THE SOUTH AFRICAN MATHEMATICS OLYMPIAD
SENIOR SECOND ROUND 2018
Solutions

1. **Answer 820**
   $2018 - (1000 + 100) - (\sqrt{100})^2 + 2 = 2018 - 1100 - 100 + 2 = 820.$

2. **Answer 012**
   From left to right, the first pile has six blocks, the second four blocks, and the third two blocks. The total number of blocks is $6 + 4 + 2 = 12$.

3. **Answer 009**
   The two smaller numbers must be positive, different, and as small as possible, that is, 1 and 2. The largest number is therefore $12 - 1 - 2 = 9$.

4. **Answer 031**
   The numbers of problems solved on the five days are $1, 2, 4, 8, 16$, and the total number of problems solved is $1 + 2 + 4 + 8 + 16 = 31$, which can also be found from the formula $2^5 - 1$.

5. **Answer 052**
   The area of a square with perimeter P is $(\frac{1}{4}P)^2 = \frac{1}{16}P^2$, and the area of a circle with circumference P is $\pi(\frac{P}{2\pi})^2 = \frac{1}{4\pi}P^2$. The ratio of the areas is therefore $\frac{16}{4\pi} = \frac{4}{\pi}$, so the area of the circle is $\frac{4}{\pi}(13\pi) = 52$.

6. **Answer 006**
   If the child's age is x, then $x + 3 = (x - 3)^2$, giving $x^2 - 7x + 6 = 0$, which has solutions $x = 6$ or $x = 1$. [The answer $x = 1$ is not valid, because then $x+3 = 4$ and $x - 3 = -2 \neq \sqrt{4}$. Remember that square roots cannot be negative, and $\sqrt{4} = +2$.]

7. **Answer 968**
   If the original triangle has base b and height h, then $\frac{1}{2}bh = 800$. The new triangle has base $1.1 \times b$ and height $1.1 \times h$, so its area is $\frac{1}{2}(1.1b)(1.1h) = 1.21 \times \frac{1}{2}bh = 1.21 \times 800 = 968$.

8. **Answer 960**
   There are ten such numbers from 51 to 141, so
   $51 + 61 + \cdots + 131 + 141 = 10 \times \frac{1}{2}(51 + 141) = 5 \times 192 = 960,$
   using the formula for the sum of an arithmetic series. [Otherwise simply add them.]

9. **Answer 100**
   Since $AB \| CD$, it follows that $A\widehat{B}C = 180° - 110° = 70°$, so $A\widehat{B}D = 70° - 30° = 40°$.

   Then $B\widehat{A}D = A\widehat{B}D = 40°$ also, since $AD = BD$, and finally $A\widehat{D}B = 180° - (40° + 40°) = 100°$.

10. **Answer 032**
    If the larger number is n, then the smaller is $n - 8$, so $4(n - 8) = 3n$, giving $n = 32$.

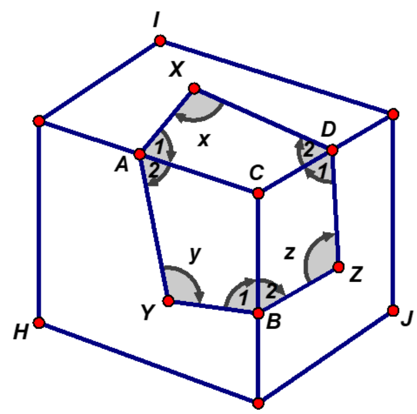
11. **Answer 477**
    From 25 to 99 there are $99 - 25 + 1 = 75$ numbers with two digits each, and from 100 to 208 there are $208 - 100 + 1 = 109$ numbers with three digits each. The total number of digits is therefore $2 \times 75 + 3 \times 109 = 477$.

12. **Answer 061**
If Thembi has k keyrings, then $k-1$ must be positive and divisible by $2, 3, 4, 5$. The smallest value of $k-1$ is the LCM of $2, 3, 4, 5$, which is 60, so $k = 61$.

13. **Answer 401**
The sum of any two sides of a triangle must be greater than the remaining side, so if the unknown side is s cm, then $150 + s > 200$. The smallest integer value is therefore $s = 51$, and the minimum perimeter is $200 + 150 + 51 = 401$.

14. **Answer 009**
If the tens and units digits of the number are T and U, then the number is $10T + U$, which is equal to $T \times U + T + U$. Therefore $9T = TU$, so $U = 9$ (since $T \neq 0$), and T can have any value from 1 to 9. [If $T = 0$, then it is not a 2-digit number.]

