SOUTH AFRICAN MATHEMATICS OLYMPIAD
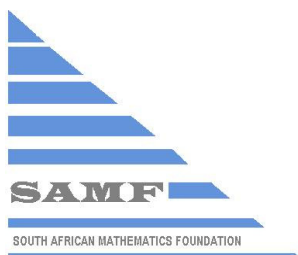
Harmony™

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SOUTH AFRICAN MATHEMATICS FOUNDATION

SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

2010 FIRST ROUND
JUNIOR SECTION: GRADES 8 AND 9

17 March 2010   **Time: 60 minutes**   **Number of questions: 20**

Instructions

1. *Do not open this booklet until told to do so by the invigilator.*
2. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
3. *Scoring rules:*
   3.1. *Each correct answer is worth 5 marks.*
   3.2. *There is no penalty for an incorrect answer or any unanswered question.*
4. *You must use an HB pencil. Rough paper, a ruler and an eraser are permitted.*
   Calculators and geometry instruments are not permitted.
5. *Diagrams are not necessarily drawn to scale.*
6. *Indicate your answers on the sheet provided.*
7. *The centre page is an information and formula sheet. Please tear it out for your use.*
8. *Start when the invigilator tells you to do so. You have 60 minutes to complete the question paper.*
9. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so
Draai die boekie om vir die Afrikaanse vraestel

PRACTICE EXAMPLES

1. $23 + 6 - 4 =$

   (A) 6   (B) 23   (C) 25   (D) 29   (E) 33

2. $\dfrac{1}{5} + \dfrac{2}{3} \times \dfrac{1}{2}$ equals

   (A) $\dfrac{1}{15}$ (B) $\dfrac{3}{11}$ (C) $\dfrac{21}{50}$ (D) $\dfrac{8}{15}$ (E) $9\dfrac{4}{5}$

1. $4 \times 5 + 3 \times 6$ is equal to

   (A) 18   (B) 38   (C) 80   (D) 192   (E) 360

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2. The units digit of the product $29 \times 37 \times 21 \times 55 \times 43 \times 39$ is

   (A) 1   (B) 2   (C) 3   (D) 5   (E) 7

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3. When $\dfrac{19}{11}$ is written as a recurring decimal, how many different digits appear?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

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4. The number of two-digit prime numbers that can be written using only digits from the list 2; 3; 5; 7 is

   (A) 1   (B) 2   (C) 4   (D) 5   (E) 6

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5. The smallest number which must be added to 2010 to arrive at a perfect square is

   (A) 10   (B) 15   (C) 20   (D) 25   (E) 30

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6. The sum of the digits of the product $5^{104} \times 4^{52}$ is

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

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7. How many different arrangements of the letters A C T O R are there if the O and the R must be next to each other?

   (A) 4   (B) 8   (C) 16   (D) 48   (E) 256

8. Did you know …

$$2! = 2 \times 1 = 2$$
$$3! = 3 \times 2 \times 1 = 6$$
$$4! = 4 \times 3 \times 2 \times 1 = 24 \quad ?$$

This means that the value of $\dfrac{11! - 9!}{11! + 9!}$ is

(A) $\dfrac{109}{111}$ (B) $\dfrac{56}{65}$ (C) $\dfrac{25}{26}$ (D) $\dfrac{138}{137}$ (E) $\dfrac{150}{151}$

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9. A jug was 60% full of water. After 20% of that water is removed the jug contains 192 ml.
The maximum amount of water (in ml) that the jug could contain is

(A) 275 (B) 300 (C) 325 (D) 350 (E) 400

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10. Five equal squares each with side 2 cm are used to make the figure alongside.
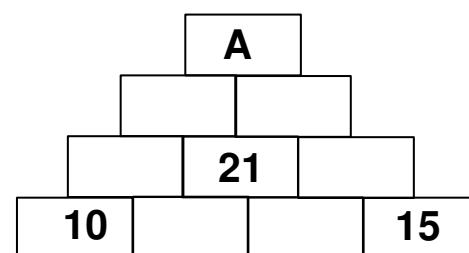M is the midpoint of PQ.

