THE OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2022 THIRD ROUND
JUNIOR SECTION: GRADES 8 AND 9

28 July 2022 **Time: 4 Hours** **Number of questions: 16**

TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form, or any geometric instruments may be used.
- Use your time wisely and do not spend all your time on only a few questions.
- Questions are not necessarily arranged in order of difficulty.
- Answers and solutions will be made available at: www.samf.ac.za

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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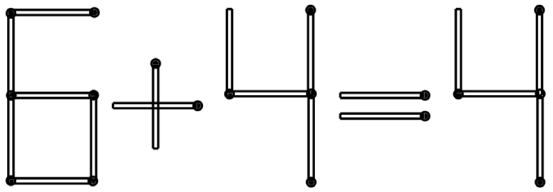
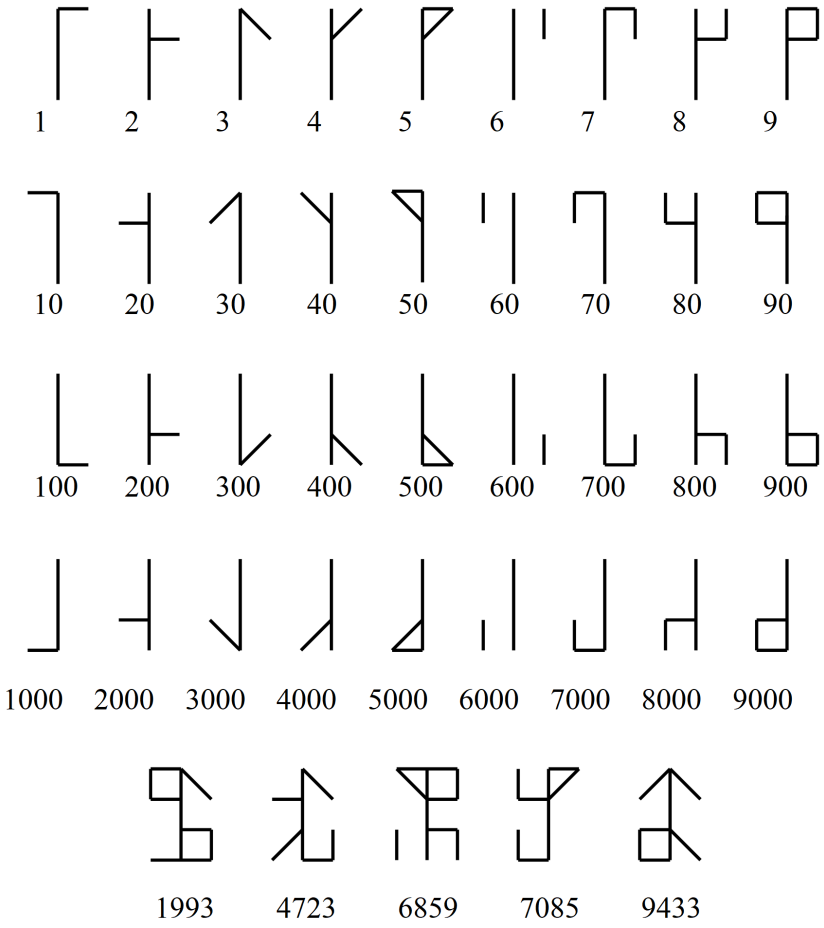
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

1) Move one matchstick and replace it elsewhere to make the statement true. The Olympiad committee could only find three possible solutions, so bonus marks if you find more!

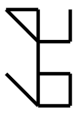


(3)

2) The Cistercian monks invented a numbering system in the 13th century which meant that any number between 1 and 9999 could be written using a single symbol.



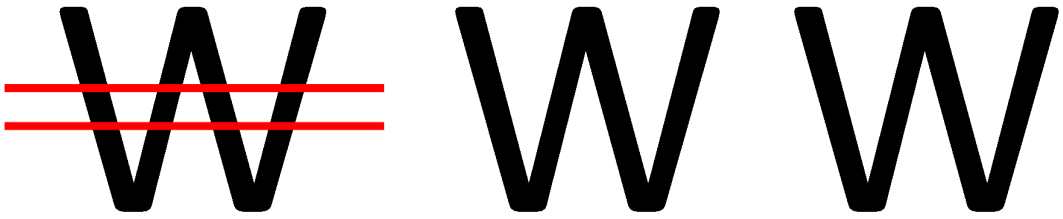
a) What is the numerical value of the Cistercian symbol?



