3-17-21

II. 2 # 12 ÷ 15, 20, 22

12)

$Ax = b$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

$A^T A \hat{x}$

$$= \begin{bmatrix} 4 & \Sigma t_i \\ \Sigma t_i & \Sigma t_i^2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} \Sigma b_i \\ \Sigma b_i t_i \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 8 & 26 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 36 \\ 112 \end{bmatrix}$$

$4C + 8D = 36$

$8C + 26D = 112$

$10D = 40$

$D = 4$

$\boxed{C = 1, D = 4}$

$36 - 32 = 4C$

$C = 1$

$$\hat{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$p = 1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} = \begin{matrix} 1 \\ 5 \\ 13 \\ 17 \end{matrix}$$

$$b = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix} \qquad e = p - b = \begin{bmatrix} 1 \\ -3 \\ +5 \\ -3 \end{bmatrix}$$

$$||e|| = 1^2 + (-3)^2 + 5^2 + (-3)^2 = \boxed{44}$$

13)

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

$C + 0 \cdot D = 0$
$C + D = 8$
$C + 3D = 8$
$C + 4D = 20$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 13 \\ 17 \end{bmatrix}$$

$C = 1$
$C + D = 5$
$C + 3D = 13$
$C + 4D = 17$

$\hat{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$C = 1, \ D = 4$ ✓

14)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix} = -1 + 3 - 5 + 3 = 0 \ ✓$$

$$\begin{bmatrix} 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix} = 3 - 15 + 12 = 0 \ ✓$$

$\boxed{44 \text{ from\#12}}$

15) $E = \|Ax - b\|^2$

$$= (C + 4D - 20)^2 + (C + 3D - 8)^2 + (C + D - 8)^2 + (C \quad)^2$$

$$\frac{\partial E}{\partial C} = \begin{pmatrix} 2(C+4D-20)(1) + \\ 2(C+3D-8)(1) + \\ 2(C+D-8)(1) + \\ 2(C)(1) \end{pmatrix} = 8C + 16D - 72 = 0$$

$$\frac{\partial E}{\partial D} = \begin{matrix} 2(C+4D-20)(4) + \\ 2(C+3D-8)(3) + \\ 2(C+D-8)(1) + \\ 0 \end{matrix} = 16C + 52D - 224 = 0$$

$/2 \rightarrow$

$$= \begin{cases} 4C + 8D - 36 \\ 8C + 26D - 112 \end{cases}$$

20)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

$$A^T A \hat{x} = A^T b$$

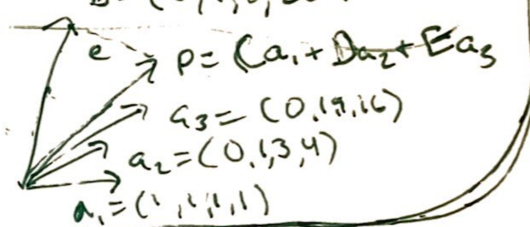
$$\Downarrow$$

$$\begin{bmatrix} 4 & 8 & 26 \\ 8 & 26 & 92 \\ 26 & 92 & 338 \end{bmatrix}\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 72 \\ 224 \\ 800 \end{bmatrix}$$

$$e = \|Ax - b\|^2$$

$$\begin{bmatrix} 2 & 4 & 13 \\ 4 & 13 & 46 \\ 13 & 46 & 169 \end{bmatrix}\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 36 \\ 112 \\ 400 \end{bmatrix}$$

$$= (C)^2 + (C+D+E-8)^2 + (C+3D+9E-8)^2 + (C+4D+16E-20)^2$$



$$\frac{\partial e}{\partial C} = \left(\begin{array}{l} 2(C)(1) + \\ 2(C+D+E-8)(1) + \\ 2(C+3D+9E-8)(1) + \\ 2(C+4D+16E-20)(1) \end{array} \right) = 8C + 16D + 52E - 72 = 0$$

$$\frac{\partial e}{\partial D} = \left(\begin{array}{l} \cancel{2(C)(0)} + \\ 2(C+D+E-8)(1) + \\ 2(C+3D+9E-8)(3) + \\ 2(C+4D+16E-20)(4) + \end{array} \right) = 16C + 52D + 184E - 224 = 0$$

$$\frac{\partial e}{\partial E} = \left(\begin{array}{l} \cancel{2(C)(0)} + \\ 2(C+D+E-8)(1) + \\ 2(C+3D+9E-8)(9) + \\ 2(C+4D+16E-20)(16) \end{array} \right) = 52C + 184D + 676E - 800 = 0$$

$$/2 \Rightarrow$$

$$= \begin{cases} 4C + 8D + 26E - 36 = 0 \\ 8C + 26D + 92E - 112 = 0 \\ 26C + 92D + 338E - 400 = 0 \end{cases}$$

22)

$1 + 2 \cdot 4 = 9$

$9 = 9$ ✓

the average point is on the average line.

m is the count of samples.

Σt_i is the sum of ts

Σb_i is the sum of bs

dividing each value will yeild $1, \frac{\sum^m t_i}{m}, \frac{\sum^n b_i}{m}$

or just $1, \bar{t}, \bar{b}$

the first value of $\hat{x}$ is C, and so

the matrix product yeilds

$$mC + \Sigma t_i D = \Sigma b_i$$

divide all by m,

$$C + \bar{t} D = \bar{b}$$