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Simplified modelling for the simulation of bolted and/or bonded joints.

Macro-element technique.

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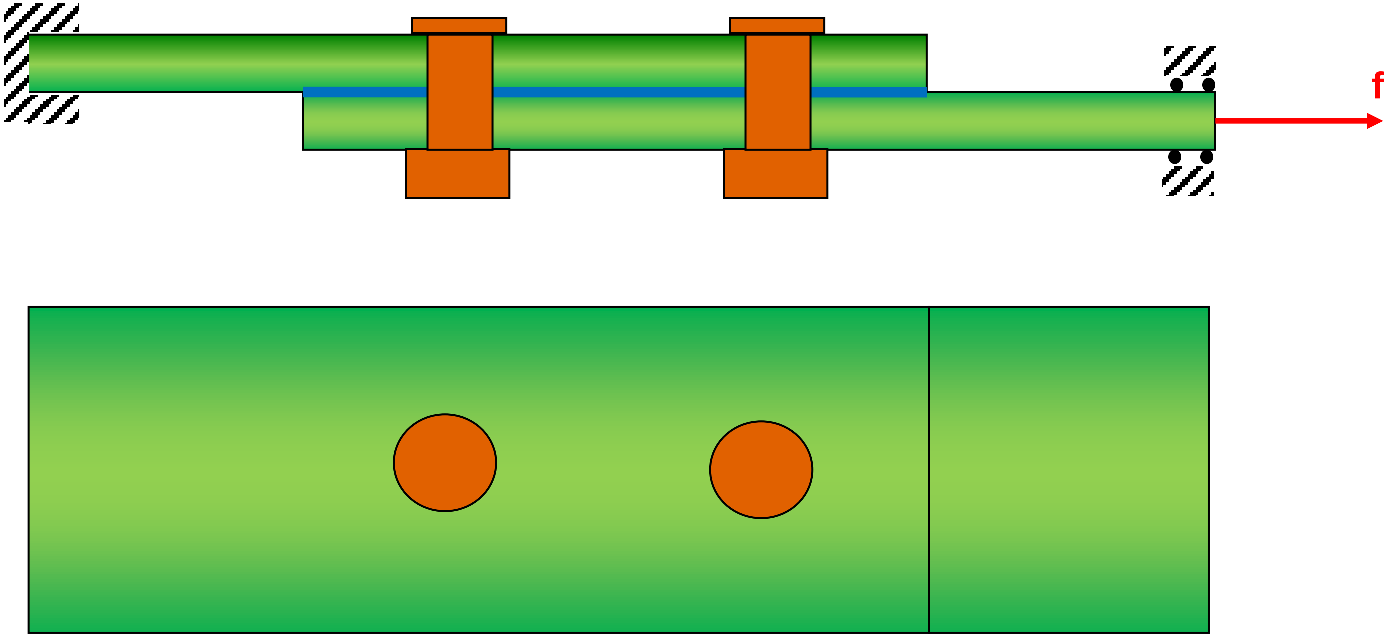
CO-WORKERS

The work presented is the result of numerous contributions from several co-workers, among them:

- Pr. Marc Sartor (INSA Toulouse)
- Pr. Jacques Huet (ISAE-SUPAERO)
- Pr. Frédéric Lachaud (ISAE-SUPAERO)
- Sébastien Schwartz (Sogeti High Tech)
- Anthony Da Veiga (Sogeti High Tech)
- Dr. Guillaume Lélias (Sogeti High Tech, ISAE-SUPAERO)
- Pr. Joseph Morlier (ISAE-SUPAERO)

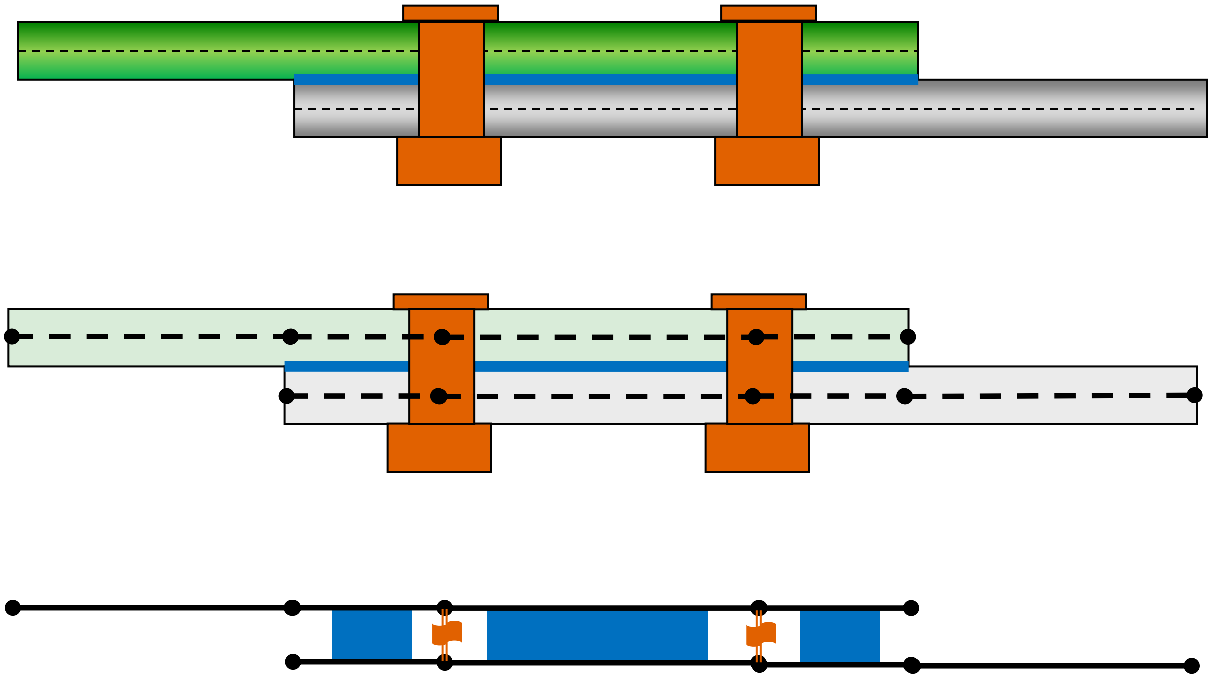
ORIGIN

Which **simplified model** to **quickly and accurately** assess the load transfer within a **hybrid (bolted / bonded) joint** in-plane loaded?



ORIGIN

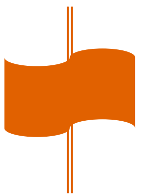
Pr. Marc Sartor (INSA Toulouse) suggested to model the joints with special elements, termed macro-elements (**ME**). **(Paroissien, 2006) (Paroissien et al., 2007a) (Paroissien et al., 2007b)**



SIMPLIFIED MODELLING

ORIGIN

Pr. Marc Sartor (INSA Toulouse) suggested to model the joints with special elements, termed macro-elements (**ME**).

kinematics			
1D-bar	bar element	**bonded-bars** element (**BBa** element)	2 DoF fastener element
1D-beam	beam element	**bonded-beams** element (**BBe** element)	6 DoF fastener element
elementary stiffness matrix	$\boldsymbol{K_{barre}}$ $\boldsymbol{K_{poutre}}$	$\boldsymbol{K_{BBa}}$ $\boldsymbol{K_{BBe}}$	$\boldsymbol{K_{F\_3}}$ $\boldsymbol{K_{F\_6}}$

SIMPLIFIED MODELLING

ORIGIN

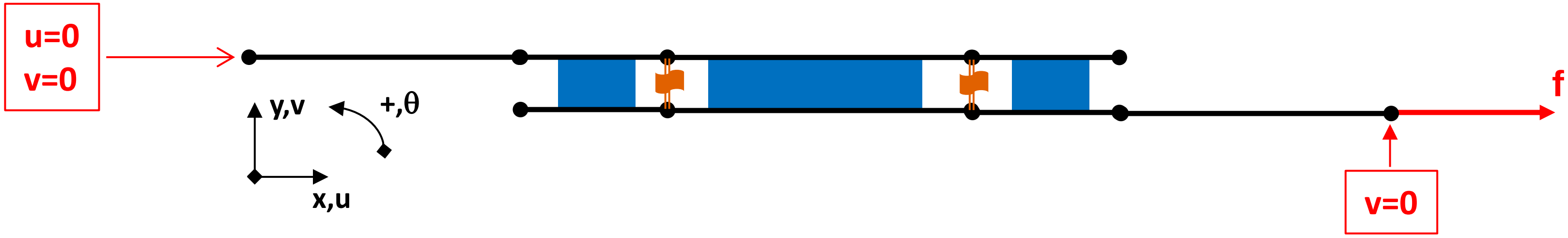
The methodology consists in:

1. **to assemble the structural stiffness matrix** from the elementary stiffness matrix $\boldsymbol{K_s}$

2. **to apply the boundary conditions**

3. **to minimize the potential energy** leading to the linear system $\boldsymbol{F_s = K_s U_s}$

The main difficulty is then the formulation of elementary stiffness matrices.

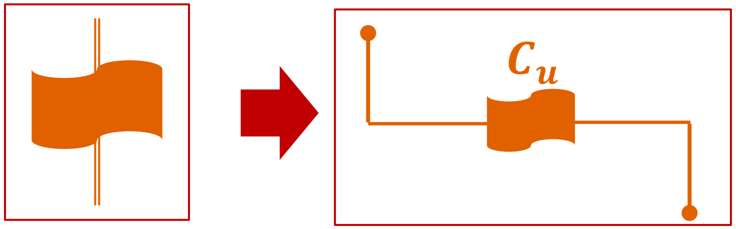


ELEMENTARY STIFFNESS MATRIX

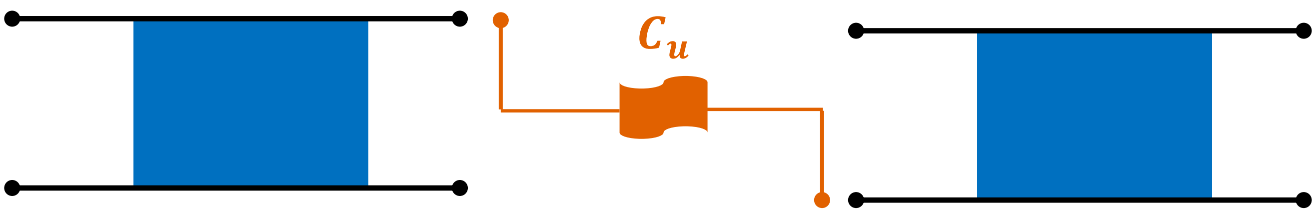
The elementary stiffness matrix of fastener macro-element depends on the chosen kinematics.

1D-bar	Shear spring

This modelling corresponds to the classical one used for the simplified stress analysis of bolted joint under 1D-bar kinematics (Ross, 1947).



$$K_{F-2} = C_u \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

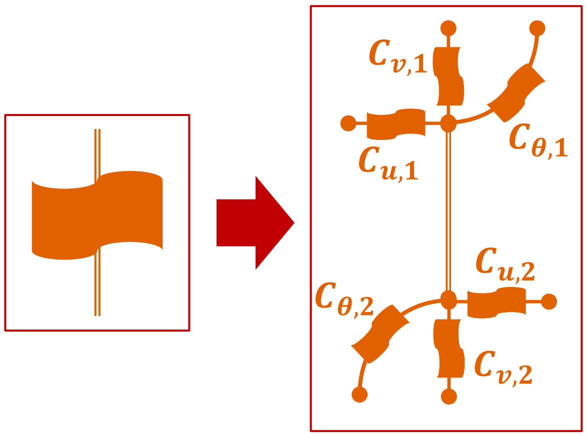


FASTENER MACRO-ELEMENT

ELEMENTARY STIFFNESS MATRIX

The elementary stiffness matrix of fastener macro-element depends on the chosen kinematics.

