i]]=i;
    return ret;
  int LCP(int i, int j) {
  int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
         j += 1 << k;
        len += 1 << k;
    return len;
  vi GetLCP(vi &sa)
    vi lcp(sa.size()-1);
    for (int i=0; i < sa.size() -1; i++) {</pre>
      lcp[i]=LCP(sa[i],sa[i+1]);
    return lcp;
};
```

## 2.4 Rabin Karp

```
const 11 M = 1000004099;
const 11 B = 31;
11 int_mod(11 a, 11 b) { return (a % b + b) % b; }
ll eleva(ll a, ll b, ll mod)
   return 1;
  else if (b == 1)
   return a;
  11 \times = eleva(a, b / 2, mod);
  if (b % 2 == 0)
    return (x * x) % mod;
  else
    return (a * ((x * x) % mod)) % mod;
bool Rabin_karp(string text, string pattern)
 int n = text.size();
  int m = pattern.size();
  if (n < m) return false;
  for (int i = 0; i < m; i++) hp = int_mod(hp * B + pattern[i], M);</pre>
  for (int i = 0; i < m; i++) ht = int_mod(ht * B + text[i], M);</pre>
  if (ht == hp) return true;
  11 E = eleva(B, m - 1, M);
  for (int i = m; i < n; i++)
   ht = int_mod(ht - int_mod(text[i - m] * E, M), M);
ht = int_mod(ht * B, M);
    ht = int_mod(ht + text[i], M);
    if (ht == hp) return true;
  return false;
```

## 3 Mathematics

### 3.1 Big Number

```
void zero_esq(string &resp)
{
   string retorno = resp;
```

```
reverse(retorno.begin(), retorno.end());
  int i = resp.size() - 1;
  while (retorno[i] == '0' and i > 0) {
    retorno.erase(i);
  reverse(retorno.begin(), retorno.end());
  resp = retorno;
string sum_big(string a, string b)
  string resp;
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
if (a.size() <= b.size()) {
  int carry = 0;</pre>
    for (int i = 0; i < a.size(); i++) {
  int x = b[i] - '0' + a[i] - '0' + carry;
  resp.push_back((char)(x % 10 + '0'));</pre>
       carry = x / 10;
    for (int i = a.size(); i < b.size(); i++) {</pre>
      int x = b[i] - '0' + carry;
       resp.push_back((char)(x % 10 + '0'));
      carry = x / 10;
    if (carry > 0) resp.push_back((char)(carry + '0'));
  else {
    int carry = 0:
    for (int i = 0; i < b.size(); i++) {
  int x = a[i] - '0' + b[i] - '0' + carry;
       resp.push_back((char)(x % 10 + '0'));
       carry = x / 10;
    for (int i = b.size(); i < a.size(); i++) {
  int x = a[i] - '0' + carry;
  resp.push_back((char)(x % 10 + '0'));</pre>
       carry = x / 10;
    if (carry > 0) resp.push_back((char)(carry + '0'));
  reverse(resp.begin(), resp.end());
  zero esq(resp);
  return resp;
string mul_big(string a, string b)
  string resp;
  resp.push_back('0');
  string temp;
  int carry = 0;
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  for (int i = 0; i < a.size(); i++) {</pre>
    temp.clear();
for (int k = 0; k < i; k++) temp.push_back('0');</pre>
    int x = a[i] - '0';
for (int j = 0; j < b.size(); j++) {
   int y = b[j] - '0';</pre>
      int novo = (x * y + carry);
      temp.push_back((novo % 10) + '0');
      carry = novo / 10;
    if (carry > 0) temp.push_back(carry + '0');
    reverse(temp.begin(), temp.end());
    carry = 0;
    resp = sum_big(temp, resp);
  zero esq(resp);
  return resp;
```

#### 3.2 Chinese Remainder

```
11 mulmod(11 a, 11 b, 11 m)
{
    11 ret = 0;
    while (b > 0) {
        if (b % 2 != 0) ret = (ret + a) % m;
        a = (a + a) % m;
        b >>= 1;
    }
    return ret;
}
11 expmod(11 a, 11 e, 11 m)
{
    11 ret = 1;
```

```
while (e > 0) {
    if (e % 2 != 0) ret = mulmod(ret, a, m);
    a = mulmod(a, a, m);
    e >>= 1;
}
return ret;
}

