THE OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2020 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

14 May 2020 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 000 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

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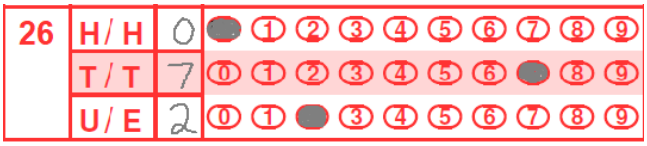
HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

DO NOT TURN THE PAGE BEFORE YOU ARE TOLD TO DO SO

Part A: Three marks each

1. The sum of the digits of a 7-digit number is 6. What is the product of the digits?

2. How many of the first one hundred positive integers are divisible by all of the numbers 2, 3, 4 and 5?

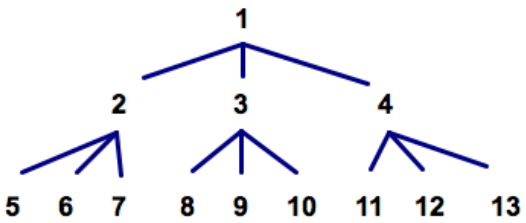
3. Kerstin opens her dictionary and says: "The sum of the number of the page I'm currently on and the number of the next page is 341". On which page is Kerstin?

4. What is the smallest positive integer that you can multiply by 2020 to get a perfect square?

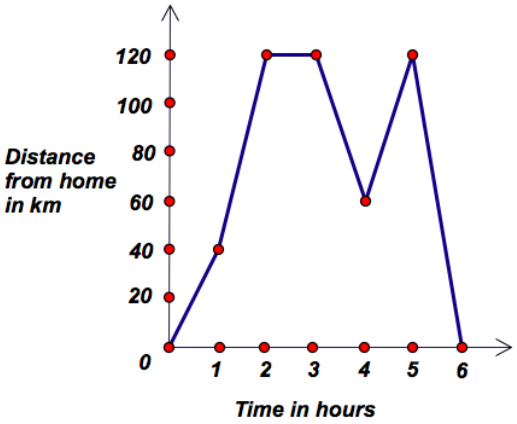
5. On four consecutive days David woke up at 5:30, 5:45, 7:30 and 7:55, respectively. On average, over the four days, how many minutes after 6:00 did he wake up?

6. One hundred students wrote the June examinations. Thirty students failed Physics, 25 failed Mathematics and 12 failed both subjects. How many students passed both subjects?

7. Consider the pyramid of numbers in the diagram. What is the second-to-last number in row six?



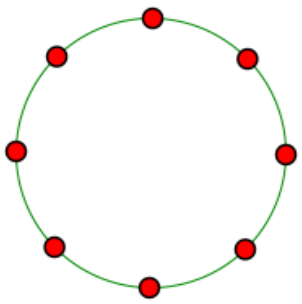
8. The graph describes a six-hour trip that Herman recently took. What was his average speed in km/h?



9. The difference between the square root and the fourth root of the same number is 12. What is the number?

10. If the repeating (recurring) decimal number $0.\dot{7}$ is represented by the fraction $\frac{a}{b}$ in lowest terms, what is $a + b$?

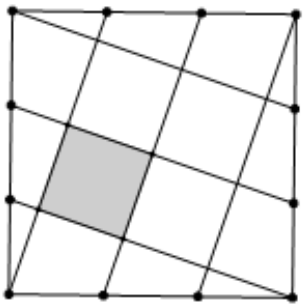
11. Andy has a number of red, green and blue counters. He places eight counters equally spaced around a circle according to the following rules:



- two red counters may not be placed next to each other,
- two green counters may not be placed diagonally opposite each other.

What is the smallest number of blue counters that Andy will need to use?

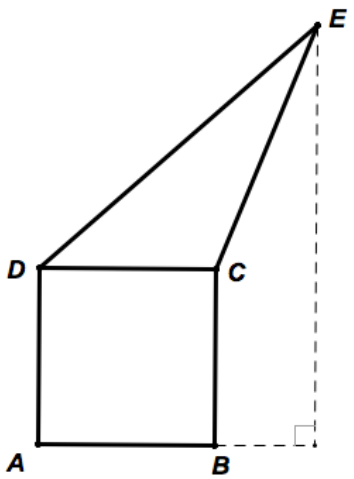
12. Each side of the large square is divided into three equal parts as shown in the figure. The area of the small shaded square is 9. What is the area of the large square?



13. Ellie spun a coin thirty times. Whenever the coin showed heads, Ellie gave two of her sweets to Pebetse. When the coin showed tails, Pebetse gave three of her sweets to Ellie. After 30 spins, both Ellie and Pebetse had the same number of sweets as they started with. How many times were heads spun?