1D- beam	6 springs + 1 RBE

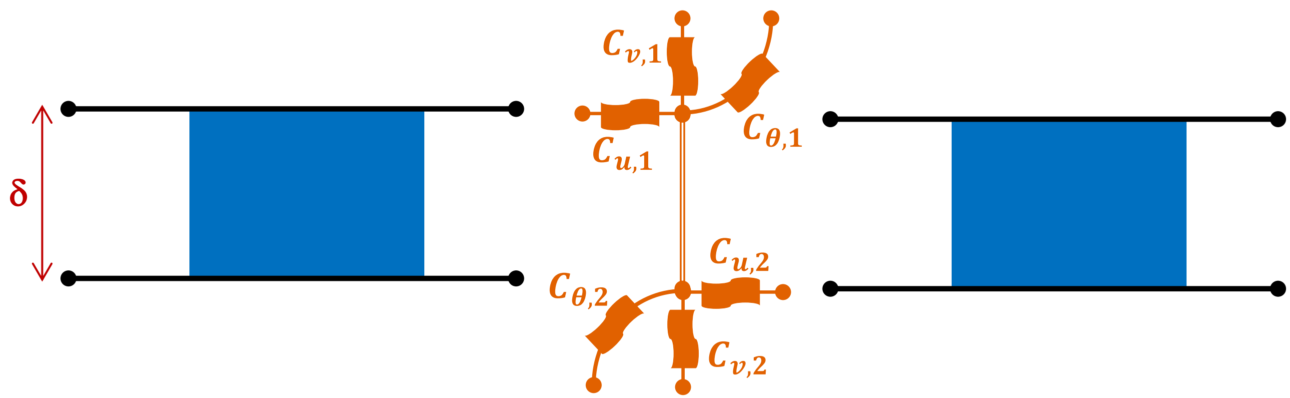


symmetrical fastener

$$K_{F-6} = \frac{1}{k}\begin{pmatrix} 2C_uC_\theta & -2C_uC_\theta & 0 & 0 & -\delta C_uC_\theta & -\delta C_uC_\theta \\ -2C_uC_\theta & 2C_uC_\theta & 0 & 0 & \delta C_uC_\theta & \delta C_uC_\theta \\ 0 & 0 & kC_v & -kC_v & 0 & 0 \\ 0 & 0 & -kC_v & kC_v & 0 & 0 \\ -\delta C_uC_\theta & \delta C_uC_\theta & 0 & 0 & 2C_\theta{}^2 + \delta^2 C_uC_\theta & -2C_\theta{}^2 \\ -\delta C_uC_\theta & \delta C_uC_\theta & 0 & 0 & 2C_\theta{}^2 & 2C_\theta{}^2 + \delta^2 C_uC_\theta \end{pmatrix}$$

(Paroissien et al., 2017)

$$\boldsymbol{k = 2C_\theta + \frac{\delta^2}{2} C_u}$$



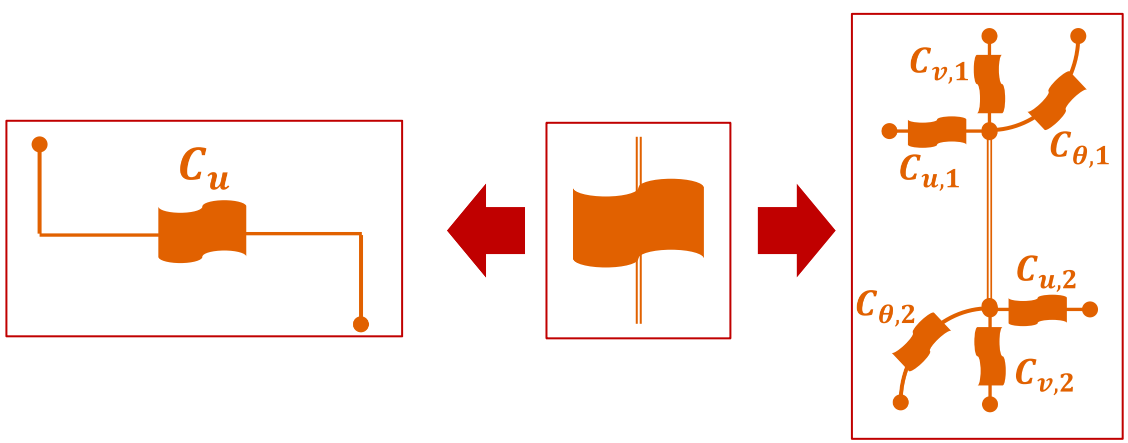
FASTENER MACRO-ELEMENT

ELEMENTARY STIFFNESS MATRIX

The elementary stiffness matrices of fastener macro-element depend on fastener stiffness C_u, C_v and C_θ.

In the literature, it exists a large number of formulae to compute these stiffnesses such as:
(Tate and Rosenfeld, 1946) (Swift, 1984) (Huth, 1986) (Cope and Lacy, 2000) (Morris, 2004)

These fastener stiffnessses are regarded as global parameters representing for several local phenomenon.
They can be assessed from experimental and numerical tests, in a tailored application field.



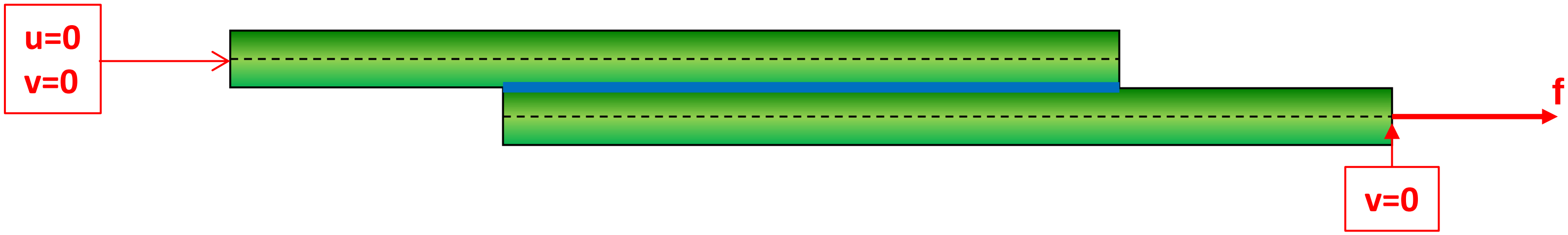
ELEMENTARY STIFFNESS MATRIX

A – Preamble

In the literature, it exists a large numbers of closed-form formulae to accurately predict the adhesive stress distribution along the overlap., such as detailed in: **(van Ingen and Volt, 1993) (Tsaï and Morton, 1994) (da Silva et al. 2009)**.

➢ it means that the choice of simplified hypotheses on which are based these formulae is judicious.

Nevertheless, the application field of these formulae is in general quite restricted. For example, Goland and Reissner provided in 1944 **(Goland and Reissner, 1944)** the adhesive shear and peel stress distribution for a balanced, simply supported, in plane loaded lap joint. But, there is not any more closed-form solution if the adherends are dissimilar or if the joint is clamped….



ELEMENTARY STIFFNESS MATRIX

A – Preamble

The judicious simplified hypotheses lead to a system of ODEs composed of the constitutive equations and of the local equilibrium equations.

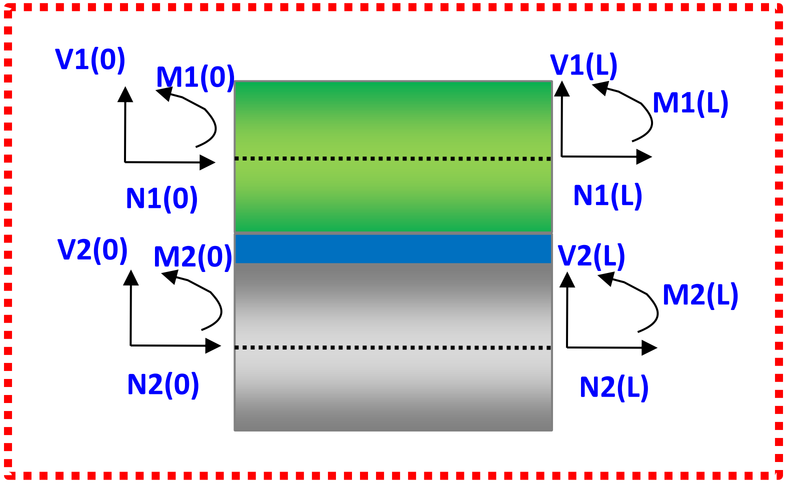
This system can be difficult to be solved in particular :

- when the **adherends** are **dissimilar**
- when the **boundary conditions** are given
- when the material are **non linear**

Goland and Reissner introduced a methodology for the stress analysis of bonded joints referred to **sandwich-type analysis**. It consists in analyzing the bonded overlap as function of internal loads and/or displacements joint extremities.

The adhesive layer is seen as **bed of springs** linked to the adherends **surface modelled as plates**.

Moreover, in the solution by Goland and Reissner takes into account for the **non linear geometrical effect** induced by the lag of the **neutral line**.



ELEMENTARY STIFFNESS MATRIX

A – Preamble

As a result, following Goland and Reissner, several other analyses have been published **to enlarge the application field**. Most of them used the sandwich type analysis, such as: **(Hart-Smith 1973b) (Williams 1975) (Bigwood and Crocombe 1989) (Oplinger 1991) (Tsaï et al. 1998) (Högberg 2004) (Nemes and Lachaud 2009) (Luo et Tong 2009) (Weißgraeber et al. 2014)**.

Other simplified stress analyses of bonded joints were published by modelling the adherends and the adhesive layer such as **2D continuum media** **(Renton and Vinson, 1977) (Allman, 1977) (Adams and Mallick, 1992)**. This type of analysis allows for the compliance of the **free stress state** at overlap ends.

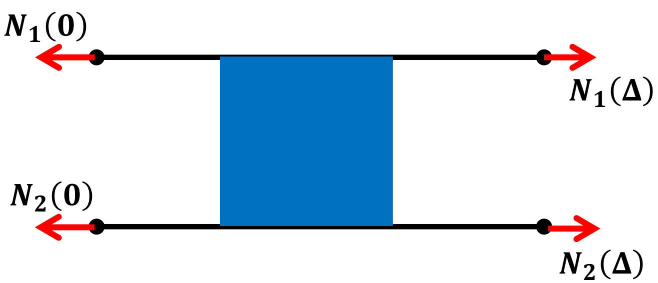
Nevertheless, in order to solve the system of ODEs in a large application field, **particular resolution schemes** have to used such as:

- « multisegment integration method » **(Mortensen, 1997)**
- the **macro-element technique** **(Gustafson et al., 2006) (Paroissien, 2006)**

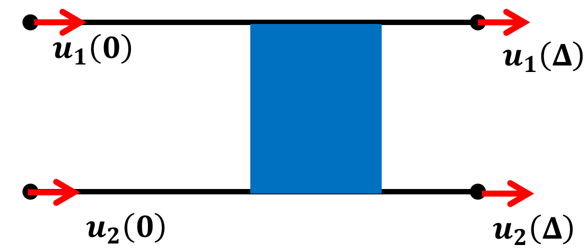
ELEMENTARY STIFFNESS MATRIX

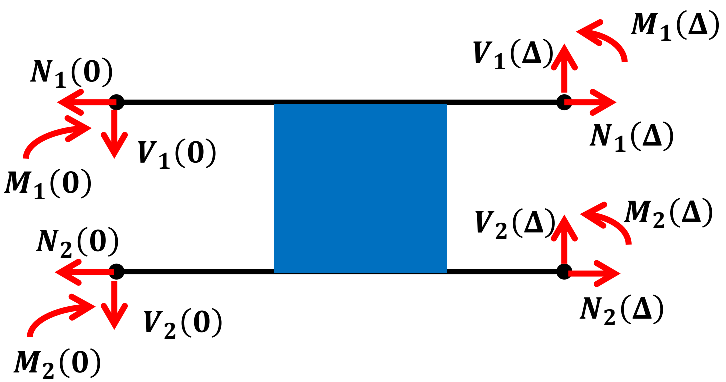
B – Principle

The elementary stiffness matrix for a bonded overlap expresses the relationships between the nodal displacements and the nodal forces.

