

SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2018 THIRD ROUND JUNIOR SECTION: GRADES 8 AND 9

26 July 2018

Time: 4 Hours

Number of questions: 15

TOTAL: 100

Instructions

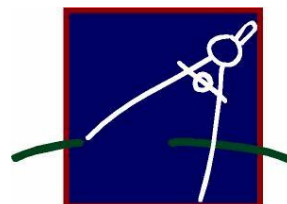
- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on only a few questions.
- Questions are not necessarily arranged in order of difficulty.
- Question 3 should be done on the given answer sheet.
- The answer sheet can also be used to assist in other questions (e.g. Q7, Q10, Q14, etc.).
- A die will be given to you to use in Question 10.
- Answers and solutions will be made available at: www.samf.ac.za

Do not turn the page until you are told to do so.

Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 FAX: (012) 392-9312
E-mail: info@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns



Question 1

Which one of the following five prime numbers is the average of the other four?

19 ; 31 ; 29 ; 17 ; 59

[3]

Question 2

What is the largest power of 2 that divides into 20^{18} without leaving a remainder?

[4]

Question 3

Futoshiki puzzles originated in Japan and the word futoshiki means inequality which refers to the fact that the value of some cells on the board are forced to be greater than an adjacent cell. Fill in the squares so that each digit from 1 to 4 occurs exactly once in each row and column. Greater-than and less-than signs indicate the relationship between the two adjacent squares. (Fill this in on the answer sheet.)

	^		v
	<	3	>
			v

[4]

Question 4

A positive integer is to be placed in each box below. Integers may be repeated, but the product of any four adjacent integers is always 120.

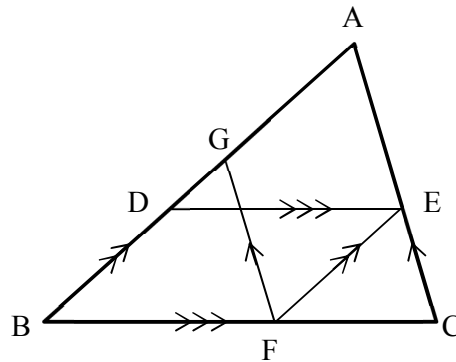
Determine all possible values for x .

		2			4			x			3		
--	--	---	--	--	---	--	--	-----	--	--	---	--	--

[5]

Question 5

In triangle ABC, let point D be on AB.
The line parallel to BC through D meets
AC at E. The line parallel to
AB through E meets BC at F. The
line parallel to AC through F meets AB at G.
Prove that $AD = BG$.



[5]

Question 6

Four closed treasure chests are placed on a table. One of them contains a gold nugget, another treasure chest contains sand, the third treasure chest contains pebbles and the fourth treasure chest contains gravel. Three of the four treasure chests are labelled. One treasure chest is labelled “Gold or Sand”, another treasure chest is labelled “Pebbles or Gravel” and another is labelled “Gold or Gravel”. The fourth treasure chest is unlabelled. Molly may choose one of the treasure chests and is aiming for the gold nugget. She has been told that all the labels are correct. Molly is allowed to look inside one of the treasure chests before making a final choice.

Determine whether it is possible for Molly to choose the treasure chest containing the gold nugget with certainty.



[6]

Question 7

In 1256 a story was recorded about the invention of chess. In the story the inventor asked the king for one grain of rice as payment for the first square on the chessboard and to double the number of grains of rice on each subsequent square. The king laughed it off as a meagre prize for a brilliant invention, only to have the court treasurers' report that the unexpected huge number would be more than the king's resources.

