SOUTH AFRICAN MATHEMATICS OLYMPIAD
2013 Junior Grade 9 Round 1
Solutions

1. **C** Simple addition

2. **D** There are 52 weeks and 1 day in a year (unless it is a leap year). So in 1985 there were 52 weeks going Tuesday to Monday, and one extra day (a Tuesday). That makes 53 Tuesdays.

3. **E** The sum of the digits must be divisible by 3, so d must be 0 or 3 or 6 or 9. Dividing the four possibilities by 11 shows that only 792 is divisible by 11.

4. **C** The recurring part of the decimal uses 6 digits, and $2013 = 6 \times 335 + 3$. So the required digit is the third in the recurring part, i.e. 8.

5. **B** We rearrange the sum as $(2 - 1) + (4 - 3) + (6 - 5) + \ldots + (50 - 49)$. Each bracket evaluates to 1, and there are $50/2 = 25$ of them, so the total must be 25.

6. **D** There are $5\times8 = 40$ tiles in all. Of these, $8 + 8 + 3 + 3 = 22$ are on an edge, so the required probability is $22/40 = 11/20$.

7. **D** We know $A = \frac{2}{3}C$, $D = 2B$ and $B + C = 180°$. That means $B = 180° - C$, so $D = 360° - 2C$, and of course $A + D = 180°$ as well. This can be written as $\frac{2}{3}C + 360° - 2C = 180°$, which gives $180° = 2C - \frac{2}{3}C = \frac{4}{3}C$, so $C = 180° \times 3 \div 4 = 135°$

8. **B** We note first that AD = BC (opposite sides of parallelogram), but then Δ ADP is isosceles. That means $D\hat{P}A = (180° - 2x)/2 = 90° - x$. Further, $B\hat{P}C = B\hat{C}P$, while $B\hat{C}P = 180° - 2x$ (cointerior). So the three angles at P add up to 180°, giving $90° - x + 72° + 180° - 2x = 180°$, whence $3x = 162°$ and so $x = 54°$.

9. **D** The difference of squares factorises as $(5675 - 4325)(5675 + 4325) = 1350 \times 10000$, which clearly ends in 5 zeros.

10. **D** Using Pythagoras in Δ APC shows AC = 15. Now in Δ ABC we have $15^2 + 36^2 = BC^2$. Since $15 = 5\times3$ and $36 = 12\times3$, $BC = 13\times3 = 39$.

11. **E** If r is the radius of the semicircles, then $r + a + r = A$, so $r = \dfrac{A-a}{2}$. The two semicircles together make just one circle, with combined area $\pi r^2 = \pi\left(\dfrac{A-a}{2}\right)^2$

12. **C** When n is very large, $n + 2$ is almost exactly equal to n, and $2n + 1$ is almost exactly equal to $2n$. Then the fraction is equivalent to $n/2n = ½$.

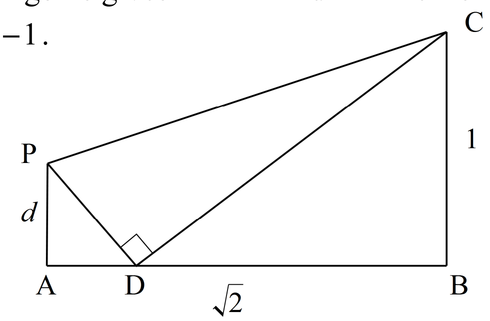
13. **B** The sum of the first 9 digits is 45. This 9-digit sequence is repeated endlessly, and after a total of 10 appearances has accumulated a total of 450. To gain the extra 10 we need to have a 1, 2, 3 and 4.

14. **C** The perpendicular bisector of the base of an isosceles triangle (as BP is for Δ ABM) divides it into two congruent triangles: so 1, 2 and 5 are all true. If $\hat{C} = \theta$, $\hat{A} = 90^\circ - \theta$ and so $B\hat{M}A = 90^\circ - \theta$ and $A\hat{B}M = 180^\circ - 2(90^\circ - \theta) = 2\theta$. That means $P\hat{B}M = \theta$, so 4 is true. Thus 3 is the dubious statement, and in fact it can only be true if $\theta = 30^\circ$.

15. **E** I have 5 options for the first book, and then for each of those I continue with 4 options for the next one, and so on. So the number of possibilities is $5 \times 4 \times 3 \times 2 \times 1 = 120$.

16. **A.** The third row of the pattern must be $2x$; xy; $5y$. Then the second row is $2x.xy$; $xy.5y$, and so we know that $2x.xy \times xy.5y = 80$. This gives $10x^3y^3 = 80$, which means $x^3y^3 = 8$ and so $xy = 2$.

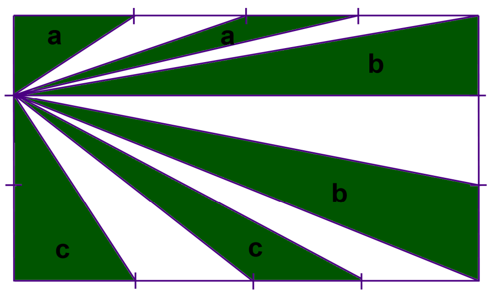
17. **A** CD is the top edge of the rectangle before folding, so is equal to AB = $\sqrt{2}$. $P\hat{D}C$ is the top left corner of the original rectangle, so is 90°. Now Pythagoras gives BD = 1 and that means $C\hat{D}B = 45^\circ$, so $P\hat{D}A$ is also 45° and then d = AD = $\sqrt{2} - 1$.



18. **C** The sum of all the numbers is $12 \times 18 = 216$, and the sum of the largest and smallest is $2 \times 28 = 56$. The sum of the other ten must therefore be $216 - 56 = 160$, so their average is 16

19. **A** If L is the length of each rectangle and W its width, then the diagram shows 2L = 3W, and also that L + W = 15. Doubling this gives 2L + 2W = 30, or 3W + 2W = 30 and therefore W = 6. Now the length of the big rectangle is 3W = 18, so its area is $18 \times 15 = 270$.

20. **C** The triangles marked **a** have one quarter the base and one third the height of the rectangle, so each is $\frac{1}{2}.\frac{1}{4}.\frac{1}{3} = \frac{1}{24}$ the area of the rectangle.
Triangles marked **b** have one third the base and the whole height of the rectangle while triangles marked **c** have one quarter the base and 2/3 the height. The total proportion occupied by these six triangles is thus



$$2.\frac{1}{2}.\frac{1}{4}.\frac{1}{3} + 2.\frac{1}{2}.\frac{1}{3}.1 + 2.\frac{1}{2}.\frac{1}{4}.\frac{2}{3} = \frac{1}{4}.\frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{1}{12} + \frac{4}{12} + \frac{2}{12} = \frac{7}{12}$$