THE OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

18 May 2021 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 000 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 4 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

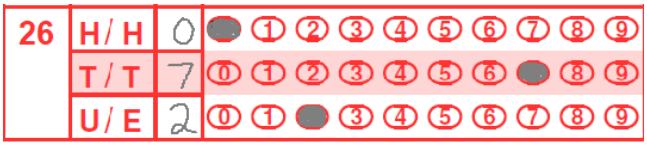
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



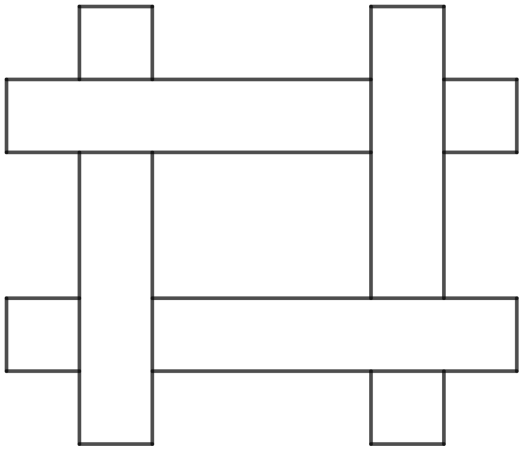
Write the digits of your answer in the the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

DO NOT TURN THE PAGE BEFORE YOU ARE TOLD TO DO SO

Part A: Three marks each

1. A scale with two pans will balance if 8 identical cakes were put in one pan and 6 identical cakes and 0.125 kg flour in the other pan. What is the mass of 16 cakes in kg?

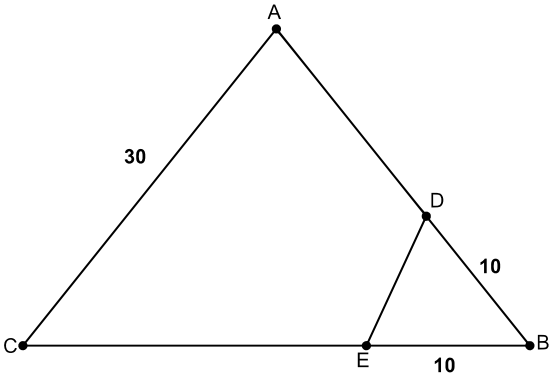
2. Four rectangular fabric strips of length 10 cm and width 1 cm are put flat on a table and overlap at right angles as shown. What area of the table, in square cm, is covered?



3. A palindromic number reads the same from left to right and from right to left, e.g. 14541. The palindrome 15951 shows on the odometer of your car. How far must you travel before the next palindromic number shows on the odometer?

4. February 14, Valentine's Day, was on a Sunday in 2021. What are the last two digits of the first year after 2021 in which Valentine's Day will fall on a Sunday?

5. A triangular corner with side length $DB = EB = 10$ is cut from the equilateral triangle ABC of side length 30. What is the perimeter of quadrilateral $ADEC$?



6. How many 3-digit numbers with digits from the set $\{2, 3, 7, 9\}$ are divisible by 3? Digits may not be repeated.

7. What is the difference between the two prime numbers that add up to 1 less than the largest perfect cube less than 100?

8. The number 126 can be written as the sum of two different prime numbers in several ways. Find the largest difference between two such prime numbers.

9. Pascal, Newton, Galileo and Fermat all took the same test. The average score of the four candidates was 16. Pascal and Newton had an average of 16, Pascal and Fermat had an average of 13, while Newton and Fermat had an average of 18. What was Galileo's score?

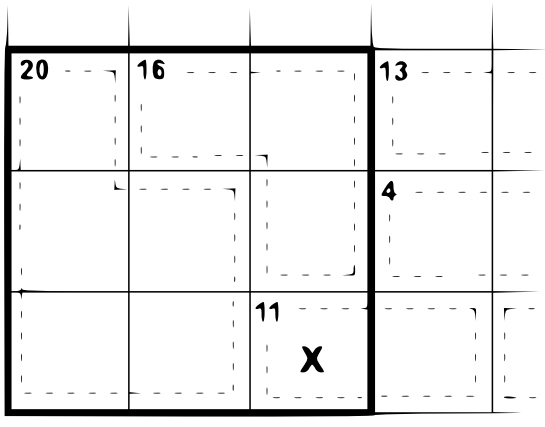
10. The number 2021 is the product of two successive prime numbers. What is the sum of these two prime numbers?

