SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 FIRST ROUND
JUNIOR SECTION: GRADE 9

15 March 2017 **Time: 60 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by five answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 5 marks.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *The centre page is an information and formula sheet. Please tear out the page for your own use.*
7. *Start when the invigilator tells you to do so.*
8. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns, ASTEMI

1. The value of $\dfrac{2017-1017}{500}$ is

(A) 1 (B) 1,5 (C) 2 (D) 2,5 (E) 3

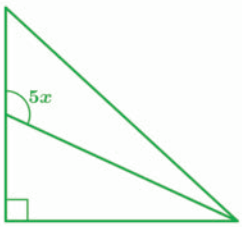
2. Ravi plans to leave on a car trip at 10h15. If he leaves 30 minutes earlier than planned, and the trip takes 2 hours 10 minutes, at what time does he reach his destination?

(A) 11h45 (B) 11h55 (C) 12h05 (D) 12h15 (E) 12h25

3. A certain tree grows $\frac{1}{2}$ m per year for 20 years and then $\frac{1}{3}$ m every year after that. If the tree is now 13 m high, how old is the tree in years?

(A) 23 (B) 25 (C) 27 (D) 29 (E) 32

4. Which one of the following could be a value of x in degrees?

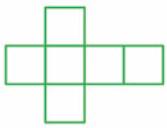


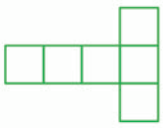
(A) 10 (B) 15 (C) 20 (D) 40 (E) 50

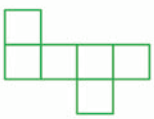
5. If the product of 6 integers is negative, at most how many of the integers can be negative?

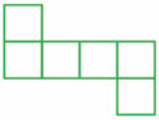
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

6. Which one of the following figures cannot be folded into a closed cube?

(A) 

(B) 

(C) 

(D) 

(E) 

7. Five sweets cost R12 more than one sweet. What is the cost of one sweet?

(A) R1 (B) R2 (C) R3 (D) R4 (E) R5

8. Let a, b and c be positive integers. If $a + b = 4$, $b + c = 8$ and $c + a = 6$. What is the value of $a \times b \times c$?

(A) 8 (B) 9 (C) 12 (D) 15 (E) 18

9. Every third visitor to a show is given a pen while every fifth visitor is given a bag. Of the first 200 visitors, how many receive a pen and a bag?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

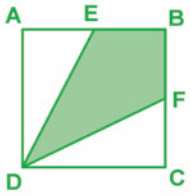
10. The diagram shows a perfectly balanced scale:

Which of the following could be placed on the right-hand side of the scale shown below to make it perfectly balanced?

(A)  (B)  (C) 

(D)  (E) 

11. ABCD is a square of side 4, and E and F are the midpoints of sides AB and BC respectively. What is the area of the quadrilateral EBFD?

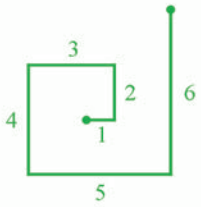


(A) 8 (B) 10 (C) 12 (D) 14 (E) 16

12. If $\sqrt{xy} = 4$ and $\sqrt[3]{xyz} = 2$ then find the value of z.

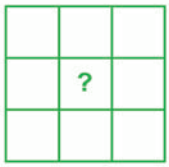
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

13. A hiker walks 1 km east, then 2 km north, then 3 km west, then 4 km south, then 5 km east and finally 6 km north. The hiker's straight-line distance from the starting point is



(A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

14. The numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are placed once each in the grid so that the average of the eight outer numbers is equal to the central number. What is the central number?

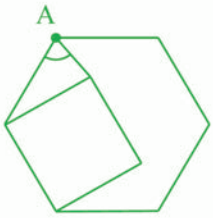


(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

15. Three randomly chosen faces of a cube are painted red. What is the probability that there are at least two adjacent red faces?

(A) 70% (B) 75% (C) 85% (D) 90% (E) 100%

16. The diagram shows a square inside a regular hexagon. What is the size of the marked angle at vertex A?



(A) 60° (B) 65° (C) 70° (D) 75° (E) 80°

17. Andy and Betty both choose an integer from 1 to 10. In how many ways can Andy's number be bigger than Betty's?

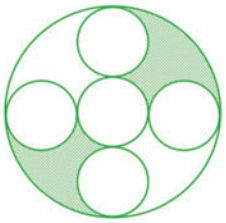
(A) 45 (B) 50 (C) 55 (D) 60 (E) 65

18. Which one of the following is divisible by all of the integers from 1 to 10 inclusive?

(A) 27×36 (B) 27×48 (C) 35×36 (D) 42×45 (E) 35×72

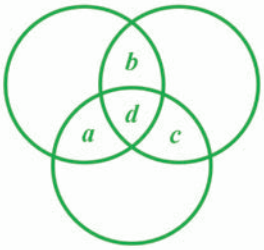
19. In the figure alongside, the area of each of the five small circles is 2017 cm^2. They are arranged in the form of a cross inside a circle whose radius is three times as large as that of each small circle.

