# OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

### **Grade NINE First Round 2020**

#### **Solutions**

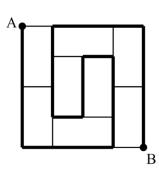
1. **E** 
$$\frac{2020}{202} = \frac{202 \times 10}{202} = 10$$

2. **A** 
$$60 \times 20 = 1200$$
 seconds

3. **B** 
$$\sqrt{2 \times 10 \times 2 \times 10} = \sqrt{20 \times 20} = \sqrt{20^2} = 20$$

- 4. **C** The train leaves at 20:40 and takes 20 minutes, arriving at its destination at 21:00.
- 5. **E**  $20 \text{ m} = 2000 \text{ cm} \text{ and } 2000 \text{ cm} \div 20 \text{ cm} = 100.$
- 6. **D** D is equal to 1 while all the others equal 2.
- 7. **E** Note that  $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$  and that  $2020 \div 20 = 101$ .
- 8. **B** Only O, I and D would look the same after reflection across a horizontal line.
- 9. **A** The sum of the five integers is  $5 \times 9 = 45$ . For one of the integers to be as large as possible, the other four need to be as small as possible. Since all the integers need to be different, the four smallest integers would need to be 1, 2, 3 and 4. The greatest possible integer would thus be 45 (1 + 2 + 3 + 4) = 35.
- 10. **B** Note that each circle is directly connected only to circles of the opposite colour. Thus, to end up on a black circle the fly must have landed on a white circle, and 5 of the 9 circles are white.
- 11. C Note that  $9 = 3^2$ ,  $10 = 2 \times 5$  and  $12 = 2^2 \times 3$ . The lowest number divisible by all three of the numbers 9, 10 and 12 is thus  $2^2 \times 3^2 \times 5 = 180$ . There are 5 multiples of 180 which are 3-digit numbers.
- By a process of trial and error the dimensions of the rectangle can be found to be 4 cm by 5 cm. The perimeter is thus 18 cm. Alternatively, letting the dimensions of the rectangle be x and y, we have xy = 20 and  $x^2 + y^2 = 41$ . Now, since  $(x + y)^2 = x^2 + 2xy + y^2$  we have  $(x + y)^2 = 41 + 40 = 81$ . Thus x + y = 9 and the perimeter is 18 cm.
- 13. A The largest amount possible is  $4 \times R100 + 3 \times R50 + 3 \times R20 + 3 \times R10 = R640$ .
- 14. **E** Since a = c + d and e = b + c we have  $a + e = 2c + d + b = 180^{\circ} + c$ . From this it follows that  $a + e > 180^{\circ}$ . Thus E is not true.

15. **C**  $7 \times 2 + 10 \times 1 = 24$  cm.



- 16. **D** Note that 8 + a + b = K, 10 + c = K, 11 + d = K, and 13 + e + f = K. We thus have: 4K = 42 + a + b + c + d + e + f = 42 + (2 + 4 + 5 + 6 + 8 + 9) = 76. Thus K = 19.
- 17. **C** There are three different possible orientations for rhombuses formed from two adjacent small triangles:

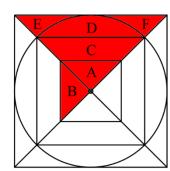


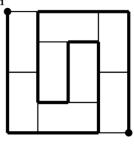


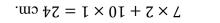


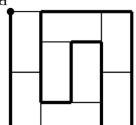
The grid contains six rhombuses in each of these three orientations, making a total of 18 rhombuses.

- 18. **B** For every 60 minutes that passed, the clock only moved forward by 55 minutes. From when the clock showed 6 a.m. to when the clock showed 5 p.m. a total of  $11 \times \frac{60}{55} = 12$  hours would have passed in real time. The clock thus stopped when the actual time was 6 p.m. The correct time now is thus 8 p.m.
- Initially  $\frac{1}{3}$  of the cards are face up. After some of the cards are turned over then  $\frac{2}{5}$  of the cards are face up. Since 3 and 5 are relatively prime then the smallest number of cards possible is  $3 \times 5 = 15$ . [Initially there were 15 cards with 5 face up and 10 face down, i.e. a 1:2 ratio. When a single face down card is turned over there are 6 face up and 9 face down, i.e. a 2:3 ratio.]
- 20. **A** Move the shaded areas as shown. The regions A, C, D, E and F represent a quarter of the largest square and thus have combined area  $\frac{1}{4} \times 20^2 = 100 \text{ cm}^2$ . B thus has area  $36 \text{ cm}^2$ . Since B is a quarter of the smallest square, the smallest square must have area  $144 \text{ cm}^2$ , and thus side length of 12 cm and perimeter 48 cm.









