HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

THIRD ROUND 2012
JUNIOR SECTION: GRADES 8 AND 9

6 SEPTEMBER 2012
TIME: 4 HOURS
NUMBER OF QUESTIONS: 15
TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- For Question 9 and Question 13 you need a number of rings that will be provided.
- There is an Answer Sheet at the end of the paper on which you need to do Question 1 and which you need to hand in.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions will be available at: www.samf.ac.za

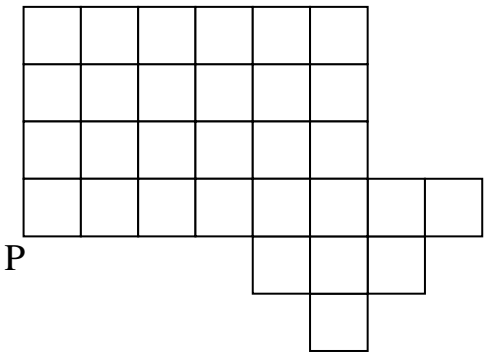
DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9323 FAX: (012) 392-9312
E-mail:ellie@samf.ac.za

Organizations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

Question 1 (This question must be done on the answer sheet)

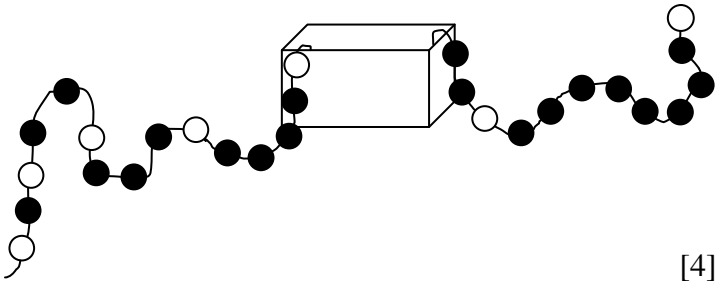
The figure is made up of squares.
Draw a straight line through P to divide the shape into two equal parts and **explain** why your line in fact works.



[4]

Question 2

This string of beads was made according to a certain pattern. How many beads are hidden in the box?

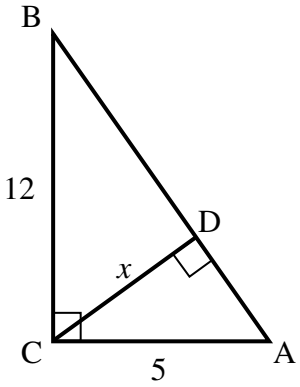


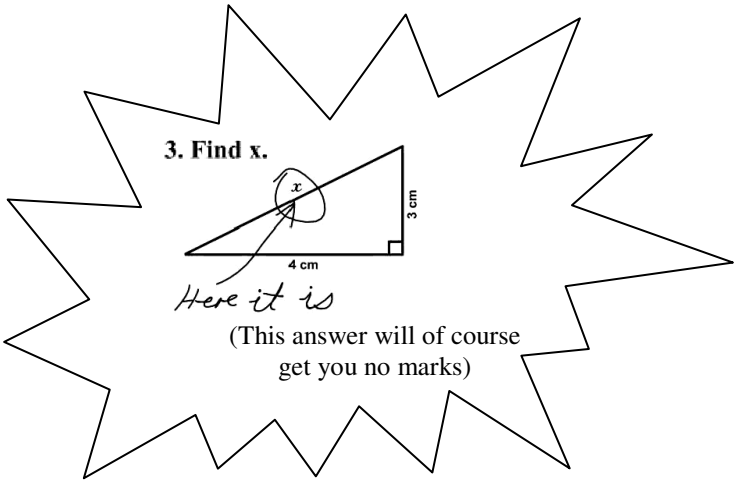
[4]

Question 3

In the right-angled triangle ABC, BC = 12 and AC = 5. DC is perpendicular to AB.

Find *x*.





[6]

Question 4

Calculate each of the following and thus state which of them are integers.

