THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

SECOND ROUND 2000: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A: (Each correct answer is worth 4 marks)

1. ANSWER: C
You only need to consider the last digits, i.e. $1 \times 3 \times 5 = 15$, therefore the last digit will be 5.

2. ANSWER: D
2000 written in prime numbers is: $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^4 \times 5^3$

3. ANSWER: D
$2{,}001 \div 2{,}000 \approx 1$ and $1{,}999 \approx 2$ $\therefore \dfrac{2{,}001 \div 2{,}000}{1{,}999} \approx \dfrac{1}{2}$

4. ANSWER: B
Equivalent fractions to $\frac{2}{3}$ are: $\frac{4}{6}$; $\frac{6}{9}$; $\frac{8}{12}$; $\frac{10}{15}$, etc. Look for one of these fraction where the difference between the numerator of this fraction and the numerator of $\frac{8}{11}$ which is 8 is the same as the difference between the denominator of this fraction and the denominator of $\frac{8}{11}$ which is 11.
The fraction that works is $\frac{6}{9}$ because $8 - 6 = 2$ and $11 - 8 = 2$.
The answer is 2.

OR $\dfrac{8-a}{11-a} = \dfrac{2}{3}$ $\Rightarrow (8-a)3 = (11-a)2$
$\therefore 24 - 3a = 22 - 2a$
$\therefore a = 2$

5. ANSWER: B
Because the remainder is 2, 98 is fully divisible by x.
$198 = 98 + 98 + 2$
If 198 is divided by x, the remainder will be 2.

OR For $x > 0$, $\dfrac{100}{x}$ has rem 2. $\therefore \dfrac{98}{x}$ has no remainder

$\therefore \dfrac{198}{x} = \dfrac{100+98}{x} = \dfrac{100}{x} + \dfrac{98}{x}$ has remainder 2.

PART B: (Each correct answer is worth 5 marks)

6. ANSWER: B

2 λ/minute = 2000 cm^3/min. In one minute, say water level is h.

Then $\text{Volume} = 100\text{ cm} \times 100\text{ cm} \times \text{h} = 2000\text{ cm}^3$

$$\therefore \text{h} = 0{,}2\text{ cm}$$

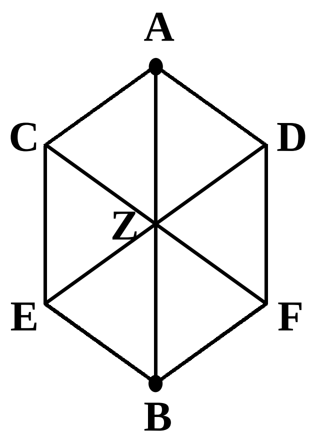
$\therefore$ The rate is 0,2 cm / min

7. ANSWER: C

The different paths from A to B are:

ACEB; ACZB; ACZEB; ACZFB,
(moving left from A through C)
ADFB; ADZB; ADZEB; ADZFB,
(moving right from A through D)
AZB; AZEB; AZFB.
(from A through Z)

Total: 4 + 4 + 3 = 11



8. ANSWER: C

For two digit numbers the sum of the two digits may only be 1, 4, 9 and 16, but 1 is not possible.

We get 4 from: 13, 22, 31 and 40
We get 9 from: 18, 27, 36, 45, 54, 63, 72, 81 and 90
We get 16 from: 79, 88 and 97
In total there are 16 two digit numbers of this type.

9. ANSWER: C

$990 = 2 \times 3 \times 3 \times 5 \times 11$

The appearance of 11 in the above suggests that 11! will be the smallest *n* since 11 is prime.

10. ANSWER: B

Total surface area (TSA) of Cube: $6(2 \times 2)\text{ m}^2 = 24\text{ m}^2$

TSA of Rectangular box: $[2(8 \times 1) + 2(4 \times 1) + 2(8 \times 4)]\text{ m}^2$

$[16 + 8 + 64]\text{ m}^2 = 88\text{ m}^2$

If 24 m^2 needs 12 λ of paint, i.e. 1 λ for every 2 m^2, 88 m^2 needs 44 λ of paint.

11. <u>ANSWER:</u> E

Let n be the number of tests written in the year.

