THE SOUTH AFRICAN

MATHEMATICS OLYMPIAD

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organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

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SECOND ROUND 1999

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

22 June 1999

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   3.1 Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 7 marks in Part C.
   3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.
7. When the invigilator gives the signal, start attempting the problems. You will have 120 minutes working time for the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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Part A: Three marks each.

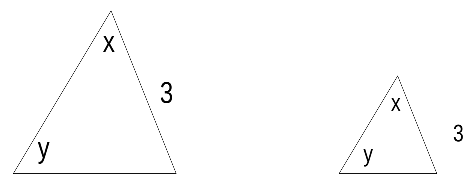
1. Three parties contest an election with 100 000 voters who all cast valid votes. The winning party is the one that obtains more votes than either of the other two parties. The smallest possible number of votes with which a party can win the election, is

(A) 33 333 (B) 33 334 (C) 50 000 (D) 50 001 (E) 66 667

2. $\frac{2^1+2^0+2^{-1}}{2^{-2}+2^{-3}+2^{-4}}$ is equal to

(A) 6 (B) 8 (C) $\frac{31}{2}$ (D) 24 (E) 512

3. One of the triangles in the figure is an enlargement of the other (i.e. the triangles are similar). If the corresponding sides have lengths as shown, the ratio of the area of the large triangle to the area of the small one is

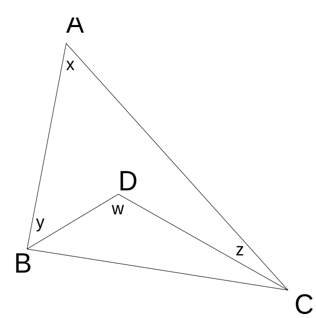


(A) $\sqrt{3}:1$ (B) $3\sqrt{3}:1$ (C) $3:1$ (D) $\frac{3}{2}\sqrt{3}:1$ (E) $\frac{1}{2}\sqrt{3}:1$

4. A given set of 20 numbers has an average of 20. Nine of these numbers have an average of 9. The average of the remaining 11 numbers is

(A) 20 (B) 31 (C) 29 (D) 10 (E) 11

5. D is an interior point of triangle ABC and x, y, z and w are the measures of the angles in degrees, as shown in the figure. An expression for x in terms of y, z and w is



(A) $w-y-z$ (B) $w-2y-2z$ (C) $2w-y-z$ (D) $180^o-w-y-z$ (E) $\frac{1}{2}w$

6. If $a-1=b+2=c-3=d+4$, then the largest of the four quantities a, b, c and d is

(A) a (B) b (C) c (D) d (E) cannot be determined

Part B: 5 marks each

7. The value of $100^2 - 99^2 + 98^2 - 97^2 + \cdots - 3^2 + 2^2 - 1^2$ is

(A) 5 050 (B) 4 950 (C) 5 000 (D) 25 000 (E) 10 100

8. If m and n are integers and $2m - n = 3$, then $m - 2n$ must be

(A) equal to -3 (B) equal to 0 (C) a multiple of 3 (D) an odd integer
(E) an even integer

9. The maximum value of x such that 2^x divides $21 \times 20 \times 19 \times \cdots \times 3 \times 2 \times 1$ is

(A) 10 (B) 20 (C) 18 (D) 12 (E) 16

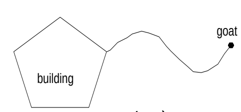
10. The number of pairs of integers $(m; n)$ which satisfy the the equation $m(m+1) = 2^n$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

11. $n!$ is defined to be the product $n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$. For example $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. The last digit of the sum $1! + 2! + 3! + 4! + \cdots + 1999!$ is

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

12. A goat is tied to a 10 metre long rope which is tied to the corner of a building which is in the shape of a regular pentagon (a 5 sided figure). If each side of the pentagon is 6m then the best approximation of the area, in m^2, that the goat can graze is



