THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

FIRST ROUND 2004: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

Thank you for entering the First Round of the Mathematics Olympiad.

Many of the solutions to the problems which are given below use trial-and-improvement methods or an investigative approach.

Most of them can also be solved using more formal mathematical methods which you will learn in due course. Sometimes we have also given a more formal solution which you might find interesting.

PRACTICE EXAMPLES:

1. $23+6-4=$

A) 6 B) 23 C) 25 D) 29 E) 33

ANSWER: C
EXPLANATION:
$23+6-4=(23+6)-4=29-4=25$

2. $\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}$ equals

A) $\frac{1}{15}$ B) $\frac{3}{11}$ C) $\frac{21}{50}$ D) $\frac{8}{15}$ E) $9\frac{4}{5}$

ANSWER: D
EXPLANATION:
$\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}=\frac{1}{5}+(\frac{2}{3}\times\frac{1}{2})=\frac{1}{5}+\frac{1}{3}=\frac{3+5}{15}=\frac{8}{15}$

QUESTIONS:

1. Which one of the following numbers is the smallest?

A) 0,068 B) 0,07 C) 0,2 D) 0,087 E) 0,2443

ANSWER: A
EXPLANATION:
If the numbers are written with the same denominator they become:
$\frac{680}{10\,000}$; $\frac{700}{10\,000}$; $\frac{2\,000}{10\,000}$; $\frac{870}{10\,000}$; $\frac{2443}{10\,000}$, which clearly indicates that $\frac{680}{10\,000}$ or 0,068 is the smallest number.

2. $0,9(0,4+0,6)$ equals

A) 0,09 B) 0,96 C) 1,9 D) 9 E) 0,9

<u>ANSWER:</u> E
<u>EXPLANATION:</u>

$0,9(0,4+0,6)$

$=0,9(1)$ BODMAS preferred to distributive.

$=0,9$

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3. Find the value of $\boxed{}$ if

$1008 \div \boxed{} = 24$

A) 32 B) 42 C) 52 D) 62 E) 72

<u>ANSWER:</u> B
<u>EXPLANATION:</u>

$1008 \div \boxed{} = 24$ can be rewritten as

$\boxed{} = \frac{1008}{24}$ divisible by 3 and 4 ($\div$ each by 12)

$\boxed{} = \frac{84}{2}$ $\therefore$ $\boxed{} = 42$

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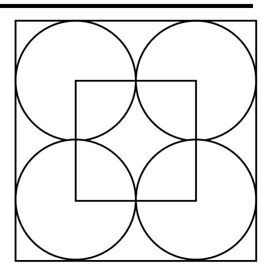
4. In the diagram, four equal circles fit perfectly inside a square; their centres are the vertices of the smaller square. The area of the smaller square is 4.
The area of the larger square is

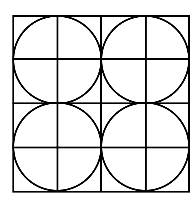


A) 4 B) 8 C) 12 D) 16 E) 20

<u>ANSWER:</u> D
<u>EXPLANATION:</u>

Equal circles have equal radii.
$\therefore$ Each side of a small square is equal to $2 \times r$
but area of a small square is
$(2 \times r)(2 \times r) = 4$ square units
$\therefore 2 \times r = 2$ $\therefore$ <u>r = 1</u>
Each side of big square is equal to $4 \times r$ i.e. $4 \times 1 = 4$
$\therefore$ Area of big square is equal to
$4 \times 4 =$ <u>16 square units</u>

Or Draw lines to divide figure in congruent (equal little squares).



There are 4 little squares in the small square (4 square units) therefore 1 little square is equal to 1 square unit. There are 16 such little squares, which gives us the total area of the big square as <u>16 square units.</u>

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5. Jackie said that 12% of the oranges were not sold.
Pam said that is the same as 360 oranges! How many oranges were sold?