11 invmul(11 a, 11 m) { return expmod(a, m - 2, m); }
11 chinese(vector<11> r, vector<11> m)
{
    int sz = m.size();
    11 M = 1;
    for (int i = 0; i < sz; i++) {
        M *= m[i];
    }
    ll ret = 0;
    for (int i = 0; i < sz; i++) {
        ret += mulmod(mulmod(M / m[i], r[i], M), invmul(M / m[i], M), M);
        ret = ret % M;
    }
    return ret;</pre>
```

#### 3.3 Matrix Exponentiation

```
vvi matmul(vvi &m1, vvi &m2)
  vvi ans;
  ans.resize(m1.size(), vi(m2.size(), 0));
  for (int i = 0; i < n; i++)
for (int j = 0; j < n; j++)</pre>
      for (int k = 0; k < n; k++) {
   ans[i][j] += m1[i][k] * m2[k][j];</pre>
         ans[i][j] %= MOD;
  return ans;
vvi matpow(vvi &m1, ll p)
  ans.resize(ml.size(), vi(ml.size(), 0));
  for (int i = 0; i < n; i++) ans[i][i] = 1;
  while (p) {
    if (p & 1) ans = matmul(ans, m1);
    m1 = matmul(m1, m1);
    p >>= 1;
// VETOR TEM N LINHAS E A MATRIZ E QUADRADA
vi mulvet(vvi &m1, vi &vet)
  vi ans;
  ans.resize(vet.size(), 0);
  for (int i = 0; i < n; i++)
    for (int i = 0; i < n, i ...,
for (int j = 0; j < n; j++) {
   ans[i] += (m1[i][j] * vet[j]);</pre>
       ans[i] %= MOD;
  return ans;
```

### 3.4 Pascal Triangle

```
unsigned long long comb[61][61];
for (int i = 0; i < 61; i++) {
   comb[i][i] = 1;
   comb[i][0] = 1;
}
for (int i = 2; i < 61; i++)
   for (int j = 1; j < i; j++)
      comb[i][j] = comb[i - 1][j] + comb[i - 1][j - 1];</pre>
```

#### 3.5 Eulers Totient Function

```
int phi(int n)
{
  int result = n;
  for (int i = 2; i * i <= n; ++i)</pre>
```

```
if (n % i == 0) {
  while (n % i == 0) n /= i;
  result -= result / i;
}
if (n > 1) result -= result / n;
return result;
```

#### 3.6 Pollard Rho

```
11 u;
11 t;
const int tamteste=5;
11 abss(11 v) { return v>=0 ? v : -v;}
11 randerson()
  ld pseudo=(ld)rand()/(ld)RAND_MAX;
  return (11) (round((1d) range*pseudo))+1LL;
ll mulmod(ll a, ll b, ll mod)
  11 ret=0;
  while (b>0)
    if(b%2!=0) ret=(ret+a)%mod;
    a=(a+a)%mod;
    b=b/2LL;
  return ret;
11 expmod(11 a, 11 e, 11 mod)
  11 ret=1;
  while (e>0)
    if(e%2!=0) ret=mulmod(ret,a,mod);
    a=mulmod(a,a,mod);
    e=e/2LL;
  return ret:
bool jeova(ll a, ll n)
  11 x = expmod(a, u, n);
  for(int i=0;i<t;i++)</pre>
    x=mulmod(x,x,n);
    if(x==1 and last!=1 and last!=(n-1)) return true;
    last=x;
  if(x==1) return false;
  return true;
bool isprime(ll n)
  u=n-1;
  while (u%2==0)
    u/=2LL;
  if(n==2) return true:
  if(n==3) return true;
  if(n%2==0) return false;
  if(n<2) return false;</pre>
  for(int i=0;i<tamteste;i++)</pre>
    11 v = randerson()%(n-2)+1;
     //cout<<"jeova "<<v<<" "<<n<<endl;
    if(jeova(v,n)) return false;
  return true;
11 gcd(l1 a, l1 b) { return !b ? a : gcd(b,a%b);}
ll calc(ll x, ll n, ll c)
  return (mulmod(x,x,n)+c)%n;
11 pollard(11 n)
```