15. **Answer 270**



Since the paths are straight lines, the pairs of angles A_1 and A_2, B_1 and B_2, and D_1 and D_2 are supplementary angles. Considering the sum of all the angles of the three quadrilaterals $XACD$, $AYBC$ and $CBZD$, we have the following:

$$\begin{aligned} x + \angle A + y + \angle B + z + \angle D + 3 \times \angle C &= 3 \times 360^\circ \\ \therefore x + 180^\circ + y + 180^\circ + z + 180^\circ + 3 \times 90^\circ &= 3 \times 360^\circ \\ \therefore x + y + z &= 270^\circ \end{aligned}$$

A much easier, shorter solution is to consider the special case of respectively moving Y to H, X to I, and Z to J, from which it immediately follows that $x + y + z = 3 \times 90^\circ = 270^\circ$. One can use this simplified method because the sum of the marked angles is clearly independent of the positions of X, Y and Z. (This strategy holds provided a part of all the indicated paths remain on the respective surfaces containing the three quadrilaterals $XACD$, $AYBC$ and $CBZD$. For example, moving X to C, Y to the corner directly below C, and Z to J would not be valid, since no part of the path XDZ would then still lie on the rectangular surface containing quadrilateral $XACD$.)

16. **Answer 039**
If $g(x) = f(x) - 5 = ax^7 + bx^3 + cx$, then $g(x)$ involves only odd powers of x, so $g(-x) = -g(x)$. Therefore $f(7) = g(7) + 5 = -g(-7) + 5 = -[f(-7) - 5] + 5 = -[-29 - 5] + 5 = 39$.

17. **Answer 048**
The horizontal distance between A and B is $11 - 3 = 8$, so the vertical distance between them is $8\sqrt{35}$, since they lie on a line of gradient $\sqrt{35}$. By Pythagoras' theorem, $AB^2 = 8^2(1 + 35) = 8^2 \times 6^2$, so $AB = 48$.

18. **Answer 003**
We are given that $2a + b = 15$, so $b = 15 - 2a$. Considering the possible values of a

from 1 to 9, we must eliminate:
$a = 1$ and $a = 2$ because $b > 9$,
$a = 5$ because $b = a$,
$a = 6$ because 636 is divisible by 4,
$a = 8$ and $a = 9$ because $b < 0$.
This leaves the three numbers 393, 474, and 717.

19. **Answer 004**
The sequence S_k contains 2019 if $2019 = 1 + mk$ for some m, that is, if k is a factor of 2018, so we need all the factors of 2018. Clearly $2018 = 2 \times 1009$, and (not so easily) 1009 is prime. (You need to test for divisibility by all primes up to 31, that is, less than $\sqrt{1009}$). Thus the four factors of 2018 are 1, 2, 1009, and 2018.

20. **Answer 010**
The sum of the roots of the equation is $-a$, so a must also be an integer. For rational roots, the discriminant $a^2 - 24a$ must be a perfect square, say $a^2 - 24a = d^2$, and we complete the square to give $(a-12)^2 = d^2 + 12^2$. The Pythagorean triples including 12 are $3 \times (3, 4, 5)$, $4 \times (3, 4, 5)$, $(5, 12, 13)$, and $(12, 35, 37)$, so $a - 12 = \pm 15, \pm 20$, ± 13, ± 37 or ± 12 (in the case that $d = 0$). This gives 10 values of a for which the equation has rational roots, and a quick check shows that in all 10 of these cases the roots are indeed integers.

21. **Answer 018**
In square metres, let x be the area covered by three layers of carpet, and z the area covered by exactly one layer. Then $x + 24 + z = 140$ and $3x + 2(24) + z = 200$. These equations simplify to $x + z = 116$ and $3x + z = 152$, so $2x = 36$ and $x = 18$. Alternatively, as with finding the number of elements in the union of three sets, it is possible to use a Venn diagram or the Principle of Inclusion and Exclusion.

22. **Answer 027**
The series ends after at most four games, and we can list the sequences of game winners leading to an overall series win for A:
AA (probability $\frac{1}{4}$);
ABA, BAA (probability $\frac{1}{8}$ each);
ABBA, BABA, BBAA (probability $\frac{1}{16}$ each).
Thus the probability of a series win for A is $\frac{1}{4} + \frac{2}{8} + \frac{3}{16} = \frac{11}{16}$, and $11 + 16 = 27$.