The area of the shaded region (in cm^2) is
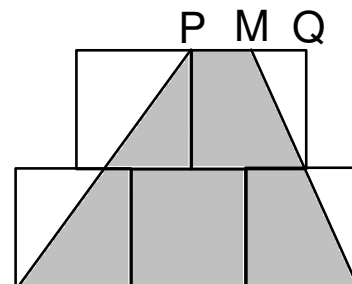
(A) 14 (B) 12 (C) 10 (D) 8 (E) 6

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11. In the given diagram, the number in any box is equal to the sum of the numbers in the two boxes immediately below it.
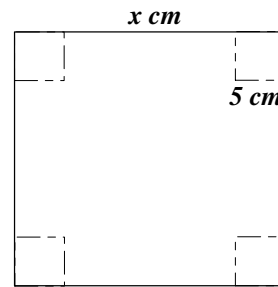
The value of A is

(A) 88 (B) 67 (C) 21 (D) 97 (E) 12

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12. If $\dfrac{a}{b} = 6 \; ; \; \dfrac{b}{c} = \dfrac{1}{4}$ and $a + c = 30$ then the value of b is

(A) 3 (B) 18 (C) 12 (D) 60 (E) 10

13. A square of side length 5 cm is removed from each corner of a square piece of cardboard of side length x cm. The sides are then turned up to make an open box. If the volume of the box is 605 cm^3, the length of a side (in cm) of the original piece of cardboard is
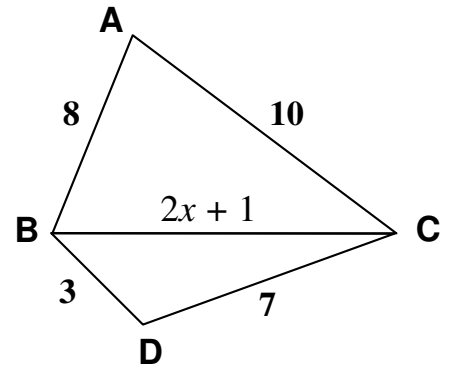
x cm

5 cm

(A) 5   (B) 605   (C) 16   (D) 10   (E) 21

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14. A car takes 2 hours to travel from Apetown to Beeville. If its speed is reduced by 30 km/h, it will take 3,2 hours.
The distance from Apetown to Beeville (in km) is

(A) 30   (B) 90   (C) 120   (D) 140   (E) 160

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15. In the figure, AB = 8 cm, AC = 10 cm, BD = 3 cm, CD = 7 cm and BC = $2x + 1$ cm.

If x is an integer, the sum of the possible values of x is

A   8   10   B $2x + 1$ C   3   7   D
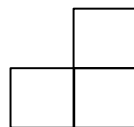
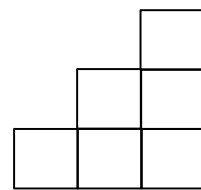(A) 10   (B) 9   (C) 8   (D) 7   (E) 6

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16. Identical matchsticks are used to make the different figures shown.
4 matchsticks make figure 1,
10 matchsticks make figure 2,
18 matchsticks make figure 3,
28 matchsticks would make figure 4, and so on.

(1)   (2)   (3)

The number of matchsticks required to make figure 20 is

(A) 420   (B) 440   (C) 460   (D) 480   (E) 500
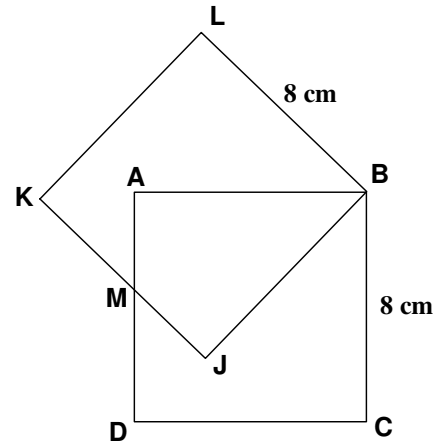
17. In a grade of 100 learners, 40% of the boys scored A symbols and 50% of the girls scored A symbols. Four more boys than girls scored A symbols.