```
11 d=1;
  11 i=1;
  11 k=1;
  11 x=2;
  11 y=x;
  11 c;
  do
    c=randerson()%n;
  }while(c==0 or (c+2)%n==0);
  while (d!=n)
    if(<u>i</u>==k)
        k *=2LL;
        y=x;
        i=0;
    x=calc(x,n,c);
    d=gcd(abss(y-x),n);
    if(d!=1) return d;
vector<ll> getdiv(ll n)
  vector<ll> ret:
  if(n==1) return ret;
  if(isprime(n))
    ret.pb(n);
    return ret;
  11 d = pollard(n);
  ret=getdiv(d);
  vector<l1> ret2=getdiv(n/d);
  for(int i=0;i<ret2.size();i++) ret.pb(ret2[i]);</pre>
  return ret;
```

#### 3.7 Sieve of Eratosthenes

```
const int MAX = 1e6;
int primes[MAX];
void gen_primes()
{
   int i, j;
   for (i = 2; i*i <= MAX; i++)
        if (primes[i])
        for (j = i; j * i < MAX; j++) primes[i * j] = 0;
}</pre>
```

### 3.8 Extended Euclidean Algorithm

```
struct ext {
  11 x;
   11 mdc;
};
); ext tmp; 

// ax + by=c, se mdc(a,b) nao divide c, nao tem solucao, caso contrario, x = x0 

// // + (b/mdc) *n, y=yo-(a/mdc) *n 

11 ee(11 a, 11 b, 11 &x, 11 &y)
   if (b == 0)

\begin{array}{l}
\mathbf{x} = 1; \\
\mathbf{y} = 0;
\end{array}

      return a;
    11 x1, y1;
    11 tmp = ee(b, a % b, x1, y1);
   x = y1;

y = x1 - (a / b) * y1;
    return tmp;
ext extended_euclid(ll a, ll b)
   ll tmp, tmp1;
   ext ret:
   ret.mdc = ee(a, b, tmp, tmp1);
    ret.x = tmp;
   ret.y = tmp1;
```

```
return ret;
```

#### 3.9 Multiplicative Inverse

## 4 Combinatorial Optimization

#### 4.1 Dinic

```
struct Edge
  int v, rev;
  Edge(int v_, int cap_, int rev_) : v(v_), rev(rev_), cap(cap_) {}
struct MaxFlow {
  vector<vector<Edge> > g;
  vector<int> level;
  queue<int> q;
  int flow, n:
  MaxFlow(int n_) : g(n_), level(n_), n(n_) {}
  void addEdge(int u, int v, int cap)
    if (u == v) return;
    Edge e(v, cap, int(g[v].size()));
    Edge r(u, 0, int(g[u].size()));
    g[u].push_back(e);
    g[v].push_back(r);
  bool buildLevelGraph(int src, int sink)
   fill(level.begin(), level.end(), -1);
   while (not q.empty()) q.pop();
level[src] = 0;
    q.push(src);
    while (not q.empty()) {
     int u = q.front();
      for (auto e = g[u].begin(); e != g[u].end(); ++e) {
       if (not e->cap or level[e->v] != -1) continue;
        level[e->v] = level[u] + 1;
        if (e->v == sink) return true;
        q.push(e->v);
    return false:
  int blockingFlow(int u, int sink, int f)
    if (u == sink or not f) return f;
    for (auto e = g[u].begin(); e != g[u].end(); ++e) {
     if (not e->cap or level[e->v] != level[u] + 1) continue;
      int mincap = blockingFlow(e->v, sink, min(fu, e->cap));
      if (mincap) {
       g[e->v][e->rev].cap += mincap;
        e->cap -= mincap;
        fu -= mincap;
    if (f == fu) level[u] = -1;
    return f - fu;
```

```
int maxFlow(int src, int sink)
{
  flow = 0;
  while (buildLevelGraph(src, sink))
    flow += blockingFlow(src, sink, numeric_limits<int>::max());
  return flow;
};
```

### 4.2 Hopcroft-Karp