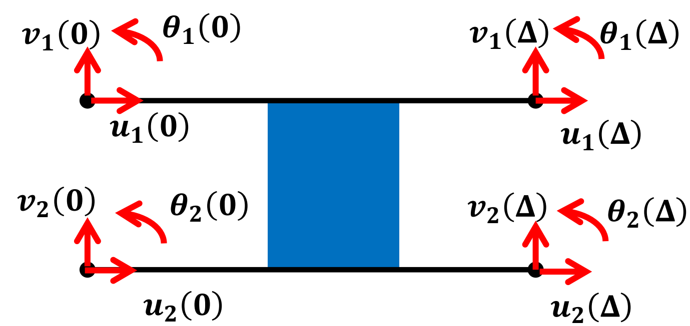


$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \end{pmatrix} = K_{BC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \end{pmatrix}$$





$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \\ -V_1(0) \\ -V_2(0) \\ V_1(\Delta) \\ V_2(\Delta) \\ -M_1(0) \\ -M_2(0) \\ M_1(\Delta) \\ M_2(\Delta) \end{pmatrix} = K_{PC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \\ v_1(0) \\ v_2(0) \\ v_1(\Delta) \\ v_2(\Delta) \\ \theta_1(0) \\ \theta_2(0) \\ \theta_1(\Delta) \\ \theta_2(\Delta) \end{pmatrix}$$

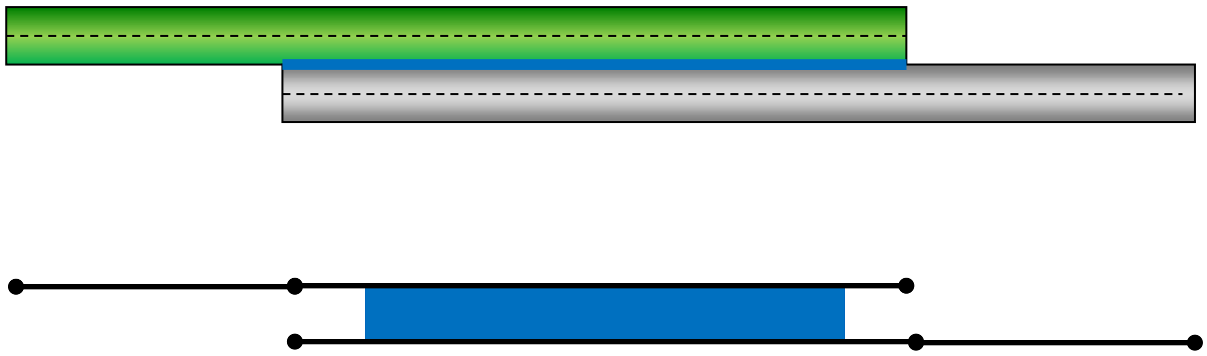


ELEMENTARY STIFFNESS MATRIX

B – Principle

Contrary to the classical FE, **the shape of interpolation functions is not assumed** a priori. The shape of interpolation functions has the shape of functions solving the ODEs.

One significant consequence is that only one ME is needed to model an entire bonded overlap (linear elastic analysis). The displacements, internal forces and adhesive stresses are obtained at any abscissa of the overlap.



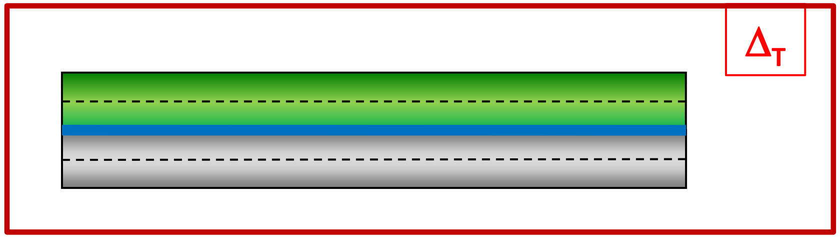
- **6 nodes**
- **6 dof in 1D-bar**
- **18 dof in 1D-beam**

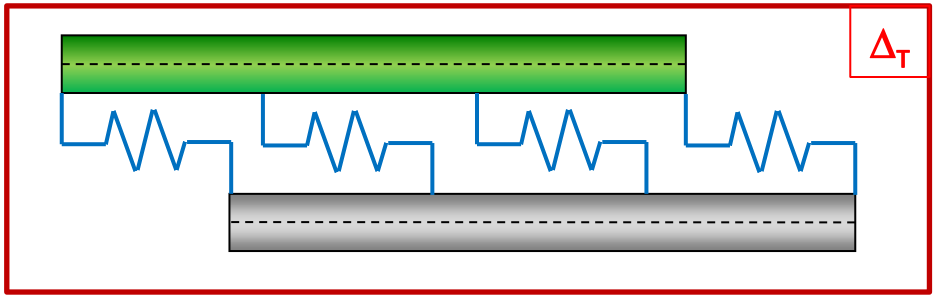
ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

The hypotheses are:

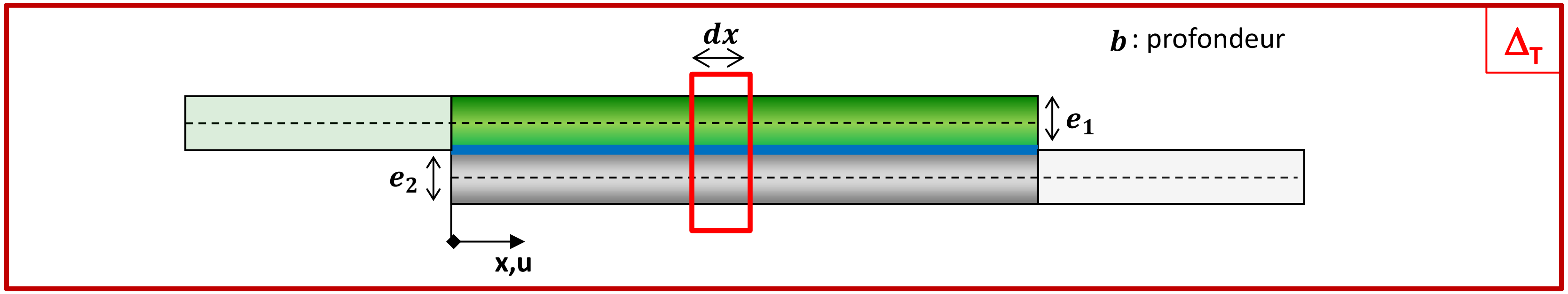
- homogeneous linear elastic material behavior
- local equilibrium of Volkersen **(Volkersen, 1938)**
- the adherends are modelled as bars, with eventually a linear variation of the shear stress with the thickness **(Tsaï et al., 1998)**
- the adhesive layer is modelled as a bed of shear springs
- the adhesive thickness $\mathbf{e_a}$ is constant
- mechanical loading and application of a uniform temperature variation Δ_T



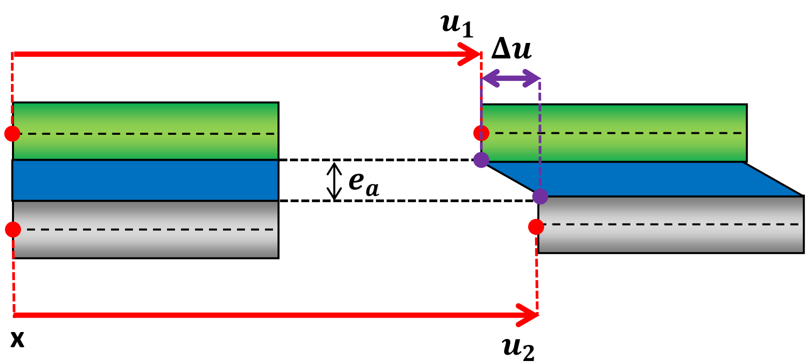


ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

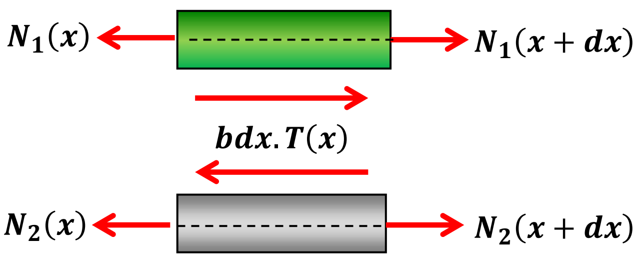


constitutive equations



$$\frac{du_j}{dx} = \frac{N_j}{A_j} + \alpha_j \Delta_T \qquad A_j = E_j b e_j \qquad T = \frac{G_a}{e_a}\Delta u$$

local equilibrium of Volkersen



$$\frac{dN_1}{dx} = -bT \qquad \frac{dN_2}{dx} = bT$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

From the constitutive equations and the local equilibrium equations, a system of ODE's in the adherend longitudinal displacements can be written and solved.

$$\begin{cases} \dfrac{dN_j}{dx} = (-1)^j bT, j = 1,2 \\ T = \dfrac{G_a}{e_a}(u_2 - u_1) \\ \dfrac{du_j}{dx} = \dfrac{N_j}{A_j} + \alpha_j \Delta_T, j = 1,2 \end{cases} \Rightarrow \begin{cases} \dfrac{d^2u_1}{dx^2} + \dfrac{G_a}{e_a}\dfrac{1}{e_1E_1}(u_2 - u_1) = 0 \\ \dfrac{d^2u_2}{dx^2} + \dfrac{G_a}{e_a}\dfrac{1}{e_2E_2}(u_2 - u_1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} u_1 = \dfrac{1}{2}[c_1 + c_2x - c_3(1+\chi)e^{-\eta x} - c_4(1+\chi)e^{\eta x}] \\ u_2 = \dfrac{1}{2}[c_1 + c_2x + c_3(1-\chi)e^{-\eta x} + c_4(1-\chi)e^{\eta x}] \end{cases}$$

$$\eta^2 = \frac{G_a}{e_a}\left(\frac{1}{e_1E_1} + \frac{1}{e_2E_2}\right) \qquad \chi = \frac{\dfrac{1}{e_1E_1} - \dfrac{1}{e_2E_2}}{\dfrac{1}{e_1E_1} + \dfrac{1}{e_2E_2}}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

The 4 integration constants $\mathbf{c_i}$ can then be expressed as functions of the 4 nodal displacements.

$$\begin{cases} \boldsymbol{u_1 = \frac{1}{2}[c_1 + c_2 x - c_3(1+\chi)e^{-\eta x} - c_4(1+\chi)e^{\eta x}]} \\ \boldsymbol{u_2 = \frac{1}{2}[c_1 + c_2 x + c_3(1-\chi)e^{-\eta x} + c_4(1-\chi)e^{\eta x}]} \end{cases} \Rightarrow \boldsymbol{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = M_e^{-1} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\varDelta) \\ u_2(\varDelta) \end{pmatrix}} \Leftrightarrow \boldsymbol{C = M_e^{-1} U_e}$$

$$\boldsymbol{M_e^{-1} = \begin{pmatrix} (1-\chi) & (1+\chi) & 0 & 0 \\ -\frac{(1-\chi)}{\varDelta} & -\frac{(1+\chi)}{\varDelta} & \frac{(1-\chi)}{\varDelta} & \frac{(1+\chi)}{\varDelta} \\ -\frac{e^{\eta\varDelta}}{2\sinh\eta\varDelta} & \frac{e^{\eta\varDelta}}{2\sinh\eta\varDelta} & \frac{1}{2\sinh\eta\varDelta} & -\frac{1}{2\sinh\eta\varDelta} \\ \frac{e^{-\eta\varDelta}}{2\sinh\eta\varDelta} & -\frac{e^{\eta-\varDelta}}{2\sinh\eta\varDelta} & -\frac{1}{2\sinh\eta\varDelta} & \frac{1}{2\sinh\eta\varDelta} \end{pmatrix}}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

From the constitutive equation of adherends, the normal forces can be deduced as functions of integration constants:

$$\begin{cases} \dfrac{N_1}{A_1} = \dfrac{du_1}{dx} - \alpha_1 \Delta_T \\ \dfrac{N_2}{A_2} = \dfrac{du_2}{dx} - \alpha_2 \Delta_T \end{cases} \Rightarrow \begin{cases} N_1(x) = \dfrac{1}{2}\left[c_2 + c_3 \eta (1+\chi) e^{-\eta x} - \eta c_4 (1+\chi) e^{\eta x}\right] A_1 - A_1 \alpha_1 \Delta_T \\ N_2(x) = \dfrac{1}{2}\left[c_2 - c_3 \eta (1-\chi) e^{-\eta x} + \eta c_4 (1-\chi) e^{\eta x}\right] A_2 - A_2 \alpha_2 \Delta_T \end{cases}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

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$$\begin{cases} \dfrac{N_1}{A_1} = \dfrac{du_1}{dx} - \alpha_1 \Delta_T \\ \dfrac{N_2}{A_2} = \dfrac{du_2}{dx} - \alpha_2 \Delta_T \end{cases} \Rightarrow \begin{cases} N_1(x) = \dfrac{1}{2}\left[c_2 + c_3\eta(1+\chi)e^{-\eta x} - \eta c_4(1+\chi)e^{\eta x}\right]A_1 - A_1\alpha_1\Delta_T \\ N_2(x) = \dfrac{1}{2}\left[c_2 - c_3\eta(1-\chi)e^{-\eta x} + \eta c_4(1-\chi)e^{\eta x}\right]A_2 - A_2\alpha_2\Delta_T \end{cases}$$

The nodal forces are then expressed as functions of integration constants such as:

