

IFSMG- PCS

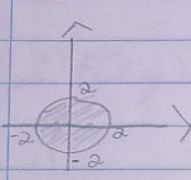
ENGENHARIA DA COMPUTAÇÃO- 3º PERÍODO

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Prova 1

1. Região II



$x^2 + y^2 = 4$   
 $y = \pm \sqrt{4 - x^2}$

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} xy \sqrt{x^2 + y^2} \, dy \, dx \Rightarrow \int_{-2}^2 x \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} y \sqrt{x^2 + y^2} \, dy \, dx \Rightarrow$$
$$\left( x \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{1}{2} \sqrt{t} \, dt \right) \Rightarrow \int_{-2}^2 x \frac{1}{2} \frac{2\sqrt{t}}{3} \bigg|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \Rightarrow \int_{-2}^2 x \frac{\sqrt{x^2 + y^2} (x^2 + y^2)}{3} \bigg|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \Rightarrow$$
$$\frac{x \sqrt{x^2 + 4} (x^2 + 4 - x^2)}{3} - \frac{x \sqrt{x^2 + 4} (x^2 + 4 - x^2)}{3} \Rightarrow \frac{x \sqrt{4} (4)}{3} \Rightarrow$$
$$\frac{x \sqrt{4} (4)}{3} \Rightarrow \frac{8x}{3} - \frac{8x}{3} \Rightarrow 0 //$$

Região III

$$\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} xy \sqrt{x^2 + y^2} \, dx \, dy \Rightarrow \int_{-2}^2 y \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x \sqrt{x^2 + y^2} \, dx \Rightarrow$$
$$\int_{-2}^2 y \frac{1}{2} \sqrt{t} \, dt \Rightarrow \int_{-2}^2 y \frac{1}{2} \frac{2\sqrt{t}}{3} \bigg|_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \Rightarrow \int_{-2}^2 y \frac{\sqrt{4-y^2} (4 - y^2)}{3} \bigg|_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \Rightarrow$$

$$\int_{-2}^2 \frac{y}{2} \frac{2(x^2+y^2)\sqrt{x^2+y^2}}{3} \frac{\sqrt{4-y^2}}{-\sqrt{4-y^2}} dy \Rightarrow \int_{-2}^2 \frac{y\sqrt{x^2+y^2}(x^2+y^2)}{3} \frac{\sqrt{4-y^2}}{-\sqrt{4-y^2}} dy \Rightarrow$$

$$\frac{y\sqrt{y^2+1-y^2}(y^2+1-y^2)}{3} - \frac{y\sqrt{y^2+1-y^2}(y^2+1-y^2)}{3} \Rightarrow \frac{y\sqrt{4}\cdot 1}{3} - \frac{y\sqrt{4}\cdot 1}{3}$$

$$\frac{4\sqrt{4}\cdot 1}{3} - \frac{4\sqrt{4}\cdot 1}{3} \Rightarrow \frac{8\sqrt{4}}{3} - \frac{8\sqrt{4}}{3} = 0 //$$

Região IV

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n \cos\theta \cdot n \sin\theta \sqrt{n^2 \cos^2\theta + n^2 \sin^2\theta} \cdot n \, d\theta \, r \Rightarrow \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n^3 \cos\theta \sin\theta \sqrt{n^2 \cos^2\theta + n^2 \sin^2\theta} \, dr \, d\theta$$

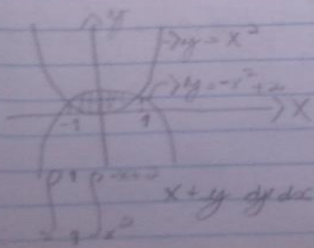
$$\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n^3 \cos\theta \sin\theta \sqrt{n^2} \, dr \, d\theta \Rightarrow \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n^3 \cos\theta \sin\theta \sqrt{n^2} \, dr \, d\theta \Rightarrow$$

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n^3 \cos\theta \sin\theta \cdot n \, dr \, d\theta \Rightarrow \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} n^4 \cos\theta \sin\theta \, dr \, d\theta \Rightarrow \left. \frac{n^4}{4} \sin^2\theta \right|_{-\pi/2}^{\pi/2} = 0$$

$$\frac{2^4}{4} - \frac{2^4}{4} = 0$$

2.