```
class MaxMatch {
  vi graph[N];
  int match[N], us[N];
  void addEdge(int u, int v) { graph[u].pb(v); }
  int dfs(int u)
   if (us[u]) return 0;
    us[u] = 1;
    for (int v : graph[u]) {
     if (match[v] == -1 or (dfs(match[v]))) {
       match[v] = u;
        return 1;
    return 0;
  int maxMatch(int n)
    memset(match, -1, sizeof(match));
    int ret = 0;
    for (int i = 0; i < n; i++) {
     memset(us, 0, sizeof(us));
     ret += dfs(i);
    return ret;
```

#### 4.3 Min Cost Max Flow

```
int flow[N][N];
vector<pair<int, int> > g[N];
int n, m, k;
inline int ent(int a) { return a * 2; }
inline int out (int a) { return a * 2 + 1; }
inline void addEdge(int a, int b, int custo, int fluxo)
  flow[a][b] += fluxo;
  g[a] push_back(make_pair(b, custo));
 g[b].push_back(make_pair(a, -custo));
int src = N - 1, tgt = N - 2;
int dis[N], pai[N];
inline int dii()
  memset (dis. INF. sizeof dis);
 memset (pai, -1, sizeof pai);
 priority_queue<pair<int, int> > q;
dis[src] = 0;
  q.push(make_pair(0, src));
  while (!q.empty()) {
    pair<int, int> foo = q.top();
    int x = foo.second, cost = -foo.first;
    if (dis[x] != cost) continue;
    for (int i = 0; i < g[x].size(); ++i) {</pre>
      int y = g[x][i].first, w = g[x][i].second;
      if (flow[x][y] <= 0) continue;</pre>
      if (dis[y] > dis[x] + w) {
  dis[y] = dis[x] + w;
        pai[v] = x;
        q.push(make_pair(-dis[y], y));
```

```
return dis[tgt] != INF;
int minCost()
  int maxFlow = 0;
  int minC = 0;
  while (dij()) {
    int u = tgt;
    int minFlow = TNF:
    while (pai[u] != -1) {
     minFlow = min(minFlow, flow[pai[u]][u]);
     u = pai[u];
    maxFlow += minFlow;
    minC += minFlow * dis[tgt];
    u = tgt;
    while (pai[u] != -1) {
      flow[pai[u]][u] -= minFlow;
      flow[u][pai[u]] += minFlow;
      u = pai[u];
  if (\max Flow != n * k) \min C = -1;
 return minC;
inline void init()
 memset(flow, 0, sizeof flow);
for (int i = 0; i < N; ++i) {</pre>
    g[i].clear();
```

## 4.4 Edmonds Karp

```
struct Edge {
  int at, where;
  ill cap;
  void init(int _at, ll _cap, int _where)
  {
    at = _at, cap = _cap, where = _where;
  }
};
struct dad {
  int at, up, down;
  dad() { at = -1; }
  dad(int _at, int _up, int _down) { at = _at, up = _up, down = _down; }
};
class MaxFlow {
  private:
    vector<vector<Edge> > g;
  ill mf, f;
    int s, t;
    vector<dad> p;

public:
```

```
void augment(int v, ll minEdge)
    if (v == s) {
      f = minEdge;
       return;
    else if (p[v].at != -1) {
      augment(p[v].at, min(minEdge, g[p[v].at][p[v].up].cap));
       g[p[v].at][p[v].up].cap -= f;
      g[v][p[v].down].cap += f;
  void init(int N)
    for (int i = 0; i < g.size(); i++) g[i].clear();
mf = 0, f = 0;</pre>
    g.resize(N);
  void addEdge(int u, int v, 11 cap)
    A.init(v, cap, g[v].size());
    Edge B;
    B.init(u, 0, g[u].size());
    g[u].pb(A);
    g[v].pb(B);
  int maxFlow(int source, int sink)
    s = source;
    t = sink;
    mf = 0;
    while (true) {
       vector<int> dist(g.size(), INF);
       dist[s] = 0;
       queue<int> q;
       q.push(s);
       p.clear();
      p.resize(g.size());
while (!q.empty())
         int u = q.front();
         int u = q.rfolt();
q.pop();
if (u == t) break;
for (int i = 0; i < g[u].size(); i++) {</pre>
           if (dist[prox.at] == INF and prox.cap > 0) {
    dist[prox.at] = dist[u] + 1;
              q.push(prox.at);
             dad paizao(u, i, prox.where);
             p[prox.at] = paizao;
      augment(t, INF);
if (f == 0) break;
      mf += f;
    return mf;
};
```