2019 JUNIOR ROUND TWO SOLUTIONS

1. 4 $\dfrac{20+1-9}{\sqrt{2^0-1+9}} = \dfrac{12}{\sqrt{9}} = \dfrac{12}{3} = 4$

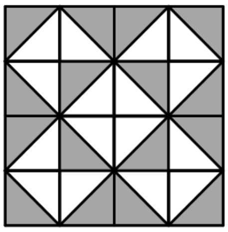
2. 10 The bottom layer contains 6 cubes. The middle layer contains 3 cubes. The top layer contains 1 cube. Thus 6 + 3 + 1 = 10.

3. 47 $3x = 180° - (20° + 19°) = 141° \quad \therefore \quad x = 47°$

4. 4 10% of 20 is 2 so 20% of 20 is 4.

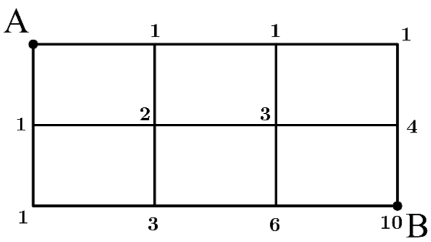
5. 5 $\sqrt{20+\sqrt{19+\sqrt{20+16}}} = \sqrt{20+\sqrt{19+6}} = \sqrt{20+5} = 5$

6. 50 Sub-dividing the diagram into smaller identical triangles shows that half the area is shaded.



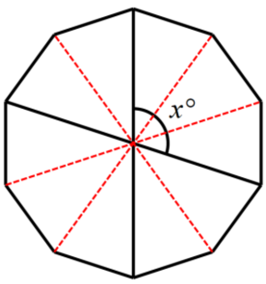
7. 57 20 + 19 = 39. 117 ÷ 39 = 3. 19 × 3 = 57

8. 10 The diagram shows the number of paths to each vertex.



9. 6 With a bit of experimentation it soon becomes clear that the only solution is $2^4 = 4^2$.

10. 108 360° ÷ 10 = 36°. 36° × 3 = 108°.



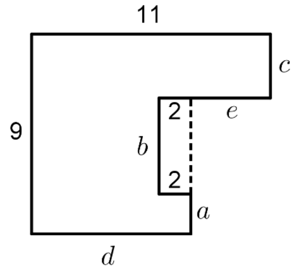
11. 10 The sum of the numbers in rows 1, 2, 3, 4 etc. forms the pattern 1, 8, 27, 64, …, i.e. the sequence of perfect cubes. If $n^3 = 1000$ then $n = 10$.

12. 288 If we let the short side of each rectangle be x then the path length is $18x$. We thus have $18x = 36$ from which $x = 2$. Rectangle ABCD has dimensions $9x$ and $8x$. The area of the rectangle is thus $18 \times 16 = 288\ cm^2$.

13. 44 $a+b+c=9$ and $d+e=11$.
Perimeter = $9+11+(a+b+c)+(d+e)+2+2=44\ cm$.

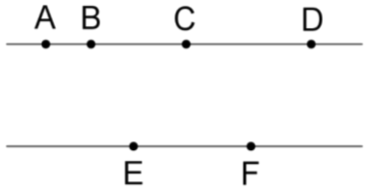
Using logical reasoning, one could simplify the calculation to

$2\times(11+9)+2\times 2=44$.



14. 11 We have $\frac{x^2+19}{2}=70$, thus $x^2+19=140$, and hence $x^2=121$.
Since a and b are both positive we have $x=11$.

15. 16 Label the six dots as shown alongside. If we chose EF as the base then there are 4 choices for the third vertex, i.e. 4 possible triangles. If the base is chosen from the top line then there are six possibilities – AB, AC, AD, BC, BD and CD. For each of these six bases there are two choices for the third vertex, E or F, making 12 triangles. The total number of triangles is thus 12 + 4 = 16.



16. 380 The right-angled triangle walked is a 3-4-5 Pythagorean triple scaled down by a factor of 2, i.e. the side lengths are 1½, 2 and 2½. Total time away from home = 1½ + 2 + ⅓ + 2½ = 6 hours 20 minutes, i.e. 380 minutes.

