# THE HARMONY GOLD SOUTH AFRICAN MATHEMATICS OLYMPIAD

## SECOND ROUND 2002: JUNIOR SECTION: GRADES 8 AND 9

## SOLUTIONS AND MODEL ANSWERS

**PART A:** (Each correct answer is worth 4 marks)

**1.** The value of $1001 \times 99 - 99$ is

A) 99 B) 990 C) 9 900 D) 99 000 E) 990 000

ANSWER: D

$$\begin{aligned}&1001 \times 99 - 99\\ &= (1000+1) \times 99 - 99\\ &= 99\,000 + 99 - 99\\ &= 99\,000\end{aligned}$$

or

$$\begin{aligned}&1001 \times 99 - 99\\ &= 1001 \times 100 - 1001 - 99\\ &= 100100 - 1001 - 99\\ &= 99\,000\end{aligned}$$

or

(*long way*)

$$\begin{aligned}&(1001 \times 99) - 99\\ &= 99\,099 - 99\\ &= 99\,000\end{aligned}$$

**2.** The value of $\frac{3}{4} - \frac{1}{2} \times \frac{1}{2}$ is

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) 0 D) $\frac{3}{4}$ E) $\frac{1}{8}$

ANSWER: A

$$\begin{aligned}&\frac{3}{4} - \frac{1}{2} \times \frac{1}{2}\\ &= \frac{3}{4} - \frac{1}{4}\\ &= \frac{2}{4} \qquad = \frac{1}{2}\end{aligned}$$

**3.** A truckload of books contains $x$ cartons. Each carton contains $y$ boxes and each box contains $z$ books. The number of books in the truckload is

A) $x+y+z$ B) $x(y+z)$ C) $xy+xz+yz$ D) $\frac{xy}{z}$ E) $xyz$

ANSWER: E

$x$ cartons contain $y$ boxes

$\therefore$ there are $xy$ boxes

$y$ boxes contain $z$ books

$\therefore$ Total number of books in truckload is $xyz$

**4.** The smallest integer $x$, for which $\frac{15}{x-1}$ is an integer, is

A) $-29$ B) $-14$ C) $-4$ D) $0$ E) $2$

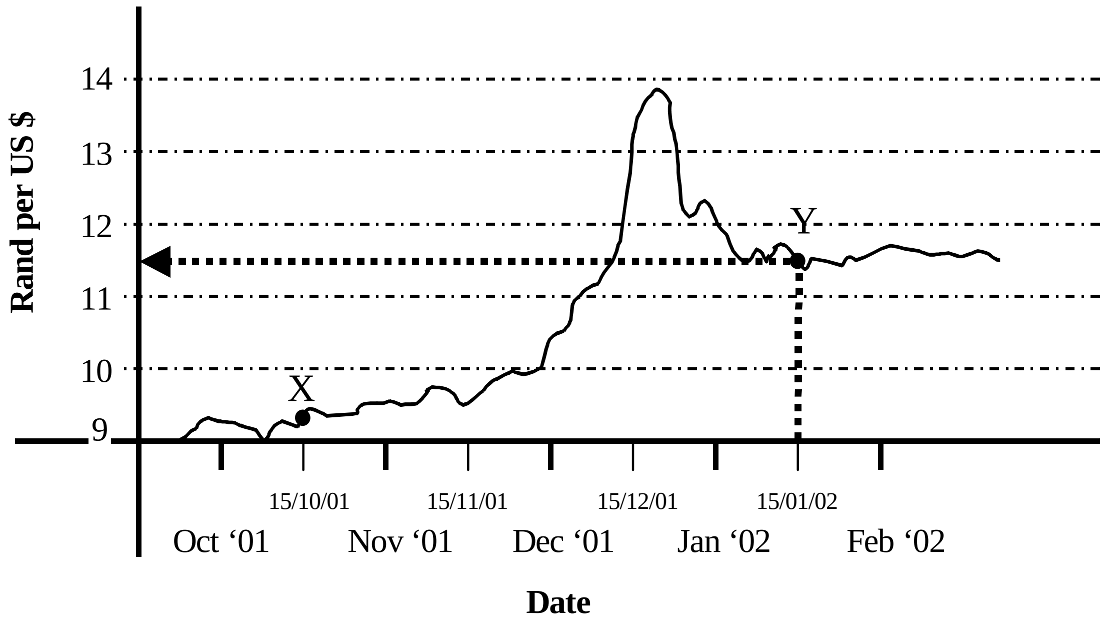
ANSWER: B

For the fraction $\frac{15}{x-1}$ to be an integer, 15 must be divisible by $x - 1$. Thus $x - 1$ have to be smaller than 15 and a factor of 15. Possible values for $x - 1$ are 15; 5; 3; 1; –1; –3; –5 and –15, which gives $x = -14$ to be the smallest integer for which $\frac{15}{x-1}$ is an integer.

