THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

Senior First Round 2018
Solutions

1. **Answer D**
(Remember that to the right of the decimal point, each decimal place is worth 10 times the next one.) In increasing order, the five given numbers are: 4.004, 4.04, 4.044, 4.4, 4.44.

2. **Answer B**
Since $x + 70 = 9 \times 8 = 72$, it follows that $x = 72 - 70 = 2$.

3. **Answer B**
(Remember that a positive integer is prime if it has exactly two factors, itself and 1.) Let n be the required number, so $n = p + q$, where p and q are both prime. Since n is odd, it follows that p is even and q is odd (or the other way round). The only even prime is 2, so we can put $p = 2$ and $q = n - 2$, which must also be prime. By trial and error, the only possibility is $n = 15$, because $15 - 2 = 13$, which is prime.

4. **Answer D**
Since 1 km = 1000 m and 1 h = 3600 s, it follows that
720 km/h = 720 × 1000/3600 m/s = 200 m/s.

5. **Answer E**
(Remember that if a product is zero, then one of the factors must be zero, and if a product is not zero, then none of the factors can be zero.) Since $abc = 1$, it follows that a, b and c are all non-zero. We can therefore divide both sides of the equation $ace = 0$ by ac to get $e = 0$. (Note that d might or might not be zero.)

6. **Answer B**
Since the sides of the triangle are in the ratio $3 : 4 : 5$, it follows that they can be written as $3k, 4k, 5k$, where k is some constant. The perimeter is then $3k+4k+5k = 12k$, which equals 60. Thus $k = 5$ and the shortest side is $3k = 3 \times 5 = 15$.

7. **Answer D**
In cm, if each side of the square has length x, then the width of each rectangle is $\frac{1}{3}x$, and the perimeter of each rectangle is $2x + \frac{2}{3}x = \frac{8}{3}x$. Thus $\frac{8}{3}x = 24$, so $x = \frac{3}{8} \times 24 = 9$, and the area is $x^2 = 9^2 = 81$.

8. **Answer B**
(Remember that for a number to be a perfect square, the exponent of each prime factor must be even (divisible by 2), and for it to be a perfect cube, the exponent of each prime factor must be divisible by 3.) Now $12n = 2^2 \times 3 \times n$ and is a perfect square, so n must be $3\times$ a perfect square, and the smallest value is $n = 3$. Next, $75np = 3 \times 5^2 \times 3 \times p = 3^2 \times 5^2 \times p$, and is a perfect cube. It follows that $p = 3 \times 5\times$ a perfect cube, and the smallest value is $3 \times 5 = 15$. Thus the smallest value of $n + p = 3 + 15 = 18$.

9. **Answer E**
The smallest circle has radius $\frac{1}{4}$ of the radius of the outer circle, so its area is $(\frac{1}{4})^2 = \frac{1}{16}$ of the area of the target.

10. **Answer D**
In two hours, or 120 minutes, Henry will have inflated $120 \times 8 \div 3 = 320$ balloons. One tenth of these have burst, which makes $\frac{1}{10} \times 320 = 32$ balloons that have burst. The number of balloons remaining is $320 - 32 = 288$.

11. **Answer C**
Suppose Michael buys C soft sweets; then he buys $9 - C$ hard sweets. The total cost in Rand is $3C + 2(9 - C) = C + 18$, which is equal to 22. Therefore $C = 4$.

12. **Answer D**
We are given $1 < 4n - 5 < 200$, so $6 < 4n < 205$ and $\frac{3}{2} < n < 51\frac{1}{4}$. Since n is an integer, it must satisfy $2 \leq n \leq 51$, which gives 50 possible values of n.

13. **Answer B**
(Remember that the sum of the interior angles of an n-sided polygon is $180(n-2)^\circ$.) In degrees, the interior angle at H is $360 - 110 = 250$. If the interior angle at A is θ, then $4(\theta + 250) = 180 \times 6 = 1080$. Therefore $4\theta = 1080 - 1000 = 80$ and $\theta = 20$.

14. **Answer D**
If we think of the side of length 8 as the base of the triangle, then the height can be anything from 0 to 5, so the area A satisfies $0 \leq A \leq \frac{1}{2} \times 8 \times 5 = 20$. Thus only 5 and 20 are possible values for the area.

15. **Answer C**
(Remember that both sides of an inequality may be multiplied or divided by a positive number without changing the direction of the inequality.) We are given $x < x^3$ and $x^3 < x^2$, so $x \neq 0$ and therefore $x^2 > 0$. Dividing by x^2 gives $\frac{1}{x} < x$ and $x < 1$. The first inequality holds when $-1 < x < 0$ or $x > 1$ (think of the graphs $y = \frac{1}{x}$ and $y = x$), and combining it with the second inequality leaves only $-1 < x < 0$. Of the given values, $x = -\frac{3}{5}$ is the only one in this interval.

16. **Answer A**
Let C be the percentage of animals that are cats, so the percentage of dogs is $100 - C$. The percentage considering themselves to be cats is then $\frac{9}{10}C + \frac{1}{10}(100 - C) = 10 + \frac{4}{5}C$. This is equal to 20, so $10 + \frac{4}{5}C = 20$, giving $\frac{4}{5}C = 10$, so $C = \frac{5}{4} \times 10 = 12.5$.

