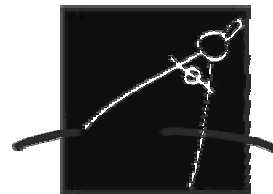


HARMONY



THE HARMONY GOLD SOUTH AFRICAN  
MATHEMATICS OLYMPIAD

**FIRST ROUND 2002: JUNIOR SECTION: GRADES 8 AND 9**

**SOLUTIONS AND MODEL ANSWERS**

Thank you for entering the First Round of the Mathematics Olympiad.

Many of the solutions to the problems which are given below use trial-and-improvement methods or an investigative approach.

Most of them can also be solved using more formal mathematical methods which you will learn in due course. Sometimes we have also given a more formal solution which you might find interesting.

**PRACTICE EXAMPLES:**

1.  $23 + 6 - 4 =$

- A) 6      B) 23      C) 25      D) 29      E) 33

ANSWER: C

$$23 + 6 - 4 = (23 + 6) - 4 = 29 - 4 = 25$$

2.  $\frac{1}{5} + \frac{2}{3} \times \frac{1}{2}$  equals

- A)  $\frac{1}{15}$       B)  $\frac{3}{11}$       C)  $\frac{21}{50}$       D)  $\frac{8}{15}$       E)  $9\frac{4}{5}$

ANSWER: D

$$\frac{1}{5} + \frac{2}{3} \times \frac{1}{2} = \frac{1}{5} + \left(\frac{2}{3} \times \frac{1}{2}\right) = \frac{1}{5} + \frac{1}{3} = \frac{3+5}{15} = \frac{8}{15}$$

**QUESTIONS:**

1. Find the missing number if  $182 \times \Delta = 2002$ .

- A) 8      B) 9      C) 10      D) 11      E) 12

ANSWER: D

$$182 = 2 \times 7 \times 13$$

$$2002 = 2 \times 7 \times 11 \times 13$$

$$\therefore 2002 = 182 \times 11$$

2. The answer to  $5 - 2 + 4 \times 3$  is  
A) 12      B) 15      C) 17      D) 19      E) 20

ANSWER: B

$$(5 - 2) + 4 \times 3$$

$$= 3 + 12$$

$$= 15$$

3. If  $n = 5$  then the value of  $(7n - 5)(n - 5)(3n + 5)$  is  
A) 0      B) 50      C) 500      D) 5000      E) 50000

ANSWER: A

$$(7n - 5)(n - 5)(3n + 5) = (7(5) - 5)(5 - 5)(3(5) + 5) = 0$$

$$\text{since } n - 5 = 5 - 5 = 0$$

4. An ant covers a distance of 90 metres in 3 hours.  
The average speed of the ant in centimetres per minute is  
A) 30      B) 40      C) 50      D) 60      E) 70

ANSWER: C

$$\text{distance (cm)} = 90 \times 10 = 9\,000$$

$$\text{time (minutes)} = 3 \times 60 = 180$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{distance}}{\text{time}} = \frac{9\,000}{180} \\ &= 50 \text{ cm / min} \end{aligned}$$

5. Given that  $\frac{9}{25} = 0,36$   
then  $\frac{0,9}{0,25}$  is equal to  
A) 0,036      B) 0,36      C) 3,6      D) 36      E) 360

ANSWER: C

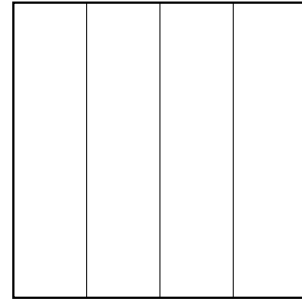
$$\frac{0,9}{0,25} = \frac{0,9 \times 100}{0,25 \times 100}$$

$$= \frac{90}{25}$$

$$= \frac{9}{25} \times 10$$

$$= 0,36 \times 10 = 3,6$$

6. A square is divided into 4 identical rectangles as shown in the diagram. The perimeter of each of the four rectangles is 30 units. What is the perimeter of the square?