**Qr** Test all given distracters to see which one fits (Beware, this method takes time)

| | | | |
|---|---|---|---|
| $x=-29$: | $\frac{15}{-29-1}=\frac{15}{-30}=\frac{1}{-2}$ | not an integer | |
| $x=-14$: | $\frac{15}{-14-1}=\frac{15}{-15}=-1$ | an integer | (smallest $x$) |
| $x=-4$: | $\frac{15}{-4-1}=\frac{15}{-5}=-3$ | an integer | |
| $x=0$: | $\frac{15}{0-1}=\frac{15}{-1}=-15$ | an integer | |
| $x=2$: | $\frac{15}{2-1}=\frac{15}{1}=15$ | an integer | |

**5.**




The above graph represents the value of one United States Dollar (US $) in South African Rand. At point X, on 15 October 2001 one US $ cost approximately R9,31. Use the graph to determine the approximate Rand value of one US $ on 15 January 2002.

A) less than R10 B) between R10 and R11

C) between R11 and R12 D) between R12 and R13

E) more than R13

ANSWER: C

Start on 15/01/02 on the *x*-axis. Make a projection from point Y (see graph above) to the *y*-axis. The Rand value of one US $ was between R11 and R12.

## PART B: (Each correct answer is worth 5 marks)

**6.** Of the 28 T–shirts in a drawer, six are red, five are blue, and the rest are white. If Bob selects T–shirts at random whilst packing for a holiday, what is the smallest number of T–shirts he must remove from the drawer to be sure that he has three T–shirts of the same colour?

A) 4 B) 13 C) 9 D) 19 E) 7

ANSWER: E

Lets consider the worse case scenario, i.e. every time you take out a T–shirts, you take out a different colour. Then, if you take out:

3, you have 3 different colours, e.g. 1 White, 1 Red and 1 Blue.
4, you have 3 different colours, and one double e.g. 2 W, 1 R and 1 B.
5, you have 3 different colours, and two doubles e.g. 2 W, 2 R and 1 B.
6, you have 3 different colours, and three doubles e.g. 2 W, 2 R and 2 B.
7, you have 3 different colours, one triplet and two doubles
e.g. 3 W, 2 R and 2 B.

Therefore, you need to select **seven** T–shirts to have three of the same colour.

**7.** Virginia is making 5-digit arrangements from the digits 2 and 4. An example of such an arrangements is 22442. She was told that the first digit cannot be a 4. How many such arrangements are there?

A) 16 B) 24 C) 20 D) 8 E) 32

ANSWER: A

You can either write all possibilities out (but it will take time), or you know that to arrange 4 digits, there are $2^4$, which is 16 possibilities.

Written out, you get the following 16 possibilities:

| | | | | |
|---|---|---|---|---|
| 22222 | 22224 | 22244 | 22444 | 24444 |
| | 22242 | 22424 | 24244 | |
| | 22422 | 24224 | 24424 | |
| | 24222 | 22442 | 24442 | |
| | | 24242 | | |
| | | 24422 | | |

**8.** Two friends Petros and Sammy have pocket money in the ratio 3 : 5. Each one spends R30. The ratio changes to 1 : 2. The total amount of pocket money the two friends started off with is

A) R210  B) R240  C) R270  D) R300  E) R330

<u>ANSWER:</u> B

For every R3 Petros has, Sammy has R5. After spending R60, Petros has R1 for every R2 Sammy has.

Let's say they started with R*x*.

Petros: $\frac{3}{8}x$  and Sammy: $\frac{5}{8}x$

$\therefore$ Petros: $\frac{3}{8}x = \frac{1}{3}(x-60)+30$

$\therefore \frac{3}{8}x - \frac{1}{3}x = -20+30 \qquad \therefore \frac{1}{24}x = 10$

so that $x$ = R240,00, the amount they have started with.