(2)

b) Write the number 2022 using a Cistercian numbering symbol. (3)

3) Two horizontal lines cut the letter W into 9 pieces.

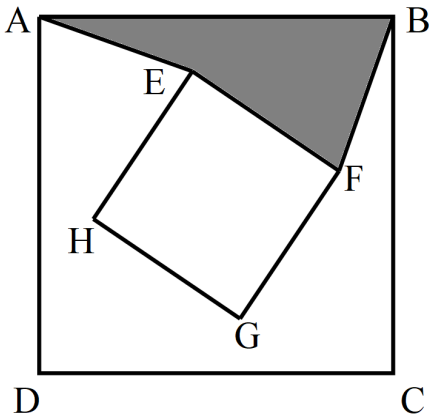


a) Using four horizontal lines, how many pieces do you get? (2)

b) Determine the smallest number of lines needed to cut the W into at least 2022 pieces. (3)

4) Four friends were racing side by side down a dusty staircase. Ambete went down two steps at a time, Bruce three steps at a time, Claire four steps at a time and Divakaran five steps at a time.
If the only steps with all four's footprints were at the top and the bottom, how many steps had just one person's footprints? (5)

5) In the figure below square ABCD has side length 12 and square EFGH has side length 6. **Both of which have the same centre**.
Interesting fact: the area of the shaded quadrilateral ABFE is always constant and does not depend on the rotation of square EFGH.



What is the area of the shaded region? (4)

6) Consider the numbers: 24, 55, 27, 64 and x

- The average of these five numbers is prime
- The median is a multiple of 3

(The median of a set of numbers is the middle number when the set is arranged in ascending order. For example, the median of 1,2, 5, 8, 10 is 5).

Calculate the sum of all the possible positive integer values of x.

(6)

7) All positive integers n for which $n(n+1)(n+2)$ is a multiple of 5 are listed in increasing order.

a) Give the first three possible values of n. (2)

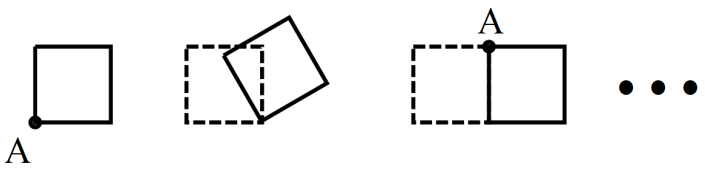
b) What is the 2022nd number in this list? (4)

8) Find the smallest integer which, when multiplied by 123 yields a product that ends in 2022.

(6)

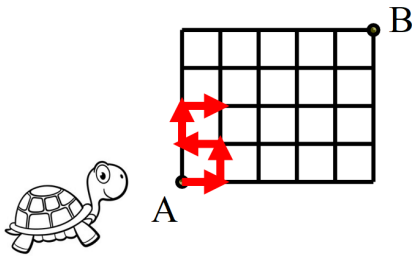
9) A square "wheel" rolls without slipping along a straight level road until it has completed one revolution. If the side length is 1 cm, how far does point A travel?

(6)



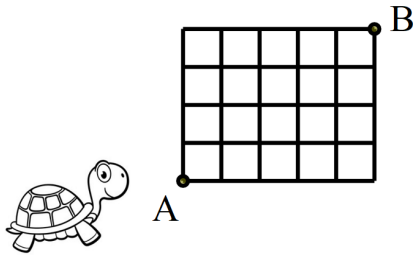
10) A tortoise moves around on a chequered grid with unit squares by changing direction each time it reaches a new square.

For example:



a) What is the shortest distance it travels from A to B on a 4×5 grid?

