THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

SECOND ROUND 1998: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

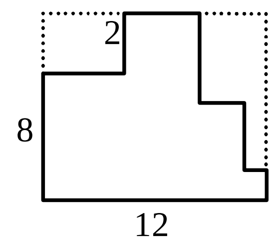
PART A: (Each correct answer is worth 3 marks)

1. ANSWER: D

$(5\times4\times3\times2)-(5\times4\times3)$

$=120-60=60$

2. ANSWER: A



$L=12$

$B=2+8=10$

$\therefore P=2(10+12)=44$

The original figure has the same perimeter as has the figure with the dotted lines.

3. ANSWER: B

$6 ☺ 2 = 6\times2+\frac{6}{2}$

$12 + 3 = 15$

4. ANSWER: A

$A=\frac{1}{2}b\times\perp h$

$\therefore 24=\frac{1}{2}\times8\times MP$

$\therefore MP=6$

$MP = 6$ and $PQ = 8$ and

$MQ^2 = MP^2 + PQ^2$ (*Pythagoras*)

$\therefore MQ=\sqrt{6^2+8^2}=10$

But perimeter $=(MQ+PQ+MP)$

$\therefore$ Perimeter is $10+8+6=24$ cm

5. ANSWER: D

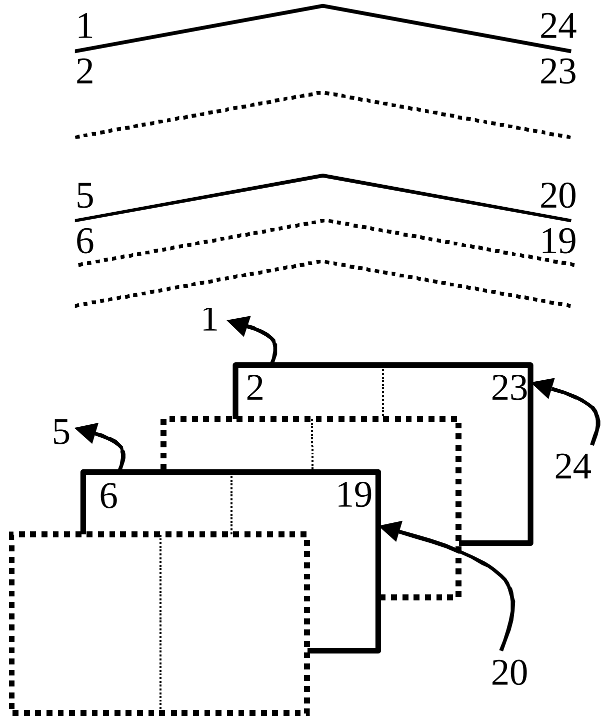
Total number of oranges is $m\times p$. Oranges lost = $mp-\frac{3}{4}mp=\frac{1}{4}mp$

and $\frac{1}{4}$ is the same as 25%

6. <u>ANSWER:</u> C

If the newspaper consists of 24 pages, then pp 1 and 2 have to be on the same sheet as pp 23 and 24. Pp 3 and 4 on same sheet together with pp 21 and 22. Thus 6 and 20 are on the same sheet together with pp 5 and 19.

Or look at it this way:



PART B: (Each correct answer is worth 5 marks)

7. <u>ANSWER:</u> D

LCM of 15 and 20

$15 = 3 \times 5$

$20 = 2 \times 2 \times 5$

LCM: $15 \times 4 = 60$

$60 \div 15 = 4$

Thus after 4 rotations with the 15 teeth gear the marked teeth will be together again.

Or

15:	15	30	45	60	75
20:	20	40	60	80	

Thus LCM = 60.

Thus after 4 rotations with the 15 teeth gear and 3 rotations with the 20 teeth gear the marked teeth will be together again.