Test: 5(R30) = R150 and 3(R30) = R90

$\frac{90}{150} = \frac{3}{5}$ and $\frac{90-30}{150-30} = \frac{60}{120} = \frac{1}{2}$

Or

Lets say Petros started with 3*x* and Sammy started with 5*x* rands.

$\therefore \frac{3x}{5x} = \frac{3}{5}$, the ratio they have started with.

They both spend R30, therefore:

$$\frac{3x-30}{5x-30} = \frac{1}{2}$$

$$\therefore 5x-30 = 6x-60$$

$$\therefore x = 30$$

Petros started with R3*x* = R3(30) = R90

Sammy started with R5*x* = R5(30) = R150

In total they have started with R240.00 pocket money.

Test: $\frac{90}{150} = \frac{3}{5}$ and $\frac{90-30}{150-30} = \frac{60}{120} = \frac{1}{2}$

**9.** There are ten learners in an environmental club. They have decided to go on shell collection trips. The vehicle with which they undertake the trips, can only take eight learners at a time. Each learner goes at least once. What is the minimum number of trips the vehicle must make so that each learner goes on the same number of trips?

A) 2 B) 3 C) 4 D) 5 E) 6

ANSWER: D

Name the learners: A; B; C; D; E; F; G; H; I; J

| | | | | |
|---|---|---|---|---|
| **Trip 1:** | **Go:** | A; B; C; D; E; F; G; H | **Stay:** | I; J |
| **Trip 2:** | **Go:** | A; B; C; D; E; F; I; J | **Stay:** | G; H |
| **Trip 3:** | **Go:** | A; B; C; D; I; J; G; H | **Stay:** | E; F |
| **Trip 4:** | **Go:** | A; B; I; J; E; F; G; H | **Stay:** | C; D |
| **Trip 5:** | **Go:** | I; J; C; D; E; F; G; H | **Stay:** | A; B |

All the learners were away for four times and stayed for one trip.

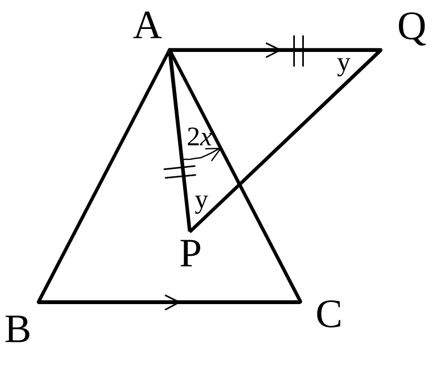
Or

8 of the 10 learners on each trip.

$LCM_{8;\,10} = 40$

Thus the minimum number of trips is $\frac{40}{8} = 5$

**10.**




In the above diagram, $\Delta ABC$ is equilateral. $\Delta APQ$ is isosceles with AQ = AP and AQ || BC. If $P\hat{A}C = 2x$, then the size of $\hat{Q}$ in terms of $x$ is

A) $60^\circ + x$ B) $180^\circ + 3x$ C) $120^\circ - x$

D) $2x$ E) $60^\circ - x$

ANSWER: E

Let $\hat{Q} = y$, then $\hat{P} = y$, because $\Delta APQ$ is isosceles

$Q\hat{A}C = A\hat{C}B$ Alternate interior angles $(AQ \| BC)$

but $A\hat{C}B = 60^\circ$ $\Delta ABC$ is equilateral

$\therefore Q\hat{A}C = 60^\circ$

$\therefore 180^\circ = 2y + 2x + 60^\circ$

$\therefore 2y = 120^\circ - 2x$

$\therefore y = 60^\circ - x$

**11.** The units digit of $2^{2000} + 2^{2001} + 2^{2002}$ is

A) 0 B) 2 C) 4 D) 6 E) 8

ANSWER: B

$2^{2000} + 2^{2001} + 2^{2002}$

$= 2^{2000}(1 + 2 + 2^2)$

$= 2^{2000}.7$

The number $2^x$, $x \in N$, has only the following possibilities for its unit digit:

