SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 SECOND ROUND
JUNIOR SECTION: GRADE 8 & 9

10 May 2017 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 4 marks in Part B and 5 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

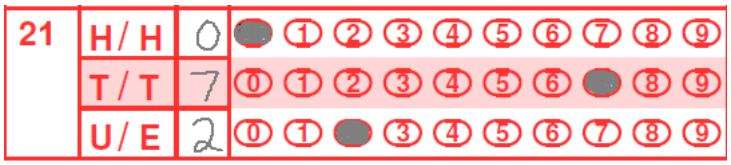
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following **example question**:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom.
The three digits that you write down will not be marked, since it is only for your convenience - only the shaded circles will be marked.

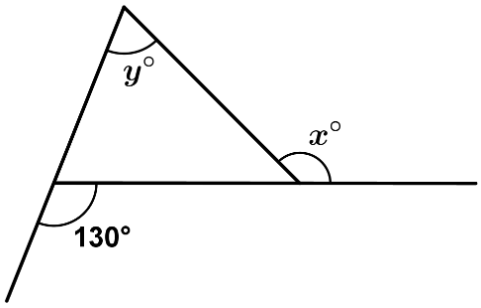
Part A: 3 marks each

1. Determine the value of $\sqrt{2+0\times1+7}$.

2. Simplify 2017 – 1027 – 201.

3. I answer 30 questions in one hour. If I spend the same amount of time on each question, how many questions do I answer in 10 minutes?

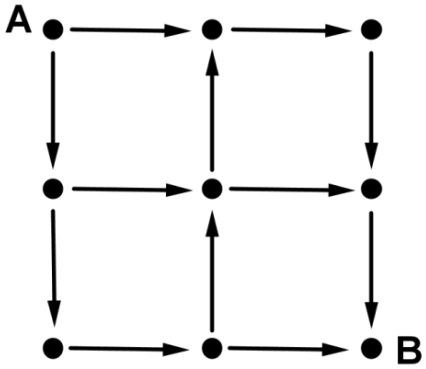
4. Determine the value of $x-y$ in degrees.



5. If $\frac{5}{7}=\frac{x}{35}=\frac{35}{y}$, then find the value of $x+y$.

Part B: 4 marks each

6. How many different ways are there from A to B following the arrows in the diagram?



7. At a party, 15 people ate hotdogs, and 12 ate hamburgers. If 10 of the people ate both, and 3 ate neither, how many people were at the party?

8. A certain amount of money was shared between Angie and Busi in the ratio 3:4. If Angie's share is R240, determine the total amount of money that was shared.

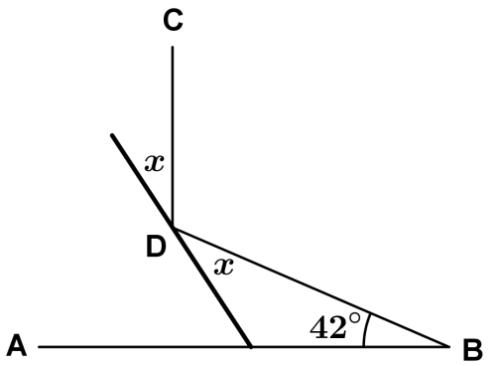
9. The average of three numbers is 12. When a fourth number is added, the average of all four numbers is 15. What is the value of the fourth number?

10. In the grid, the products of the numbers represented by the letters in each of the rows and columns are given. If x, y, z, and w are different positive whole numbers, what is the value of $\frac{xw}{yz}$?

x	y	8
z	w	68
16	34	

11. Dimitri takes 12 minutes to walk around the sides of a square field. How many minutes will he need to walk at the same speed around a square field that has an area four times greater?

12. A beam of light shines from point B, reflects off a mirror at point D, and reaches point C so that DC is perpendicular to AB. Find the size of x in degrees.



13. If $\frac{3+1}{3}\times\frac{4+1}{4}\times\frac{5+1}{5}\times\ldots\times\frac{x+1}{x} = 6$, determine the value of x.