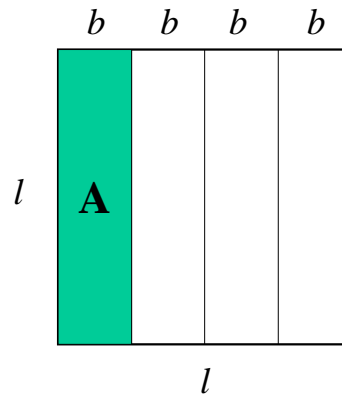
A) 36      B) 40      C) 44      D) 48

ANSWER: D

$$4 \times b = l$$

$$\begin{aligned} \text{Perimeter of A} &= 2l + 2b \\ &= 2(4 \times b) + 2b \\ &= 10b \\ 10b &= 30 \\ \therefore b &= 3 \quad \therefore l = 12 \end{aligned}$$

$$\begin{aligned} \text{Perimeter of square} &= 4l \\ &= 4(12) = 48 \end{aligned}$$



7. Allison, Nomsa and Jan shared a sum of money in the ratio of 4 : 3 : 1 respectively. Allison received R70 more than Nomsa. The total amount that was shared initially was

A) R640    B) R560    C) R480    D) R400    E) R320

ANSWER: B

Allison : Nomsa : Jan

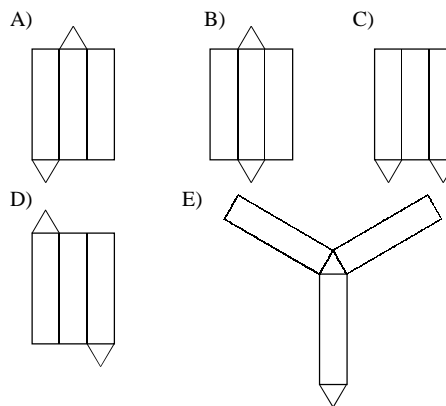
4 : 3 : 1

Total number of parts =  $4 + 3 + 1 = 8$

Difference between Allison and Nomsa: 1 part = R70

$$\begin{aligned} \therefore \text{Total amount} &= 8 \times \text{R70} \\ &= \text{R560} \end{aligned}$$

8. Which of the following nets will **NOT** form a closed triangular prism?



ANSWER: C

The net of C has 2 triangular 'ends' at the same side. The prism will therefore not be closed.

9. The product of two numbers is 504 and each of the numbers is divisible by 6. Neither of the two numbers is 6. What is the larger of the two numbers?

A) 48      B) 84      C) 72      D) 60      E) 42

ANSWER: E

2	504
2	252
2	126
3	63
3	21
7	7
	1

$$\begin{aligned}\therefore 504 &= (2 \times 3 \times 2) \times (2 \times 3 \times 7) \\ &= 12 \times 42\end{aligned}$$

10. Two windmills pour water into a 1 000 ℓ tank. One windmill pours water into the tank at a rate of 20 ℓ per minute. The other windmill pours water into the tank at 20 ℓ in 3 minutes. How many minutes will it take to fill the tank?

A)  $37\frac{1}{2}$       B) 75      C)  $112\frac{1}{2}$       D) 150      E) 175

ANSWER: A

Windmill 1: rate = 20 ℓ/min.

Windmill 2: rate =  $20 \ell / 3 \text{ min} = 6,667 \ell / \text{min}$

Windmill 1 + Windmill 2: rate =  $26,667 \ell / \text{min}$

Volume of tank: 1 000 ℓ

$$\begin{aligned}
 \text{time} &= \frac{\text{volume}}{\text{rate}} \\
 &= \frac{1\,000\ \ell}{26,667\ \ell / \text{min}} \\
 &= 37\frac{1}{2}\ \text{min}
 \end{aligned}$$

Or

Time	Windmill 1		Windmill 2	Total
	rate: 20 ℓ/min		rate: 20 ℓ/3 min	
After 50 min	1 000 ℓ	+	333,3 ℓ	= 1 333,3 ℓ
After 40 min	800 ℓ	+	266,7 ℓ	= 1 066,7 ℓ
After 30 min	600 ℓ	+	200 ℓ	= 800 ℓ
After 35 min	700 ℓ	+	233,3 ℓ	= 933,3 ℓ
After 37,5 min	750 ℓ	+	250 ℓ	= 1 000 ℓ