17. **Answer B**
The number of ways of choosing three points from nine is $\dfrac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$. The only choices that do not give rise to triangles are when the three points lie in a line. There are eight of these lines: three horizontal, three vertical, and two diagonal. Thus the number of triangles is $84 - 8 = 76$.

18. **Answer B**
Since side $AC = AB = 8$, it follows that $CE = AE - AC = 20 - 8 = 12$. Triangle CDE, which has angles $30^\circ, 60^\circ, 90^\circ$, forms half of an equilateral triangle. Therefore $CD = \frac{1}{2}CE = \frac{1}{2} \times 12 = 6$, so $BD = BC + CD = 8 + 6 = 14$.

19. **Answer E**
The number of ways of choosing three coins out of six is $\dfrac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$. Without

the R5 coin, the maximum total is $1+2+2=5$ rands. Thus for a greater total the R5 coin must be taken, together with any two of the remaining five coins, which can be done in $\dfrac{5\times 4}{2\times 1}=10$ ways. The probability that the sum is greater than R5 is therefore $\frac{10}{20}=\frac{1}{2}$.

20. **Answer A**
The circle has area $\pi\times 1^2=\pi$, so the sides of the square are of length $\sqrt{\pi}$. If M is the midpoint of PQ, then $OM=\frac{1}{2}\sqrt{\pi}$, and by Pythagoras' theorem $PM=\sqrt{1-\frac{1}{4}\pi}=\frac{1}{2}\sqrt{4-\pi}$. Finally, $PQ=2PM=\sqrt{4-\pi}$.

---

18. **Antwoord B**
Ons het sye $AC = AB = 8$, en dit volg dan dat $CE = AE - AC = 20 - 8 = 12$. Driehoek CDE, met hoeke $30°, 60°, 90°$, is die helfte van 'n gelyksydige driehoek. Dus is $CD = \frac{1}{2}CE = \frac{1}{2} \times 12 = 6$, sodat $BD = BC + CD = 8 + 6 = 14$.

19. **Antwoord E**
Die aantal maniere waarop drie muntstukke uit ses gekies kan word, is $\dfrac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$. Sonder die R5-muntstuk, is die maksimumwaarde $1 + 2 + 2 = 5$ Rand. Vir 'n groter totaal moet die R5-muntstuk dus gebruik word, saam met enige van die oorblywende vyf muntstukke, wat op $\dfrac{5 \times 4}{2 \times 1} = 10$ maniere gedoen kan word. Die waarskynlikheid dat die som groter is as R5, is dus $\frac{10}{20} = \frac{1}{2}$.

20. **Antwoord A**
Die oppervlakte van die sirkel is $\pi \times 1^2 = \pi$, sodat die sye van die vierkant se lengtes gelyk is aan $\sqrt{\pi}$. As M die middelpunt van PQ is, dan is $OM = \frac{1}{2}\sqrt{\pi}$, en volgens die stelling van Pythagoras, is $PM = \sqrt{1 - \frac{1}{4}\pi} = \frac{1}{2}\sqrt{4 - \pi}$. Dus, $PQ = 2PM = \sqrt{4 - \pi}$.

---

9. **Antwoord E**
Die radius van die kleinste sirkel is $\frac{1}{4}$ van die radius van die buitenste sirkel, en dus is sy oppervlakte $(\frac{1}{4})^2 = \frac{1}{16}$ van die oppervlakte van die teiken.

10. **Antwoord D**
Na twee uur, of 120 minute, sou Harry $120 \times 8 \div 3 = 320$ ballonne opgeblaas gehad het. Een tiende hiervan het gebars, sodat $\frac{1}{10} \times 320 = 32$ ballonne gebars het. Die getal ballonne wat oor is, is $320 - 32 = 288$.

11. **Antwoord C**
Veronderstel dat Michael C sagte lekkers gekoop het; en dus $9 - C$ harde lekkers. Die totale koste, in Rand, is $3C + 2(9 - C) = C + 18$, wat gelyk is aan 22. Dus, $C = 4$.

12. **Antwoord D**
Ons het $1 < 4n - 5 < 200$, sodat $6 < 4n < 205$ en $\frac{3}{2} < n < 51\frac{1}{4}$. Omdat n 'n heelgetal is, moet dit $2 \leq n \leq 51$ bevredig, wat 50 moontlike waardes vir n gee.

13. **Antwoord B**
(Onthou dat die som van die binnehoeke van 'n veelhoek met n sye gelyk is aan $180(n-2)°$.) In grade is die binnehoek by H dan $360 - 110 = 250$. As θ die binnehoek by A is, dan is $4(\theta + 250) = 180 \times 6 = 1080$. Dus $4\theta = 1080 - 1000 = 80$ en $\theta = 20$.

14. **Antwoord D**
As ons die sy met lengte 8 as die basis van die driehoek beskou, dan kan die hoogte enige waarde van 0 tot 5 wees, sodat die oppervlakte A dan $0 \leq A \leq \frac{1}{2} \times 8 \times 5 = 20$ bevredig. Dus is slegs 5 en 20 moontlike waardes vir die oppervlakte.