23. **Answer 021**
Since $YE \times ME = TTT = T \times 3 \times 37$, we know that one of the factors (say YE) is divisible by 3 and the other factor ME is divisible by 37, so $ME = 37$ or 74. If $ME = 74$, then $E = 4$, so $YE = 24$ or 54 or 84, which are all too large, since $74 \times 24 > 1000$.Thus $ME = 37$ and YE can only be 27, since 57 and 87 are too large. Therefore the equation is $27 \times 37 = 999$ and $E + M + T + Y = 7 + 3 + 9 + 2 = 21$.

24. **Answer 200**
Firstly $\tan(A\widehat{B}E) = \frac{EA}{AB} = \frac{1}{2}$, so

$$\tan(E\widehat{B}F) = \tan(45^\circ - \alpha) = \frac{1-\frac{1}{2}}{1+1.\frac{1}{2}} = \frac{1}{3}.$$

For convenience, let $x = 20\sqrt{3}$. Then $\triangle ABC$ has area $2x^2$ and $\triangle ABE$ has area x^2. By Pythagoras' theorem, $BE = x\sqrt{5}$, so $EF = \frac{1}{3}x\sqrt{5}$ and the area of $\triangle BEF$ is $\frac{5}{6}x^2$. Finally, the area of $\triangle CEF$ equals $2x^2 - x^2 - \frac{5}{6}x^2 = \frac{1}{6}x^2 = 200$.
The fact that $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3}) = 45^\circ$ can be proved in other ways, for example, by joining the points $(2;1)$ and $(1;3)$ to the origin and to each other to form an isosceles right-angled triangle. This breaks the right angle xOy into the angles $\tan^{-1}(\frac{1}{2})$, 45°, and $\tan^{-1}(\frac{1}{3})$.

Alternatively, drop a perpendicular from F to AC to intersect it at D. Then $\triangle CFA|||\triangle CBA$ and $\triangle EFD|||\triangle BEA$. Therefore, if we let $FD = y$, then $CD = y$ and $DE = 2y$. Hence $CE = 3y = 20\sqrt{3}y$ and so $y = \dfrac{20}{\sqrt{3}}$. Thus, area of $\triangle CFE = \dfrac{1}{2} \times 20\sqrt{3} \times \dfrac{20}{\sqrt{3}} = 200.$

25. **Answer 970**
Firstly, let $a = (\sqrt{3}+\sqrt{2})^2 = 5 + 2\sqrt{6}$, so we must estimate a^3. If $b = 5 - 2\sqrt{6}$, then $a + b = 10$ and $ab = 1$, so

$$1000 = (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a+b) + b^3 = a^3 + 30 + b^3.$$

It follows that $a^3 = 970 - b^3 = 970 - a^{-3}$, which is just less than 970.

Alternatively, instead of trying to compute $(\sqrt{3}+\sqrt{2})^6$ directly, we compute something slightly larger and easier to compute, because many terms cancel. Namely, using the binomial theorem, we compute $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6 = 2[27 + 15(18 + 12) + 8] = 970$. Since $0 < \sqrt{3} - \sqrt{2} < 1$, 970 is the smallest integer larger than $(\sqrt{3}+\sqrt{2})^6$.

24. **Antwoord 200**

Eerstens: $\tan(A\widehat{B}E) = \frac{EA}{AB} = \frac{1}{2}$, sodat

$$\tan(E\widehat{B}F) = \tan(45^\circ - \alpha) = \frac{1 - \frac{1}{2}}{1 + 1.\frac{1}{2}} = \frac{1}{3}.$$

Geriefshalwe laat ons $x = 20\sqrt{3}$. Dan is $\triangle ABC$ se oppervlakte $2x^2$ en $\triangle ABE$ se oppervlakte x^2. Uit Pythagoras se stelling is $BE = x\sqrt{5}$, sodat $EF = \frac{1}{3}x\sqrt{5}$ en die oppervlakte van $\triangle BEF$ is $\frac{5}{6}x^2$. Die oppervlakte van $\triangle CEF$ is dan gelyk aan $2x^2 - x^2 - \frac{5}{6}x^2 = \frac{1}{6}x^2 = 200$.
Die feit $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3}) = 45^\circ$ kan op ander maniere bewys word, bv. deur die punte $(2;1)$ en $(1;3)$ met die oorsprong en met mekaar te verbind om 'n gelykbenige reghoekige driehoek te vorm. Nou is die regte hoek xOy opgebreek in die hoeke $\tan^{-1}(\frac{1}{2})$, 45°, en $\tan^{-1}(\frac{1}{3})$.