Part B: Four marks each

11. What is the least possible perimeter of a triangle whose side lengths are consecutive square numbers?

12. Two empty containers, P and Q, have the same volume. Water flows into P at a rate of 4 litres per minute, and into Q at a rate of 6 litres per minute. After a certain time container P can still take another 60 litres but Q has overflowed by 10 litres. What is the volume of each container?

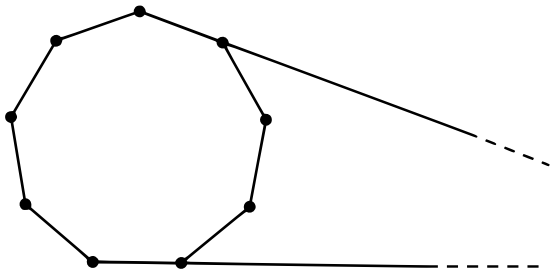
13. Killer Sudoku consists of a 9 by 9 block of small squares. It is divided into 3 by 3 blocks that consists of 9 small squares each. The digits 1 to 9 must be put in these small squares. No digit may be repeated in any row or column or 3 by 3 block of 9 small squares. One 3 by 3 block is shown below. The number in the upper left corner of the group of small squares enclosed by dotted lines is the sum of the digits in the small squares in that group. With which digit should the X be replaced?



14. A man can commute either by train or by bus. If he goes to work by train in the morning, he comes home by bus in the afternoon; and if he comes home in the afternoon by train, he took the bus in the morning. In a particular month the man took the bus to work in the morning 8 times, came home by bus in the afternoon 15 times, and commuted by train (either morning or afternoon) 9 times. How many days in this month did the man work?

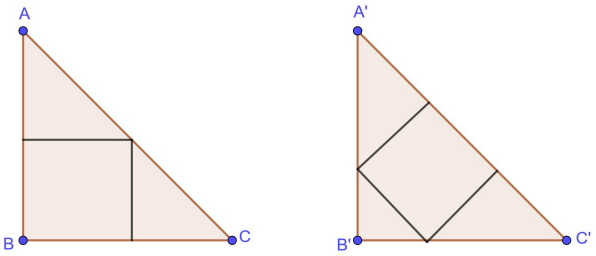
15. A committee has 4 meetings and at each meeting there are 3 attendees. No two committee members attend more than 1 meeting together. What is the least number of members there can be on the committee?

16. What is the size of the angle that is formed by extending two sides of a regular nonagon,as shown in the sketch, to eventually meet?



17. Every child in the school has one or more unique numbers for the school's lucky draw. The numbers start at 1 and continue up to 1000. What is the probability, multiplied by 1000, that the first number drawn at random has no seven and no nine among its digits?

18. Two squares are inscribed differently in the same given isosceles right triangle as shown. If it is done as in the first figure, then one finds that the area of the square is 441. What is the area of the square inscribed in the same triangle as shown in the second figure?



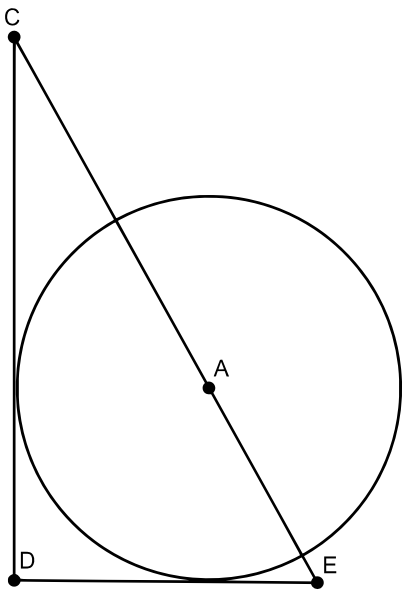
19. In a chess tournament, each of eight participants plays one game against each of the others. A win is worth 1 point, a draw is $\frac{1}{2}$ a point and a loss 0 points. At the end of the tournament each of the participants has a different total score. The total score of the participant in second place is equal to the sum of the scores of the participants in the bottom four places. If the participant in eighth place drew one match and lost the rest, what is the maximum score the participant in fifth place could have achieved?

