THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

Senior First Round 2022
Solutions

1. **Answer B**
   $2022^0 + 2 \times 2022 = 1 + 4044 = 4045$.

2. **Answer D**
   Either just count on your fingers, or proceed as follows. The n-th number is $43 - 4(n-1) = 47 - 4n$, and the last positive number in the sequence is $47 - 4 \times 11 = 3$, so $n = 11$.

3. **Answer E**
   $\sqrt{\frac{1}{9} + \frac{1}{16}} = \sqrt{\frac{16+9}{9 \times 16}} = \sqrt{\frac{25}{144}} = \frac{5}{12}$.

4. **Answer A**
   The powers $3^0, 3^1, 3^2, 3^3$ have units digits $1, 3, 9, 7$, respectively. This eliminates four of the five choices, leaving only 5 as the answer. Alternatively, by inspection we see that the sequence $1, 3, 9, 7$ repeats for higher powers of 3, so 5 never appears.

5. **Answer B**
   $2022 = 2 \times 1011 = 2 \times 3 \times 337$, which is the prime factorization. The numbers between 100 and 1000 that divide exactly into 2022 are 337 and $2 \times 337 = 674$, and their sum is $3 \times 337 = 1011$.

6. **Answer E**
   The solution is $x - 10 = \pm 2022$, say $x_1 - 10 = +2022$ and $x_2 - 10 = -2022$. Adding these two gives $x_1 + x_2 - 20 = 0$, so $x_1 + x_2 = 20$.

7. **Answer C**
   By inspection there is a solution $x = 2$, since $2 \times 3 = 6$, which gives the answer $12 = 3 \times 4$.

8. **Answer D**
   In the notation of exponents, $\sqrt[p]{2022} = 2022^{\frac{1}{p}}$ for any positive integer p. Since area $=$ length $\times$ width, it follows that

   $$\sqrt[n]{2022} = 2022^{\frac{1}{6}} \times 2022^{\frac{1}{12}} = 2022^{\frac{1}{6}+\frac{1}{12}} = 2022^{\frac{1}{4}} = \sqrt[4]{2022}, \text{ so } n = 4.$$

9. **Answer A**
   The sum of the angles of a quadrilateral is $360°$, so (in degrees) $2x + 2y = 360° - 65° - 125° = 170°$, giving $x + y = 85°$. Finally $z = 180° - (x + y) = 180° - 85° = 95°$.

10. **Answer C**
    The number can have either two or three digits. For two-digit numbers, since $6 = 1 \times 6 = 2 \times 3$, the possibilities are $16, 23, 32, 61$. For three-digit numbers, the expression $6 = 1 \times 1 \times 6$ gives three possibilities by rearranging the factors, and $6 = 1 \times 2 \times 3$ gives six possibilities. The total is therefore $4 + 3 + 6 = 13$.

11. **Answer E**
There are eight shaded small squares, which is half of the number in the big square, so the area of the big square is $128 \times 2 = 256$ cm^2, and therefore its sides are of length $\sqrt{256} = 16$ cm. The perimeter of the large square is therefore $4 \times 16 = 64$ cm.

12. **Answer A**
Suppose there are n games in the season and Thomas's total score is S just before the last game. We are given $\frac{1}{n}(S+19) = 18$, so $S+19 = 18n$. Similarly, $S+35 = 20n$, and subtracting gives $2n = 16$, so $n = 8$.

13. **Answer B**
The shaded regions consist of one full circle and four quarter circles, so the combined area is the same as that of two circles. The combined area of the unshaded regions is therefore the same as that of three circles (either $5-2$ or $4 \times \frac{3}{4}$), and the ratio of shaded to unshaded is $2:3$.

14. **Answer E**
The vertical matchsticks form 20 rows with 11 sticks in each row, and the horizontal matchsticks form 21 rows with 10 sticks in each row. The total number of matchsticks is therefore $20 \times 11 + 21 \times 10 = 430$.

15. **Answer B**
For the first drawing, there are ten letters in the bag, including four A's, so the probability of drawing A is $\frac{4}{10} = \frac{2}{5}$. For the second drawing, there are nine letters, including one R, so the probability of drawing R second is $\frac{1}{9}$, and similarly the probability of drawing C third is $\frac{1}{8}$. Thus the probability of drawing ARC in that order is $\frac{2}{5} \times \frac{1}{9} \times \frac{1}{8} = \frac{1}{180}$.

16. **Answer B**
By factorizing into primes (or otherwise), we see that the highest common factor of $80, 128, 112$ is 16. Thus the maximum number of cookies is 16 (and each cookie has 5 chocolate chips, 7 pecan nuts and 8 Smarties).

17. **Answer D**
If the paths are of width x m, then $(8-x)(6-x) = \frac{1}{2}(8 \times 6)$, giving $x^2 - 14x + 24 = 0$. This factorizes to $(x-2)(x-12) = 0$, and $x = 12$ is impossible, so $x = 2$.

18. **Answer D**
The edges of the cubes have lengths $1, 2, 3$, so the total surface area is $6(1^2+2^2+3^2) = 84$. To obtain the minimum surface area after gluing, the middle cube must be glued to the largest cube, and the smallest cube glued to both of the larger ones. The gluing reduces the outer surface by $2 \times (2^2+1^2+1^2) = 12$, and the remaining surface area is 72.

19. **Answer A**
Factorizing gives $496 = 16 \times 31 = 2^4(2^5-1) = 2^9 - 2^4$, so $m = 9$ and $n = 4$, and $m + n = 13$.

20. **Answer C**
The full hexagon is formed from six equilateral triangles, each of side length $\frac{1}{2} \times 12 =$

6 and therefore height $3\sqrt{3}$ (using Pythagoras' theorem). The six unshaded triangles can be combined into three equilateral triangles, congruent to those above. The shaded area of the saw blade is therefore equal to the area of three triangles, that is, $3 \times (\frac{1}{2} \times 6 \times 3\sqrt{3}) = 27\sqrt{3}$.
