Computer Vision Mathematics: Homework 3

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1 Linear Algebra Solutions

- 1.7 # 19: A diagonal entry s_{jj} of a symmetric matrix cannot be smaller than all the λ's. Suppose this were false, then $S - s_{jj}I$ would have positive eigenvalues.

  There are two ways we can come to this conclusion.

  1. $S = Q\Lambda Q^T$
     $S - s_{jj}I = Q\Lambda Q^T - s_{jj}I$
     $S - s_{jj}I = Q\Lambda Q^T - Q(s_{jj}I)Q^T$
     $S - s_{jj}I = Q(\Lambda - s_{jj}I)Q^T$
     Let $\Lambda_2 = \Lambda - s_{jj}I$
     $S = Q\Lambda_2 Q^T$
  2. Equation 5 from 1.6: $(A + sI)x = \lambda x + sx = (\lambda + s)x$
     Applying to this problem:$(S - s_{jj}I)x = \lambda x - s_{jj}x = (\lambda - s_{jj})x$

  Therefore

  $$\lambda_i - s_{jj} > 0|_{i=0\ldots n}$$

  If this is the case, then the matrix is Symmetric Positive Definite.

  However, from 1.7 # 18, we know that a symmetric positive definite matrix cannot have a 0 along its diagonal. Therefore, the first statement must be true.