Or

Volume of the tank = 1000 ℓ

Windmill 1: rate = 20 ℓ/min

Windmill 2: rate = 20 ℓ/3 min

For both windmills we have  $\frac{20\ \ell}{1\ \text{min}} + \frac{20\ \ell}{3\ \text{min}} = \frac{80\ \ell}{3\ \text{min}}$

Let the time taken to fill a 1000 ℓ tank be  $x$ . i.e. tempo:  $\frac{1000\ \ell}{x}$

$$\text{Now } \frac{1000\ \ell}{x} = \frac{80\ \ell}{3\ \text{min}}$$

$$\therefore x = \frac{1000\ \ell \times 3\ \text{min}}{80\ \ell}$$

$$= \frac{300\ \ell \cdot \text{min}}{8\ \ell} = 37,5\ \text{min}$$

Note:	
Tank 1	Tank 2
Min 1: 20 ℓ	
Min 2: 20 ℓ	
Min 3: 20 ℓ	20 ℓ
60 ℓ	20 ℓ

11. The Ancient Romans used the following different numerals in their number system:

$$I = 1$$

$$C = 100$$

$$V = 5$$

$$D = 500$$

$$X = 10$$

$$M = 1\,000, \text{ etc.}$$

$$L = 50$$

They used these numerals to make up numbers as follows:

$$I = 1$$

$$VI = 6$$

$$II = 2$$

$$VII = 7$$

$$III = 3$$

$$VIII = 8$$

$$IV = 4$$

$$IX = 9$$

$$V = 5$$

$$X = 10, \text{ etc.}$$

So, for example, XCIX is 99. What is the value of XLVI?

A) 42      B) 44      C) 46      D) 64      E) 66

ANSWER: C

$$L = 50$$

$$XL = 40 \text{ (i.e. 10 from 50 = 40)}$$

$$VI = 6 \text{ (i.e. 1 added to 5 = 6)}$$

$$XLVI = 46$$

## 12. Did you know?

- An equilateral triangle is a regular polygon with 3 equal sides, and each interior angle is  $60^\circ$ .
- A square is a regular polygon with 4 equal sides, and each interior angle is  $90^\circ$ .

If a regular polygon has  $n$  sides, then the formula to find the size of each

interior angle is  $\frac{(n-2) \times 180^\circ}{n}$ . If each interior angle of a regular

polygon measures  $150^\circ$ , then the number of sides ( $n$ ) is

A) 6      B) 9      C) 10      D) 11      E) 12

ANSWER: E

$$\text{interior } \angle = \frac{(n-2) \times 180^\circ}{n}$$

so if  $\angle = 150^\circ$ , then

$$150^\circ = \frac{(n-2) \times 180^\circ}{n}$$

$$\begin{aligned} 150^\circ n &= (n-2) \times 180^\circ \\ &= 180^\circ n - 360^\circ \end{aligned}$$

$$\text{Hence: } 180^\circ n - 150^\circ n = 360^\circ$$

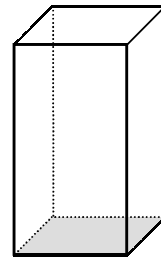
$$n(180^\circ - 150^\circ) = 360^\circ$$

$$30^\circ n = 360^\circ$$

$$\therefore n = 12$$

- 13.** A solid right prism has a square base. The height is twice the length of the side of the base. The surface area of this prism is  $160 \text{ cm}^2$ .

If  $1 \text{ cm}^3$  of the prism has a mass of 250 grams, then the mass of the prism in kilograms is



- A) 28      B) 32      C) 36      D) 40      E) 44

ANSWER: B

Surface Area = 2 bases + 4 sides.

Let the side of the base be  $s$  cm.

The height of the prism will then be  $2s$  cm.

So the Surface Area =  $2s^2 + 4(s \times 2s)$

$$= 2s^2 + 8s^2$$

$$= 10s^2$$

But the Surface Area = 160

$$\therefore 10s^2 = 160$$

$$\therefore s^2 = 16$$

$$\therefore s = 4 \text{ cm}$$

The volume =  $s \times s \times 2s$  (length  $\times$  breadth  $\times$  height)

$$= 2s^3$$

$$= 2 \times 4^3$$

$$= 128 \text{ cm}^3$$

1  $\text{cm}^3$  has a mass of 250 g =  $\frac{1}{4}$  kg

$\therefore$  The prism has a mass of  $128 \times \frac{1}{4} = 32$  kg



14. The diagrams show three scales. On each scale there are different objects on each side which balance each other, as shown.