8. <u>ANSWER:</u> D

Each one is the power of 2 minus the previous expression:

(A) $2^2 - 2 + 1 = 4 - 2 + 1 = 3$ Prime

(B) $2^3 - (2^2 - 2 + 1) = 8 - 3 = 5$ Prime

(C) $2^4 - (2^3 - 2^2 + 2 - 1) = 16 - 5 = 11$ Prime

(D) $2^5 - (2^4 - 2^3 + 2^2 - 2 + 1) = 32 - 11 = 21$ Not Prime

(E) $2^6 - (2^5 - 2^4 + 2^3 - 2^2 + 2 - 1) = 64 - 21 = 43$ Prime

9. <u>ANSWER:</u> C

The algebraic method:

Take length as x and width as y

$\therefore Area_{orig} = l \times b$ $Perimeter_{orig} = 2(l + b)$

$\therefore A_{orig} = xy$ $P_{orig} = 2(x + y)$

For $2x$ and $2y$:

$A_{new} = 2x \times 2y = 4xy$ $P_{new} = 2(2x + 2y) = 2[2(x + y)]$

$\therefore$ New area is 4 times the original area

$\therefore$ New perimeter is double the original perimeter

or

Take a few examples say:

For $l = 2$ *and* $b = 1$:

$A = 2$ *and* $P = 6$

For $l = 4$ *and* $b = 2$:

$A = 8$ *and* $P = 12$

Rule has to work for every special case of length and width

10. <u>ANSWER:</u> C

Top layer has 1 orange

2nd layer has 4 oranges

3rd layer has 9 oranges

4th layer has 16 oranges

5th layer has 25 oranges

6th layer has 36 oranges

Thus total is $1 + 4 + 9 + 16 + 25 + 36 = 91$

11. <u>ANSWER:</u> E

$n = 30^\circ + (180^\circ - m)$ $(ext.\angle\ of\ \Delta)$

$n + m = 210^\circ$

$but\ m = \frac{2}{3}n \qquad or\ \ n = \frac{3}{2}m$

$\therefore m + \frac{3}{2}m = 210^\circ$

$\therefore \frac{5}{2}m = 210^\circ \qquad \therefore m = 84^\circ$

12. <u>ANSWER:</u> B

For each pair of two colours there are 4 possibilities:

$3 \times 4 = 12$

Red & Green	Red & Yellow	Green & Yellow
RRRRG	*YYYYR*	*GGGGY*
RRRGG	*YYYRR*	*GGGYY*
RRGGG	*YYRRR*	*GGYYY*
RGGGG	*YRRRR*	*GYYYY*

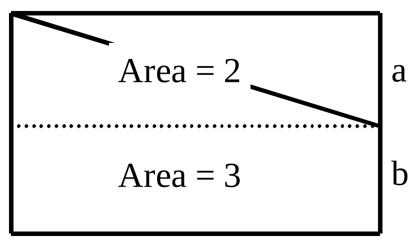
There is a maximum number of 12 learners in the class.

13. <u>ANSWER:</u> E

Upper rectangle has area 2.

Lower rectangle has area 3.

Therefore the ratio a:b is 2:3



14. <u>ANSWER:</u> E

$3^1 = 3$ If looking at the last digits

$3^2 = 9$ you get the following pattern:

$3^3 = 27$ $3; 9; 7; 1; 3; \ldots etc$

$3^4 = 81$

$3^5 = 243\, etc.$

Note that when the exponent is a multiple of 4, the answer ends in 1.

1998 is 2 less than 2000 (which is a multiple of 4), and hence ends in 1.

Therefore 3^{1998} ends in 9, and $\frac{9}{5}$ gives a remainder of 4.

PART C: (Each correct answer is worth 7 marks)

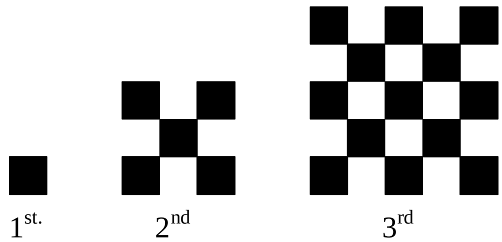
15. ANSWER: D

$$\text{Fig.}1 \rightarrow \frac{1}{2}(1^2+1)=1$$

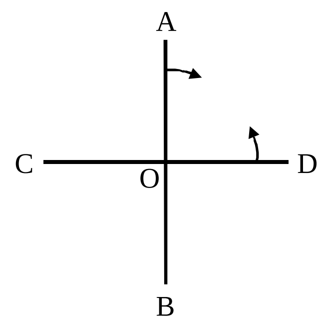
$$\text{Fig.}2 \rightarrow \frac{1}{2}(3^2+1)=5$$