(a) $((1 \div 2) \div 3) \div 4$

(b) $(1 \div 2) \div (3 \div 4)$

(c) $1 \div ((2 \div 3) \div 4)$

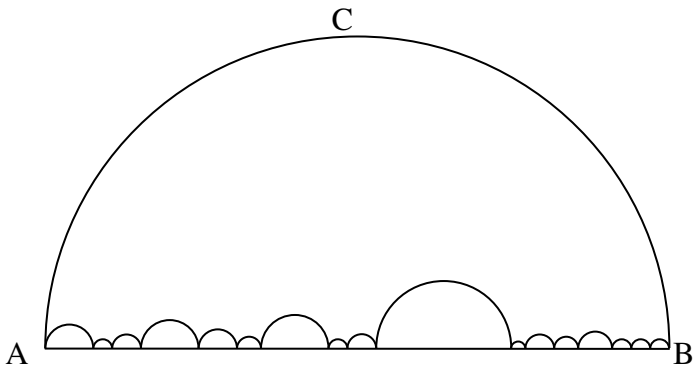
(d) $(1 \div (2 \div 3)) \div 4$

(e) $1 \div (2 \div (3 \div 4))$

[6]

Question 5

When you travel from A to B, you can either travel along the big semi-circle (i.e. via C) or you can travel along all the smaller semi-circles.



Which is the shorter route and why?

[6]

Question 6

The Luhn algorithm is used in computer systems to check the validity of a credit card number as follows:

- Starting from the right, colour each second digit in the number red.
- Count the number of red digits that are 5 or greater. Let this number be R.
- Add up all the digits in the number, and add each red digit again.
- Finally add the number R.
- If the final sum is divisible by 10, then the credit card number is valid.

For example: 4532 0972 5042 9185 R = 4

Sum = (4+5+3+2+0+9+7+2+5+0+4+2+9+1+8+5) + (4+3+0+7+5+4+9+8) + 4 = 110

Since 110 is divisible by 10 we have a valid credit card number.

Find the missing digit in the following valid credit card numbers.

(a) 4565 3328 9132 662**X** (2)

(b) 5232 7198 3402 **Y**781 (4)

[6]

Question 7

Fred cycles up a hill at a constant speed of 5 km/h (Yes it's very steep). He cycles down the hill at a constant speed of 45 km/h.

(a) What is his average speed? (4)

(b) If he is able to increase his speed on the downhill, what is the maximum average speed he will be able to achieve? (2)

[6]

Question 8

The picture shows a gift that you can buy in curio shops. It is a calendar which tells you the date and consists of two loose cubes which can be moved and rotated in any way. There must always be two numbers on display and in this case the date is 16 February.

(Don't worry about the month which is displayed below the cubes)

What numbers must be on the six faces of each of the cubes so that all the necessary days of any month can be displayed?

[6]

Question 9

A certain type of ring has an outer diameter of 58 mm and an inner diameter of 40 mm and a thickness of 1 mm.

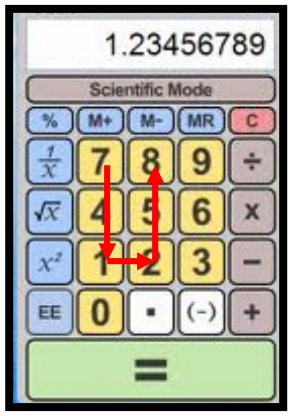
If one stacks enough rings on top of each other, it is possible to stand another ring vertically on top of the pile in such a way that the ring doesn't touch the ground.

What is the minimum number of rings you need to stack on top of each other so that the vertical ring just doesn't touch the ground?

(You have been given some rings (washers) to help you with this question. They ***do not have the same dimensions*** *as in the question but you can build the model with them if you need to)*

[8]

Question 10



Did you know that if you form a four digit number using any four non-zero digits on the corners of any rectangle on a calculator the number will always be divisible by 11!!

In the example in the picture we have 7128.

Look : $\frac{7128}{11} = 648$ which is an integer.

5236 also works: $\frac{5236}{11} = 476$ which is an integer.

It even works if you rotate your calculator!!

(i.e. if you use numbers like 2365 or 4697)

(a) Prove it.

(b) Prove that it even works if you rotate the calculator 90 degrees clockwise.

[8]

Question 11

N students are playing an elimination game with a paintball gun. The one student that doesn't get shot is the winner! The method of elimination works as follows: All students stand in a circle. A paintball gun is given to student 1, and he has to shoot the student to his left (who is then eliminated). He then passes the paintball gun to the next "alive" student on his left, who, in turn will shoot the student on his left, and pass the paintball gun to the next "alive" student on his left, and so on.

For example, let N = 5. Call them students 1, 2, 3, 4 and 5.

- 1 shoots 2, and passes the paintball gun to 3.
- 3 shoots 4, and passes the paintball gun to 5.
- 5 shoots 1, and passes the paintball gun to 3 (2 is already "dead")
- 3 shoots 5, and 3 is the last one left and so he is the winner!

Who will be left with the paintball gun (not shot) and therefore the winner for N = 100?