Year average before last test: $89 = \dfrac{\text{sum of all tests}}{\text{number of test written}} = \dfrac{s}{n-1}$

$\therefore s = 89n - 89 \quad (1)$

Year average after last test: $90 = \dfrac{s+97}{n} \quad \therefore s = 90n - 97 \quad (2)$

$\therefore 90n - 97 = 89n - 89$ [From (1) and (2)]

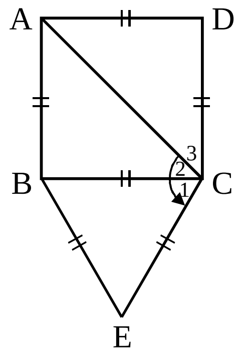
$\therefore n = 8$

12. <u>ANSWER:</u> A

$\hat{C}_2 = \frac{1}{2}$ *of* $90^\circ = 45^\circ$

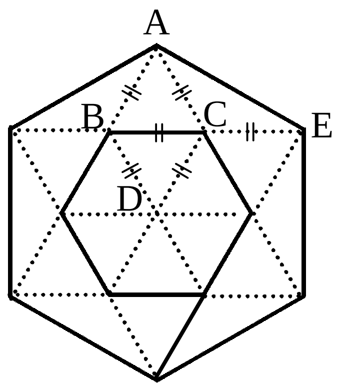
$\hat{E}_1 = 60^\circ$

$\therefore A\hat{C}E = 105^\circ$



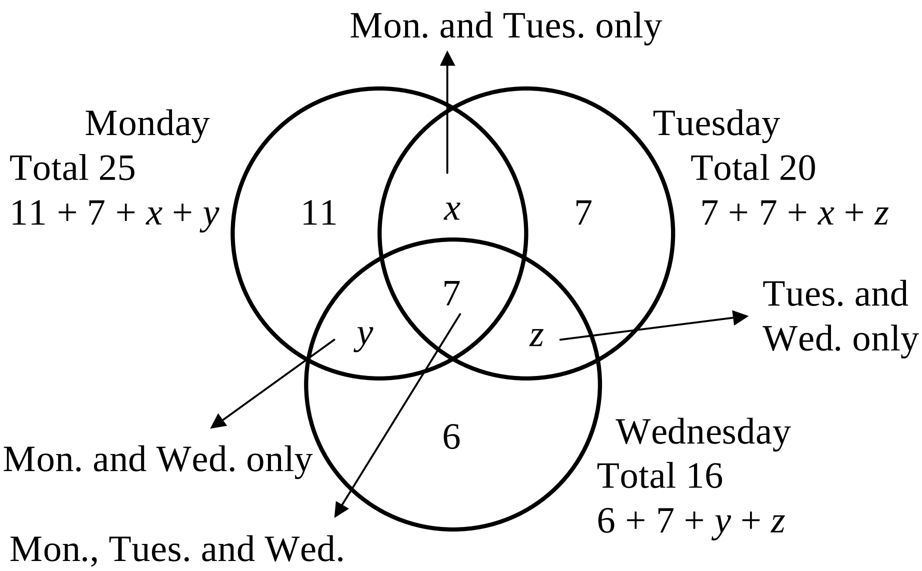
13. <u>ANSWER:</u> A

Use the following repeatedly:
Area of ΔABC = area of ΔBCD
(simply fold ΔABC onto ΔBCD).
Now Area of ΔABC = area of ΔACE
(equal base, same height).
So all the small triangles are equal in area.
The small hexagon has 6 small triangles and the big one has 18. So the small hexagon covers $\frac{6}{18} = \frac{1}{3}$ of the big one.



14. ANSWER: D

	Total	Only one day	All 3 days
Monday	25	11	7
Tuesday	20	7	7
Wednesday	16	6	7



$x+y=7$ (1) $\quad\therefore y-z=1$ (4)(*from* 1 *and* 2)

$x+z=6$ (2) $\quad x-y=3$ (5)(*from* 2 *and* 3)

$y+z=3$ (3) $\quad x-z=4$ (6)(*from* 4 *and* 5)

$2x=10$ (*from* 2 *and* 6)

$\therefore x=5, y=2, z=1$

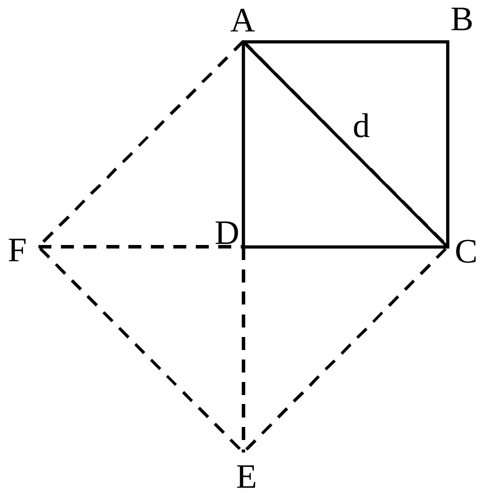
The total learners in the class is 11 + 7 +6 + 7 + 5 + 2 + 1 = 39

OR:

One day:	$11+7+6$		24
Two days	$\frac{7+6+3}{2}$	Divided by 2 because each number has been counted twice	8
Three days	7		7
Total			39

15. <u>ANSWER:</u> A

Construct a square using d as side as shown..



