Junior Second Round 2012: SOLUTIONS

1.	**E**	If Arthur gets as much as 6 for each subject, he still only achieves a total of $6 \times 8 = 48$. To score an extra 2 points he needs to convert 2 of these 6s to 7s.
2.	**E**	He gives away 50% to the friend, then 10% of the remaining 50%, which is 5%, to charity. So he keeps $100 - 50 - 5 = 45\%$.
3.	**E**	For divisibility by 5 the last digit (Y) must be either 0 or 5; but then it must be 0 since the number is divisible by 4. Now the sum of the digits is $X + 14$, which must be divisible by 9, so we require $X = 4$. That means $X + Y = 4 + 0$.
4.	**B**	Since PS||QR, $x = \mathrm{Q\hat{R}P}$. Each exterior angle of the hexagon is 60°, so $\hat{\mathrm{Q}} = 120°$, and then in the isosceles triangle QPR, $\mathrm{Q\hat{R}P} = 30°$. OR: join RT, and then Δ PRT is equilateral, so that $\mathrm{R\hat{P}T} = 60°$. But the triangle is bisected by PS, so $x = 30°$.
5.	**A**	Removing the brackets shows we have $1 - 2 + 3 - 4 + \ldots - 100$, which is $(1 - 2) + (3 - 4) + \ldots + (99 - 100) = (-1) \times 50 = -50$.
6.	**D**	Let the top left number be x; then the one to its right is $x + 1$ and the one below it is $x + 7$. So the sum of the four numbers is $x + (x + 1) + (x + 7) + (x + 8) = 4x + 16$. This is 312, so $4x = 296$ and $x = 74$.
7.	**A**	The equations rearrange to become $x - 1 = xy$ and $y + 1 = xy$. That means $x - 1 = y + 1$ (both equal xy) and therefore $x - y = 2$.
8.	**E**	Let the three primes be p, q and r. The prime factorisation of N must then be pqr (none of them can be squared). Then the factors of N are 1, p, q, r, pq, qr, pr and pqr, which makes 8 in all.
9.	**D**	$bd < cd$ means $b < c$. If $bd < ab$ then $d < a$ and $cd < bc$ shows $d < b$. $ab < bc < ae$ requires $b < e$. Thus every letter except d is shown to exceed at least one of the others, so d must be the smallest.
10.	**C**	If the radii are r, $2r$ and $3r$, then the area of the outer ring is $\pi(3r)^2 - \pi(2r)^2 = 5\pi r^2$, while the area of the central circle is just πr^2. So the ratio of the areas is 5:1, which is the ratio of the likelihoods.
11.	**C**	If x is the number of people with chocolates only, then $2x$ have toffees only, and so $x + 2x = 3x$ have both and then $4x$ is the number of all the people who have chocolates. That means the required proportion is $3x/4x$ OR let x be the number of people with both; it is also the number of people with toffees only plus the number of people with chocolates only. Then $\frac{2}{3}x$ is the number with toffees only, $\frac{1}{3}x$ is the number with chocolates only, and the proportion we want is $\dfrac{x}{x+\frac{1}{3}x} = \dfrac{3}{3+1} = \dfrac{3}{4}$
12.	**B**	If Olga's speed is v, then Dimitri's speed is $2v$ and the average of these speeds is $1.5v$. The distance covered by Olga is $20v$, and is the same for all three people, so Boris's time is $\dfrac{20v}{1.5v} = \dfrac{40}{3}$ minutes.
13.	**D**	Imagine the cards are numbered 1, 2, 3, …. Alice turns over the even ones, of which there are 50. Then Brenda turns over every third remaining card, i.e. one third of the first 48 of those cards, so another 16. That means that $100 - 50 - 16 = 34$ cards have not been turned over.

14. **C** If b is the cost of a chocolate bar and c the cost of a packet of chips, then $2b + 3c =$ R 19 and also $3c - 2b =$ R 3. Subtracting these shows $4b = 16$ so that $b = 4$. Finally, Jane pays for one more chocolate bar than John, so her total bill is R 19 + R 4 = R 23.00.

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15. **B** The distance travelled by Peter is $2\pi r$ where r is the radius of the circle. The length of each side of the square is $r\sqrt{2}$, so in the time that Peter travels, Quentin covers $\frac{2\pi r}{r\sqrt{2}} = \pi\sqrt{2}$ sides of the square. This is $\sqrt{2\pi^2}$, and since π is a little more than 3, this is a little more than $\sqrt{18}$, i.e. between 4 and 5. So Quentin is on his fifth side, i.e. between A and B.

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16. **C** The value of the terms in the sequence changes after 1 or 1+2 or 1+2+3 or .. . terms. The largest such number not exceeding 81 is $78 = 1 + 2 + 3 + ... + 12$, so by the 81st term the sequence has completed its twelve 12s and is now having terms equal to 13.

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17. **A** Suppose there are $5x$ seniors and $3x$ juniors. Then there are $\frac{3}{5}.3x$ junior boys and $\frac{2}{5}.5x$ senior boys, which is $\frac{19}{5}x$ boys altogether. Since this must be a whole number, we need x to be at least 5, which means that the total number of pupils in the school is 5.5 + 3.5 = 40.

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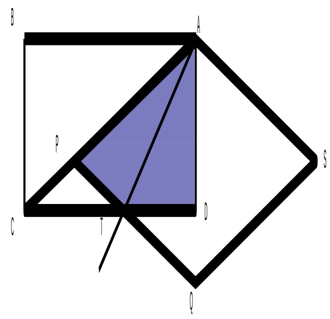
18. **C** Since AC = $\sqrt{2}$ and AP = 1, PC = $\sqrt{2}-1$.
But then PT is also $\sqrt{2}-1$ because ΔPCT is isosceles and right-angled. The area of ΔAPT is $\frac{1}{2}$AP.PT and thus $\frac{1}{2}\times 1\times(\sqrt{2}-1)$, while the area of the shaded quadrilateral is twice that, by symmetry, and therefore $\sqrt{2}-1$.



OR: with PT = $\sqrt{2}-1$ the area of Δ PCT is
$\frac{1}{2}\left(\sqrt{2}-1\right)^2 = \frac{1}{2}(2-2\sqrt{2}+1) = \frac{3}{2}-\sqrt{2}$, and the shaded area is
ΔACD – ΔPCT = $\frac{1}{2}-\left(\frac{3}{2}-\sqrt{2}\right) = \sqrt{2}-1$

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19. **E** If the digits of X are h, t, u, then Y has digits u, t, h and adding the numbers gives $101h + 101u + 20t$, which is $101(h + u) + 20t$. Since $1535 = 101 \times 15 + 20 \times 1$, we must have $h + u = 15$ and $t = 1$, and therefore $h + u + t = 16$.

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20. **B** The areas of the parts are as indicated.
But Δ ADQ = 2.ΔAQC means DQ = 2.QC, so DQ = $\frac{2}{3}$.
Now by Pythagoras AQ = $\sqrt{1^2+\left(\frac{2}{3}\right)^2} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$
while QC = $\frac{1}{3}$. So the required ratio is $\sqrt{13}$