OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 FIRST ROUND
SENIOR SECTION: GRADE 10 - 12

11 March 2021 **Time: 60 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by five answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 5 marks.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns, ASTEMI

1. One half of 2^{22} is equal to

   (A) 1^{22} (B) 2^{11} (C) 1^{11} (D) 2^{21} (E) 4^{11}

2. The difference between the squares of two numbers is 80. The sum of the two numbers is 16. What is the positive difference between the two numbers?

   (A) 1 (B) 5 (C) 20 (D) 10 (E) 2

3. What is the value of a if $3\sqrt{3} + 2\sqrt{11} - (3\sqrt{11} - \sqrt{3}) = 4\sqrt{3} + a\sqrt{11}$?

   (A) -1 (B) 1 (C) 4 (D) 2 (E) 3

4. The expression $\dfrac{2023^2 - 2021^2}{2} - \dfrac{2023^2 - 2021^2}{4044}$ simplifies to

   (A) 0 (B) 2023 (C) 4044 (D) 4042 (E) 4046

5. For how many integer values of n is $\dfrac{250}{n}$ a positive integer?

   (A) 8 (B) 10 (C) 16 (D) 24 (E) 30

6. A toy shop has one bag that contains 50% red and 50% yellow marbles, another bag contains 25% red and 75% yellow marbles and a third bag contains 21% red, 40% blue and 39% yellow marbles. All the bags have the same number of marbles. What percentage of the marbles will be red if all the marbles are put in one bag?

   (A) 18 (B) 96 (C) 32 (D) 50 (E) 21

7. A fish has a mass of 75 kg plus one quarter of its mass while the fisherman has a mass of 100 kg plus one fifth of his mass. What is the positive difference between their masses, in kg?

   (A) 15 (B) 20 (C) 25 (D) 1 (E) 30

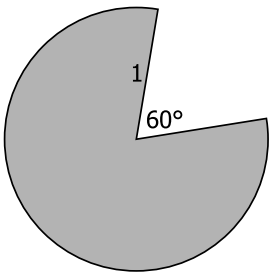
8. The operation $\diamondsuit$ is defined by $x \diamondsuit y = 3x - 8y + xy$. For how many real numbers y does $24 = 8 \diamondsuit y$?

A) 0 (B) 1 (C) 3 (D) 4 (E) more than 4

9. Each side of a cube is increased by 50%. By what percentage is the volume of the cube increased?

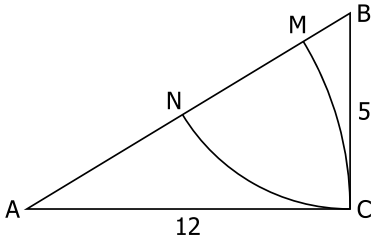
A) 50 (B) 237.5 (C) 153.5 (D) 300 (E) 150

10. In a cellphone game, the "monster" is the shaded sector of a circle of radius 1, as shown in the figure. The missing piece (the mouth) has a central angle of 60°. What is the perimeter of the "monster"?



A) $\pi + 2$ (B) 2π (C) $\frac{5}{3}\pi$ (D) $\frac{5}{6}\pi + 2$ (E) $\frac{5}{3}\pi + 2$

11. In right-angled $\triangle ABC$ with sides 5 and 12, two arcs of circles are drawn, one with centre A and radius 12 and the other one with centre B and radius 5. M and N lie on the hypotenuse. What is the length of MN?



A) 2 (B) $\frac{13}{5}$ (C) 3 (D) 4 (E) $\frac{24}{5}$

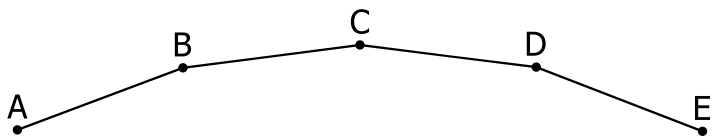
12. Let $p > 1$. What is the value of $\sqrt[3]{p\sqrt[3]{p\sqrt[3]{p}}}$?