What is the area of the shaded region?



(A) 1008 cm^2 (B) 2017 cm^2 (C) 4034 cm^2 (D) 6051 cm^2 (E) 7060 cm^2

20. Three circular carpets can cover a combined floor area of 200 m^2. If we overlap the carpets they cover a floor area of 140 m^2. If the area covered by exactly two layers (a, b and c) is 24 m^2, determine the floor area covered by three layers (d).



(A) 14 m^2 (B) 18 m^2 (C) 22 m^2 (D) 26 m^2 (E) 30 m^2

Formula and Information Sheet

1.1 The natural numbers are: 1; 2; 3; 4; 5; …

1.2 The whole numbers are: 0; 1; 2; 3; 4; 5; …

1.3 The integers are: …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$a \times a \times a \times a \times \ldots \times a = a^n$ (n factors of a)

(a is the base and n is the index (exponent))

3.2 Factorial notation:

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$1 \times 2 \times 3 \times \ldots \times n = n!$

3.3 $1 + 2 + 3 + 4 \ldots + n = \frac{1}{2}n(n+1)$

4 Area of a

4.1	triangle is:	$\frac{1}{2} \times$ (base $\times$ height) $= \frac{1}{2}(b.h)$
4.2	rectangle is:	length $\times$ width $= lw$ length $\times$ breadth $= lb$
4.3	square is:	side $\times$ side $= s^2$
4.4	rhombus is:	$\frac{1}{2} \times$ (product of diagonals)
4.5	trapezium is:	$\frac{1}{2} \times$ (sum of parallel sides) $\times$ height
4.6	circle is:	πr^2 (r = radius)

5 Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh$ (h = height)

5.2 sphere is: $4\pi r^2$

6 Perimeter of a:

6.1 rectangle is: $2 \times \text{length} + 2 \times \text{breadth}$

$2l + 2b$

or $2l + 2w$ (w = width)

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s \times s \times s = s^3$

8.2 rectangular prism is: $l \times b \times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section × perpendicular height

or area of base × perpendicular height

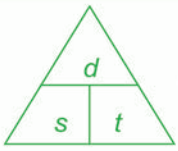
9.2 Surface area of a right prism is: (perimeter of base × h) + (2 × area of base)

10. Sum of the interior angles of a polygon is: $180^\circ(n-2)$ [n = number of sides]

11. Distance = speed × time ($d = s \times t$)

Speed = distance ÷ time ($s = \frac{d}{t}$)

Time = distance ÷ speed ($t = \frac{d}{s}$)

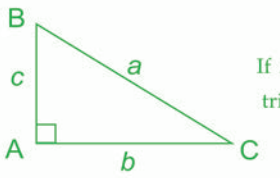


$d = s \times t$

$s = \frac{d}{t}$

$t = \frac{d}{s}$

12. Pythagoras:



If ΔABC is a right-angled triangle, then $a^2 = b^2 + c^2$

13. Conversions:

1 cm³ = 1 mℓ ; 1000 cm³ = 1 ℓ

1000 m = 1 km ; 1000 g = 1 kg ; 100 cm = 1 m

5. Buite-oppervlakte van 'n

5.1 regte prisma is: $2lb + 2lh + 2bh$ (h = hoogte)

5.2 sfeer is: $4\pi r^2$

6 Omtrek van 'n:

6.1 reghoek is: 2×lengte + 2×breedte
$2l + 2b$

6.2 vierkant is: $4s$

7. Omtrek van 'n sirkel is: $2\pi r$

8. Volume van 'n:

8.1 kubus is: $s \times s \times s = s^3$

8.2 reghoekige prisma is: $l \times b \times h$

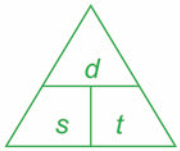
8.3 silinder is: $\pi r^2 h$

9.1 Volume van 'n regte prisma is: oppervlakte van dwarssnit × hoogte
of oppervlakte van basis × hoogte

9.2 Buite-oppervlakte van 'n regte prisma is:
(omtrek van basis × h) + (2 × oppervlakte van basis)

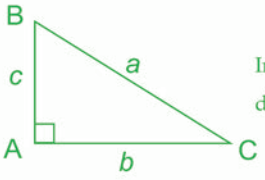
10. Som van die binnehoeke van 'n veelhoek is: $180^\circ(n-2)$ [n = aantal sye]

