THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

FIRST ROUND 1999

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

29 APRIL 1999

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 7 marks in Part C.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.
7. When the invigilator gives the signal, start attempting the problems. You will have 60 minutes working time for the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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Part A: Three marks each.

1. The value of $\frac{0,1+0,01}{1+0,1}$ is

(A) $\frac{1}{11}$ (B) 0,1 (C) 0,2 (D) $\frac{1}{9}$ (E) 0,11

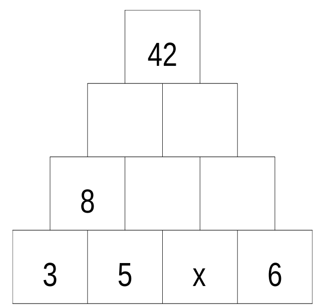
2. Three parties contested an election for 400 seats in parliament. Party A won 35% of the seats and party B won 20% of the seats. How many seats did party C win?

(A) 345 (B) 200 (C) 350 (D) 250 (E) 180

3. For long distance telephone calls, Telkom charges 30,9 cents per metering unit of 13,6 seconds. A long distance call of 3 minutes will cost you about

(A) R3.00 (B) R4.00 (C) R5.00 (D) R6.00 (E) R7.00

4. In the figure the number 8 is obtained by adding the two numbers directly below it. The other numbers in the top three rows can be obtained in the same way. The value of x is



(A) 7 (B) 3 (C) 5 (D) 4 (E) 6

5. If $n = 1999$, which of

$$2n,\quad n+2,\quad 3n^2,\quad 2^n,\quad n^3$$

is the largest?

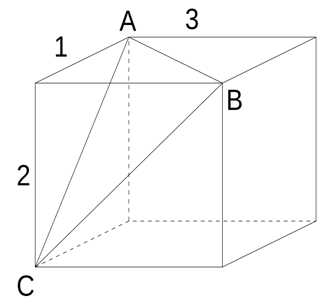
(A) $2n$ (B) $n+2$ (C) $3n^2$ (D) 2^n (E) n^3

Part B: 5 marks each

6. If a cube has a volume, in cm^3, which is numerically the same as its total surface area, in cm^2, then the length, in cm, of one side of the cube is

(A) 1 (B) 8 (C) 3 (D) 4 (E) 6

7. The diagram shows a rectangular box with side lengths as shown. The length of the *shortest* side of triangle ABC is



(A) 2 (B) $\sqrt{5}$ (C) 3 (D) $\sqrt{10}$ (E) $\sqrt{13}$

8. The value of

$$1+2(1+2(1+2(1+2(1+2(1+2(1+2(1+2(1+2(1+2(1+2))\cdots))$$

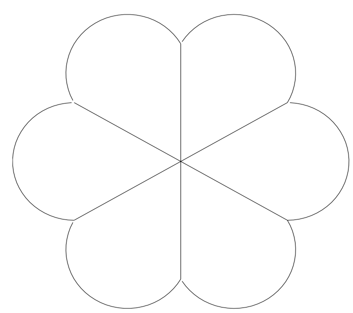
is

(A) $2^{10}+1$ (B) $2^{11}-1$ (C) $2^{11}+1$ (D)$2^{12}-1$ (E) $2^{12}+1$

9. Let $x=\sqrt{0,25}$, $y=\sqrt[3]{0,124}$ and $z=\sqrt[4]{0,0626}$. Then

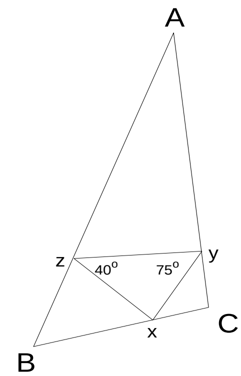
(A) $y<z<x$, (B) $x<z<y$, (C) $z<y<x$, (D) $x<y<z$, (E) $y<x<z$

10. A geometrical flower is made by drawing a semicircle on each side of a regular hexagon with sides of length 2, as shown. The area of the flower is



(A)$6\sqrt{3}+3\pi$ (B) $9+3\pi$ (C) $6+4\pi$ (D) $2+6\pi$ (E)$4\sqrt{3}+4\pi$

11. In ΔABC, $AY = AZ$, $BX = BZ$ and $CX = CY$, and the sizes of the two angles are as shown. The angle at A is



(A) 50^o (B) 60^o (C) 55^o (D) $52\frac{1}{2}^o$ (E) $57\frac{1}{2}^o$

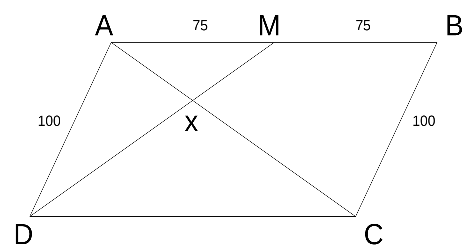
12. The number of natural numbers less than 400 that are not divisible by 17 or 23 is

(A) 360 (B) 376 (C) 359 (D) 382 (E) 358

13. Between 12:00 and 13:00 there are two times when the hands on a clock are exactly at right angles. How many minutes apart are these two times?

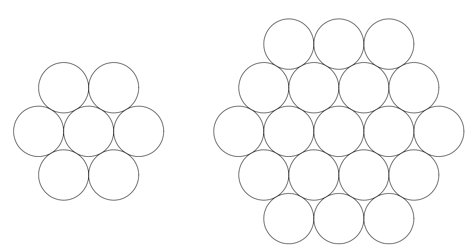
(A) 30 (B) $32\frac{8}{11}$ (C) $32\frac{1}{2}$ (D) $31\frac{5}{12}$ (E) $31\frac{2}{3}$

14. $ABCD$ is a parallelogram. $AD = BC = 100$ and $AM = MB = 75$. The ratio of the areas of triangles AMX and CDX is



(A) $\frac{3}{10}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{2}{9}$

15. The diagram show shapes made with the same size coins. The first six-sided shape has 2 coins along each side, and the second has 3 coins along each side. How many coins do you need to make up a six-sided shape with 21 coins along each side?



(A) 441 (B)820 (C) 1071 (D) 1141 (E) 1261

Part C: 7 marks each

16. The remainder if $1+2+2^2+2^3+\cdots+2^{1999}$ is divided by five is

(A) 0 (B) 1 (C)2 (D) 3 (E) 4

17. A two digit number is divided by the sum of its digits. The largest possible remainder is

(A) 9 (B) 13 (C) 15 (D) 16 (E) 17

18. A South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A) $\frac{1}{2^{14}}$ (B) $\frac{7}{2^{13}}$ (C) $\frac{1}{2^{13}}$ (D) $\frac{13}{2^{14}}$ (E) $\frac{13}{2^{13}}$

19. The square $ABCD$ has sides of length 2 units. M is the midpoint of AB and P is a variable point on BC. The smallest value of $DP+PM$ is

(A) $\sqrt{13}$ (B) $\sqrt{2}+\sqrt{5}$ (C) $2\sqrt{3}$ (D) $1+2\sqrt{2}$ (E) $\sqrt{15}$

20. The population of the village of Blaauwklippen at one time was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square. The original population is a multiple of

(A) 3 (B) 7 (C) 9 (D) 11 (E) 17