OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Senior First Round 2020

Solutions

1. **Answer D**
   $20 \times n = 2020$, so $n = 2020 \div 20 = 101$.

2. **Answer B**
   M and A each has a vertical axis of symmetry, C and E each has a horizontal axis of symmetry, and I and H both have horizontal and vertical axes of symmetry. The only letter without an axis of symmetry is L.

3. **Answer C**
   Since it is impossible to get both 5 and 6 on one throw, it follows that the probability of getting 5 or 6 is $P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

4. **Answer D**
   $\sqrt{(2023+3)(2023-3)+9} = \sqrt{2023^2 - 9 + 9} = \sqrt{2023^2} = 2023$.

5. **Answer C**
   There are ten brackets, each equal to -1, so the expression is equal to $(-1)^{10} = +1$.

6. **Answer A**
   Working in rands, suppose the sums received are $2k$, $3k$, $4k$; then $4k = 76$, so $k = 19$. The total amount shared out is $2k + 3k + 4k = 9k = 9 \times 19 = 171$.

7. **Answer D**
   A sector of 45° represents one-eighth of a circle, since $45^\circ = \frac{1}{8} \times 360^\circ$. The mass of bananas sold was therefore $\frac{1}{8} \times 120 = 15$ kg, so the mass of the other types of fruit was $120 - 15 = 105$ kg.

8. **Answer E**
   The first digit cannot be 1, since the last digit cannot be 10. That leaves eight choices for the first digit (from 2 to 9), which determines the last digit. Combined with ten choices for the middle digit, that gives a total of $8 \times 10 = 80$ numbers.

9. **Answer C**
   A 10% increase means multiplying by $1 + \frac{10}{100} = 1.1$. Since both length and width are multiplied by 1.1, it follows that the area is multiplied by $1.1^2 = 1.21 = 1 + \frac{21}{100}$. This is the same as a 21% increase in area.

10. **Answer A**
    Let $Y\widehat{R}S = x$. Then $\widehat{S} = x$, since triangle YRS is isosceles. Also $\widehat{A} = 2x$ and $\widehat{T} = 3x$ (given). By summing the angles in triangle TAS, it follows that $x + 2x + 3x = 180^\circ$, so $6x = 180^\circ$ and $x = 30^\circ$.

11. **Answer B**
    The sum of the exterior angles of any polygon is 360°, so for a regular 720-sided polygon each exterior angle is equal to $\frac{360}{720}^\circ = \frac{1}{2}^\circ$. Each interior angle is therefore equal to $180 - \frac{1}{2} = 179\frac{1}{2}$ degrees.

12. **Answer B**
$9^{999}+1=(9+1)(9^{998}-9^{997}+\cdots+9^2-9+1)$. The first factor is 10 and the second factor is odd, because it is the sum of an odd number of odd numbers. Therefore $9^{999}+1$ is divisible by 10 but not by any higher power of 10.
Alternatively, by the Binomial Theorem $9^{999}=(10-1)^{999}=-1+(999)10+$ terms in higher powers of 10, which leads to the same result.

13. **Answer D**
The units digit of $743\ 589 \times 301\ 647$ is the same as the units digit of 9×7, which is 3. Therefore the remainder after division by 10 or by 5 is 3.

14. **Answer C**
Each of the eight players wrestles with the six players in the other three teams. Since each match involves two players, it follows that the number of matches is $6 \times 8 \div 2 = 24$.

15. **Answer B**
The base angles of the first isosceles triangle are $8°$; for the second triangle they are $8+8=16°$ (exterior angle of triangle); for the third, they are $16+16-8=24°$, and so on, increasing by $8°$ each time. In the n-th triangle, the base angles are both $(8n)°$, which must be less than $90°$. Therefore $n < \frac{90}{8} = 11\frac{1}{4}$, so $n \leq 11$, since n is a natural number.

16. **Answer E**
Suppose Peter rides at k km/h and Jack at $\frac{3}{4}k$ km/h. If they meet at a point d km from Peter's house, then $d = kx$ and $13 - d = (\frac{3}{4}k)(3x) = \frac{9}{4}kx = \frac{9}{4}d$. Therefore $13 = d + \frac{9}{4}d = \frac{13}{4}d$, so $d = 4$.

17. **Answer E**
By considering the vertical distance on each side of the figure, we see that $a+c+e=g=7$ (given). From the horizontal distance at top and bottom, it follows that $h-b+d=f$, so $6-b+3=f$, or $b+f=9$. The perimeter is equal to $(a+c+e)+(b+f)+d+g+h=7+9+3+7+6=32$.

