THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

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FIRST ROUND 2000

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

12 APRIL 2000

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

ANSWERS

1. E
2. B
3. E
4. E
5. A
6. B
7. E
8. A
9. D
10. B
11. D
12. C
13. A
14. B
15. C
16. D
17. E
18. B
19. D
20. D

SOLUTIONS

1. $45 \times 100 \times 100$ square metres cost R180 000. Therefore 1 square metre cost $\frac{18\cancel{0000}}{45\cancel{0000}} = \frac{2}{5} = 0,4$ rands.

Answer: E

2. The expression is

$$\frac{(10-9)+(8-7)+(6-5)+(4-3)+(2-1)}{1+(-2+3)+(-4+5)+(-6+7)+(-8+9)}$$
$$= \frac{1+1+1+1+1}{1+1+1+1+1} = \frac{5}{5} = 1$$

Answer: B

3. The average is $\frac{1}{2}\left(\frac{1}{8}+\frac{1}{10}\right) = \frac{1}{2}\left(\frac{5+4}{40}\right) = \frac{9}{80}$

Answer: E

4. The numbers are (A) $3,14160000\ldots$ (B) $3,14161416\ldots$ (C) $3,1416416\ldots$ (D) $3,14161616\ldots$ (E) $3,141666\ldots$

The first four decimal places are the same in all five of these numbers, but the fifth decimal place is the highest in (E).

Answer: E

5. If $\frac{x}{y} = 0,75 = \frac{4}{3}$ then $\frac{y}{x} = \frac{4}{3}$ and $\frac{x+2y}{x} = 1 + 2 \times \frac{y}{x} = 1 + 2 \times \frac{4}{3} = \frac{11}{3}$

Answer: A

6. $g(-\frac{1}{2}) = \frac{1}{-\frac{1}{2}} = -2$. Therefore $f(g(-\frac{1}{2})) = f(-2) = 2(-2) - 1 = -5$.

Answer: B

7. $p^2 - q^2 = (p-q)(p+q)$ where $p^2 - q^2 = 1$ and $p + q = 4$. Therefore $1 = (p-q) \times 4$ and $p - q = \frac{1}{4}$.

Answer: E

8. If 400g costs R10,00 then 100g costs R$\frac{10}{4}$ = R2,50, if 500g costs R13,00 then 100g costs R2,60, and if 800g costs R16,00 then 100g costs R2,00. Therefore the most expensive way to buy 2kg of fish is to buy four 500g packets costing $4 \times R13 = R52$.

The cheapest is to buy two 800g packets and one 400g packet costing a total of $2 \times R16 + R10 = R42$. The difference is R10.

Answer: A

9. 732 000 new telephone lines in 366 days is 732 000 phone lines in 366x24x60 minutes. Therefore 1 new telephone line in $\frac{366 \times 24 \times 60}{732000}$ minutes. That is 1 new line in every $\frac{366 \times 24 \times 60}{732\ 000} = \frac{72}{100}$ minutes. That is approximately three quarters of a minute or 45 seconds. (D) is the closest.

Answer: D

10. The sum of the original three numbers is $3 \times 18 = 54$, and the sum of the new set of three numbers is $3 \times 23 = 69$. Therefore the largest number was increased by $69 - 54 = 15$. Therefore the number that was replaced is $38 - 15 = 23$.

Answer: B

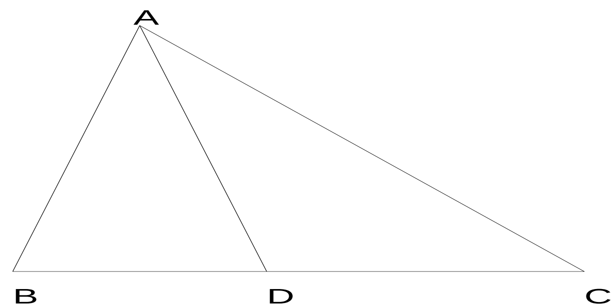
11. The lengths of all the sides were multiplied by 3. Therefore the lengths of the base and of the perpendicular height were each multiplied by 3. But the area of a triangle is half the product of the lengths of the base and the perpendicular height. Therefore the area is $3 \times 3 = 9$ times that of the smaller triangle. The number of ml of paint needed is $9 \times 4 = 36$.

Answer: D

12. Notice that the marks add up to 100. From Sections A and B Ellie obtains $5 \times 3 + \frac{90}{100}(10 \times 5) = 60$ points. In order to obtain another 20 points from Section C she needs to answer at least 3 questions correctly.