$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \end{pmatrix} + \begin{pmatrix} -A_1\alpha_1 \\ -A_2\alpha_2 \\ A_1\alpha_1 \\ A_2\alpha_2 \end{pmatrix}\Delta_T = N_e \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \Leftrightarrow F_e + F_{therm} = N_e C$$

$$N_e = \frac{1}{2}\begin{pmatrix} 0 & -A_1 & -\eta(1+\chi)A_1 & \eta(1+\chi)A_1 \\ 0 & -A_2 & \eta(1-\chi)A_2 & -\eta(1-\chi)A_2 \\ 0 & A_1 & \eta(1+\chi)e^{-\eta\Delta}A_1 & -\eta(1+\chi)e^{\eta\Delta}A_1 \\ 0 & A_2 & -\eta(1-\chi)e^{-\eta\Delta}A_2 & \eta(1-\chi)e^{\eta\Delta}A_2 \end{pmatrix}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

For the 1D-bar kinematics, it is then possible to obtain the expressions for the components of the elementary stiffness matrix $\mathbf{K_{BBa}}$ **(Paroissien, 2006) (Paroissien, 2007a)**.

$$C = M_e^{-1} U_e$$

$$F_e + F_{therm} = N_e C$$

$$F_e + F_{therm} = \underbrace{\boxed{N_e M_e^{-1}}}_{K_{BBa}} U_e$$

$$K_{BBa} = \frac{1}{1+\chi_A}\frac{A_2}{\Delta}\begin{pmatrix} \frac{\eta\Delta}{\tanh\eta\Delta}+\frac{1}{\chi_A} & 1-\frac{\eta\Delta}{\tanh\eta\Delta} & -\frac{\eta\Delta}{\sinh\eta\Delta}-\frac{1}{\chi_A} & \frac{\eta\Delta}{\sinh\eta\Delta}-1 \\ 1-\frac{\eta\Delta}{\tanh\eta\Delta} & \frac{\eta\Delta}{\tanh\eta\Delta}+\chi_A & \frac{\eta\Delta}{\sinh\eta\Delta}-1 & -\frac{\eta\Delta}{\sinh\eta\Delta}-\chi_A \\ -\frac{\eta\Delta}{\sinh\eta\Delta}-\frac{1}{\chi_A} & \frac{\eta\Delta}{\sinh\eta\Delta}-1 & \frac{\eta\Delta}{\tanh\eta\Delta}+\frac{1}{\chi_A} & 1-\frac{\eta\Delta}{\tanh\eta\Delta} \\ \frac{\eta\Delta}{\sinh\eta\Delta}-1 & -\frac{\eta\Delta}{\sinh\eta\Delta}-\chi_A & 1-\frac{\eta\Delta}{\tanh\eta\Delta} & \frac{\eta\Delta}{\tanh\eta\Delta}+\chi_A \end{pmatrix}$$

$$\chi_A = \frac{A_2}{A_1}$$

SOLUTION

C – 1D-bar

The resolution of $\boldsymbol{F_S = K_S U_S}$ allows for the determination of the nodal displacement vector: $\boldsymbol{U_S}$

For each ME, the vector of nodal displacement is then known: $\boldsymbol{U_e}$

Thus , for each ME, the integration constants are obtained such as: $\boldsymbol{C = M_e^{-1} U_e}$

As result, the internal loads, the displacements and adhesive shear stress are obtained at any x:

$$u_1 = \frac{1}{2}\left[c_1 + c_2 x - c_3(1+\chi)e^{-\eta x} - c_4(1+\chi)e^{\eta x}\right]$$

$$u_2 = \frac{1}{2}\left[c_1 + c_2 x + c_3(1-\chi)e^{-\eta x} + c_4(1-\chi)e^{\eta x}\right]$$

$$T = \frac{G_a}{e_a}(u_2 - u_1)$$

$$N_1(x) = \frac{1}{2}\left[c_2 + c_3\eta(1+\chi)e^{-\eta x} - \eta c_4(1+\chi)e^{\eta x}\right]A_1 - A_1\alpha_1\Delta_T$$

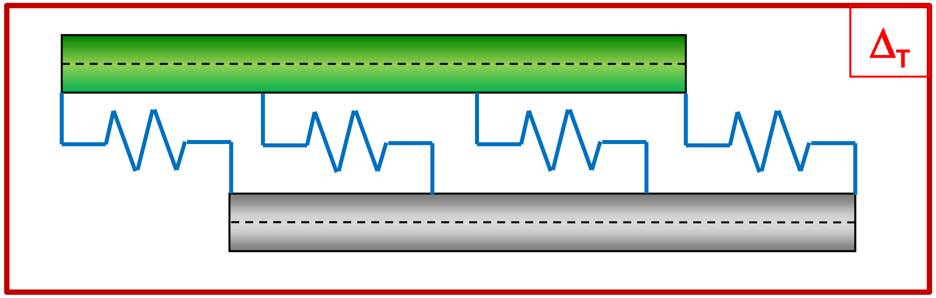
$$N_2(x) = \frac{1}{2}\left[c_2 - c_3\eta(1-\chi)e^{-\eta x} + \eta c_4(1-\chi)e^{\eta x}\right]A_2 - A_2\alpha_2\Delta_T$$

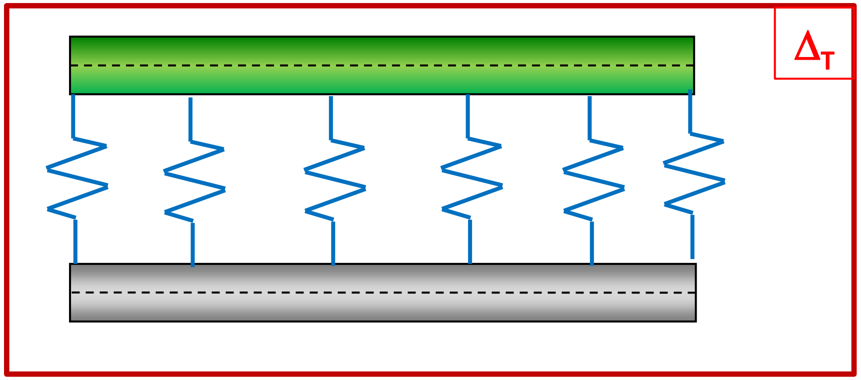
ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

The hypotheses are:

- linear elastic material behavior
- local equilibrium of Goland and Reissner **(Goland and Reissner, 1944)**
- the adherends are modelled as laminated Euler-Bernoulli beams, with eventually a linear variation of the shear stress with the thickness **(Tsaï et al., 1998)**
- the adhesive layer is modelled as a bed of shear and peel springs
- the adhesive thickness $\mathbf{e_a}$ is constant
- mechanical loading and application of a uniform temperature variation Δ_T

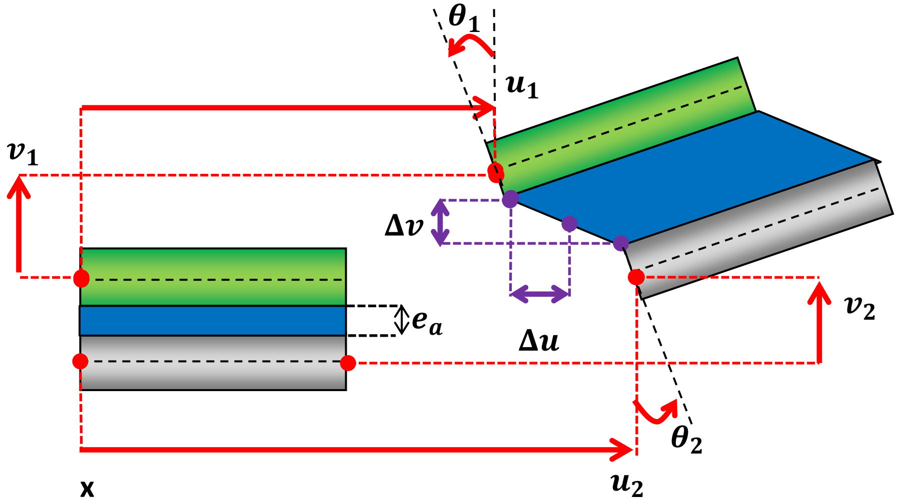




ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

constitutive equations

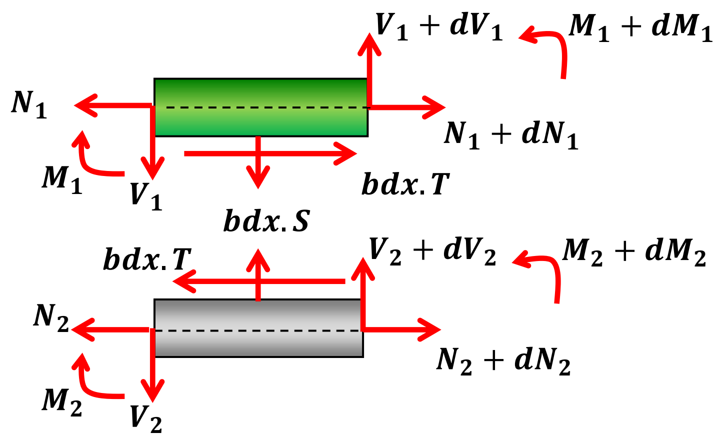


$$N_j = A_j \frac{du_j}{dx} - B_j \frac{d\theta_j}{dx} \qquad M_j = -B_j \frac{du_j}{dx} + D_j \frac{d\theta_j}{dx}$$

$$\theta_j = \frac{dv_j}{dx}$$

$$T = \frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right) \qquad S = \frac{E_a}{e_a}\Delta v$$

local equilibrium of Goland and Reissner



$$\frac{dN_j}{dx} = (-1)^j bT$$

$$\frac{dV_j}{dx} = (-1)^{j+1} bS$$

$$\frac{dM_j}{dx} + V_j + b\frac{e_j}{2}T = 0$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

Contrary to the 1D-bar case, the closed-form expressions for the components of the elementary stiffness matrix are not obtained. However, the resolution scheme consisting in:

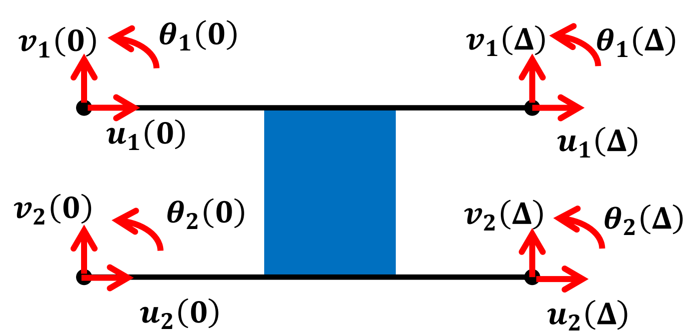
- determining the shape of solutions in displacements as functions of integration constants
- Deducing the shape of internal loads as functions of integration constants

can be applied.

