

THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION.
Sponsored by HARMONY GOLD MINING.

SECOND ROUND 2005 JUNIOR SECTION: GRADES 8 AND 9 10 MAY 2005 TIME: 120 MINUTES NUMBER OF QUESTIONS: 20

Instructions:

- 1. Do not open this booklet until told to do so by the invigilator.
- 2. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
- 3. Scoring rules:
 - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in part B and 6 marks in part C.
 - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
- 4. You must use an HB pencil. Rough paper, a ruler and a rubber are permitted.

Calculators and geometry instruments are not permitted.

- 5. Diagrams are not necessarily drawn to scale.
- 6. The centre page is an information and formula sheet. Please tear it out for your use.
- 7. Indicate your answers on the sheet provided.
- 8. Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.
- 9. Answers and solutions will be available in June 2005 at www.samf.ac.za

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO. DRAAI DIE BOEKIE OM VIR DIE AFRIKAANSE VRAESTEL

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PRACTICE EXAMPLES

1. 23 + 6 – 4 =

A) 6 B) 23 C) 25 D) 29 E) 33

2. $\frac{1}{5} + \frac{2}{3} \times \frac{1}{2}$ equals

A) $\frac{1}{15}$ B) $\frac{3}{11}$ C) $\frac{21}{50}$ D) $\frac{8}{15}$ E) $9\frac{4}{5}$

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PART A: four marks each

1	Calculate:	$1 - \frac{1}{2} \times \frac{1}{2}$
٠.	Calculate.	$\frac{1-7}{2}$

- A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) $\frac{1}{8}$ E) $\frac{3}{8}$
- 2. In the magic square below, the sum of the numbers in each of the rows, in each of the diagonals and in each of the columns is equal. Find x.

9	x	7
8	10	

- A) 11
- B) 12 C) 13 D) 14 E) 15

- 3. Terry received an initial discount of 20% on the marked price of a used car. The salesman, who was very sympathetic, then gave Terry a further discount of 10% on the already discounted price. According to Terry's calculations, this was the same as a single discount of:
 - A) 44%
- B) 40%
- C) 36%
- D) 32%
- E) 28%
- The sum of any two-digit number and the number formed by 4. interchanging the digits of that number is always a multiple of:
 - A) 7
- B) 9
- C) 11
- D) 13
- E) 15
- 5. The sum of two numbers is -8. The product of the same two numbers is 7.

The numbers are:

- (-1 and -7) A)
- B)
- (1 and 7) C) (-1 and 7)
- D) (1 and -7)
- E) (1 and -9)

PART B: five marks each

6. **Evaluate:**

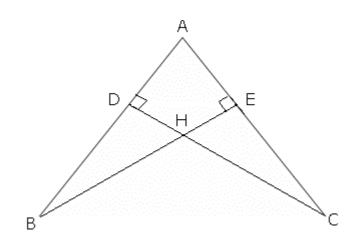
$$4,32^2 - 3,32^2 + 1,36$$

- A) 7
- B) 8
- C) 9 D) 10 E) 11

7. Calculate:

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{2005}\right)$$

- A) $\frac{1}{2004}$ B) $\frac{3}{2004}$ C) $\frac{1}{2005}$ D) $\frac{2}{2005}$ E) $\frac{3}{2005}$
- A rocket is travelling towards Mars at 60 000 km/h. If Mars is 8. 1.2×10^8 km away, how long will the rocket take to get there?
 - Just less than 1 month.
 - Just less than 2 months. B)
 - Just less than 3 months. C)
 - Just less than 4 months. D)
 - Just less than 5 months.
- In the figure ${}^{B\hat{H}\,C}$ is equal to: 9.



- A)
- $3 \, \hat{A}$
- $360^{\circ} \hat{A}$ B)
- **C)** $180^{0} \hat{A}$
- D)

- E)
- **Given** $-6 < x < 10, -2 < y < -\frac{1}{2}$ and $a < \frac{x}{y} < b$ 10.

Find: $a \times b$

- A) 0 B) -50 C) -60 D) -120 E)

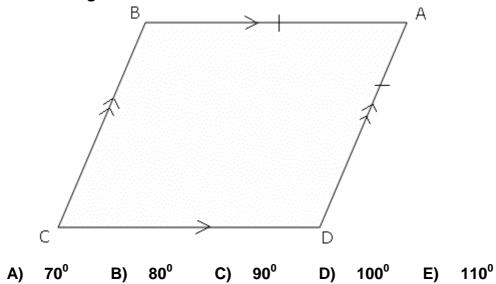
- 11. Veli looks at a calendar for the year 20xy. He notices that April 20xy has exactly four Mondays and four Fridays. 28 April 20xy would then fall on a:
 - A) Saturday
- B) Sunday
- C) Monday

- D) Friday
- E) Wednesday
- 12. The surface area of a cube is 6 cm². If 100 cubes are put together end to end as shown, what is the surface area of the resulting shape (in cm²)?

-- -- --

(100 cubes)

- A) 402
- B) 400
- C) 202
- D) 200
- E) 50
- 13. Consider a rhombus. If the sizes of any two angles that are not opposite to each other differ by 40°, what is the size of one of the smaller angles?