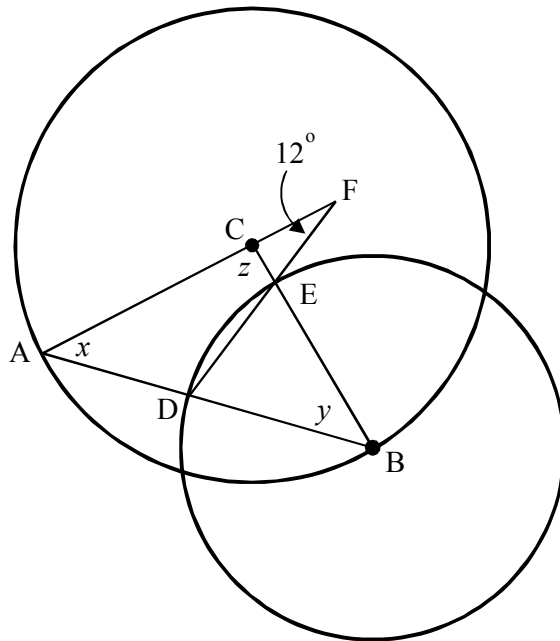
If the number of grains of rice on the last square is 9 223 372 036 854 775 808, what is the total number of grains of rice on the whole chessboard?

[6]

Question 8

- Let C be the centre of a circle. Points A and B are both points on the circle.
- Let B be the centre of a circle that intersects line AB at D and line BC at E.
- Lines DE and AC are extended to meet in F.

If $\angle AFD = 12^\circ$, find the sizes of the angles in $\triangle ABC$.



[7]

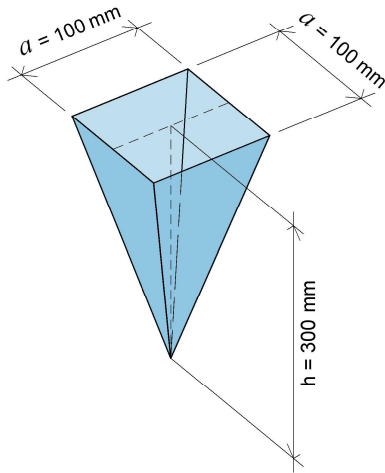
Question 9

A rain gauge in the shape of an inverted pyramid has dimensions:

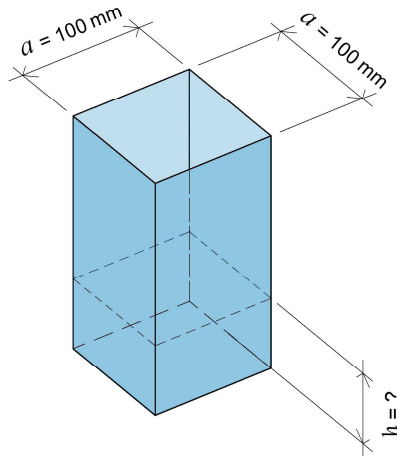
- Total height: 300 mm
- Top side-length: 100 mm

A square-based right prism has dimensions:

- Total height: 300 mm
- Base side-length: 100 mm



Rain gauge



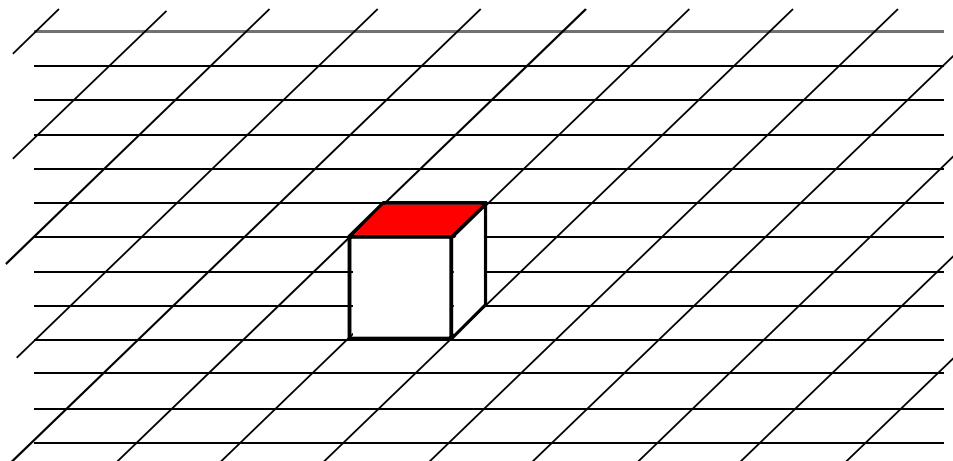
Square-based right prism

Hint: Volume of a pyramid = $\frac{\text{Area of base} \times \perp \text{ height}}{3}$