16. **D** Let op dat 
$$8 + a + b = K$$
,  $10 + c = K$ ,  $11 + d = K$ , en  $13 + e + f = K$ . Ons het dus:  $4K = 42 + a + b + c + d + e + f = 42 + (2 + 4 + 5 + 6 + 8 + 9) = 76$ . Dus  $K = 19$ .

klein driehoeke: Daar is drie verskillende oriëntasies van ruite gevorm deur twee aangrensende .71  $\mathbf{C}$ 







.02

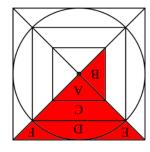
.81

.21

18 ruite gee. Die rooster bevat ses ruite in elk van hierdie oriëntasies, wat 'n totaal van

dus gestop toe die werklike tyd 6 nm. was. Die werklike tyd is dus nou 8 nm. het, het 'n totaal van 11  $\times \frac{60}{55} = 12$  ure in werklike tyd verloop. Die horlosie het aangeskuif. Vandat die horlosie 6 vm. getoon het totdat die horlosie 5 nm. getoon Vir elke 60 minute wat verloop het, het die horlosie met slegs 55 minute

2:3 verhouding.] onder kaart omgedraai word dan is daar 6 gesig na bo en 9 gesig na onder, d.i. 'n na bo en 10 gesig na onder, d.i. 'n 1:2 verhouding. Wanneer 'n enkele gesig na aantal moontlike kaarte  $3 \times 5 = 15$ . [In die begin was daar 15 kaarte met 5 gesig is, is  $\frac{2}{5}$  van die kaarte gesig na bo. Aangesien 3 en 5 relatief priem is is die kleinste In die begin is  $\frac{1}{3}$  van die kaarte gesig na bo. Na sommige van die kaarte omgedraai a .91



48 cm hê. 144 cm² hê en dus 'n sylengte van 12 cm en omtrek van die kleinste vierkant is moet die kleinste vierkant 'n area van B het dus 'n area van van 36 cm². Aangesien B 'n kwart van grootste vierkant en het dus 'n area van  $\frac{1}{4} \times 20^2 = 100 \text{ cm}^2$ . gebiede A, C, D, E en F verteenwoordig 'n kwart van die Verskuif die ingekleurde gebiede soos aangetoon. Die

# OLD MUTUAL SUID-AFRIKAANSE

# MISKUNDE OLIMPIAE

# Graad NEGE Eerste Ronde 2020

# **sgnissolqO**

1. 
$$\mathbf{E} = \frac{2020}{202} = \frac{202 \times 10}{202} = 10$$

2. **A** 
$$60 \times 20 = 1200$$
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3. **a** 
$$\sqrt{2 \times 10 \times 2 \times 10} = \sqrt{20 \times 20} = \sqrt{20^2} = 20$$

5. **E** 
$$20 \text{ m} = 2000 \text{ cm} \text{ en } 2000 \text{ cm} \div 20 \text{ cm} = 100.$$

7. **Example 19.1** Let op dat 
$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$
 en dus 2020 ÷ 20 = 101.

9. A Die som van die vyf heelgetalle is 
$$5 \times 9 = 45$$
. Vir een van die heelgetalle om so groot as moontlik te wees moet die ander vier so klein as moontlik wees. Aangesien al die heelgetalle verskillend moet wees moet die vier kleinste heelgetalle 1, 2, 3 en 4 wees. Die grootste moontlike heelgetal is dus  $45 - (1 + 2 + 3 + 4) = 35$ .

11. C Let op dat 
$$9 = 3^2$$
,  $10 = 2 \times 5$  en  $12 = 2^2 \times 3$ . Die kleinste getal deelbaar deur al drie van die getalle 9, 10 en 12 is dus  $2^2 \times 3^2 \times 5 = 180$ . Daar is 5 veelvoude van 180 wat 3-syfer getalle is.

Deur inspeksie kan gesien word dat die afmetings van die reghoek 4 cm by 5 cm is. Die omtrek is dus 18 cm. Alternatiewelik, laat die afmetings van die reghoek x en y wees en dan het ons xy = 20 en  $x^2 + y^2 = 41$ . Aangesien  $(x + y)^2 = x^2 + 2xy + y^2$  het ons  $(x + y)^2 = 41 + 40 = 81$ . Dus x + y = 9 en die omtrek is 18 cm.

13. A Die grootste bedrag is 
$$4 \times R100 + 3 \times R50 + 3 \times R20 + 3 \times R10 = R640$$
.

14. If Aangesien 
$$a = c + d$$
 en  $e = b + c$  het ons  $a + e = 2c + d + b = 180^{\circ} + c$ . Uit hierdie volg dat  $a + e > 180^{\circ}$ . Dus is E nie waar nie.