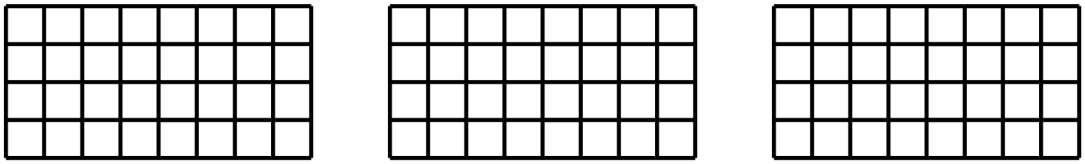
(2)



b) If the tortoise travels from the lower left to the upper right corner of a 2022×4044 grid, what is the shortest distance it travels?

(5)

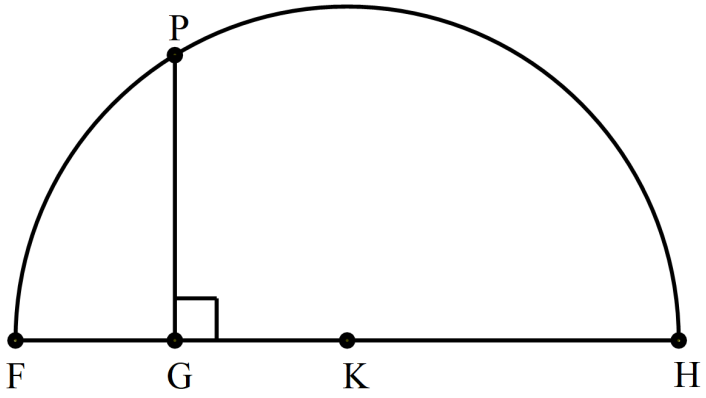
(some blank 4×8 grids have been drawn below to help you make some deductions)



11) Here's a cool way to find the square root of a number!

Suppose you want to find $\sqrt{12}$:

- You draw a line segment of length 12 units. (GH in the diagram).
- You extend GH by a length of 1 unit. (FG in the diagram).
- With a diameter FH and K as a centre you draw a semicircle.
- You construct PG perpendicular to FH to touch the circle at P.
- Then $\mathrm{PG} = \sqrt{12}$.



Prove that this construction method works for finding the square root of any number.

(7)

12) Let a sequence $a_0, a_1, \ldots$ be defined by

$$a_0 = 2022 \qquad a_{n+1} = \frac{1 + a_n}{1 - a_n} \qquad \text{for } n = 0, 1, \ldots$$

What is the value of a_{2022} ?

(6)

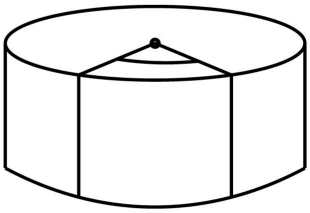
13) A two-digit number is divided by the sum of its digits (they are not the same).

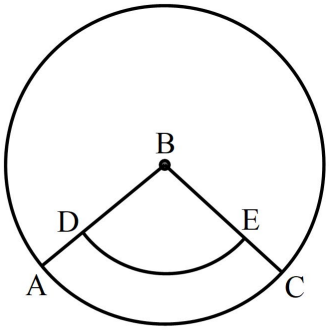
What is the largest possible remainder?

(7)

14) You want to divide a birthday cake in a non-traditional way.
If area of BDE = area of DACE, what is the ratio of AD : BD?

(7)