A) $p^{\frac{13}{81}}$ (B) $p^{\frac{1}{3}}$ (C) $p^{\frac{1}{9}}$ (D) $p^{\frac{13}{27}}$ (E) p

13. A sequence is formed by repeatedly multiplying by a constant factor. The sequence starts as follows: $54, 36, 24, 16, \ldots$. The tenth term of this sequence can be written in the form $\frac{2^a}{3^b}$. Find the difference between a and b.

A) -1 (B) 1 (C) 3 (D) 4 (E) 7

14. The sketch below represents a section of a regular polygon with side lengths 1. The size of $\angle BDC$ is 10°. What is the perimeter of the polygon?



A) 4 (B) 15 (C) 18 (D) 20 (E) 25

15. Abbey and Zoliswa are running to get fit for a road race. Abbey starts running two minutes before Zoliswa. Abbey runs at 11 km/h and Zoliswa runs at 13 km/h. How many minutes will it take Zoliswa to catch up with Abbey?

A) 2 (B) 15 (C) 11 (D) 4 (E) 10

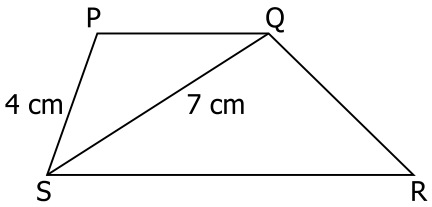
16. What is the units (ones) digit of $(1846^{82})(1007^{82})$?

A) 2 (B) 4 (C) 6 (D) 8 (E) 0

17. Three balls marked 1, 2 and 3, are put in a bag. A ball is drawn at random, its number is recorded, and then the ball is returned into the bag. This is done three times, and each ball is equally likely to be drawn on each occasion. What is the probability that the sum of the numbers on the three balls drawn, is 6?

A) $\frac{1}{27}$ (B) $\frac{1}{3}$ (C) $\frac{7}{27}$ (D) $\frac{2}{9}$ (E) $\frac{1}{9}$

18. Quadrilateral $PQRS$ with $PQ \parallel SR$ and $PQ = 5$ was created by joining two similar triangles. What is the perimeter of the quadrilateral?

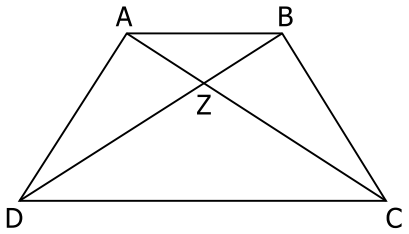


A) $\frac{77}{5}$ (B) 22 (C) 25 (D) $\frac{122}{5}$ (E) $\frac{129}{5}$

19. A country uses the alphabet $\{A, B, C, D, E, ..., Z\}$ and the digits $\{0, 1, 2, ..., 9\}$. Its current licence plate system consists of two letters followed by four digits. The country wants to change to a licence plate system that will consist of four letters followed by three digits. In both cases the letters and digits may be repeated. By what factor will the number of possible licence plate numbers increase?

A) $\frac{26^2}{10}$ (B) $\frac{26}{10}$ (C) $\frac{26^3}{10^3}$ (D) $\frac{26^3}{10^2}$ (E) $\frac{26^2}{10^2}$

20. The figure shows quadrilateral $ABCD$ with $AB = 30$, $CD = 54$ and $AB \parallel DC$. The diagonals, AC and BD, are equal with lengths 56. What is the area of $\triangle ABZ$?



A) 150 (B) $75\sqrt{7}$ (C) $5\sqrt{7}$ (D) $25\sqrt{5}$ (E) $7\sqrt{5}$

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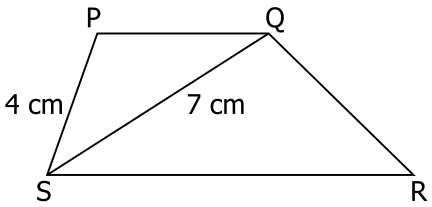
18. Vierhoek $PQRS$ met $PQ \parallel SR$ en $PQ = 5$ is gevorm deur twee gelykvormige driehoeke te verbind. Wat is die omtrek van die vierhoek?

