THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

FIRST ROUND 2003: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

Thank you for entering the First Round of the Mathematics Olympiad.

Many of the solutions to the problems which are given below use trial-and-improvement methods or an investigative approach.

Most of them can also be solved using more formal mathematical methods which you will learn in due course. Sometimes we have also given a more formal solution which you might find interesting.

PRACTICE EXAMPLES:

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1. $23+6-4=$

A) 6 B) 23 C) 25 D) 29 E) 33

ANSWER: C

$23+6-4=(23+6)-4=29-4=25$

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2. $\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}$ equals

A) $\frac{1}{15}$ B) $\frac{3}{11}$ C) $\frac{21}{50}$ D) $\frac{8}{15}$ E) $9\frac{4}{5}$

ANSWER: D

$\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}=\frac{1}{5}+(\frac{2}{3}\times\frac{1}{2})=\frac{1}{5}+\frac{1}{3}=\frac{3+5}{15}=\frac{8}{15}$

QUESTIONS:

1. The value of $1{,}1 \times 10$ is

A) 1 B) 11 C) 1,1 D) 11,1 E) 110

ANSWER: B
EXPLANATION:
x 10 means the original number becomes ten times larger. (Move the decimal comma one place to the right.)

2. $4^3 \div 2^2$ is equal to

A) 2 B) 3 C) 4 D) 16 E) 32

ANSWER: D
EXPLANATION:

$4^3 = 4 \times 4 \times 4$ $\quad$ $2^2 = 2 \times 2$

$= 64$ $\quad$ $= 4$

$\therefore \frac{64}{4} = 16$

3. **Did you know?** In the decimal number system (base 10) ten different digits, 0 to 9, are used to write all the numbers. In the binary number system (base 2) two different digits are used, i.e. 0 and 1.

Which one of the following numbers is not a valid number in the octal number system (base 8)?

A) 128 B) 127 C) 126 D) 125 E) 124

ANSWER: A
EXPLANATION:
When working in base 2: numbers are written in terms of 0 and 1;
When working in base 3: we use 0, 1 and 2;
When working in base 4: we use 0, 1, 2 and 3;
When working in base 5: we use 0, 1, 2, 3 and 4;
When working in base 10: we use 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9;
and so on.

When working in base 8: we use 0, 1, 2, 3, 4, 5, 6 and 7.

4. At a recent Athletics Championship Sipho broke the old record of 11,01 seconds for the 100 metre race by 0,04 seconds.
The new record for the race in seconds is

A) 10,01 B) 10,04 C) 10,61 D) 11,05 E) 10,97

ANSWER: E
EXPLANATION:

$$\begin{array}{r} 11,01 \\ -0,04 \\ \hline 10,97 \end{array}$$

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5. What is the units digit of 4^{2003}?

A) 0 B) 2 C) 4 D) 6 E) 8

ANSWER: C
EXPLANATION:

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

All even powers of 4 give a units digit of 6 and all odd powers of 4 give a units digit of 4.
The units digit of 4^{2003} will therefore be 4.

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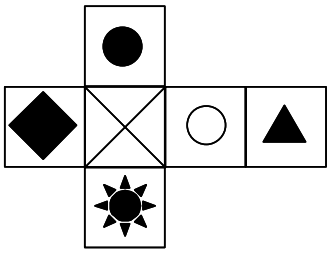
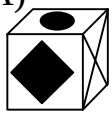
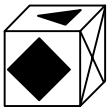
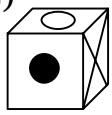
6. The following net is given:



Which one of the following cubes cannot be folded from this net?

