SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade NINE First Round 2022

Solutions

1. **A** $2 \times 5^2 - 5 \times 2^2 = 2 \times 25 - 5 \times 4 = 50 - 20 = 30$

2. **B** One tenth and one fifth in decimal form are 0,1 and 0,2 respectively. Of those given, the only fraction lying between 0,1 and 0,2 is 0,18.

3. **D** $\frac{20 \times 22}{2^0 \times 2^2} = \frac{20 \times 22}{1 \times 4} = 5 \times 22 = 110$

4. **E** If $\frac{\sqrt[3]{p}}{4} = 1$ then $\sqrt[3]{p} = 4$, thus $p = 64$.

5. **B** $M\hat{S}E = 80°$ (vertically opposite), $M\hat{E}S = 80°$ (isosceles triangle), thus $E\hat{M}N = 30°$ (exterior angle of a triangle).

6. **C** Each postcard requires 4 drawing pins. However, since there are 24 overlaps we will only need $25 \times 4 - 24 = 76$ drawing pins.
Alternatively, for 1, 2 and 3 postcards we need 4, 7 and 10 drawing pins respectively. In general, for n postcards we require $3n + 1$ drawing pins.
For 25 postcards we thus need $3 \times 25 + 1 = 76$ drawing pins.

7. **A** Area of shaded region $= 30 \times 4 + 40 \times 2 - 4 \times 2 = 192\ u^2$.

8. **C** Each square and triangle has side length $30\ cm \div 4 = 7{,}5\ cm$. There are 24 sides each of length $7{,}5\ cm$, so the amount of wire used is $7{,}5\ cm \times 24 = 180\ cm$.

9. **C** The two shaded semicircles can fit into the two unshaded semicircles in square ABCD. The shaded area is thus equivalent to the area of the square, i.e. 144 units^2.

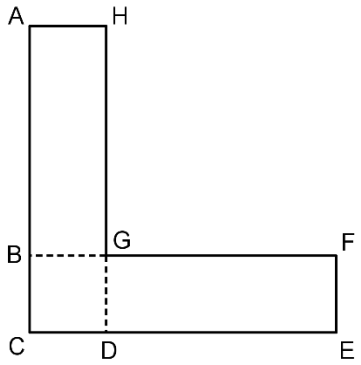
10. **B** C is clearly the largest. Note that (A) is equivalent to (D). Comparing these fractions (with numerator of 1), (B) has the largest denominator and will thus be the smallest fraction.

11. **A** The ratio of the two diameters, and hence the two circumferences, is $\frac{5}{3}$.
Wheel A would thus need to make $60 \times \frac{5}{3} = 100$ revolutions to cover the same distance as wheel B.

12. **B** The area of the smallest square is $5 \times 5 = 25$ square units. The sum of the areas of the four regions is $4 \times 25 = 100$. So the area of the largest square is 100. Hence, the side length of the largest square is 10.

13. **D** At 9:00 the angle between the hour hand and the minute hand is 90°.
At 9:10 the minute hand has moved $\frac{1}{6} \times 360° = 60°$ while the hour hand has moved $\frac{1}{6} \times 30° = 5°$. The obtuse angle between the two hands at 9:10 is thus $90° + 60° - 5° = 145°$.

14. **E** Let the longer side of each rectangle be x. Then the perimeter of the L-shape is: $Per = AC + CE + EF + FG + GH + HA = AC + CE + (EF + GH) + (FG + HA) = 4x$. Thus $4x = 20$, from which we have $x = 5$. The longer side of each rectangle is thus 5 units, and since the area of each of these rectangles is 10 units^2, the width must be 2 units. The perimeter of each rectangle is thus 14 units.



15. **E** Let us call the five people A, B, C, D and E. Suppose E is not in any of the two teams. Then A could be paired with either B, C or D (i.e. 3 possibilities). Once A is paired, the second team is automatically also paired (e.g. if A is paired with C then B and D would be the second team). Thus, there are 3 ways to form the teams if E is excluded. Likewise, there would be 3 ways to form the teams if any particular individual is excluded. Hence, the total number of ways to form the two teams is $5 \times 3 = 15$.

16. **A** $1\%\ of\ P = \frac{1}{100} \times \frac{1}{4} \times 20^{22} = \frac{1}{400} \times 20^{22} = \frac{20^{22}}{20 \times 20} = 20^{20}$.

17. **D** Let x be the required number of litres of Brand A. The volume of Brand B required would thus be $50 - x$. The total cost of the blend would thus be $15x + 10(50 - x)$. Solving the equation $50 \times 12 = 15x + 10(50 - x)$ yields $x = 20$.

18. **C** There are 100 numbers (300 – 399) that have a 3 in the hundreds position, 10 numbers (330 – 339) that have a 3 in the tens position, and 10 numbers (303, 313, 323, … , 393) that have a 3 in the units position. In total there are thus 100 + 10 + 10 = 120 instances of the digit 3.

19. **E** The sum of the digits of the 5-digit number would be 1+2+3+4+5=15 which is divisible by 3. Thus, no matter the arrangement of the 5 digits, the 5-digit number will be divisible by 3. If the last digit of the 5-digit number is either 2 or 4 then the 5-digit number would be divisible by both 3 and 2, and hence divisible by 6. Hence, the required probability is 2/5.

20. **D**