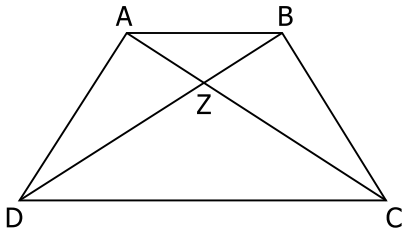


A) $\frac{77}{5}$ (B) 22 (C) 25 (D) $\frac{122}{5}$ (E) $\frac{129}{5}$

19. 'n Land gebruik die alfabet $\{A, B, C, D, E, ..., Z\}$ en syfers $\{0, 1, 2, ..., 9\}$. Die huidige nommerplaatstelsel bestaan uit twee letters gevolg deur vier syfers. Die land wil verander na 'n stelsel wat bestaan uit vier letters gevolg deur drie syfers. In albei gevalle mag die letters en syfers herhaal word. Met watter faktor sal die aantal beskikbare nommerplate toeneem?

A) $\frac{26^2}{10}$ (B) $\frac{26}{10}$ (C) $\frac{26^3}{10^3}$ (D) $\frac{26^3}{10^2}$ (E) $\frac{26^2}{10^2}$

20. Die figuur toon vierhoek $ABCD$ met $AB = 30$, $CD = 54$ en $AB \parallel DC$. Die hoeklyne, AC en BD, is gelyk met lengtes 56. Wat is die oppervlakte van $\triangle ABZ$?

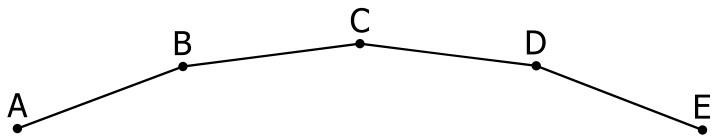


A) 150 (B) $75\sqrt{7}$ (C) $5\sqrt{7}$ (D) $25\sqrt{5}$ (E) $7\sqrt{5}$

13. 'n Ry word gevorm deur herhaaldelik met 'n konstante faktor te vermenigvuldig. Die ry begin as volg: $54, 36, 24, 16, \ldots$. Die tiende term van die ry kan in die vorm $\frac{2^a}{3^b}$ geskryf word. Vind die verskil tussen a en b.

A) -1 (B) 1 (C) 3 (D) 4 (E) 7

14. Die figuur hieronder toon 'n deel van 'n reëlmatige veelhoek met sylengtes 1. Die grootte van $\angle BDC$ is $10°$. Wat is die omtrek van die veelhoek?



A) 4 (B) 15 (C) 18 (D) 20 (E) 25

15. Abbey en Zoliswa hardloop om fiks te word vir 'n padwedloop. Abbey trek twee minute voor Zoliswa weg. Abbey hardloop teen 11 km/h en Zoliswa hardloop teen 13 km/h. Hoeveel minute sal dit Zoliswa neem om Abbey in te haal?

A) 2 (B) 15 (C) 11 (D) 4 (E) 10

16. Wat is die enesyfer van $(1846^{82})(1007^{82})$?

A) 2 (B) 4 (C) 6 (D) 8 (E) 0

17. Drie balle, gemerk 1, 2 en 3, word in 'n sak gesit. 'n Bal word ewekansig uitgehaal, sy nommer word neergeskryf en die bal word teruggesit in die sak. Dit word drie keer gedoen en elke bal het dieselfde kans om uitgehaal te word. Wat is die waarskynlikheid dat die som van die getalle op die drie balle wat uitgehaal is, gelyk is aan 6?

A) $\frac{1}{27}$ (B) $\frac{1}{3}$ (C) $\frac{7}{27}$ (D) $\frac{2}{9}$ (E) $\frac{1}{9}$

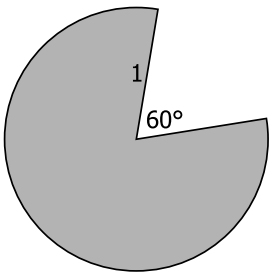
8. Die bewerking $\diamondsuit$ word gedefinieer deur $x \diamondsuit y = 3x - 8y + xy$. Vir hoeveel reële getalle y is $24 = 8 \diamondsuit y$?

