# SOUTH AFRICAN MATHEMATICS OLYMPIAD
## Grade EIGHT First Round 2021
### Solutions

1.    **C**    $20{,}21 + 20 + 2{,}1 = 42{,}31$

2.    **B**    $\sqrt{20 + 20 + 20 + 21} = \sqrt{81} = 9$

3.    **E**    3 hours and 20 minutes after 20:21 is 23:41

4.    **A**    The closest multiple of 4 to 2021 is 2020, and a quarter of 2020 is 505.

5.    **D**    Numbers between 10 and 30 that are not even: 11, 13, 15, 17, 19, 21, 23, 25, 27, 29
Removing the primes leaves: 15, 21, 25, 27
Removing the multiples of 3 leaves: 25

6.    **B/E**    Both B and E can be folded to form a cube.

7.    **D**    Since the average age is 20, the sum of their ages must be 100. The fifth person's age is thus $100 - (10 + 15 + 20 + 22) = 33$.

8.    **A**    $400 \div 8 = 50$ minutes

9.    **B**    Determine the total number of pathways to each vertex:
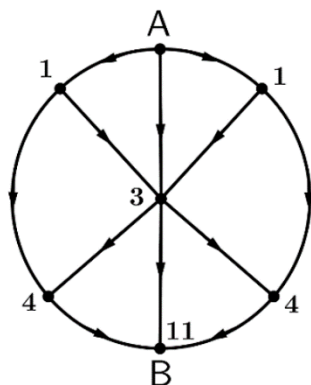


10.    **E**
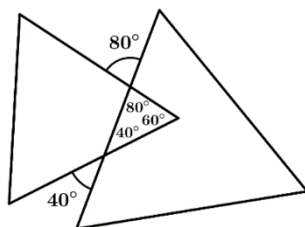


11.    **A**    Shaded area $= 10 \times 6 - \frac{1}{2} \times 5 \times 6 - \frac{1}{2} \times 10 \times 3 = 30$ cm².

12.    **C**    $4x = 180° - (30° + 50°)$. Thus $x = 25°$.

13. **A** The 50<sup>th</sup> shape contains 50 upward pointing triangles, each containing 3 sticks, as well as a top horizontal row of 49 sticks, making 199 sticks in total.

14. **C** The first nine perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81. Using two of these in combination, the largest possible sum is $81 + 16 = 97$.

15. **D** The last number in the $n$th group is simply $3n$. The last number in the 100<sup>th</sup> group is thus 300. The 100<sup>th</sup> group is thus $\{298; 299; 300\}$ and $298 + 299 + 300 = 897$.

16. **B** Note that if the area of a square decreases by a factor of 4 then the side length decreases by a factor of 2. From bottom to top the squares thus have side lengths of 16 cm, 8 cm, 4 cm and 2 cm, giving a total height of $16 + 8 + 4 + 2 = 30$ cm.

17. **D** The total distance completed is the first half of the run plus 3/5 of the second half.
Thus: $\frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

18. **B** Let the code be $ABCDE$. Since $A + B + C + D = 19$ and $A + B + C = 15$ it follows that $D = 4$. Also, since $B + C + D = 15$ it follows that $A = 4$. Similarly, $B = E = 4$. The code is thus $44744$ and the sum of the digits is 23.

19. **E** The maximum sum of the digits of a four-digit number is $4 \times 9 = 36$. We thus only need to consider 11, 22 and 33 as CC. Since the number ABBA has two A's and two B's, the sum of its digits has to be an even number. CC must thus be 22. If C = 2 and A + B + B + A = 22. Then A + B = 11, thus A + B + C = 11 + 2 = 13.

20. **C** We can establish a pattern by working out the first few terms:
$T_1 = 2018$ ; $T_2 = (2 + 0 + 1 + 8)^2 = 121$ ; $T_3 = (1 + 2 + 1)^2 = 16$ ;
$T_4 = (1 + 6)^2 = 49$ ; $T_5 = (4 + 9) = 169$ ; $T_6 = (1 + 6 + 9)^2 = 256$
$T_7 = (2 + 5 + 6)^2 = 169$ ; …
The sequence is thus: 2018 ; 121 ; 16 ; 49 ; 169 ; 256 ; 169 ; 256 ; 169 ; 256 ; …
The 100<sup>th</sup> term is thus 256.