SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2018 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

16 May 2018 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 000 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

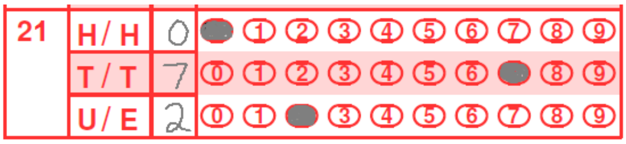
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

21. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



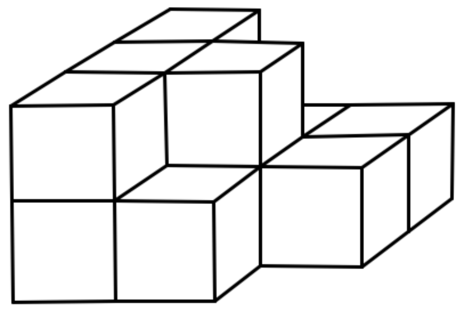
Write the digits of your answer in the the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

Part A: Three marks each

1. Calculate $2018 - (1000 + 100) - (\sqrt{100})^2 + 2$.

2. How many identical wooden cubes were used to build the structure in the picture? (The structure contains no hidden 'holes' in the parts one cannot see from the front).



3. Three different positive integers add up to 12. What is the largest possible value of the largest number?

4. Herman spent five days preparing for a test. On the first day he solved one problem, and on each consecutive day he solved twice as many problems as on the day before. How many problems did Herman solve altogether while preparing for the test?

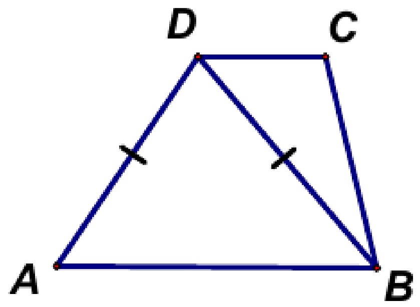
5. A square and a circle have equal perimeters. If the area of the square is 13π, what is the area of the circle?

6. A child's age, when increased by 3 years, gives a perfect square and when decreased by 3 years, gives the square root of this perfect square. How old is the child?

7. A triangle has area 800. If the base of this triangle is increased by 10% and the altitude to this base is also increased by 10%, what is the area of the resulting triangle?

8. What is the sum of all the integers between 50 and 150 that end in 1?

9. In trapezium $ABCD$, sides AB and CD are parallel, while diagonal BD is equal to side AD. $D\widehat{C}B = 110°$ and $C\widehat{B}D = 30°$. What is the size of angle $A\widehat{D}B$ in degrees?



10. If three times the larger of two numbers is four times the smaller and the difference between the two numbers is 8, what is the larger of the two numbers?

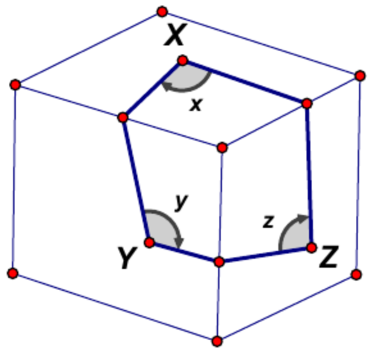
11. I write down all the consecutive integers from 25 to 208 inclusive: $25, 26, 27, \ldots 208$. How many digits have I written down in total? (Note that the three numbers 99, 100 and 101 have 8 digits in total.)

12. Thembi wants to arrange her collection of keyrings in rows and columns so that each row is complete. When she tries to arrange them in rows of 2, 3, 4 or 5 there is always one left over. If Thembi has more than 1 keyring, what is the least number of keyrings that she can have?

13. The lengths of two sides of a triangle are 200 cm and 150 cm. The length of the third side is also an integer (in cm). What is the minimum perimeter (in cm) of this triangle? (Ignore the case when the three vertices of the triangle lie on a straight line.)

14. The number 49 is special because it is possible to add the product and the sum of the digits and get 49 again, i.e. $4 \times 9 + (4 + 9) = 36 + 13 = 49$. How many such 2-digit positive integers are there in total?

15. An ant lives on the surface of a cube that has points X, Y and Z on three of the faces. The ant travels between X, Y and Z along the shortest paths between each pair of points. What is the sum of the three indicated angles x, y and z on the ant's path in degrees?



Part B: Five marks each

16. Let $f(x) = ax^7 + bx^3 + cx + 5$ where a, b and c are constants. If $f(-7) = -29$, determine $f(7)$.

17. Points A and B with respective coordinates $(3; a)$ and $(11; b)$ lie on the same line $y = \sqrt{35}x + k$. What is the distance between A and B?

18. I have a 3-digit number aba with a and b different. The 3-digit number is not divisible by 4 and the sum of the three digits is 15. How many different such 3-digit numbers aba are there?

19. For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence $1; 4; 7; 10; \ldots$. For how many values of k does S_k contain the term 2019?

