THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

SECOND ROUND 1999: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

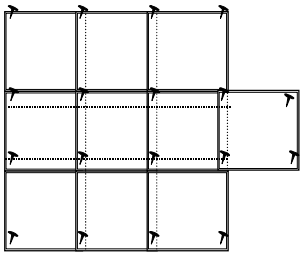
PART A: (Each correct answer is worth 3 marks)

1. ANSWER: D
$(1053 \times 15) - 450 = (1053 \times 15) - (30 \times 15) = 1023 \times 15$, so ♥ = 1023.

2. ANSWER: B
The dates of the year move forward one day each year with the exception of a leap year when it moves 2 days forward. Note that by a rule of the Gregorian calendar 2000 is a leap year. The other leap years are 2004 and 2008. Add 11 + 3 = 14 days. Therefore 27 April 2010 will also be on a Tuesday.

3. ANSWER: C
Let the white area be x. Now Area of figure ABED – area of BCEFG = (area of big rectangle – area ΔBCE) minus
(area of small rectangle – area ΔBCE) = $(80 - x) - (60 - x) = 20$ cm^2

4. ANSWER: D
To save on pins he should arrange the pictures in the shape of a 3×3 square and the last square can be attached to any one picture edge. In this way he uses only 18 pins.



5. ANSWER: E
Put BC = x and AB = $2x$. Now AC may be either x or $2x$ since ΔABC is isosceles. (But AC cannot be x since the sum of any two sides of a triangle is always more than the third side).
So $2x + 2x + x = 200$, $x = 40$ that gives AC = 80 mm

6. ANSWER: B
Two dots are always 500 units apart. So 657 – 500 = 157.

PART B: (Each correct answer is worth 5 marks)

7. ANSWER: B
If the votes were divided equally among the three, each would get 33 333 giving a total of 99 999. The remaining vote will decide the winner with 33 334 votes

8. ANSWER: C

The pattern (number of black squares out of the total number of squares) for the first few diagrams is $\frac{1}{1};\frac{3}{4};\frac{6}{9};\frac{10}{16};\frac{15}{25}$. The numerators are the triangular numbers: e.g. 6 = 1 + 2 + 3 and 10 = 1 + 2 + 3 + 4 which is the same as $\frac{4(4+1)}{2}$. The denominator is a square e.g. $9 = 3 \times 3$ and $16 = 4 \times 4$. The 50^{th} triangular number is 1 + 2 + 3 + ... + 50 = $\frac{50(50+1)}{2} = 1275$. Dividing by 50×50 gives $\frac{1275}{2500}$ or 51%.

9. ANSWER: A

Let length = *l* and breadth = 1 when closed. Now when opened length = 2 and breadth = *l*. We have $\frac{l}{1} = \frac{2}{l} \quad \therefore l^2 = 2 \quad \therefore l = \sqrt{2}$ if $b = 1$ and therefore the ratio of length to breadth is $\sqrt{2}:1$

10. ANSWER: C

$42 = 2 \times 3 \times 7$ and $462 = 2 \times 3 \times 7 \times 11$. Now *n* must be a multiple of 11 and it can be combined with 2, 3 or 7 each taken once in a particular factor. Clearly $88 = 2 \times 2 \times 11$ does not qualify since it requires two separate twos. (Some candidates may note immediately that 88 is not a factor of 462)

11. ANSWER: A

The black area is exactly half the rectangle's area. Area = 42 cm^2 (A).

(Area of triangles $= \frac{1}{2}\text{BE} \times \text{h} + \frac{1}{2}\text{EF} \times \text{h} + \frac{1}{2}\text{FC} \times \text{h}$

$= \frac{1}{2}(\text{BE} + \text{EF} + \text{FC}) \times \text{h} = \frac{1}{2}\text{BC} \times \text{h}$)

12. ANSWER: C

Since the answer starts with 2 it is clear that *a* can only be 2 or 3 since if the answer ranges between 207 and 297, *a*21 can only be 221 or 321. Now $2b7 \times 321 = 79\,287$. The tens digit in the answer is 8. There is no carry over from the units digit since $1 \times 7 = 7$. Now for the tens digit we have $20 \times 7 = 40$; we seek a further 40 to make 80. Thus **b** = 4. So $a + b = 7$.

(If by inspection it was decided that *a*21 is 321, then you could have done the following calculation: $79\,287 \div 321 = 247$. If *a*21 is tried by inspection as 221 no answer is possible.)

