SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 FIRST ROUND
SENIOR SECTION: GRADE 10 - 12

10 March 2017 **Time: 60 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by five answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 5 marks.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns, ASTEMI

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1+\dfrac{1}{3+\dfrac{1}{2}}$ is

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{6}{7}$ (E) $\frac{9}{7}$

3. The tens digit of $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

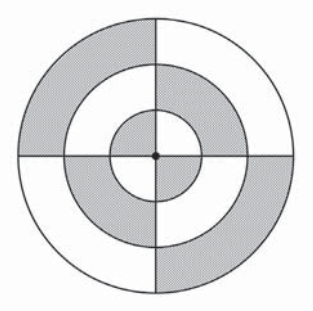
(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

1. The value of $1009^2 - 1008^2$ is

(A) 2017 (B) 2016 (C) 2015 (D) 17 (E) 1

2. The radii of the three concentric circles shown, are 2, 4 and 6 respectively. The diameters cut each circle into quarters. What is the area of the shaded region?

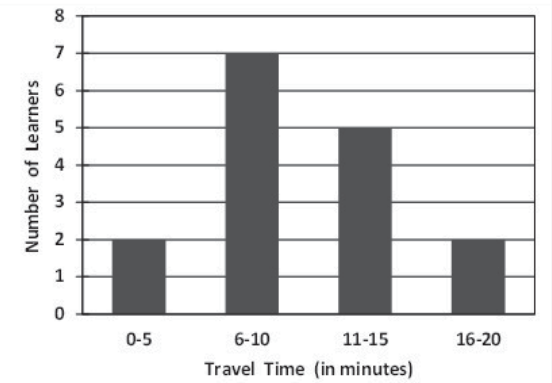


(A) 8π (B) 12π (C) 16π (D) 18π (E) 24π

3. Which number from the set $\{-2; -1; 0; 1; 2\}$ is the smallest value of n for which $3^n.3^3$ is a perfect square?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

4. The graph shows the travel time of learners from home to school. How many learners travel longer than 10 minutes?



(A) 2 (B) 5 (C) 7 (D) 8 (E) 15

5. A straight line passes through the points $(2;3)$ and $(4;7)$. Which one of the following points is also on the line?

(A) $(3;5)$ (B) $(1;2)$ (C) $(4;5)$ (D) $(0;2)$ (E) $(2;4)$

6. The length of each side of a triangle is a different even integer. If the triangle has non-zero area, what is the minimum perimeter that it can have?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 22

7. In a list of six consecutive positive integers the sum of the three smallest numbers is N and the sum of the three largest numbers is M. Which one of the following is true?

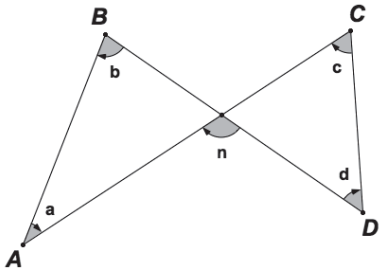
(A) $N = M + 9$ (B) $M = N + 9$ (C) $M + N = 9$ (D) $3M + 3N = 9$

(E) $3M - 3N = 12$

8. The coordinates of a point in the Cartesian plane are $(a; 2 - a)$, where a is a real number. Which one of the following is true for the position of this point?

(A) It cannot be in the first quadrant
(B) It cannot be in the second quadrant
(C) It cannot be in the third quadrant
(D) It cannot be in the fourth quadrant
(E) It can be in any quadrant

9. AC and BD are straight lines. What is the sum of the angles a, b, c and d?

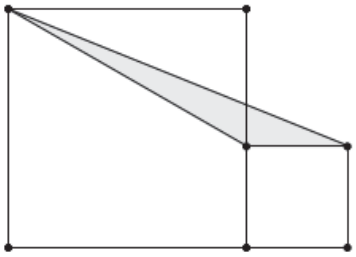


(A) n (B) $2n$ (C) $180° - n$ (D) $360° - n$ (E) $360° - 2n$

10. There are 8 red, 7 blue and 6 green balls in a box. One ball is taken out at random. What is the probability that it is neither red nor green?

(A) $\frac{6}{21}$ (B) $\frac{8}{21}$ (C) $\frac{8}{13}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

11. The side length of the large square is 6 and the side length of the small square is 2. Find the area of the shaded triangle.



