THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

# FIRST ROUND 2000: JUNIOR SECTION: GRADES 8 AND 9

## SOLUTIONS AND MODEL ANSWERS

**PRACTICE EXAMPLES:**

**1.** ANSWER: C

$$23+6-4=(23+6)-4=29-4=25$$

**2.** ANSWER: D

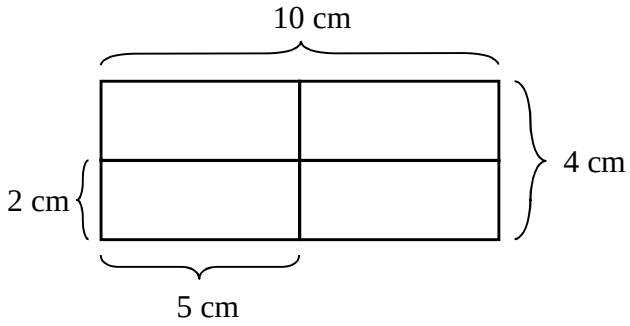
$$\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}=\frac{1}{5}+(\frac{2}{3}\times\frac{1}{2})=\frac{1}{5}+\frac{1}{3}=\frac{3+5}{15}=\frac{8}{15}$$

**QUESTIONS:**

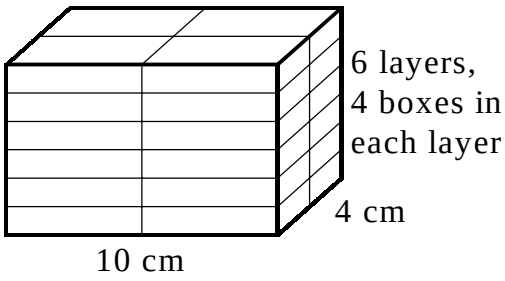
**1.** ANSWER: B

$$\frac{1}{2}-\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}-(\frac{1}{2}\times\frac{1}{2})=\frac{1}{2}-\frac{1}{4}=\frac{2-1}{4}=\frac{1}{4}$$

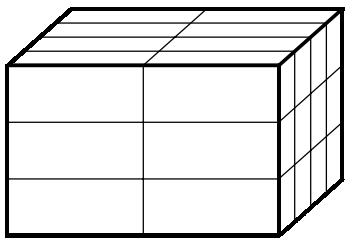
**2.** ANSWER: A




If a top view of the Box A is taken, it is clear that 4 boxes identical to Box B will fit into Box A. Because Box B is 1 cm and Box A is 6 cm high, 6 such layers will fit into Box A. Therefore 24 boxes, identical to Box B will fill Box A exactly.



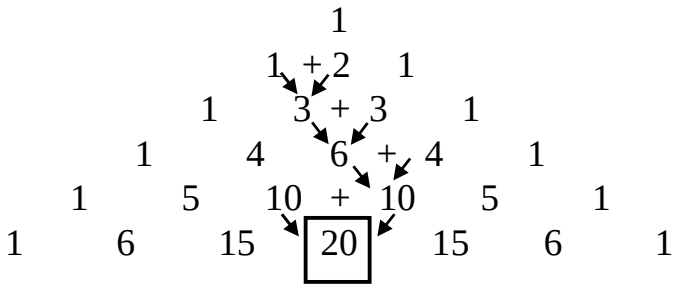

Another way to fill Box A could be:




There are more ways to fill Box A.
Try to find it.

**3.** ANSWER: B

The pattern works as follows: If 2 numbers in a line next to each other are added the number in the following line between these two numbers is produced.




Thus 10 + 10 = 20, which means the missing number in the square is 20.

