SOUTH AFRICAN MATHEMATICS OLYMPIAD
2014 SECOND ROUND: JUNIOR DIVISION
SOLUTIONS

1. $\sqrt{201-4-(2\times0+1)^4} = \sqrt{197-1^4} = \sqrt{196} = 14$

2. With OA = OB (radii), $O\hat{B}A = O\hat{A}B = 70°$.

   Now $C\hat{B}A + O\hat{A}B = 180°$ (cointerior), so $C\hat{B}O = 40°$.

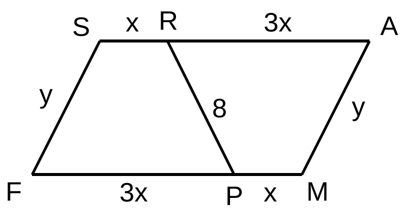
   But then $O\hat{C}B = 40°$ and $C\hat{O}B = 100°$

3. Join AC. By Pythagoras' Theorem $AC^2 = 16 + 49 = 65$. But $AC^2 = AB^2 + BC^2$, therefore $65 = AB^2 + 1$, and so AB = 8.

4. $315 = 3^2 \times 5 \times 7$, so K must be $5 \times 7 = 35$

5. The hands coincide at noon, then again shortly after 1 o'clock, a little more than that after 2 o'clock, and so on. The time after 10 o'clock at which they coincide is shortly before 11 o'clock, but the time after 11 o'clock at which they coincide is actually 12 o'clock. So the hands coincide at noon and at midnight and after each of 10 'hours' in between. This is 22 times.

6. Clearly SR = PM and RA = FP.
   Let SF = AM = y and SR = PM = x.
   Thus RA = FP = $3x$.
   Perimeter of SAMF = 52, i.e. $8x + 2y = 52$.
   Now perimeter of RAMP = $4x + y + 8$
   $= ½(8x + 2y) + 8 = 34$.

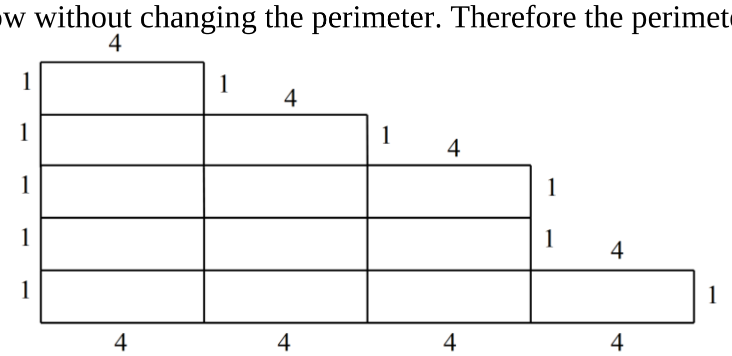


7. All powers of odd numbers are odd. The sum of four odd numbers is always even, thus it is divisible by 2.

8. If the length of a small part is x, the other lengths are x and $1.5x$ with sum $3.5x$. Since this is 77, $3.5x = 77$, so $7x = 7\times11\times2$ and therefore x is 22; then the largest part is $22 \times 1.5 = 33$.

9. There are $2\times 8 = 16$ ends altogether. Whichever end I pick first, there is only 1 of the remaining ends that belongs to the same piece of string, and the chance that I pick that as the second end is 1/15, i.e. $k = 15$.

10. The diagram can be rearranged as below without changing the perimeter. Therefore the perimeter will be $8 \times 4 + 10 \times 1 = 42$



11. All the triangles in the top rectangle have the same height, whether white or shaded, so the sum of the areas of the shaded triangles depends on the sum of their bases, which is the same as for the white triangles. Thus the shaded triangles occupy one half of the rectangle, the white triangles the other half. The same applies to the bottom rectangle. So the shaded area is in total one half of the rectangle ABCD, which means it is $9 \times 6/2 = 27$