$$\text{Fig.}3 \rightarrow \frac{1}{2}(5^2+1)=13$$

$$\text{Fig.}15 \rightarrow \frac{1}{2}(29^2+1)=421$$



16. ANSWER: C



AB: 360° in 2 min

$\therefore 180^\circ$ / min

CD: 1080° in 1 min

$\therefore 1080^\circ$ / min

After x min they meet:

AB has covered $180x$ degrees and CD $1080x$ degrees:

$180x+1080x=90^\circ$ (Total degrees moved are 90°)

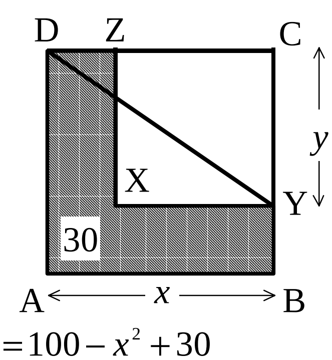
$\therefore 1260x=90^\circ$

$\therefore x=\dfrac{1}{14}$ min

$\therefore x=4\dfrac{2}{7}$ sec $\quad (\dfrac{1}{14}\div 60)$

17. ANSWER: B

Let x be the side length of square ABCD and y the side length of square XYCZ



Then: $x^2=y^2+30 \quad ...1$

$(Area\ ABCD = Area\ XYCZ + 30)$

and $DY^2=x^2+y^2 \quad ...2 \quad (Pyth. \Delta DYC)$

$\therefore 100=x^2+y^2 \quad (DY=10,\ given)$

$\therefore y^2=100-x^2$

In 1: $x^2=100-x^2+30$

$\therefore 2x^2=130$

$\therefore x^2=65 \quad \therefore x=\sqrt{65}$

$\therefore CD=\sqrt{65}$

18. ANSWER: B

$$18s + 6m = 5l \quad ...eq.1$$

$$2m + 1l = 10s \quad ...eq.2$$

$$?s = l$$

$$(3\times eq.2) \qquad 6m + 3l \quad = 30s$$

$$Minus\ eq.1: \qquad \underline{18s + 6m \qquad = 5l}$$

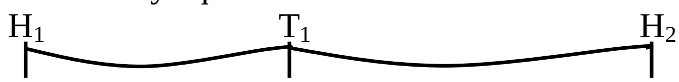
$$-18s \qquad +3l \quad = 30s - 5l$$

$$\therefore 8l = 48s \quad \therefore l = 6s$$

Thus 6 small marbles weigh as much as 1 large marble

19. ANSWER: D

Take 2 houses with a tree between them, the total distance the children have to walk to the tree is exactly equal to the distance between the two houses.



If the tree was not between the houses, the total distance the children have to walk to the tree would be more than the distance from one house to the other.

The tree that we are looking for, therefore has to be between the 2 houses for the shortest distance. Thus we need to find a tree so that there are exactly as many houses on both sides of the tree. The answer is D.

20. ANSWER: E Follow steps 1 to 6

Girls	Distance (A)	Speed (S)	Time (t)
A	100	S_a 1	$t_a = \frac{100}{S_a}$ 2
B	90	$S_b = \frac{90}{t_a}$ 3	
B	100	$S_b = \frac{90}{t_a}$	$t_b = \frac{100t_a}{90}$ 4
C	$A_c = \frac{72}{t_a} \times t_a = 72$ m 6	$S_c = \frac{80}{t_b} = \frac{80}{100t_a} \times 90 = \frac{72}{t_a}$ 5	

Or B ran $\frac{9}{10}$ of the distance of A and C ran $\frac{8}{10}$ of the distance of B. Thus C ran $\frac{8}{10} \times \frac{9}{10} = \frac{72}{100}$ the distance of A. Thus C ran 72 m when A finished (100 m), therefore C was 28 m behind A.

THE END