15. **Antwoord C**
(Onthou dat albei kante van 'n ongelykheid vermenigvuldig of gedeel kan word deur 'n positiewe getal sonder om die rigting van die ongelykheid te verander.) Ons het $x < x^3$ en $x^3 < x^2$, sodat $x \neq 0$ en dus is $x^2 > 0$. Deling deur x^2 gee $\frac{1}{x} < x$ en $x < 1$. Die eerste ongelykheid is waar as $-1 < x < 0$ of $x > 1$ (dink aan die grafieke $y = \frac{1}{x}$ en $y = x$), en gekombineer met die tweede ongelykheid laat dit dan $-1 < x < 0$. Van die gegewe waardes, is $x = -\frac{3}{5}$ die enigste wat in hierdie interval is.

16. **Antwoord A**
Laat C die persentasie diere wees wat katte is, sodat die persentasie honde $100 - C$ is. Die persentasie wat hulself as katte beskou is dan $\frac{9}{10}C + \frac{1}{10}(100 - C) = 10 + \frac{4}{5}C$. Dit is gelyk aan 20, sodat $10 + \frac{4}{5}C = 20$, wat $\frac{4}{5}C = 10$ gee, sodat $C = \frac{5}{4} \times 10 = 12.5$.

17. **Antwoord B**
Die aantal kere wat drie punte uit die nege gekies kan word, is $\dfrac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$. Die enigste keuses wat nie driehoeke sal gee nie, is wanneer die drie punte in 'n lyn lê. Daar is agt van hierdie lyne: drie horisontaal, drie vertikaal, en twee diagonaal. Dus is die aantal driehoeke $84 - 8 = 76$.

DIE SUID-AFRIKAANSE WISKUNDE OLIMPIADE

Senior Eerste Rondte 2018

Oplossings

1. **Antwoord D**
   (Onthou dat aan die regterkant van die desimale punt, elke desimale plek 10 maal die waarde van die volgende een is.) In stygende orde is die vyf gegewe getalle: 4.004, 4.04, 4.044, 4.4, 4.44.

2. **Antwoord B**
   Omdat $x + 70 = 9 \times 8 = 72$, volg dit dat $x = 72 - 70 = 2$.

3. **Antwoord B**
   (Onthou dat 'n positiewe getal priem is as dit presies twee faktore het, die getal self en 1.) Laat n die vereiste getal wees, dan is $n = p + q$, waar p en q albei priem is. Omdat n onewe is, volg dit dat p ewe is en q onewe (of andersom). Die enigste ewe priemgetal is 2, en dus stel ons $p = 2$ en $q = n - 2$, wat ook priem moet wees. Deur probeer-en-tref, is die enigste moontlikheid $n = 15$, omdat $15 - 2 = 13$, wat priem is.

4. **Antwoord D**
   Omdat 1 km = 1000 m en 1 h = 3600 s, volg dit dat
   720 km/h = 720 × 1000/3600 m/s = 200 m/s.

5. **Antwoord E**
   (Onthou dat as 'n produk nul is, dan is een van die faktore nul, en as die produk nie nul is nie, is geen van die faktore nul nie.) Omdat $abc = 1$, volg dit dat a, b en c almal nie-nul is. Ons kan dus aan albei kante van die vergelyking $ace = 0$ deur ac deel om $e = 0$ te kry. (Let op dat d nul of nie-nul kan wees.)

6. **Antwoord B**
   Omdat die sye van die driehoek in die verhouding $3 : 4 : 5$ is, volg dit dat ons hulle kan skryf as $3k, 4k, 5k$, waar k 'n konstante is. Die omtrek is dan $3k+4k+5k = 12k$, wat gelyk is aan 60. Dus is $k = 5$ en die kortste sy is $3k = 3 \times 5 = 15$.

7. **Antwoord D**
   As die lengte van elke sy van die vierkant x is, dan is die breedte van elke reghoek $\frac{1}{3}x$, en die omtrrek van elke reghoek $2x+\frac{2}{3}x = \frac{8}{3}x$. Dus is $\frac{8}{3}x = 24$, sodat $x = \frac{3}{8}\times 24 = 9$, en die oppervlakte is $x^2 = 9^2 = 81$.

8. **Antwoord B**
   (Onthou dat as 'n getal 'n volkome vierkant is, dan is die eksponent van elke priemfaktor ewe (deelbaar deur 2), en as dit 'n volkome derdemag is, is die eksponent van elke priemfaktor deelbaar deur 3.) Nou is $12n = 2^2 \times 3 \times n$ 'n volkome vierkant, sodat n $3\times$ 'n volkome vierkant is, en die kleinste waarde is $n = 3$. Dan volg dat $75np = 3\times 5^2 \times 3 \times p = 3^2 \times 5^2 \times p$, wat 'n volkome derdemag is. Dus is $p = 3\times 5\times$ 'n volkome derdemag is, en die kleinste waarde is $3\times 5 = 15$. Dus is die kleinste waarde van $n + p = 3 + 15 = 18$.