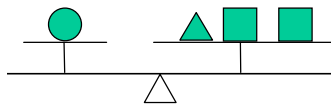


Diagram 1

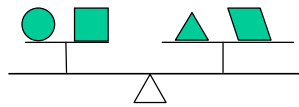


Diagram 2

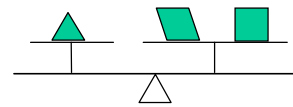






Diagram 3

How many -shaped objects will balance a -object?

- A) 3      B) 4      C) 5      D) 6      E) 7

ANSWER: D

Let  = C  
 = T  
 = S  
 = P

$$C = T + S + S \text{ (Diagram 1)} \quad \text{eq. 1}$$

$$\text{but } T = P + S \text{ (Diagram 3)} \quad \text{eq. 2}$$

$$\therefore C = P + S + S + S \text{ (from eq. 1 \& 2)} \quad \text{eq. 3}$$

$$C + S = T + P \text{ (Diagram 2)} \quad \text{eq. 4}$$

$$\therefore C + S = P + P + S \text{ (from eq. 4 \& 2)} \quad \text{eq. 5}$$

$$\therefore C = P + P \quad \text{eq. 6}$$

$$P = S + S + S \text{ (from eq. 3 \& 6)} \quad \text{eq. 7}$$

$$\therefore C = S + S + S + S + S + S \text{ (from eq. 6 \& 7)}$$

15.  $\frac{1}{3}; a; b; \frac{1}{2}$

These numbers are arranged from smallest to largest. The difference between any two adjacent (next to each other) numbers is the same.

The value of  $b$  is

- A)  $\frac{5}{12}$       B)  $\frac{7}{18}$       C)  $\frac{4}{9}$       D)  $\frac{5}{6}$       E)  $\frac{1}{4}$

ANSWER: C

Let the common difference be  $r$ .

$$\text{i.e. } a - \frac{1}{3} = r$$

$$\therefore \frac{1}{2} - \frac{1}{3} = 3r$$

$$\therefore 3r = \frac{1}{6}$$

$$\therefore r = \frac{1}{18} \quad (1)$$

$$\text{now from (1): } b = \frac{1}{2} - r$$

$$= \frac{1}{2} - \frac{1}{18}$$

$$= \frac{9-1}{18} = \frac{4}{9}$$

Or

$$\begin{aligned}a - \frac{1}{3} &= b - a & \text{and} & & b - a &= \frac{1}{2} - b \\ \therefore 2a &= b + \frac{1}{3} & & & \therefore -a &= \frac{1}{2} - 2b \\ \therefore a &= \frac{1}{2}b + \frac{1}{6} & & & \therefore a &= 2b - \frac{1}{2} \\ \therefore \frac{1}{2}b + \frac{1}{6} &= 2b - \frac{1}{2} & \text{eq.1} & & & \\ \therefore 3b + 1 &= 12b - 3 & \text{eq.1} \times 6 & & & \\ \therefore 9b &= 4 & & & & \\ \therefore b &= \frac{4}{9} & & & & \end{aligned}$$

Or

Multiply through by 18. Then 6; 18a; 18b; 9 corresponds to 6; 7; 8; 9.

$$\therefore 18b = 8, \text{ hence } b = \frac{4}{9}.$$

**16.** Given the set of six numbers below:

11; 31; 19; 3; 10; 6

Three numbers are selected from the above set and added together. The remaining three numbers are also added together. These possible two sums are then multiplied to get a product. The maximum product is

A) 1 200   B) 1 400   C) 1 500   D) 1 600   E) 1 800

ANSWER: D

Method 1:

A product of two numbers  $x$  and  $y$  can be maximized when  $xy$  represents the area of a square.

Thus add 11; 19 and 10 together as well as 31; 6 and 3 to get a sum of 40 in both cases.