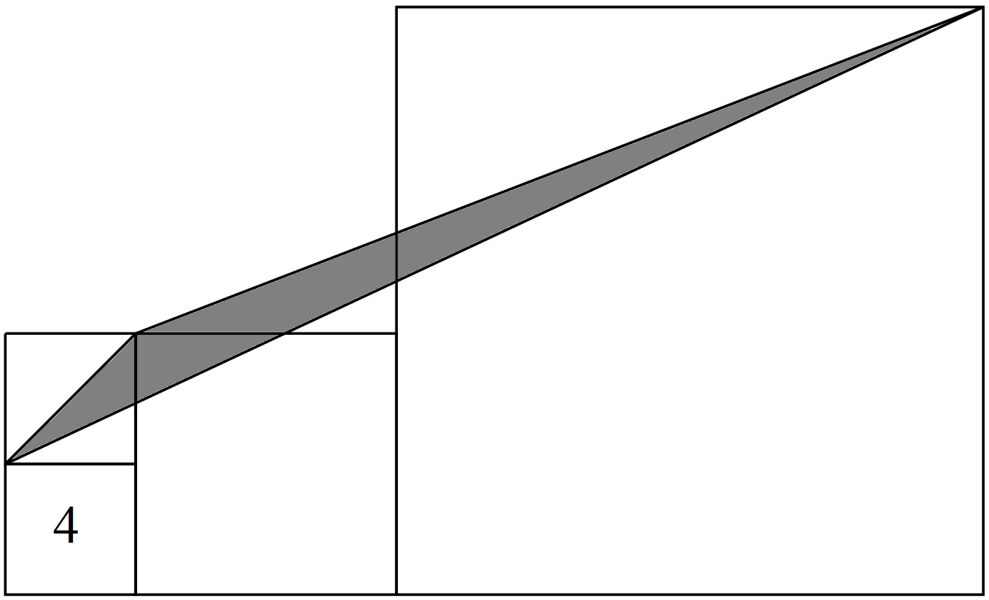




15) Four squares are drawn below, with the area of the bottom left square being 4 units^2.

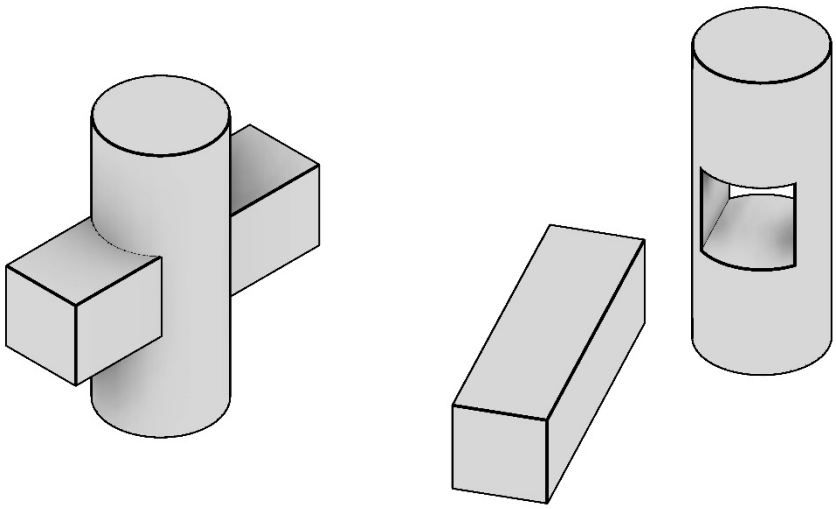
Determine the area of the shaded triangle.

(10)



16) You have a cylinder, with diameter $\sqrt{2}$.

A square hole with side length 1 is cut through the centre of a cylinder so that a square tube fits exactly in the hole as shown. Determine the volume of the part that was removed from the cylinder.



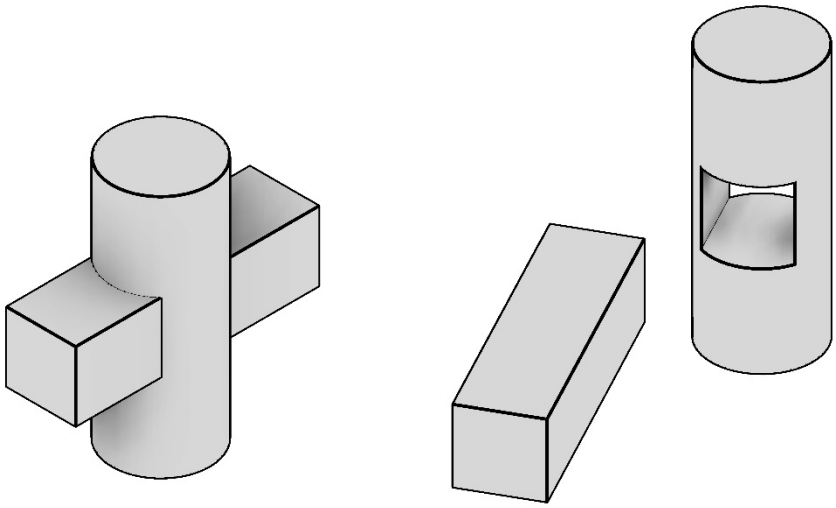
Use the actual model to touch,
to understand the problem,
to investigate and plan strategies,
so that you can solve the problem.
(Please note: Model was not made on scale)

(10)

THE END

16) Jy het ‘n silinder met ‘n deursnee van $\sqrt{2}$.