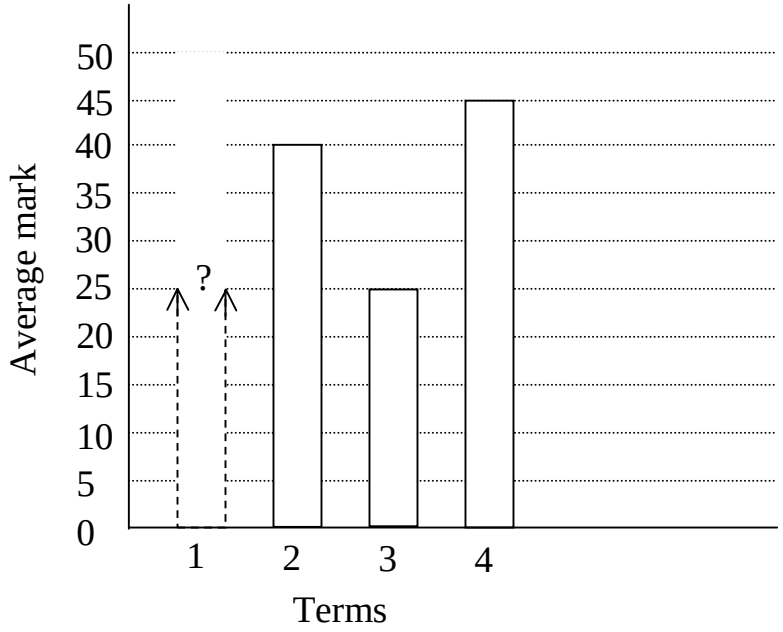
A) 
B) 
C) 
D) 
E) 

ANSWER: B
EXPLANATION:
Check the distractors:

A: The three visible sides are adjacent as required.
B: The triangle is adjacent to the cross.
C: The three visible sides are adjacent as required.
D: The three visible sides are adjacent as required.
E: The three visible sides are adjacent as required.

B: Not possible from given net because the triangle has to be opposite the cross.

7. Susan's average mark for the four terms (numbered 1 to 4 on the horizontal axis) of 2002 was 70%. The mark for each term is out of 50 and carries the same weight when calculating the average. Use the graph to determine what her percentage was for the first term of 2002.



A) equal to or more than 70%
B) equal to or less than 20%
C) between 20% and 30%
D) between 40% and 50%
E) between 50% and 70%

ANSWER: E
EXPLANATION:
Average is 70%, which is 70 out of 100. For the four terms, the total is therefore 280 out of 400.

Second term:	40 out of 50 equals 80 out of 100
Third term:	25 out of 50 equals 90 out of 100
Fourth term:	45 out of 50 equals 50 out of 100

First term: $280 - (80 + 50 + 90)$
$= 60$ out of 100 (which is between 50% and 70%)

8. If $20x - 25$ is expressed in the form $a(4x + b)$,
then the value of $a + b$ is

A) -20 B) -10 C) 0 D) 10 E) 20

ANSWER: C
EXPLANATION:

$$\begin{aligned} \text{If } 20x - 25 &= a(4x + b) \\ &= 4ax + ab \\ \text{then } 4a &= 20 \\ \therefore a &= 5 \\ \text{and } ab &= -25 \\ \therefore 5(b) &= -25 \\ \therefore b &= -5 \\ \therefore a + b &= 5 + (-5) \\ &= 0 \end{aligned}$$

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9. **Did you know?** In a magic square the sum of the numbers in every horizontal row, vertical column and diagonal are all the same.

In this magic square,
the row total is equal to 15.

x		6
	5	1
		M

The value of x is

A) 1 B) 2 C) 3 D) 5 E) 8

ANSWER: B
EXPLANATION:

$$\begin{aligned} 6 + 1 + M &= 15 \\ \therefore M &= 8 \\ \therefore x + 5 + 8 &= 15 \\ \therefore x &= 2 \end{aligned}$$

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10. How many numbers between 800 and 1000 are divisible by both 7 and 8?

A) 2 B) 3 C) 4 D) 5 E) 6

ANSWER: B

EXPLANATION:

The first multiple of 8 after 800 is 808 and the first multiple of 7 after 800 is 805. A number divisible by both 7 and 8 should be divisible by 56. The first such number is 840 obtainable by looking at the sequences:

808	805
816	812
824	819
832	826
840	833
	840

Hence the next multiple of 56 is 840+56 = 896 and the one after that is 896+56 = 952. Since 952+56 = 1008 > 1000, it follows that there are 3 numbers between 800 and 1000 that are divisible by both 7 and 8.

