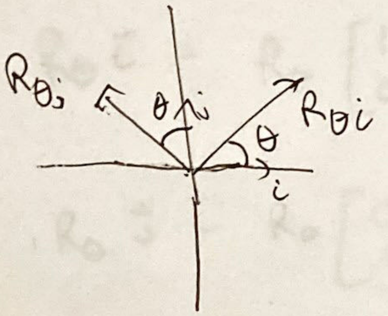
Rotation Matrixes

in 2D



$i = \begin{bmatrix}1\\0\end{bmatrix}$ $j = \begin{bmatrix}0\\1\end{bmatrix}$

R_θ: Matrix that rotates vectors counter clockwise by the angle θ.

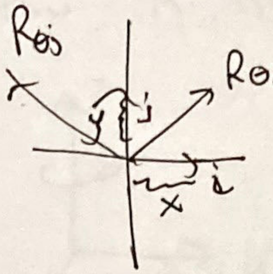
✱ The rotation is determined by what it does to the basis vectors

Given any 2×2 matrix A, the first column of $A = Ai = A\vec{e}_1$

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}a_{11}\\a_{21}\end{bmatrix}$$

the second column of $A =$

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}a_{12}\\a_{22}\end{bmatrix}$$



$$\frac{x}{1} = \cos\theta \Rightarrow x = \cos\theta$$

$$\frac{y}{1} = \sin\theta \Rightarrow y = \sin\theta$$

$$R_\theta \vec{i} = R_\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$R_\theta \vec{j} = R_\theta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

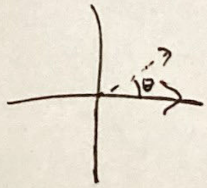
$$R_\theta = \begin{bmatrix} R_\theta \vec{i} & R_\theta \vec{j} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

<u>Example</u>

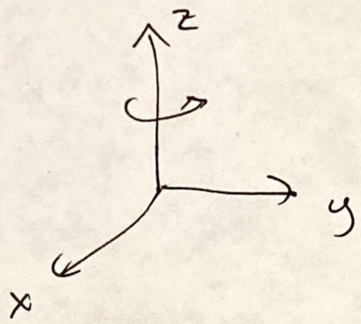
$$\theta = 30^\circ = \frac{\pi}{6}$$

$$R_\theta = \begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$R_\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0.866 \\ 0.5 \end{bmatrix} \rightarrow$$



Rotation in $\mathbb{R}^3$



$R_z(\theta)$ = counter clock-wise rotation about z-axis in XY plane