Then:  $40 \times 40 = 1\,600$

Method 2:

There are 20 ways to choose 3 numbers from 6. The pairs of 3 are:

1)	$11 + 31 + 19$ $= 61$	AND	$3 + 10 + 6$ $= 19$
2)	$11 + 31 + 3$ $= 45$		$19 + 10 + 6$ $= 35$
3)	$11 + 31 + 10$ $= 51$		$19 + 3 + 6$ $= 28$
4)	$11 + 31 + 6$ $= 48$		$19 + 3 + 10$ $= 32$
5)	$11 + 19 + 3$ $= 33$		$31 + 10 + 6$ $= 47$
6)	$11 + 19 + 6$ $= 36$		$31 + 10 + 3$ $= 44$
7)	$11 + 19 + 10$ $= 40$		$31 + 6 + 3$ $= 40$
8)	$11 + 3 + 10$ $= 24$		$31 + 19 + 6$ $= 56$
9)	$11 + 3 + 6$ $= 20$		$31 + 19 + 10$ $= 60$
10)	$11 + 10 + 6$ $= 27$		$31 + 19 + 3$ $= 53$

Answer is

$$(11 + 19 + 10) \times (31 + 6 + 3)$$

$$40 \times 40 = 1\,600$$

- 17.** The digit 3 is written at the right of a certain 2 digit number forming a 3 digit number. The new number is 372 more than the original 2 digit number. The sum of the digits of the original 2 digit number is
- A) 4      B) 5      C) 6      D) 7      E) 8

ANSWER: B

Let the two digit number be  $xy$ .

With the 3 written on the right of the two digit number, the number becomes  $xy3$

Now  $xy3$

$-xy$

$\overline{372}$

$\therefore y = 1$ , then  $x13$

$-x1$

$\overline{372}$

$\therefore x = 4$

So:  $413$

$-41$

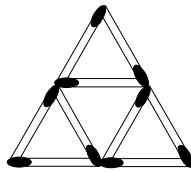
$\overline{372}$

18. Matchsticks of equal length are used to make the following figures.

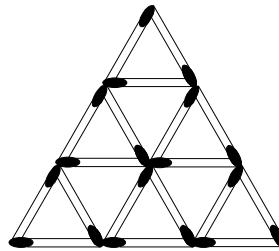
i)



ii)



iii)



3 matchsticks are used for figure i).

9 matchsticks are used for figure ii).

18 matchsticks are used for figure iii).

How many matchsticks are needed for a similar figure which has 10 matchsticks along each side?

- A) 84      B) 108      C) 135      D) 165      E) 180

ANSWER: D

Given: Diagrams 1, 2 and 3 above.

Try own diagram 4.

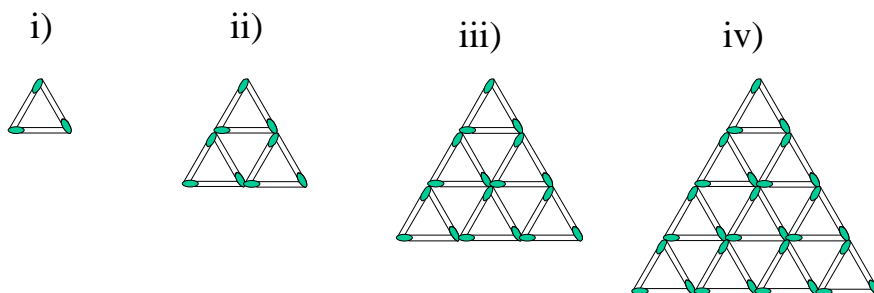


Figure	No. of matchsticks	Pattern
i)	3	$1 \times 3$
ii)	9	$3 \times 3$
ii)	18	$6 \times 3$
iv)	30	$10 \times 3$
v)	45	$15 \times 3$
vi)	63	$21 \times 3$
vii)	84	$28 \times 3$
viii)	108	$36 \times 3$
ix)	135	$45 \times 3$
x)	165	$55 \times 3 = 165^*$

\* = answer

Observe the pattern:

1; 3; 6; 10; 15; 21; 28; 36; 45; 55; etc.  
 $\xrightarrow{+2} \xrightarrow{+3} \xrightarrow{+4} \xrightarrow{+5} \xrightarrow{+6} \xrightarrow{+7} \xrightarrow{+8} \xrightarrow{+9} \xrightarrow{+10}$

i.e. triangular numbers generated by  $\frac{n}{2}(n+1)$  where  $n$  represents the position in the sequence.