- If you empty the rain gauge (filled to the brim) into the empty square-based right prism, what height will the water in the prism reach?
- If the rain gauge is filled to a height of 150 mm, and emptied into the empty square-based right prism, what height will the water in the prism reach?

[8]

Question 10 (A die will be provided for you to use on the Answer Sheet)



On some square of an infinite grid of squares there is a cube which covers the square exactly. The top face is red and the other faces are all white. In one move, the cube can be turned over any edge so that it covers a neighbouring square.

Determine whether it is possible to move the cube such that it can be returned to the initial square with the red face at the **bottom** after exactly:

- a) 2018 moves
- b) 2019 moves

Prove your answers.

[8]

Question 11

In an auditorium each row (from row two onwards) has one more seat than the row in front of it. In an exam, students may sit in any row but must leave at least one seat open between themselves and the next student in that row.

If there are p seats in the first row and 20 rows in the auditorium, find the maximum number of students that can be seated in terms of p .

[8]

Question 12

Solve the ALPHAMETIC: (Remember: Each different letter represents a different digit.)

$$\begin{array}{r}
 \text{D A Y S} \\
 + \quad \text{T O O} \\
 \hline
 = \text{S H O R T}
 \end{array}$$

[8]

Question 13

We define the digit product of a number as the product of its digits. E.g. the digit product of 327 is $3 \times 2 \times 7 = 42$, the digit product of 4210 is $4 \times 2 \times 1 \times 0 = 0$.

A digit product chain is formed by successively calculating the digit product of each digit product in the chain. E.g. 327 goes to 42, which goes to 8. We end as soon as a single digit number is reached.

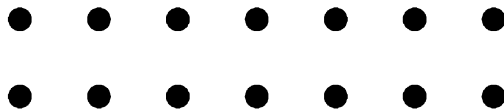
- a) Write down the digit product chain of 77.
- b) Prove that for any positive integer its digit product chain must always end.

[8]

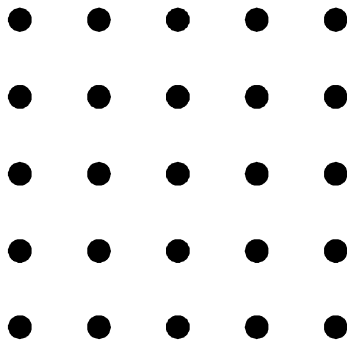
Question 14

Alice and Bongile play a game on a grid of equally spaced dots. Alice starts the game with the first move. They take turns to draw either a vertical or a horizontal line between two adjacent points (diagonals are not allowed). The first player to close a 1 by 1 square loses.

- a) If they play on the 2 by 7 grid below, which player has a winning strategy? Describe the strategy.



- b) If instead they play on the 5 by 5 grid below, which player has a winning strategy? Describe the strategy.



- c) If they play on a 2018 by 2019 grid, which player has a winning strategy? Describe the strategy.

[10]

Question 15

A computer outputs the values of the expression $(n+1) \times 2^n$ for $n = 1$, $n = 2$, $n = 3$, etc. What is the largest number of consecutive values that are perfect squares?

Prove your answer.

[10]

Total: 100

THE END

Turn over for the Answer Sheet

ANSWER SHEET / ANTWOORDBLAD

This answer sheet needs to be handed in / Hierdie antwoordblad moet ingehandig word

Name / Naam:

School / Skool:

Question 3 /Vraag 3 (Please indicate which one is your final answer to be marked) /

(Dui asb. jou finale antwoord aan)

Use for any question you need it! / Gebruik vir enige vraag waarvoor jy dit nodig het!