A) 0 (B) 1 (C) 3 (D) 4 (E) meer as 4

9. Elke sy van 'n kubus word met 50% vermeerder. Met watter persentasie vermeerder die volume van die kubus?

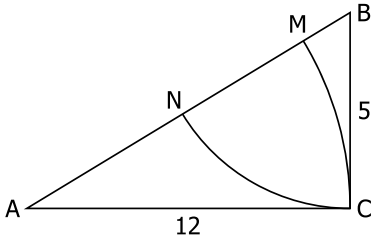
A) 50 (B) 237.5 (C) 153.5 (D) 300 (E) 150

10. In 'n selfoonspeletjie is die "monster" die ingekleurde sektor van 'n sirkel met radius 1, soos aangetoon in die figuur. Die vermiste deel (die mond) het 'n hoek van 60° by die middelpunt. Wat is die omtrek van die "monster"?



A) $\pi + 2$ (B) 2π (C) $\frac{5}{3}\pi$ (D) $\frac{5}{6}\pi + 2$ (E) $\frac{5}{3}\pi + 2$

11. In die reghoekige $\triangle ABC$ met sye 5 en 12, is twee sirkelboë getrek, een met middelpunt A en radius 12, die ander met middelpunt B en radius 5. M en N is beide op die skuinssy. Wat is die lengte van MN?



A) 2 (B) $\frac{13}{5}$ (C) 3 (D) 4 (E) $\frac{24}{5}$

12. Laat $p > 1$. Wat is die waarde van $\sqrt[3]{p\sqrt[3]{p\sqrt[3]{p}}}$?

A) $p^{\frac{13}{81}}$ (B) $p^{\frac{1}{3}}$ (C) $p^{\frac{1}{9}}$ (D) $p^{\frac{13}{27}}$ (E) p

1. Die helfte van 2^{22} is gelyk aan

   (A) 1^{22} (B) 2^{11} (C) 1^{11} (D) 2^{21} (E) 4^{11}

2. Die verskil tussen die kwadrate van twee getalle is 80. Die som van die twee getalle is 16. Wat is die positiewe verskil tussen die twee getalle?

   (A) 1 (B) 5 (C) 20 (D) 10 (E) 2

3. Wat is die waarde van a as $3\sqrt{3} + 2\sqrt{11} - (3\sqrt{11} - \sqrt{3}) = 4\sqrt{3} + a\sqrt{11}$?

   (A) -1 (B) 1 (C) 4 (D) 2 (E) 3

4. Die uitdrukking $\dfrac{2023^2 - 2021^2}{2} - \dfrac{2023^2 - 2021^2}{4044}$ vereenvoudig na

   (A) 0 (B) 2023 (C) 4044 (D) 4042 (E) 4046

5. Vir hoeveel heelgetalwaardes van n is $\dfrac{250}{n}$ 'n positiewe heelgetal?

   (A) 8 (B) 10 (C) 16 (D) 24 (E) 30

6. 'n Speelgoedwinkel het 'n sak met 50% rooi en 50% geel albasters, 'n ander sak met 25% rooi en 75% geel albasters en 'n derde sak met 21% rooi, 40% blou en 39% geel albasters. Al die sakke bevat almal ewe veel albasters. Watter persentasie albasters sal rooi wees as al die albasters in een sak gegooi word?

   (A) 18 (B) 96 (C) 32 (D) 50 (E) 21

7. 'n Vis se massa is 75 kg plus een kwart van sy massa terwyl die visserman se massa 100 kg plus een vyfde van sy massa is. Wat is die positiewe verskil tussen hulle massas, in kg?

   (A) 15 (B) 20 (C) 25 (D) 1 (E) 30

OLD MUTUAL SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 EERSTE RONDTE
SENIOR AFDELING: GRAAD 10-12

11 Maart 2021 **Tyd: 60 minute** **Aantal vrae: 20**

Instruksies

1. *Hierdie is 'n veelvuldige-keuse vraestel. Na elke vraag is vyf antwoorde, genommer A, B, C, D en E. Net een van hulle is reg.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 5 punte.*
   2.2. *Daar is geen penalisering vir foutiewe antwoorde of vrae wat nie beantwoord is nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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