THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

FIRST ROUND 1998: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A: (Each correct answer is worth 3 marks)

1. ANSWER: E

$\frac{7}{5} \Rightarrow 5\overline{)7}^{1,4}$

$\underline{5}$

20 or

2. ANSWER: C

$50\%\ \text{of}\ 50 \Rightarrow \frac{50}{100} \times 50 = \frac{1}{2} \times 50 = 25$

3. ANSWER: A

$12{,}34 - 1{,}234 \Rightarrow 12{,}340 - 1{,}234 = 11{,}106$

4. ANSWER: C

The scale is divided in 2 kg increments.

5. ANSWER: E

$\frac{1 \times 9 \times 9 \times 8}{1 + 9 + 9 + 8} = \frac{648}{27} = 24$

PART B: (Each correct answer is worth 5 marks)

6. ANSWER: D

There are 6 rows and 7 columns.

Thus $6 \times 7 = 42$ desks

7. ANSWER: B

23 and 31 are wrong by 4. 25 and 29 are wrong by 2. 27 is the correct number of balloons in the bunch.

8. ANSWER: C

$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$

9. ANSWER: C

The different rectangles are 1×7; 2×6; 3×5 and 4×4. Remember a square is also a rectangle.

(All squares are rectangles but not all rectangles are squares)

10. ANSWER: B

There are 100 cm in 1 m, thus $100 \div 5 = 20$ but 2 sets of triangles are formed which means there are $20 \times 2 = 40$ triangles in total.

11. ANSWER: D

$\frac{1}{2} = \frac{6}{12}$ and $\frac{2}{3} = \frac{8}{12}$; thus $\frac{7}{12}$ lies between $\frac{1}{2}$ and $\frac{2}{3}$.

12. ANSWER: B

The total length (w) is equal to $x + y + x + y + x + p$ thus

$w = 3x + 2y + p$

$\therefore\ p = w - 3x - 2y$

13. ANSWER: C

$45 \div 7 \approx 6$, but if you start or end with a Monday, it will be possible to fit another Monday in. Therefore the total number of Mondays is 7.

14. ANSWER: E

There are 360 degrees in a revolution. $\hat{AOB} + \hat{COD} = 30^\circ + 60^\circ = 90^\circ$, 90° of 360° is $\frac{1}{4}$ of the total area of the circle

15. ANSWER: B

$\hat{PXY} = 90^\circ$; $\hat{XPY} = x^\circ$

$\therefore 2x^\circ + 26^\circ + 90^\circ = 180^\circ$

$\therefore 2x^\circ = 64^\circ$

$\therefore x = 32$

PART C: (Each correct answer is worth 7 marks)

16. ANSWER: B

There are 10 routes.

You can either draw it or

you can describe the route by the letters S and E for the southern and eastern direction. In order for you the get from the school to the community centre you have to go twice in a southern direction and three times in an eastern direction, e.g. SSEEE. There are 10 different ways to do this, namely:

SSEEE	ESSEE	EESSE	EEESS
SESEE	ESESE	EESES	
SEESE	ESEES		
SEEES			

17. ANSWER: C

Look at combinations of powers of 2 and 3:

$2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $2^5 = 32$; $2^6 = 64$

$3^1 = 3$; $3^2 = 9$; $3^3 = 27$; $3^4 = 81$

Consider the combination which will give an answer of 41 which is:

$32 + 9 = 2^5 + 3^2 = 41$; thus $x + y = 5 + 2 = 7$

18. ANSWER: B

$3^1 = 3$; $3^2 = 9$; $3^3 = 27$; $3^4 = 81$; $3^5 = 243$; $3^6 = 729$ etc.

A pattern for the last digits is formed namely: 3, 9, 7, 1, 3, 9 etc. The power to hundred (the hundredth power of 3) will have a last digit which is the same as the digit for the power to four which is 1.

19. ANSWER: A

For 20 wins the score will be 80 and for 20 losses the score will be –120.

For 10 wins (score 40) together with 10 losses (score – 60) the total score is –20. For 15 wins (score 60) together with 5 losses (score – 30) the total score is 30, which gives the answer. He therefore missed 5 times.

20. ANSWER: D

Suppose he started with Rx, then the equation(s) will be:

Money left after shop 1: $x - (\frac{1}{2}x + 2) = \frac{1}{2}x - 2$

Money left after shop 2: $(\frac{1}{2}x - 2) - [\frac{1}{2}(\frac{1}{2}x - 2) + 1] = \frac{1}{4}x - 2$

Money left after shop 3: $(\frac{1}{4}x - 2) - [\frac{1}{2}(\frac{1}{4}x - 2) + 1] = \frac{1}{8}x - 2$

Money left after shop 4: $\frac{1}{2}(\frac{1}{8}x - 2) = 3 \quad \therefore \frac{1}{16}x - 1 = 3$

$\therefore x - 16 = 48 \quad \therefore x = 64$

or

Try all 5 possibilities until you get 64 which is working:

$\frac{1}{2} \times 64 = 32;\ 32 + 2 = 34,\ \therefore 64 - 34 = 30$ left over,

$\frac{1}{2} \times 30 = 15;\ 15 + 1 = 16,\ \therefore 30 - 16 = 14$ left over,

$\frac{1}{2} \times 14 = 7;\ 7 + 1 = 8,\ \therefore 14 - 8 = 6$ left over,

$6 - (\frac{1}{2} \times 6) = 6 - 3 = 3$ left over, which is the answer.