1. From an initial $\triangle A_0B_0C_0$ a sequence $\triangle A_1B_1C_1$, $\triangle A_2B_2C_2, \ldots$ is formed such that at each stage A_{k+1}, B_{k+1} and C_{k+1} are the points where the incircle of $\triangle A_kB_kC_k$ touches the sides B_kC_k, C_kA_k and A_kB_k.

   (a) Express $A_{k+1}\widehat{B_{k+1}}C_{k+1}$ in terms of $A_k\widehat{B_k}C_k$.

   (b) Deduce that, as k increases, $A_k\widehat{B_k}C_k$ tends to $60°$.

2. Find all natural numbers with the property that, when the first digit is moved to the end, the resulting number is $3\frac{1}{2}$ times the original one.

3. Find all solutions $x,\ y \in \mathbb{Z}$, $x,\ y \geq 0$ to the equation

$$1 + 3^x = 2^y.$$

4. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ which satisfy

$$f(m + f(n)) = f(m) + n$$

   for all $m,\ n \in \mathbb{Z}$.

5. A circle and a point P higher than the circle lie in the same vertical plane. A particle moves along a straight line under gravity from P to a point Q on the circle. Given that the distance travelled from P in time t is equal to $\frac{1}{2}gt^2 \sin\alpha$, where α is the angle of inclination of the line PQ to the horizontal, give a geometrical characterization of the point Q for which the time taken from P to Q is a minimum.

6. Six points are connected in pairs by lines, each of which is either red or blue. Every pair of points is joined. Determine whether there must be a closed path having four sides all of the same colour. (A path is closed if it begins and ends at the same point).

In questions 3 and 4, $\mathbb{Z}$ denotes the set of all integers.