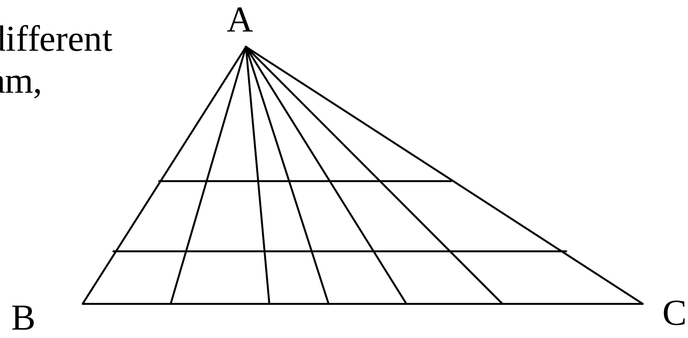
$(2^1 = 2;$ $2^2 = 4;$ $2^3 = 8;$ $2^4 = 16)$

i.e. 2; 4; 8; and 6

Since $2^{2000} = 2^{4n}$ for some $n$, this number has 6 as its unit digit.

Multiplying $2^{2000}$ with 7, yields $(6 \times 7 = 42)$, hence 2 as its unit digit.

**12.** The total number of different triangles in the diagram, including $\Delta ABC$, is



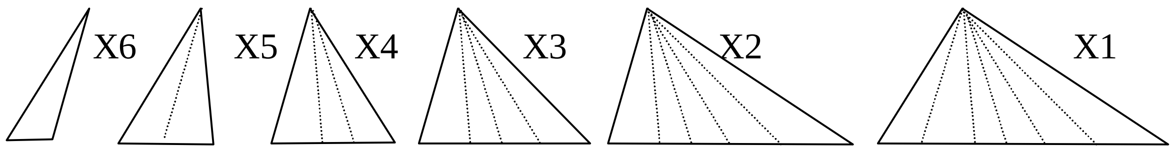

A) 21 B) 42 C) 63 D) 84 E) 105

<u>ANSWER:</u> C

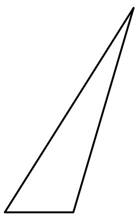
Count all the different triangles. Work with a plan, say from the smallest to the biggest:

e.g. (only some of the triangle sizes)

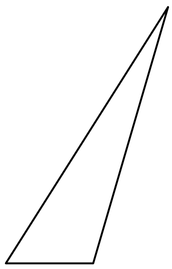
Top row triangles:



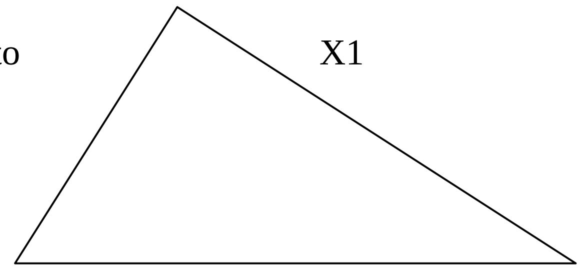

Second row triangles:



X6 … etc.



X6 … etc. up to



X1

If all the different triangles have been identified and calculated, you get 63 different triangles.

**13.** A 3-digit number has $x$ as its units digit; $(x-1)$ as the tens digit and $\frac{1}{2}x$ as its hundreds digit. The number in terms of $x$ is

A) $5x-1$ B) $7x-1$ C) $111x-1$

D) $61x-10$ E) $\frac{5}{2}x-1$

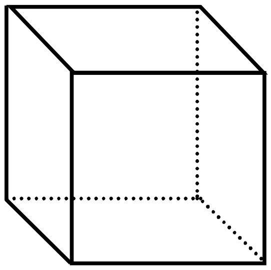
<u>ANSWER:</u> D

3 – digit number: $x+10(x-1)+100.\frac{1}{2}x$

$$=x+10x-10+50x$$

$$=61x-10$$

**14.** A company makes solid blocks with square bases. The volume of this block is 640 $cm^3$. Its height is 10 cm. The cost of painting all the faces of this block at 15c per $cm^2$ is




A) R64,00 B) R67,20 C) R70,40 D) R73,60 E) R60,20

<u>ANSWER:</u> B

Volume: $\text{V}=\text{area of base}\times h$

$$\therefore 640\ cm^3=\text{area of base}\times 10\ cm$$

$$\text{area of base}=\frac{640\ cm^3}{10\ cm}=64\ cm^2$$

$\text{Area of base}=\text{length of base}\times\text{length of base}$

$$\therefore 64\ cm^2=l\times l$$

$$\therefore l=8\ cm$$

Surface area of block: $S=[2\times\text{base area}]+[\text{base perimeter}\times h]$

$$\therefore S=(2\times 64)+(32\times 10)\ cm^2$$

$$\therefore S=(128+320)\ cm^2$$

$$\therefore S=448\ cm^2$$

$\therefore$ Cost: $448\times 15c=\text{R}67{,}20$

**15.** In an alien language, ***jalez borg farn*** means "**good maths skills**". ***Nurf klar borg*** means "**maths in harmony**" and ***darko klar farn*** means "**good in gold**". What is "**harmony gold**" in this language?