A 10x10 grid with a 5x5 subgrid highlighted in black. The top row of the grid contains labels 1, 2, 4, 8, 16, 32, 64, and empty cells for 128 and 256. The 5x5 subgrid is located in the bottom-left corner, starting from the 6th column and 6th row.

ANSWER SHEET / ANTWOORDBLAD

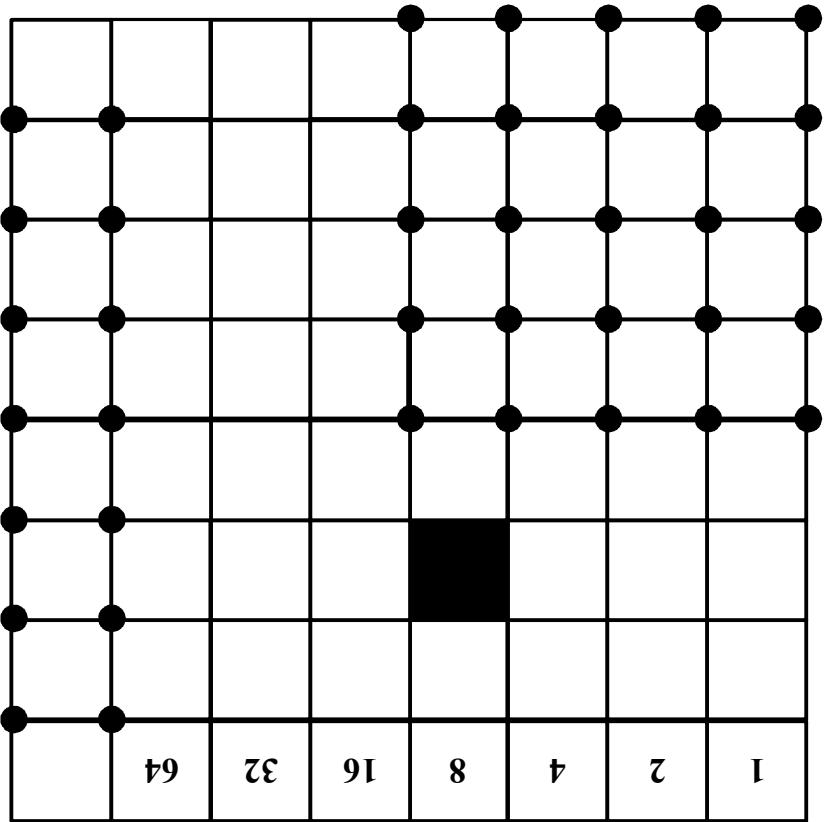
This answer sheet needs to be handed in / Hierdie antwoorblad moet ingehandig word

<p>Name / Naam:</p> <hr style="border: 0; border-top: 1px solid black; margin-top: 10px;"/>	<p>School / Skool:</p> <hr style="border: 0; border-top: 1px solid black; margin-top: 10px;"/>
--	---

Question 3 /Vraag 3 (Please indicate which one is your final answer to be marked) /

(Dui asb. jou finale antwoord aan)

Use for any question you need it! / Gebruik vir enige vraag waarvoor jy dit nodig het!



Blaai om vir die Antwoordblad

Vraag 15

'n Rekenaar bereken die waardes van die uitdrukking $(n+1) \times 2^n$ vir $n = 1$, $n = 2$, $n = 3$, ens. Wat is die grootste aantal opeenvolgende waardes wat perfekte vierkante is?

Bewys jou antwoord.

[10]

Totaal: 100

DIE EINDE

Vraag 13

Definieer die syferproduk van 'n getal as die produk van sy syfers. Bv. die syferproduk van 327 is $3 \times 2 \times 7 = 42$ en 4210 se syferproduk is $4 \times 2 \times 1 \times 0 = 0$.

'n Syferproduk ketting word gevorm deur opeenvolgend die syferproduk van elke syferproduk in die ketting te bereken. Bv. 327 word 42, wat dan 8 word. Ons eindig sodra 'n enkelsyfer getal bereik word.

a) Skryf die syferproduk ketting van 77 neer.

