SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

10 May 2017 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

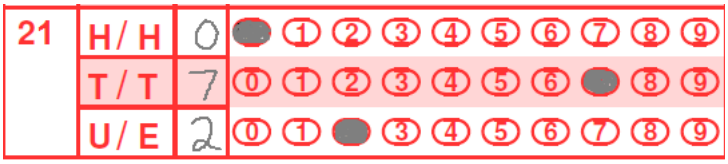
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following **example question**:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom.
The three digits that you write down will not be marked, since it is only for your convenience - only the shaded circles will be marked.

Part A: Three marks each

1. A train travels at constant speed of 100 km/h. How many kilometres does it travel in 12 minutes?

2. Twelve identical blue socks and twelve identical black socks are mixed up in a drawer. How many socks must be taken out without looking, to be certain of getting two matching socks?

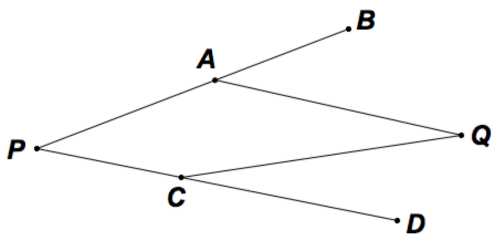
3. A quadrilateral has an area of 16 square units. If each of the sides is made three times longer, determine the area of the enlarged quadrilateral.

4. What is the average of the first 100 positive even integers?

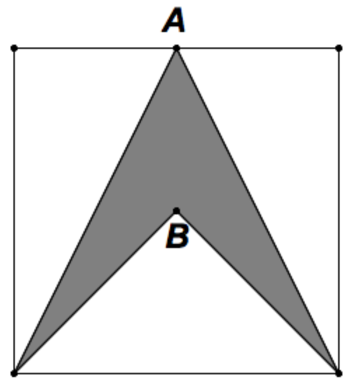
5. If $x^2 = 2017$, what is the value of $(x-40)(x+40)$?

6. How many digits does the number $N = 2^{12} \times 5^8$ have?

7. Angles $B\widehat{A}Q$ and $D\widehat{C}Q$ are respectively 42° and 38°. Calculate the sum of $B\widehat{P}D$ and $A\widehat{Q}C$ in degrees.



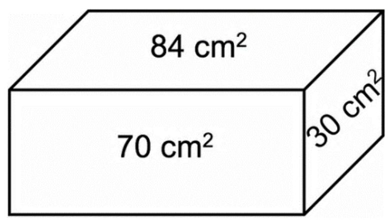
8. A shaded arrowhead is inscribed in a square of side length 4 as shown. Point A is the midpoint of a side of the square and point B is at the centre of the square. What is the area of the shaded arrowhead?



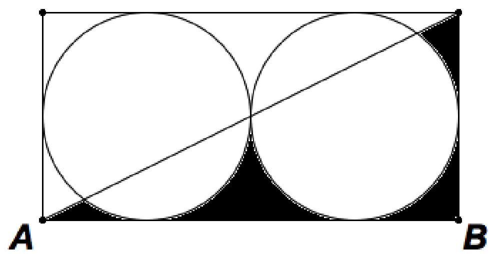
9. In the Fibonacci sequence, $1; 1; 2; 3; 5; \ldots$, each term after the second is the sum of the previous two terms. How many of the first 100 terms of the Fibonacci sequence are odd?

10. If $x \oplus y = (x+y)^2 - (x-y)^2$, calculate $\sqrt{5} \oplus \sqrt{5}$.

11. The areas of the faces of a rectangular box are $84\ \text{cm}^2$, $70\ \text{cm}^2$ and $30\ \text{cm}^2$ respectively. What is the volume of the box in cm^3?



12. The area of the shaded region in the rectangle is $(100 - 25\pi)\ \text{cm}^2$. Calculate the length of side AB in cm.