20. Find the value of

$$66 \times \left(\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+21} \right).$$

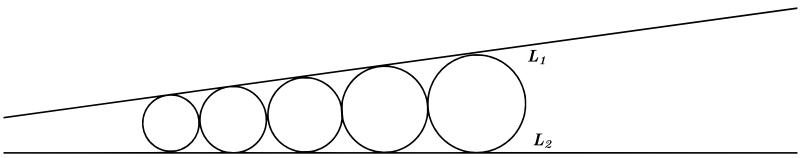
Part C: Six marks each

21. The lengths of the legs of a right triangle are 18 and 63. $\angle CDE$ is 90°. What is the radius of the circle that is tangent to both legs of the triangle and has its centre on the hypotenuse as shown in the figure?

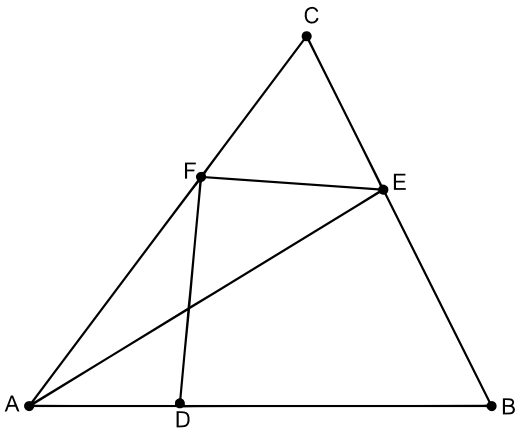


22. Jo has 20 litres of each of two different solutions of salt. The concentration of solution A is 20% and for solution B it is 45%. How many litres of solution B must be added to 10 litres of solution A to obtain a solution with a concentration of 30%? Round your answer to the nearest integer.

23. Five circles are tangent to one another consecutively and to the lines L_1 and L_2. If the radius of the largest circle is 288 and that of the smallest one is 128, what is the radius of the middle circle?



24. Triangle ABC in the figure has area 450. Points D, E and F, all distinct from A, B and C, are on sides AB, BC and CA respectively, and $\dfrac{AD}{DB} = \dfrac{2}{3}$. Triangle ABE and quadrilateral $DBEF$ have equal areas. What is the area of each?



25. If $n! = n(n-1)(n-2)(n-3)\cdots 3 \times 2 \times 1$, determine the largest positive integer n such that 1005! is divisible by 10^n.

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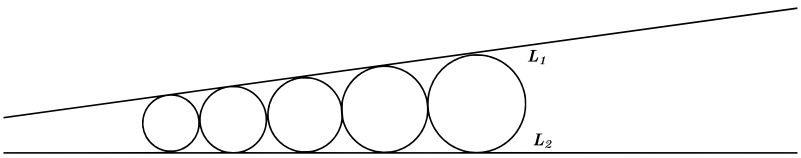
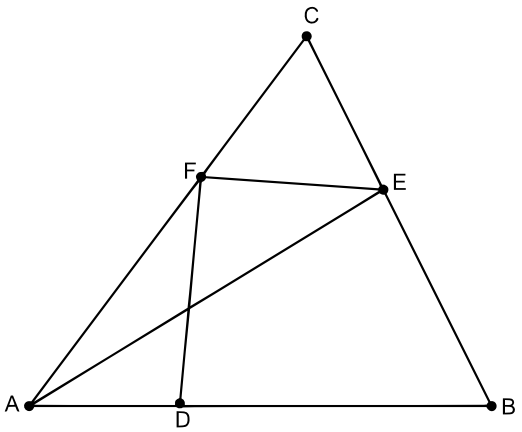
23. Vyf sirkels raak opeenvolgend aan mekaar en ook aan die lyne L_1 en L_2. Die radius van die grootste sirkel is 288 en die radius van die kleinste een is 128. Wat is die radius van die sirkel in die middel?



24. Die oppervlakte van driehoek ABC in die figuur is 450. Punte D, E en F, almal verskillend van A, B en C, is onderskeidelik op sye AB, BC en CA, terwyl $\frac{AD}{DB} = \frac{2}{3}$. Driehoek ABE en vierhoek $DBEF$ het gelyke oppervlaktes. Wat is die oppervlakte van elk?



