SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2014 THIRD ROUND
JUNIOR SECTION: GRADES 8 AND 9

12 September 2014 **Time: 4 Hours** **Number of questions: 15**

TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions will be available at: www.samf.ac.za

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Societ
SA Akademie vir Wetenskap en Kuns

Question 1

What is the largest 5-digit number that is a multiple of 9, but which has no two digits the same?

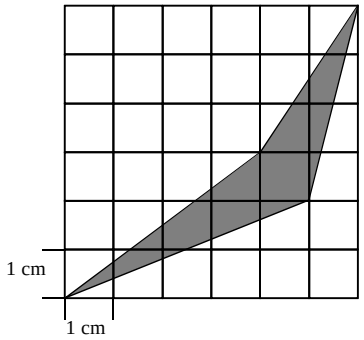
[4]

Question 2

Find the area of the shaded region.

The grid is made up of $1\,cm \times 1\,cm$ squares.

Vertices of the shape lie on the corners of squares.



[4]

Question 3

5 factorial, written as 5! is defined as $5! = 5 \times 4 \times 3 \times 2 \times 1$; also $3! = 3 \times 2 \times 1$.

In general: $n! = n \times (n-1) \times (n-2) \times ... \times 3 \times 2 \times 1$.

If $10! = (a!) \times (b!) \times (c!)$ and $a \neq b \neq c \neq 0$, find a, b and c.

[6]

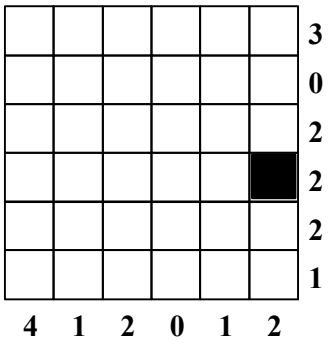
Question 4

On the six-by-six (6×6) grid we need to place

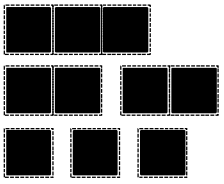
One (3×1) rectangle

Two (2×1) rectangles and

Three (1×1) squares, so that none of the above figures touch each other, not even diagonally.



The rectangles may be placed vertically or horizontally within the grid.



One (3×1) rectangle

Two (2×1) rectangles

Three (1×1) squares

The numbers at the bottom and on the right of the grid show how many squares in the corresponding rows and columns are occupied by the figures.

One square, which is part of a rectangle or one of the square figures is indicated on the grid.

Show on the grid where all six figures are located by colouring in the squares.

[6]

Question 5

The sum of four positive integers is 396. If 5 is added to the first number, 5 is subtracted from the second, the third is multiplied by 5 and the fourth is divided by 5, we get four equal positive integers. Find the four original integers.

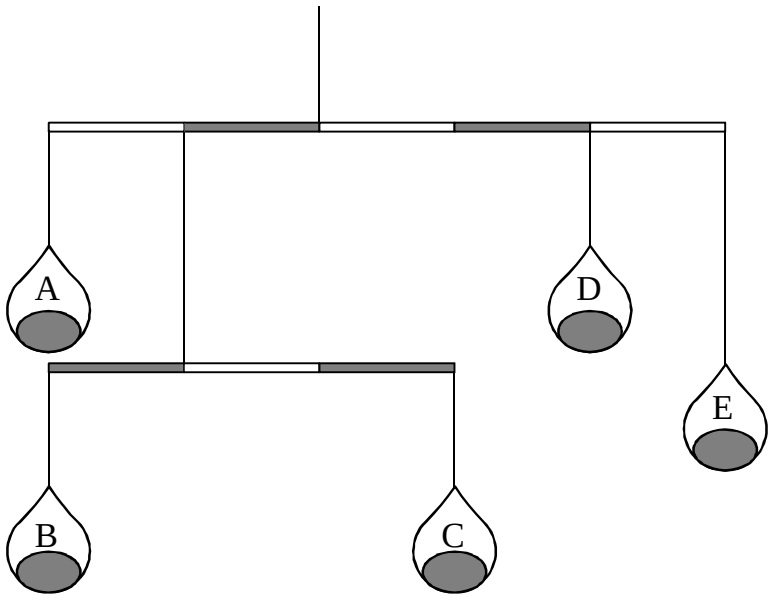
[5]

Question 6

You are given a set of scales.