14. The average amount a waitress earns in tips is worked out by dividing the total amount earned over a certain period by the number of days worked. Marianne earned an average of R78 over a 20 day period.

What is the total amount she must earn in tips over the next five days if she wishes to achieve an average of R90 per day for the 25 day period?

- A) R600
- B) R690
- C) R780
- D) R510
- E) R90

A and B are the digits of a two-digit number. A > B; A and B differ by x. When this two-digit number is divided by the sum of its digits, the quotient is 7 and the remainder is x. What is the value of x?

- A) 1
- B)
- C) 5
- D) 2
- E) 4

PART C: six marks each

- A three-digit number has (2x-1) as its units digits and x as its 16. hundreds digit. This number is represented by 112x + 29. Find, in terms of x, the tens digit.
 - A) x+1
- B) x+2
- **C)** x + 3
- **D)** x + 4
- **E)** x + 5
- **17.** In the following multiplication, each * stands for a digit. These digits are not necessarily different.

The second three-digit number is:

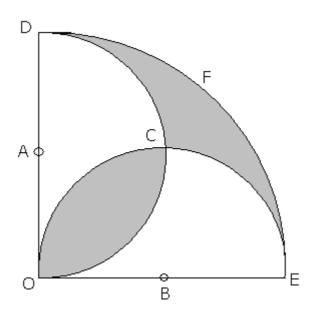
- A) 133
- B) 189
- C) 181
- D) 145
- E) 147
- Sipho purchased six identical Mandela gold coins from a nonreputable dealer. He discovered that there were three counterfeit (false) coins, which were lighter in mass than the real coins. Using only the coins that he has bought and a scale balance, what is the least number of weighings that Sipho must do, to identify at least one counterfeit coin?



- A) 1
- B) 2 C) 3 D) 4
- E) 5

- 19. Eric finds the sum of the digits for every possible eight-digit number. Which sum does he find occurs most often?
 - A) Both 27 and 28
- B) 41
- C) 32

- D) Both 36 and 37
- E) 39
- 20. A quadrant of a circle, with centre 0 and radius 4 cm is drawn.



Two semi-circles are drawn as shown. What is the area of the shaded region?

- A)
- 4π **B)**
- $4\pi 2$ **C)**
- **C)** $4\pi 4$
- **D)** $4\pi 6$

E) $4\pi - 8$

Formula and Information Sheet

- 1.1 The natural numbers are 1; 2; 3; 4; 5; ...
- 1.2 The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; ...
- The integers are \dots ; -4; -3; -2; -1; 0; 1; 2; 3; 4; 5; \dots 1.3
- 2. In the fraction a is called the numerator and the denominator.
- **Exponential notation:** 3.1

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

$$3\times3\times3\times3\times3\times3=3^6$$

$$a \times a \times a \times a \times \dots \times a = a^n$$
 (*n* factors of *a*)

(a is the base and n is the index (exponent))

3.2 **Factorial notation:**

$$1\times2\times3\times4=4!$$

$$1 \times 2 \times 3 \times \dots \times n = n!$$

- Area of a
- length \times width = Iw4.1 rectangle is: length \times breadth = lb

side \times side = s^2 4.2 square is:

- ----
- $\frac{1}{2}$ × (product of diagonals) rhombus is: 4.3
- $\frac{1}{2}$ × (sum of parallel sides) × height trapezium is: 4.4 -----
- 4.5 circle is: πr^2 (r = radius)

5.1 rectangular prism is:
$$2lb + 2lh + 2bh$$
 $(h = height)$

5.2 sphere is:
$$4\pi r^2$$

6 Perimeter of a:

6.1 rectangle is:
$$2 \times length + 2 \times breadth$$

$$2l + 2b$$

or
$$2l + 2w$$
 ($w = width$)

7. Circumference of a circle is:
$$2\pi r$$

8. Volume of a:

8.1 cube is:
$$s \times s \times s = s^3$$

8.2 rectangular prism is:
$$l \times b \times h$$

8.3 cylinder is:
$$\pi r^2 h$$

9.1 Volume of a right prism is: area of cross-section \times perpendicular height

or area of base \times perpendicular height Surface area of a right prism is: perimeter of base \times h + 2 \times area

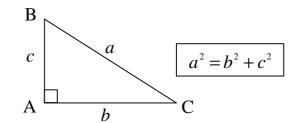
9.2 Surface area of a right prism is: perimeter of base \times h + 2 \times area of base

10. Sum of the interior angles of a polygon is:

$$180^{\circ}(n-2)$$
 ($n = \text{number of sides}$)

11. Distance is: speed x time
$$(d = s \times t)$$

12 Pythagoras:



 ΔABC is a right-angled triangle

13. Conversions:

$$1 \text{ cm}^3 = 1 \text{ m}\ell$$
 ; $1000 \text{ cm}^3 = 1 \ell$

$$1000 \text{ m} = 1 \text{ km}$$
 ; $1000 \text{ g} = 1 \text{ kg}$; $100 \text{ cm} = 1 \text{ m}$