SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade EIGHT First Round 2017

Solutions

1. **B** 10h15 plus 2 hours 10 minutes is 12h25.

2. **C** $\frac{2017-1017}{500}=\frac{1000}{500}=2$

3. **C** Rotating and aligning the shaded segments shows that one quarter of the shape is shaded.

4. **D** Over the first 20 years the tree grows a total of 10 m. The further 3 m of growth, at a rate of $\frac{1}{3}$ m per year, would have taken 9 more years. 20 + 9 = 29 years.

5. **C** The angle $5x$ is bigger than 90° but smaller than 180°. Of the five options given, 20 is the only value of x for which $5x$ falls in this interval.

6. **E** It is not possible to fold this net without two faces overlapping. It is thus not possible to form a closed cube with six faces with this net.

7. **D** $\frac{21}{20}+7=\frac{105}{100}+7=1{,}05+7=8{,}05$.

8. **E** Since the top two squares sum to 3 and the bottom two squares sum to 12, the sum of all four small squares must be 15. Since $8+x$ also represents the sum of all four small squares, $x=15-8=7$.

9. **A** The Least Common Multiple of 3 and 5 is 15. Every 15th visitor will thus receive a pen and a bag. There are 13 multiples of 15 between 1 and 200.

10. **B** The sequence repeats after every five shapes. 2017 = 5 × 403 + 2. The 2017th shape is thus the second shape in the repeating sequence.

11. **D** Since $1=1^2$ and $196=14^2$, there are 14 perfect squares from 1 to 200. $\frac{14}{200}=\frac{7}{100}=7\%$.

12. **C** If five sweets cost R12 more than one sweet, then R12 is the cost of the four additional sweets. Each sweet thus costs R12 ÷ 4 = R3.

13. **A** Triangles AED and CFD are identical, and each has area $\frac{1}{2}\times 4\times 2=4$. The area of quadrilateral EBFD is simply the area of square ABCD minus the areas of the two triangles. Thus, Area of EBFD $=4^2-2\times 4=8$.

14. **E** From the balanced scale we can remove two white squares and one shaded square from each side. From this it becomes clear that one white square is equivalent to two shaded squares. One shaded square and two white squares is thus equivalent to five shaded squares.

15. **B** Since both guesses are no more than 10 out, from Yandi's guess of 53 we know Xavier's number lies between 43 and 63 inclusive. From Zanele's guess of 71 we know that Xavier's number lies between 61 and 81 inclusive. The only number of those given that lies in both of these intervals is 62.

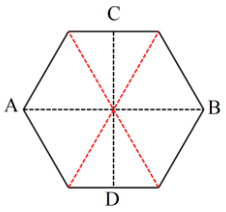
16. **B** From $\sqrt{xy} = 4$ we have $xy = 16$. From $\sqrt[3]{xyz} = 2$ we have $xyz = 8$.
Thus $16z = 8$ and hence $z = \frac{1}{2}$.

17. **A** The shaded triangle has base 2 cm and perpendicular height $6 - 2 = 4$ cm.
It's area is thus $\frac{1}{2} \times 2 \times 4 = 4$ cm^2.

18. **A** Let the 5-digit code be PQRST. Since P + Q + R + S = 17 and Q + R + S = 12, it follows that P = 5. By using similar reasoning one can show that Q, S and T also equal 5. Since P + Q + R = 12 it follows that R = 2. The code is thus 55255, and the sum of the five digits is 22.

19. **A** There are 100 possible outcomes. Of these there are 10 ways in which they can both choose the same number. Of the remaining 90 possibilities, half will have Andy's number bigger than Betty's, and half will have Betty's number bigger than Andy's. Alternatively, If Andy chooses 10 then there are 9 numbers that are smaller. If Andy chooses 9 then there are 8 numbers that are smaller. Continuing this logic there will be 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45 cases where Andy's number is bigger than Betty's.

20. **D** As shown in the diagram, the hexagon consists of six equilateral triangles with a base of $\frac{AB}{2}$ cm and a height of $\frac{CD}{2}$ cm.



The area of the hexagon is then $6 \times \frac{1}{2} \times \frac{AB}{2} \times \frac{CD}{2} = \frac{3}{4} \times AB \times CD$

The area of the rectangle is then $123 \div \frac{3}{4} = 164$ cm^2.