THE HARMONY GOLD SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

FIRST ROUND 2002

SENIOR SECTION: GRADES 10, 11 AND 12

19 MARCH 2002

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   3.1 Each correct answer is worth 5 marks.
   3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

DRAAI DIE BOEKIE OM VIR AFRIKAANS

Private Bag X11, ARCADIA, 0007 TEL: (012)328-5082
FAX: (012)328-5091 E-mail: ellieo@mweb.co.za

PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129

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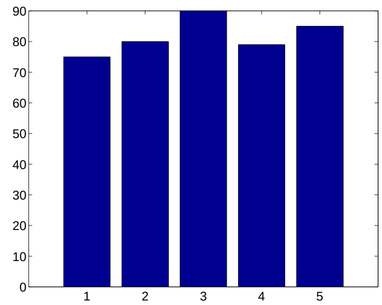
1. The times of five Harmony Gold Mining marathon runners are shown in the graph. The difference between the times of the fastest and slowest runners is approximately



(A) 15 (B) 20 (C) 6 (D) 12 (E) 1

2. $2002 - 2001 + 2000 - 1999 + \cdots + 2 - 1$ equals

(A) 2002 (B) 0 (C) -1 (D) 1001 (E) -1001

3. If each number in a set of ten numbers is increased by 20, then the average of the set

(A) remains the same (B) is increased by 20 (C) is increased by 200
(D) is increased by 10 (E) is increased by 2

4. In a supermarket a 50 mℓ tube of toothpaste costs R2,99 and a 100 mℓ tube costs R5,09. Approximately what percentage do you pay more if you buy two 50 mℓ tubes instead of one 100 mℓ tube?

(A) 20 (B) 18 (C) 15 (D) 12 (E) 10

5. If $f(a) = a - 2$ and $g(p, q) = p^2 + q$ then $g(3, f(4))$ equals

(A) 13 (B) 28 (C) 7 (D) 8 (E) 11

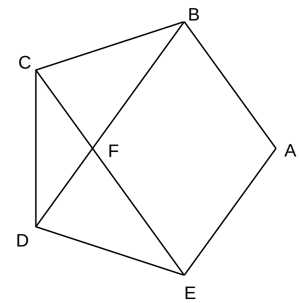
6.

$$\begin{array}{rr} & 2\text{P}4 \\ + & \text{Q}5 \\ + & \text{R}7 \\ \hline & 406 \end{array}$$

In the given addition sum, each letter stands for a different digit from 0 to 9. Which of the following values is not a possible value for R?

(A) 1 (B) 3 (C) 6 (D) 8 (E) 9

7. The figure shows a regular pentagon $ABCDE$. The size of angle $\widehat{DFC}$ is



(A) $36°$ (B) $108°$ (C) $72°$ (D) $100°$ (E) $54°$

8. A cyclist notices that her average speed when she has covered exactly half the total distance of her race is 30 km/h. What should her average speed over the second half of the race be if she wants to finish the race with an average speed of 40 km/h?

(A) 45 km/h (B) 50 km/h (C) 60 km/h (D) 54 km/h
(E) cannot be determined from the available information

9. How many two-digit numbers are exactly seven times the sum of their digits?

(A) 4 (B) 5 (C) 6 (D) 7 (E) more than 7

10. The remainder when 7^{2002} is divided by 10 is

(A) 1 (B) 3 (C) 7 (D) 9 (E) 5

11.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 3 & & 5 & & & \\ & & 7 & & 9 & & 11 & & \\ & 13 & & 15 & & 17 & & 19 & \\ 21 & & 23 & & 25 & & 27 & & 29 \end{array}$$

Consecutive odd numbers are arranged in rows as shown. If the rows are continued in the same pattern, then the middle number of row 51 is

(A) 2601 (B) 2500 (C) 2704 (D) 2809 (E) 2401

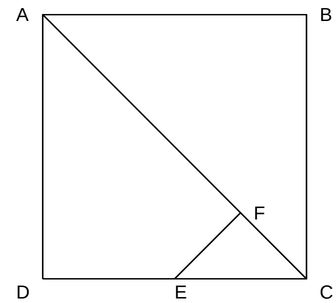
12. Given that $x + y = -1$, the largest value of xy is

(A) 0 (B) 2 (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) -1

13. If p and q are positive integers, how many pairs (p, q) satisfy $2p + 3q = 25$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) more than 5

14. In the given diagram $ABCD$ is a square with sides $7\sqrt{2}$ cm. E is a point on DC such that $EC = 2\sqrt{2}$ cm and angle $\widehat{CFE} = 90°$. The area of $AFED$ in cm^2 is



(A) 2 (B) 49 (C) 7 (D) 45 (E) 47

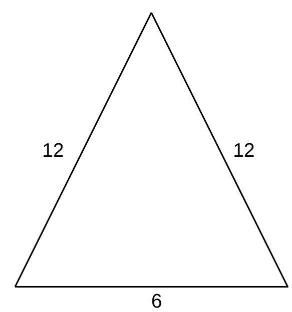
15. If $n! = n \times (n-1) \times (n-2) \times \cdots \times 1$, then the number of perfect squares in the infinite sequence

$$1!\ ,\ 1!+2!\ ,\ 1!+2!+3!\ ,\ \ldots,\ 1!+2!+3!+4!+\cdots+n!\ ,\ \ \ldots$$

is

(A) 0 (B) 2 (C) 3 (D) 5 (E) more than 5

16. The radius of the circle passing through the vertices of the triangle is



(A) $\dfrac{8\sqrt{15}}{5}$ (B) $\dfrac{7\sqrt{15}}{5}$ (C) $3\sqrt{15}$ (D) $3\sqrt{5}$ (E) $3\sqrt{2}$.

17. How many three-digit positive integers are there such that the product of their digits is a prime number? (Remember 1 is not a prime number.)

(A) 15 (B) 12 (C) 17 (D) 20 (E) 21

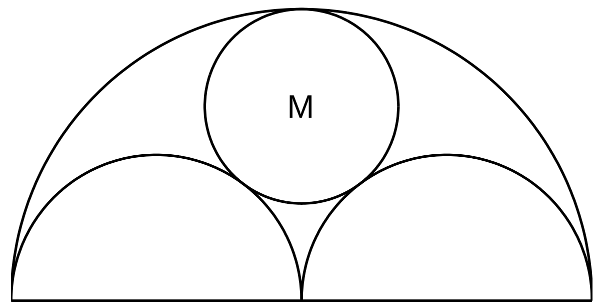
18. If a, b and c are integers with $0 < a < b < c$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, the value of c is

(A) 6 (B) 8 (C) 4 (D) 2 (E) impossible to determine

19. Of the numbers below, the largest one that divides every term of the sequence $2^5 - 2,\ 3^5 - 3,\ \ldots,\ n^5 - n,\ \ldots$, is

(A) 1 (B) 2 (C) 3 (D) 6 (E) 5

20. Two semi-circles (each of radius $\frac{1}{2}$) touch each other, and a larger semi-circle of radius 1 touches both of them. The radius of the circle M which touches all three semi-circles is



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{5}}$ (E) $\frac{2}{\sqrt{6}}$