(A) 1 (B) 3 (C) 2 (D) 6 (E) 4

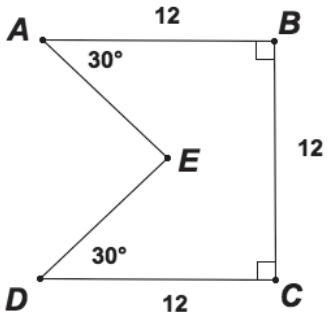
12. In a group of 50 girls each girl wears either a red or a yellow shirt and either black or grey pants. If 14 girls wear a red shirt with black pants, 31 girls wear yellow shirts, and 18 girls wear grey pants, then the number of girls who wear a yellow shirt with grey pants is

(A) 5 (B) 7 (C) 9 (D) 11 (E) 13

13. A container has a mass of 36 kg when it is a quarter full and when it is one-third full it has a mass of 40 kg. What is the mass of the empty container in kg?

(A) 24 (B) 36 (C) 48 (D) 18 (E) 20

14. What is the perimeter of $ABCDE$?

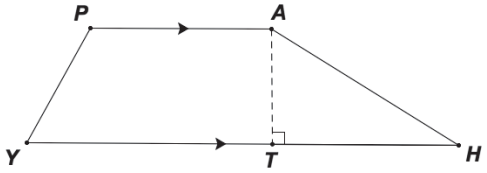


(A) $36 + 6\sqrt{3}$ (B) 48 (C) $36 + 8\sqrt{3}$ (D) $36 + 12\sqrt{3}$ (E) 60

15. If $a + b + c = 12$, $3a + 2b + c = 6$ and $5a + 3b + 2c = 9$, then the value of c is

(A) -9 (B) 12 (C) 9 (D) 6 (E) 3

16. In the diagram $PA /\!\!/ YH$, $PA = 12$, $PY = 10$, $YT = 18$ and $HA = 17$. The area of $PYHA$ is



(A) 160 (B) 180 (C) 225 (D) 264 (E) 300

17. The *floor* of a real number x, denoted by $\lfloor x \rfloor$, is the largest integer less than or equal to the number, e.g. $\lfloor 3.2 \rfloor = 3$. The *ceiling* of a real number x, denoted by $\lceil x \rceil$, is the smallest integer greater than or equal to the number, e.g. $\lceil -4.7 \rceil = -4$. What is the value of $\left\lceil \sqrt{23} \right\rceil + \lfloor -3.2 \rfloor$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

18. For any positive integer, n, let $f(n)$ denote the sum of its digits. For example, $f(23) = 2 + 3 = 5$. How many positive two-digit integers, n, are there such that $\dfrac{n}{f(n)} > 8$?

(A) 5 (B) 12 (C) 15 (D) 17 (E) 61

19. 30 litres of a certain fruit juice contains 6% pure juice concentrate mixed in water. Another 20 litres is made using 5% pure juice concentrate. When both these mixtures are poured into a single container, the percentage pure juice concentrate will be

(A) 5.4 (B) 5.6 (C) 5.7 (D) 5.8 (E) 5.9

20. Find the value of $\dfrac{1}{1 \times 3} + \dfrac{1}{3 \times 5} + \dfrac{1}{5 \times 7} + \cdots + \dfrac{1}{99 \times 101}$.

(A) $\dfrac{50}{99}$ (B) $\dfrac{51}{99}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{50}{101}$ (E) $\dfrac{51}{101}$

18. Vir enige positiewe heelgetal, n, laat $f(n)$ die som van die syfers van n aandui. Byvoorbeeld, $f(23) = 2 + 3 = 5$. Hoeveel positiewe twee-syferheelgetalle, n, is daar sodanig dat $\dfrac{n}{f(n)} > 8$?