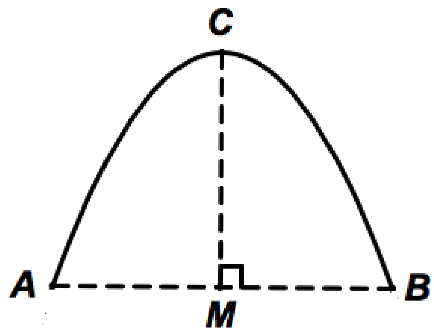
13. Let r be the result of doubling both the base and the exponent of 3^b (with b non-zero). If r equals the product of 3^b and x^b, calculate x.

14. Half of the children at Peter's birthday party drank red cooldrinks, one third drank green cooldrinks while 15 children drank no cooldrinks. None of the children drank both colours of cooldrink. How many children were at the party?

15. How many whole numbers from 1 to 100 have exactly five different factors?

Part B: Five marks each

16. A parabolic arch has a height of 16 m and a span AB of 40 m. What is the height, in m, of the arch at a point 5 m from the centre M?

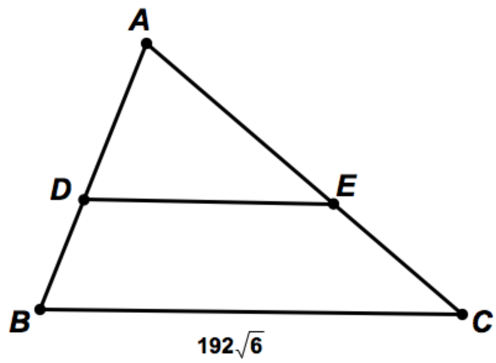


17. Sixty equal pieces of string are each cut into two at a randomly selected point. In how many of these 60 cases would you expect the longer piece of the cut string to be at least twice as long as the shorter piece?

18. In a family with three children at least one child is a girl. The probability that at least two of the children are girls is $\frac{x}{56}$. Assuming that the probability of having a girl is equal to the probability of having a boy, find the value of x.

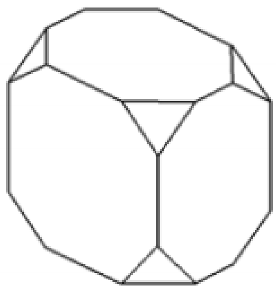
19. At the end of the 2016 school year, 5% of learners at O.R. Tambo High School left the school. At the beginning of 2017, 189 new learners joined the school. As a result of these two changes the number of learners in 2017 is 10% higher than in 2016. How many learners left O.R. Tambo High School at the end of 2016?

20. Side BC of triangle ABC is $192\sqrt{6}$. Inside the triangle a line segment DE is drawn parallel to BC forming trapezium $DBCE$ with area one-third that of the triangle ABC. What is the length of DE?



Part C: Six marks each

21. The eight corners of a cube are cut off to form eight small triangular surfaces, giving rise to a new solid with 24 vertices, as illustrated in the picture. Each pair of vertices of this new solid is then joined by a line segment. How many of these line segments pass through the interior of the solid and do not lie on the surface?



22. The population of a small town in 2015 was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, in 2017, with an additional increase of 100, the population is again a perfect square. What is the sum of the digits of the original population in 2015?

23. For $-1 < r < 1$, $r \neq 0$, let $S(r)$ denote the sum of the infinite geometric series

$$12 + 12r + 12r^2 + 12r^3 + \cdots.$$

If $S(r)S(-r) = 2016$, find $S(r) + S(-r)$.

24. A certain card shuffling machine always rearranges cards in the same way. If 13 playing cards are inserted in the order

$$\text{Ace}, 2, 3, 4, 5, 6, 7, 8, 9, 10, \text{Jack}, \text{Queen}, \text{King},$$

then the machine returns them in the order

$$6, \text{Ace}, 7, 4, 2, 9, \text{King}, \text{Jack}, 10, \text{Queen}, 8, 5, 3.$$

What is the least number of times that the cards can be fed through the machine to get the original order back?

25. If M is the midpoint of side BC of $\triangle ABC$, $A\widehat{C}B = 30°$ and $B\widehat{M}A = 45°$, calculate the size of $B\widehat{A}C$ in degrees.