A) 2 400 B) 2 640 C) 3 000 D) 3 600 E) 4 320

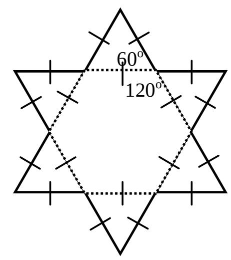
ANSWER: B
EXPLANATION:
Let the number of oranges = $\boxed{}$
12% of $\boxed{}$ = 360
$\therefore$ 1% of $\boxed{}$ = 30
$\therefore$ 100% of $\boxed{}$ = 3000
$\therefore$ Number sold: 3000 – 360 = 2640

or
then: 88% of 3000
is equal to $88 \times 30 = 2640$

6. If $b = 3a$ and $c = 2b$, then $a + b + c$ is equal to
A) $6a$ B) $8a$ C) $10a$ D) $12a$ E) $14a$

ANSWER: C
EXPLANATION:
If $b = 3a$ and $c = 2b = 2(3a) = 6a$
Then $a + b + c = a + 3a + 6a = 10a$.

7. A regular six point star is formed by extending the sides of a regular hexagon.
If the perimeter of the star is 96 cm then the perimeter of the hexagon (in cm) is



A) 30 B) 36 C) 42 D) 48 E) 54

ANSWER: D
EXPLANATION:
The extended sides of the regular hexagon form 6 equilateral triangles.
Take one side as x, then $12x = 96$ cm and therefore $x = 8$ cm.
Perimeter of hexagon: $6x = 6 \times 8 = 48$ cm

8. The last digit of $2^{2004} - 2$ is
A) 0 B) 1 C) 2 D) 3 E) 4

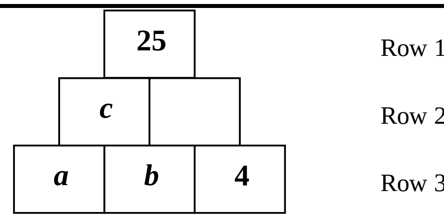
ANSWER: E
EXPLANATION:

	Product	Last digit
$2^1 = 2$	= 2	2
$2^2 = 2 \times 2$	= 4	4
$2^3 = 2 \times 2 \times 2$	= 8	8
$2^4 = 2 \times 2 \times 2 \times 2$	= 1<u>6</u>	6
$2^5 = 2 \times 2 \times 2 \times 2 \times 2$	= 32	2
$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$	= 64	4
$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	= 128	8
$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	= 25<u>6</u>	6

Note that the last digit of the product forms a repeating sequence (pattern). E.g. the 4th number ends in 6 as well as the 8th number, 3rd and 6th in 4, etc.

2^{2004} = ends in a 6 because 2004 is a multiple of 4.
$\therefore 2^{2004} - 2$ ends in 4.

9. The game Pyramaths works as follows: 2 adjoining blocks' sum is equal to the block above the 2 adjoining blocks, *e.g.* $a + b = c$.



If the sum of the numbers in row 3 is 17, then the value of $\boldsymbol{a}$ is

A) 2 B) 3 C) 4 D) 5 E) 7

<u>ANSWER:</u> D
<u>EXPLANATION:</u>
$a + b = c$ given
but $a + b = 13$ because $a + b + 4 = 17$ (given)
therefore: $c = 13$
The value of the block next to c is 12, because $13 + 12 = 25$
and $b = 8$ because $8 + 4 = 12$
thus $a = 5$ because $5 + 8 = 13$

10. In a mathematics class, the learners voted to have a new operation on numbers called "super op" and used the symbol # for the operation.