14. None of the learners in a certain school know the rules of Sodoku. On 15 January a teacher teaches these rules to three learners. Every day after this, every leaner who already learnt the rules, teaches them to two learners who do not know them yet. How many learners in the school know the rules of Sodoku by midnight on 20 January?

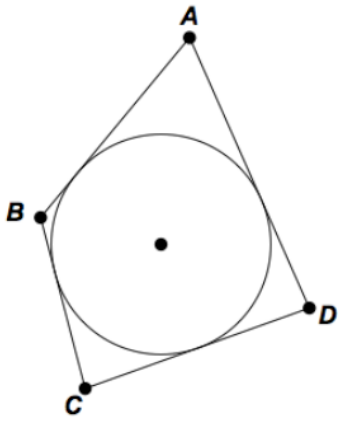
15. Square $ABCD$ has side length 60. Triangle CDE and square $ABCD$ have equal areas. What is the distance of point E to the extended line AB?



Part B: Four marks each

16. You have two friends, Odo and Priscilla. Everything Odo says on odd numbered days of the month is a lie while everything Priscilla says on prime numbered days of the month is a lie. On all other days they tell the truth. One day in January you meet these two friends sitting under a tree in the park. Priscilla says "I have already lied on 13 days this month." Odo says "I have lied on at least 13 days this month", to which Priscilla responds, "In fact, he has lied on less than 11 days this month". Which day of the month is it?

17. If a quadrilateral $ABCD$ has an area of 60 and an inscribed circle with radius 3, what is the perimeter of quadrilateral $ABCD$?



18. At a certain university, 52 percent of all students pass; 48 percent of the male students pass and 55 percent of the female students pass. If the ratio of the number of male students who pass to the number of female students who pass is expressed as a fraction $\frac{m}{f}$ in lowest terms, determine $m + f$.

19. If $f(n+1) = \dfrac{2f(n)+1}{2}$ for every positive integer n, and $f(1) = 2$, determine $f(101)$.

20. Tom has 3 blue and 4 red balls in a bag. Dick takes 2 balls out of the bag and keeps them. Harry then takes out one of the balls left in the bag. If the probability of Harry taking out a blue ball is $\dfrac{x}{210}$, what is the value of x?

Part C: Five marks each

21. A supermarket has 128 crates of apples. Each crate contains at least 120 and at most 144 apples. What is the largest integer n such that at least n crates must contain the same number of apples?

22. If $x = \sqrt{38 - 12\sqrt{2}}$, calculate the value of $x^2 - 12x + 40$.

23. How many triangles with positive areas in the xy-plane, and vertices with integer coordinates $(x; y)$, are there satisfying
$1 \leq x \leq 4$ and $1 \leq y \leq 4$?

24. A circle C has radius 5 and its centre is at $(-3; 2)$. Point A has coordinates $(x; 2)$. Q is the midpoint of AB and traces out a circle as B moves around the circumference of circle C. For what value of x will the centre of the smaller circle formed by Q lie on the circumference of circle C?

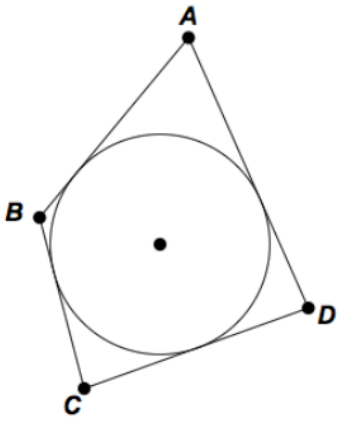
25. On level ground, a car travels at 63 km/h. Going uphill, it slows down to 56 km/h. Going downhill, it speeds up to 72 km/h. A trip from A to B by this car takes 4 hours. The return trip from B to A takes 4 hours and 40 minutes. What is the distance between A and B?

23. Hoeveel driehoeke met positiewe oppervlaktes, en hul hoekpunte heelgetalkoördinate $(x; y)$ in die xy-vlak, is daar sodat
$1 \leq x \leq 4$ en $1 \leq y \leq 4$?

24. 'n Sirkel C het 'n radius van 5 en sy middelpunt is by $(-3; 2)$. Punt A se koördinate is $(x; 2)$. Q is die middelpunt van AB en stippel 'n sirkel uit soos wat B op die omtrek van sirkel C beweeg. Vir watter waarde van x sal die middelpunt van die sirkel gevorm deur Q op die omtrek van sirkel C lê?

25. Op 'n gelyke pad ry 'n motor teen 63 km/h. Teen 'n opdraande ry dit teen 56 km/h en teen 'n afdraande teen 72 km/h. 'n Rit van A na B met hierdie motor neem 4 uur. Die terugrit van B na A neem 4 uur 40 minute. Wat is die afstand tussen A en B?

