THE HARMONY SOUTH AFRICAN
MATHEMATICS OLYMPIAD

FIRST ROUND 2005

SENIOR SECTION: GRADES 10, 11 AND 12

15 MARCH 2005

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.

2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. Scoring rules:

    3.1 Each correct answer is worth 5 marks.

    3.2 There is no penalty for an incorrect answer or any unanswered questions.

4. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**

5. Diagrams are not necessarily drawn to scale.

6. Indicate your answers on the sheet provided.

7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.

8. Answers and solutions are available at: http://science.up.ac.za/samo/

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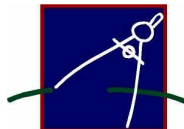
PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

    (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

2. The circumference of a circle with radius 2 is

    (A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129
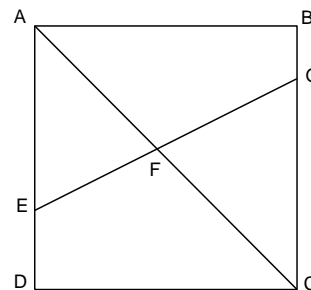
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1. $0.001 \div 0.05$ equals

(A) $\dfrac{1}{55}$ (B) $\dfrac{1}{500}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{50}$ (E) 2.

2. $\dfrac{1}{\frac{1}{3}+\frac{1}{4}}$ equals
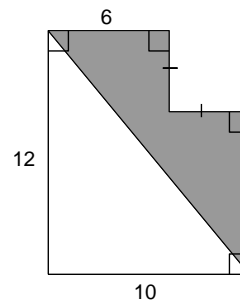
(A) $\dfrac{7}{12}$ (B) $\dfrac{2}{7}$ (C) $\dfrac{7}{2}$ (D) $\dfrac{1}{6}$ (E) $\dfrac{12}{7}$.

3. $ABCD$ is a square. If angle $\widehat{AFG} = 120°$, then angle $\widehat{AEF}$ equals

(A) 45°   (B) 50°   (C) 60°   (D) 75°   (E) 80°.

4. The area of the shaded region is

(A) 54   (B) 56   (C) 44   (D) 40   (E) 60.

5. If $\dfrac{h}{x} = 12$, $\dfrac{h}{y} = 8$, and $x + y = 5$, then h equals

(A) 24   (B) 4   (C) 48   (D) 15   (E) 12.

6. Two different numbers from the list -8; -5; -3; 0; 2; 4 are multiplied together. The least possible result is

(A) -40   (B) -32   (C) -20   (D) 0   (E) 8.

7. The wheels of a car have radius 70 cm. The number of revolutions that each wheel makes while the car travels 100 m is approximately

(A) 71   (B) 37   (C) 51   (D) 46   (E) 23.

8. If $2^a 2^b 2^c = 256$, then the average of a, b and c is

(A) $\dfrac{256}{3}$ (B) $\dfrac{8}{3}$ (C) $\dfrac{4}{3}$ (D) 8   (E) impossible to determine.

9. If $0 < x < y < 1$, which one of the following is not necessarily true?

(A) $(x - y)^2 < x^2$ (B) $(x - y)^2 < y^2$ (C) $x^2 < 2y^2$ (D) $x^3 - y^3 < x^2$
(E) $y^3 - x^3 < y^3$
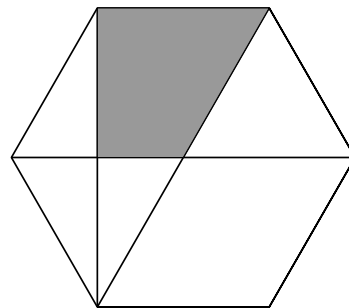
10. The proportion of the area of the regular hexagon that is shaded is

(A) $\dfrac{1}{5}$ (B) $\dfrac{9}{40}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2\sqrt{3}}$ (E) $\dfrac{1}{6}$.

11. The digits of the numbers from 20 to 59 inclusive are typed in one long line. Below that the digits of the numbers from 21 to 60 are typed:

$$202122232425 \quad \cdots \quad 59$$
$$212223242526 \quad \cdots \quad 60.$$

The number of times that a digit on the top line has the same digit directly below it is

(A) 44   (B) 36   (C) 54   (D) 32   (E) 40.

12. Adam and Brian find a jumble of n ropes lying on the floor. Each takes hold of one loose end. The probability that they are both holding the same rope is

(A) $\dfrac{1}{n-1}$　　　(B) $\dfrac{1}{2n-1}$　　　(C) $\dfrac{1}{n}$　　　(D) $\dfrac{1}{2n}$　　　(E) $\dfrac{2n-1}{2n}$.

13. Which one of the following has the largest value?

(A) 5×5^5　　　(B) 5^{5^5}　　　(C) $(5^5)^5$　　　(D) 5^{55}　　　(E) $(5 \times 5)^5$

14. The number 2005 is made up of exactly two zeros and two other digits whose sum is 7. The number of 4-digit numbers with these properties is

(A) 9　　　(B) 11　　　(C) 24　　　(D) 6　　　(E) 18.

15. If x is a positive number such that $x + \dfrac{1}{x} = 4$, then the value of $x^3 + \dfrac{1}{x^3}$ is

(A) 60　　(B) 52　　(C) 64　　(D) 12　　(E) not uniquely determined.

16. $PQRS$ is a straight line. There are six triangles in this figure, and the smallest one, which is shaded, has area equal to $\dfrac{1}{4}$ of the area of the largest. If the smallest triangle has area 1, the average area of all the triangles is
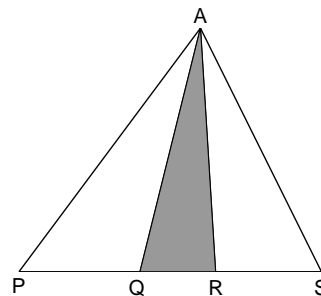
(A) $\dfrac{8}{3}$　　　(B) $\dfrac{5}{3}$　　　(C) $\dfrac{13}{6}$　　　(D) $\dfrac{3}{2}$　　　(E) $\dfrac{7}{3}$.

17. Consider the pattern shown below:

$$
\begin{array}{llllll}
\text{row 1:} & & 1 & & & \\
\text{row 2:} & & 3 & 5 & & \\
\text{row 3:} & & 7 & 9 & 11 & \\
\text{row 4:} & & 13 & 15 & 17 & 19 \\
& \text{etc.} & & & &
\end{array}
$$

The number at the end of row 80 is

(A) 6479   (B) 6319   (C) 6481   (D) 6379   (E) 6531.

18. Ann is three times as old as Carol will be next year. In 12 years' time Betty will be three times as old as Ann is now, and she is now eight times as old as Carol is. Betty's age is now

(A) 12   (B) 9   (C) 24   (D) 18   (E) 20.

19. The right-angled triangle has two circles touching its sides as shown. If the angle at R is $60°$ and the radius of the smaller circle is 1, then the radius of the larger circle is
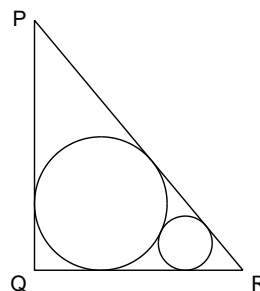
(A) $2\sqrt{3}$ (B) 2   (C) $2\sqrt{2}$ (D) 3   (E) $\sqrt{3}$.

20. Statistics show that 10% of smokers get lung cancer, and 90% of lung cancer patients are smokers. If 20% of the population smokes, then the percentage of the population having lung cancer is

(A) $\dfrac{9}{5}$ (B) 2   (C) 3   (D) 9   (E) $\dfrac{20}{9}$.