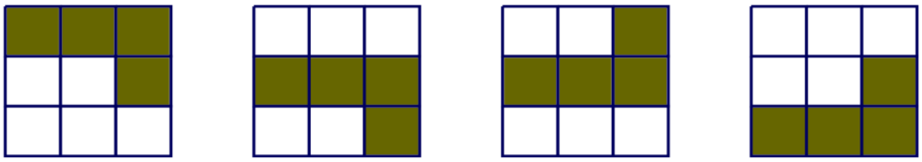
12. The sequence is 1, 4, 3, -1, -4, -3, 1, 4, 3, -1, -4, -3, … The sequence repeats every 6 terms and the sum of the 6 repeating terms is zero. The sum of the first 96 terms (i.e. 16 multiples of 6) is thus zero. The sum of the first 100 terms would therefore be 0 + 1 + 4 + 3 + (-1) = 7.

13. We can pair off the two-digit numbers where each number in a pair is the reverse of the other, except that the numbers with both digits the same have no partner. There are 9 two-digit numbers with both digits the same, and 9 two-digit numbers that are single-digit when reversed. Since there are 90 two-digit numbers altogether, there must be (90–9 – 9)/2 = 36 pairs and therefore that number of two-digit numbers whose reverse is bigger.

14. Let the new radius be R. Then $\pi R^2 = 4.\pi.5^2$, which means R = 10.
But then R has increased from 5 to 10, so the circumference has changed from $2\pi.5$ to $2\pi.10$, which is an increase of 10π, at a rate of π cm per minute. This must take 10 minutes.

15. Let N be the 2-digit number '*ab*'. Then $N = 10a + b$. The reverse of N, i.e. '*ba*' is $10b + a$.
The sum of '*ab*' and '*ba*' is thus $11a + 11b$ i.e. $11(a + b)$. Since 11 is prime, for $11(a + b)$ to be a perfect square $a + b = 11$ (since a and b must each be less than 10). There are thus 8 possible values for N, namely 29, 38, 47, 56, 65, 74, 83, 92.

16. With its long part horizontal the L-piece can be placed in 8 different ways: two with the short leg going down, and two with the short leg up (see diagram) and then 4 others with the short leg on the left of the grid.



It can also be placed in 8 ways with the long leg vertical. For each position of the L-piece there are 5 possible positions for the round piece. This gives $(8 + 8) \times 5 = 80$ possibilities.

17. If the middle cell in the last column contains a, we can see that $3 + x + a = y + a + 4$, which easily rearranges to give $x - y = 1$.

1		y
3	x	a
		4

18. The square SAMF has side length 13. Since the area of the smaller inner square is a quarter of the larger inner square, its side length must be half that of the larger. Let the side length of the smaller inner square be x; then the larger inner square has side length $2x$. Since the overlap has area 4 and is also a square (from symmetry) it has side length 2. We thus have the side length of square SAMF equal to $(2x - 2) + x = 13$, which gives $x = 5$. The shaded rectangle therefore has dimensions 8 by 3, so its area is 24.

19. Let the hats be labelled **a**, **b**, **c**, **d** according to their owner's name. If everyone has the wrong hat, then hat **a** cannot go with person A; suppose it goes to person B. Then to avoid either C or D getting the right hat, it must be that hat **c** goes to person D, and hat **d** to person C. Continuing in this way we draw up a list of all possibilities and find there are 9 altogether.

a	**b**	**c**	**d**	**a**	**b**	**c**	**d**	**a**	**b**	**c**	**d**
B	A	D	C	C	A	D	B	D	A	B	C
B	D	A	C	C	D	A	B	D	C	A	B
B	C	D	A	C	D	B	A	D	C	B	A

20. $2014 = 8 \times 251 + 6$, which means 2014 is in column **d** immediately after 2013 in column **e**. The numbers in column **e** have the form $8k - 3$ where $k = 1$ (for 5) up to 252 (for 2013). Of these 252 numbers, every third one is divisible by 3, and $252 = 3 \times 84$. So there are 84 multiples of 3 in column **e**.

NUMERICAL ANSWERS

1. 14
2. 100
3. 8
4. 35
5. 22
6. 34
7. 2
8. 33
9. 15
10. 42
11. 27
12. 7
13. 36
14. 10
15. 8
16. 80
17. 1
18. 24
19. 9
20. 84