SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade NINE First Round 2021

Solutions

1. **C** $20{,}21 + 20 + 2{,}1 = 42{,}31$

2. **E** 3 hours and 20 minutes after 20:21 is 23:41

3. **A** The closest multiple of 4 to 2021 is 2020, and a quarter of 2020 is 505.

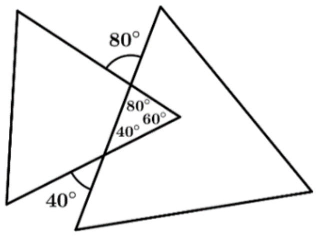
4. **B** $\sqrt{\sqrt{20 + 20 + 20 + 21}} = \sqrt{\sqrt{81}} = \sqrt{9} = 3$

5. **D** Numbers between 10 and 30 that are not even: 11, 13, 15, 17, 19, 21, 23, 25, 27, 29
Removing the primes leaves: 15, 21, 25, 27
Removing the multiples of 3 leaves: 25

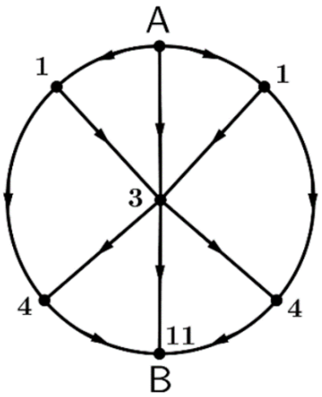
6. **D** The perimeter of the house is the same length as the perimeter of the outer square.
The outer square has an area of 400 m^2, and thus a side length of 20 m.
The perimeter of the house is thus $20 \times 4 = 80$ m.

7. **C** The first nine perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81. Using two of these in combination, the largest possible sum is 81 + 16 = 97.

8. **C**



9. **C** Determine the total number of pathways to each vertex:



10. **D** While the other four statements *may* be true, $b = h$ will *always* be true since $b = d$ (vertically opposite angles) and $d = h$ (corresponding angles on parallel lines).

11. **A** Let the code be $ABCDE$. Since $A + B + C + D = 19$ and $A + B + C = 15$ it follows that $D = 4$. Also, since $B + C + D = 15$ it follows that $A = 4$. Similarly, $B = E = 4$. The code is thus 44744 and the sum of the digits is 23.

12. **A** Since the code is palindromic it is of the form A B C C B A. Since the 3rd digit is twice the 1st digit we can express the code as A B 2A 2A B A. Since the 5th digit is one more than the 4th digit we can write A (2A+1) 2A 2A (2A+1) A. Finally, since the 2nd digit is 7 we have 2A+1 = 7, thus A = 3. The code is thus 376673, and the sum of the digits is 32.

13. **B** Since the triangle is equilateral, and $\frac{60°}{360°} = \frac{1}{6}$, the area of each of the three shaded sectors is $\frac{1}{6}$ of the area of the circle. Total shaded area = $\pi(4)^2 \times \frac{1}{6} \times 3 = 8\pi$ cm^2.

14. **D** The total distance completed is the first half of the run plus 3/5 of the second half. Thus: $\frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

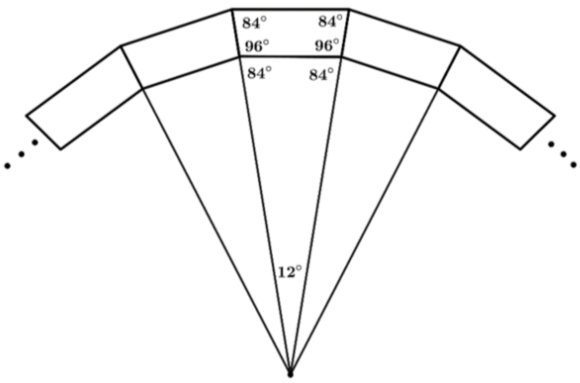
15. **B** Note that if the area of a square decreases by a factor of 4 then the side length decreases by a factor of 2. If the smallest square has side length x then the side lengths of the four squares are x, $2x$, $4x$ and $8x$.
We thus have $x + 2x + 4x + 8x = 30$, i.e. $15x = 30$. Therefore $x = 2$, from which the largest square will have side length 16 cm.

16. **B** $\frac{1}{2021} + \frac{1}{2022} + \frac{1}{2023} = \frac{20^2+20+1}{2023} = \frac{421}{2023}$

17. **E** The area of triangle AFD is half the area of the square. The area of triangle AED is a quarter of the area of the square. It thus follows that the area of the shaded region is a quarter of the area of the square, i.e. 3 cm^2.

18. **E** If each square has side length x then $x^2 + x^2 = \left(2\sqrt{2}\right)^2$. Thus $2x^2 = 8$ from which we have $x = 2$. The rectangle is thus 6 by 8 and hence has a diagonal of 10.

19. **A** $360° \div 12° = 30$.



20. **E** Since m and n are positive integers, both m^3 and $\frac{n^2}{2}$ are positive and less than 45. The only cubes less than 45 are 1, 8 and 27. If $m = 1$ then $\frac{n^2}{2} = 44$ and $n^2 = 88$, which is not possible since n has to be a positive integer. Thus $m \neq 1$. Using a similar argument we can show that $m \neq 2$. If $m = 3$ then $m^3 = 27$ and $\frac{n^2}{2} = 18$, making $n = 6$. Thus $m + n = 3 + 6 = 9$.