25. As $n! = n(n-1)(n-2)(n-3)\cdots 3 \times 2 \times 1$, bepaal die grootste positiewe heelgetal n sodat 1005! deelbaar is deur 10^n.

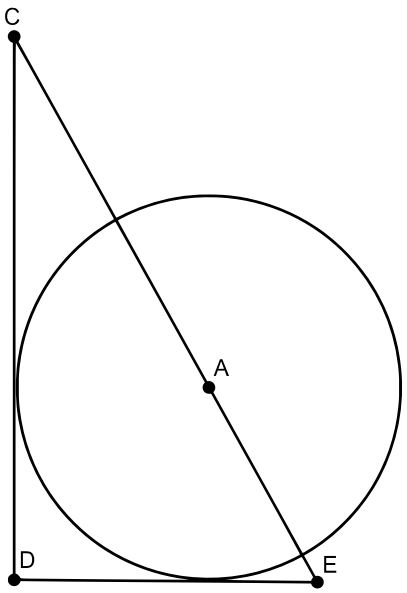
19. In 'n skaaktoernooi speel elk van die agt deelnemers een spel teen elke ander deelnemer. 'n Wen tel 1 punt, gelykop is 'n $\frac{1}{2}$ punt en verloor is 0 punte. Aan die einde van die toernooi het al die deelnemers verskillende totale. Die totale punt van die deelnemer in die tweede plek is dieselfde as die som van die punte van die deelnemers in die onderste vier plekke. Die deelnemer in die agtste plek het een spel gelykop gespeel en al die ander verloor. Wat is die hoogste punt wat die deelnemer in die vyfde plek kon behaal?

20. Vind die waarde van

$$66 \times \left(\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+21} \right).$$

Afdeling C: Ses punte elk

21. Die lengtes van die reghoeksye van 'n reghoekige driehoek is 18 en 63 en $\angle CDE$ is 90°. Wat is die radius van die sirkel wat aan albei reghoeksye raak en waarvan die middelpunt op die skuinssy is, soos aangetoon in die figuur?

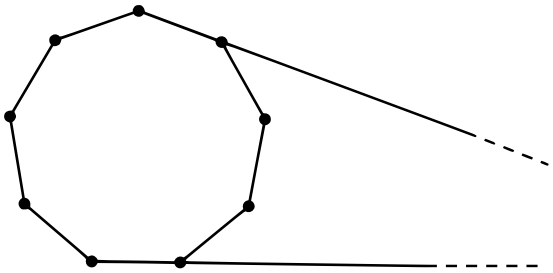


22. Jo het 20 liter van elk van twee verskillende soutoplossings. Die konsentrasie van oplossing A is 20% en vir oplossing B is dit 45%. Hoeveel liter van oplossing B moet by 10 liter van oplossing A gevoeg word om 'n oplossing met 'n konsentrasie van 30% te kry? Rond jou antwoord tot die naaste heelgetal.

14. 'n Man kan werk toe ry met 'n bus of met 'n trein. As hy in die oggend met die trein ry, kom hy in die middag met die bus huis toe; en as hy in die middag met die trein huis toe kom, het hy in die oggend met bus gery. In 'n sekere maand het die man 8 keer met die bus werk toe gegaan en in die middag 15 keer bus gery huis toe. Hy het 9 keer (in die oggend of in die middag) met die trein gery. Hoeveel dae het die man hierdie betrokke maand gewerk?

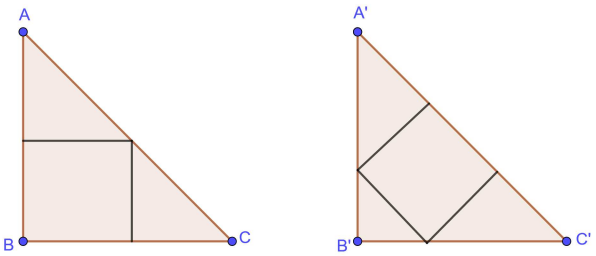
15. 'n Komitee het 4 vergaderings gehad en by elke vergadering was 3 komiteelede teenwoordig. Geen twee komiteelede het meer as 1 vergadering saam bygewoon nie. Wat is kleinste aantal lede wat op hierdie komitee kan wees?