The difficulty here is the identification of a set of 12 free integration constants among the 27 appearing during the employed athematical path.

$$\boldsymbol{F_e + F_{therm} = N_e M_e^{-1} U_e = K_{PC} U_e}$$

$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \\ -V_1(0) \\ -V_2(0) \\ V_1(\Delta) \\ V_2(\Delta) \\ -M_1(0) \\ -M_2(0) \\ M_1(\Delta) \\ M_2(\Delta) \end{pmatrix} + \begin{pmatrix} -N_1^{\Delta_T} \\ -N_2^{\Delta_T} \\ N_1^{\Delta_T} \\ N_2^{\Delta_T} \\ 0 \\ 0 \\ 0 \\ 0 \\ M_1^{\Delta_T} \\ M_2^{\Delta_T} \\ -M_1^{\Delta_T} \\ -M_2^{\Delta_T} \end{pmatrix} = K_{PC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \\ v_1(0) \\ v_2(0) \\ v_1(\Delta) \\ v_2(\Delta) \\ \theta_1(0) \\ \theta_2(0) \\ \theta_1(\Delta) \\ \theta_2(\Delta) \end{pmatrix}$$



ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

Another resolution scheme is developed since 2014. It is based on the resolution of the system of 1st order ODEs in the internal loads and displacements, making use of the exponential matrix **(Paroissien et al., 2018a) (Paroissien et al., 2018b)**.

$$\frac{d\boldsymbol{N}_j}{dx} = (-1)^j b \frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right)$$

$$\frac{dV_j}{dx} = (-1)^{j+1} b \frac{E_a}{e_a}(v_1 - v_2)$$

$$\frac{dM_j}{dx} = -V_j - b\frac{e_j}{2}\frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right)$$

$$\frac{du_j}{dx} = \frac{D_j N_j + B_j M_j}{\Delta_j}$$

$$\frac{dv_j}{dx} = \theta_j$$

$$\frac{d\theta_j}{dx} = \frac{A_j M_j + B_j N_j}{\Delta_j}$$

$$\Rightarrow \quad \frac{dX}{dx} = AX \qquad X = \begin{pmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \\ u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{pmatrix}$$

$$\Rightarrow \quad \begin{cases} \Phi_A(X=0) = expm(A.0) \\ \Phi_A(X=\Delta) = expm(A.\Delta) \end{cases}$$

$$\Rightarrow \quad \boxed{M_e, N_e, K_{PC} = N_e {M_e}^{-1}}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D poutre

This resolution scheme allows for the fast formulation of the elementary stiffness matrix when the simplified hypotheses are modified. For example: Timoshenko beam, local equilibrium of Hart-Smith **(Hart-Smith, 1973)** or of Luo and Tong **(Luo et Tong, 2009)**.

$$\frac{dN_j}{dx} = (-1)^j b \frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right)$$

$$\frac{dV_j}{dx} = (-1)^{j+1} b \frac{E_a}{e_a}(v_1 - v_2)$$

$$\boxed{\frac{dM_j}{dx} = -V_j - b\frac{e_j}{2}\frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right)}$$

$$\boxed{\frac{dM_j}{dx} = -V_j - b\frac{e_j}{2}\frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right) - \theta_j N_j}$$

$$\boxed{\frac{dM_j}{dx} = -V_j - b\frac{(e_j + e_a)}{2}\frac{G_a}{e_a}\left(u_2 - u_1 - \frac{e_2}{2}\theta_2 - \frac{e_1}{2}\theta_1\right)}$$

$$\frac{du_j}{dx} = \frac{D_j N_j + B_j M_j}{\Delta_j}$$

$$\boxed{\frac{dv_j}{dx} = \theta_j} \longrightarrow \boxed{\frac{dv_j}{dx} = \frac{V_j}{H_i} + \theta_j}$$

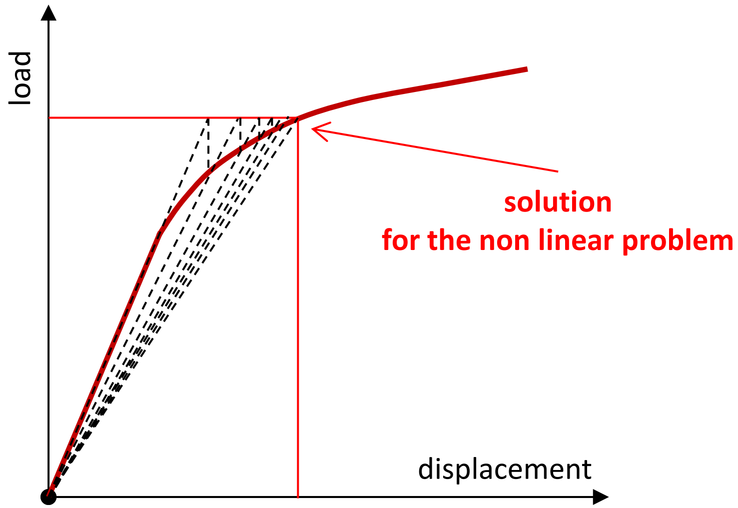
$$\frac{d\theta_j}{dx} = \frac{A_j M_j + B_j N_j}{\Delta_j}$$

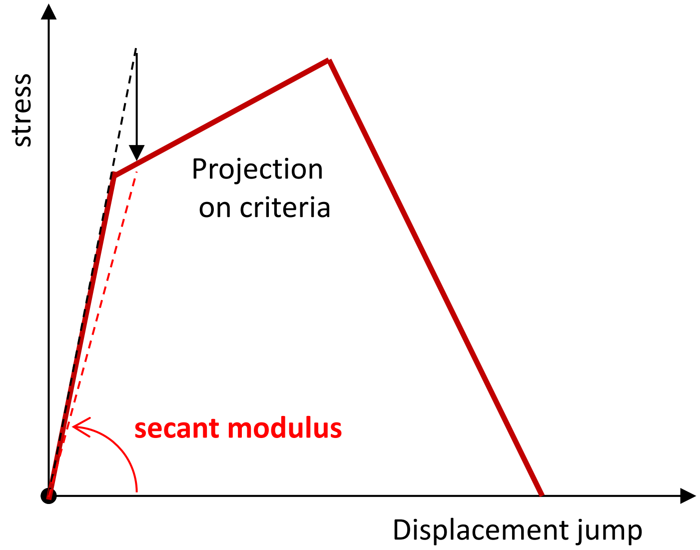
NONLINEAR MATERIAL BEHAVIOR

NONLINEAR COMPUTATION

The non linear material behavior of the adhesive layer and of fasteners can be taken into account thanks to computation scheme based on the Newton-Raphson iterative scheme using the secant stiffness matrix (Lélias et al., 2015).

In the case of non linear adhesive material, a mesh along the overlap is then required in order to be able to update the elementary stiffness matrix of each ME.



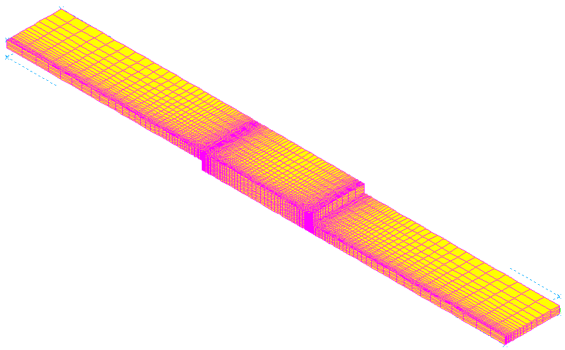


Various behavior laws are available (elasto-plastic, damaging evolution (CZM)) or will be available (visco-eastic, visco-plastic).

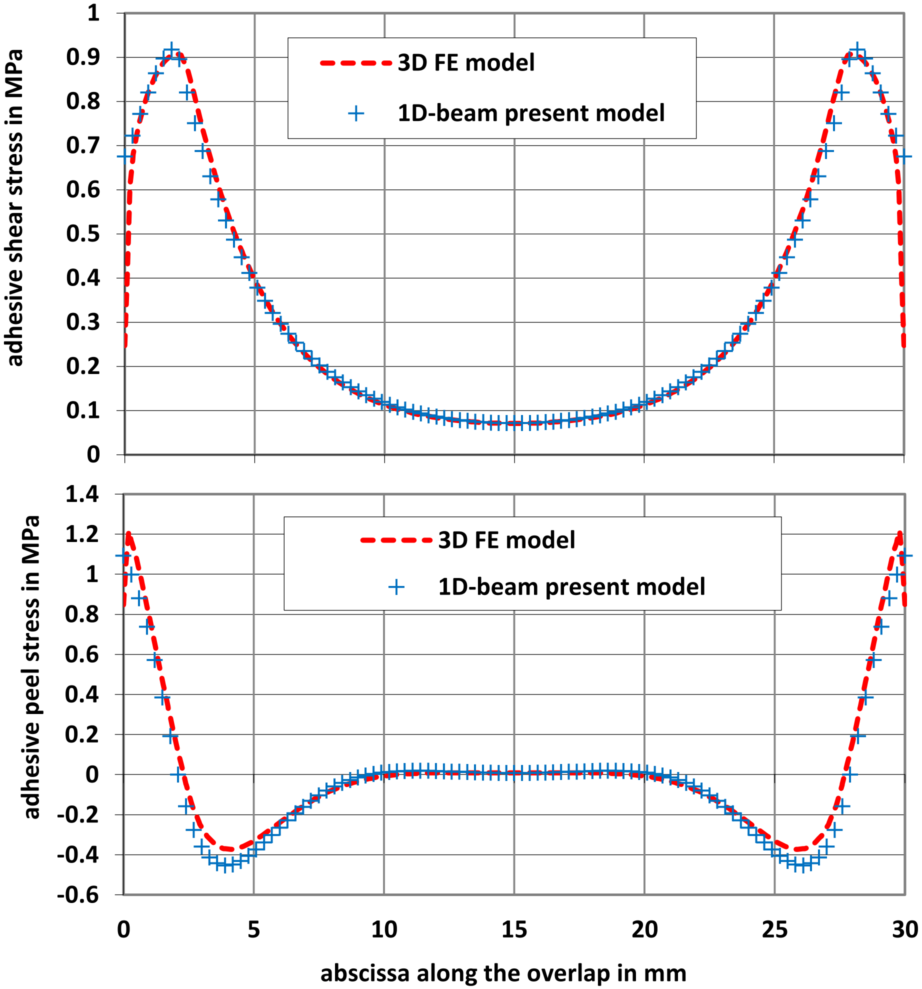
CPU TIME REDUCTION

Unbalanced single-lap bonded joint with an elastic perfectly plastic adhesive material in plane loaded (Paroissien et al., 2013a).

The adhesive stresses are read on the **converged** 3D FE model on the neutral line of the adhesive layer.

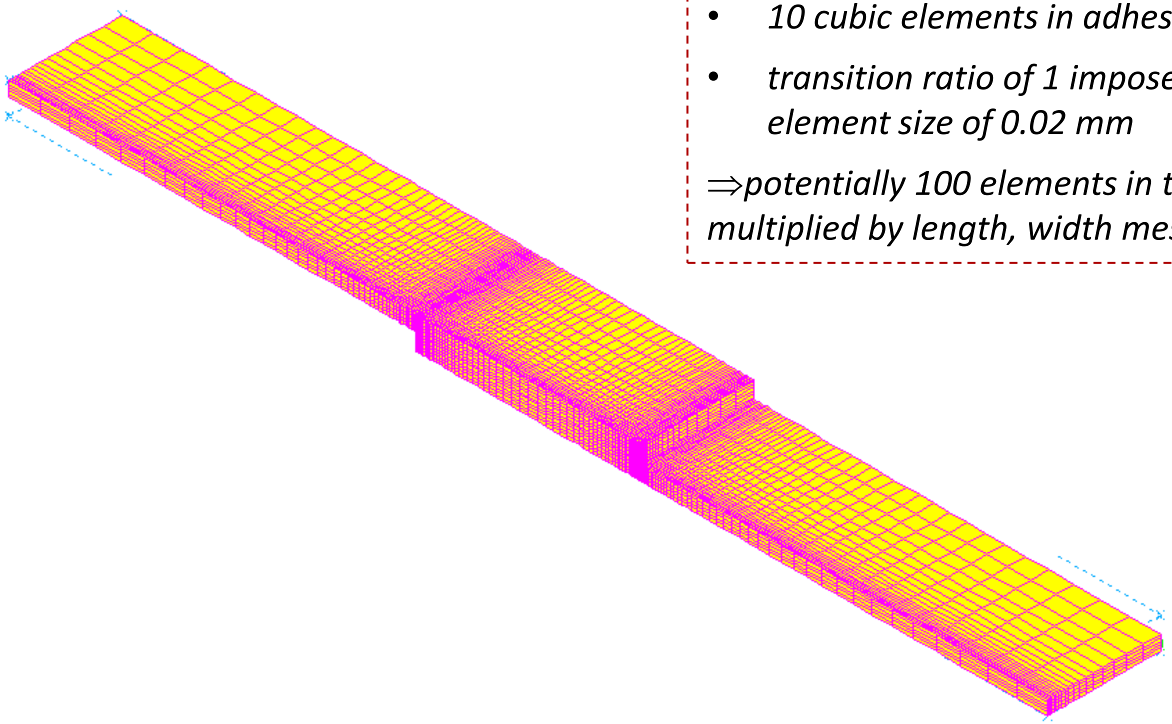


Benefit in CPU time: x50



CPU TIME REDUCTION

The simulation of the mechanical behavior of bonded joints using the FEM is time consumming due to the relative difference in thickness between the adherends and the adhesive layers.