They defined it as: $a(\#)b = \frac{1}{a} + \frac{1}{b} + ab$. The value of $\frac{1}{3}(\#)6$ is

A) $5\frac{1}{6}$ B) 18 C) $2\frac{1}{2}$ D) 2 E) $8\frac{1}{3}$

<u>ANSWER:</u> A
<u>EXPLANATION:</u>

$$\frac{1}{3}(\#)6 = \frac{1}{\frac{1}{3}} + \frac{1}{6} + (\frac{1}{3})(6) \qquad = 3 + \frac{1}{6} + 2 \qquad = 5\frac{1}{6}$$

11. Vishnu has displayed his technology project as a mobile and hung it from the classroom ceiling. It is perfectly balanced (figure 1).

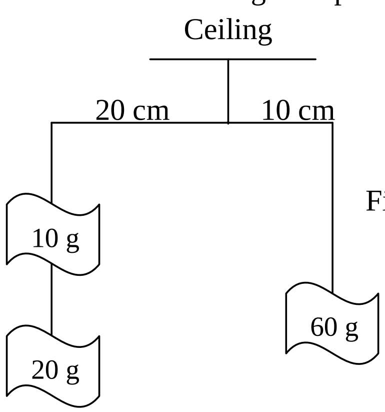


Figure 1

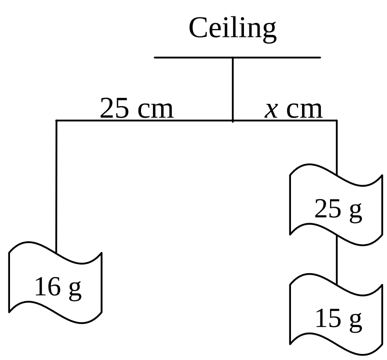


Figure 2

Sipho wants to display his project in the same way (figure 2).
What must the length (x) of the wire be for his mobile to be perfectly balanced?
[Ignore the mass of the wire]

A) 5 B) 10 C) 15 D) 20 E) 25

ANSWER: B
EXPLANATION:

On figure 1: 20 cm is balanced by 30 g (10 g + 20 g) and
10 cm is balanced by 60 g. **i.e. $20 \times 30 = 10 \times 60$ for balance**
On figure 2: 25 cm is balanced by 16 g and
x cm is balanced by 40 g (25 g + 15 g).
If 16 g balances 25 cm, 40 g balances x cm.

$\therefore 16 \times 25 = 40 \times x$
$\therefore 400 = 40x$
$\therefore x = 10$ cm

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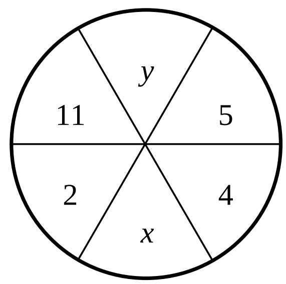
12. In the diagram, the numbers occupying opposite sectors are related in the same way.
The relationship of y in terms of x is



A) $y = 3x - 1$
B) $y = 2x + 1$
C) $y = 2x + 2$
D) $y = x - 1$
E) $y = 2x - 1$

ANSWER: A
EXPLANATION:
Method: Trial – and – improvement

$11 \neq 2 \times 4 + 1$
$5 = 2 \times 2 + 1$
$\therefore y \neq 2x + 1$ (doesn't work for both)
B), C), D) and E) could all be ruled out.

$11 = 3 \times 4 - 1$
$5 = 3 \times 2 - 1$
$\therefore y = 3x - 1$
A) works with both 11 and 5.

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13. Kelly took 60 minutes to cycle 25 kilometres after which she increased her average speed by 5 kilometres per hour.

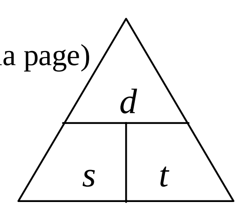
How long will it take her to cover the next 25 kilometres if she maintains the new average speed?