Alternatiewelik, kan ons 'n loodlyn vanaf F op AC trek om AC in D te sny. Dan is $\triangle CFA|||\triangle CBA$ en $\triangle EFD|||\triangle BEA$. As $FD = y$, dan is $CD = y$ en $DE = 2y$. Dus is $CE = 3y = 20\sqrt{3}y$ en $y = \dfrac{20}{\sqrt{3}}$ en die oppervlakte van $\triangle CFE = \dfrac{1}{2} \times 20\sqrt{3} \times \dfrac{20}{\sqrt{3}} = 200$.

25. **Antwoord 970**

Eerstens, laat $a = (\sqrt{3} + \sqrt{2})^2 = 5 + 2\sqrt{6}$, en nou kan ons a^3 skat. As $b = 5 - 2\sqrt{6}$, is $a + b = 10$ en $ab = 1$, sodat

$$1000 = (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a+b) + b^3 = a^3 + 30 + b^3.$$

Dit volg nou dat $a^3 = 970 - b^3 = 970 - a^{-3}$, wat net kleiner is as 970.

Alternatiewelik, kan ons iets makliker en net groter as $(\sqrt{3} + \sqrt{2})^6$ probeer bereken, waar meer terme kan uitkanselleer. Deur die binomiaalstelling te gebruik, bereken ons dan $(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6 = 2[27 + 15(18 + 12) + 8] = 970$. Omdat $0 < \sqrt{3} - \sqrt{2} < 1$, is 970 die kleinste heelgetal groter as $(\sqrt{3} + \sqrt{2})^6$.

18. **Antwoord 003**
Uit $2a + b = 15$ het ons $b = 15 - 2a$. Uit die moontlike waardes van a van 1 tot 9, moet ons sekere waardes elimineer:
$a = 1$ en $a = 2$, want $b > 9$,
$a = 5$, want $b = a$,
$a = 6$, want 636 is deelbaar deur 4,
$a = 8$ en $a = 9$, want $b < 0$.
Dit laat die drie getalle, naamlik 393, 474, en 717.

19. **Antwoord 004**
Die ry S_k bevat 2019 as $2019 = 1 + mk$ vir sommige m, i.e., as k 'n faktor van 2018 is, en gevolglik benodig ons al die faktore van 2018. Ons weet dat $2018 = 2 \times 1009$, en (dalk nie so maklik nie) dat 1009 priem is. (Jy moet vir deelbaarheid deur alle priemgetalle toets tot by 31, m.a.w. kleiner as $\sqrt{1009}$). Dus is die vier faktore van 2018 gelyk aan 1, 2, 1009 en 2018.

20. **Antwoord 010**
Die som van die wortels van die vergelyking is $-a$, sodat a ook 'n heelgetal moet wees. Vir rasionale wortels is die diskriminant $a^2 - 24a$ 'n volkome vierkant, $a^2 - 24a = d^2$, en met vierkantsvoltooiing het ons dan $(a-12)^2 = d^2 + 12^2$. Die Pythagorasdrietalle wat 12 insluit, is $3 \times (3,4,5)$, $4 \times (3,4,5)$, $(5,12,13)$, en $(12,35,37)$, sodat $a - 12 = \pm 15, \pm 20$, ± 13, ± 37 of ± 12 (in die geval waar $d = 0$). Dit gee 10 waardes vir a waarvoor die vergelyking rasionale wortels het, en 'n vinnige toets vir elke geval toon dat al hierdie wortels inderdaad heelgetalle is.

21. **Antwoord 018**
Laat x die oppervlakte, in vierkante meter, wees van die deel wat bedek is deur drie lae mat. Laat z die oppervlakte wees van die deel wat deur slegs een laag mat bedek word. Dan is $x + 24 + z = 140$ en $3x + 2(24) + z = 200$. Hierdie vergelykings vereenvoudig na $x + z = 116$ en $3x + z = 152$, sodat $2x = 36$ en $x = 18$.
Alternatiewelik, soos ons die aantal elemente vind in die vereniging van drie versamelings is dit moontlik om 'n Venn diagram te gebruik of the Beginsel van Insluiting en Uitsluiting.

