

SOUTH AFRICAN MATHEMATICS OLYMPIAD

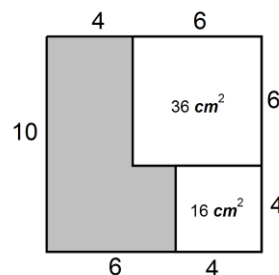
Grade NINE First Round 2016

Solutions

1. **D** The last digit is found by subtracting 8 from 15
2. **D** Of the nine positions that the blue Smartie could occupy, only one is in the centre; so the probability is $\frac{1}{9}$
3. **C** $2 - 0 + 1 \times 6 = 2 - 0 + 6 = 8$
4. **D** If he spends x minutes on planning, then his total time spent in the garden is $x + 25 + x + x - 55$ and is 180 minutes. This means $3x - 30 = 180$, so $x = 70$
5. **A** The number of brothers is $1 \times 0 + 3 \times 1 + 2 \times 2 = 7$ while the number of girls asked is $1 + 3 + 2 = 6$. So the average is $7/6$
6. **D** The others are, in order, 2; 1; 2,5 and 0,7
7. **D** If you buy one item and get one at half price you are getting two items for the price of one and a half items. Since $2:1\frac{1}{2}$ is equivalent to $4:3$, the average cost per item is the same as four items for the price of three.

8. **B**

Perimeter = $10 + 6 + 4 + 2 + 6 + 4 = 32$ cm.



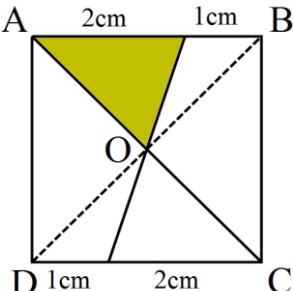
9. **A** At each level we have two choices, so we have $2 \times 2 \times 2 = 8$ options.
10. **B** The second horse eats $P\%$ of 60% and this is the same as 40% : so $P\%$ is $\frac{2}{3}$, i.e. $P = 66\frac{2}{3}$
11. **B** $\frac{\left(\frac{1}{8}\right)^2}{\left(\frac{1}{2}\right)^8} = \frac{\frac{1}{64}}{\frac{1}{256}} = \frac{1}{64} \times \frac{256}{1} = 4$
12. **E** In 8 hours the tank filled up a further 64% : that is 8% per hour. So in the first two hours it filled up 16% , meaning it started with $36 - 16 = 20\%$

13. **A** Let the middle cell contain y . Then the middle column has total $x + y + 28$, while the bottom left to top right diagonal has total $31 + y + 33$, in other words $x + 28 = 31 + 33$. That easily shows $x = 36$.

14. **E** 120 % of a is $\frac{6}{5}a$ while 80 % of b is $\frac{4}{5}b$.

We thus have $\frac{6}{5}a = \frac{4}{5}b$ from which it follows that $\frac{a}{b} = \frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$

15. **C** $CM = MB = 8$. Then by Pythagoras in $\triangle OMC$, $OC = 15$. But then $AP = 15 - 3 = 12$, and Pythagoras in $\triangle OAP$ shows $OP = 20$.

16. **C**
- 

We note that the two lines cutting the square must meet at its centre; call that O , and draw the diagonal DB . It is now clear that the height of the shaded triangle is 1,5 cm, while its base is 2 cm, so its area is $\frac{1}{2} \times 1,5 \times 2 = 1,5 \text{ cm}^2$.

17. **D** Considering the interior angles of the quadrilateral, we see their total is $x + (180^\circ - c) + (180^\circ - b) + a = 360^\circ$, so $360^\circ + x - c - b + a = 360^\circ$, which means $x = c + b - a$.

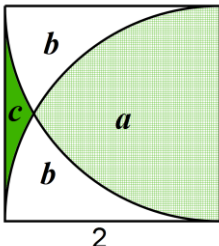
18. **A** Let the five marks be a, b, c, d and e .

$$\frac{a+b+c+d+e}{5} = 80 \quad \therefore \quad a+b+c+d+e = 400$$

$$\text{But } \frac{a+b+c+d}{4} = 75 \quad \therefore \quad a+b+c+d = 300, \text{ and so } e = 100$$

19. **C** If the radius is r cm then $\frac{\text{area smaller circle}}{\text{area larger circle}} = \frac{\pi r^2}{\pi \cdot 12^2} = \frac{r^2}{12^2} = \frac{403}{900} \approx \frac{4}{9}$

$$\text{So } r^2 \text{ is close to } \frac{2^2 \cdot 12^2}{3^2} \text{ which means } r \text{ is } \frac{2 \cdot 12}{3} = 8$$

20. **A**
- 

Let the regions have areas a, b and c as shown. Then $a + b$ is a quarter circle with area $\frac{1}{4} \cdot \pi \cdot 2^2$, i.e. π . But $a + 2b + c$ is the whole square, i.e. 4. Now $a - c = 2(a + b) - (a + 2b + c) = 2\pi - 4$.