SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade EIGHT First Round 2019

Solutions

1. **E** $\dfrac{20+19}{20-19} = \dfrac{39}{1} = 39$

2. **C** $\sqrt{2\times 0+1\times 9} = \sqrt{9} = 3$

3. **C** At a rate of 7 pages in 10 seconds it will print 21 pages in 30 seconds (half a minute).

4. **E** $2+0+1\times 9 = 2+0+9=11$ which is odd.

5. **D** $\dfrac{2019}{20+19} \approx \dfrac{2020}{40} \approx 50$

6. **B** If all the shaded parts are placed in the same row it can be seen that exactly one quarter of the figure is shaded.

7. **E** Since $5 \times 4 = 20$, multiplying any positive whole number by 5 and then by 4 must result in a multiple of 20 and must thus end in a zero.

8. **E** Triangle ABC is isosceles. Thus angle B = 75° and angle A = 30°.
It follows that x = 50° + 30° = 80°. One can alternatively use the sum of the angles in quadrilateral BCED.

9. **C** The sequence of dots contained within the shapes is 1, 3, 6, 10, … i.e. the triangular numbers. The 10^{th} triangular number is 1 + 2 + 3 + … + 9 + 10 = 55.

10. **C** Note that $1 = 1^2$ and $400 = 20^2$. There are thus 20 perfect squares from 1 to 400.

11. **B** Each cube has volume 56/7 = 8 cm^3. The side length of each cube is thus 2 cm, and each face has area of 4 cm^2. There are 30 visible faces with a total area of 30 x 4 = 120 cm^2.

12. **D** We can immediately see that q = ¼ and u = 2 and p = 8. Thus r = ½ and s = 1/16. Thus r + s = 8/16 + 1/16 = 9/16.

13. **A** For the sum to be zero, each bracket has to be zero. Thus x = 20 and y = –19, and hence $x + y = 1$.

14. **A** $\dfrac{1}{a}+\dfrac{1}{a}=1 \quad \therefore \quad a=2 \quad ; \quad \dfrac{1}{b}+\dfrac{1}{b}+\dfrac{1}{b}=1 \quad \therefore \quad b=3$

Thus: $\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{c}=1 \quad \therefore \quad \dfrac{3}{6}+\dfrac{2}{6}+\dfrac{1}{c}=1 \quad \therefore \quad c=6$

15. **D** There are 9 triangles. The 9 equal angles at the centre of the picture are thus each 360°/9 = 40°. Each of the angles at the tips is thus 180° – 40° – 90° = 50°.

16. **B** Let the number of spiders be x. The number of zebras is $2x$ and the number of bees is $3x$. The number of legs is thus $8(x) + 4(2x) + 6(3x)$, i.e. $34x$. $34x = 102$, thus $x = 3$. Alternatively, the ratio of zebras to spiders to bees is 2:1:3. The ratio of their legs is thus 8:8:18. 8 + 8 + 18 = 34. 102/34 = 3.

17. **B** B and E have the same height, but B jumped higher than E, so B will have the better score. B and C jumped the same height, but C is taller than B, so B will again have the better score. E jumped higher than D in spite of being shorter than D, hence E performed better than D. So B who performed better than E, also performed better than D. A is taller than B but jumped a lower height, thus B has the highest score overall.

18. **D** Set up a table summarising the information:

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Donald	L	T	L	T	L	T	T
Herman	T	L	T	T	L	L	T

Note that Donald lies on Wednesdays, so his statement "*Today is Wednesday*" could not have been made on Wednesday. His statement must thus have been made on a day, other than Wednesday, on which he was lying, i.e. either Monday or Friday. Monday doesn't work since Herman's response "*Yes, it is*" is a lie, but he tells the truth on Monday. The statement must thus have been made on Friday when both Donald and Herman are lying.

19. **A** The sum of a 2-digit number and two 1-digit numbers can be at most 9 + 9 + 99 = 117. This means that C = 1 and hence the sum of the three numbers is 111. Note that 88 + 9 + 9 = 106, which is less than 111. Since B needs to be greater than 8 this means that B = 9. 111 – 99 = 12, which means A = 6. We thus have A + B + C = 6 + 9 + 1 = 16.

20. **A** Consider the first 18 songs played where every 3rd song is a song that Waheeda doesn't like, and all the others are songs that she likes. Waheeda would then have played all the songs that she doesn't like. Hence, the 19th, 20th and 21st songs would have to be songs that she likes. Waheeda would have to play 21 songs to be sure that there would be 3 consecutive songs played that she likes. (If the number of songs played is 20 or less, one could spread the 6 songs that she doesn't like across the list (i.e. every 3rd song) in such a way that there are no 3 consecutive songs that she likes.)