16. Hoe groot is die hoek wat gevorm word as die verlengdes van twee sye van 'n reëlmatige negehoek, soos aangetoon in die figuur, uiteindelik ontmoet?



17. Elke kind in die skool het een of meer unieke nommers gekry vir die skool se gelukstrekking. Die getalle begin by 1 en gaan tot by 1000. Wat is die waarskynlikheid, vermenigvuldig met 1000, dat die eerste getal wat ewekansig getrek word geen sewe en geen nege as 'n syfer sal hê nie?

18. Twee vierkante is op verskillende maniere ingeskrewe in dieselfde reghoekige gelykbenige driehoek soos aangetoon. In die figuur aan die linkerkant is die oppervlakte van die vierkant 441. Wat is oppervlakte van die vierkant in die figuur aan die regterkant?



7. Wat is die verskil tussen die twee priemgetalle waarvan die som 1 minder is as die grootste derdemag kleiner as 100?

8. Die getal 126 kan op verskillende maniere as die som van twee verskillende priemgetalle geskryf word. Wat is die grootste verskil tussen twee sulke priemgetalle?

9. Pascal, Newton, Galileo en Fermat het al vier dieselfde toets geskryf. Die gemiddeld vir die toets was 16. Pascal en Newton se gemiddeld was 16, Pascal en Fermat se gemiddeld was 13, terwyl Newton en Fermat se gemiddeld 18 was. Wat was Galileo se punt?

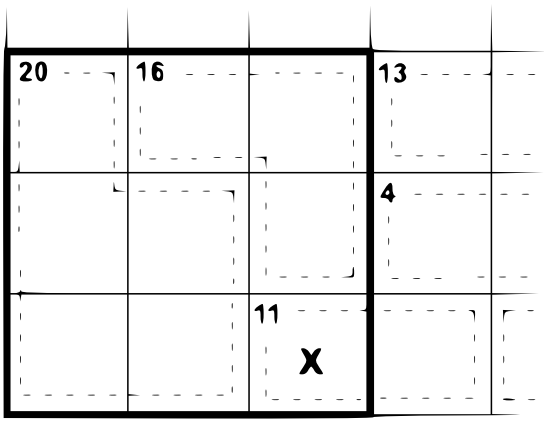
10. Die getal 2021 is die produk van twee opeenvolgende priemgetalle. Wat is die som van hierdie twee priemgetalle?

Afdeling B: Vier punte elk

11. Wat is die kleinste moontlike omtrek van 'n driehoek waarvan die sylengtes opeenvolgende kwadrate is?

12. Twee leë houers, P en Q, het dieselfde volume. Water vloei in P in teen 'n tempo van 4 liter per minuut, en in Q teen 'n tempo van 6 liter per minuut. Na 'n sekere tyd is vasgestel dat houer P nog 60 liter water kan neem, terwyl daar reeds 10 liter water uit houer Q oorgeloop het. Wat is die volume van elke houer?

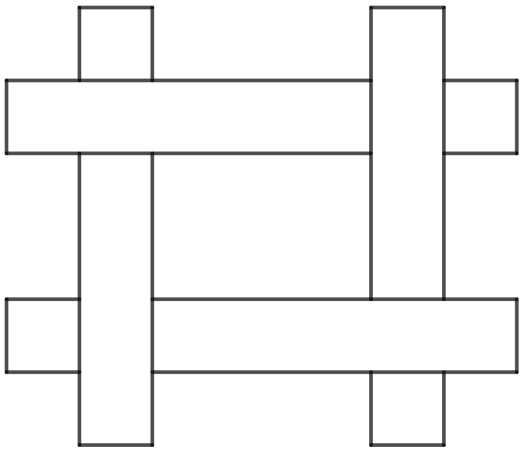
13. "Killer Sudoku" bestaan uit 'n groot 9×9-blok van klein vierkantjies. Dit is verdeel in nege kleiner 3×3-blokke wat elk uit 9 klein vierkantjies bestaan. Die syfers 1 tot 9 word in hierdie klein vierkantjies geskryf. Geen syfer mag herhaal word in enige ry of kolom of 3×3-blok nie. Die getal in die boonste linkerhoek van 'n groep, afgebaken deur die stippellyne soos aangedui in die figuur, is die som van die syfers in die vierkantjies van die spesifieke groep. Met watter syfer moet die X vervang word?