22. **Antwoord 027**
Die reeks eindig na 'n maksimum van vier spelle, en ons kan dan 'n lys maak van die spelwenners wat lei tot 'n totale reeksoorwinning vir A:
AA (waarskynlikheid $\frac{1}{4}$);
ABA, BAA (waarskynlikheid $\frac{1}{8}$ each);
ABBA, BABA, BBAA (waarskynlikheid $\frac{1}{16}$ elk.
Dus is die waarskynlikheid van 'n reeksoorwinning vir A gelyk aan $\frac{1}{4} + \frac{2}{8} + \frac{3}{16} = \frac{11}{16}$, en $11 + 16 = 27$.

23. **Antwoord 021**
Omdat $YE \times ME = TTT = T \times 3 \times 37$, weet ons dat een van die faktore (bv. YE) deelbaar is deur 3 en die ander faktor ME deelbaar is deur 37, sodat $ME = 37$ of 74. As $ME = 74$, dan is $E = 4$, en $YE = 24$ of 54 of 84, wat almal te groot is, want $74 \times 24 > 1000$. Dus is $ME = 37$ en YE kan slegs 27 wees omdat 57 en 87 albei te groot is. Die vergelyking is dan $27 \times 37 = 999$ en $E + M + T + Y = 7 + 3 + 9 + 2 = 21$.

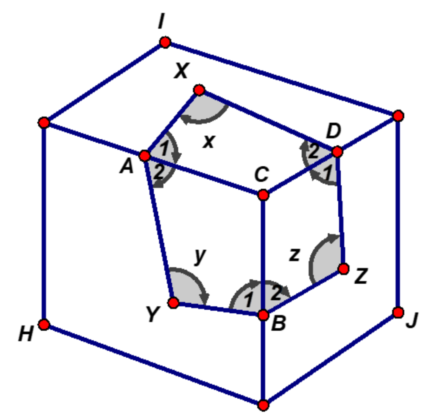
11. **Antwoord 477**
Vanaf 25 tot 99 is daar $99 - 25 + 1 = 75$ getalle met twee syfers elk, en vanaf 100 tot 208 is daar $208 - 100 + 1 = 109$ getalle met drie syfers elk. Die totale aantal syfers is dus $2 \times 75 + 3 \times 109 = 477$.

12. **Antwoord 061**
Thembi het k sleutelringe, en $k-1$ is positief en deelbaar deur $2, 3, 4, 5$. Die kleinste waarde van $k-1$ is die KGV van $2, 3, 4, 5$, wat gelyk is aan 60, en dus is $k = 61$.

13. **Antwoord 401**
Die som van enige twee sye van 'n driehoek moet groter wees as die derde sy en as die onbekende sy s cm is, is $150 + s > 200$. Die kleinste heelgetalwaarde is dan $s = 51$, en die kleinste omtrek is $200 + 150 + 51 = 401$.

14. **Antwoord 009**
As die tiene- en enesyfers van die getal onderskeidelik T en U is, is die getal $10T+U$, wat gelyk is aan $T \times U + T + U$. Dan is $9T = TU$, sodat $U = 9$ (omdat $T \neq 0$), en T kan enige waarde van 1 tot 9 hê. [As $T = 0$, dan is dit nie 'n 2-syfergetal nie.]

15. **Antwoord 270**



Aangesien die paaie reguit lyne is, is die pare hoeke A_1 en A_2, B_1 en B_2, en D_1 en D_2 supplementêre hoeke. Beskou die som van al die hoeke van die drie vierhoeke $XACD$, $AYBC$ en $CBZD$. Dan het ons die volgende:

$$\begin{aligned} x + \angle A + y + \angle B + z + \angle D + 3 \times \angle C &= 3 \times 360^\circ \\ \therefore x + 180^\circ + y + 180^\circ + z + 180^\circ + 3 \times 90^\circ &= 3 \times 360^\circ \\ \therefore x + y + z &= 270^\circ \end{aligned}$$

'n Veel makliker en korter oplossing is om die spesiale geval te beskou waar respektiewelik Y na H, X na I, en Z na J geskuif word, waaruit dit dan onmiddellik volg dat $x + y + z = 3 \times 90^\circ = 270^\circ$. Ons kan hierdie vereenvoudigde metode gebruik want die som van die aangeduide hoeke is duidelik onafhanklik van die posisies van X, Y en Z. (Hierdie strategie werk mits 'n gedeelte van alle aangeduide paaie op die onderskeie vlakke bly wat die drie vierhoeke $XACD$, $AYBC$ and $CBZD$ bevat. Byvoorbeeld, om X na C, Y na die hoekpunt direk onder C, en Z na J te skuif sal nie geldig wees nie, aangesien geen gedeelte van die pad XDZ dan meer op die reghoekige vlak sal lê wat die vierhoek $XACD$ bevat nie.)