19. In a certain town some people were affected by a 'flu' epidemic. In the first month 20 % of the population contracted the flu whilst 80 % were healthy. In the following month 20 % of the sick people recovered and 20 % of the healthy people contracted the disease.

What fraction of the population is healthy at the end of the second month?

- A) 0,68    B) 0,60    C) 0,52    D) 0,44    E) 0,36

ANSWER: A

$$\begin{aligned}
 & \frac{4}{5} \times \frac{1}{5} + \frac{4}{5} - 20\% \times \frac{4}{5} \\
 &= \frac{4}{5} \times \frac{1}{5} + \frac{4}{5} - \left(\frac{1}{5} \times \frac{4}{5}\right) \\
 &= \frac{4}{25} + \frac{4}{5} - \frac{4}{25} \\
 &= \frac{1}{25} + \frac{16}{25} \\
 &= \frac{17}{25} = 0,68
 \end{aligned}$$

20. Mpho, Barry, Sipho, Erica and Fatima are sitting on a park bench. Mpho is not sitting on the far right. Barry is not sitting on the far left. Sipho is not sitting at either end. Erica is sitting to the right of Barry, but not necessarily next to him. Fatima is not sitting next to Sipho. Sipho is not sitting next to Barry.

Who is sitting at the far right?

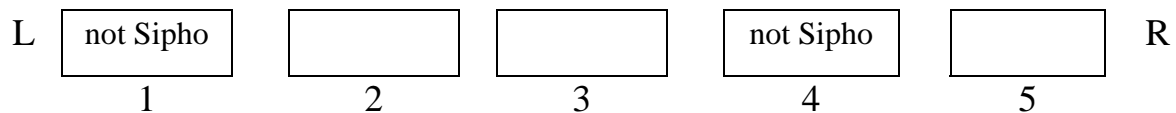
- A) Mpho    B) Barry    C) Sipho    D) Fatima    E) Erica

ANSWER: E

This is essentially a trial-and-improvement problem, in which we can test whether the 6 statements are satisfied by placing “x” on the far right. Let’s number the statements:

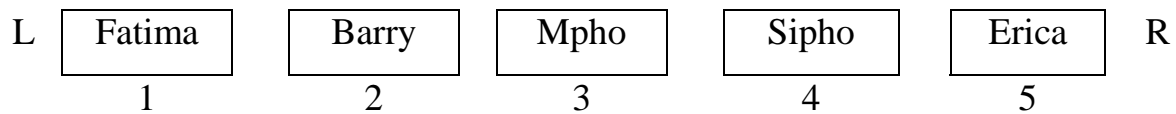
1. Mpho (A) not far right.
2. Barry (B) not far left.
3. Siphon (C) not at either end.
4. Erica (E) to the right of Barry.
5. Fatima (D) not next to Siphon.
6. Siphon not next to Barry.

Statement 1 eliminates Mpho (A). Statement 4 eliminates Barry (B), because nobody can be to the right of the far right. Statement 3 eliminates Siphon (C). This leaves us to consider Erica or fatima. Considering Fatima on the right, we have:



So Siphon must be in the second or third seat, and Barry can’t be in the first seat, but can’t be next to Siphon, so Siphon must be in seat 2, with Barry in seat 4. However Erica cannot be to the right of Barry, because Fatima is already there. SO; FATIMA (D) CANNOT BE ON THE FAR RIGHT.

No consider Erica on the far right, with the others as shown:



All six statements are satisfied by this arrangement, and so: ERICA (E) IS ON THE FAR RIGHT.

## THE END

**ANSWER POSITIONS:****JUNIOR FIRST ROUND 2002**

<b>PRACTICE EXAMPLES</b>	<b>POSITION</b>
1	C
2	D

<b>NUMBER</b>	<b>POSITION</b>
1	D
2	B
3	A
4	C
5	C
6	D
7	B
8	C
9	E
10	A
11	C
12	E
13	B
14	D
15	C
16	D
17	B
18	D
19	A
20	E

<b>DISTRIBUTION</b>	
A	3
B	4
C	5
D	5
E	3
<b>TOTAL</b>	<b>20</b>