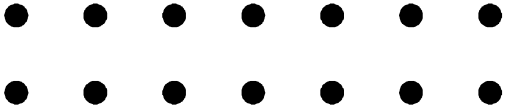
b) Bewys dat vir enige positiewe heelgetal sy syferproduk ketting altyd moet eindig.

[8]

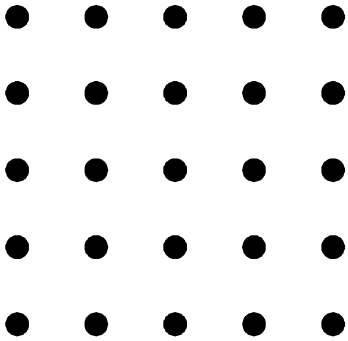
Vraag 14

Alice en Bongile speel 'n spel op 'n rooster van eweredig gespasieerde kolletjies. Alice begin die spel met die eerste beurt. Hulle maak beurt om of 'n vertikale of 'n horisontale lyn tussen twee aangrensende punte te trek (diagonale is nie toelaatbaar nie). Die eerste speler om 'n 1 by 1 vierkant te voltooi, verloor.

a) As hulle op die onderstaande 2 by 7 rooster speel, watter speler het 'n wenstrategie? Beskryf die strategie.



b) As hulle in plaas daarvan op die onderstaande 5 by 5 rooster speel, watter speler het 'n wenstrategie? Beskryf die strategie.

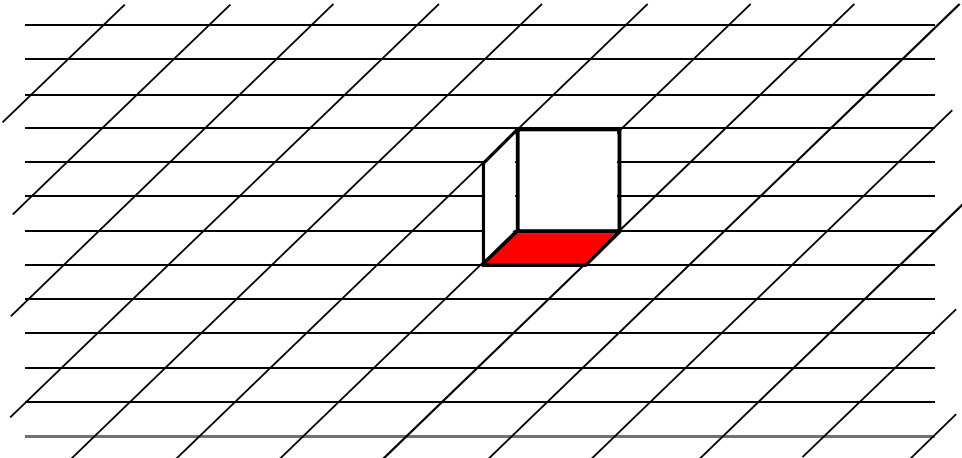


c) As hulle op 'n 2018 by 2019 rooster speel, watter speler het 'n wenstrategie?

Beskryf die strategie.

[10]

Vraag 10 ('n Dobbelsteentjie sal vir jou gegee word om op die Antwoordblad te gebruik)



Op 'n sekere vierkant van 'n oneindige rooster van vierkante is daar 'n kubus wat die vierkant presies bedek. Die boonste kant is rooi en die ander kante is almal wit. In enige skuif kan die kubus na enige kant toe gedraai word sodat dit 'n naburige vierkant bedek.

Bepaal of dit moontlik is om die kubus te skuif sodat dit kan terugkeer na die oorspronklike vierkant met die rooi kant na **onder** na presies:

- a) 2018 skuife
- b) 2019 skuife

Bewys jou antwoord.

