SOLUTIONS Junior Round Two 2013

1. $\left(1+\frac{4}{7}\right)\div\left(1-\frac{3}{14}\right)=\frac{11}{7}\times\frac{14}{11}=2$

2. If x is the original number, Oleg obtains $2x + 1$ and Ravi gets $3(x – 2)$. If these are equal, $2x + 1= 3x – 6$ and this easily gives $x = 7$.

3. The rectangular arrangement must have dimensions 12×1 or 6×2 or 4×3, with the latter having the least perimeter of $(4\times2 + 3\times2)\times2 = 28$

4. The prime factorisation of 2013 is $3\times11\times61$, and so the largest proper factors of 2013 are 11×61 and 3×61. The sum of these is $14\times61 = 854$.

5. To keep the total number of boxes as small as possible we use as many large boxes as possible, and thus have the number of small boxes no more than half the number of big boxes. If there are x small boxes there are then $2x$ big boxes and therefore $6.x + 12.2x = 30x$ bottles, and for this to be 240 we must have $x = 8$. Then the total number of boxes is $8 + 2.8 = 24$.

6. $103^2 + 101^2 – 100^2 – 102^2 = 103^2 – 102^2 + 101^2 – 100^2 = (103 – 102)(103 + 102) + (101 – 100)(101 + 100) = 1.205 + 1.201 = 406$

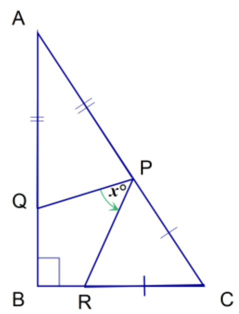
7. $ab = 2$ requires EITHER $a = 1$, $b = 2$; then $bc = 12$ requires $c = 6$ and ac is indeed 6
OR $a = 2$, $b = 1$; then $bc = 12$ requires $c = 12$ and so ac cannot be 6.
So we must have $a = 1$, $b = 2$, $c = 6$ and their sum is 9.

8. Since 224 is just less than $225 = 15^2$, and $39 < 49 = 7^2$, we see that x and y are each one of the integers 7, 8, 9, ..., 14. When we add them, we can get any results from 7 + 7 (at least) through 7 + 14 and 8 + 7 etc. up to 14 + 14 (at most), i.e. from 14 to 28. This is 15 different results.

9. Adding the two equations shows $5A + 5B = 20 = 5(A + B)$, so $A + B = 4$, and then $4A + 4B = 4(A + B) = 16$.

10. With three shirts, four skirts and B belts, Jane would have $3 \times 4 \times B$ possible combinations. Since she has at least 50 combinations, $3 \times 4 \times B$ is at least 50, and that requires B to be at least 5.

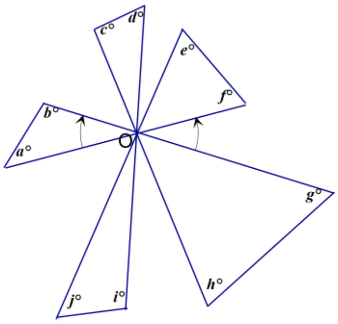
11. $A\hat{P}Q=\frac{1}{2}(180°-A)$ and $R\hat{P}C=\frac{1}{2}(180°-C)$.
$Q\hat{P}R=180°-A\hat{P}Q-R\hat{P}Q$
$=180°-90°+\frac{1}{2}A-90°+\frac{1}{2}C=\frac{1}{2}(A+C)$
But of course $A + C = 90°$, so $x = 45$.



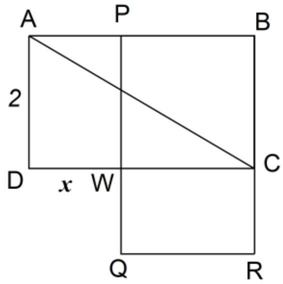
12. Each of the numbers is even, and so to be a square has to be divisible by 4. But then the number formed by the last two digits (i.e. 66) must be divisible by 4, and that never happens. Thus there can be no squares in the sequence.