14. The sum of the digits of a 2-digit number is 9. If the digits are written down in the reverse order, the new number is 9 more than the original number. What is the original number?

15. Determine the units digit of the product $\left(5^{2017}+1\right)\times\left(5^{2016}+1\right)\times\left(5^{2015}+1\right)\times\ldots\times\left(5^{1}+1\right)$.

16. Given that $n!$ means $1 \times 2 \times 3 \times \ldots \times n$ (so $5! = 5 \times 4 \times 3 \times 2 \times 1$), find the value of $\frac{10!}{9!}+\frac{9!}{8!}+\frac{8!}{7!}+\frac{7!}{6!}+\frac{6!}{5!}+\frac{5!}{4!}+\frac{4!}{3!}+\frac{3!}{2!}+\frac{2!}{1!}$.

17. The number A8307300B is divisible by 45. Find the largest positive difference between A and B.

18.

			2						Row 1
		4		6					Row 2
	8		10		12				Row 3
14		16		18		20			Row 4
22		24		26		28		30	Row 5

If the pattern is continued then determine the middle number in the 25th row.

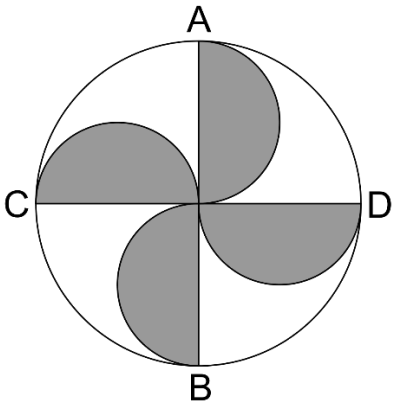
19. On a 24-hour digital clock displaying hours, minutes and seconds, e.g. 19:23:52, how many times in each 24-hour period do all six digits change at the same time?

20. What is the smallest positive 2-digit whole number which when divided by 7 has a remainder of 2 and when divided by 6 has a remainder of 3?

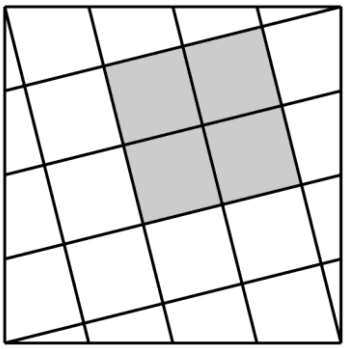
Part C: 5 marks each

21. Nivar's water bottle is 90% full. After drinking 40% of the water, 486 ml remain. How many millilitres of water does the bottle hold when it is full?

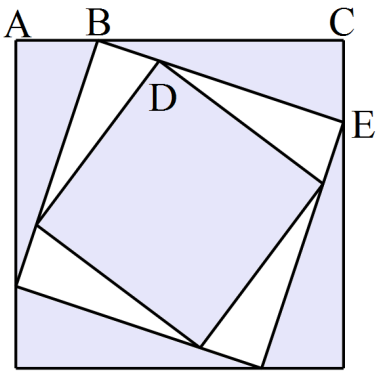
22. In the circle shown, diameters AB and CD are perpendicular, and each of the four shaded regions is a semicircle. If the ratio of the shaded area to the unshaded area is $\frac{s}{u}$ in simplest form, then what is the value of $s+u$?



23. Each side of a square of length 17 cm is divided into four equal parts. Some of these division points are connected as shown in the diagram. Find the area of the shaded region in cm^2.



24. The diagram shows three squares such that the vertices of the inner squares are positioned a quarter of the length along the sides they touch. For example, $AB=\frac{1}{4}AC$ and $BD=\frac{1}{4}BE$. If the area of the largest square is 64 cm^2, find the area of the smallest square in cm^2.



25. Travelling at an average speed of 40 km/h, we will be 1 hour late to our destination. Travelling at an average speed of 60 km/h, we will be 1 hour early. At what average speed, in km/h, should we travel in order to arrive on time?