[8]

Vraag 11

In 'n ouditorium het elke ry (van tweede ry af) een sitplek meer as die vorige ry. In 'n eksamen mag studente in enige ry sit, maar moet ten minste een sitplek oop laat tussen hui self en die volgende student in daardie ry.

As daar p sitplekke in die eerste ry en 20 rye in die ouditorium is, bepaal die maksimum aantal studente wat geplaas kan word, in terme van p .

[8]

Vraag 12

Los die ALPHAMETIC op: (Onthou: Elke verskillende letter stel 'n verskillende syfer voor.)

$$\begin{array}{r} \text{D A V S} \\ + \quad \text{T O O} \\ \hline = \text{S H O R T} \end{array}$$

[8]

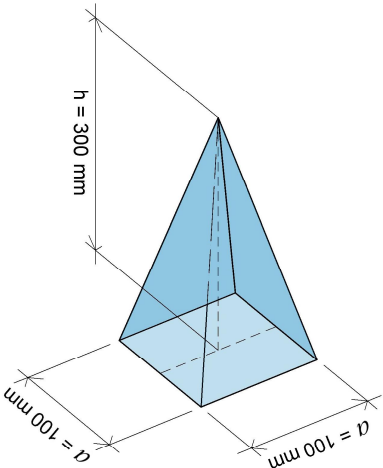
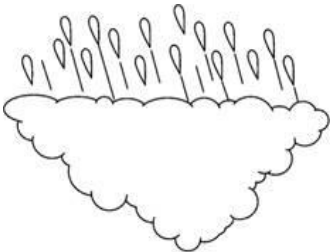
Vraag 9

'n Reënmeter in die vorm van 'n omgekeerde piramide het afmetings:

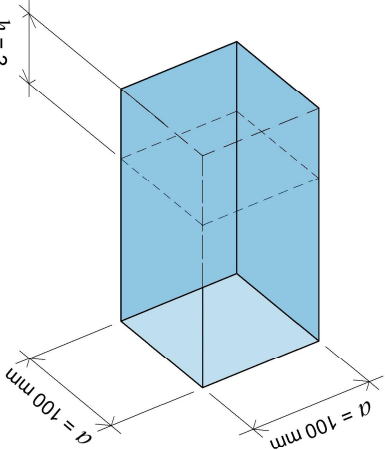
- Totale hoogte: 300 mm
- Boonste sylengte: 100 mm

'n Reghoekige prisma met vierkantige basis het afmetings:

- Totale hoogte: 300 mm
- Basis sylengte: 100 mm



Reënmeter



Reghoekige prisma met vierkantige basis

$$\text{Wenk: Volume van 'n piramide} = \frac{\text{Oppervlakte van basis} \times \perp \text{ hoogte}}{3}$$

- As jy die reënmeter (propvol) in die leë reghoekige prisma met vierkantige basis leegmaak, watter hoogte sal die water in die prisma bereik?
- As die reënmeter tot 'n hoogte van 150 mm gevul word en dan in die leë reghoekige prisma met vierkantige basis leeggemaak word, watter hoogte sal die water in die prisma bereik?

Vraag 7

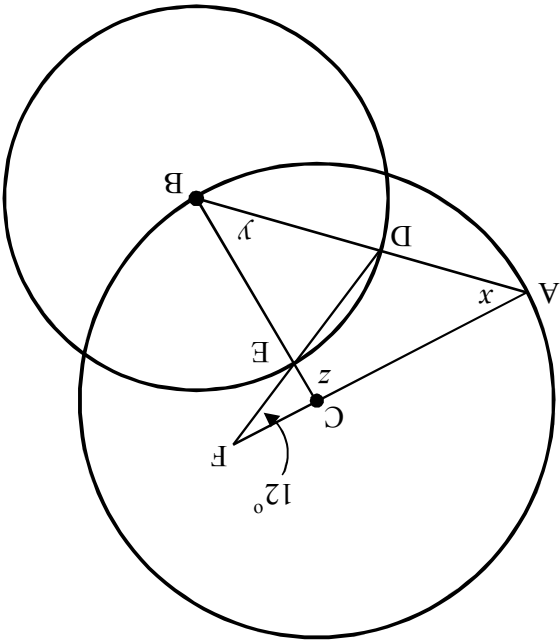
'n Storie is in 1256 opgeteken oor die ontstaan van skaak. In die storie vra die uitvinder vir die koning een korrel rys vir die eerste blokkie op die skaakbord as betaling en om die aantal korrels rys te verdubbel vir elke daaropvolgende blokkie. Die koning het dit afgelaag as 'n skamele prys vir 'n briljante uitvinding, slegs om uit te vind dat die hof se tesouriersverslag lui dat die onverwagte groot getal meer sou wees as die koning se bronne.