A) ***klar darko***

B) ***borg nurf***

C) ***jalez klar***

D) ***darko nurf***

E) ***farn borg***

ANSWER: D

a) jalez borg farn → good maths skills<br>
b) Nurf klar borg → maths in harmony<br>
c) darko klar farn → good in gold<br>
From a) and b): borg → maths<br>
From b) and c): klar → in<br>
□ nurf → harmony<br>
From a) and c): farn → good<br>
Hence darko → gold

Also, from all the above, the sentences are translated in reverse order, e.g. the last word in the alien language becomes the first word in English.

"**harmony gold**" → darko nurf

## PART C: (Each correct answer is worth 6 marks)

**16.** As a result of poor attendance at soccer matches it was decided to decrease the ticket price by 20%. At the next match the number of tickets sold increased by 20%. Compared to the previous match, the income from the sale of tickets

A) increased by 20%

B) decreased by 20%

C) increased by 4%

D) decreased by 4%

E) remained the same

<u>ANSWER:</u> D

Let ticket price (original) be $x$

And number attended be $y$

Income (1) $= xy$ (i)

New ticket price $= x \times 0,8$

Number attended at new price $= y \times 1,2$

Income (2) $= x \times 0,8 \times y \times 1,2$

$= xy \times 0,96$ (ii)

Comparing (i) and (ii)

Income decreased by from 1 to 0,96 thus by 4%

**17.** The mean (average) of $n$ numbers is $p$. When the number $q$ is removed from the list of numbers of which the average was taken, the mean (average) increases by 2. The value of $q$ is

A) $p-2n$
B) $p-n+2$
C) $2p-n$
D) $2p-n+2$
E) $p-2n+2$

ANSWER: B

Total = $pn$

New total = $pn - q$

Number of numbers remaining = $n - 1$

New average = $p + 2$

Therefore the equation is:

$$\frac{pn-q}{n-1} = p+2$$

$$\therefore pn-q = pn-p+2n-2$$

$$\therefore q = p-2n+2$$

**18.**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $^{1}$R | $^{2}$B | $^{3}$X | $^{4}$S | $^{5}$O | $^{6}$P | $^{7}$E | $^{8}$D | $^{9}$M | $^{10}$Z | $^{11}$L | $^{12}$K | $^{13}$A |
| $^{14}$G | $^{15}$C | $^{16}$T | $^{17}$N | $^{18}$J | $^{19}$F | $^{20}$U | $^{21}$H | $^{22}$V | $^{23}$W | $^{24}$Q | $^{25}$Y | $^{26}$I |

In the above table each letter of the alphabet is given a value.

The algebraic expression $4x-3$ is used as a key to convert the letters **P S R X B O E** into the word **H A R M O N Y**. Which one of the following keys is used to convert **S R X B** into **G O L D** ?

A) $x+2$ B) $2x-1$ C) $3x+2$ D) $5x+1$ E) $2x+1$

<u>ANSWER:</u> C

First you need to work out how the key works.

**Letters in code** **Letters in word**

P = 6 ⟶ H = 21

S = 4 ⟶ A = 13

etc.

Let the "letter in code' be $x$

Then for $\text{P} = 6, 4x-3 \;= 4\times 6-3$

$= 24-3$

$= 21 = \text{H}$

And for $\text{S} = 4, 4x-3 \;= 4\times 4-3$

$= 16-3$

$= 13 = \text{A}$

Thus the key is a "formula" which converts the code-number of "Letter in code" into the code-number of "Letter in word".

Now consider **S R X B** ⟶ **G O L D**

S = 4 ⟶ G = 14

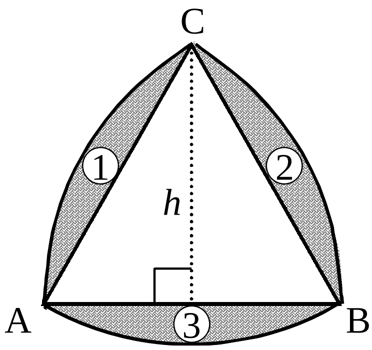
R = 1 ⟶ O = 5

X = 3 ⟶ L = 11

B = 2 ⟶ D = 8

It is not too difficult to see that the key here is $3\times x+2 = 3x+2$

**19.** ABC is an equilateral triangle with sides of 2 units. Using A, B and C as centres of circles, arcs BC, AC and AB are drawn.




