# SOUTH AFRICAN MATHEMATICS OLYMPIAD

*Organised by the*
**SOUTH AFRICAN MATHEMATICS FOUNDATION**

## 2013 SECOND ROUND
## SENIOR SECTION: GRADES 10, 11 AND 12

**14 May 2013** **Time: 120 minutes** **Number of questions: 20**

**Instructions**

1. *The answers to all the questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for wrong answers or unanswered questions.*
3. *Use an HB pencil. Scrap paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

***Do not turn the page until you are told to do so.***
***Draai die boekie om vir die Afrikaanse vraestel.***

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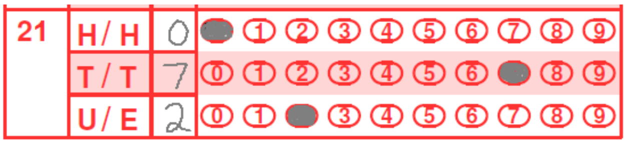
SA Akademie vir Wetenskap en Kuns

# HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

**21.** If $3x - 216 = 0$, determine the value of $x$.

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:




Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

## Part A: Four marks each

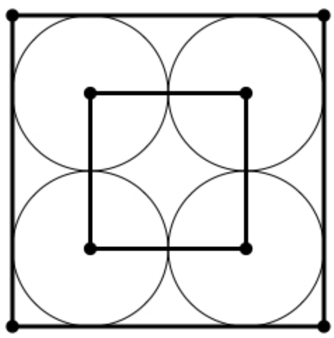
**1.** A child's age, increased by 3, gives a perfect square, and when decreased by 3 the age is the square root of that perfect square. How old is the child?

**2.** Calculate the value of:

$$\frac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}}$$

**3.** If $10^{101} - 1$ is written as an ordinary number, find the sum of the digits of this number.

**4.** In the diagram, the congruent circles are tangent to the large square and each other as shown; and their centres are the vertices of the small square. The area of the small square is 4. Find the area of the large square.



**5.** I wrote down the integers $25, 26, 27, \ldots, 208$. How many **digits** did I write down?

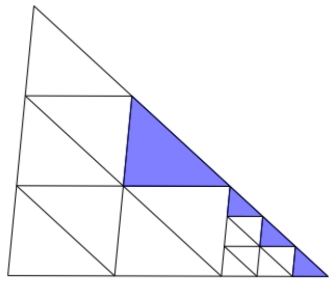
## Part B: Five marks each

**6.** If the number $A1234567B$ is divisible by 45, determine the value of $A + B$.

**7.** Find the value of $(2013)^2 - (2)(2013)(2000) + (2000)^2$.

**8.** Calculate the value of $2013 - 2009 + 2005 - 2001 + 1997 - 1993 + \cdots + 29 - 25$.

**9.** In the diagram, the sides of the larger triangle are divided into three equal parts to produce smaller congruent triangles as shown. This process is repeated for the smaller triangle on the bottom right. If the area of the largest triangle is 81, what is the total area of the shaded triangles?

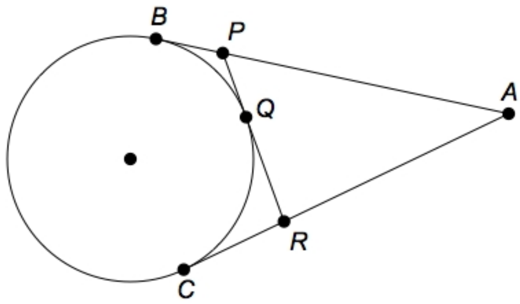


**10.** A bag contains 65 marbles of the same size. There are 20 red ones, 20 green ones, 20 blue ones, and another 5 that are either yellow or white. Lindiwe removes marbles from the bag without looking. What is the smallest number of marbles that she must remove to ensure that she has 10 of the same colour?

**11.** Determine the number of pairs $(x; y)$ of integer solutions for $2^{2x} - 3^{2y} = 55$.

**12.** In how many ways can five students $A$, $B$, $C$, $D$ and $E$ line up in one row if students $B$, $C$ and $D$ must stay together?

**13.** Two tangents are drawn to a circle from a point $A$, which lies outside the circle; they touch the circle at points $B$ and $C$ respectively. A third tangent intersects $AB$ in $P$ and $AC$ in $R$, and touches the circle at $Q$. If $AB = 20$ and $PQ = 3$, find the perimeter of triangle $APR$.



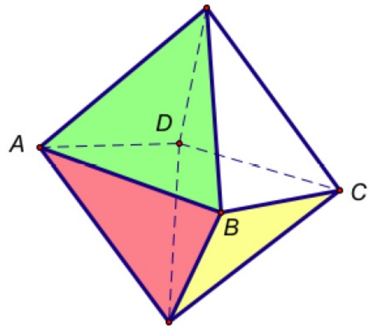

**14.** The function $f(x)$ is defined for all real numbers $x$. If $f(a+b) = f(ab)$ for all $a$ and $b$, and $f(1) = 2$, find the value of $f(2013)$.

**15.** Jack, John and James are identical triplets. It is impossible to distinguish them by appearance. Jack and John always tell the truth, but James always lies — everything he says is false. You know that the triplets are between 20 and 30 years old, 20 and 30 included. One day you meet two of the triplets and ask them how old they are. A says ‘We are between 20 and 29 years old, 20 and 29 included’. B makes the following statements: ‘We are between 21 and 30 years old, 21 and 30 included’ and ‘One of us present is lying’. How old are they?

## Part C: Six marks each

**16.** John takes 300 steps to walk from point $A$ to point $B$ in a flat field. Each step is of length $\frac{1}{\sqrt{2}}$ meters, and he makes a $90^\circ$ turn after every step except after the last one. He makes 99 left turns and 200 right turns in total. He stops at point $B$. What is the maximum possible distance from $A$ to $B$?

**17.** Determine the volume of a regular tetrahedron with each edge of length $3\sqrt{2}$. A regular tetrahedron is a 3-dimensional object bounded by 8 congruent, equilateral triangular faces. As illustrated it can also be viewed as two pyramids with square bases stuck together at $ABCD$.




**18.** What is the biggest possible area of a quadrilateral with sides of length 1, 4, 7 and 8?

**19.** Erica noted that a train to Muizenberg took 8 minutes to pass her. A train in the opposite direction to Cape Town took 12 minutes to pass her. The trains took 9 minutes to pass each other. Assuming each train maintained a constant speed, and given that the train to Cape Town was 150m long, what was the length of the train to Muizenberg?

**20.** In triangle $ABC$ the angle at $A$ is $60^\circ$ and the inscribed circle touches $AB$ at the point $D$. If $AD = 5$ and $DB = 3$, find the length of $BC$.