(A) 5 (B) 12 (C) 15 (D) 17 (E) 61

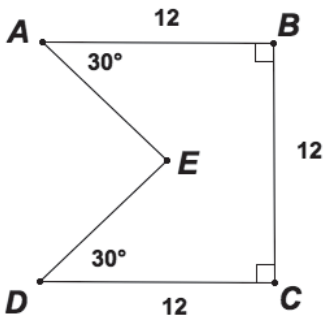
19. 30 liter van 'n sekere vrugtesap bevat 6% suiwer sapkonsentraat gemeng met water. Nog 20 liter word gemaak met 5% suiwer sapkonsentraat. Wanneer beide mengsels in een houer gegooi word, sal die persentasie suiwer sapkonsentraat gelyk wees aan

(A) 5.4 (B) 5.6 (C) 5.7 (D) 5.8 (E) 5.9

20. Vind die waarde van $\dfrac{1}{1\times 3} + \dfrac{1}{3\times 5} + \dfrac{1}{5\times 7} + \cdots + \dfrac{1}{99\times 101}$.

(A) $\dfrac{50}{99}$ (B) $\dfrac{51}{99}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{50}{101}$ (E) $\dfrac{51}{101}$

14. Wat is die omtrek van $ABCDE$?

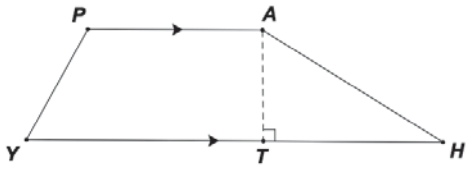


(A) $36+6\sqrt{3}$ (B) 48 (C) $36+8\sqrt{3}$ (D) $36+12\sqrt{3}$ (E) 60

15. As $a+b+c=12$, $3a+2b+c=6$ en $5a+3b+2c=9$, dan is die waarde van c

(A) -9 (B) 12 (C) 9 (D) 6 (E) 3

16. In die diagram is $PA /\!\!/ YH$, $PA=12$, $PY=10$, $YT=18$ en $HA=17$. Die oppervlakte van $PYHA$ is



(A) 160 (B) 180 (C) 225 (D) 264 (E) 300

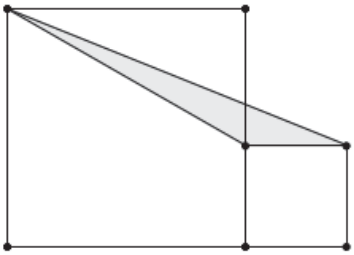
17. Die *vloer* van 'n reële getal x, aangedui deur $\lfloor x \rfloor$, is die grootse heelgetal kleiner of gelyk aan die getal, bv. $\lfloor 3.2 \rfloor = 3$. Die *plafon* van 'n reële getal x, aangedui deur $\lceil x \rceil$, is die kleinste heelgetal groter of gelyk aan die getal, bv. $\lceil -4.7 \rceil = -4$. Wat is die waarde van $\left\lceil \sqrt{23} \right\rceil + \lfloor -3.2 \rfloor$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

10. Daar is 8 rooi, 7 blou en 6 groen balle in 'n houer. Een bal word ewekansig uitgehaal. Wat is die waarskynlikheid dat dit nóg rooi nóg groen is?

(A) $\frac{6}{21}$ (B) $\frac{8}{21}$ (C) $\frac{8}{13}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

11. Die sylengte van die groot vierkant is 6 en die sylengte van die klein vierkant is 2. Vind die oppervlakte van die ingekleurde driehoek.



(A) 1 (B) 3 (C) 2 (D) 6 (E) 4

12. In 'n groep van 50 meisies dra elkeen 'n rooi of 'n geel hemp en 'n swart of 'n grys broek. As 14 meisies rooi hemde met swart broeke dra, 31 meisies geel hemde dra en 18 meisies grys broeke dra, dan is die aantal meisies wat geel hemde met grys broeke dra

(A) 5 (B) 7 (C) 9 (D) 11 (E) 13

13. A houer het 'n massa van 36 kg wanneer dit 'n kwart vol is en wanneer dit 'n derde vol is, het dit 'n massa 40 kg. Wat is die massa van die leë houer in kg?

(A) 24 (B) 36 (C) 48 (D) 18 (E) 20

5. 'n Reguitlyn gaan deur die punte $(2;3)$ en $(4;7)$. Watter een van die volgende punte lê ook op die lyn?

(A) $(3;5)$ (B) $(1;2)$ (C) $(4;5)$ (D) $(0;2)$ (E) $(2;4)$

6. Die lengtes van die sye van 'n driehoek is drie verskillende ewe heelgetalle. Indien die oppervlakte van die driehoek nie nul is nie, wat is die minimum omtrek wat dit kan hê?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 22

7. In 'n lys van ses opeenvolgende positiewe heelgetalle is die som van die drie kleinste getalle N en is die som van die drie grootste getalle M. Watter een van die volgende is waar?