17. 100 There are at most 366 days in a year. Suppose none of the first 366 students has their birthday on the same day of the year. Then the last student's birthday *has to* fall on the same day of the year as the birthday of one of the first 366 students.

18. 9 $\frac{20}{19}\left(\frac{1}{20}+\frac{1}{18}\right)=\frac{20}{19}\left(\frac{18+20}{20\times18}\right)=\frac{1}{19}\left(\frac{38}{18}\right)=\frac{1}{9}$

19. 80 From symmetry considerations the shaded area is simply the area of the rectangle minus the area of the two circles divided by 2, so shaded area = $(16\times 8-32\pi)/2$, i.e. $64-16\pi$. Thus $x+y=64+16=80$.

20. 3 Note that $n!$ for $n\geq 5$ will always have 2 and 5 as factors, and will thus always end in zero. The required units digit is thus simply the units digit of $1!+2!+3!+4!=1+2+6+24=33$. The units digit is thus 3.

21. 16 For every 3/5 of a lap that A completes, B completes 2/5 of a lap. Since 3/5 +2/5 =1, together they complete a full lap and meet every time that A completes 3/5 of a lap. Hence, during A's first 10 laps, the number of times they would meet would be the whole number of times that 3/5 divides into 10, i.e. 16 times.

Alternatively, image the track has length 500 metres. A will lap B every 300 metres. If A completes 10 laps, i.e. 5000 metres, then the number of times they would meet is the integer part of 5000/300, i.e. 16.

22. 50 The 3-digit palindromes have the form XYX where X is the 1st and 3rd digit, and Y is the middle digit. The 4-digit palindromes have the form XYYX where X is the digit in the 1st and 4th positions and Y is the digit in the 2nd and 3rd positions. For every 3-digit palindrome XYX, one can repeat the middle digit to obtain the form XYYX. The number of 3-digit palindromes is the same as the number of 4 digit palindromes.

23. 66 Note that $6 = 2 \times 3$ and $180 = 2^2 \times 3^2 \times 5$. Thus, possible pairs of numbers for A and B are:

$2 \times 3 \times 5 = 30$ and $2^2 \times 3^2 = 36$

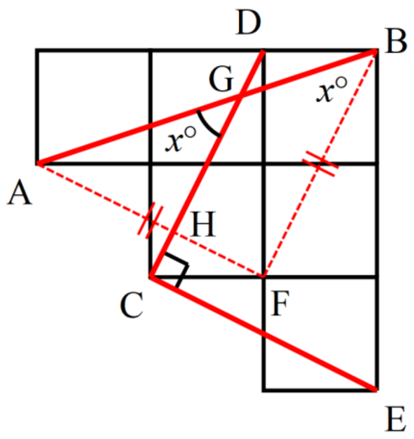
$2^2 \times 3 = 12$ and $2 \times 3^2 \times 5 = 90$

$2 \times 3^2 = 18$ and $2^2 \times 3 \times 5 = 60$

The smallest possible value of A + B is thus 30 + 36 = 66

24. 4 The sum of the weights of the right-hand side of the first scale and the left-hand side of the second scale equals the sum of the weights of the left-hand side of the first scale and the right-hand side of the second scale. So, 7A + 3C = 4A + 2C + 4B. Subtracting 4A and 2C from both sides of this equation shows that 3A + 1C = 4B.

25. 45 Translate DC to BF and draw in AF. We now see that $A\hat{G}C = A\hat{B}F$ (corresponding angles on parallel lines). Since AF = BF and AF is perpendicular to BF it follows that triangle AFB is a right-angled isosceles triangle, and hence that $B\hat{A}F = x = 45°$.