**4.** ANSWER: D

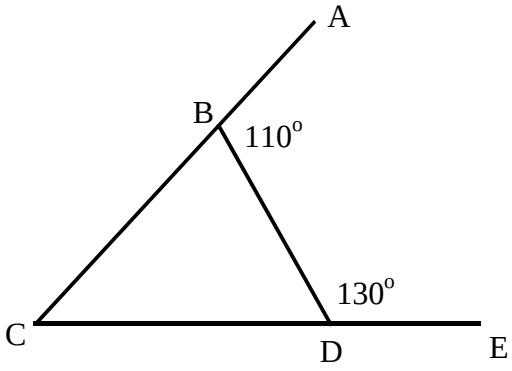
$C\hat{B}D = 70^\circ$ [*CBA is a straight line*]

$C\hat{D}B = 50^\circ$ [*CDE is a straight line*]

*but* $C\hat{B}D + C\hat{D}B + B\hat{C}D = 180^\circ$

[interior angles of a triangle is $180^\circ$]

$\therefore \hat{C} = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$




**5.** ANSWER: E

$0{,}3 \times 0{,}4 + 0{,}3 \times 0{,}9$

$= (0{,}3 \times 0{,}4) + (0{,}3 \times 0{,}9)$

$= 0{,}12 + 0{,}27$

$= 0{,}39$

**6.** ANSWER: E

$2 \Theta [(2 \times 3) + (3 \times 5)]$

$= 2 \Theta 21$

$= (2 \times 2) + (3 \times 21)$

$= 67$

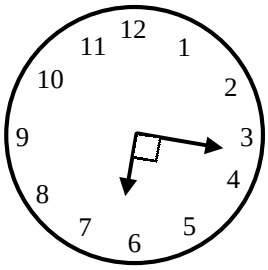
**7.** ANSWER: A

$\frac{10^4}{5^4} = \frac{(2 \times 5)^4}{5^4} = \frac{2^4 \times 5^4}{5^4} = 2^4 = 16$

*or* $\frac{10^4}{5^4} = \frac{10 \times 10 \times 10 \times 10}{5 \times 5 \times 5 \times 5} = \frac{10\,000}{625} = 16$

*or* $\frac{10^4}{5^4} = \left(\frac{10}{5}\right)^4 = 2^4 = 16$

**8.** ANSWER: B




The times the hour hand and the minute hand of a clock form a right angle with each other between 06:00 and 12:00 are:
±06:17; ±06:49; ±07:22; ±07:54; ±08:28; ±09:00; ±09:33; ±10:05; ±10:38; ±11:11; ±11:49 (11 times)

**Note:** This happens twice every hour, except between 08:00 and 10:00 when it happens only three times and not four times as expected. This is because at 09:00 exactly the hands form a right angle. Thus between 06:00 and 12:00 there are $(6\times 2)-1=11$ times.

**9.** ANSWER: B

The total of the original three numbers is $18\times 3=54$. Now the total becomes $23\times 3=69$. The total has increased by 15 because a number was replaced by 38. So, the original number must be $38-15=23$

**Or**

$$\frac{a+b+c}{3}=18 \qquad eq.1$$

but $\frac{a+b+38}{3}=23$ $\qquad eq.2$

*from eq.* 2 *it follows that*

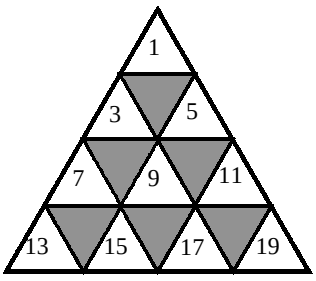
$a+b=(3\times 23)-38=31$

$\therefore a+b=31$ $\qquad eq.3$

*when* $a+b=31$ *is replaced in eq.*1 *we get*:

$\frac{31+c}{3}=18$ *and therefore*: $c=(3\times 18)-31=23.$

**10.** ANSWER: A




When the numbers in each row are added we get the following sequence: 1; 8; 27; 64 which is the sequence of cube numbers, each time the cube of the row number, therefore the sum of the numbers in the $100^{th}$ row will be $100^3$ = 1 000 000.

**11.** ANSWER: C

Four fifths of the container takes 8 hours to be filled, so clearly one fifth takes 2 hours. The remainder of the container is one fifth and therefore it will take 2 hours to be filled.

**Or**

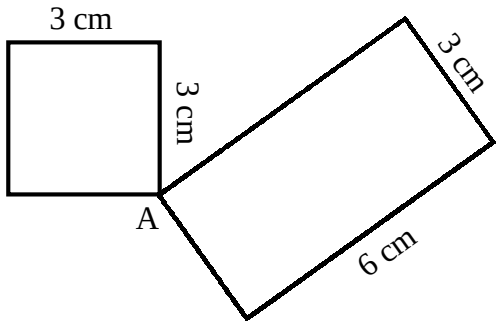
8 hours to fill $\frac{4}{5}$ of the container

$\therefore \frac{1}{5}$ of the container still needs to be filled

$\therefore \frac{1}{5} = \frac{1}{4} of \frac{4}{5}$ and therefore it will take

$\frac{1}{4} of\ 8 = 2$ hours to fill the remainder of the container.