Example of single-lap bonded joint in 3D:

- *adherend thickness: 2 mm / adhesive thickness: 0.2 mm*
- *10 cubic elements in adhesive thickness = 0.02 mm each*
- *transition ratio of 1 imposed at the adhesive interface, an element size of 0.02 mm*

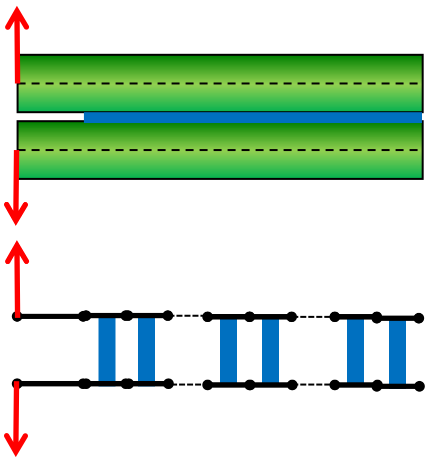
⇒potentially 100 elements in the adherend thickness, to be multiplied by length, width mesh parameters…

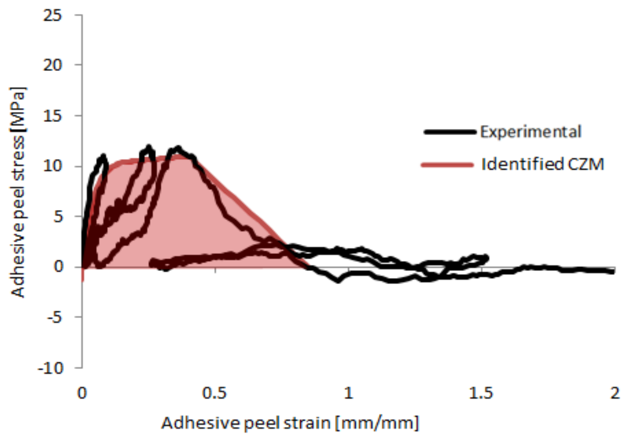
APPLICATION CASES

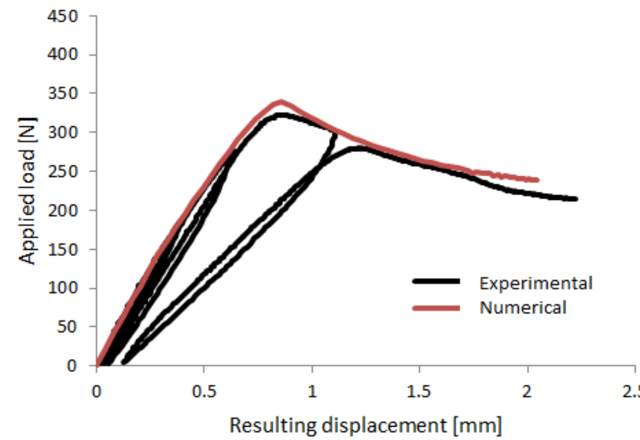
ASSESSMENT OF CZM FOR THIN ADHESIVE LAYERS

The ME technique has successively been used for the assessment of CZM for thin adhesive layers **(Lélias, 2016) (Lélias et al., 2018)**.

DCB

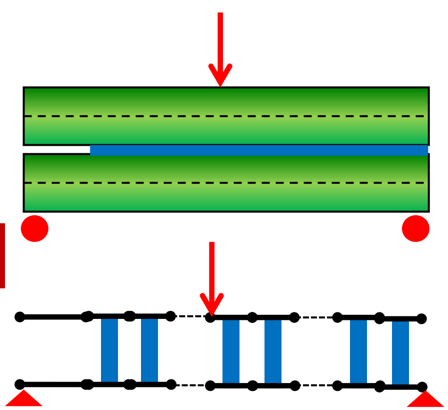


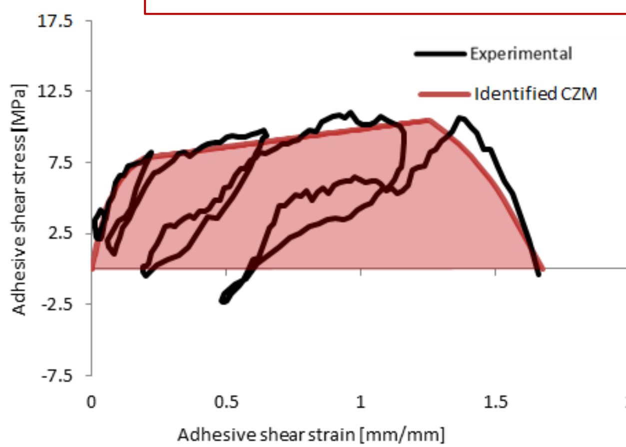


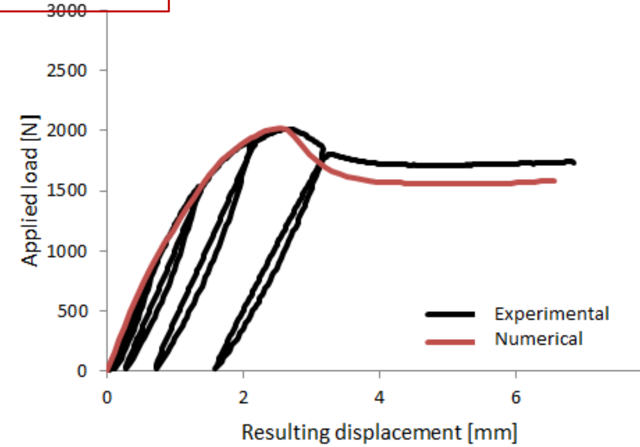


various geometries // lego game

ENF



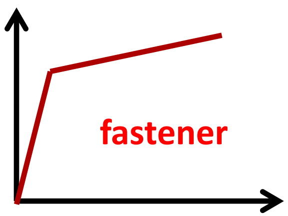


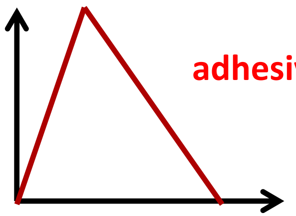


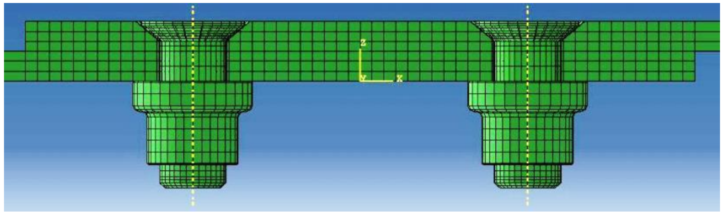
APPLICATION CASES

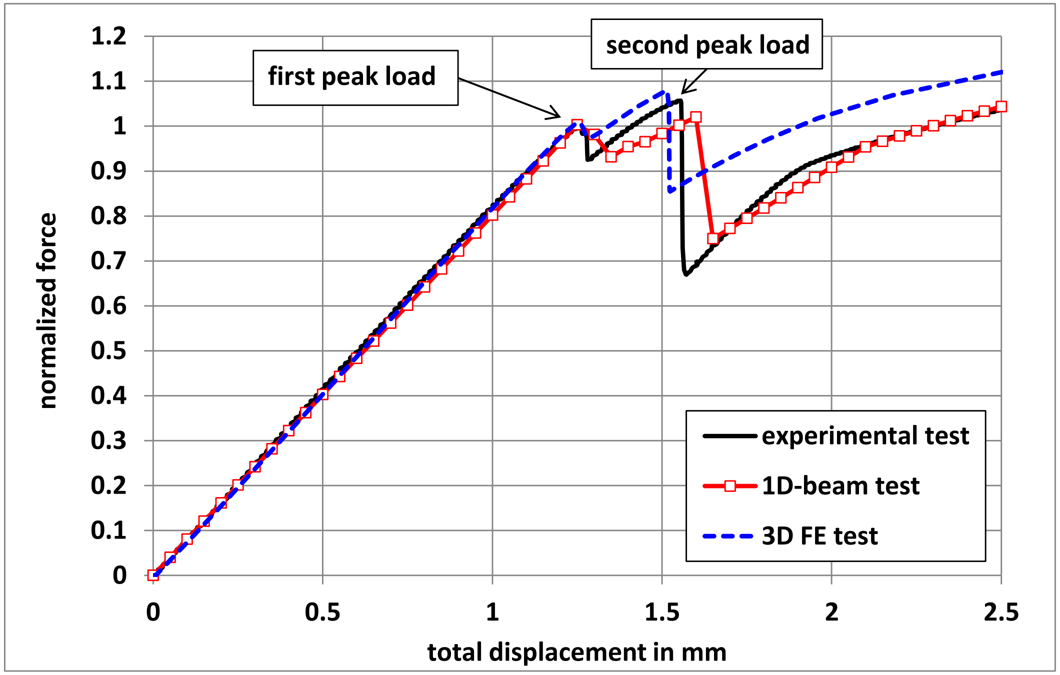
HYBRIDE (BOLTED/BONDED) JOINTS

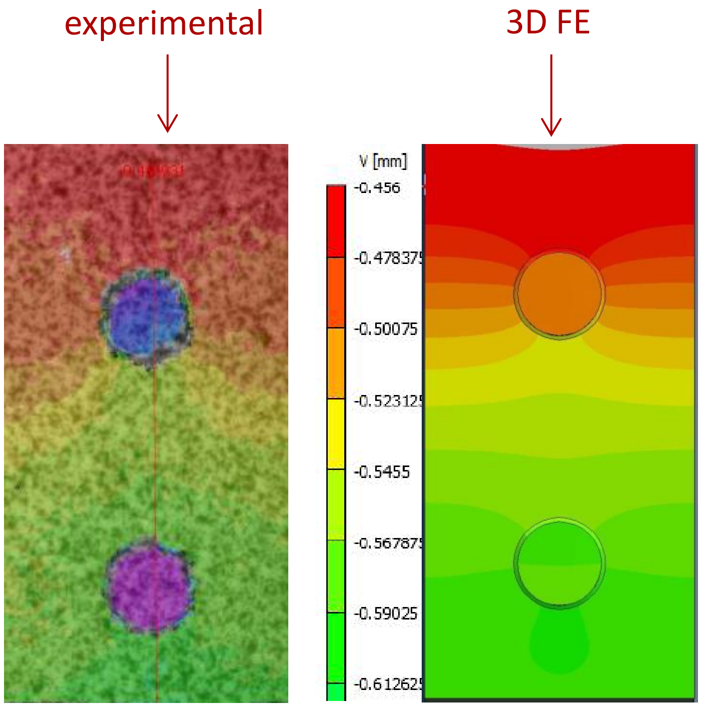
Comparison between experimental test, 3D FE test and 1D-beam ME test of a single lap HBB joint in-plane loaded **(Paroissien et al., 2017)**.







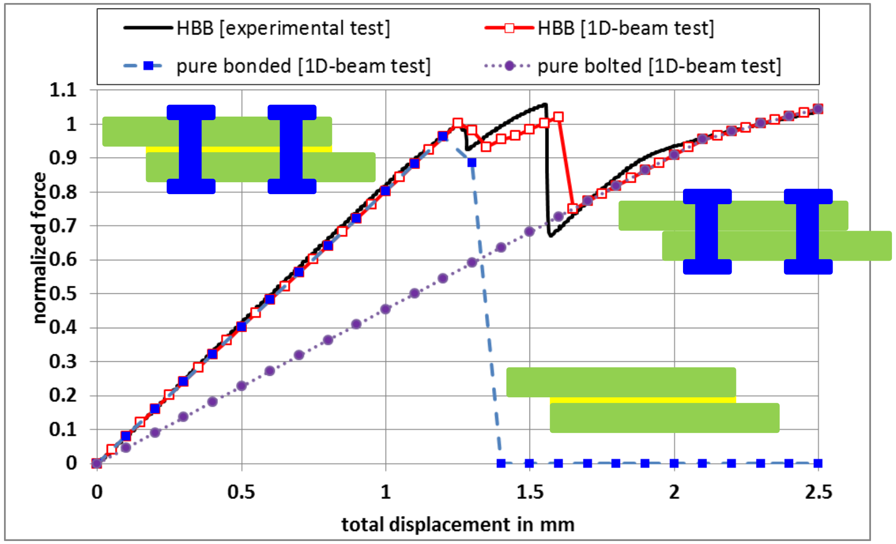


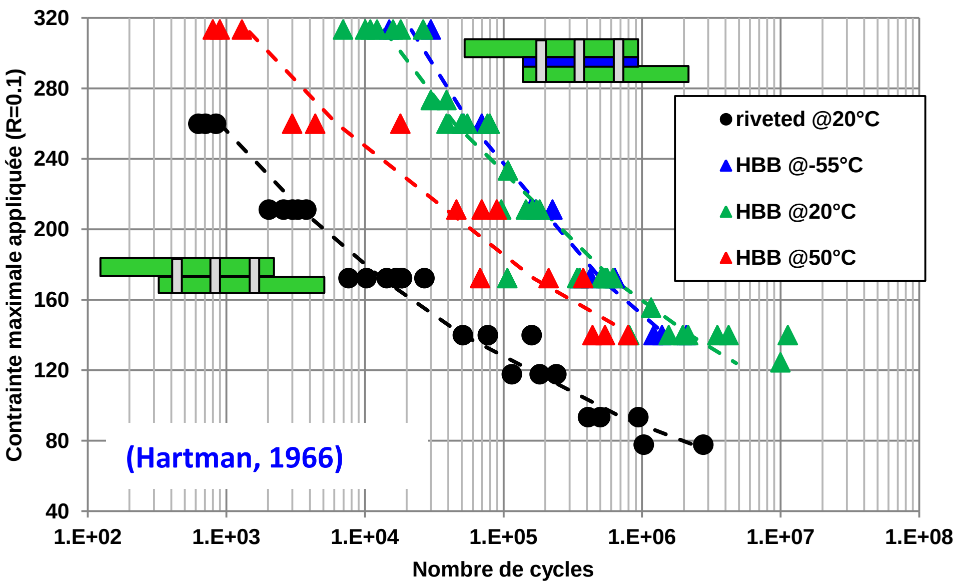


APPLICATION CASES

HYBRID (BOLTED/BONDED)

Static and fatigue performance better than pure bolted or pure bonded, if the adhesive is judiciously chosen.