A) 35 min B) 40 min C) 45 min D) 50 min E) 55 min

ANSWER: D
EXPLANATION:
New speed: 25 + 5 = 30 km/h
Using distance, time and speed formula: (see formula page)

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{25\,km}{30\,km/h}$$

$= 0{,}83\,h$ $= 50$ minutes



$d = s \times t$

$s = \dfrac{d}{t}$

$t = \dfrac{d}{s}$

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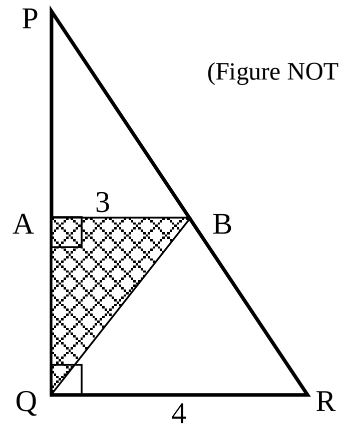
14. The area of the shaded triangle is $4\frac{1}{2}\,cm^2$. Angles PQR and QAB are right angles (90^o).
QR = 4 and AB = 3.



(Figure NOT drawn to scale)

The size of angle ABQ is

A) 15° B) 30°
C) 45° D) 60° E) 75°

ANSWER: C
EXPLANATION:
The shaded area is a right-angled triangle.
Area of the shaded triangle is $4\frac{1}{2}\,cm^2$.

$\therefore$ Area of ΔABQ $= \frac{1}{2}bh = \frac{1}{2}$ AB.AQ $= \frac{1}{2}$(3.AQ) $= 4\frac{1}{2}$

$\therefore$ 3.AQ = 9 cm $\therefore$ AQ = AB = 3 cm
$\therefore$ ΔABQ is isosceles

$\therefore$ Angles PQR and QAB are both 45^o because $B\hat{A}Q = 90^\circ$ (sum of the interior angles of any triangle is 180^o).

15. Two 3-digit numbers are multiplied.
A star (*) represents any digit.
The second 3-digit number is

```
  **5   1st 3-digit number
  1**   2nd 3-digit number
  2**5
 13*00
 ***00
 **77*
```

A) 140 B) 189 C) 180
D) 155 E) 147

ANSWER: E
EXPLANATION:

1)
```
 **5
 1*□
2**5
```
□ × 5 = 5
$\therefore$ The last digit must be odd
$\therefore$ (A) and (C) can be ruled out.

2)
```
  **5
  1□*
 2**5
13*00
```
□ × 5 = 0
$\therefore$ The second digit must be even
$\therefore$ (D) can be ruled out.

3)
```
  **5
  1**
 2*□5
13*00
***00
**77*
```
□ + 0 + 0 = 7
$\therefore$ □ = 7

4) If we try 189 for the second number, then:
```
**5
189
945
```
which cannot be true, (B) can be ruled out.

$\therefore$ Answer is E, 147. (You can check your answer by looking for the 1st 3-digit number.) [1st 3-digit number: 325]

16. Below is a list of numbers with their corresponding codes. Determine the three digit number **w**.

Number	**Code**
589	524
724	386
1346	9761
w	485

A) 945 B) 543 C) 425 D) 623 E) 925

<u>ANSWER:</u> E

<u>EXPLANATION:</u>

Number	1	2	3	4	5	6	7	8	9
Code	9	8	7	6	5	1	3	2	4

How?

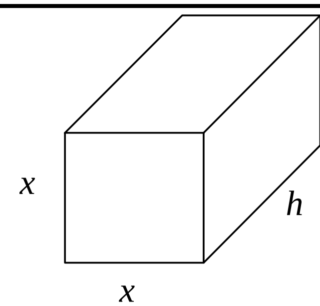
Number	Code
5 8 9	5 2 4

For w = ?, code: 485, use the above conversion table:

Code	to	number
4	$\longrightarrow$	9
8	$\longrightarrow$	2
5	$\longrightarrow$	5

$\therefore$ w = 925

17. A rectangular right prism has the dimensions x cm by x cm by h cm.
The surface area of the prism is $14x^2$ cm^2.



Find h in terms of x.