17. As vierhoek $ABCD$ 'n oppervlakte van 60 en 'n ingeskrewe sirkel met radius 3 het, wat is die omtrek van vierhoek $ABCD$?



18. Aan 'n sekere universiteit slaag 52 persent van die studente; 48 persent van die manstudente slaag en 55 persent van die damestudente slaag. As die verhouding van die getal manstudente wat slaag tot die getal damestudente wat slaag as 'n breuk $\frac{m}{f}$ in eenvoudigste vorm uitgedruk is, bepaal $m + f$.

19. As $f(n+1) = \frac{2f(n)+1}{2}$ vir elke positiewe heelgetal n, en as $f(1) = 2$, bepaal $f(101)$.

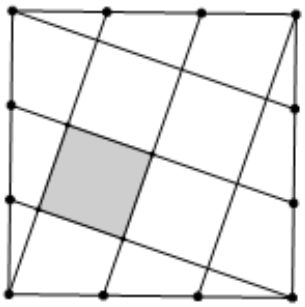
20. Tom het 3 blou en 4 rooi balle in 'n sak. Dick haal 2 balle uit die sak en sit hulle eenkant. Dan haal Harry een van die balle wat nog in die sak is, uit. As die waarskynlikheid dat Harry 'n blou bal uitgehaal het $\frac{x}{210}$ is, wat is die waarde van x?

Afdeling C: Vyf punte elk

21. 'n Winkel het 128 kratte appels. Elke krat bevat ten minste 120 en hoogstens 144 appels. Wat is die grootste heelgetal n sodat ten minste n kratte dieselfde hoeveelheid appels sal bevat?

22. As $x = \sqrt{38 - 12\sqrt{2}}$, bereken die waarde van $x^2 - 12x + 40$.

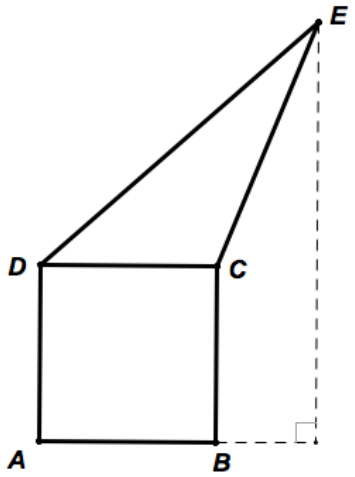
12. Elke sy van die groot vierkant is in drie gelyke dele verdeel soos aangetoon in die figuur. Die oppervlakte van die klein ingekleurde vierkant is 9. Wat is die oppervlakte van die groot vierkant?



13. Ellie spin 'n muntstuk dertig keer. Wanneer die kop na bo wys, gee Ellie twee lekkers aan Pebetse. Wanneer die stert na bo wys, gee Pebetse drie lekkers aan Ellie. Na 30 spinbeurte het Ellie en Pebetse dieselfde hoeveelheid lekkers as waarmee hulle begin het. Hoeveel keer het kop na bo gewys?

14. Nie een van die kinders in 'n sekere skool ken die reëls vir Sodoku nie. Op 15 Januarie leer 'n onderwyser die reëls aan drie kinders. Elke dag hierna leer elke kind wat reeds die reëls geleer het, die reëls aan twee kinders wat dit nog nie ken nie. Hoeveel kinders in die skool ken die reëls teen middernag van 20 Januarie?

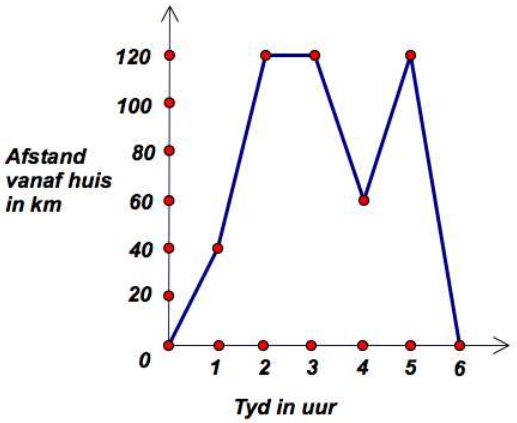
15. Vierkant $ABCD$ het 'n sylengte van 60. Driehoek CDE en vierkant $ABCD$ het gelyke oppervlaktes. Wat is die afstand van punt E na die verlengde lyn AB?



Afdeling B: Vier punte elk

16. Jy het twee vriende, Odo en Priscilla. Alles wat Odo op onewe dae van die maand sê, is 'n leuen terwyl alles wat Priscilla op priemgetaldae van die maand sê leuens is. Op al die ander dae praat hulle die waarheid. Een dag in Januarie ontmoet jy hierdie twee vriende waar hulle onder 'n boom in 'n park sit. Priscilla sê "Ek het alreeds op 13 dae van hierdie maand gejok." Odo sê "Ek het op ten minste 13 dae van hierdie maand gejok" waarop Priscilla antwoord, "Eintlik het hy op minder as 11 dae van hierdie maand gejok". Watter dag van die maand is dit?

