THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCES AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

SECOND ROUND 1997

SENIOR SECTION: GRADES 10, 11 AND 12

(STANDARDS 8, 9 AND 10)

10 JUNE 1997

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 For each correct answer: 5 marks
   - 3.2 For no answer: 0 marks
   - 3.3 For each wrong answer: 0 marks.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Answer on the sheet provided.
7. When the invigilator gives the signal, start the problems. You will have 120 minutes working time for the question paper.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers

$$0,5129; 0,9; 0,89; \text{ and } 0,289$$

is

(A) 1,189 (B) 0,8019 (C) 1,428 (D) 1,179 (E) 1,4129.

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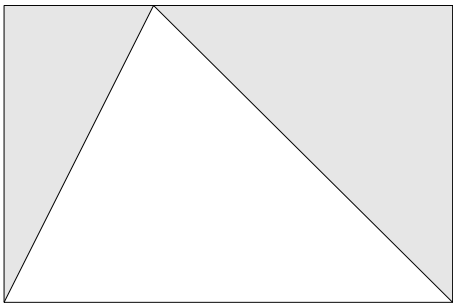
1. When half a number is decreased by 8 the result is 25. The original number is

(A) 50 (B) 58 (C) 82 (D) 41 (E) 66

2. There are 6 black socks, 14 green socks and 8 blue socks in a drawer. What is the smallest number of socks that must be taken out, all at once, in order to be sure to get a matching pair?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 8

3. The rectangle shown has length 12 and width 7. The area of the shaded part is

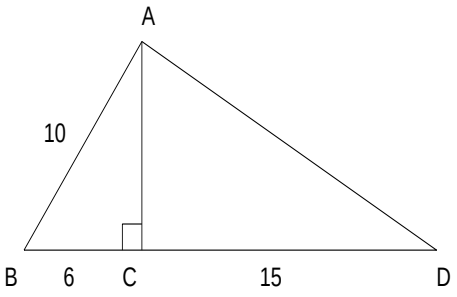


(A) $\frac{81}{2}$ (B) 42 (C) 45 (D) $\frac{85}{2}$ (E) $17\sqrt{5}$

4. Marie has saved R10,02. She finds that this amount is made up of equal numbers of 2c, 5c, 10c, 50c and R1 coins. The total number of coins she has is

(A) 5 (B) 15 (C) 24 (D) 25 (E) 30

5. The length of AD is

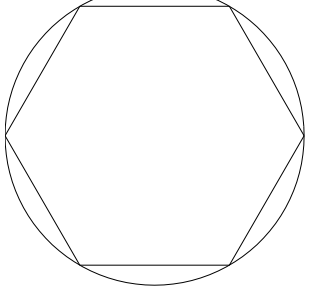


(A) 17 (B) $\sqrt{296}$ (C) $\sqrt{312}$ (D) $\sqrt{341}$ (E) 19

6. The shortest side of a rectangular box is 50 cm and the longest side is 90 cm. Of the following, the number which could represent the volume of the box, in cm^3, is

(A) 4500 (B) 180 000 (C) 90 000 (D) 360 000 (E) 450 000

7. A regular hexagon is drawn inside a circle. The ratio of the circumference of the circle to the perimeter of the hexagon is



(A) $4 : \pi$ (B) $2 : \sqrt{3}$ (C) $\sqrt{2} : 1$ (D) $\pi : 3$ (E) $\pi : \sqrt{6}$

8. The sum of the digits of a two-digit number is subtracted from the number. The answer is 45. For how many two-digit numbers is this result possible?

