SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2012 SECOND ROUND
SENIOR SECTION: GRADES 10, 11 AND 12

23 May 2012 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *For each incorrect answer one mark is deducted. There is no penalty for unanswered questions.*
3. *You must use an HB pencil. Scrap paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9323 Email: ellie@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1 + \dfrac{1}{3 + \dfrac{1}{2}}$ is

(A) $\dfrac{6}{5}$ (B) $\dfrac{7}{6}$ (C) $\dfrac{9}{2}$ (D) $\dfrac{6}{7}$ (E) $\dfrac{9}{7}$

3. The tens digit of the product $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

Part A: Four marks each

1. The cost of producing a set of CDs consists of a R1000 fixed cost, plus R6 per CD. The cost in rand of producing a set of n CDs is

(A) $1000 + 6n$ (B) $106 + n$ (C) $6 + 100n$ (D) $106n$ (E) $600n$

2. If $(x+3)(x+a) = x^2 + bx - 12$ for some numbers a and b, then b is equal to

(A) -2 (B) -1 (C) 1 (D) 2 (E) 3

3. $\sqrt{5} \times \sqrt[3]{5}$ equals

(A) $5^{1/2}$ (B) $5^{1/3}$ (C) $5^{2/3}$ (D) $5^{4/3}$ (E) $5^{5/6}$

4. The diameter of a circle equals the radius of another circle. The ratio of their areas is

(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 1 : 8 (E) 1 : 16

5. An ordinary die is rolled. From the following choices, the number thrown is most likely to be

(A) odd (B) even (C) prime (D) a factor of 12 (E) a factor of 18

Part B: Five marks each

6. If the equations $ax + 3y = 5$ and $2x + by = 3$ represent parallel lines, then the product ab is equal to

(A) -5 (B) -1 (C) 2 (D) 3 (E) 6

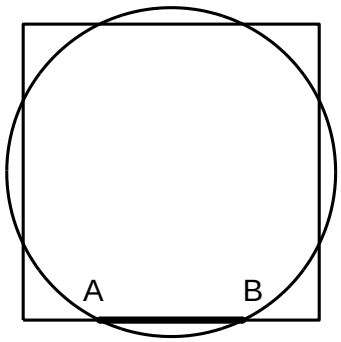
7. The largest prime factor of $7^{99} + 7^{100} + 7^{101}$ is

(A) 7 (B) 13 (C) 17 (D) 19 (E) 29

8. The largest number in the following list is

(A) 31^{11} (B) 17^{14} (C) 2^{56} (D) 32^{10} (E) 127^8

9. A circle of radius 1 and a square have the same centre and the same area. The length of chord AB is



(A) $\sqrt{4-\pi}$ (B) $\sqrt{\pi-2}$ (C) $4-2\sqrt{\pi}$ (D) $\sqrt{\pi}-1$ (E) $4-\pi$

10. Which number in the list is the best approximation to

$$2012^2 - 2011^2 + 2010^2 - 2009^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2 \ ?$$

(A) 5×10^5 (B) 1×10^6 (C) 2×10^6 (D) 5×10^6 (E) 2×10^7

11. In our cafeteria, each muffin costs an integer number of rands as does each sandwich. If every girl in the class buys one muffin and every boy buys one sandwich, they will spend one rand less than if every boy buys one muffin and every girl buys one sandwich. There are more girls in the class than boys. How many more girls are there?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

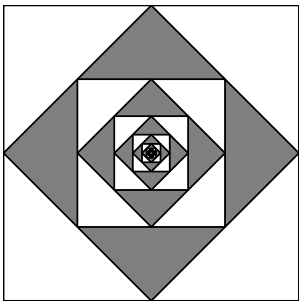
12. A password consists of four distinct digits such that their sum is 19 and such that exactly two of these digits are prime, for example 0397. The number of possibilities for the password is

(A) 168 (B) 176 (C) 180 (D) 192 (E) 216

13. When $10^{99} - 99$ is evaluated, the sum of the digits is

(A) 2 (B) 512 (C) 728 (D) 865 (E) 874

14. The diagram shows a sequence of squares, decreasing in size and alternating between white and grey. Each new square is drawn by connecting the midpoints of the sides of the larger square that precedes it. If this process is continued indefinitely, then the fraction of the area that will ultimately be grey is



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{5}{16}$ (D) $\frac{2}{5}$ (E) $\frac{3}{8}$

15. A solid cube, painted all around, is cut into 64 identical cubes. One of these cubes is chosen at random and rolled. The probability that none of the five faces showing is painted, is

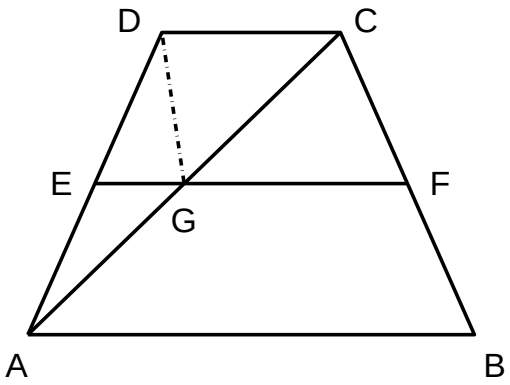
(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{3}{16}$ (E) $\frac{5}{16}$

Part C: Six marks each

16. Boat A is 100 km east of boat B. Simultaneously, boat A sails directly west at constant speed 9 km/h and boat B sails directly south at constant speed 12 km/h. The closest boat A will get to boat B is

(A) 75 km (B) 78 km (C) 80 km (D) 82 km (E) 85 km

17. In trapezium ABCD we have that AB ∥ DC and AD = BC. E and F are the midpoints of AD and BC, respectively. If AC = 2 EF, AB = 2 and DC = 1, then DG is equal to



(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{3}$ (C) $\sqrt{\frac{7}{2}}$ (D) $\frac{\sqrt{7}}{2}$ (E) $\frac{3}{\sqrt{7}}$

18. We define $n!$ as the product of the integers from 1 to n, and 0! is defined as 1. Note that $40585 = 4! + 0! + 5! + 8! + 5!$ There is one three-digit number that can be written in this way. The sum of its three digits is

(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

19. The largest positive root of $x^4 + 2x^3 - 22x^2 + 2x + 1 = 0$ is

(A) $3 + \sqrt{2}$ (B) $2 + \sqrt{3}$ (C) $1 + \sqrt{7}$ (D) $1 + \sqrt{8}$ (E) $2 + \sqrt{2}$

20. For each positive integer n, define $f(n)$ such that $f(n)$ is a positive integer, $f(n+1) > f(n)$ and $f(f(n)) = 3n$. The value of $f(10)$ is

(A) 12 (B) 15 (C) 19 (D) 21 (E) 30
