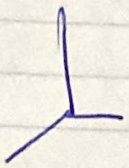
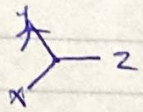
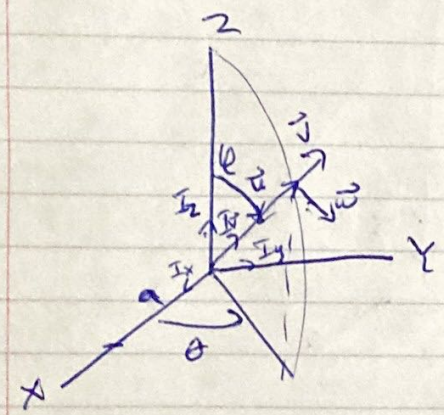
Stable Reference Frame XYZ (world frame)



Camera Reference Frame



* cam looks in Z dir



$\vec{r}$ = position vector of cam in stable frame

$\vec{u} = \frac{-\vec{r}}{|\vec{r}|} = -\hat{i}_r$

$\vec{v} = \hat{i}_\theta$ (direction of increasing θ)

$\omega = \hat{i}_{\varphi}$ (direction of moving φ)

Goal: rotate camera frame so Z axis is aligned with $\vec{u}$, X axis with $\vec{v}$, Y axis with $\vec{\omega}$.

$$\vec{u} = -\hat{i}_r$$
$$= -\sin\varphi\cos\theta\,\hat{i}_x - \sin\varphi\sin\theta\,\hat{i}_y - \cos\varphi\,\hat{i}_z$$

$$\vec{v} = \hat{i}_\theta$$
$$= -\sin\theta\,\hat{i}_x + \cos\theta\,\hat{i}_y$$

$$\vec{\omega} = \hat{i}_{\varphi} = \vec{u}\times\vec{v}$$
$$= \cos\varphi\cos\theta\,\hat{i}_x + \cos\varphi\sin\theta\,\hat{i}_y - \sin\varphi\,\hat{i}_z$$

$\vec{r}_{xy}$ = projection of $\vec{r}$ onto xy plane
$= a\hat{i}_x + b\hat{i}_x$

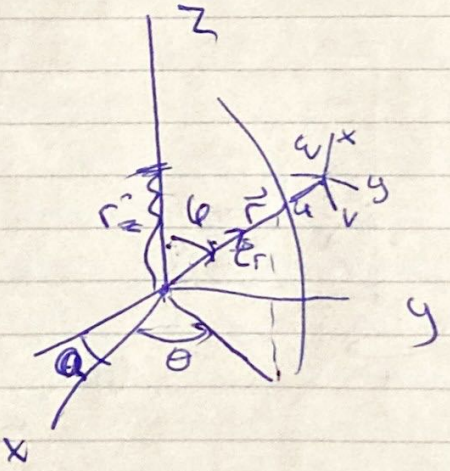
$\tan\theta = b/a$ if $a=0$

$\tan^{-1}x$ returns a value between $-90°$ and $90°$

if $a>0$
$\theta = \tan^{-1}\left(\frac{b}{a}\right) \in (-90, 90)$

if $a<0$

$0 \le \theta \le 2\pi$
$0 \le \varphi \le \pi$



$x = x_r\vec{v} + x_w\vec{w}$
where $x_r = \vec{x}\cdot\vec{v}$
$x_w = \vec{x}\cdot\vec{w}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right) + \pi$

if $a = 0$
if $b>0$, $\theta = \pi/2$
$b<0$, $\theta = -\pi/2$ or $3\pi/2$

$\cos\varphi = \frac{\vec{r}_z}{|\vec{r}|} \Leftrightarrow \varphi = \cos^{-1}\left(\frac{r_z}{|r|}\right.$