Homogeneous Transform – the construction used to cast the rotation and translation of the general transform into a single matrix form ~~to operator~~

$${}^A_BT = \begin{bmatrix} {}^A_BR & {}^AP_{Borg} \\ 0 \ 0 \ 0 & 1 \end{bmatrix}$$

$${}^AP = {}^A_BT \cdot {}^BP = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x_{{}^AP_{Borg}} \\ r_{21} & r_{22} & r_{23} & y_{{}^AP_{Borg}} \\ r_{31} & r_{32} & r_{33} & z_{{}^AP_{Borg}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{{}^BP} \\ y_{{}^BP} \\ z_{{}^BP} \\ 1 \end{bmatrix} =$$

Special case of homogeneous transform:

- Translation Operator: we keep orientation the same, but move to another point in space

$${}^AP_2 = TRANS({}^A\hat{Q}, |Q|)\,{}^AP_1$$

$$TRANS(\hat{Q}_1, |Q|) = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation Operator: keep position same, change orientation

$${}^AP_2 = ROT(\hat{K}, \theta)\,{}^AP_1$$

Ex. $$ROT(\hat{z}, \theta) = \begin{bmatrix} \cos\theta, & -\sin\theta & 0, & 0 \\ \sin\theta, & \cos\theta & 0, & 0 \\ 0 & 0 & 1, & 0 \\ 0 & 0 & 0, & 1 \end{bmatrix}$$

Transformation Operator T :
- only 1 coordinate system needed
- T rotates and translates a vector $^{A}P_1$ to $^{A}P_2$

$$^{A}P_2 = T\,^{A}P_1$$

Transformation Arithmetic

~~for multiple transformations or multiple frames, we can see~~

$$\left.\begin{aligned} ^{B}P &= {}^{B}_{C}T\,^{C}P \\ ^{A}P &= {}^{A}_{B}T\,^{B}P \end{aligned}\right\} \longrightarrow \; ^{A}P = {}^{A}_{B}T\,{}^{B}_{C}T\,^{C}P$$

$$^{A}_{C}T = {}^{A}_{B}T\,{}^{B}_{C}T = \left[\begin{array}{ccc|c} & {}^{A}_{B}R\,{}^{B}_{C}R & & {}^{A}_{B}R\,^{B}P_{CORG} + {}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Inverting a Transform
Page 34 (Craig Intro to Robotics)

Roll, Pitch, Yaw

$$^{A}_{B}R_{rpy}(\gamma, \beta, \alpha) = ROT(^{A}\hat{Z}_A, \alpha)\,ROT(^{A}\hat{Y}_A, \beta)\,ROT(^{A}\hat{X}_A, \gamma)$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$