The black and white rods on which the pans are hanging are all of exactly the same length and the weight of the rods and pans can be ignored.

Assign the weights of 3 kg; 4 kg; 5 kg; 6 kg and 7 kg to the scales A, B, C, D and E such that the scales balance.



[5]

Question 7

Bangkok is East of Johannesburg and New York is West of Johannesburg.

Flight time from Johannesburg to Bangkok is 11 hours and 40 minutes and the time difference is 7 hours.

Flight time Johannesburg to New York is 15 hours and the time difference is 8 hours.

Flight time New York to Bangkok is 17 hours.

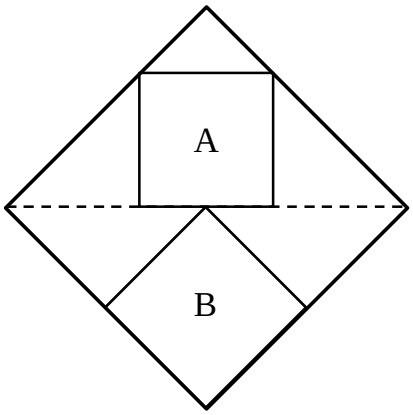
If I depart from Johannesburg at 06h00 (SA time) and fly to New York, stop over there for 24 hours and then fly to Bangkok, what will the local Bangkok time be when I arrive there?

[6]

Question 8

The diagram shows two squares A and B inside a bigger square.

Find the ratio of the area of A to the area of B.



[6]

Question 9

The number N is the product of two primes.

The sum of the positive divisors of N that are less than N is 2014.

Find the value of N.

[6]

Question 10

Jack and Gill play a game on a grid of white squares. They take turns in colouring in any size squares (i.e. 1×1 or 2×2 or 3×3 etc.) which are still white.

The last player who is able to colour in a square, wins.

(a) If Gill starts the game on a 5 by 3 board, can she definitely win?

If so, what is her strategy to force a win?

(b) If Gill starts, can she always win on a 20 by 14 grid?

If so, describe her strategy.

(c) Show that on an odd-by-odd grid of squares, if Gill starts, she can always win.

Describe the strategy she will use to force a win.

[10]

Question 11

For a positive integer n, the n^{th} triangular number is $T(n) = \dfrac{n(n+1)}{2}$.

(a) There is one positive integer, k, such that:

$T(4) + T(k) = T(10)$

Determine k.

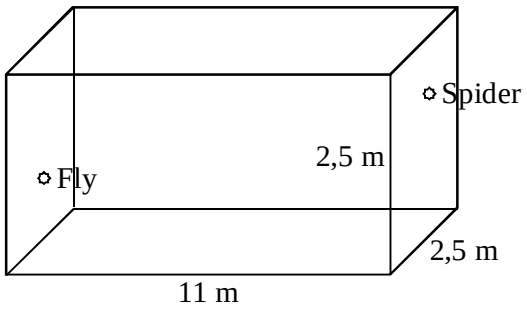
(b) Determine the smallest value of the integer b, where $b > 2014$, such that

$T(b+1) - T(b) = T(x)$ for some positive integer x.

[8]

Question 12

In a rectangular room (a cuboid) with dimensions 11 m by 2,5 m by 2,5 m, a spider is located in the middle of one 2,5 m by 2,5 m wall 0,5 metres below the ceiling.



A fly is in the middle of the opposite wall, 0,5 metres above the floor.
If the fly remains stationary, what is the shortest total distance the spider must crawl along the walls, ceiling, and/or floor in order to capture the fly?