The number of boys in the grade is

(A) 80   (B) 70   (C) 60   (D) 50   (E) 40

18. ABCD and BJKL are two identical squares with sides of length 8 cm. M is the midpoint of AD and also of JK.

The area of the hexagon BCDMKL (in cm^2) is

(A) 64   (B) 80   (C) 96   (D) 112   (E) 128

19. The sum of eleven consecutive even numbers is p.
The largest of the numbers, in terms of p, is

(A) $\dfrac{p}{5}+5$ (B) $\dfrac{p}{11}+5$ (C) $\dfrac{p}{5}+10$ (D) $\dfrac{p}{11}+10$ (E) $\dfrac{p}{6}+10$

20. Anne says Barbara is lying.
Barbara says Catherine is lying.
Catherine says Barbara is lying.
Diane says Anne is lying.

How many girls are lying?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

Formula and Information Sheet

1.1 The natural numbers are 1; 2; 3; 4; 5; …

1.2 The whole numbers are 0; 1; 2; 3; 4; 5; …

1.3 The integers are …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\dfrac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$$2\times2\times2\times2\times2 = 2^5$$
$$3\times3\times3\times3\times3\times3 = 3^6$$
$$a\times a\times a\times a\times \ …\ \times a = a^n \qquad (n \text{ factors of } a)$$
(a is the base and n is the index (exponent))

3.2 Factorial notation:

$$2! = 1\times2 = 2$$
$$3! = 1\times2\times3 = 6$$
$$4! = 1\times2\times3\times4 = 24$$
$$n! = 1\times2\times3\times \ …\ \times n$$

4 Area of a

4.1	triangle is:	$\dfrac{1}{2}\times (\text{base} \times \text{height}) = \dfrac{1}{2}(b.h)$
4.2	rectangle is:	length $\times$ width = lw length $\times$ breadth = lb
4.3	square is:	side $\times$ side = s^2
4.4	rhombus is:	$\dfrac{1}{2}\times (\text{product of diagonals})$
4.5	trapezium is:	$\dfrac{1}{2}\times(\text{sum of parallel sides})\times \text{height}$
4.6	circle is:	πr^2 (r = radius)

5 Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh$ (h = height)

5.2 sphere is: $4\pi r^2$

6 Perimeter of a:

6.1 rectangle is: $2\times$length $+ 2\times$breadth
$2l + 2b$
or $2l + 2w$ (w = width)

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s\times s\times s = s^3$

8.2 rectangular prism is: $l\times b\times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section $\times$ perpendicular height
or   area of base $\times$ perpendicular height

9.2 Surface area of a right prism is: (perimeter of base $\times$ h) + (2 $\times$ area of base)

10. Sum of the interior angles of a polygon is: $180°(n-2)$ [n = number of sides]
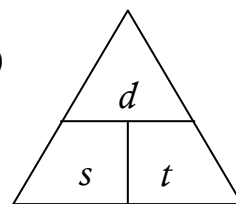
11. Distance $=$ speed $\times$ time   ($d = s \times t$)

Speed $=$ distance $\div$ time   ($s = \dfrac{d}{t}$)

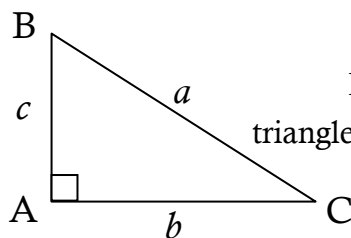Time $=$ distance $\div$ speed   ($t = \dfrac{d}{s}$)

$d = s \times t$

$s = \dfrac{d}{t}$

$t = \dfrac{d}{s}$

12 Pythagoras:

If $\triangle ABC$ is a right-angled triangle, then $a^2 = b^2 + c^2$

13. Conversions:
$1\text{ cm}^3 = 1\text{ m}\ell$; $1000\text{ cm}^3 = 1\ \ell$
$1000\text{ m} = 1\text{ km}$; $1000\text{ g} = 1\text{ kg}$; $100\text{ cm} = 1\text{ m}$