The shaded area is

A) $2\pi - 3\sqrt{3}$ B) $\pi - \sqrt{3}$ C) $\dfrac{\pi}{2}$ D) $2\sqrt{3} - \pi$ E) $\pi$

<u>ANSWER:</u> A

Area 1, area 2 and area 3 are equal.

$$\text{Area of } \Delta + \text{Area } 1 = \frac{1}{6}\pi r^2 \qquad \left(\frac{1}{6} \text{ of area of circle because } 60^\circ = \frac{1}{6} \times 360^\circ\right)$$

$$= \frac{1}{6}\pi \times 4$$

$$= \frac{2}{3}\pi \qquad (i)$$

Area $\Delta$:

$$h^2 = 2^2 - 1^2 = 3$$

$$\therefore h = \sqrt{3} \qquad (ii)$$

$$\text{Area } 1 = \frac{2}{3}\pi - \sqrt{3} \qquad (i) - (ii)$$

$$\therefore \text{Area of shaded regions} = 3\left(\frac{2}{3}\pi - \sqrt{3}\right)$$

$$= 2\pi - 3\sqrt{3}$$

**20.** Five children, Amelia, Bongani, Charles, Divine and Edwina, were in the classroom when one of them broke a window. The teacher asked each of them to make a statement about the event, knowing that three of them always lie and two always tell the truth. Their statements were as follows:

| | |
|---|---|
| **Amelia**: | "Charles did not break it, nor did Divine." |
| **Bongani**: | "I didn't break it, nor did Divine." |
| **Charles**: | "I didn't break it, but Edwina did." |
| **Divine**: | "Amelia or Edwina broke it." |
| **Edwina**: | "Charles broke it." |

Who broke the window?

A) Amelia B) Bongani C) Charles

D) Devine E) Edwina

<u>ANSWER:</u> C

This problem requires you to organize a confusing picture in a systematic way.

It involves:

1. A fact: one of them broke a window
2. A fact: three of them always lie, and two always tell the truth
3. Statements: there are five of them, of which three are false and two are true.

The approach take here is:.

1. Assume a particular learner broke the window
2. If so, which statements ate true, and which are false?
3. Are three false, and two true?
   If so, your assumption is correct,
   if not, your assumption is incorrect,
   so try another pupil.

<u>First test:</u>

1. Assume that Amelia broke the window
2. Then: Amelia's statement is True
   Bongani's statement is True
   Charles' statement is False
   Divine's statement is True
3. Three statements are True, so your assumption is incorrect. Try Bongani!

Second test:

1. Assume that Bongani broke the window
2. Then: Amelia's statement is True
   Bongani's statement is False
   Charles' statement is False
   Divine's statement is False
   Edwina's statement is False
3. Four statements are False, so your assumption is incorrect. Try Charles!

Third test:

1. Assume that Charles broke the window
2. Then: Amelia's statement is False
   Bongani's statement is True
   Charles' statement is False
   Divine's statement is False
   Edwina's statement is True
3. Three statements are False, and two are True, so your assumption is correct.

So Charles broke the window.

(For fun, work through the other two learners and check that neither Divine nor Edwina were the culprits)

## ANSWER POSITIONS: JUNIOR SECOND ROUND 2002

| PRACTICE EXAMPLES | POSITION |
|---|---|
| 1 | C |
| 2 | D |

| NUMBER | POSITION |
|---|---|
| 1 | D |
| 2 | A |
| 3 | E |
| 4 | B |
| 5 | C |
| 6 | E |
| 7 | A |
| 8 | B |
| 9 | D |
| 10 | E |
| 11 | B |
| 12 | C |
| 13 | D |
| 14 | B |
| 15 | D |
| 16 | D |
| 17 | E |
| 18 | C |
| 19 | A |
| 20 | C |

| DISTRIBUTION | |
|---|---|
| A | 3 |
| B | 4 |
| C | 4 |
| D | 5 |
| E | 4 |
| **TOTAL** | **20** |