‘n Vierkantige opening met ‘n sylengte van 1 word deur die middel van ‘n silinder uitgesny sodat ‘n vierkantige staaf presies in die opening pas (soos aangetoon). Bepaal die volume van die gedeelte van die silinder wat verwyder is.



Gebruik die werklike model om te voel,

die probleem te verstaan,

oplossingstrategieë te ondersoek en

om dan die probleem op te los!

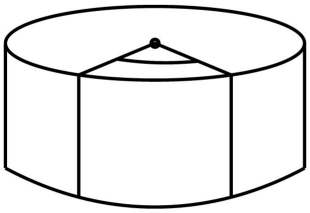
(Let wel: Die model is nie op skaal gemaak nie)

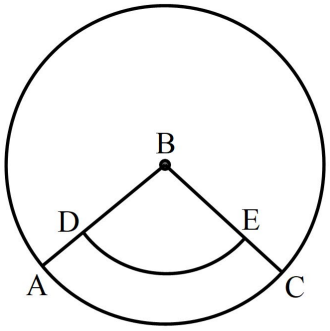
(10)

DIE EINDE

14) Jy wil ‘n verjaardagkoek op ‘n nie-tradisionele manier verdeel.
Indien die area BDE = area DACE, bepaal die verhouding AD : BD.

(7)

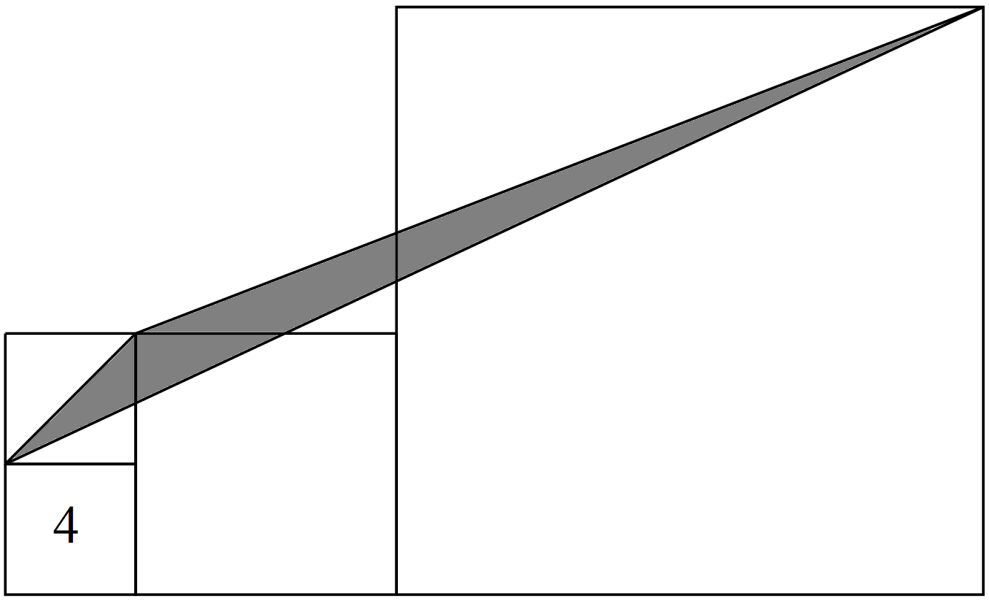




15) Vier vierkante is hieronder geteken. Die area van die linkerkantste onderste vierkant is 4 eenhede^2.

Bepaal die area van die ingekleurde driehoek.

