THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

SECOND ROUND 1998

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

26 MAY 1998

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 7 marks in Part C.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.
7. When the invigilator gives the signal, start attempting the problems. You will have 120 minutes working time for the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers

$$0,5129; \quad 0,9; \quad 0,89; \text{ and } 0,289$$

is

(A) 1,189 (B) 0,8019 (C) 1,428 (D) 1,179 (E) 1,4129.

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PART A: 3 marks each

1. Successive discounts of 10% and 20% are equivalent to a single discount of

(A) 30% (B) 15% (C) 72% (D) 28% (E) None of these.

2. m and n are integers with $m > n$. The number of integers between (but not including) m and n, is

(A) $m - n$ (B) $m - n - 1$ (C) $m - n + 1$ (D) $m + n$ (E) $m + n - 1$

3. Recall that 1 litre is 1 000 cubic centimetres.
A hosepipe is 20m long with an inside diameter of 15mm. The amount of water (in litres) that it takes to fill the hosepipe is closest to

(A) 0,45 (B) 3,5 (C) 4,5 (D) 35 (E) 45

4. The perimeter of a square is four times the perimeter of another square. What is the ratio of their areas?

(A) 2:1 (B) 4:1 (C) 8:1 (D) 16:1 (E) 64:1

5. If n is a natural number then we define $n!$ to be the product $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. For example $4! = 4 \times 3 \times 2 \times 1 = 24$. If $6! = a! \times b!$ where $a > 1$ and $b > 1$, then $a + b$ is

(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

6. How many black tiles will be required to build the 15th figure in the given pattern?

fig 1

fig 2

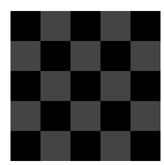
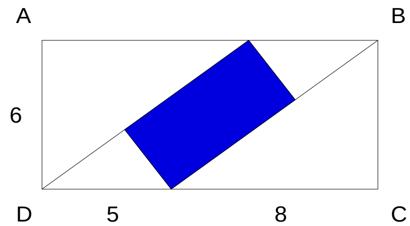


fig 3

(A) 403 (B) 365 (C) 481 (D) 421 (E) 225

Part B: 5 marks each

7. $ABCD$ is a rectangle. The area of the shaded rectangle is



(A) 10 (B) 14 (C) 15 (D) 16 (E) 18

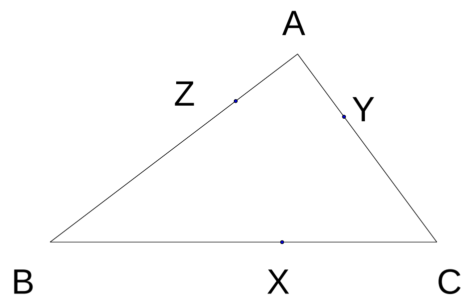
8. Let p be a solution of the equation $x^6 - 2 = 0$. Then the value of $(p-1)(p^{12} + p^{13} + \cdots + p^{41})$ is

(A) 124 (B) 144 (C) 192 (D) 212 (E) 252

9. If m and n are natural numbers such that $\sqrt{7+\sqrt{48}} = \sqrt{m} + \sqrt{n}$ then $m^2 + n^2$ equals

(A) 25 (B) 37 (C) 29 (D) 40 (E) 41

10. In $\triangle ABC$, $AY = AZ$, $BZ = BX$ and $CX = CY$. The length of BC is a, that of CA is b and BA is c. The length of AY is

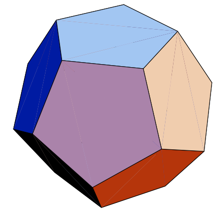


(A) $\frac{1}{2}(a+c-b)$ (B) $\frac{1}{2}(a+b-c)$ (C) $\frac{1}{2}(b+c-a)$ (D) $\frac{1}{4}(2b+a-c)$
(E) $\frac{1}{4}(2b+c-a)$

11. $ABCDEFGH$ is a regular octagon (an 8-sided figure). The angle HBC is

(A) $112\frac{1}{2}^o$ (B) 108^o (C) 90^o (D) 105^o (E) $97\frac{1}{2}^o$

12. The dodecahedron illustrated is a regular solid with 12 faces, each of which is a regular pentagon (5-sided figure). The faces are joined along edges and meet at vertices. For this solid the number of vertices plus the number of edges is



(A) 20 (B) 42 (C) 50 (D) 60 (E)72

13. Agnes said 'Billy always speaks the truth'.
Billy said 'Agnes is lying'.
Charlotte said 'Agnes and Billy are both lying'.
Who is speaking the truth?

(A) Only Agnes (B) Only Billy (C) Only Charlotte
(D) Both Billy and Charlotte (E) None of them.

14. The number of pairs of integers $(x;y)$ that are solutions to the equation $6x+15y=85$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3.

Part C: 7 marks each

15. $ABCD$ is a trapezium with AD parallel to BC. The diagonals AC and BD intersect at O. If the area of triangle AOD is x, and the area of triangle BOC is y then the area of the trapezium is

(A) $2(x+y)$ (B) $4\sqrt{xy}$ (C) $\frac{8xy}{x+y}$ (D) $(\sqrt{x}+\sqrt{y})^2$ (E) None of these.

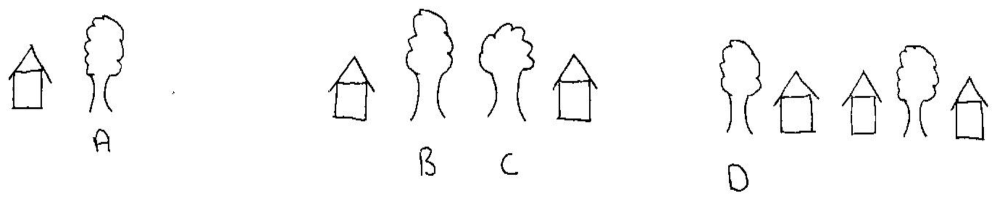
16. If $s = 1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots$, then $1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\cdots$ equals

(A) $\frac{1}{2}s$ (B) $\frac{3}{4}s$ (C) $s-\frac{1}{4}$ (D) $s-\frac{1}{2}$ (E) Cannot be determined

17. f is a function for which $f(1)=1$ and $f(n)=n+f(n-1)$ for each natural number $n \geq 2$. The value of $f(19)$ is

(A) 171 (B) 190 (C) 210 (D) 241 (E) 255

18. The houses and trees in the diagram are all in a straight line. In each of the six houses lives a child. At which tree should the children meet so that the sum of the distances they walk to that tree is a minimum?



(A) A (B) B (C) C (D) D (E) Impossible to determine.

19. The tune 'Twinkle, Twinkle Little Star' has 7 notes in its first line, $CCGGAAG$. All notes are held the same length of time. If the notes are rearranged at random, how many different melodies can be composed?

(A) 5040 (B) 210 (C) 105 (D) 72 (E) 12

20. $P(x)$ is a polynomial of degree 1998 such that $P(k) = \frac{1}{k}$ for $k = 1, 2, \ldots, 1999$. The value of $P(2\ 000)$ is

(A) $\frac{1}{1\ 000}$ (B) $\frac{1}{999}$ (C) $\frac{1}{2000}$ (D) $\frac{1}{1\ 999}$ (E) $\frac{1}{4\ 000}$