20. For how many real numbers a does the quadratic equation $x^2 + ax + 6a = 0$ have only integer roots for x?

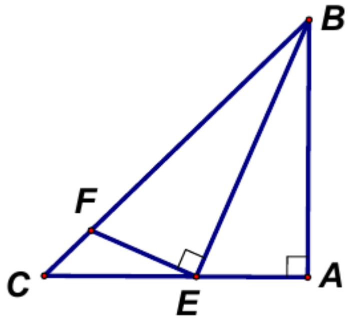
Part C: Six marks each

21. Three carpets have a combined area of 200 m^2. By overlapping the three carpets to cover a total floor area of 140 m^2, the area which is covered by exactly two layers of carpet is 24 m^2. What area of floor is covered by exactly three layers of carpet?

22. Teams A and B are playing a series of games. The probabilities of either team to win any game are equal. Either team A must win two games or team B must win three games to win the series. If the probability of team A winning the series is expressed in simplest form as $\frac{x}{y}$, calculate the value of $x + y$.

23. In the equation $(YE) \times (ME) = TTT$, each of the letters represents a different digit from the set $\{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$. Determine $E + M + T + Y$.

24. Right angled, isosceles triangle ABC has $\widehat{A} = 90°$ with E the midpoint of AC. F is a point on BC such that FE is perpendicular to EB. $AB = AC = 40\sqrt{3}$. What is the area of triangle CEF?



25. What is the smallest integer larger than $(\sqrt{3} + \sqrt{2})^6$?

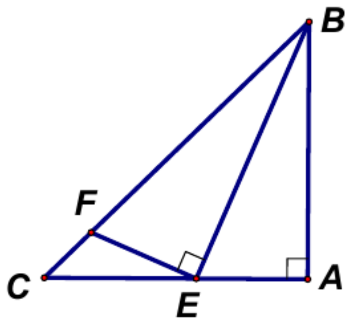
Deel C: Ses punte elk

21. Drie matte het 'n gesamentlike oppervlakte van 200 m^2. Die drie matte lê gedeeltelik oormekaar en bedek saam 'n totale vloeroppervlakte van 140 m^2. Die oppervlakte wat deur presies twee lae mat bedek word, is 24 m^2. Watter vloeroppervlakte word deur presies drie lae mat bedek?

22. Spanne A en B speel 'n reeks wedstryde. Die waarskynlikheid vir enige span om enige wedstryd te wen, is dieselfde. Span A moet twee wedstryde wen óf span B moet drie wedstryde wen om die reeks te wen. Die waarskynlikheid dat span A die reeks sal wen, word in eenvoudigste vorm uitgedruk as $\frac{x}{y}$. Bereken die waarde van $x + y$.

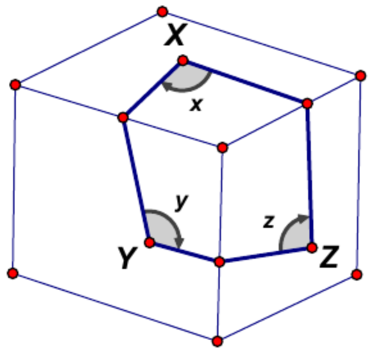
23. In die vergelyking $(YE) \times (ME) = TTT$ verteenwoordig elke letter 'n ander syfer in die versameling $\{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$. Bepaal $E + M + T + Y$.

24. In die reghoekige gelykbenige driehoek ABC is $\widehat{A} = 90°$ met E die middelpunt van AC. F is 'n punt op BC sodat FE loodreg is op EB. $AB = AC = 40\sqrt{3}$. Wat is die oppervlakte van driehoek CEF?



25. Wat is die kleinste heelgetal groter as $(\sqrt{3} + \sqrt{2})^6$?

15. ’n Mier woon op die oppervlakte van ’n kubus met punte X, Y en Z op drie van die vlakke. Die mier beweeg tussen X, Y en Z langs die kortste paaie tussen elke paar punte. Wat is die som van die drie aangeduide hoeke x, y en z op die mier se pad (in grade)?



Deel B: Vyf punte elk

16. Laat $f(x) = ax^7 + bx^3 + cx + 5$ waar a, b en c konstantes is. As $f(-7) = -29$, bepaal $f(7)$.

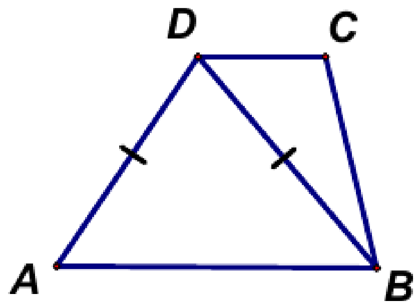
17. Punte A en B met koördinate $(3; a)$ en $(11; b)$ respektiewelik, lê op dieselfde lyn $y = \sqrt{35}x + k$. Wat is die afstand tussen A en B?

18. Ek het ’n 3-syfergetal aba met a en b verskillend. Die 3-syfergetal is nie deelbaar deur 4 nie en die som van die drie syfers is 15. Hoeveel verskillende 3-syfergetalle aba met hierdie eienskappe is daar?

