SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 THIRD ROUND
JUNIOR SECTION: GRADES 8 AND 9

27 July 2017 **Time: 4 Hours** **Number of questions: 15**

TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form and no geometric instruments may be used.
- Use your time wisely and do not spend all your time on a few questions.
- Questions are not necessarily arranged in order of difficulty.
- Question 7 should be done on the given answer sheet.
- Answers and solutions will be available at: www.samf.ac.za

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

Question 1

Find the longest string of digits such that every two consecutive digits form a 2-digit square number.

[3]

Question 2

Consider the following sum:

1 + 11 + 101 + 1001 + 10001 + é + 10000é 0001

The fourth number has two zeroes between the ones.
The last number has fifty zeroes between the ones.
Find the sum of the digits in the answer.

[4]

Question 3

Find the smallest positive integer whose digits have a product of 1728.

[4]

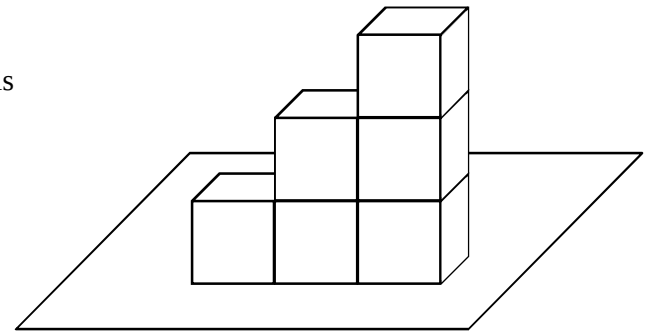
Question 4

In standard dice the sum of the numbers on opposite faces add up to seven.

In other words the 1 is opposite the 6, the 2 is opposite the 5 and the 3 is opposite the 4.

In the diagram, six dice are placed on a table.

What is the maximum possible sum of the numbers on the 21 visible faces?



[4]

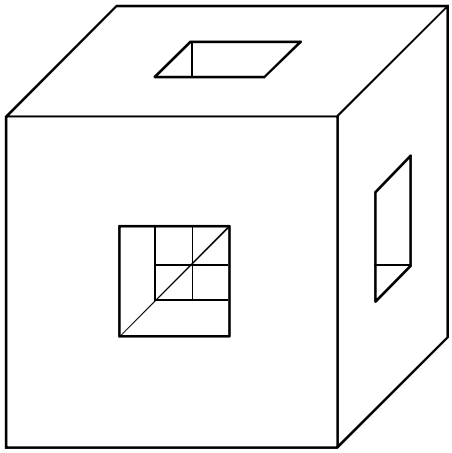
Question 5

Consider all positive integers that consist of a string of 7s, for example, 77777.

What is the smallest such number that is divisible by 99?

[6]

Question 6



In the figure, a wooden cube has edges of length 3.

Square holes of side 1, centred on each face are cut right through the cube.

The edges of the holes are parallel to the edges of the cube.

Find the entire surface area, including the inside areas.

[6]

Question 7 (Answer this question on the answer sheet)

	2	**2**					**6**		
		7	**4**			**A**			
	12								**3**
						3		**2**	
				2			**B**		**6**
			5	**3**	**2**	**2**	**2**		
2								**4**	
6				**4**					
				5					

The above grid can be divided into rectangles in such a way that each rectangle contains exactly one number (A and B are placeholders for numbers).

The number in each rectangle is equal to the rectangleôs area.
(the area of a small square is 1 square unit)

a) Divide the grid into rectangles in the manner described above.

b) What are the values of A and B?

[8]

Question 8

Here is a quick way to square numbers that end in a 5:

$35^2 = 1225$	$3\times4 = 12$ and $5^2 = 25$.	Then the answer is 1225.
$75^2 = 5625$	$7\times8 = 56$ and $5^2 = 25$.	Then the answer is 5625.
$225^2 = 50625$	$22\times23 = 506$ and $5^2 = 25$.	Then the answer is 50625.

a) Use this method to calculate 305^2

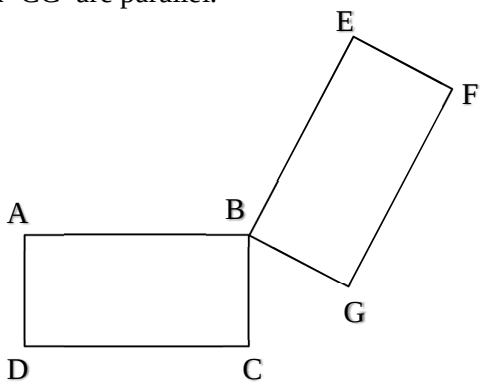
b) Prove that this method works in general.

[8]

Question 9

Two congruent rectangles ABCD and BEFG are given as shown.

Prove that AE and CG are parallel.



[8]

Question 10

Two busses start travelling at the same time, Bus 1 from city A to City B, and Bus 2 from City B to City A using the same road. Both busses travel at constant speeds.