11. Afstand is: spoed x tyd $(d = s \times t)$
Spoed = afstand ÷ tyd $(s = \frac{d}{t})$
Tyd = afstand ÷ spoed $(t = \frac{d}{s})$



$d = s \times t$
$s = \frac{d}{t}$
$t = \frac{d}{s}$

12. Pythagoras:



Indien ΔABC 'n reghoekige driehoek is, dan sal $a^2 = b^2 + c^2$

13. Omskakelings:
1 cm³ = 1 mℓ; 1000 cm³ = 1 ℓ
1000 m = 1 km; 1000 g = 1 kg ; 100 cm = 1 m

Formule- en Inligtingblad

1.1 Die natuurlike getalle is: 1; 2; 3; 4; 5; …

1.2 Die telgetalle is: 0; 1; 2; 3; 4; 5; …

1.3 Die heelgetalle is: …, –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In die breuk $\frac{a}{b}$, word a die teller en b die noemer genoem.

3.1 Eksponensiële notasie:

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$a \times a \times a \times a \times \ldots\ldots \times a = a^n$ (n faktore van a)

(a is die grondtal en n is die indeks (eksponent))

3.2 Fakulteitnotasie:

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$1 \times 2 \times 3 \times \ldots. \times n = n!$

3.3 $1 + 2 + 3 + 4 \ldots. + n = n(n + 1)/2$

4 Oppervlakte van 'n:

4.1 driehoek is: $\frac{1}{2} \times$ (basis $\times$ loodregte hoogte) = $\frac{1}{2}(b.h)$

4.2 reghoek is: lengte $\times$ breedte = lb

4.3 vierkant is: sy $\times$ sy = s^2

4.4 ruit (rombus) is: $\frac{1}{2}$ (produk van die diagonale)

4.5 trapesium is: $\frac{1}{2}$(som van ewewydige sye)$\times$hoogte

4.6 sirkel is: πr^2 (r = radius)

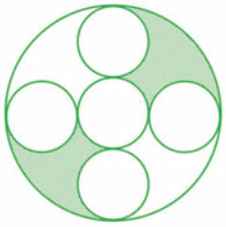
17. Andy en Betty kies elkeen ‘n heelgetal tussen 1 en 10. Op hoeveel maniere kan Andy se getal groter as Betty s’n wees?

(A) 45 (B) 50 (C) 55 (D) 60 (E) 65

18. Watter een van die volgende getalle is deelbaar deur alle heelgetalle vanaf 1 tot 10 inklusief?

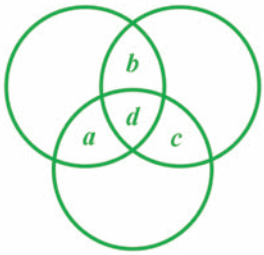
(A) 27×36 (B) 27×48 (C) 35×36 (D) 42×45 (E) 35×72

19. In die figuur hiernaas, is die oppervlakte van elkeen van die vyf klein sirkels 2017 cm^2. Hulle word gerangskik in die vorm van ‘n kruis binne-in ‘n sirkel met radius drie keer so groot soos die van elke klein sirkel. Wat is die oppervlakte van die gearseerde gebied?



(A) 1008 cm^2 (B) 2017 cm^2 (C) 4034 cm^2 (D) 6051 cm^2 (E) 7060 cm^2

20. Drie sirkelvormige matte kan ‘n totale vloeroppervlakte van 200 m^2 bedek. Indien ons die matte laat oorvleuel, bedek hulle ‘n vloeroppervlakte van 140 m^2. Indien die oppervlakte bedek deur presies twee matte (a, b en c) 24 m^2 is, bepaal die vloeroppervlakte wat deur al drie matte (d) gelyktydig bedek word.

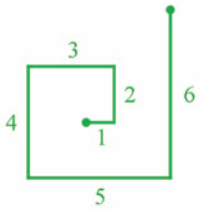


(A) 14 m^2 (B) 18 m^2 (C) 22 m^2 (D) 26 m^2 (E) 30 m^2

12. Indien $\sqrt{xy} = 4$ en $\sqrt[3]{xyz} = 2$ bepaal die waarde van z.

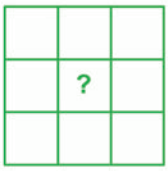
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

13. 'n Stapper loop 1 km oos, dan 2 km noord, dan 3 km wes, dan 4 km suid, dan 5 km oos en laastens 6 km noord. Die reglynige afstand tussen die stapper se beginpunt en eindpunt is



(A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

14. Die getalle 1, 2, 3, 4, 5, 6, 7, 8 en 9 word elkeen een keer in die rooster langsaan geplaas op so 'n wyse dat die gemiddeld van die agt buitenste getalle gelyk is aan die middelste getal. Wat is die middelste getal?