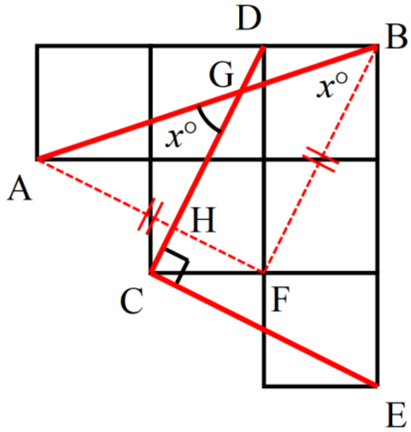
23. 66 Let op dat $6 = 2 \times 3$ en $180 = 2^2 \times 3^2 \times 5$. Dus, moontlike pare van getalle vir A en B is:

$$2 \times 3 \times 5 = 30 \text{ en } 2^2 \times 3^2 = 36$$
$$2^2 \times 3 = 12 \text{ en } 2 \times 3^2 \times 5 = 90$$
$$2 \times 3^2 = 18 \text{ en } 2^2 \times 3 \times 5 = 60$$

Die kleinste moontlike waarde van A + B is dus 30 + 36 = 66

24. 4 Die som van die gewigte aan die regterkant van die eerste skaal en die linkerkant van die tweede skaal is gelyk aan die som van die gewigte aan die linkerkant van die eerste skaal en die regterkant van die tweede skaal. Dus, 7A + 3C = 4A + 2C + 4B.
Deur 4A en 2C van beide kante van die vergelyking af te trek kry ons 3A + 1C = 4B.

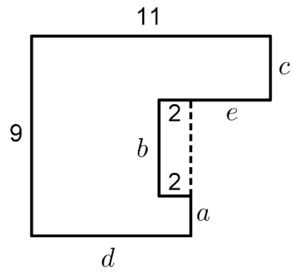
25. 45 Verskuif DC na BF en trek AF. Ons sien nou dat $A\hat{G}C = A\hat{B}F$ (ooreenkomstige hoeke van parallelle lyne). Aangesien AF = BF en AF loodreg is tot BF volg dit dat driehoek AFB 'n reghoeke gelykbenige driehoek is, en daarom is $B\hat{A}F = x = 45°$.



13. 44 $a + b + c = 9$ en $d + e = 11$.
Omtrek = $9 + 11 + (a + b + c) + (d + e) + 2 + 2 = 44\ cm$.

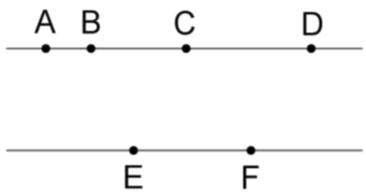
Die berekening kan vereenvoudig word na

$2 \times (11 + 9) + 2 \times 2 = 44$.



14. 11 Ons het $\frac{x^2 + 19}{2} = 70$, dus $x^2 + 19 = 140$, en daarom is $x^2 = 121$.
Aangesien a en b beide positief is, het ons $x = 11$.

15. 16 Noem die ses punte soos hier langsaan aangetoon. As ons EF kies as die basis dan is daar 4 keuses vir die derde hoek, d.i. 4 moontlike driehoeke. As die basis gekies word vanaf die boonste lyn dan is daar ses moontlikhede – AB, AC, AD, BC, BD en CD. Vir elk van hierdie ses basisse is daar twee keuses vir die derde hoek, E of F, wat 12 driehoeke gee. Die totale aantal driehoeke is dus 12 + 4 = 16.



16. 380 Die reghoekige driehoek waarin gestap word, is ’n 3-4-5 Pythagoras drietal verklein met ’n faktor van 2, d.i. die sylengtes is 1½, 2 en 2½. Totale tyd weg van die huis is 1½ + 2 + ⅓ + 2½ = 6 ure 20 minute, d.i. 380 minute.

17. 100 Daar is op die meeste 366 dae in ’n jaar. Veronderstel dat geen van die eerste 366 studente hul verjaarsdag op dieselfde dag het nie. Dan moet die laaste student se verjaarsdag val op dieselfde dag as een van die eerste 366 studente se verjaarsdag.

18. 9 $\frac{20}{19}\left(\frac{1}{20} + \frac{1}{18}\right) = \frac{20}{19}\left(\frac{18+20}{20\times 18}\right) = \frac{1}{19}\left(\frac{38}{18}\right) = \frac{1}{9}$

19. 80 Uit simmetrie is die verdonkerde gebied die oppervakte van die reghoek minus die oppervlakte van die twee sirkels gedeel deur 2, so ingekleurde oppervlakte = $(16 \times 8 - 32\pi)/2$, d.i. $64 - 16\pi$. Dus is $x + y = 64 + 16 = 80$.