A) $3x$ B) $\frac{x}{2}$ C) $4x$ D) $2x$ E) x

<u>ANSWER:</u> A

<u>EXPLANATION</u>

$$\begin{aligned}
\text{Surface area} &= x.x + x.x + x.h + x.h + x.h + x.h \\
&= x^2 + x^2 + 4xh \\
&= 2x^2 + 4xh \\
2x^2 + 4xh &= 14x^2 \\
2x^2 + 4xh &= 2x^2 + 12x^2 \\
4xh &= 12x^2 \\
4xh &= 4x.3x \\
h &= 3x
\end{aligned}$$

18. Zama used six digits, 3; 7; 4; 6; 2 and 5 to make two-digit numbers e.g. 37; 44 etc.

If 7 cannot be used as the ten's digit and 3 cannot be used as the units' digit, then the sum of all possible two digit numbers is

A) 1 000 B) 1 040 C) 1 080 D) 1 120 E) 1 160

<u>ANSWER:</u> D
<u>EXPLANATION</u>
For every tens digit there are 5 units digits that can be paired with it, e.g.:

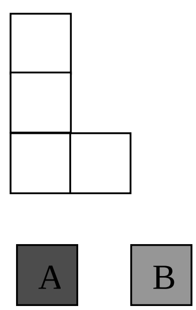
Sum (tens column) $= (20 + 30 + 40 + 50 + 60) \times 5$
$= 1\ 000$

Sum (units column) $= (2 + 4 + 5 + 6 + 7) \times 5$
$= 120$

$\therefore$ Sum $= 1\ 120$

T	**U**
~~7~~	~~3~~
2	2
2	4
2	5
2	6
2	7
3	2
3	4
.	.
etc.	
100	120

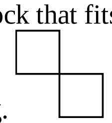
19. Four one-centimetre squares are joined as in the figure alongside. One or two one-centimetre squares, called A and B respectively, but interchangeable and not unique, may be added to this figure.



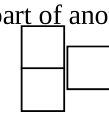
What is the maximum number of ways this can be done to create different figures which are symmetrical about a line of symmetry?

A) 8
B) 9
C) 10
D) 11
E) 12

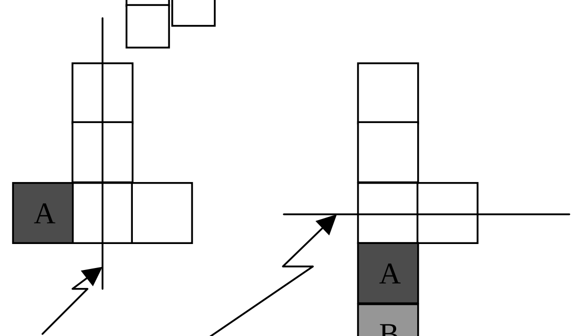
Example: The sides of the added blocks must fit closely against the sides of any other block (i.e. you are not allowed to add a block with only their vertices as the common point or one block that fits to a



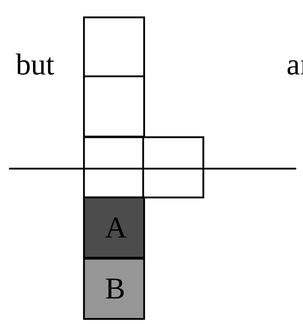
part of another block, e.g. and



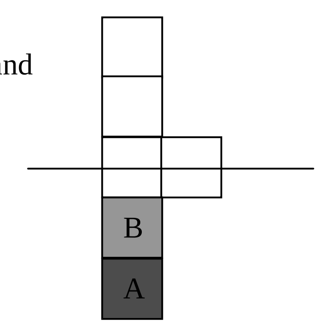
are not allowed)