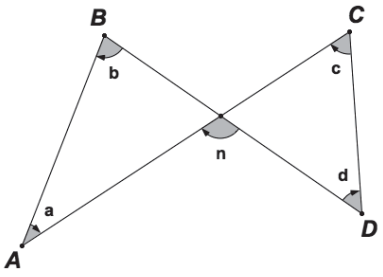
(A) $N = M + 9$ (B) $M = N + 9$ (C) $M + N = 9$ (D) $3M + 3N = 9$

(E) $3M - 3N = 12$

8. Die koördinate van 'n punt in die Cartesiese vlak is $(a; 2 - a)$, waar a 'n reële getal is. Watter een van die volgende is waar vir die posisie van die punt?

(A) Dit kan nie in die eerste kwadrant wees nie
(B) Dit kan nie in die tweede kwadrant wees nie
(C) Dit kan nie in die derde kwadrant wees nie
(D) Dit kan nie in die vierde kwadrant wees nie
(E) Dit kan in enige kwadrant wees

9. AC en BD is reguitlyne. Wat is die som van die hoeke a, b, c en d?

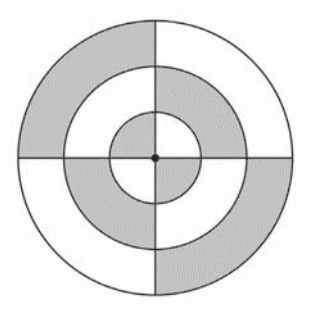


(A) n (B) $2n$ (C) $180° - n$ (D) $360° - n$ (E) $360° - 2n$

1. Die waarde van $1009^2 - 1008^2$ is

(A) 2017 (B) 2016 (C) 2015 (D) 17 (E) 1

2. Die radiusse van die konsentriese sirkels in die skets is onderskeidelik 2, 4 en 6. Die middellyne sny elke sirkel in kwarte. Wat is die oppervlakte van die gearseerde gedeelte?

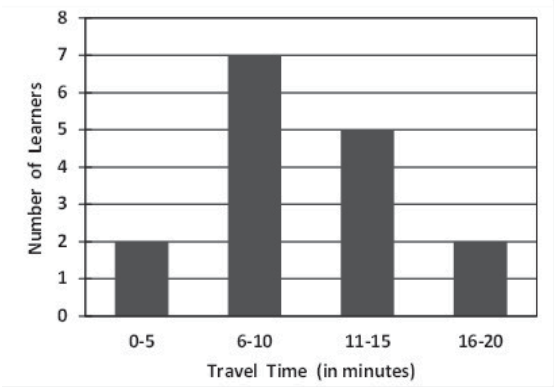


(A) 8π (B) 12π (C) 16π (D) 18π (E) 24π

3. Watter getal in die versameling $\{-2; -1; 0; 1; 2\}$ is die kleinste waarde van n waarvoor $3^n.3^3$ 'n volkome vierkant is?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

4. Die grafiek toon leerders se reistye van die huis na die skool. Hoeveel leerders reis langer as 10 minute?



(A) 2 (B) 5 (C) 7 (D) 8 (E) 15

OEFENVOORBEELDE

1. As 'n desimale getal is 6.28% gelyk aan

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. Die waarde van $1 + \dfrac{1}{3 + \dfrac{1}{2}}$ is

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{6}{7}$ (E) $\frac{9}{7}$

3. Die tienesyfer van $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

MOENIE OMBLAAI VOORDAT JY GEVRA WORD OM DIT TE DOEN NIE

SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2017 EERSTE RONDTE
SENIOR AFDELING: GRAAD 10-12

10 Maart 2017 **Tyd: 60 minute** **Aantal vrae: 20**

Instruksies

1. *Hierdie is 'n veelvuldige-keuse vraestel. Na elke vraag is vyf antwoorde, genommer A, B, C, D en E. Net een van hulle is reg.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 5 punte.*
   2.2. *Daar is geen penalisering vir foutiewe antwoorde of vrae wat nie beantwoord is nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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Organisasies betrokke: AMESA, SA Wiskundevereniging,
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