20. 3 Let op dat $n!$ Vir $n \geq 5$ altyd 2 en 5 as faktore het en sal dus altyd eindig op nul. Die eenheidssyfer is dus die eenheidssyfer van $1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$. Die eenheidssyfer is dus 3.

21. 16 Vir elke 3/5 van ’n rondte wat A voltooi, voltooi B 2/5 van ’n rondte. Aangesien 3/5 +2/5 =1, voltooi hulle saam ’n volle rondte en ontmoet elke keer wat A 3/5 van ’n rondte voltooi. Dus gedurende A se eerste 10 rondtes is die hoeveelheid keer wat hulle ontmoet die heeltallige waarde wat 3/5 in 10 in deel, d.i. 16 keer.

Alternatiewelik, veronderstel die baan het lengte 500 meter. A sal elke 300 meter verby B gaan. As A 10 rondtes voltooi, d.i. 5000 meter, dan is die aantal keer wat hulle sal ontmoet die heeltallige deel van 5000/300, d.i. 16.

22. 50 Die 3-syfer palindroom het die vorm XYX waar X die 1ste en 3de syfer is, en Y die middelste syfer. Die 4-syfer palindroom het die vorm XYYX waar X die syfer in die 1ste en 4de posisies is en Y is die syfer in die 2de en 3de posisies. Vir elke 3-syfer palindroom XYX kan ons die middelste syfer herhaal om die vorm XYYX te kry. Die aantal 3-syfer palindrome is dieselfde as die aantal 4-syfer palindrome.

2019 JUNIOR RONDE TWEE OPLOSSINGS

1. 4 $\dfrac{20+1-9}{\sqrt{2^0-1+9}} = \dfrac{12}{\sqrt{9}} = \dfrac{12}{3} = 4$

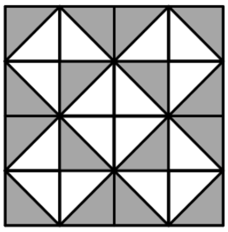
2. 10 Die onderste laag bevat 6 kubusse. Die middelste laag bevat 3 kubusse. Die boonste laag bevat 1 kubus. Dus 6 + 3 + 1 = 10.

3. 47 $3x = 180° - (20° + 19°) = 141° \quad \therefore \quad x = 47°$

4. 4 10% van 20 is 2 dus: 20% van 20 is 4.

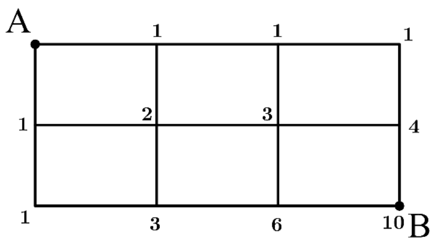
5. 5 $\sqrt{20+\sqrt{19+\sqrt{20+16}}} = \sqrt{20+\sqrt{19+6}} = \sqrt{20+5} = 5$

6. 50 Onderverdeling van die diagram in kleiner identiese driehoeke toon dat die helfte van die area ingekleur is.



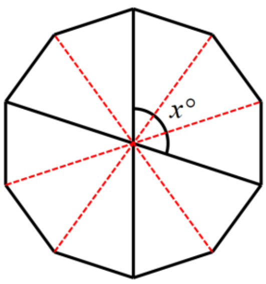
7. 57 20 + 19 = 39. 117 ÷ 39 = 3. 19 × 3 = 57

8. 10 Die diagram toon die aantal paaie na elke kruispunt.



9. 6 Met 'n bietjie eksperimentasie sien ons dat die enigste oplossing $2^4 = 4^2$ is.

10. 108 360° ÷ 10 = 36°. 36° × 3 = 108°.



11. 10 Die som van die getalle in rye 1, 2, 3, 4 ens. vorm die patroon 1, 8, 27, 64, …, d.i. die ry van perfekte derdemagte. As $n^3 = 1000$ dan $n = 10$.

12. 288 As ons die korter sy van elke reghoek x maak, dan is die padlengte $18x$. Ons het dus $18x = 36$ en dus is $x = 2$. Reghoek ABCD het afmetings $9x$ en $8x$. Die oppervlakte van die reghoek is dus $18 \times 16 = 288\ cm^2$.