but



and

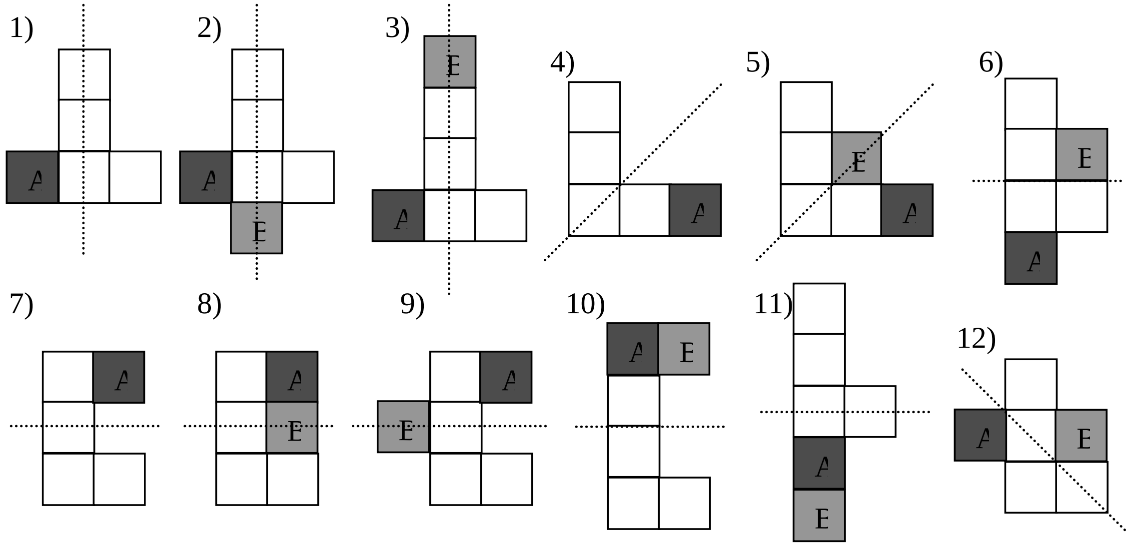


are regarded as the same solution for the purpose of this question.

ANSWER: E

EXPLANATION

The following (12) ways are all possibilities:



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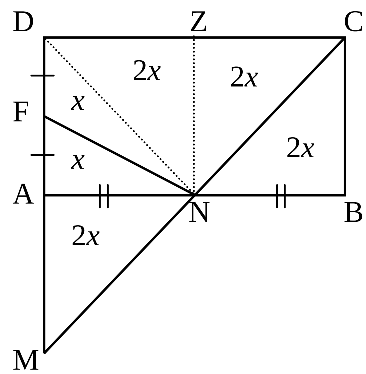
20. ABCD is a rectangle. N is the midpoint of AB. F is the midpoint of DA. DA produced meets CN produced at M.



The area of ΔFNM, as a fraction of the area of rectangle ABCD is

A) $\frac{1}{8}$ B) $\frac{3}{8}$ C) $\frac{5}{8}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

ANSWER: B

EXPLANATION:

Let area of $\Delta FNA = x$

then area of $\Delta DFN = x$

(Complete by symmetry as per diagram)

$$\therefore \frac{\text{Area of } \Delta\text{FNM}}{\text{Area of ABCD}} = \frac{3x}{8x} = \frac{3}{8}$$

Or:

Draw lines DN and ZN (DZ = ZC). Through symmetry (or otherwise) you will be able to see that:

$$\text{Area } \Delta\text{ADN} = \frac{1}{4}\text{area ABCD}$$

$$\text{and area } \Delta\text{CNB} = \frac{1}{4}\text{area ABCD}$$

$$\text{but area } \Delta\text{AMN} = \text{area } \Delta\text{CNB}$$ (it can be proved through congruency (not Gr. 8))