13. $\frac{2^{1000}+2^{1008}}{2^{1001}+2^{1001}}=\frac{2^{1000}+2^{1008}}{2.2^{1001}}=\frac{2^{1000}+2^{1008}}{2^{1002}}=\frac{2^{1000}}{2^{1002}}+\frac{2^{1008}}{2^{1002}}=\frac{1}{2^2}+2^6$, and the closest integer to this is $2^6 = 64$.

14. If O is the vertex at which all the triangles meet, then each triangle has an angle at O which is vertically opposite (and equal to) an angle which does not belong to any triangle. The sum of all these vertex angles of the triangles is therefore half the revolution at O, which is 180°. Now the sum we seek is the sum of all the angles of all five triangles, minus the sum of their vertex angles.
This is $5 \times 180° - 180° = 720°$.

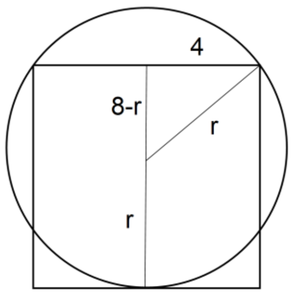


15. PBCW is a square of side 2, and rectangles APWD and WCRQ are congruent. Let DW = x cm.
Considering areas, we must have $2x \times 2$ = area PBCW = 4, so $x = 1$.
Now in Δ ADC Pythagoras gives $AC^2 = 2^2 + (2+1)^2 = 13$.



16. If r is the radius of the circle, then joining the centre of the circle to one vertex of the square shows

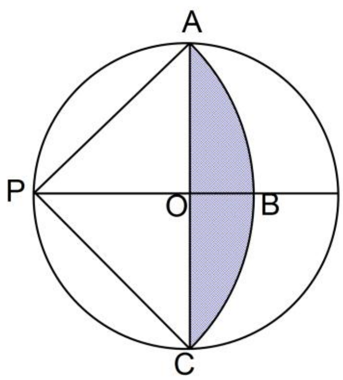
$r^2 = (8 - r)^2 + 4^2$ (by Pythagoras) and hence
$r^2 = 64 - 16r + r^2 + 16$
so that $16r = 80$ and $r = 5$.



17. Multiples of 4 must have last two digits divisible by 4, and in this case that means they must be 12 or 24 or 32 or 44 or 52. These are the two-digit numbers which qualify. Any permissible digit placed before them will give a qualifying three-digit number, and there are five such starting digits allowed (from 1 to 5). So we have 5 permissible two-digit numbers and 5×5 three-digit numbers, which is 30 numbers in all.

18. If 114 and 70 are in the same column, then the difference between them is divisible by the number of columns: so m is a factor of 44. Similarly we know that 207 and 152 are in the same column, so m is a factor of $207 - 152 = 55$. Thus m is a common factor of 44 and 55, and so must be 11.

19. With PO = OA (radii) and $P\hat{O}A = 90°$, $A\hat{P}O = 45°$. Similarly $C\hat{P}O = 45°$ and so $A\hat{P}C = 90°$, and then AP = $\sqrt{2}$.OA.
The area of segment ABC is a quarter of a circle of radius PA minus the area of the right-angled triangle APC, which is $\frac{1}{4}.\pi\left(4\sqrt{2}\right)^2 - \frac{1}{2}.AP^2$. Now AP = $4\sqrt{2}$, and so this area simplifies to $8\pi - 16$.
The shaded area required is the whole circle minus two segments, which is $\pi.4^2 - 2(8\pi - 16) = 32$ m^2.



20. If the cube of N ends in 8 then N must end in 2. But then N is $10m + 2$ for some integer m, and $N^3 = (10m + 2)^3 = 1000m^3 + 600m^2 + 120m + 8$. The penultimate (i.e. tens) digit of this must be the penultimate digit of the term $120m$, and can be 8 only if $m = 4$ or 9. Thus the two smallest integers having cubes ending in 88 are 42 and 92, whose sum is 134.

ANSWERS

1.	2
2.	7
3.	28
4.	854
5.	24
6.	406
7.	9
8.	15
9.	16
10.	5
11.	45
12.	0
13.	64
14.	720
15.	13
16	5
17.	30
18.	11
19.	32
20.	134