THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

SECOND ROUND 2001

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

29 May 2001

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

KEER DIE BOEKIE OM VIR AFRIKAANS

Private Bag X11, ARCADIA, 0007 TEL: (012-) 328-5082 FAX: (012-) 328-5091

PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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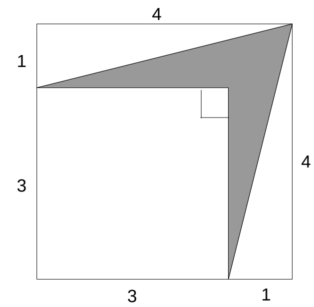
Part A: Four marks each.

1. The area of the shaded region in the square is



(A) 1 (B) 3 (C) 4 (D) 5 (E) 2

2. A Peregrine Falcon migrated from Europe to South Africa in three weeks. Its average speed in km/hour was approximately

(A) 20 (B) 2 (C) 200 (D) 100 (E) 400

3. A boy has as many sisters as he has brothers. Each of his sisters has twice as many brothers as she has sisters. How many children are there in the family?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

4. If x is $x\%$ of y and y is $y\%$ of z, where x is a positive real number, what is the value of z?
(A) 100 (B) 200 (C) 10 000 (D) It does not exist (E) It cannot be determined

5. Each of the figures below shows a square of side 1 with inscribed circles. In which of the figures do the circles have the greatest total area?

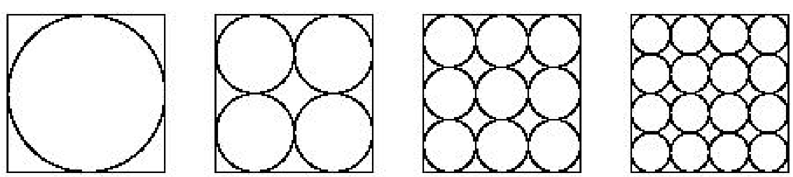


Fig. 1 Fig. 2 Fig. 3 Fig. 4

(A) Figure 1 (B) Figure 2 (C) Figure 3 (D) Figure 4
(E) They all have the same area

Part B: Five marks each

6. Of the following triangles given by the lengths of their sides, which one has the greatest area?

(A) 5, 12, 12 (B) 5, 12, 11 (C) 5, 12, 14 (D) 5, 12, 15 (E) 5, 12, 13

7. By placing a 2 at both ends of a number, its value is increased by 2317. The sum of the digits of the original number is

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

8. How many numbers in the list $1, 2, 3, \ldots, 2001$ are perfect squares and also perfect cubes of whole numbers?

(A) Three (B) One (C) Four (D) More than four (E) Two

9. If two regular six-sided dice are thrown, the probability that the sum will be a prime number is

(A) $\frac{5}{36}$ (B) $\frac{1}{6}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$ (E) $\frac{2}{9}$

10. If $f(x) = x^{2001}(x-1)^{2001}$, then $f(9) - f(3)f(4)$ equals

(A) 9 (B) 0 (C) 3 (D) 4 (E) 2

11. The sum of the divisors of 24 is 60. The sum of the reciprocals of the divisors of 24 is

(A) $\frac{5}{2}$ (B) $\frac{5}{4}$ (C) $\frac{1}{60}$ (D) 2 (E) $\frac{8}{3}$

12. If

$$x = 3 + \cfrac{1}{3 + \cfrac{1}{x}} \text{ and } y = 3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{y}}}$$

where x and y are positive real numbers, then $x - y$ equals

(A) 3 (B) $3 + \frac{1}{y}$ (C) $\cfrac{1}{3 + \cfrac{1}{y}}$ (D) 0 (E) Cannot be determined

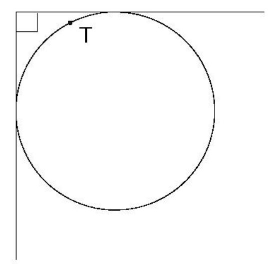
13. If $n + 1 = 2000^2 + 2001^2$ then $\sqrt{2n+1}$ equals

(A) $3000\sqrt{2}$ (B) 3999 (C) 4001 (D) $\sqrt{4000^2+1}$ (E) $\sqrt{4000^2+3}$

14. The sum of the first p terms of a sequence of numbers is $p(p+1)(p+2)$. The 10th term of the sequence is

(A) 1320 (B) 396 (C) 600 (D) 114 (E) 330

15. A circle touches two lines perpendicular to each other as shown in the diagram. Point T is 8 cm from one line and 9 cm from the other. The radius of the circle, in cm, is



(A) 5 (B) 17 (C) 39 (D) 35 (E) 29

Part C: Six marks each

16. Starting with the numbers $1, 2, 3, \ldots, 500$, a new sequence is formed by deleting numbers so that the sum of any two numbers of the new sequence is never a multiple of seven. What is the maximum length of the new sequence?

(A) 213 (B) 217 (C) 216 (D) 284 (E) 287

17. The integers from 1 to 2001 are written in order around a circle. Starting at 1, every 6th number is marked (that is 1, 7, 13, 19, etc.). This process is continued until a number is reached that has already been marked. How many unmarked numbers remain?

(A) None (B) 1668 (C) 1669 (D) 1004 (E) 1334

18. Let n be a fixed positive integer. The maximum value of A such that

$$x^n + x^{n-2} + x^{n-4} + \cdots + \frac{1}{x^{n-4}} + \frac{1}{x^{n-2}} + \frac{1}{x^n} \geq A$$

for all positive real numbers x, is

(A) n^2 (B) $n+1$ (C) 0 (D) $n(n+1)$ (E) $n^2(n+1)$

19. Which one of the numbers below can be expressed as the sum of the squares of 6 odd integers?

(A) 1998 (B) 1996 (C) 2000 (D) 2004 (E) 2002

20. During a recent period of time, eleven days had some rain. A morning rain was always followed by a clear afternoon. An afternoon rain was always preceded by a clear morning. A total of nine mornings and twelve afternoons were clear. How many days had no rain at all?

(A) Six (B) Four (C) Three (D) Five (E) Seven