

SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade EIGHT First Round 2016

Solutions

1. **D** The last digit will be found by subtracting 8 from 15
2. **E** $2 \times 0 \times 1 \times 6 = 0$, and $\sqrt{0} = 0$
3. **D** 0,125 is half of 0,25, so result is 0,5
4. **D** Of the nine positions that the blue Smartie could occupy, only one is in the centre; so the probability is $\frac{1}{9}$
5. **A** The angles of the triangle add up to 180° , so $\hat{C} + 5\hat{C} + 120^\circ = 180^\circ$, i.e. $6\hat{C} = 60^\circ$ so $\hat{C} = 10^\circ$.
6. **B** If $x - y > x$ then $-y > 0$, so $y < 0$
7. **C** The multiples of 7 include 98 ($= 7 \times 14$), 105, 112, ..., 994 ($= 7 \times 142$), 1001, ... Of these, $142 - 14 = 128$ are between 100 and 1000.
8. **A** The number of years is $\frac{3000000}{7400} = \frac{30 \times 100}{7,4} \approx 400$
9. **C** Adding the frequencies shows the number of girls to be $1 + 3 + 2 = 6$
10. **B** Looking at the difference of successive pairs of small primes, we note that $3 - 2 = 1$ which is not prime, $5 - 2 = 3$ and $5 - 3 = 2$, and then larger numbers differing by 2 (or more). So the smallest sum comes from $5 + 2 = 7$
11. **E** In 8 hours the tank filled up a further 64%: that is 8% per hour. So in the first two hours it filled up 16 %, meaning it started with $36 - 16 = 20\%$
12. **A** At each level we have two choices, so we have $2 \times 2 \times 2 = 8$ options.
13. **E** There are 6 digits in the repeating part of the decimal form. 100 is $6 \times 16 + 4$, so the 100th digit will be the 4th in the repeating part 076923, which is 9

14. **B** The Jelly Tots to be counted are the ones not at either end of each row; the number of these for the stages shown are 0, 1, 3, 5, so we see that we need the 25th number in this sequence, which is $25 \times 2 - 3 = 47$.

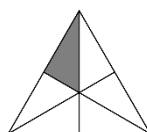
15. **D** Statements A, B and E can be made irrespective of whether Devious Steve is lying or telling the truth. Statement C can also be made whether he is telling the truth or lying. However, if statement D is true, then Steve is a liar, which contradicts the truth of statement D. Similarly if statement D is false, then Steve is a truth-teller, which contradicts the falsity of statement D.

16. **B** There are 9 ways to select the nonzero hundreds' digit, then 1 way to select the units' digit (same as the hundreds' digit), and then 10 ways to select the tens' digit. Hence there are $9 \times 1 \times 10 = 90$ three-digit palindromes.

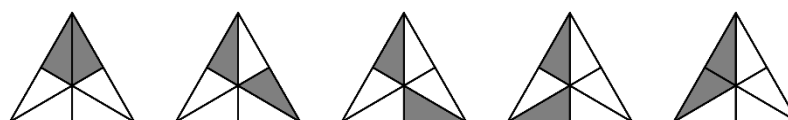
17. **A** Let the middle cell contain y . Then the middle column has total $x + y + 28$, while the bottom left to top right diagonal has total $31 + y + 33$, in other words $x + 28 = 31 + 33$. That easily shows $x = 36$.

18. **B** $\left(\frac{1}{8}\right)^2 \times 2^8 = \frac{1}{64} \times 256 = 4$ OR $\left(\frac{1}{2^3}\right)^2 \times 2^8 = \frac{1}{2^6} \times 2^8 = 2^2 = 4$

19. **E** Suppose the first chosen triangle is:



Then there are five possibilities for the second triangle:



Only the first, third and fifth of these have an axis of symmetry, which is three of the five. This is true no matter which triangle is chosen first. The probability of the resulting figure having an axis of symmetry is thus $\frac{3}{5}$.

20. **C**
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We note that the two lines cutting the square must meet at its centre; call that O, and draw the diagonal DB. It is now clear that the height of the shaded triangle is 1.5 cm, while its base is 2 cm, so its area is $\frac{1}{2} \times 1.5 \times 2 = 1.5 \text{ cm}^2$.