13. ANSWER: A

The first generation back was a female; the second was a male and female; the third was a female, male and female, etc. The number of ancestors in each generation is 1, 2, 3, 5, 8, ... These are the famous Fibonacci numbers where every number in the pattern is the sum of the previous two. The tenth back is 89 (A) not counting the original male honeybee.

14. ANSWER: B

Let $D\hat{B}C = x$. Then $\hat{C} = x + 36°$ (AC = AB).

Also $B\hat{D}C = x + 36°$ (BC = BD).

In ΔBDC: $x + 36° + x + 36° + x = 180°$. So $x = 36°$ and $A\hat{B}C = 72°$.

PART C: (Each correct answer is worth 7 marks)

15. ANSWER: A

Area of circle = πr^2 and circumference = $2\pi r$ so we need only compare r^2 with $2r$. Since r is positive, r^2 is bigger than $2r$ only if r is bigger than 2. So (A) is true. Out of interest we examine the other options. (B) is false since area is smaller than circumference when r is smaller than 2. (C) is false since they are equal when $r = 2$. (D) is false since doubling the circumference means doubling the radius and consequently the area increases 4 times as the area is linked to r^2. (E) is clearly false in the same way as (B).

16. ANSWER: A

The outer faces of the big cube consist of a 3×3 square with a 1×1 square removed giving an area of $9 - 1 = 8$ m^2. Six faces have a total of 48 m^2 area. Each square hole is made up of four 1×1 squares. On the six faces the area is $6 \times 4 \times 1 = 24$ m^2. Total area = 72 m^2

17. ANSWER: C

Look at the situation from the woman's point of view. On this day she saved 10 minutes even though she came 1 hour early. That is she saved 5 minutes in each direction.

So she had been walking for 60 min – 5 min = 55 min.

18. ANSWER: E

If we work out the first few powers of 2 we see that they end in 2, 4, 8 and 6 in that order. Every power of 2 which is a multiple of 4 ends in 6. So 2^{1999} ends in a 8 (1999 is 1 before a multiple of 4 that is 2000). Similarly every power of 3 which is a multiple of 4 ends in 1 so 3^{2000} ends in 1. The sum of the two ends in 8 + 1 = 9.

19. ANSWER: D

We need only consider all 4-digit numbers (since 10 000 is out) and even include numbers written as say 0020 as 20. There are 3 positions for the 99:

(a.) 99*xy*, here there are 9 possibilities for *x* (leaving out 9) and 10 for *y* (0 to 9) a total of $10 \times 9 = 90$.

(b.) *x*99*y*, here there are 9 possibilities for *x* and 9 for *y* giving a total of $9 \times 9 = 81$.

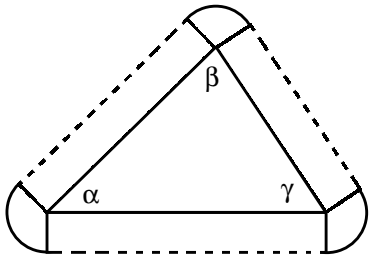
(c.) *xy*99, same as case (a.) 90 possibilities.

Total = 90 + 90 + 81 = 261 possibilities.

20. ANSWER: E

Solution 1: Since the ant returns to its start and faces the same direction as when it started, it must have turned a full circle of radius 1 along its tour. This circle has circumference 2π. Also it has covered the straight lines on the triangle's perimeter which is 18 cm. Total distance = $18 + 2\pi$ cm.

Solution 2: The path of the ant consists of the dotted lines which are just the same as the triangles sides (on the drawn rectangles) as well as the circular pieces at each corner at the triangle. These three pieces actually make a full circle: let the angles of the triangle be α, β and γ. The angles in the circular pieces are $180° - \alpha$, $180° - \beta$ and $180° - \gamma$. Their sum is $540° - (\alpha+\beta+\gamma) = 540° - 180° = 360°$ which is a full circle of circumference 2π.



THE END

ANSWER POSITIONS: JUNIOR SECOND ROUND 1999

PRACTICE EXAMPLES	POSITION
1	D
2	C
3	A

NUMBER	POSITION
1	D
2	B
3	C
4	D
5	E
6	B
7	B
8	C
9	A
10	C
11	A
12	C
13	A
14	B
15	A
16	A
17	C
18	E
19	D
20	E

DISTRIBUTION	
A	5
B	4
C	5
D	3
E	3
TOTAL	**20**