They meet for the first time 7 km from City A. After both buses reach their destinations (City B and A respectively, possibly at different times), they immediately start travelling back along the same road and with the same speeds.

They meet again 4 km from City B.

Find the distance between Cities A and B.

[8]

Question 11

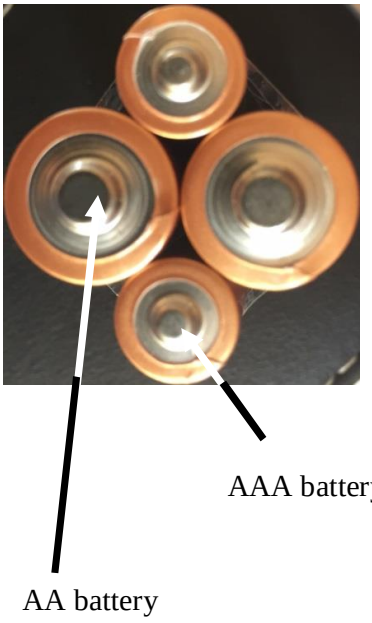
a) What is the largest prime number p such that $275!$ is divisible by p^3?

b) What is the largest integer n such that $15!$ is divisible by n^3?

[8]

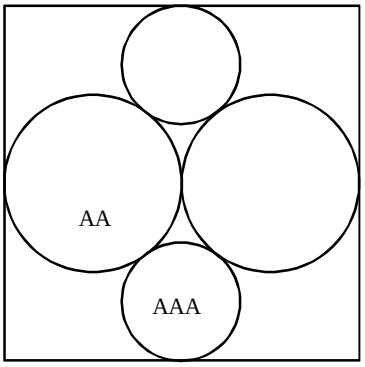
Question 12

Two AA-batteries (bigger) and two AAA-batteries (smaller) are put together in such a way that it perfectly fits into a square of length 30 mm.



Diagrammatically it can be shown in the sketch alongside.

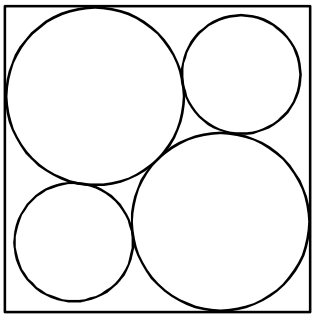
(the circles indicate the batteries that are fitted in a square of length 30 mm)



a) Determine the radius of an AA-size battery.

b) Determine the radius of an AAA-size battery.

c) Determine the size of the smallest square in which the 4 batteries can be fitted in with the arrangement as shown alongside. (Leave your answer in surd form)



[8]

Question 13

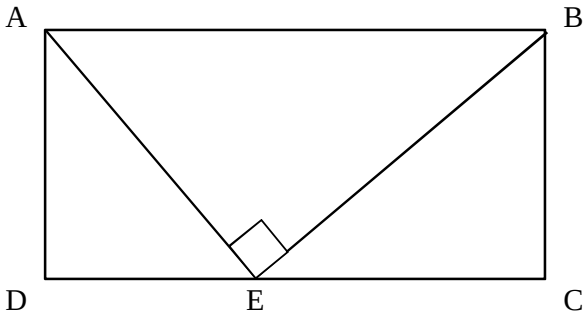
Two players play a game, starting with a positive integer written on a board. Each player's turn consists of subtracting any positive factor of the number on the board from the number, and replacing the number on the board with the answer (new number). The player that first writes the number zero on the board, loses.

If the starting number on the board is 2017, can either player force a win?

If yes, which player? Explain the winning strategy. [8]

Question 14

Let ABCD be a rectangle and let E be a point on CD such that AE and BE are perpendicular. It is given that the lengths of the line segments AB, BC, CE, ED, AD, AE and BE are all integers.



a) If CE = 9, determine the lengths of the other line segments and prove that there are no other solutions.

b) Show that it is impossible that CE = 7.

[9]

Question 15

a) What is the greatest possible length of a strictly increasing sequence of positive integers such that each term, from the second term onwards, is an integer multiple of the previous one, and all terms are smaller than 2017?

b) How many sequences of this length are there?

[8]

Total: 100

THE END

ANSWER SHEET/ ANTWOORDBLAD

This answer sheet needs to be handed in / Hierdie antwoorblad moet ingehandig word

Name:	**School:**
_____	_____

Question 7 / Vraag 7

Rough work / Rofwerk

	2	**2**					**6**		
		7	**4**			**A**			
	12								**3**
						3		**2**	
				2			**B**		**6**
			5	**3**	**2**	**2**	**2**		
2								**4**	
6				**4**					
				5					

Final Answer / Finale Antwoord

	2	**2**					**6**		
		7	**4**			**A**			
	12								**3**
						3		**2**	
				2			**B**		**6**
			5	**3**	**2**	**2**	**2**		
2								**4**	
6				**4**					
				5					

A = _____

B = _____