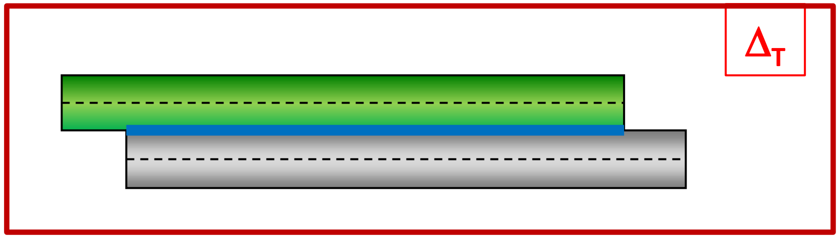
Fatigue strength prediction from ME output:

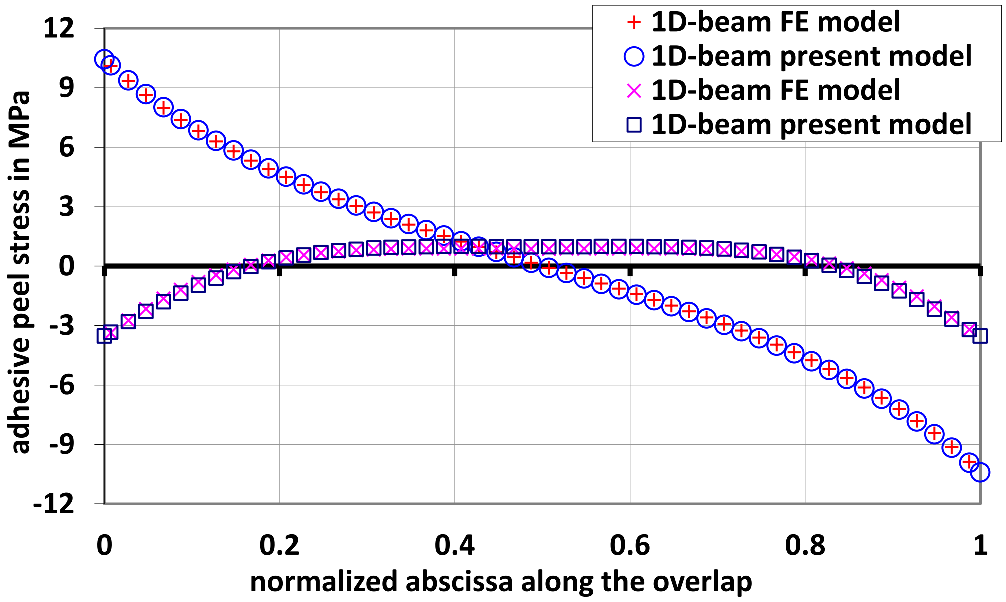
- for the holes with classical semi-empirical uni-axial approaches **(Müller, 1995)**
- for the adhesive layer through the progressive degradation of CZM **(Khomarishad, 2011)**

APPLICATION CASES

BONDED JOINTS UNDER THERMAL LAODING

Comparison between 1D FE model and 1D ME model **(Paroissien et al., 2013b)**.
The 1D FE model is built with beam and spring elements.
Unbalanced single-lap joints under pure thermal loading under membrane and bending.



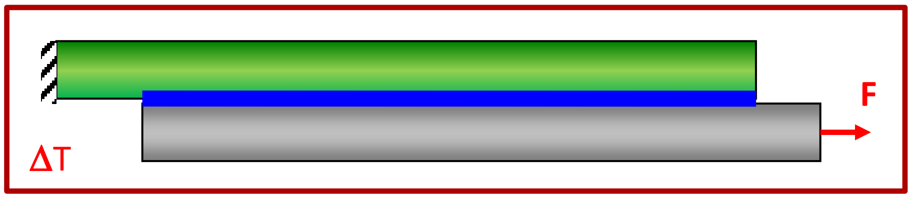


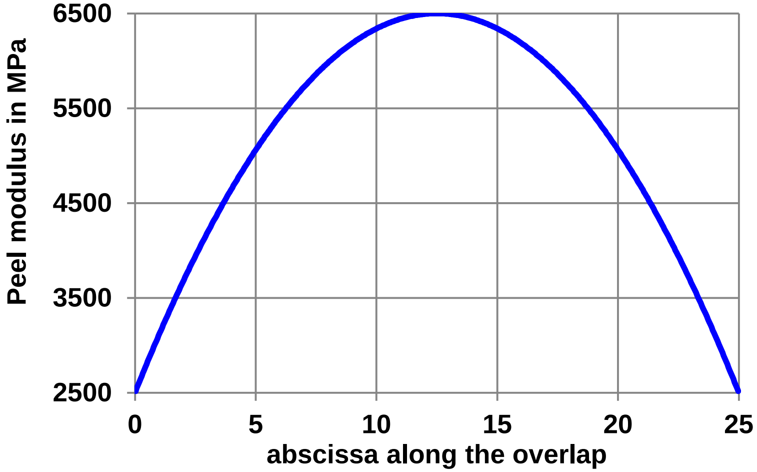
FUNCTIONNALY GRADED ADHESIVE JOINTS

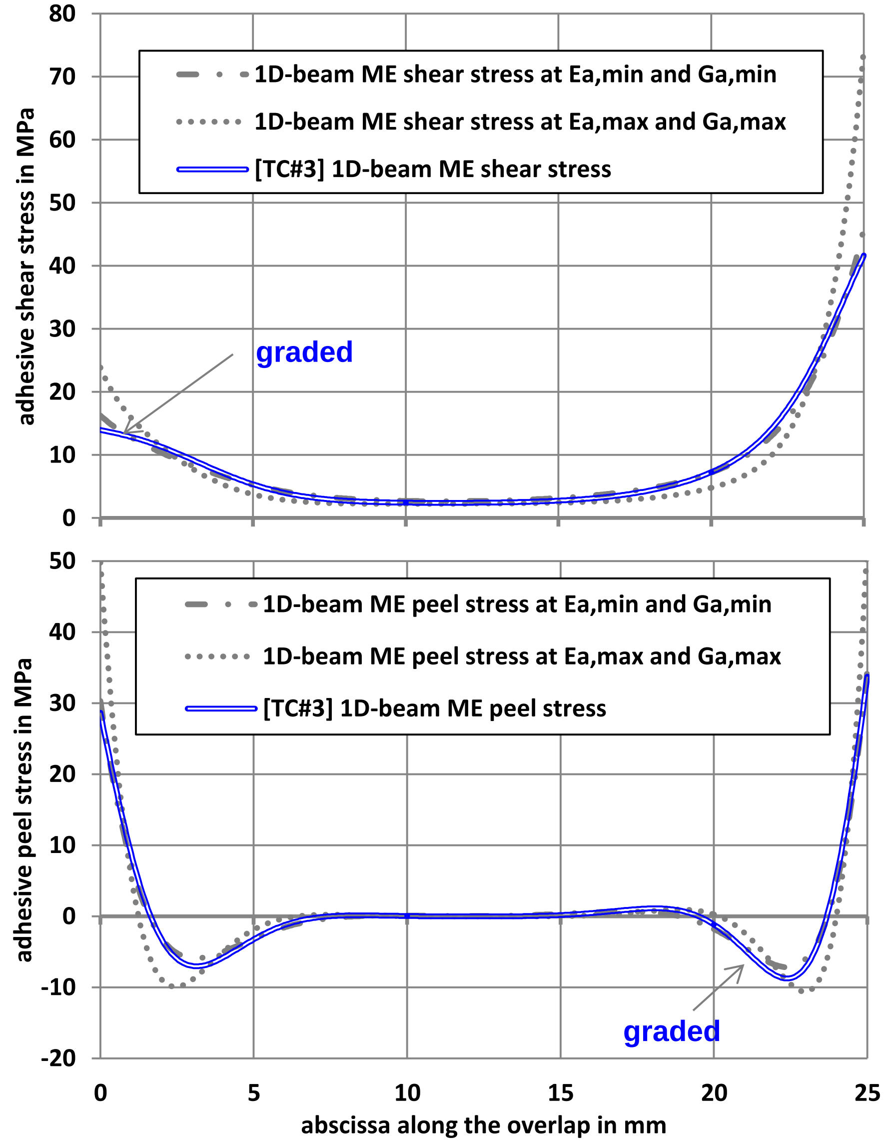
in collaboration with University of Porto

Unbalanced single-lap joints under combined mechanical and thermal loading with a symmetrical parabolic graduation **(Paroissien et al., 2018b)**.

- reduction of peak stresses
- the simplified model offers a solution to optimize the graduation

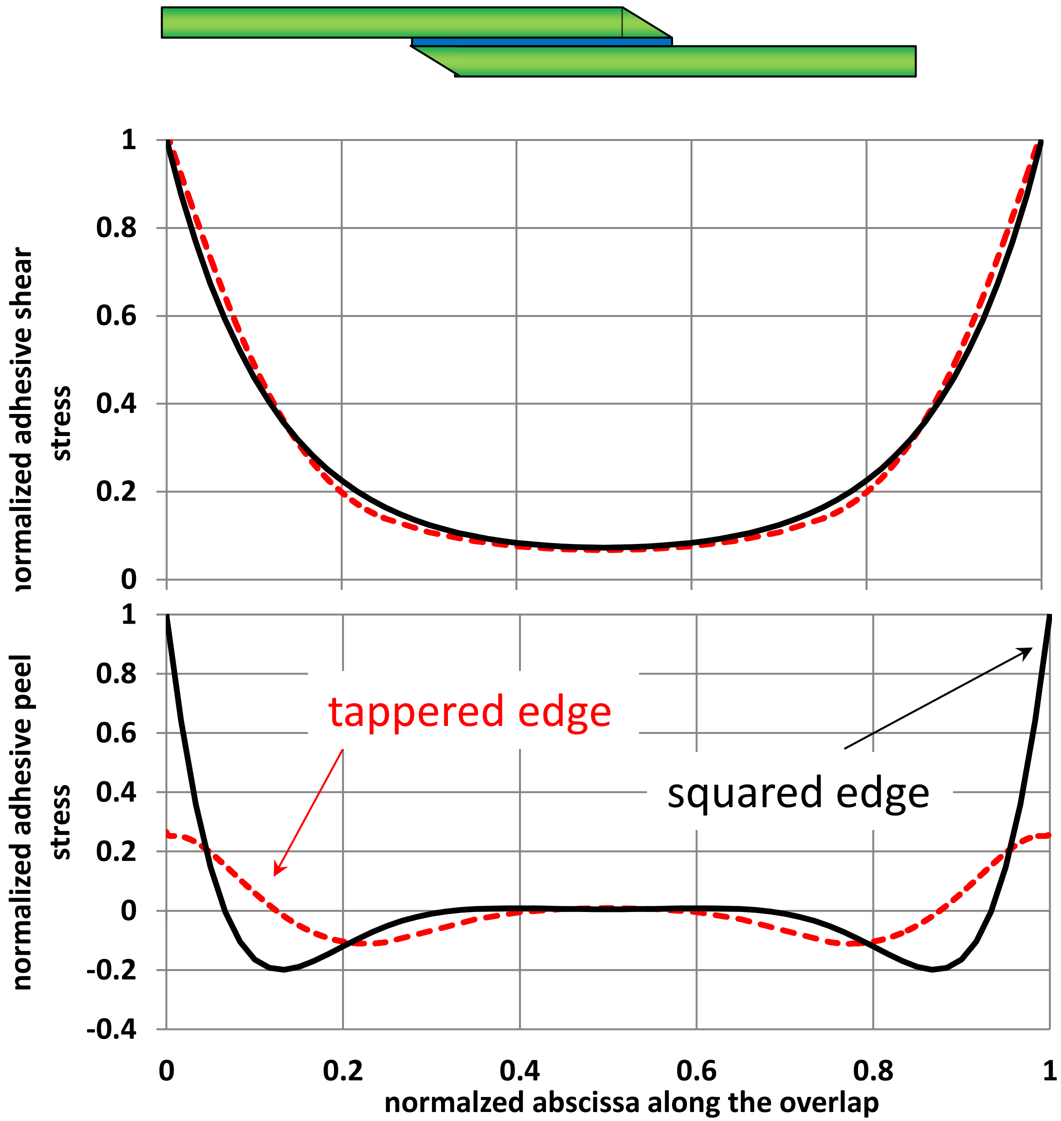






REDUCTION OF PEAK STRESSES

single-lap joint with tapered adherend
- reduction of peak peel stresses



IN PROGRESS

Prediction of failure at adherend/adhesive interface [PRACCOMET]

