# THE SOUTH AFRICAN
# MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

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**FIRST ROUND 2001**

**SENIOR SECTION: GRADES 10, 11 AND 12**
(STANDARDS 8, 9 AND 10)

**28 March 2001**

**TIME: 60 MINUTES**

**NUMBER OF QUESTIONS: 20**

## ANSWERS

**1.** C
**2.** D
**3.** E
**4.** E
**5.** B
**6.** D
**7.** A
**8.** E
**9.** B
**10.** B
**11.** D
**12.** A
**13.** D
**14.** B
**15.** D
**16.** C
**17.** D
**18.** C
**19.** B
**20.** D

## SOLUTIONS

**1.** (This problem tests estimation.) 12 kg is about the weight of a small child or a dog, 120 kg is about the weight of a very big man, 1200 kg (or 1,2 tonne) is the weight of a hippo or a small car, 120 tonne is about the weight of a medium-size airliner, and 1200 tonne is about the weight of two railway engines.

Answer: C

**2.**

$$(\frac{2001}{5})^2 - (\frac{1999}{5})^2 = (\frac{2001}{5} - \frac{1999}{5})(\frac{2001}{5} + \frac{1999}{5}) = \frac{2}{5} \times 800 = 320.$$

Answer: D

**3.** Two bags cost R 6,00 and one bottle costs 60 cents more, so the percentage extra is $0,60 \div 6 \times 100$.

Answer: E

**4.** The sum of the interior angles in a polygon with $n$ sides is $(180n - 360)^\circ$, and a pentagon has five sides, so the angle sum is $540^\circ$. In a regular pentagon all the angles are the same, so each angle is equal to $540/5 = 108^\circ$. In an equilateral triangle each angle is equal to $60^\circ$. Thus $G\widehat{A}F = (360 - 60 - 108 - 60)^\circ$.

Answer: E

**5.** $2001 \times \frac{1}{2001} = 1 < 2001 - \frac{1}{2001} < 2001 + \frac{1}{2001} < 2001, 2001 < 2001 \div \frac{1}{2001} = 2001 \times 2001$.

Answer: B

**6.** The area of a semicircle of radius $r$ is $\frac{1}{2}\pi r^2$ and the area of a triangle with base $2r$ and height $r$ is $\frac{1}{2}(2r)r = r^2$, so the difference is $(\frac{\pi}{2} - 1)r^2$. Now substitute $r = 6$.

Answer: D

**7.** $279 = 100 + (3 \times 50) + 29$, so the total cost is R 4,10 + 4× R 1,40.

Answer: A

**8.** If $x = 0,454545\cdots$, then $100x = 45,454545\cdots = 45 + x$. Therefore $x = \frac{45}{99} = \frac{5}{11}$.

Answer: E

**9.** Choose any one corner; then take the three corners at the ends of the diagonals of the three faces meeting at the first corner.

Answer: B

**10.** The three small rectangles have length $a$; if their width is $x$, then $a = 2x$ and $b = a + x = 3x$.

Answer: B

**11.** If each side of the square is of length $s$, then the sides of the rectangle are $1,2\,s$ and $0,8\,s$, so its area is $0,96\,s^2$. (Notice that exactly one out of (A), (C), and (D) must be true, so the other two need not be considered.)

Answer: D

**12.** The three outer small triangles are isosceles. Let $A\widehat{Z}Y = A\widehat{Y}Z = \alpha$ and $B\widehat{Y}X = B\widehat{X}Y = \beta$ and $C\widehat{X}Z = C\widehat{Z}X = \gamma$. The angle sums at $Y, X, Z$ give $\alpha + \beta = 115^\circ$ and $\beta + \gamma = 105^\circ$ and $\gamma + \alpha = 140^\circ$, from which it follows that $\alpha = 75^\circ$ and $\beta = 40^\circ$ and $\gamma = 65^\circ$. Finally, $\widehat{C} = 180^\circ - 2\gamma$.

Answer: A

**13.** The first newcomer needs 20 lines and the second one needs 21.

Answer: D

**14.** If there are $m$ men, then there are $m - 1$ spaces between them. Each space contains at least 3 women, so the number of women is at least $3(m-1)$. Since the number of women is $2001 - m$, it follows that $2001 - m \geq 3(m-1)$, giving $2004 \geq 4m$.

Answer: B

**15.** $a^{1/10} = 2^3 = 8$, while $b^{1/10} = 3^2 = 9$. (Again only (A), (D) or (E) need to be considered.)

Answer: D

**16.** For example, it is not true for 1, 2, 3.

Answer: C

**17.** The midpoint in time corresponds to the midpoint in distance, since the speed is constant. Therefore the fraction at 07:10 is $\frac{1}{2}(\frac{1}{6} + \frac{3}{4}) = \frac{1}{2}(\frac{2}{12} + \frac{9}{12})$.

Answer: D

**18.** The equation gives $(m+1)(n+1) = 13 \times 7$, so $m = 12$ and $n = 6$, or the other way around. (No other factorization of 91 gives permissible values of $m$ and $n$.)

Answer: C

**19.** The three triangles are all similar and right-angled, with sides in the ratio $3 : 4 : 5$. Therefore $|BE| = \frac{3}{5}|AE| = \frac{9}{5}$.

Answer: B

**20.** First give Nic three oranges, and Sudan and Vishnu two each. Then the remaining two oranges can (a) both go to Nic or to Sudan (two possibilities) or (b) be shared between two of the three (three possibilities). Alternatively, count the cases systematically.

Answer: D