You can also see that area $\Delta\text{ANF} = \text{area } \Delta\text{DFN}$ (equal sides, same height)

$$\therefore \text{area } \Delta\text{ANF} = \frac{1}{2}\text{area } \Delta\text{ADN}$$

$$= (\frac{1}{2}).(\frac{1}{4}) = \frac{1}{8}\text{area ABCD}$$

$$\text{Area } \Delta\text{FNM} = \text{area } \Delta\text{ANF} + \text{area } \Delta\text{ANM} = (\frac{1}{8} + \frac{1}{4})\text{ area ABCD}$$

$$= \frac{3}{8}\text{area ABCD}$$

THE END

Formula and Information Sheet

1.1	The natural numbers are 1; 2; 3; 4; 5; …	
1.2	The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; …	
1.3	The integers are	…; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …
2.	In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.	

3.1 Exponential notation:

$$2\times 2\times 2\times 2\times 2 = 2^5$$

$$3\times 3\times 3\times 3\times 3\times 3 = 3^6$$

$$a\times a\times a\times a\times \ldots \times a = a^n \qquad (n \text{ factors of } a)$$

(a is the base and n is the index (exponent))

3.2 Factorial notation:

$$1\times 2\times 3\times 4 = 4!$$

$$1\times 2\times 3\times \ \ldots \times n = n!$$

4 Area of a

4.1	triangle is:	$\frac{1}{2}\times$ (base $\times$ height) = $\frac{1}{2}(b.h)$
4.2	rectangle is:	length $\times$ width = lw length $\times$ breadth = lb
4.3	square is:	side $\times$ side = s^2
4.4	rhombus is:	$\frac{1}{2}\times$ (product of diagonals)
4.5	trapezium is:	$\frac{1}{2}\times$(sum of parallel sides)$\times$height
4.6	circle is:	πr^2 (r = radius)

5 Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh\,(h = \text{height})$

5.2 sphere is: $4\pi r^2$

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6 Perimeter of a:

6.1 rectangle is: $2\times\text{length} + 2\times\text{breadth}$

$2l + 2b$

or $2l + 2w \qquad (w = \text{width})$

6.2 square is: $4s$

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7. Circumference of a circle is: $2\pi r$

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8. Volume of a:

8.1 cube is: $s\times s\times s = s^3$

8.2 rectangular prism is: $l\times b\times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section × perpendicular height

or area of base × perpendicular height

9.2 Surface area of a right prism is: (perimeter of base × h) + (2 × area of base)

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10. Sum of the interior angles of a polygon is: $180^\circ(n-2) \qquad [n = \text{number of sides}]$

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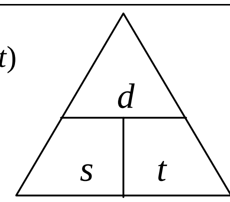
11. Distance = speed × time $(d = s \times t)$

Speed = distance ÷ time $(s = \dfrac{d}{t})$

Time = distance ÷ speed $(t = \dfrac{d}{s})$



$d = s\times t$

$s = \dfrac{d}{t}$

$t = \dfrac{d}{s}$

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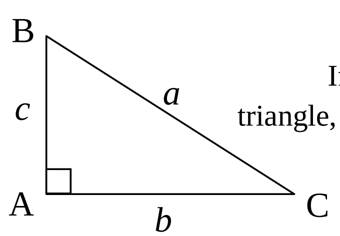
12 Pythagoras:



If ΔABC is a right-angled triangle, then $a^2 = b^2 + c^2$

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13. Conversions:

1 cm^3 = 1 mℓ ; 1000 cm^3 = 1 ℓ

1000 m = 1 km ; 1000 g = 1 kg ; 100 cm = 1 m

ANSWER POSITIONS: JUNIOR FIRST ROUND 2004

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	A
2	E
3	B
4	D
5	B
6	C
7	D
8	E
9	D
10	A
11	B
12	A
13	D
14	C
15	E
16	E
17	A
18	D
19	E
20	B

DISTRIBUTION	
A	4
B	4
C	2
D	5
E	5
TOTAL	**20**