- PhD Thesis by Thiago Birro (2017-2020) supervised by [1]Frédéric Lachaud, [2]Maëlenn Aufray and [1]Éric Paroissien
- funded by Région Occitanie and ISAE-SUPAERO (APR2017 UFT MiP)
- Collaboration with CIRIMAT: **TACCOS**
- ➢ **experimental and numerical modelling of adherend/adhesive interface behavior**

Assessment of constitutive behavior of thin adhesive layer [S3PAC]

- PhD Thesis by Agathe Jaillon (2017-2020) supervised by [1]Frédéric Lachaud, [3]Julien Jumel and [1]Éric Paroissien
- funded by BPI France, Région Occitanie and Région Nouvelle Aquitaine
- FUI (21) S3PAC
- ➢ **experimental and numerical modelling of cohesive behavior as function of adhesive thickness**

[1]Institut Clément Ader (ICA), Université de Toulouse, ISAE-SUPAERO, INSA, IMT MINES ALBI, UTIII, CNRS, France
[2]CIRIMAT, Université de Toulouse, CNRS, INPT, UPS, France
[3]Université de. Bordeaux, Arts et Metiers ParisTech, CNRS, I2M, UMR 5295, France

TO COME

Macro-element of bonded plates [SCODyn]

- PhD Thesis by Benjamin Ordonneau (2018-2021) supervised by [1]Michel Salaün and [1]Éric Paroissien
- funded by CETIM and DGA
- ➢ **formulation of elementary stiffness and mass matrices of bonded overlap under 3D loading**

Dual functionalisation strength / fragmentation [SIMPACOS]

- PhD Thesis by Lorraine Silva (2018-2021) supervised by [1]Christine Espinosa and [4]Lucas FM da Silva
- funded by ED MEGeP
- ➢ **particle-based numerical simulation to predict strength and controlled fragmentation for space structures**

[1]Institut Clément Ader (ICA), Université de Toulouse, ISAE-SUPAERO, INSA, IMT MINES ALBI, UTIII, CNRS, France
[4]Department of Mechanical Engineering, Faculty of Engineering, University of Porto, Portugal

REFERENCES

(Paroissien, 2006)
E. Paroissien, 2006. Contribution aux Assemblages Hybrides (Boulonnés/Collés) – Application aux Jonctions Aéronautiques. PhD Dissertation (in French), Université de Toulouse III.

(Paroissien et al., 2007a)
Paroissien, E., Sartor, M., Huet, J., 2007a. Hybrid (bolted/bonded) joints applied to aeronautic parts: Analytical one-dimensional models of a single-lap joint. In: Advanced in Integrated Design and Manufacturing in Mechanical Engineering II. S Tichkiewitch, M Tollenaere, and P Ray (Eds.), 95-110, Springer, Dordrecht, The Netherlands.

(Ross, 1947)
R.D. Ross, 1947. An electrical computer for the solution of shear-lag and bolted joints problems. Technical Note, NACA, TN 1281.

(Paroissien et al., 2007b) Paroissien, E., Sartor, M., Huet, J., Lachaud, F., 2007b. Analytical two-dimensional model of a hybrid (bolted/bonded) single-lap joint, J. Aircraft, 44, 573-582.

(Paroissien et al., 2017)
Paroissien, E., Lachaud, F., Da Veiga, A., Barrière, P., 2017. Simplified Stress Analysis of Hybrid (Bolted/Bonded) Joints. Int. J. Adhes. Adhes., 77, 183-197.

(Morris, 2004)
Morris, G.M., 2004. Defining a standard formula and test-method for fastener flexibility in lap-joints. MS Thesis Dissertation, Delft University of Technology, The Netherlands.

(Tate and Rosenfled, 1946)
Tate, M.B., Rosenfeld, S.J., 1946. Preliminary investigation of the loads carried by individual bolts in bolted joints. Technical Note, NACA, TN 1051.

(Swift, 1984)
Swift, T., 1984. Fracture analysis of stiffened structure, Damage Tolerance of metallic structures: analysis methods and application
ASTM STP 842, J.B. Chang & J.L. Rudd, Eds., American Society for Testing and Materials, 69-107.

(Huth, 1986)
Huth, H., 1986. Influence of Fastener flexibility on the prediction of load transfer and fatigue life for multiple row joints. Fatigue in mechanically fastened composite and metallic joints, ASTM STP 927, John M. Potter, Ed., Philadelphia, PA, 221-250.

(Cope and Lacy, 2000)
Cope, D.A., Lacy, T.E. Stress intensity determination in lap joints with mechanical fasteners. Proceedings of 41st AIAA / ASME / ASCE / AHS / ASC Structures, Structural Dynamics, and Materials conference, AIAA-2000-1368, 3-6 April 2000, Atlanta, Georgia.

(van Ingen and Vlot, 1993)
van Ingen, J.W., Vlot, A. Stress analysis of adhesively bonded single lap joints. (Report LR-740). Delft University of Technology, The Netherlands.

REFERENCES

(Tsaï and Morton, 1994)
Tsaï, M.Y., Morton, J., 1994. An evaluation of analytical and numerical solutions to the single-lap joint. Int. J. Solids Struct., 31, 2537-2563.
(da Silva et al. 2009)
Da Silva, L.F.M., das Neves, P.J.C., Adams, R.D., Spelt, J.K., 2009. Analytical models of adhesively bonded joints-Part I: Literature survey. Int. J. Adhes. Adhes., 29, 319-330.
(Goland and Reissner, 1944)
Goland, M., Reissner, E., 1944. The stresses in cemented joints, J. Appl. Mech., 11, A17-A27.
(Hart-Smith, 1973)
Hart-Smith, L.J., 1973. Adhesive bonded single lap joints. NASA Technical Report, CR-112236, Douglas Aircraft Company, Long Beach, California.
(Williams, 1975)
Williams, J.H., 1975. Stresses in adhesive between dissimilar adherends. Journal of Adhesion, 7, 97-107 (1975)
(Bigwood and Crocombe, 1989)
Bigwood, DA, Crocombe AD, 1989. Elastic analysis and engineering design formulae for bonded joints. Int. J. Adhes. Adhes., 9(4), 229-242.
(Oplinger, 1991)
Oplinger, DW, 1991. A Layered Beam Theory for Single-Lap Joints, Technical Report, US AMTL, MTL91-23.
(Tsaï et al., 1998)
Tsaï, M.Y., Oplinger, D.W., Morton, J., 1998. Improved Theoretical Solutions for Adhesive Lap Joints Int. J. Solids Struct., 35(12), 1163-1185.
(Högberg, 2004)
Högberg, JL, 2004. Mechanical behavior of single-layer adhesive joints – An Integrated approach. Licensing Graduate Thesis, Department of Applied Mechanics, Chalmers University of Technology, Sweden.
(Luo and Tong, 2009)
Luo, Q., Tong, L., 2009. Analytical solutions for non linear analysis of composite single lap adhesive joints. Int. J. Adhes. Adhes., 29, 144-154.
(Nemes and Lachaud ,2009)
Nemes, O, Lachaud, F, 2009. Modeling of cylindrical adhesively bonded joints, J. Adhes. Sci. Technol., 23(10-11), 1383-1393.
(Weißgraeber et al., 2014)
Weißgraeber, P, Stein, N, Becker, W, 2014. A general sandwich-type model for adhesive joints with composite adherends. Int. J. Adhes. Adhes., 55, 56–63.
(Renton and Vinson, 1977)
Renton, W.J., Vinson, J.R., 1977. Analysis of adhesively bonded joints between panels of composite materials. J. Appl. Mech., 101-106.

REFERENCES

(Allman, 1977)
Allman, D.J., 1977. A theory for elastic stresses in adhesive bonded lap joints. Quarterly J. Mech. Appl. Math., 30, 415-436.

(Adams and Mallick, 1992)
Adams, R.D., Mallick, V., 1992. A method for the stress analysis of lap joints. J. Adhesion, 38, 199-217.

(Mortensen, 1997)
Mortensen, F., 1997. Development of tools for engineering analysis and design of high-performance FRP-composite structural elements. PhD Thesis, Aalborg University, Denmark.

(Gustafson et al., 2006)
Gustafson, P.A., Bizard, A., M. Waas., 2006. A macroscopic joint finite element for a symmetric double lap joint. Proceedings of the American Society of Composites 21st Annual Technical Conferences, No. 24, American Society of Composites.

(Volkersen, 1938)
Volkersen, O., 1938. Die Nietkraftverteilung in Zugbeanspruchten Nietverbindungen mit konstanten Laschenquerschnitten, Luftfahrforschung. 15(24), 41-47.

(Paroissien et al., 2018a)
Paroissien, E., Lachaud, F., Morlier, J., Schwartz, S., 2018. A direct method for the assessment of cohesive zone models for thin adhesive layers loaded in mode I, mode II, and mixed-mode I/II. Rev. Adhes. Adhes., Open Access Supplement, DOI: 10.7569/RAA.2018.097301.

(Paroissien et al., 2018b)
Paroissien, E., da Silva, L.F.M., Lachaud, F., 2018. Simplified stress analysis of functionally graded single-lap joints subjected to combined thermal and mechanical loads. Accepted for publication in July 2018 into Composite Structures

(Lélias et al., 2015)
Lélias, G., Paroissien, E., Lachaud, F., Morlier, J., Schwartz, S., Gavoille, C., 2015. An extended semi-analytical formulation for fast and reliable mode I/II stress analysis of adhesively bonded joints. Int. J. Solids Struct., 62, 18-38.

(Lélias, 2016)
Lélias, G., 2016. Mechanical behavior of adhesively bonded joints: Modeling, simulation and experimental characterization. PhD Thesis, University of Toulouse 3, Toulouse, France.

(Lélias et al., 2018)
Lélias, G., Paroissien, E., Lachaud, F., Morlier, J. 2018. Experimental Characterization of Cohesive Zone Models for Thin Adhesive Layers Loaded in Mode I, Mode II, and Mixed-Mode I/II by the use of a Direct Method. Submitted in April 2018 to International Journal of Solids and Structures.

REFERENCES

(Hartman, 1966)
Hartman, A., 1966. Fatigue tests on single-lap joints in clad 2024-T3 aluminium alloy manufactured by a combination of riveting and adhesive bonding. NLR, TN M2170.

(Müller, TU Delft, 1995)
Müller, R.P.G., 1995. An Experimental and Analytical Investigation on the Fatigue Behaviour of Fuselage Riveted Lap Joints. PhD Thesis Dissertation, Delft University of Technology, The Netherlands.

(Khomarishad et al. 2010)
Khomarishad ,H., Crocombe, A.D., Katman, K.B., Ashcroft, I.A., 2010. Predicting fatigue damage in adhesively bonded joints using a cohesive zone model. Int. J. Fatigue, 32, 1146-1158.

(Paroissien et al., 2013a)
Paroissien, E, Gaubert, F, Da Veiga, A, Lachaud, F, 2013. Elasto-Plastic Analysis of Bonded Joints with Macro-Elements. J. Adhes. Sci. Technol., 27(13), 1464-1498.

(Paroissien et al., 2013b)
Paroissien, E, Lachaud, F, Jacobs T, 2013. A simplified stress analysis of bonded joints using macro-elements. In: Advances in Modeling and Design of Adhesively Bonded Systems, Kumar S. and Mittal K.L. (Eds), 93-146, Wiley-Scrivener, Beverly, Massachusetts.