BINÔMIO de NEWTON

1. Número Binomial

* $\binom{n}{p} = C_{n,p} = \dfrac{n!}{p!\,(n-p)!}$ com $n \geq p$.

* $\binom{n}{0} =$ $\qquad \binom{n}{1} =$ $\qquad \binom{n}{n} =$

2. Igualdade em Números Binomiais

$$\binom{n}{p} = \binom{n}{q} \iff \begin{cases} p = q & \text{Idênticos} \\ \text{ou} \\ p + q = n & \text{complementares} \end{cases}$$

Ex: $\binom{16}{3} = \binom{16}{x+1}$ $\begin{cases} 3 = x+1 \rightarrow x = 2 \\ 3 + x + 1 = 16 \rightarrow x = 12 \end{cases}$

Como pode estar na prova para confundir vocês: $C_{16,3} = C_{16,x+1}$

Ex: $\binom{5}{2} = \binom{5}{p}$

Ex: $C_{15,\,2x+1} = C_{15,\,x+2}$

3. TRIÂNGULO DE PASCAL

$$\begin{array}{llllll}
\binom{0}{0} \\
\binom{1}{0} & \binom{1}{1} \\
\binom{2}{0} & \binom{2}{1} & \binom{2}{2} \\
\binom{3}{0} & \binom{3}{1} & \binom{3}{2} & \binom{3}{3} \\
\binom{4}{0} & \binom{4}{1} & \binom{4}{2} & \binom{4}{3} & \binom{4}{4} \\
\binom{5}{0} & \binom{5}{1} & \binom{5}{2} & \binom{5}{3} & \binom{5}{4} & \binom{5}{5} \ \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\binom{n}{0} & \binom{n}{1} & \binom{n}{2} & \ldots & \binom{n}{n-1} & \binom{n}{n}
\end{array}$$

$$\begin{array}{llllll}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1
\end{array}$$

*** PROPRIEDADES ***

* Stiffel $\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$

* $\binom{n}{0}\ (\ )\ (\ )\ (\ )\ (\ )\ (\ )$ $\to$ $(\ )$

* $\binom{n}{n}\ (\ )\ (\ )\ (\ )\ (\ )$ $\to$ $(\ )$

$\binom{n}{0} + (\ ) + (\ ) + \binom{n}{n} = 2^n$

4- BINÔMIO DE NEWTON $\rightsquigarrow (a \pm b)^n \rightsquigarrow$ a e b são fatores e n expoente de Newton.

Pai de todos os Binômios: $(a+b)^2$

$(a+b)^2 = a^2 + 2ab + b^2 \rightarrow (a+b)^2 = 1a^2b^0 + 2a^1.b^1 + 1a^0b^2 \rightarrow (a+b)^2 = \underbrace{\binom{2}{0}a^2b^0}_{T_1} + \underbrace{\binom{2}{1}a^1b^1}_{T_2} + \underbrace{\binom{2}{2}a^0b^2}_{T_3} = \sum_{p=0}^{2}\binom{2}{p}a^{2-p}b^p$

$T_{p+1} = \binom{2}{p}a^{2-p}.b^p$ ou $T_{p+1} = \binom{2}{p}a^p.b^{2-p}$

$(a+b)^3$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \rightarrow (a+b)^3 = 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3 \rightarrow (a+b)^3 = \underbrace{\binom{3}{0}a^3b^0}_{T_1} + \underbrace{\binom{3}{1}a^2b^1}_{T_2} + \underbrace{\binom{3}{2}a^1.b^2}_{T_3} + \underbrace{\binom{3}{3}a^0b^3}_{T_4} \rightarrow$

$\rightarrow (a+b)^3 = \sum_{p=0}^{3}\binom{3}{p}a^{3-p}.b^p$ $T_{p+1} = \binom{3}{p}a^{3-p}.b^p$ ou $T_{p+1} = \binom{3}{p}a^pb^{3-p}$

$(a+b)^4$

$(a+b)^4 = \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4 \rightarrow (a+b)^4 = \sum_{p=0}^{4}\binom{4}{p}a^{4-p}.b^p \rightarrow T_{p+1} = \binom{4}{p}a^{4-p}.b^p$ ou $\binom{4}{p}a^p.b^{4-p}$

$(2x-y)^6$

* $T_{p+1} = \binom{6}{p}(2x)^{6-p}(-y)^p$ ou $\binom{6}{p}(2x)^p.(-y)^{6-p}$
* $\sum_{p=0}^{6}\binom{6}{p}(2x)^{6-p}.(-y)^p = (2x-y)^6$

Achar termo 5° segundo expoentes decrescentes de y.