8. Die grafiek beskryf 'n rit van ses uur wat Herman onderneem het. Wat was sy gemiddelde spoed in km/h?

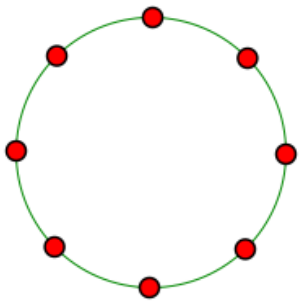


9. Die verskil tussen die vierkantswortel en vierdemagswortel van dieselfde getal is 12. Wat is die getal?

10. As die repeterende desimale getal $0.\dot{7}$ deur die breuk $\frac{a}{b}$ in eenvoudigste vorm voorgestel word, wat is $a + b$?

11. Andy het 'n aantal rooi, groen en blou tellers. Hy pak agt tellers eweredig in 'n sirkel volgens die volgende reëls:

- twee rooi tellers mag nie langs mekaar wees nie;
- twee groen tellers mag nie regoor mekaar wees nie.



Wat is die kleinste hoeveelheid blou tellers wat Andy moet gebruik?

Afdeling A: Drie punte elk

1. Die som van die syfers van 'n 7-syfergetal is 6. Wat is die produk van die syfers?

2. Hoeveel van die eerste honderd positiewe heelgetalle is deelbaar deur al die getalle 2, 3, 4 en 5?

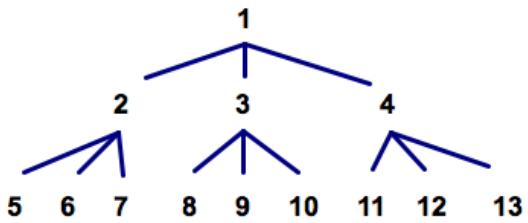
3. Kerstin maak haar woordeboek oop en sê: "Die som van die nommer van die bladsy waarop ek tans besig is en die nommer van die volgende bladsy is 341". Op watter bladsy is Kerstin?

4. Wat is die kleinste positiewe heelgetal waarmee jy 2020 kan vermenigvuldig om 'n volkome vierkant te kry?

5. Op vier agtereenvolgende oggende word David onderskeidelik om 5:30, 5:45, 7:30 en 7:55 wakker. Hoeveel minute na 6:00 word hy, gemiddeld oor die vier dae, wakker?

6. Eenhonderd studente het die Junie-eksamen geskryf. Dertig studente druip Fisika, 25 druip Wiskunde en 12 druip albei vakke. Hoeveel studente het albei vakke geslaag?

7. Beskou die getalpiramide in die diagram. Wat is die voorlaaste getal in ry ses?

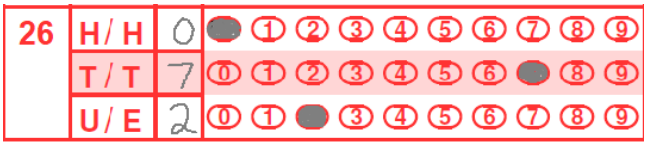


HOE OM DIE ANTWOORDBLAD TE VOLTOOI

Al die antwoorde is heelgetalle van 1 tot 999. Beskou die volgende voorbeeldvraag:

26. As $3x - 216 = 0$, bepaal die waarde van x.

Die antwoord is 72, en dus moet jy die blok vir vraag 26 op die antwoordblad as volg voltooi: kleur 0 in honderde-ry in, 7 in die tiene-ry, en 2 in die ene-ry:



Skryf die syfers van jou antwoord in die oop blokkies links in die betrokke ry, soos in die voorbeeld aangetoon; honderde, tiene en ene van bo na onder. Die drie syfers wat jy neergeskryf het, word nie nagesien nie; dit is vir jou eie gerief — slegs die ingekleurde sirkels word gemerk.

MOENIE OMBLAAI VOORDAT JY VERSOEK WORD OM DIT TE DOEN NIE

SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

DIE OLD MUTUAL SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2020 TWEEDE RONDTE
SENIOR AFDELING: GRAAD 10-12

14 Mei 2020 **Tyd: 120 minute** **Aantal vrae: 25**

Instruksies

1. *Die antwoorde op al die vrae is heelgetalle van 000 tot 999. Elke vraag het slegs een korrekte antwoord.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 3 punte in Afdeling A, 5 punte in Afdeling B en 6 punte in Afdeling C.*
   2.2. *Geen punte word afgetrek vir foutiewe antwoorde of onbeantwoorde vrae nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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