Região II



$$x^2 = -y^2 + 2 \Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$$

$$\int_{-1}^1 \int_{x^2}^{-x^2+2} (x+y) \, dy \, dx \Rightarrow \int_{-1}^1 \left[ xy + \frac{y^2}{2} \right]_{x^2}^{-x^2+2} dx \Rightarrow$$

$$\int_{-1}^1 \left[ \frac{x(-x^2+2) + (-x^2+2)^2}{2} \right] - \left[ \frac{(x \cdot x^2) + (x^2)^2}{2} \right] dx \Rightarrow$$

$$\int_{-1}^1 \left( \frac{-x^3+2x+4+x^4-4x^2}{2} \right) - \frac{x^3-x^4}{2} dx \Rightarrow \int_{-1}^1 (-2x^3-2x^2+2x+2) dx$$

$$\left[ \frac{-2x^4}{4} - \frac{2x^3}{3} + \frac{2x^2}{2} + 2x \right] \Big|_{-1}^1 \Rightarrow \left( \frac{-2}{4} - \frac{2}{3} + 1 + 2 \right) - \left( \frac{2}{4} + \frac{2}{3} + 1 - 2 \right)$$

$$\frac{-2}{4} + 2 - \frac{2}{3} + 2 = \frac{2 \times 3}{3} + \frac{2}{3} \Rightarrow \frac{8}{3}$$

### Região III

$$f(x,y) = x+y$$

$$y = x^2 \Rightarrow x = \pm \sqrt{y}$$

$$y = -x^2+2 \Rightarrow x = \pm \sqrt{2-y}$$

$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} x+y \, dx \, dy + \int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} x+y \, dx \, dy \Rightarrow$$

$$\int_0^1 \left[ \frac{x^2}{2} + yx \right]_{-\sqrt{y}}^{\sqrt{y}} + \int_1^2 \left[ \frac{x^2}{2} + yx \right]_{-\sqrt{2-y}}^{\sqrt{2-y}} dy \Rightarrow$$

$$\int_0^1 \left( \frac{y}{2} + y\sqrt{y} \right) - \left( \frac{y}{2} - y\sqrt{y} \right) dy + \int_1^2 \left( \frac{2-y}{2} + y\sqrt{2-y} \right) - \left( \frac{2-y}{2} - y\sqrt{2-y} \right) dy$$

$$\int_0^1 y\sqrt{y} + y\sqrt{y} \, dy + \int_1^2 y\sqrt{2-y} + y\sqrt{2-y} \, dy \Rightarrow \int_0^1 2y\sqrt{y} \, dy + \int_1^2 2y\sqrt{2-y} \, dy$$

$$\int_0^1 2y \cdot y^{\frac{1}{2}} \, dy + 2 \int_1^2 -(-u+2)\sqrt{y} \, du \quad \boxed{u=2-y}$$

$$\int_0^1 2y^{\frac{3}{2}} \, dy + 2 \left( - \int_0^1 (-u+2)\sqrt{u} \, du \right) \Rightarrow$$

$$\frac{4}{5} \cdot 1 - \left( \frac{4}{5} \cdot 0 \right) + 2 \left( - \int_0^1 -u^{\frac{3}{2}} + \int_0^1 2\sqrt{u} \right) - 2u^{\frac{1}{2}}$$

$$\frac{4}{5} + 2 \left( - \left[ \frac{2}{5} u^{\frac{5}{2}} \right]_0^1 + \left[ \frac{2 \cdot 2}{3} u^{\frac{3}{2}} \right]_0^1 \right)$$

$$\frac{4}{5} + 2 \left( \frac{-2}{5} - 0 + \frac{4}{3} - 0 \right) \Rightarrow \frac{4}{5} + 2 \left( \frac{2}{5} + \frac{4}{3} \right) \Rightarrow \frac{4}{5} + 2 \left( \frac{14}{15} \right) \Rightarrow \frac{4+28}{15}$$

$$\frac{12+28}{15} \Rightarrow \frac{40}{15} = \frac{8}{3}$$