$T_{p+1} = \binom{6}{p}(2x)^p(-y)^{6-p} \rightarrow p+1 = 5 \rightarrow p = 4 \rightarrow$

$T_5 = \binom{6}{4}(2x)^4(-y)^{6-2} = \binom{6}{4}2^4x^4.y^4$

$\left(x^2 - \frac{1}{x}\right)^7$

* $T_{p+1} =$
* $\sum\binom{7}{p}(x^2)^{7-p}(-1/x)^p \rightarrow$

$\sum\binom{7}{p}x^{2(7-p)}.(-1)^p.(x)^{-p} \rightarrow$

$\sum\binom{7}{p}x^{14-2p-p}.(-1)^p \rightarrow$

$\sum\binom{7}{p}x^{14-3p}(-1)^p$

* T_{p+1} p/ expoentes cresc de $-1/x$

$T_{p+1} = \binom{7}{p}(x^2)^{7-p}.(-1/x)^p$

$(x+5)^{20}$

* Σ
* T_{p+1} p/ exp. cresc. de x:
* $T_{17} = \binom{20}{16}x^{16}.5^4$

glaucoleyser

5. COMUMENTE COBRADO NOS CONCURSOS.

5.1 - Soma de todos os coeficientes numéricos de um desenvolvimento de B.N.:

Ex: $(2x+3)^2 = 4x^2 + 12x + 9$: $\Sigma coef = 25$

* BASTA TROCAR x por 1 → $(2.1+3)^2 = 25$

Ex: $(x+2y+z)^2 = x^2 + 2x(2y+z) + 4y^2 + 4yz + z^2 =$

$= x^2 + 4xy + 2xz + 4y^2 + 4yz + z^2$: $\Sigma Coef = 16$

* BASTA TROCAR x, y e z por 1 → $(1+2.1+1)^2 = 16$

5.2 Determinar o valor de um termo com uma das variáveis elevada por um determinado expoente.

1º Sempre achar o TERMO GERAL do B.N.

2º Igual o expoente, no T_{p+1}, ao valor pedido na questão.

Ex: $(5x+2)^{20}$ → $T_{p+1} = \binom{20}{p}(5x)^{20-p} . 2^p$ → Achar o valor do termo que possua a variável x^{18}.

Ex: $\left(x^2 + \frac{1}{x}\right)^{10}$

a) Ache o valor do termo em x^{12}

b) Ache o valor do termo INDEPENDENTE de x.

5.3 SOMATÓRIOS

* $\sum_{i=1}^{3} i = i + i + i = 1 + 2 + 3 = 6$
* $\sum_{j=0}^{4} 2j = 2j + 2j + 2j + 2j = 2.0 + 2.1 + 2.2 + 2.3 + 2.4 =$
* $\sum_{j=0}^{2} 3^j = 3^j + 3^j + 3^j \rightarrow 3^0 + 3^1 + 3^2 = 13$
* $\sum_{p=0}^{2} \binom{2}{p} x^{2-p} . 3^p = (x+3)^2$
* $\sum_{p=0}^{5} \binom{5}{p} 2^p . x^{5-p} =$
* $\sum_{p=0}^{7} \binom{7}{p} 2^{7-p} . (-2)^p =$
* $\sum_{p=0}^{5} \binom{5}{p} x^p =$
* $\sum_{p=0}^{10} \binom{10}{p} (-1)^{10-p} =$
* $\sum_{p=0}^{n} \binom{n}{p} =$