18. **Answer B**
The largest number must use all nine digits, so it must start in a corner. The biggest corner number is 5, and by choosing the largest possible neighbouring digit each time, the largest number is 594 836 271.

19. **Answer D**
By inspection $S_n = n^2$, and the first differences S_2-S_1, S_3-S_2, $S_4-S_3, \ldots$ increase by 2 each time. Since the first differences C_2-C_1, C_3-C_2, $C_4-C_3, \ldots$ increase twice as fast, it follows that $C_n = 2n^2+bn+c$, where b and c must be determined. Then $C_1 = 2+b+c = 1$ and $C_2 = 8+2b+c = 6$, which give $b=-1$ and $c=0$, so $C_n = 2n^2 - n$. Therefore

$$\frac{S_n}{C_n} = \frac{n^2}{2n^2-n} = \frac{n}{2n-1}.$$

The only fraction not of this form is $\frac{30}{61}$.

20. **Answer D**

If N and S denote the top and bottom points of the circle, then $AN = BN = \sqrt{2}$ and $N\widehat{B}S = 90°$. The left-hand shaded region consists of semicircle ANS plus triangle BNS minus quarter-circle BNS, so its area is

$$\tfrac{1}{2}\pi 1^2 + \tfrac{1}{2}(2)(1) - \tfrac{1}{4}\pi(\sqrt{2})^2 = \tfrac{\pi}{2} + 1 - \tfrac{\pi}{2} = 1,$$

so the total area of the two shaded regions is 2.

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word. Dan is $C_1 = 2 + b + c = 1$ en $C_2 = 8 + 2b + c = 6$ wat $b = -1$ en $c = 0$ gee, sodat $C_n = 2n^2 - n$. Dus is

$$\frac{S_n}{C_n} = \frac{n^2}{2n^2 - n} = \frac{n}{2n - 1}.$$

Die enigste breuk wat nie in hierdie formaat is nie is $\frac{30}{61}$.

20. **Antwoord D**

As N en S die boonste en onderste punte van die sirkel aandui, dan is $AN = BN = \sqrt{2}$ en $N\hat{B}S = 90°$. Die linkerkantste ingekleurde deel bestaan uit die halfsirkel ANS saam met driehoek BNS minus die kwartsirkel BNS, sodat sy oppervlakte gelyk is aan

$$\tfrac{1}{2}\pi 1^2 + \tfrac{1}{2}(2)(1) - \tfrac{1}{4}\pi(\sqrt{2})^2 = \tfrac{\pi}{2} + 1 - \tfrac{\pi}{2} = 1,$$

en die totale oppervlakte van die ingekleurde dele is dan 2.

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11. **Antwoord B**
Die som van die buitehoeke van enige veelhoek is 360°, en dus is elke buitehoek van 'n reëlmatige veelhoek met 720 sye gelyk aan $\frac{360}{720}^\circ = \frac{1}{2}^\circ$. Elke binnehoek is dus gelyk aan $180 - \frac{1}{2} = 179\frac{1}{2}^\circ$.

12. **Antwoord B**
$9^{999} + 1 = (9+1)(9^{998} - 9^{997} + \cdots + 9^2 - 9 + 1)$. Die eerste faktor is 10 en die tweede faktor is onewe omdat dit die som is van 'n onewe getal onewe getalle. Dus is $9^{999} + 1$ deelbaar deur 10 maar nie deur enige groter mag van 10 nie.
Alternatiewelik, volgens die Binomiaalstelling is $9^{999} = (10-1)^{999} = -1 + (999)10 +$ terme in groter magter van 10, wat tot dieselfde resultaat lei.

13. **Antwoord D**
Die enesyfer van $743\ 589 \times 301\ 647$ is dieselfde as die enesyfer van 9×7, wat gelyk is aan 3. Dus is die res na deling deur 10 of deur 5 gelyk aan 3.

14. **Antwoord C**
Elk van die 8 spelers druk arm teen die 6 spelers van die ander 3 spanne. Twee spelers is betrokke by elke armdrukwedstryd, en dit volg dan dat daar $6 \times 8 \div 2 = 24$ wedstryde is.

15. **Antwoord B**
Die basishoeke van die eerste gelykbenige driehoek is 8°; vir die tweede driehoek is hulle $8+8 = 16^\circ$ (buitehoek van 'n driehoek); vir die derde is hulle $16+16-8 = 24^\circ$, ensovoorts, en neem vir elke driehoek met 8° toe. In die n-de driehoek is elke basishoek $(8n)^\circ$ wat minder as 90° moet wees. Dus is $n < \frac{90}{8} = 11\frac{1}{4}$, sodat $n \leq 11$, omdat n 'n natuurlike getal is.

