SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2014 SECOND ROUND
JUNIOR SECTION: GRADE 8 & 9

13 May 2014 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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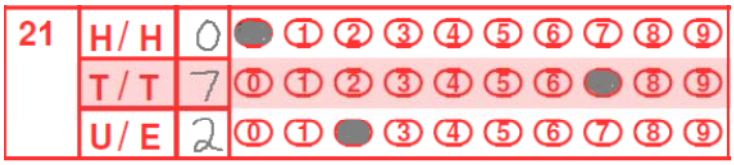
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

21. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:

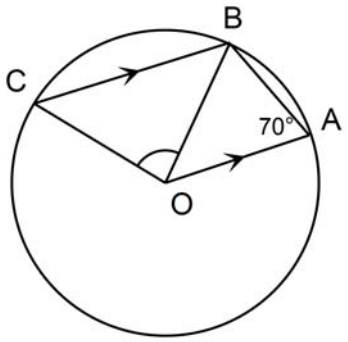


Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

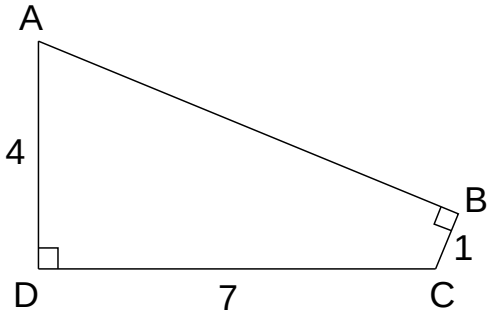
Part A (4 marks each)

1. What is the value of $\sqrt{201-4-(2\times 0+1)^4}$?

2. O is the centre of the circle through A, B and C.
CB is parallel to OA and $O\hat{A}B = 70°$.
What is the size of $C\hat{O}B$ in degrees?



3. $A\hat{D}C$ and $A\hat{B}C$ are right angles. What is the length of AB?

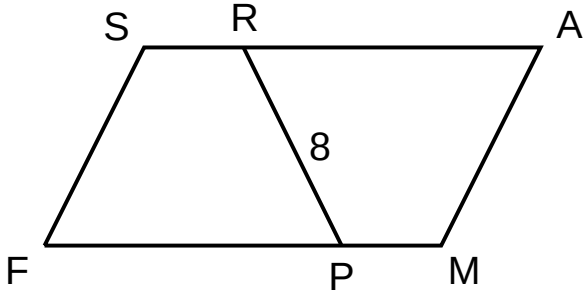


4. What is the smallest natural number K so that $315 \times K$ is a perfect square?

5. How many times in a 24-hour day do the hands on a 12-hour clock point in exactly the same direction?

Part B (5 marks each)

6. In the diagram, SAMF is a parallelogram. Point R cuts SA in the ratio 1:3 while point P cuts FM in the ratio 3:1. The length of RP is 8. If the perimeter of SAMF is 52, find the perimeter of RAMP.

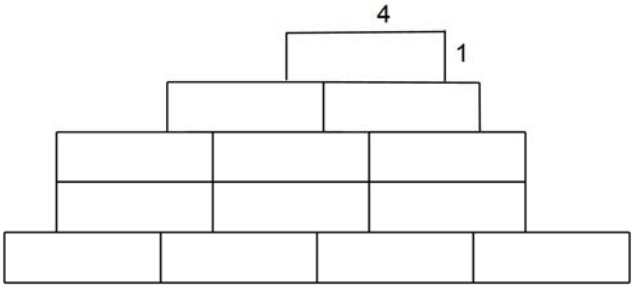


7. What is the smallest prime number that divides exactly into $3^{12} + 5^{13} + 7^{14} + 11^{15}$?

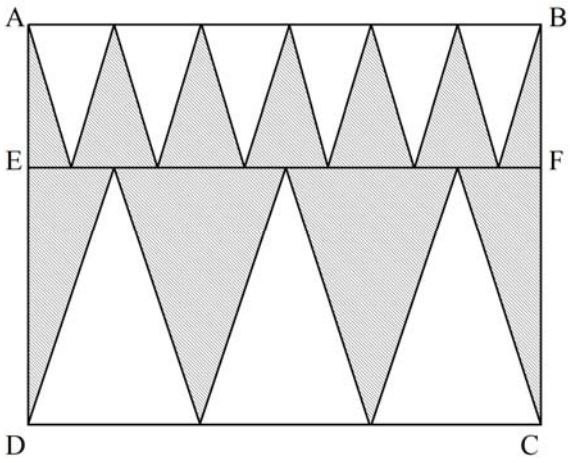
8. Divide 77 into three parts so that one of the parts is one-and-a-half times each of the other two. What is the value of the largest part?

9. 8 pieces of string are lying on the floor and I pick up two of the ends at random. If the probability that I am holding the two ends of the same piece of string is $\frac{1}{k}$, what is the value of k?

10. Identical rectangles of length 4 and breadth 1 are arranged on a plane as shown. Find the total perimeter of the shape.



11. In the figure shown, ABCD and CDEF are rectangles, AB = 9 and BC = 6. Determine the area of the shaded region in square units.



12. In the sequence of numbers 1, 4, 3, … each term after the first two is calculated as the term preceding it minus the term preceding that. So, for example, the third term is the second term minus the first term, i.e. 3 = 4 – 1. Find the sum of the first 2014 terms of the sequence.

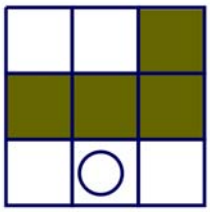
13. How many positive two-digit numbers become bigger when their digits are reversed?

14. A circle starts with a radius of 5 cm and its circumference is increasing at a rate of π cm each minute. How many minutes will it take for the area to become four times as big as at the start?

15. How many two-digit numbers N have the property that the sum of N and the number formed by reversing the digits of N is a perfect square?

Part C (6 marks each)

16. A game requires an L-shaped piece to be placed on a 3×3 grid and then a round piece to be placed in any one of the remaining squares. The L-shaped piece can be picked up and turned over or rotated but must exactly cover 4 squares.



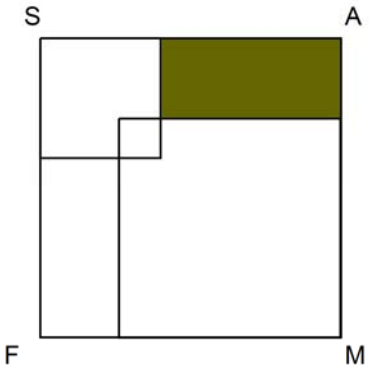
In how many different ways can the pieces be placed?

17. In the grid alongside, the sum of each row and of each column is the same.

1		y
3	x	
		4

What must be the value of $x - y$?

18. The square SAMF has an area of 169. It contains two overlapping squares. The smaller of these squares has an area that is one quarter of the larger of them, and the area of their overlap is 4. What is the area of the shaded region?



19. Four people put their hats on the table as they arrive. When they leave, each person picks up one hat. It so happens that no-one has picked up his own hat. In how many ways can this have happened?

20. The numbers 1 to 2014 are arranged in columns as shown:

a	b	c	d	e
1	2	3	4	
	8	7	6	5
9	10	11	12	
	16	15	14	13
	…	…	…	

How many multiples of 3 are in column e?