(A) 240 (B) 100 (C) 215 (D) 220 (E) 230

13. How many integers greater than 5 000 can be formed with the digits 7, 6, 5, 4 and 3, using each digit at most once?

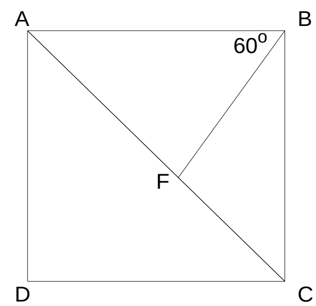
(A) 174 (B) 144 (C) 84 (D) 192 (E) 202

14. We define a relatively prime date to be a day for which the number of the month and the number of the day have no common factors other than 1. For example 22/5 (22 May) is such a day because 22 and 5 have no common factors other than 1. The month with the smallest number of relatively prime days is

(A) February (B) March (C) December (D) August (E) June

Part C: 7 marks each

15. The square $ABCD$ has sides of length 2. The area of triangle FBC is



(A) $2\sqrt{3}-3$ (B) $(\sqrt{3}+1)/4$ (C) $\sqrt{3}-1$ (D) $2\sqrt{2}-2$ (E) $\sqrt{3}/2$

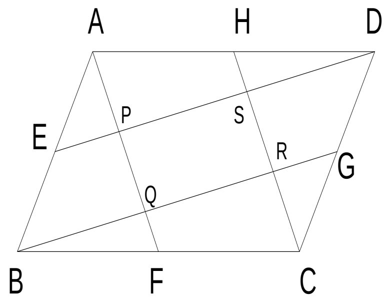
16. Mr Parker asks the 5 learners in his mathematics class how many of them attended the choir practice last night.

Petra replied:	none of us attended
Peter replied:	one of us attended
Paul replied:	two of us attended
Patricia replied:	three of us attended
Pumulla replied:	four of us attended

Mr Parker knows that the ones who did not attend lie about it and that the ones who did attend, tell the truth. How many of his mathematics learners attended the choir practice?

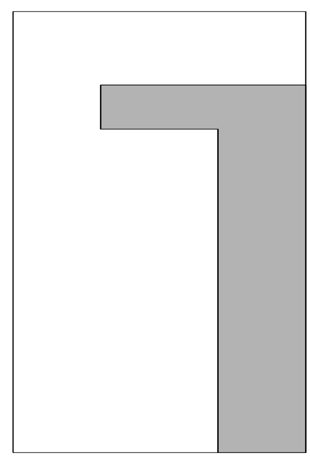
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

17. If $ABCD$ is a parallelogram and E, F, G and H are the midpoints of the sides, then the ratio of the area of ABCD to the area of PQRS is



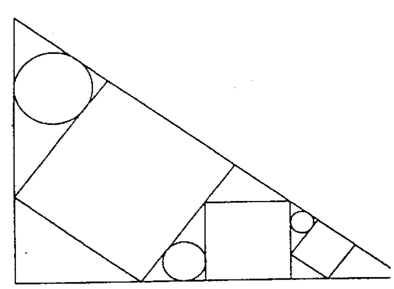
(A) $3:1$ (B) $3\frac{1}{2}:1$ (C) $4:1$ (D) $4\frac{1}{2}:1$ (E) $5:1$

18. A rectangular piece of paper is cut into two pieces by cutting along segments parallel to its sides as in the figure (not drawn to scale). The result is a 6 sided and an 8 sided piece of paper. The lengths of the sides of the 8 sided piece of paper are 1, 2, 3, 4, 5, 6, 7 and 8 units in some order. The maximum area, in square units, of the 6 sided (shaded) piece of paper is



(A) 24 (B) 27 (C) 30 (D) 33 (E) 36

19. The figure shows three squares and circles inscribed in a right-angled triangle. If the smallest and the largest circles have radii 19 and 99 respectively, what is the radius of the other circle?



(A) $\frac{1}{2}(19+99)$ (B) $\sqrt{19 \times 99}$ (C) $\sqrt{99^2 - 19^2}$ (D) $(\sqrt{19} - \sqrt{99})^2$ (E) $\frac{2}{\frac{1}{19}+\frac{1}{99}}$

20. A girl holds six pieces of string in her hand with the ends of the strings sticking out above and below her hand. Another girl ties the upper ends together in pairs, and then does the same for the lower ends. If she ties the pairs in a random manner, what is the probability that all six pieces of string will form a single ring?

(A) $\frac{8}{15}$ (B) $\frac{1}{15}$ (C) $\frac{7}{15}$ (D) $\frac{1}{32}$ (E) $\frac{1}{225}$