(A) 0 (B) 1 (C) 2 (D) 9 (E) 10

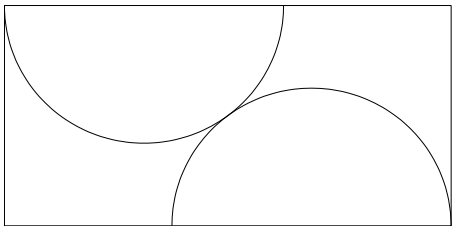
9. The sum of the squares of 4 consecutive natural numbers is 5334. The smallest of these 4 numbers is

(A) 37 (B) 41 (C) 33 (D) 35 (E) 29

10. The number of natural numbers that leave a remainder of 41 when divided into 1997 is

(A) 6 (B) 4 (C) 3 (D) 8 (E) 11

11. A taylor wants to cut two semi-circles as shown, with the same diameter from a cloth measuring 80 cm by 160 cm. The diameters, in cm, of the largest semi-circles are

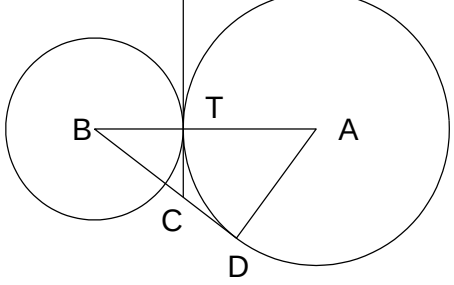


(A) 120 (B) 90 (C) 80 (D) 100 (E) 125

12. The number of solutions to the equation $\sqrt{x^4 + 16} = x^2 - 4$ is

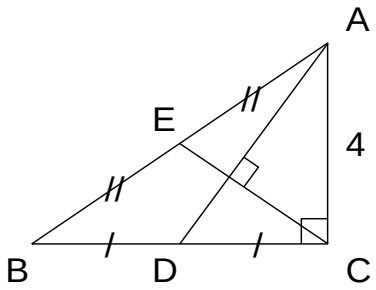
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. Two circles with centres at A and B, touch at T. BD is the tangent at D and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is



(A) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $\frac{7}{4}$ (E) $\frac{8}{5}$

14. E is the midpoint of AB and D is the midpoint of BC. Right angles are as shown. If AC has length 4, then the length of AB is



(A) 30 (B) $2\sqrt{13}$ (C) $3\sqrt{6}$ (D) $3\sqrt{5}$ (E) $4\sqrt{3}$

15. If

$$\frac{23}{30} = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n},$$

where $a_1, a_2, \ldots, a_n$ are natural numbers, then the smallest value of n is

(A) 30 (B) 2 (C) 3 (D) 4 (E) 23

16. If

$$a = \frac{1996}{1995 \times 1997}, \quad b = \frac{1997}{1996 \times 1998} \text{ and } c = \frac{1}{1997}$$

then

(A) $a < b < c$ (B) $c < b < a$ (C) $b < a < c$ (D) $c < a < b$ (E) $b < c < a$

17. A piece of string is cut into two pieces. The point at which the string is cut was chosen at random. What is the probability that the longer piece is at least three times as long as the shorter piece?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

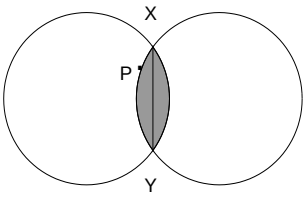
18. If x and y are natural numbers and $19x + 97y = 1997$ then the smallest value of $x + y$ is

(A) 21 (B) 23 (C) 38 (D) 41 (E) 47

19. If $a + b = 5$ and $ab = 2$, then $a^4 + b^4$ is equal to

(A) 433 (B) 437 (C) 609 (D) 625 (E) 641

20. Two circles with the same radii intersect at X and Y. XY has length 3 and subtends an angle of 120^o at P. The area of the shaded region is



(A) $2(\pi - \frac{1}{4}\sqrt{3})$ (B) $3(\pi - \sqrt{3})$ (C) $\frac{1}{2}(3\pi - \sqrt{3})$ (D) $2(\pi - \frac{2}{3}\sqrt{3})$ (E) $2(\pi - \frac{3}{4}\sqrt{3})$.