**12.** ANSWER: D




The perimeter of the 3 x 3 square is 12 cm. The perimeter of the 6 x 3 rectangle is 18 cm. For the ants to meet again at point A each ant has to complete the route around the perimeter in full. The first time the two ants will meet again at point A after their departure will be when the multiple of 12 and 18 are the same, we are therefore on the look for the LCM (Lowest common multiple) of 12 and 18. The answer is 36 cm..

**Or** another way would have been to list the multiples of 12 and 18 to see when it gives the same number for the first time.

| | |
|---|---|
| 12 | 18 |
| 24 | **36** |
| **36** | 54 |
| 48 | **72** |
| 60 | 90 |
| **72** | 108 etc. |

**13.** ANSWER: A

The total of the row totals has to be the same as the total of the column totals, giving:

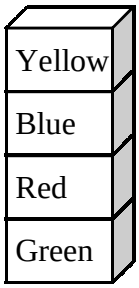
$20 + 14 + 12 = 17 + P + Q$

$P + Q = 46 - 17$

thus $P + Q = 29$

| | | | |
|---|---|---|---|
| *A* | *B* | *C* | 20 |
| *D* | *E* | *F* | 14 |
| *G* | *H* | *I* | 12 |
| 17 | *P* | *Q* | |

**14.** ANSWER: D

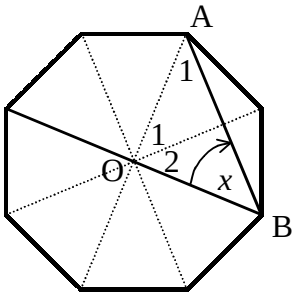



Any colour can be placed in the bottom position; there are 4 ways to do it. Any other colour can be placed on top of it; there are 3 ways to do it. The next position can be filled in 2 ways and the last in 1 way.
Therefore the number of different ways the four different coloured cubes can be stacked are $4 \times 3 \times 2 \times 1 = 24$.

**15.** ANSWER: B

There are 50 multiples of 2 from 1 to 100, both numbers included, namely 2, 4, 6, etc. There are 33 multiples of 3 from 1 to 100, both numbers included, namely 3, 6, 9, etc. When added up, 83 numbers were counted, of which some were counted twice, namely all the numbers which are both multiples of 2 and 3, e.g. 6, 12, 18, etc. The numbers that are both multiples of 2 and 3, at the same time, are also multiples of 6, therefore if the number of multiples of 6 are removed from the above number once, it will cover the doubles. There are 16 multiples of 6 from 1 to 100, both numbers included, namely 6, 12, 18, etc. Therefore 83 – 16 = 67 numbers have been removed and 33 remained.

**16.** ANSWER: C




Construction lines could be drawn as shown in the figure. The angles at O adds up to $360^o$ of which each angle is $\frac{1}{8}$ thereof, which is $45^o$.

$O_1 + O_2 = 45^\circ + 45^\circ = 90^\circ$

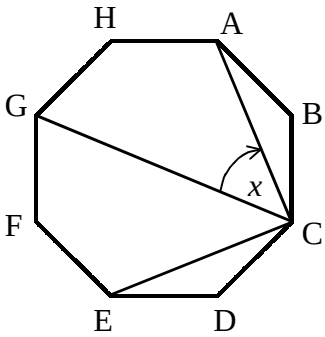
$O_1 + O_2 + \hat{A}_1 + x = 180^\circ$ [interior angles of $\Delta$]

but $\hat{A}_1 = x$ [OA = OB]

$\therefore 2x + O_1 + O_2 = 180^\circ \quad \therefore 2x = 180^\circ - 90^\circ$

$\therefore 2x = 90^\circ \quad \therefore x = 45^\circ$

**Or** by using symmetry:




$C\hat{A}B = A\hat{C}B$ [AB = BC]