Now all 5 triangles are identical in the sketch. Clearly square ABCD has 2 triangles and square ACEF has 4 triangles and an area of d^2. Therefore square ABCD has area $\frac{1}{2}d^2$

OR

$$AD^2 + DC^2 = d^2, \text{ but } AD = DC$$

Using Pythagoras: $\therefore 2AD^2 = d^2 \qquad \therefore AD^2 = \frac{d^2}{2}$

$$\text{Area of square} = AD^2 = \frac{d^2}{2} = \frac{1}{2}d^2$$

PART C: (Each correct answer is worth 6 marks)

16. <u>ANSWER:</u> D

If n is even, say $n = 2k$, then $n^2 = 4k^2$ and $n^2 - 1 = 4k^2 - 1$ so that only n^2 is divisible by 4. If n is odd, say $n = 2m - 1$, then $n^2 = 4m^2 - 4m + 1$ and $n^2 - 1 = 4m^2 - 4m$ so that only $n^2 - 1$ is divisible by 4.

17. <u>ANSWER:</u> E

Suppose Pam ate P_M sweets on Monday and P_T sweets on Tuesday. Then the total number of sweets were $\frac{3}{4}P_M + P_M + \frac{2}{3}P_T + P_T = \frac{7}{4}P_M + \frac{5}{3}P_T = 31$

Since ate P_M and P_T are integers, the value of P_M is divisible by 4 and the value of P_T is divisible by 3. Looking at possibilities we get:

If $P_M = 0$, then $\frac{5}{3}P_T = 31$ has no integer solution.

If $P_M = 4$, then $\frac{5}{3}P_T - 7 = 31 - 7 = 24$ has no integer solution.

If $P_M = 8$, then $\frac{5}{3}P_T - 14 = 31 - 14 = 17$ has no integer solution.

If $P_M = 12$, then $\frac{5}{3}P_T - 21 = 31 - 21 = 10$ has $P_T = 6$ as only solution.

Therefore Sam ate $\frac{7}{4}P_M + \frac{5}{3}P_T = \frac{7}{4}(4) + \frac{5}{3}(6) = 9 + 4 = 13$ sweets.

18. ANSWER: A

If the train takes t hours when travelling at 36 km/h, then it takes $t+\frac{1}{2}$ hours when travelling at 27 km/h. Calculating the distance both ways then gives: $t \times 36 = (t+\frac{1}{2}) \times 27$. An easy calculation now shows that $t+\frac{3}{2}$ so that the distance between Springs and Soweto is $\frac{3}{2} \times 36 = 54$ km.

OR

	Distance	Speed $(= \frac{\text{Distance}}{\text{Time}})$	Time $(= \frac{\text{Distance}}{\text{Speed}})$
First journey	x km	36 km/h	$\frac{x}{36}$ hours
Second journey	x km	27 km/h	$\frac{x}{27}$ hours

$\frac{x}{36} - \frac{9}{60} = \frac{x}{27} - \frac{39}{60}$

$\therefore \frac{x}{36} - \frac{x}{27} = -\frac{1}{2}$

$\therefore 3x - 4x = -54$ (×108 LCM)

$\therefore x = 54$ km

19. ANSWER: C

Since the dot on the coin starts horizontally, then moves down to the line on which the coin rolls and never moves backwards, the graph (C) describes the correct path.

20. ANSWER: E

The possibilities for the statements to be true (T) or false (F) are summarized as follows:

		Possible truth values			
I:	The digit is 1	T	T	T	F
II:	The digit is 2	T	T	F	F
III:	The digit is not 3	T	F	T	T
IV:	The digit is not 4	F	T	T	T

Of these, the first two are impossible since the digit cannot be 1 and 2 at the same time. The last two are both possible and in both IV is true, that is, (E) is correct. (You should convince yourself that (A), (B), (C) and (D) are not necessarily correct for both the last two possibilities.)

THE END

ANSWER POSITIONS: JUNIOR SECOND ROUND 2000

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	C
2	D
3	D
4	B
5	B
6	B
7	C
8	C
9	C
10	B
11	E
12	A
13	A
14	D
15	A
16	D
17	E
18	A
19	C
20	E

DISTRIBUTION	
A	4
B	4
C	5
D	4
E	3
TOTAL	**20**