As die aantal korrels rys op die laaste blokkie 9 223 372 036 854 775 808 is, wat is die totale aantal korrels rys op die hele skaakbord?

[6]

Vraag 8

- Laat C die middelpunt van 'n sirkel wees. Punte A en B is beide punte op die omtrek van die sirkel.
 - Laat B die middelpunt wees van 'n sirkel wat lyn AB by D sny en lyn BC by E sny.
 - Lyn DE en AC word verleng om in F te ontmoet.
- As $\angle AFD = 12^\circ$, bepaal die groottes van die hoeke in $\triangle ABC$.

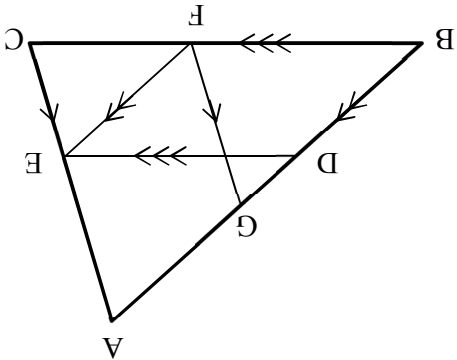


[7]

Vraag 5

In driehoek ABC, laat punt D op AB wees. Die lyn deur D ewewydig aan BC ontmoet AC by E. Die lyn deur E ewewydig aan AB ontmoet BC by F. Die lyn deur F ewewydig aan AC ontmoet AB by G.

Bewys dat $AD = BG$.

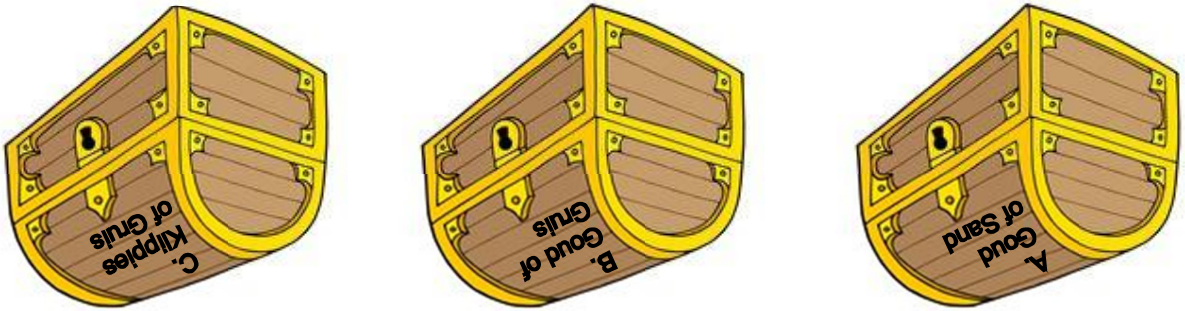


[5]

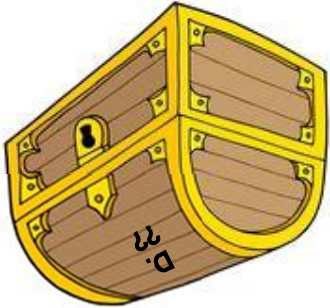
Vraag 6

Vier geslote skatkiste word op 'n tafel geplaas. Een van hulle bevat 'n klont goud, 'n ander skatkis bevat sand, die derde skatkis bevat klippies en die vierde skatkis bevat gruis. Drie van die vier skatkiste is gemerk. Een skatkis is gemerk "Goud of Sand", 'n ander skatkis is gemerk "Klippies of Gruis" en 'n ander een is gemerk "Goud of Gruis". Die vierde skatkis is ongemerk. Molly mag een van die skatkiste kies en haar doelwit is om die klont goud te kry. Daar is aan haar gesê dat al die etikette korrek is. Molly word toegelaat om in een van die skatkiste te kyk, voordat sy haar finale besluit neem.

