



THE HARMONY
SOUTH AFRICAN
MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION in collaboration
with, AMESA, SAMS and the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN
KUNS

Sponsored by HARMONY GOLD MINING

Third Round 2005
Junior Section: Grades 8 and 9
Date: 12 September 2004

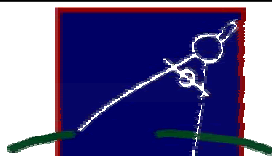
Instructions

- Answer all the questions.
- All working details and explanations must be shown.
Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks:
 - Questions 1 to 10 are worth 6 marks each.
 - Questions 11 to 15 are worth 8 marks each.
- The neatness in your presentation of the solutions will be taken into account.
- The time allocated is 3 hours.
- No calculator of any form may be used.
- Answers and solutions are available at:

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1. The area of one side of a rectangular box is 126 cm^2 . The area of another side of the rectangular box is 153 cm^2 . The area of the top of the rectangular box is 238 cm^2 .

What is the volume of the box?

(6 Marks)

2. In a Hockey tournament, each team played each other team once.

The final league table was:

	WINS	DRAWS	LOSSES	POINTS
BULLS	1	2	0	4
CHEETAHS	1	1	1	3
LIONS	1	1	1	3
SHARKS	1	0	2	2

If Sharks beat Cheetahs, then which of the statements are true?

- (i) Lions defeated Cheetahs, but lost to Sharks.
- (ii) Bulls won against either Cheetahs or Lions.
- (iii) In matches against Sharks, Cheetahs were more successful than Lions.
- (iv) In matches against Lions, Bulls were more successful than Cheetahs.
- (v) Lions were undefeated, except against Cheetahs.

(6 Marks)

3. A sequence has first term 12, after which every term is the sum of the squares of the digits of the preceding term. Thus the second term is $1^2 + 2^2 = 5$, the third term $5^2 = 25$, the fourth term $2^2 + 5^2 = 29$, and so on.

Find the 2005th term of the sequence.

(6 Marks)

4. A 4 by 4 “Antimagic square” is an arrangement of the numbers 1 to 16 in a square grid such that the totals of each of the four rows and columns and the main diagonals are 10 consecutive numbers in some order. The diagram shows an incomplete antimagic square. Complete the square.

4	5	7	14
6	13	3	
11	12	9	
10			

(6 Marks)

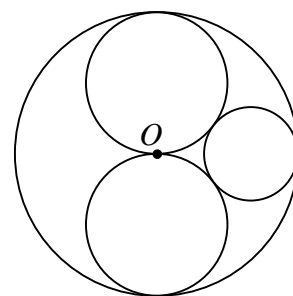
5. Each letter in the addition sum shown below stands for a different digit, with S standing for 3.

$$\begin{array}{r} SO \\ + MANY \\ \hline SUMS \end{array}$$

What is the value of $Y \times O$?

(6 Marks)

6. A circle of radius 2 cm with centre O , contains three smaller circles as shown in the diagram; two of them touch the outer circle, and touch each other at O , and the third touches each of the other circles.



Determine the radius of the third circle, in centimeters.

(6 Marks)

7. Find the smallest natural number which when multiplied by 123 yields a product that ends in 2005.

(6 Marks)

8. Let $A = 200420042004 \times 2005200520052005$
and $B = 200520052005 \times 20042004$

Find the value of $\frac{A}{B}$ in simplest form.

(6 Marks)

9. Alfred, Brigitte, Carodene, Dolly, and Effie play a game in which each is either a dog or a mouse. A dog's statement is always false while a mouse's statement is always true.

Alfred says that Brigitte is a mouse.

Carodene says that Dolly is a dog.

Effie says that Alfred is not a dog.

Brigitte says that Carodene is not a mouse.

Dolly says that Effie and Alfred are different kinds of animals.

How many dogs are there?

(6 Marks)

10. We have assigned different positive integers to different letters and then multiplied their values together to make the values of words.

For example, if $C = 4$; $A = 8$ and $T = 12$, then $CAT = 4 \times 8 \times 12 = 384$.

Given that $HILL = 15$

$PHOTO = 8470$

$HILLS = 195$

$HIPHOP = 3300$

Find the value of PITSTOP.

(6 Marks)

11. Consider the following number pattern using only the odd natural numbers:

			1			(Row 1)	
		3		5		(Row 2)	
	7		9		11	(Row 3)	
13		15		17		19	(Row 4)

and so on.

Numbers

A

B

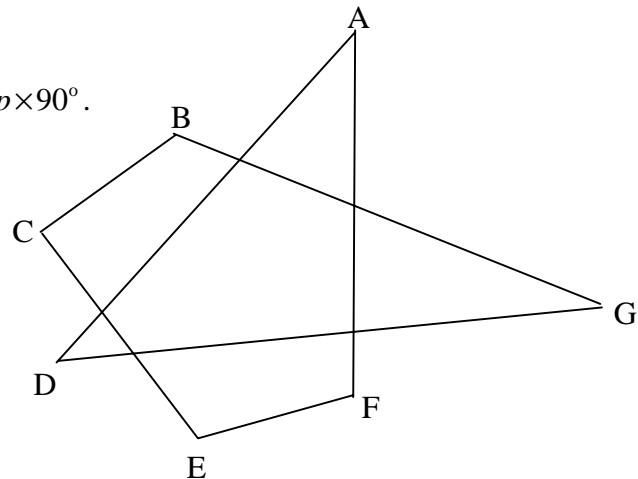
C

are taken from the pattern, where A and B are two adjacent numbers in a certain row, and C is the number in the next row, directly below, and between, A and B. If $A + B + C = 2093$, find the value of C.

(8 Marks)

12. In the diagram, $\hat{A} + \hat{B} + \hat{C} + \hat{D} + \hat{E} + \hat{F} + \hat{G} = p \times 90^\circ$.

Find p .

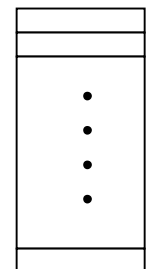


(8 Marks)

13. Paul and James go out for a cycle and are 16 km from home when Paul runs into a tree damaging his bicycle beyond repair. They decide to return home and that Paul will start on foot and James will start on his bicycle. After some time, James will leave his bicycle beside the road and continue on foot, so that when Paul reaches the bicycle he can mount it and cycle the rest of the distance. Paul walks at 4 km per hour and cycles at 10 km per hour, while James walks at 5 km per hour and cycles at 12 km per hour. For what length of time should James ride the bicycle, if they are both to arrive home at the same time?

(8 Marks)

14. The diagram shows a large rectangle whose perimeter is 300 cm. It is divided up as shown into a number of identical rectangles, each of perimeter 58 cm. Each side of these rectangles is a whole number of centimeters. Show that there are exactly two ways of splitting up the rectangle as described above.



(8 Marks)

15. A complete sentence of 20 words is concealed here. If you start with Sal and then follow the jumps of a chess knight from word to word along the correct route, a simple problem will be revealed. Sue is in her teens, so how old is Sal?

was	now	old	was	now
as	Sue	Sal	half	as
is	a	old	is	when
as	Sue	Sal	third	as

(8 Marks)

Total: 100

THE END