Afdeling A: Drie punte elk

1. 'n Balans met twee panne sal balanseer as daar 8 identiese koekies in die een pan is en 6 identiese koekies en 0.125 kg meelblom in die ander pan. Wat is die massa van 16 koekies in kg?

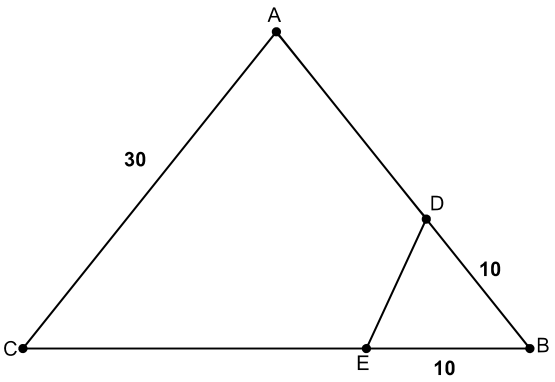
2. Vier reghoekige stroke materiaal met 'n lengte van 10 cm en breedte van 1 cm word plat op 'n tafel neergesit en oorvleuel reghoekig, soos aangetoon. Wat is die oppervlakte van die tafel, in vierkante cm, wat deur die materiaal bedek word?



3. 'n Palindromiese getal lees van links na regs dieselfde as van regs na links, bv. 14541. Die huidige kilometerlesing van jou motor is die palindroom 15951. Hoe ver moet jy ry voordat die volgende palindromiese kilometerlesing vertoon word?

4. Valentynsdag, 14 Februarie, was in 2021 op 'n Sondag. Wat is die laaste twee syfers van die eerste jaar na 2021 wanneer Valentynsdag weer op 'n Sondag sal wees?

5. 'n Driehoekige hoekpunt, met $DB = EB = 10$, word van die gelyksydige driehoek ABC, met sylengte 30, afgesny. Wat is die omtrek van vierhoek $ADEC$?



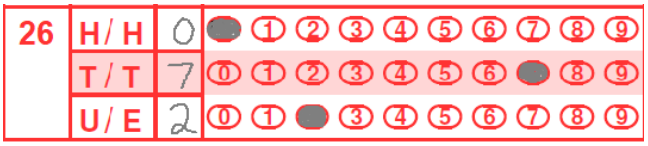
6. Hoeveel 3-syfergetalle met syfers uit die versameling $\{2, 3, 7, 9\}$ is deelbaar deur 3? Syfers mag nie herhaal word nie.

HOE OM DIE ANTWOORDBLAD TE VOLTOOI

Al die antwoorde is heelgetalle van 1 tot 999. Beskou die volgende voorbeeldvraag:

26. As $3x - 216 = 0$, bepaal die waarde van x.

Die antwoord is 72, en dus moet jy die blok vir vraag 26 op die antwoordblad as volg voltooi: kleur 0 in honderde-ry in, 7 in die tiene-ry, en 2 in die ene-ry:



Skryf die syfers van jou antwoord in die oop blokkies links in die betrokke ry, soos in die voorbeeld aangetoon; honderde, tiene en ene van bo na onder. Die drie syfers wat jy neergeskryf het, word nie nagesien nie; dit is vir jou eie gerief — slegs die ingekleurde sirkels word gemerk.

MOENIE OMBLAAI VOORDAT JY VERSOEK WORD OM DIT TE DOEN NIE

DIE OLD MUTUAL SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 TWEEDE RONDTE
SENIOR AFDELING: GRAAD 10-12

18 Mei 2021 **Tyd: 120 minute** **Aantal vrae: 25**

Instruksies

1. *Die antwoorde op al die vrae is heelgetalle van 000 tot 999. Elke vraag het slegs een korrekte antwoord.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 3 punte in Afdeling A, 5 punte in Afdeling B en 6 punte in Afdeling C.*
   2.2. *Geen punte word afgetrek vir foutiewe antwoorde of onbeantwoorde vrae nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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Organisasies betrokke: AMESA, SA Wiskundevereniging,
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