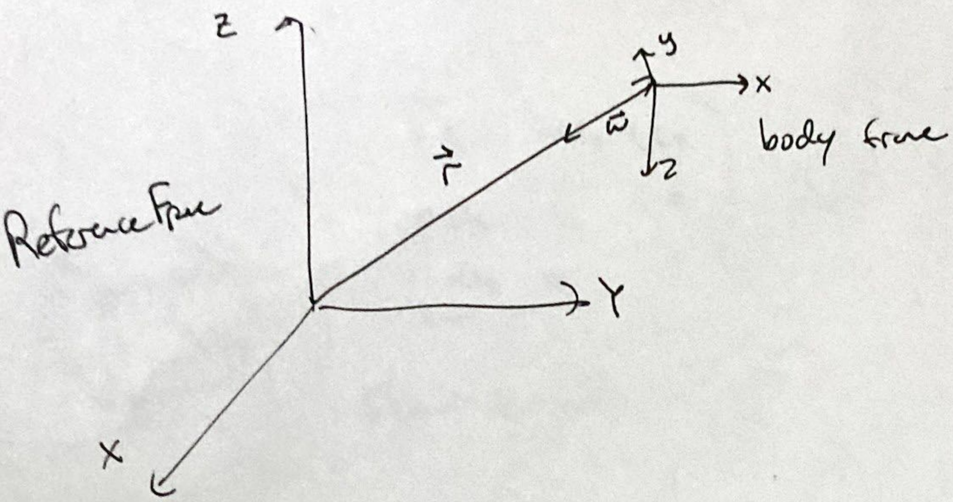
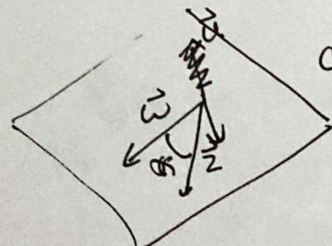
# angle - axis method w/ Quaternions




$\vec{\omega} = \frac{\vec{r}}{\|\vec{r}\|}$ unit vector pointing to origin




$\cos\alpha = \vec{z} \cdot \vec{\omega}$

$\alpha = \cos^{-1}(\vec{z} \cdot \vec{\omega})$

$0 \le \alpha \le \pi$

$$\vec{u} = \frac{\vec{z} \times \vec{\omega}}{\|\vec{z} \times \vec{\omega}\|}$$

$$\vec{q} = \left(\cos\frac{\alpha}{2}\right)\mathbb{1} + \left(\sin\frac{\alpha}{2}\right)\vec{u}$$

$$\vec{z} \times \vec{\omega} = \begin{bmatrix} 0 & -z_3 & z_2 \\ z_3 & 0 & -z_1 \\ -z_2 & z_1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

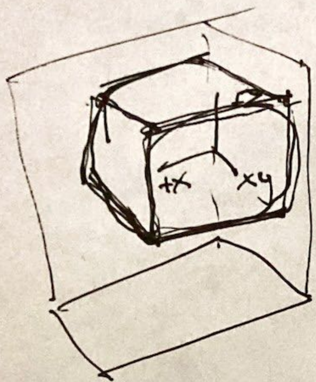
rotate z into $\omega$ :

$$q \vec{z} q^{-1} = \vec{\omega}$$

rotation matrix formulation:

$$\vec{u} = \frac{\vec{z} \times \vec{\omega}}{\|\vec{z} \times \vec{\omega}\|}$$

$$S = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$




skew-symmetric

$$R_z = \vec{\omega} \quad \text{when} \quad R = I + (\sin\alpha)S + (1-\cos\alpha)S^2$$

$$R_z = z + (\sin\alpha)Sz + (1-\cos\alpha)S^2 z$$

$$= z + (\sin\alpha)(\hat{u} \times \vec{z}) + (1-\cos\alpha)$$

$$(u_1 i + u_2 j + u_3 k)$$