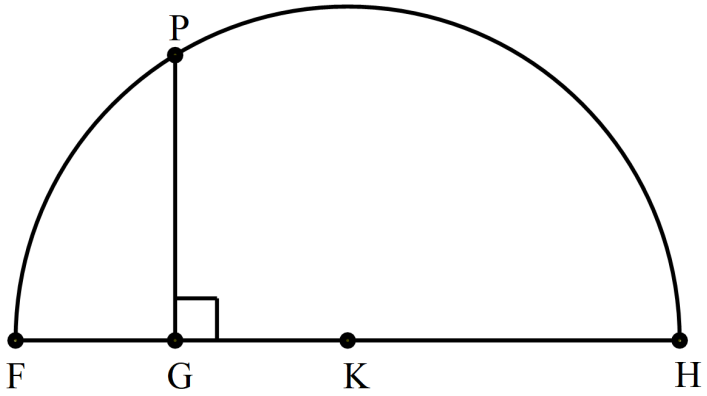
(10)



11) Hier is ‘n baie “cool” manier om die vierkantswortel van ‘n getal te bepaal!

Veronderstel jy wil die waarde van $\sqrt{12}$ bepaal:

- Teken ‘n lynsegment van 12 eenhede lank. (GH in die skets).
- Verleng GH met een eenheid lengte (FG in die skets).
- Teken ‘n semi-sirkel met FH die middellyn en K die middelpunt.
- Konstrueer nou PG loodreg op FG om die sirkel by P te raak.
- Dan sal $\text{PG} = \sqrt{12}$.



Bewys dat hierdie konstruksie-metode, om die vierkantswortel van enige getal te bepaal, sal werk.

(7)

12) Die ry $a_0, a_1, \ldots$ word gedefinieer deur:

$$a_0 = 2022 \qquad a_{n+1} = \frac{1 + a_n}{1 - a_n} \qquad \text{for } n = 0, 1, \ldots$$

Bepaal die waarde van a_{2022}.

(6)

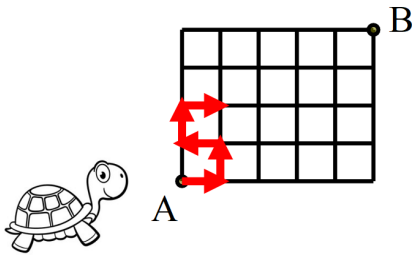
13) ‘n Twee-syfer getal word deur die som van sy twee syfers gedeel.
(die syfers is nie dieselfde nie)

Bepaal die grootste moontlike res.

(7)

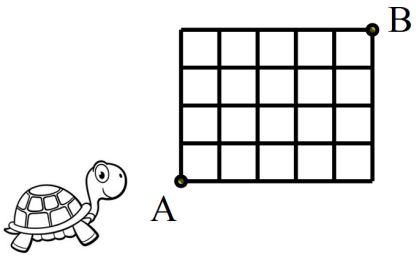
10) ‘n Skilpad beweeg rond op ‘n geruite blok wat uit eenheidsvierkante bestaan. Elke keer wanneer die skilpad by ‘n nuwe vierkant kom, verander dit van rigting.

Byvoorbeeld:



a) Wat is die kortste afstand vir die skilpad om van punt A na punt B op ‘n 4×5 ruit te beweeg?

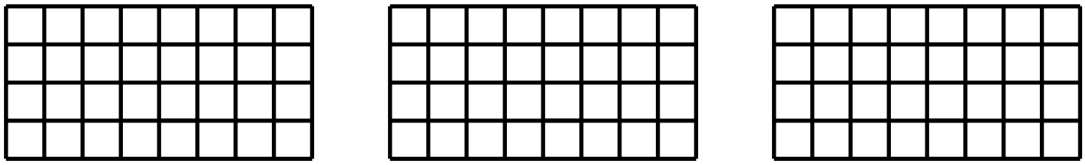
(2)



b) Wat sal die kortste afstand wees, indien die skilpad van die onderste linkerhoek na die boonste regterhoek, op ‘n 2022×4044 ruit beweeg.

(5)

(Hier is ‘n paar 4×8 ruite om jou met jou afleidings te help)



6) Beskou die getalle: 24, 55, 27, 64 en x

- Die gemiddeld van hierdie vyf getalle is ‘n priemgetal
- Die mediaan is ‘n veelvoud van 3

(Die mediaan van ‘n versameling getalle is die middelste getal wanneer die getalle in stygende orde gerangskik word. Byvoorbeeld, die mediaan van 1,2, 5, 8, 10 is 5).