16. **Antwoord E**
Veronderstel dat Peter teen k km/h en Jack teen $\frac{3}{4}k$ km/h ry. Laat hulle by 'n punt d km van Peter se huis ontmoet, dan is $d = kx$ en $13 - d = (\frac{3}{4}k)(3x) = \frac{9}{4}kx = \frac{9}{4}d$. Dus is $13 = d + \frac{9}{4}d = \frac{13}{4}d$, sodat $d = 4$.

17. **Antwoord E**
Beskou die vertikale lengte aan elke kant van die figuur. Dan is dit duidelik dat $a + c + e = g = 7$ (gegee). Uit die horisontale afstand bo en onder volg dit dat $h - b + d = f$, sodat $6 - b + 3 = f$, of $b + f = 9$. Die omtrek is dan gelyk aan $(a + c + e) + (b + f) + d + g + h = 7 + 9 + 3 + 7 + 6 = 32$.

18. **Antwoord B**
Die grootste getal moet al 9 syfers bevat en dus moet dit in 'n hoek begin. Die grootste hoekgetal is 5, en deur elke keer die grootste moontlike aanliggende syfer te kies, is die grootste getal 594 836 271.

19. **Antwoord D**
Deur inspeksie is $S_n = n^2$, en die eerste verskille is $S_2 - S_1$, $S_3 - S_2$, $S_4 - S_3, \ldots$ neem elke keer met 2 toe. Die eerste verskille in $C_2 - C_1$, $C_3 - C_2$, $C_4 - C_3, \ldots$ neem twee so vinnig toe en dus volg dit dat $C_n = 2n^2 + bn + c$, waar b em c bepaal moet

OLD MUTUAL SUID-AFRIKAANSE WISKUNDE OLIMPIADE

Senior Eerste Rondte 2020
Oplossings

1. **Antwoord D**
   $20 \times n = 2020$, en dus is $n = 2020 \div 20 = 101$.

2. **Antwoord B**
   M en A het elk ’n vertikale simmetrie-as, C en E het elk ’n horisontale simmetrie-as, en I en H het albei horisontale en vertikale simmetrie-asse. Die enigste letter sonder ’n simmetrie-as is L.

3. **Antwoord C**
   Dit is onmoontlik om ’n 5 en ’n 6 met een gooi te kry, en dus is die waarskynlikheid om ’n 5 of ’n 6 te kry $P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

4. **Antwoord D**
   $\sqrt{(2023+3)(2023-3)+9} = \sqrt{2023^2 - 9 + 9} = \sqrt{2023^2} = 2023$.

5. **Antwoord C**
   Daar is tien hakies, elkeen gelyk aan -1, sodat die uitdrukking gelyk is aan $(-1)^{10} = +1$.

6. **Antwoord A**
   As ons in Rand werk kan ons veronderstel dat die bedrae wat ontvang is $2k$, $3k$, $4k$ is; en dat $4k = 76$, sodat $k = 19$. Die totale bedrag wat verdeel word, is $2k+3k+4k = 9k = 9 \times 19 = 171$.

7. **Antwoord D**
   ’n Sektor van $45°$ is een agtste van ’n sirkel, want $45° = \frac{1}{8} \times 360°$. Die massa van die piesangs wat verkoop is, was dus $\frac{1}{8} \times 120 = 15$ kg, sodat die massa van die res van die vrugte $120 - 15 = 105$ kg was.

8. **Antwoord E**
   Die eerste syfer kan nie 1 wees nie omdat die laaste syfer nie 10 kan wees nie. Daar is dus 8 moontlikhede vir die eerste syfer (van 2 tot 9), wat dan die laaste syfer bepaal. Kombinaeer dit met tien keuses vir die middelste syfer en dit gee dan ’n totaal van $8 \times 10 = 80$ getalle.

9. **Antwoord C**
   ’n 10% toename beteken vermenigvuldiging met $1 + \frac{10}{100} = 1.1$. Omdat beide die lengte en breedte met 1.1 vermenigvuldig word, volg dit dat die oppervlakte met $1.1^2 = 1.21 = 1 + \frac{21}{100}$ vermenigvuldig word. Dit is dieselfde as ’n toename van 21% in die oppervlakte.

10. **Antwoord A**
    Laat $Y\widehat{R}S = x$. Dan is $\widehat{S} = x$, omdat driehoek YRS gelykbenig is. En $\widehat{A} = 2x$ en $\widehat{T} = 3x$ (gegee). Deur die hoeke in driehoek TAS bymekaar te tel, volg dit dat $x + 2x + 3x = 180°$, sodat $6x = 180°$ en $x = 30°$.