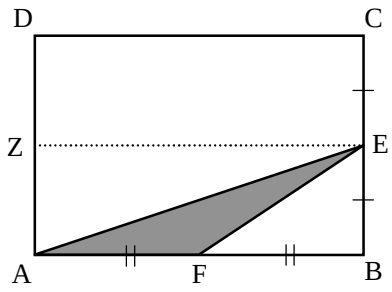
$\hat{B} = D\hat{C}B = 135^\circ$ [interior angles of octagon]

$\therefore A\hat{C}B = \left(\frac{180^\circ - 135^\circ}{2}\right) = 22{,}5^\circ$

$D\hat{C}E = A\hat{C}B = 22{,}5^\circ$ and $A\hat{C}G = G\hat{C}E = x$

$\therefore 2x + 2(22{,}5^\circ) = 135^\circ \quad \therefore x = 45^\circ$

**17.** ANSWER: E




Draw construction line ZE

ZE = AB, ZE//AB

Area $\Delta AFE$ = Area $\Delta FBE = 7\ \text{cm}^2$

[*Equal base* ($AF = FB$), *equal height* ($BE$)]

Area $\Delta ABE$ = Area $\Delta AZE = 14\ \text{cm}^2$

[*Equal base* ($BE = AZ$), *equal height* ($AB$)]

Area AZEB = Area DZEC $= 28\ \text{cm}^2$

[*Equal base* ($AB = DC$), *equal height* ($BE = CE$)]

$\therefore$ Area ABCD $= 56\ \text{cm}^2$

**18.** ANSWER: D

Cost: Ruler: R$x$, Pencil R$y$, Eraser R$z$

$3x + y - 2z = 0$ $\quad eq.1$

$x + 2y + 3z = 25$ $\quad eq.2$

$9x + 3y - 6z = 0$ $\quad 3 \times (eq.1)$

$2x + 4y + 6z = 50$ $\quad 2 \times (eq.2)$

$11x + 7y + 0 = 50$ $\quad eq.3$

From eq. 3, possible values for $x$ are:

If $x = 1$, no whole number value possible for $y$. $(7y = 39)$

If $x = 2$, the value for $y$ will be 4. $(7y = 28)$

If $x = 3$, no whole number value possible for $y$. $(7y = 17)$

If $x = 4$, no whole number value possible for $y$. $(7y = 6)$

In $eq.1$: $3(2) + 4 - 2z = 0$

$\therefore 2z = 10 \qquad \therefore z = 5$

Answer: 1 eraser costs R5,00

**19.** ANSWER: C

$800 - 10x$ increased by 10% gives:

$1{,}1 \times (800 - 10x) = 880 - 11x$

$\therefore 600 - 6x = 880 - 11x$

$\therefore 5x = 280$

$\therefore x = 56$

**20.** ANSWER: D

The value of the $n^{th}$ odd number is: $2n - 1$

$(e.g.\ the\ 3^{rd}\ odd\ number\ is\ 2(3) - 1 = 5,\ etc.)$

The value of the $n^{th}$ odd number plus the first odd number is:

$(2n - 1) + 1 = 2n$

The value of the $(n\text{-}1)^{th}$ odd number plus the second odd number is:

$(2n - 3) + 3 = 2n$

There are $\frac{n}{2}$ such combinations each equal to $2n$.

$\therefore \frac{n}{2}(2n) = 2304$

$\therefore n^2 = 2304$

$\therefore n = 48$

**THE END**

# ANSWER POSITIONS: JUNIOR FIRST ROUND 2000

| PRACTICE EXAMPLES | POSITION |
|---|---|
| 1 | C |
| 2 | D |

| NUMBER | POSITION |
|---|---|
| 1 | B |
| 2 | A |
| 3 | B |
| 4 | D |
| 5 | E |
| 6 | E |
| 7 | A |
| 8 | B |
| 9 | B |
| 10 | A |
| 11 | C |
| 12 | D |
| 13 | A |
| 14 | D |
| 15 | B |
| 16 | C |
| 17 | E |
| 18 | D |
| 19 | C |
| 20 | D |

| DISTRIBUTION | |
|---|---|
| A | 4 |
| B | 5 |
| C | 3 |
| D | 5 |
| E | 3 |
| **TOTAL** | **20** |