

**SOUTH AFRICAN MATHEMATICS OLYMPIAD**  
**2013 Junior Grade 8 Round 1**  
**Solutions**

1. **D** There are 52 weeks and 1 day in a year (unless it is a leap year). So in 1985 there were 52 weeks going Tuesday to Monday, and one extra day (a Tuesday). That makes 53 Tuesdays.
2. **E**  $3001^2 = 9006001$ , which has four 0s.
3. **E** The sum of the digits must be divisible by 3, so  $d$  must be 0 or 3 or 6 or 9. Dividing the four possibilities by 11 shows that only 792 is divisible by 11.
4. **B** We rearrange the sum as  $(2 - 1) + (4 - 3) + (6 - 5) + \dots + (50 - 49)$ . Each bracket evaluates to 1, and there are  $50/2 = 25$  of them, so the total must be 25.
5. **C** When  $n$  is very large,  $n + 2$  is almost exactly equal to  $n$ , and  $2n + 1$  is almost exactly equal to  $2n$ . Then the fraction is equivalent to  $n/2n = \frac{1}{2}$ .
6. **D** There are  $5 \times 8 = 40$  tiles in all. Of these,  $8 + 8 + 3 + 3 = 22$  are on an edge, so the required probability is  $22/40 = 11/20$ .
7. **D** We know  $A = \frac{2}{3}C$ ,  $D = 2B$  and  $B + C = 180^\circ$ . That means  $B = 180^\circ - C$ , so  $D = 360^\circ - 2C$ , and of course  $A + D = 180^\circ$  as well. This can be written as  $\frac{2}{3}C + 360^\circ - 2C = 180^\circ$ , which gives  $180^\circ = 2C - \frac{2}{3}C = \frac{4}{3}C$ , so  $C = 180^\circ \times 3 \div 4 = 135^\circ$
8. **E** We note first that  $PC = 5$ . Now  $\hat{PCA} = \hat{CAD}$  (alternate), which means that  $\hat{PCA} = \hat{PAC}$  and so  $\triangle PAC$  is isosceles. Then  $PA = PC = 5$ .
9. **C** The recurring part of the decimal uses 6 digits, and  $2013 = 6 \times 335 + 3$ . So the required digit is the third in the recurring part, i.e. 8.
10. **B** If the radii are  $R$  and  $r$ , then  $2\pi R - 2\pi r = 6$ . This can be written as  $2\pi(R - r) = 6$ , so that  $R - r = \frac{6}{2\pi} = \frac{3}{\pi}$ . Since  $\pi$  is slightly more than 3, this fraction is slightly less than 1.
11. **A** Simplifying inside each bracket shows we seek  $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \dots \frac{99}{100}$ , which cancels down to  $\frac{2}{100} = \frac{1}{50}$
12. **E** The weight of a whole tank-full of petrol is  $58 \text{ kg} - 34 \text{ kg} = 24 \text{ kg}$ . The weight of two thirds of a tank-full of petrol is  $24 \times \frac{2}{3} = 16 \text{ kg}$ , so that the tank is going to have total weight  $34 + 16 = 50 \text{ kg}$ .
13. **C** The sum of all the numbers is  $12 \times 18 = 216$ , and the sum of the largest and smallest is  $2 \times 28 = 56$ . The other ten must therefore total  $216 - 56 = 160$ , so their average is 16
14. **A** If A lost one game and won another, their win must have come before their loss. So their first-round match was against D, which means the other two teams B and C played each other in the first round, and that must be where C was beaten since B came through to the final against A.

15. **B** The terms increase by 6 each time, so each is equal to a multiple of 6 minus 1.  $1000 \div 6$  is 166 remainder 4, and so has 166 multiples of 6 before it, each leading, after subtraction of 1, to a term in the given sequence.
16. **B** The two isosceles triangles give us  $\hat{ACM} = x$  and  $\hat{BCM} = y$ . Now adding the angles of  $\triangle ABC$  shows  $2x + 2y = 180^\circ$ , so  $x + y = 90^\circ$
17. **B** Imagine dividing the females into groups of 5; there will be  $f/5$  such groups, and each must have at least one male, which requires at least  $f/5$  males. So  $m \geq \frac{f}{5}$ , i.e.  $5m \geq f$
18. **B** The first 9 digits contain one 1. The next 90 represent 45 two-digit numbers, from 10 to 54, and the 100<sup>th</sup> digit will be 5. A 1 appears in each of 10, 11, 12 ...19 (and twice in 11), and then again in 21, 31, 41, 51. This makes 16 1's altogether.
19. **D** Since the given factors are all prime, any factor of the product is formed by either using or not using the 11, using or not using the 13, etc. So for each prime factor there are two possibilities: it is used or it is not used. That gives  $2 \times 2 \times 2 \times 2 = 16$  possible factors of the product.
20. **C** If the amount began as 100 it became 110 after the first year, which is an increase of 10. Subsequent increases bring the value to 120, then 130, and so the increase in the third year is 10 but out of 120: this is a percentage of  $\frac{10 \times 100}{120} = \frac{100}{12} = 8.33$