[8]

Question 12

Put the positive integers from 5 to 9 into the blocks (with no repeats)

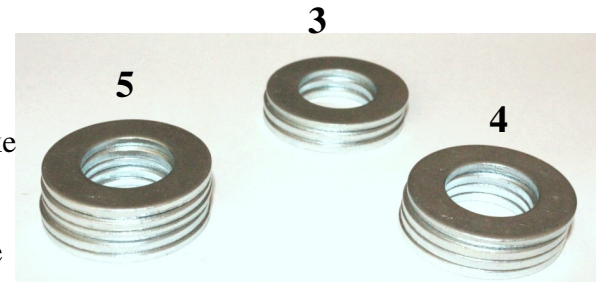
such that the resulting product will be the greatest.

$\square\square\square \times \square\square$

[8]

Question 13

The age-old game of NIM is a game for two players. Three piles of rings lie on the table with any number of rings in each pile. The players take it in turns to take any number of rings from any pile and the last player to take a ring loses. There is a winning strategy for this game.



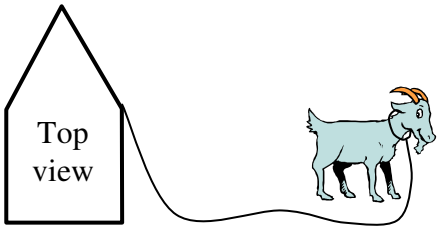
If the piles have 3, 4 and 5 rings in them respectively, explain the winning strategy and whether you should go first or second.

[8]

Question 14

(a) A goat is tethered to the corner of a shed which consists of a square and an equilateral triangle. The square has side length 2 m and the rope is 5 m long. What is the maximum area that the goat can graze outside of its shed?

Give your answer in terms of π.

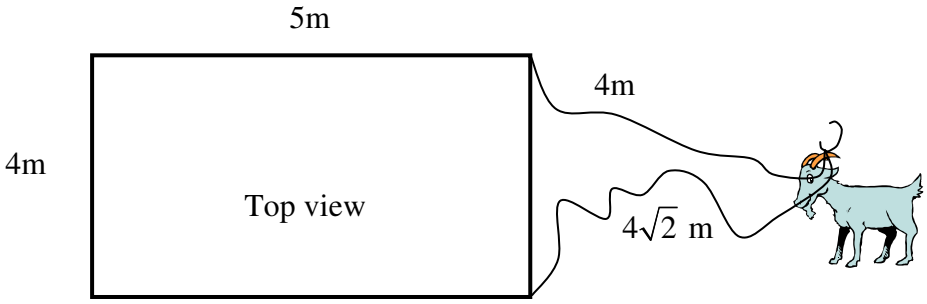


(5)

(b) This time the goat is tethered to the corner of a rectangular shed 4m by 5m, but with two ropes of length $4\sqrt{2}$ m and 4 m as shown.

What is the area the goat can graze?

Again, give your answer in terms of π. (5)



[10]

Question 15

2012 is a special number. Look at its property in terms of powers of two:

$20 - 12 = 8 = 2^3$ and $20 + 12 = 32 = 2^5$

Which is the next year with this property?

[8]

Total: 100

THE END

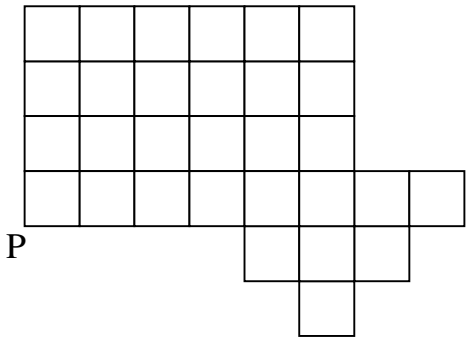
Please turn over for the Answer Sheet to answer Question 1

ANSWER SHEET

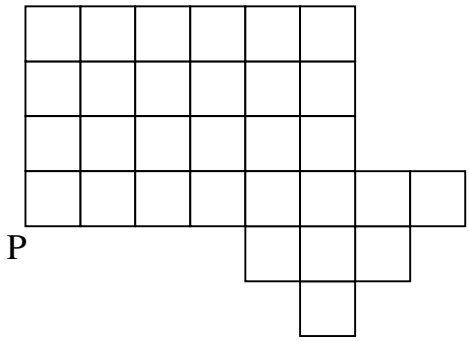
This Answer Sheet needs to be handed in!

Question 1

Practice Grid



Final answer



Explanation: _____

_____

_____

_____

_____

_____

Name: _____

School: _____

Grade: _____