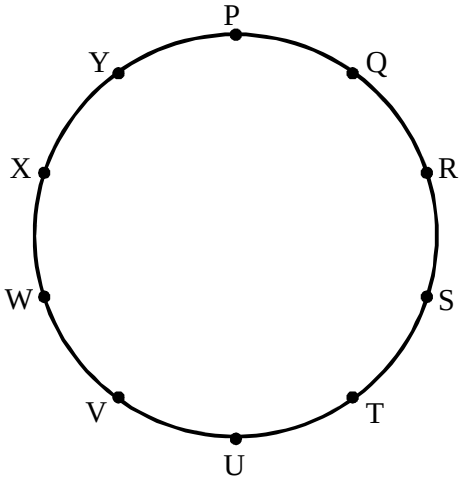
Note: The answer is NOT 13,5 m

[8]

Question 13

Ten points, P, Q, R, ⋯, Y, are equally spaced around a circle of radius one unit.

What is the <u>difference</u> in the lengths of the lines PQ and PS?



[8]

Question 14

In the sum $\frac{a}{b}+\frac{c}{d}$ each letter represents a distinct digit picked from 1 to 9.

Find the biggest sum less than 1.

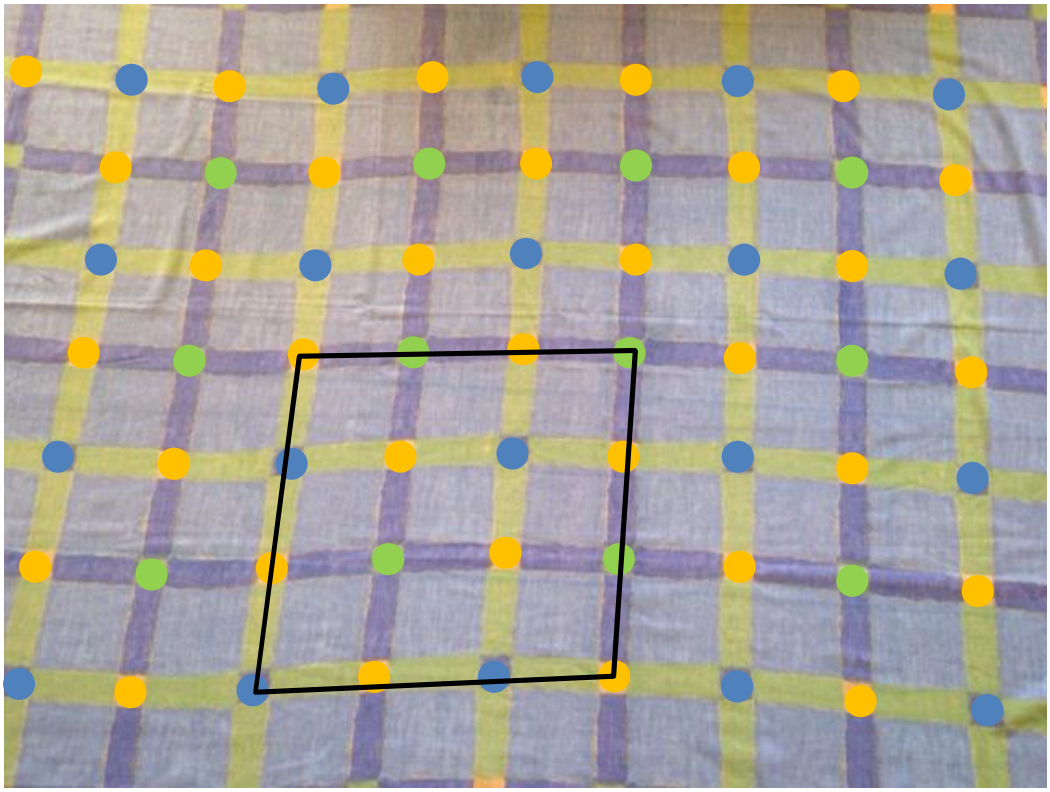
[8]

Question 15

The table cloth in the picture consists of squares with differently coloured circles at their vertices.

In black indicated, you can see a 3-by-3 square containing 8 yellow circles.

How many yellow circles are there on a *n*-by-*n* square?



[10]

Total: 100

THE END