Bepaal of dit vir Molly moontlik is om met sekerheid die skatkis met die klont goud te kies.



2



[6]

Vraag 1

Watter een van die volgende vyf priemgetalle is die gemiddeld van die ander vier?

19 ; 31 ; 29 ; 17 ; 59

[3]

Vraag 2

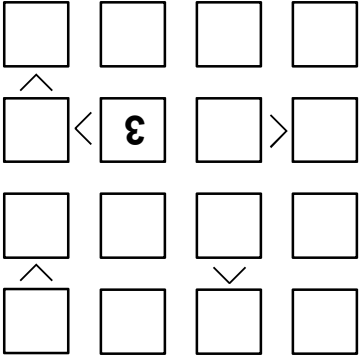
Wat is die grootste mag van 2 wat in 20^{18} deel sonder om 'n res te laat?

[4]

Vraag 3

Futoshiki raaisels het hul oorsprong in Japan en die woord futoshiki beteken ongelikheid wat verwys na die feit dat die waarde van sommige blokkies op die bord geforseer word om groter te wees as 'n aangrensende blokkie. Vul die blokkies in sodat elke syfer van 1 tot 4 presies een keer voorkom in elke ry en kolom. Groter-as en kleiner-as tekens dui die verhouding aan tussen die twee aangrensende blokkies.

(Vul hierdie op die antwoordblad in.)



[4]

Vraag 4

'n Positiewe heelgetal moet in elke blokkie hieronder geplaas word. Heelgetalle mag herhaal word, maar die produk van enige vier aangrensende heelgetalle is altyd 120.

Bepaal alle moontlike waardes vir x .

		2				4			x			3		
--	--	---	--	--	--	---	--	--	-----	--	--	---	--	--

[5]

SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

SOUTH AFRICAN MATHEMATICS FOUNDATION

26 Julie 2018

Tyd: 4 Ure

Aantal vrae: 15

2018 DERDE RONDE JUNIOR AFDELING: GRAAD 8 EN 9

TOTAAL: 100

Instrukties

- Beantwoord al die vrae.
- Alle berekeninge en motiverings moet getoon word. Antwoorde sonder motivering sal nie volpunte verdien nie.
- Die nethheid van jou oplossings mag in ag geneem word.
- Diagramme is nie noodwendig volgens skaal geteken nie.
- Geen sakrekenaar, in welke vorm ook al, of enige meetkundige instrumente, mag gebruik word nie.
- Gebruik jou tyd oordeelkundig en moenie al jou tyd op slegs 'n paar vrae spandeer nie.
- Vrae is nie noodwendig in volgorde van maklik na moeilik gerangskik nie.
- Vraag 3 moet op die antwoordblad aan die einde van die vraestel beantwoord word.
- Die antwoordblad kan ook gebruik word om te help met ander vrae (bv. Vr.7, Vr.10, Vr.14, ens.).
- 'n Dobbelsteentjies sal vir jou gegee word om by Vraag 10 te gebruik.
- Die antwoorde en oplossings sal beskikbaar wees by: www.samf.ac.za

Moenie omblaai voordat daar vir jou gesê word om dit te doen nie.
Turn the booklet over for the English paper.

PRIVAATSAK X173, PRETORIA, 0001
TEL: (012) 392-9372 FAKS: (012) 392-9312
E-pos: info@samf.ac.za

Organisasies betrokke: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns



LIBERTY