16. **Antwoord 039**
As $g(x) = f(x) - 5 = ax^7 + bx^3 + cx$, dan bevat $g(x)$ slegs onewe magte van x, en is $g(-x) = -g(x)$. Dus $f(7) = g(7) + 5 = -g(-7) + 5 = -[f(-7) - 5] + 5 = -[-29 - 5] + 5 = 39$.

17. **Antwoord 048**
Die horisontale afstand tussen A en B is $11 - 3 = 8$, sodat die vertikale afstand tussen hulle gelyk is aan $8\sqrt{35}$, omdat hulle op 'n lyn met helling $\sqrt{35}$ lê. Volgens die stelling van Pythagoras is $AB^2 = 8^2(1 + 35) = 8^2 \times 6^2$, sodat $AB = 48$.

DIE SUID-AFRIKAANSE WISKUNDE OLIMPIADE
SENIOR TWEEDE RONDTE 2018
Oplossings

1. **Antwoord 820**
   $2018 - (1000 + 100) - (\sqrt{100})^2 + 2 = 2018 - 1100 - 100 + 2 = 820.$

2. **Antwoord 012**
   Van links na regs is daar onderskeidelik ses blokkies, vier blokkies en twee blokkies op elke stapel. Die totale aantal blokkies is $6 + 4 + 2 = 12$.

3. **Antwoord 009**
   Die twee kleiner getalle moet positief, verskillend en so klein as moontlik wees, i.e., 1 en 2. Die grootste getal is dus $12 - 1 - 2 = 9$.

4. **Antwoord 031**
   Die hoeveelheid wat gedurende die vyf dae opgelos word, is $1, 2, 4, 8, 16$, en die totale hoeveelheid probleme wat opgelos is, is $1 + 2 + 4 + 8 + 16 = 31$, wat ook deur die formule $2^5 - 1$ verkry kan word.

5. **Antwoord 052**
   Die oppervlakte van 'n vierkant met omtrek P is $(\frac{1}{4}P)^2 = \frac{1}{16}P^2$ en die oppervlakte van 'n sirkel met omtrek P is $\pi(\frac{P}{2\pi})^2 = \frac{1}{4\pi}P^2$. Die verhouding van die oppervlaktes is dus $\frac{16}{4\pi} = \frac{4}{\pi}$, en dus is die oppervlakte van die sirkel $\frac{4}{\pi}(13\pi) = 52$.

6. **Antwoord 006**
   As die kind se ouderdom x is, is $x + 3 = (x - 3)^2$, sodat $x^2 - 7x + 6 = 0$, met oplossings $x = 6$ of $x = 1$. [Die antwoord $x = 1$ is nie geldig nie, omdat ons dan het dat $x + 3 = 4$ en $x - 3 = -2 \neq \sqrt{4}$. Onthou dat vierkantwortels nie negatief kan wees nie, en $\sqrt{4} = +2$.]

7. **Antwoord 968**
   As die oorspronklike driehoek basis b het en hoogte h, dan is $\frac{1}{2}bh = 800$. Die nuwe driehoek se basis is $1.1 \times b$ en die hoogte is $1.1 \times h$, en dus is die oppervlakte $\frac{1}{2}(1.1b)(1.1h) = 1.21 \times \frac{1}{2}bh = 1.21 \times 800 = 968$.

8. **Antwoord 960**
   Van 51 tot 141 is daar tien sulke getalle, en is
   $51 + 61 + \cdots + 131 + 141 = 10 \times \frac{1}{2}(51 + 141) = 5 \times 192 = 960,$
   deur die formule vir die som van 'n rekenkundige reeks te gebruik. [Andersins kan dit eenvoudig net bymekaar getel word.]

9. **Antwoord 100**
   $AB \| CD$, en dus volg dit dat $A\widehat{B}C = 180° - 110° = 70°$, sodat $A\widehat{B}D = 70° - 30° = 40°$.

   Dan is $B\widehat{A}D = A\widehat{B}D = 40°$, omdat $AD = BD$, en dus is $A\widehat{D}B = 180° - (40° + 40°) = 100°$.

10. **Antwoord 032**
    Laat die groter getal n wees sodat die kleiner een dan gelyk is aan $n - 8$, met $4(n - 8) = 3n$, wat $n = 32$ gee.