Bepaal die som van al die moontlike positiewe heelgetalwaardes van x.

(6)

7) Alle positiewe heelgetalle n waarvoor $n(n+1)(n+2)$ ‘n veelvoud van 5 is, word in stygende orde gerangskik.

a) Gee die eerste drie moontlike waardes van n. (2)

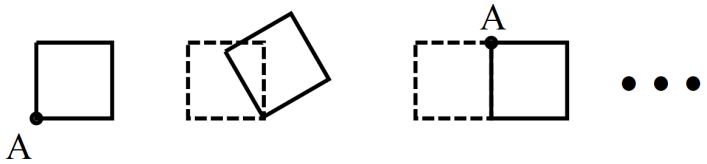
b) Wat is die 2022^{ste} getal in die lys? (4)

8) Bepaal die kleinste heelgetal wat, wanneer dit met die syfer 123 vermenigvuldig word, ‘n produk wat eindig met 2022 lewer.

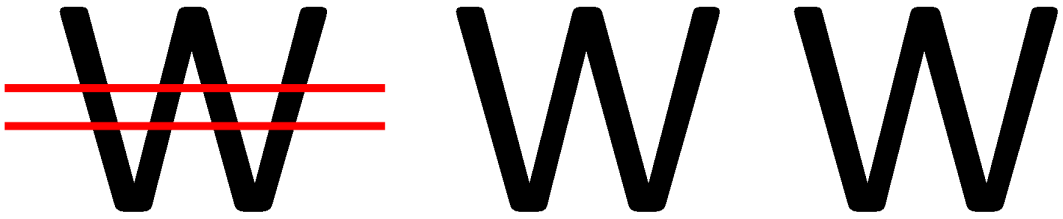
(6)

9) ‘n Vierkantige “wiel” rol op ‘n reguit gelykpad, sonder om te gly, totdat dit een volledige omwenteling gemaak het. Indien die vierkant ‘n sylengte van 1 cm het, hoe ver het punt A beweeg?

(6)



3) Twee horisontale lyne verdeel die letter W in 9 dele.

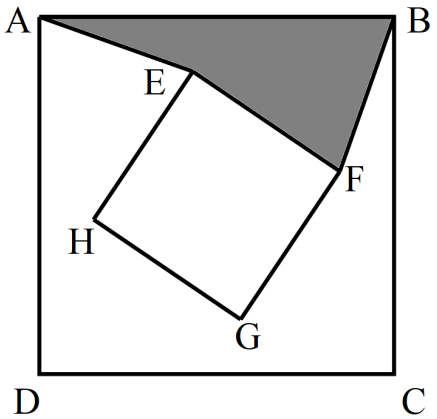


a) Hoeveel dele sal jy kry as jy vier horisontale lyne gebruik? (2)

b) Bepaal die kleinste moontlike aantal lyne om die W in ten minste 2022 dele te verdeel. (3)

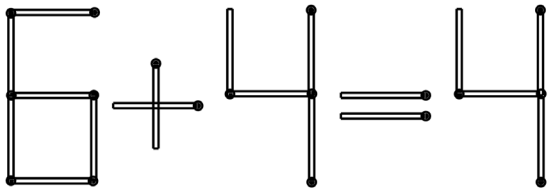
4) Vier vriende hardloop by ‘n stowwerige trap af. Ambete het twee trappe op ‘n slag afgehardloop. Bruce het drie trappe op ‘n slag, Claire het vier trappe op ‘n slag en Divakaran het vyf trappe op ‘n slag afgehardloop. Indien slegs die boonste en onderste trap al vier vriende se voetspore bevat, hoeveel trappe het slegs een persoon se voetspoor op? (5)

5) In die skets hieronder het vierkant ABCD ‘n sylengte van 12 en vierkant EFGH ‘n sylengte van 6. **Beide vierkante het dieselfde middelpunt.** Interessant om op te let dat die oppervlakte van vierhoek ABFE altyd konstant bly en onafhanklik van die rotasie van vierkant EFGH is.