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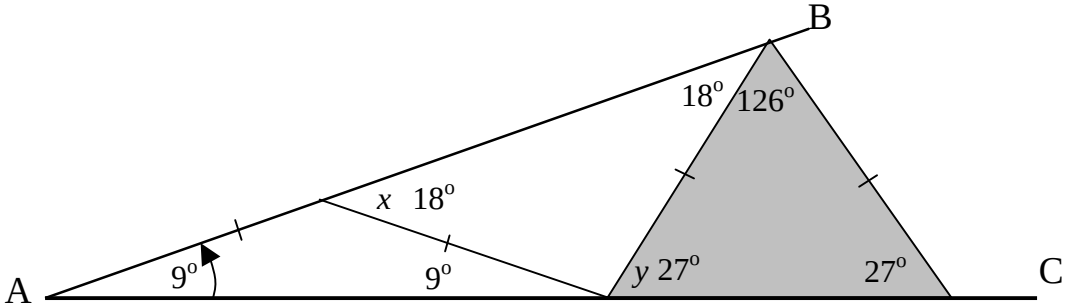
11. Isosceles triangles have been drawn between AB and AC with $B\hat{A}C = 9^o$.



What is the size of the largest angle in the shaded triangle?

A) 72^o B) 81^o C) 90^o D) 126^o E) 144^o

ANSWER: D

EXPLANATION:

$\Delta 1$: $x = 9^o + 9^o = 18^o$ *ext.∠ of* Δ

$\Delta 2$: $y = 9^o + 18^o = 27^o$ *ext.∠ of* Δ

$\Delta 3$: largest angle $= 180^o - (27^o + 27^o) = 126^o$

12. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) \quad . \quad . \quad . \quad \left(1-\frac{1}{2003}\right)$ is equal to

A) $\frac{1}{2003}$ B) $\frac{2}{2003}$ C) $\frac{202}{2003}$ D) $\frac{24}{2003}$ E) $\frac{2002}{2003}$

ANSWER: A
EXPLANATION:

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \ldots \times \frac{\cancel{2002}}{2003}$$

$$= \frac{1}{2003}$$

13. Nomava visits a shop with one 20c coin and ten 50c coins. The shopkeeper can offer change but has only two 20c coins and nine R1 coins. She buys one item and receives the correct change. Which one of the following is a possible price for this item?

A) R4,60 B) R5,40 C) R1,40 D) R4,40 E) R5,10

ANSWER: A
EXPLANATION:
If the price of the item is R4,60 then Nomava's change would be R5,00 – R4,60 = 40c. This is possible since the shopkeeper has two 20c coins.

If the price of the item is R5,40 then Nomava's cannot buy the item since she has only R5,20.

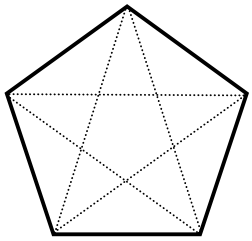
If the price of the item is R1,40 then Nomava's change would be R1,50 – R1,40 = R0,10c or R1,70 – R1,40 = R0,30c. Since the shopkeeper has only 20c coins and R1 coins, this is not possible.

If the price of the item is R4,40 then Nomava's change would be R4,50 – R4,40 = R0,10c or R4,70 – R4,40 = R0,30c. Since the shopkeeper has only 20c coins and R1 coins, this is not possible.

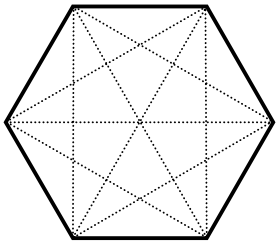
If the price of the item is R5,10 then Nomava's change would be R5,20 – R5,10 = 10c. Since the shopkeeper has only 20c coins and R1 coins, this is not possible.

14. **Did you know?**

i) A regular polygon with five sides (pentagon) has five diagonals, indicated by the dotted lines.



ii) A regular polygon with six sides (hexagon) has nine diagonals, indicated by the dotted lines.