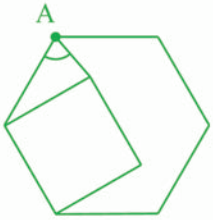


(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

15. Drie willekeurige gekose aansigte van 'n kubus word rooi geverf. Wat is die waarskynlikheid dat daar tenminste twee aangrensende rooi aansigte is?

(A) 70% (B) 75% (C) 85% (D) 90% (E) 100%

16. Die diagram toon 'n vierkant binne-in 'n reëlmatige heksagoon. Wat is die grootte van die gemerkte hoek by A?



(A) 60° (B) 65° (C) 70° (D) 75° (E) 80°

7. Vyf lekkers kos R12 meer as een lekker. Wat is die koste van een lekker?

(A) R1 (B) R2 (C) R3 (D) R4 (E) R5

8. Laat a, b en c positiewe heelgetalle wees. Indien $a + b = 4$, $b + c = 8$ en $c + a = 6$, wat is die waarde van $a \times b \times c$?

(A) 8 (B) 9 (C) 12 (D) 15 (E) 18

9. Aan elke derde besoeker by 'n vertoning word 'n pen gegee terwyl aan elke vyfde een 'n sak gegee word. Hoeveel van die eerste 200 besoekers kry beide 'n pen en 'n sak?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

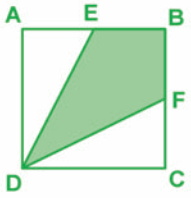
10. Die diagram stel 'n perfek gebalanseerde skaal voor:

Watter een van die volgende kan aan die regterkant van die skaal geplaas word om die skaal perfek te laat balanseer?

(A) 
(B) 
(C) 

(D) 
(E) 

11. ABCD is 'n vierkant van sylengte 4. E en F is die middelpunte van sye AB en BC onderskeidelik. Wat is die grootte van die oppervlakte van vierhoek EBFD?



(A) 8 (B) 10 (C) 12 (D) 14 (E) 16

1. Die waarde van $\dfrac{2017-1017}{500}$ is

(A) 1 (B) 1,5 (C) 2 (D) 2,5 (E) 3

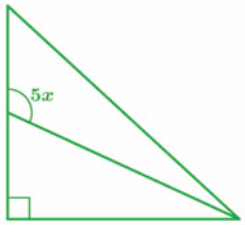
2. Ravi beplan om op 'n reis te vertrek om 10h15. Indien hy 30 minute vroeër as beplan vertrek, en die reis neem hom 2 ure en 10 minute, hoe laat kom hy by sy bestemming aan?

(A) 11h45 (B) 11h55 (C) 12h05 (D) 12h15 (E) 12h25

3. 'n Sekere boom groei $\frac{1}{2}$ m per jaar vir 20 jaar lank en $\frac{1}{3}$ m elke jaar daarna. Indien die boom tans 13 m hoog is, hoe oud is die boom in jare?

(A) 23 (B) 25 (C) 27 (D) 29 (E) 32

4. Watter een van die volgende kan x se grootte in grade wees?

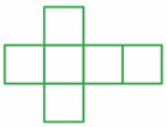


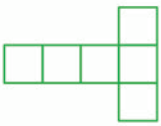
(A) 10 (B) 15 (C) 20 (D) 40 (E) 50

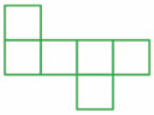
5. Indien die produk van 6 heelgetalle negatief is, wat is die maksimum hoeveelheid van die getalle wat negatief kan wees?

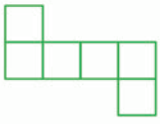
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

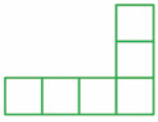
6. Watter een van die volgende figure kan nie in 'n geslote kubus ingevou word nie?

(A) 

(B) 

(C) 

(D) 

(E) 

SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 EERSTE RONDTE
JUNIOR AFDELING: GRAAD 9

15 Maart 2017 **Tyd: 60 minute** **Aantal vrae: 20**

Instruksies

1. *Hierdie is 'n veelvuldige-keuse vraestel. Na elke vraag is vyf antwoorde, genommer A, B, C, D en E. Net een van hulle is reg.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 5 punte.*
   2.2. *Daar is geen penalisering vir foutiewe antwoorde of vrae wat nie beantwoord is nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Die binneblad is 'n inligtings- en formuleblad. Skeur dit asseblief uit vir jou gebruik.*
7. *Begin sodra die toesighouer die teken gee.*
8. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

PRIVAATSAK X173, PRETORIA, 0001
TEL: (012) 392-9372 E-pos: info@samf.ac.za

Organisasies betrokke: AMESA, SA Wiskundevereniging,
SA Akademie vir Wetenskap en Kuns, ASTEMI