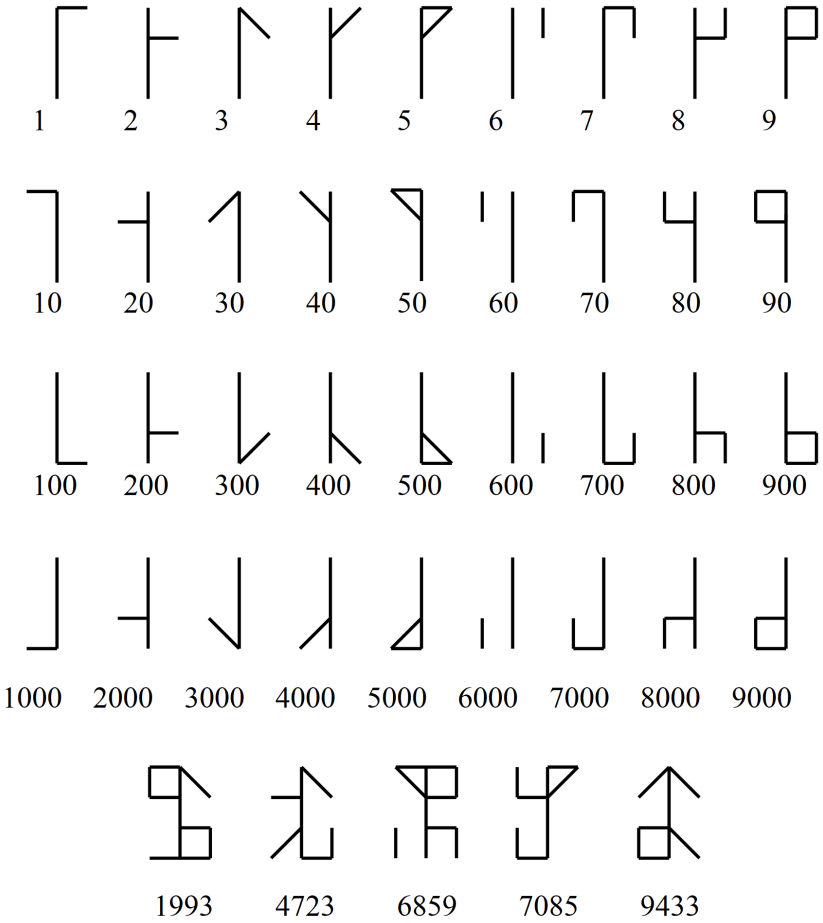
Bepaal die oppervlakte van die ingekleurde deel. (4)

1) Verplaas een vuurhoutjie om hierdie stelling waar te maak.
Die Olimpiade-komitee kon slegs drie moontlike oplossings vind.
Jy sal dus ‘n bonuspunt verdien indien jy meer kan vind!

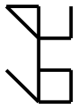


(3)

2) Die Cisterciënzer monnike het ‘n getalsisteem in die 13de eeu ontwikkel waarmee enige getal tussen 1 en 9999 as ‘n enkel simbool geskryf kon word.



a) Wat is die numeriese waarde van hierdie Cisterciënzer simbool?



(2)

b) Skryf die getal 2022 as ‘n Cisterciënzer getalsimbool. (3)

DIE OLD MUTUAL SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2022 DERDE RONDE
JUNIOR AFDELING: GRAAD 8 EN 9

28 Julie 2022 **Tyd: 4 Ure** **Aantal vrae: 16**

TOTAAL: 100

Instruksies

- Beantwoord al die vrae.
- Alle berekeninge en motiverings moet getoon word. Antwoorde sonder motivering sal nie volpunte verdien nie.
- Die netheid van jou oplossings mag in ag geneem word.
- Diagramme is nie noodwendig volgens skaal geteken nie.
- Geen sakrekenaar, in welke vorm ook al, of enige meetkunde instrumente, mag gebruik word nie.
- Gebruik jou tyd oordeelkundig en moenie al jou tyd op slegs ‘n paar vrae spandeer nie.
- Vrae is nie noodwendig in volgorde van maklik na moeilik gerangskik nie.
- Die antwoorde en oplossings sal beskikbaar wees by: www.samf.ac.za

Moenie omblaai voordat daar vir jou gesê word om dit te doen nie.
Turn the booklet over for the English paper.

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SA Akademie vir Wetenskap en Kuns