The number of diagonals that can be drawn in a regular polygon with twenty sides (icosagon), is

A) 190 B) 180 C) 170 D) 380 E) 19

ANSWER: C

EXPLANATION:

A pentagon has 5 sides and 5 diagonals. A hexagon has 6 sides and 9 diagonals. If n = number of sides of a polygon, then the number of diagonals is: $\frac{n(n-3)}{2}$.

From every vertex, $n-3$ diagonals can be drawn (lines drawn to the vertices closest to the vertex in question don't result in diagonals). $n(n-3)$ such diagonals can therefore be drawn but that means between any two points (vertices) two diagonals are drawn (e.g. from A to D and from D to A). The number of diagonals in this n–sided polygon after we have divided by 2 is therefore $\frac{n(n-3)}{2}$. The formula $\frac{n(n-1)}{2}-n$, which is basically the same formula in a different format (discuss), can also be used to determine the number of diagonals.

A polygon with 20 sides has

$\frac{20(17)}{2}=170$ diagonals.

15. **Did you know?**

Formula for speed: $\text{average speed} = \dfrac{\text{total distance}}{\text{total time}}$

The distance from Sipho's home to Harmony Gold mine is 120 km.
Sipho travels at 60 km/h from his home to the mine.
He is in a hurry to get back to his home in the afternoon, and travels at 120 km/h.
What is his average speed for the journey there and back in km/h?

A) 110 B) 100 C) 95 D) 90 E) 80

ANSWER: E
EXPLANATION:
Time Sipho took from his home to the mine:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{120\ km}{60\ km} = 2\ \text{hours}$$

Time to get back:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{120\ km}{120\ km} = 1\ \text{hour}$$

Total time = 3 hours

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{240\ km}{3\ h} = 80\ \text{km/h}$$

16. A store sold 213 bicycles during the year 2002. For the first few months they sold 20 bicycles per month, then for some months they sold 16 bicycles per month and in the remaining month(s) they sold 25 bicycles per month. For how many months did they sell only 16 bicycles per month?

A) 5 B) 6 C) 7 D) 8 E) 9

ANSWER: D
EXPLANATION:

	First months 20 per month	Middle months 16 per month	Last months 25 per month	Sales
No. of months	a	b	$12 - (a + b)$	
Trial 1 (e.g.)	1	4	7	259
Trial 2 (e.g.)	3	4	5	249
…				
Successful trial	3	8	1	213

There must be an odd number of 'last months' to achieve the odd total of 213 sales. Repeated trials will lead to the solution $b = 8$.

Algebraically one can argue:

$$20a + 16b + 25[12 - (a + b)] = 213$$

$$\therefore \; 20a + 16b + 300 - 25a - 25b = 213$$

$$\therefore \; -5a - 9b = -87$$

$$\therefore \; -9b = 5a - 87$$

$$\therefore \; b = \frac{-5a + 87}{9},$$

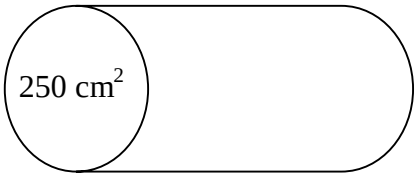
with $1 \le a \le 12$ and $1 \le b \le 12$,

and a and b natural numbers.

This solves uniquely to $a = 3$ and $b = 8$.
They have therefore sold 16 bicycles per month for 8 months.

17. **Did you know?**
1 litre is the same as 1000 cm^3.

The area of the cross-section of a pipe is 250 cm^2. Water flows through the pipe at a rate of 3 litres per second.