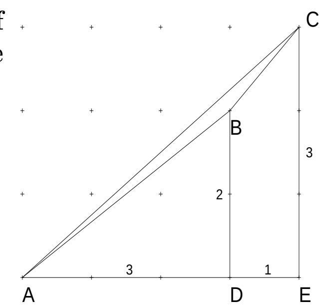
Answer: C

13. The three angles of ΔABD are each equal to $60°$. Triangles ABC and ADC have the same height. So if the area of ΔABC is twice that of ΔADC then the $BC = 2DC$. Therefore $DC = BD$ and ΔADC is isoscecles. But the exterior angle at D is $60°$. Hence $\widehat{DAC} = 30°$ and $\widehat{BAC} = 90°$.



Answer: A

14. There are various constructions that can be used to find the area of ΔABC. One possibility is shown in the figure, which also shows the lengths of the various sides.
The area of triangle ACE is $\frac{1}{2}(4)(3) = 6$.
The area of triangle ABD is $\frac{1}{2}(3)(2) = 3$.
The area of trapezium $BDEC$ is $\frac{1}{2}(2+3)(1) = 5/2$.
Therefore the area of ΔABC is $6 - 3 - 5/2 = 1/2$.



Answer: B

15. Suppose that there are x learners in each class. Then the number of girls in the entire grade 11 is $\frac{1}{3}x + \frac{3}{5}x = \frac{14}{15}x$. Therefore the number of boys is $2x - \frac{14}{15}x = \frac{16}{15}x$. Hence the required ratio is $7\left(\frac{2x}{15}\right)$ to $8\left(\frac{2x}{15}\right)$ which is 7:8. Alternatively, choose a number which is divisible by 3 and by 5, for example 15. Let $x = 15$. Then there are 14 girls and 16 boys. Again the ratio is 7:8.

Answer: C

16. The result of the heating process is that water evaporates, the sugar stays behind. So originally there are $\frac{10}{100} \times 2 = 0,2$kg of sugar in the solution. This mass of sugar stays the same. When the solution is 85% water then 15% is sugar. So 15% of the solution weighs 0,2kg. Therefore 100% of the solution weighs $\frac{100}{15} \times 0,2 = \frac{4}{3}$kg.

Answer: D

17. 10^{20} when written out in full consists of a one followed by 20 zeroes. Therefore $10^{20} - 2$ consists of 19 nines and ends in an eight. The sum of these digits is $19 \times 9 + 8 = 20 \times 9 - 1 = 179$.

Answer: E

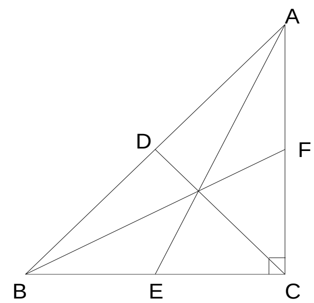
18. The lowest common multiple of 8 and 5 is 40. Therefore they will be off together every 40th day. The year 2000 has 366 days. Hence 31 December 2000 is 366-20 = 346 days after 20 January. We need the highest multiple of 40 which is less than 346. That is 320 which is 26 less than 346. The required date then is 31-26=5. The 5th day of December.

Answer: B

19. The angle at C is $90°$ and D is the midpoint of the hypothenuse. Therefore D is the centre of the circle through A, B and C. The radius of this circle is $DC = 2$, and $AB = 4$. Now use Pythagoras' theorem three times:

$$\begin{aligned} AE^2 &= AC^2 + EC^2 = AC^2 + \frac{1}{4}BC^2 \\ BF^2 &= BC^2 + CF^2 = BC^2 + \frac{1}{4}AC^2 \end{aligned}$$

Then $AE^2 + BF^2 = \frac{5}{4}\left(AC^2 + BC^2\right) = \frac{5}{4}AB^2 = \frac{5}{4} \times 16 = 20.$



Answer: D

20. 6275 is a 4-digit number which stands for $6 \times 10^3 + 2 \times 10^2 + 7 \times 10 + 5$. $2^{2\ 000}$ has m digits. Therefore $2^{2\ 000} = a \times 10^{m-1}$ plus lower powers of 10, where a is a nonzero digit. $5^{2\ 000}$ has n digits. Therefore $5^{2\ 000} = b \times 10^{n-1}$ plus lower powers of 10, where b is also a nonzero digit. Notice that $2^{2\ 000} \times 5^{2\ 000} = 10^{2\ 000}$. But $2^{2\ 000} \times 5^{2\ 000}$ is also $(a \times 10^{m-1} + \cdots)(b \times 10^{n-1} + \cdots)$. Hence $10^{2\ 000} = (ab) \times 10^{m+n-2}$ it follows that ab can only be equal to 10. So

$$\begin{aligned} 10^{2\ 000} &= 10 \times 10^{m+n-2} \\ &= 10^{m+n-1}, \end{aligned}$$

and $m + n = 2\ 001$.

Answer: D