The speed at which the water flows through the pipe in cm/s is

A) 15 B) 1,2 C) 8 D) 6 E) 12

ANSWER: E
EXPLANATION:
In one second 3000 cm^3 pass any point in the pipe. The cross-section of the cylinder of water is 250 cm^2.
Therefore the length is $\frac{3000\ \text{cm}^3}{250\ \text{cm}^2} = 12\ \text{cm}$.
Algebraically one can say:
If the water flows at x cm/s, then $250x = 3000,\ \therefore x = 12$
or

$$\text{Rate of flow } (f) = 3\text{l/s}$$

but $1\text{l} = 1000\ \text{cm}^3$

$$\therefore \text{Rate of flow} = 3000\ \text{cm}^3/\text{s}$$

$$= \frac{\text{volume}}{\text{time}}$$

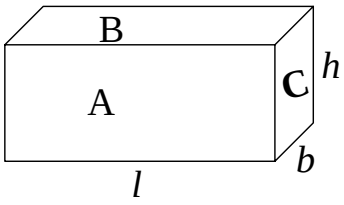
$$\text{length of pipe} = \frac{\text{volume of pipe}}{\text{area of base}} = \frac{\text{Rate} \times \text{time}}{\text{area}}$$

$$= \frac{3000\ \text{cm}^3/\text{s}}{250\ \text{cm}^2} \times \text{time}$$

$$\text{But speed of water through pipe} = \frac{\text{length of pipe}}{\text{time}}$$

$$= 12\ \text{cm/s}$$

18. The areas of the faces of the rectangular box are A, B and C.



If the volume of the box is V,
then $A \times B \times C$ is equal to

A) V B) $2V^2$ C) V^2 D) $\sqrt{V}$ E) $\sqrt[3]{V}$

ANSWER: C
EXPLANATION:

$$A = l \times h$$

$$B = l \times b$$

$$C = b \times h$$

$$\begin{aligned} A \times B \times C &= l \times h \times l \times b \times b \times h \\ &= (l \times b \times h) \times (l \times b \times h) \\ &= V \times V = V^2 \end{aligned}$$

19. In the formula $M = \frac{10n}{1+2n}$, n is any positive integer.

If n increases (gets bigger and bigger), M will

A) decrease
B) increase
C) stay the same
D) first increase and then decrease
E) first decrease and then increase

ANSWER: B
EXPLANATION:

n	1+2n	10n	$M = \frac{10n}{1+2n}$
1	3	10	3,33
2	5	20	4,00
3	7	30	4,28
4	9	40	4,44
5	11	50	4,54
6	13	60	4,61
.	.	.	.
.	.	.	.
.	.	.	.
			5

So M is increasing to 5.

This can also be seen from the fact that

$$M = \frac{10n}{1+2n} = \frac{5(2n+1)-5}{1+2n} = 5 - \frac{5}{1+2n}$$

So, the larger n gets, the less we subtract from 5 and the closer the value for M gets to 5. Therefore M increases.

20. A meal made with four eggs and 60 g cheese contains 560 calories. Another meal made with six eggs and 20 g cheese also contains 560 calories.

How many calories does one (1) egg contain?

A) 60 B) 70 C) 80 D) 90 E) 100

ANSWER: C

EXPLANATION:

From the second meal we see that 18 eggs and 60 g cheese contain 3×560 calories. But from the first meal 60 g cheese and 4 eggs contain 560 calories. The difference shows us that 14 eggs contain $2\times560=1120$ calories, i.e. one egg contains $\frac{1120}{14}=80$ calories.

Or algebraically:

Let the number of calories in each egg = e

And the number of calories in each gram of cheese = k

First meal: $4e+60k=560$

$\therefore 2e+30k=280$ ... (1)

(1)$\times3$ $\therefore 6e+90k=840$ ... (3)

Second meal: $6e+20k=560$ ... (2)

Comparing (3) and (2) the difference is:

$70k=280$

$\therefore 10k=40$

$\therefore 20k=80$

Using this information in (2):

$6e+80=560$

$\therefore 6e=480$

$\therefore e=80$

One egg contains 80 calories.

THE END

ANSWER POSITIONS: JUNIOR FIRST ROUND 2003

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	B
2	D
3	A
4	E
5	C
6	B
7	E
8	C
9	B
10	B
11	D
12	A
13	A
14	C
15	E
16	D
17	E
18	C
19	B
20	C

DISTRIBUTION	
A	3
B	5
C	5
D	3
E	4
TOTAL	**20**