19. Vir elke positiewe heelgetal k, laat S_k die toenemende rekenkundige ry heelgetalle wees met eerste term 1 en gemene verskil k. Byvoorbeeld, S_3 is die ry $1; 4; 7; 10; \ldots$. Vir hoeveel waardes van k bevat S_k die term 2019?

20. Vir hoeveel reële getalle a het die kwadratiese vergelyking $x^2 + ax + 6a = 0$ slegs heelgetalwortels vir x?

9. In trapesium $ABCD$, is sye AB en CD ewewydig, terwyl hoeklyn BD gelyk is aan sy AD. $D\widehat{C}B = 110°$ en $C\widehat{B}D = 30°$. Wat is die grootte van $A\widehat{D}B$ in grade?



10. Drie keer die grootste van twee getalle is vier keer die kleiner getal, en die verskil tussen die twee getalle is 8. Wat is die waarde van die grootste getal?

11. Ek skryf al die opeenvolgende heelgetalle van 25 tot en met 208 neer: $25, 26, 27, \ldots 208$. Hoeveel syfers het ek altesaam neergeskryf? (Let op dat die drie getalle 99, 100 en 101 saam 8 syfers het.)

12. Thembi wil haar versameling sleutelringe in rye en kolomme rangskik sodat elke ry volledig is. As sy probeer om dit in rye van 2, 3, 4 of 5 te rangskik, is daar altyd een oor. As Thembi meer as 1 sleutelring het, wat is die kleinste getal sleutelringe wat sy kan hê?

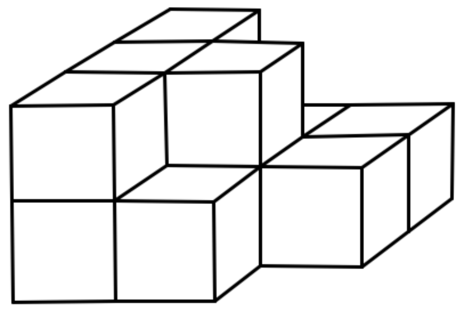
13. Die lengtes van twee sye van 'n driehoek is 200 cm en 150 cm. Die lengte van die derde sy is ook 'n heelgetal (in cm). Wat is die kleinste omtrek (in cm) van hierdie driehoek? (Ignoreer die geval waar die drie hoekpunte van die driehoek op 'n reguit lyn lê.)

14. Die getal 49 is spesiaal omdat dit moontlik is om die produk en die som van die syfers bymekaar te tel en weer 49 te kry. Bv. $4 \times 9 + (4+9) = 36 + 13 = 49$. Hoeveel sulke positiewe 2-syferheelgetalle is daar in totaal?

Deel A: Drie punte elk

1. Bereken $2018 - (1000 + 100) - (\sqrt{100})^2 + 2$.

2. Hoeveel identiese houtkubusse is gebruik om die struktuur te bou soos dit in die figuur aangetoon word? (Die struktuur het geen versteekte 'holtes' in die dele wat nie van die voorkant gesien kan word nie.)



3. Die som van drie verskillende positiewe heelgetalle is 12. Wat is die grootste moontlike waarde van die grootste getal?

4. Herman het vyf dae spandeer om vir 'n toets voor te berei. Op die eerste dag het hy een probleem opgelos, en op elke daaropvolgende dag het hy twee keer soveel probleme as die vorige dag opgelos. Hoeveel probleme het Herman altesaam opgelos terwyl hy vir die toets voorberei het?

5. 'n Vierkant en 'n sirkel het gelyke omtrekke. As die oppervlakte van die vierkant gelyk is aan 13π, wat is die oppervlakte van die sirkel?

6. Wanneer 'n kind se ouderdom met 3 jaar sou toeneem, is dit 'n volkome vierkant en wanneer dit met 3 jaar sou afneem, is dit die vierkantwortel van hierdie volkome vierkant. Hoe oud is die kind?

7. Die oppervlakte van 'n driehoek is 800. As die basis van hierdie driehoek met 10% toeneem en die hoogte op hierdie basis ook met 10% toeneem, wat is die oppervlakte van die nuwe driehoek?

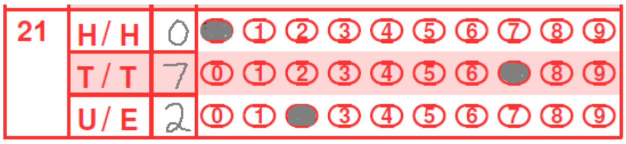
8. Wat is die som van al die heelgetalle tussen 50 en 150 wat op 1 eindig?

HOE OM DIE ANTWOORDBLAD IN TE VUL

Die antwoorde van al die vrae is heelgetalle van 0 tot 999. Beskou die volgende voorbeeldvraag:

21. As $3x - 216 = 0$, bepaal die waarde van x.

Die antwoord is 72, en dus moet jy die blok vir vraag 21 op die antwoordblad as volg voltooi: kleur die 0 in die honderde-ry in, 7 in die tiene-ry, en 2 in die ene-ry:



Skryf die syfers van jou antwoord in die oop blokkies aan die linkerkant van die betrokke rye, soos aangetoon in die voorbeeld; honderde, tiene en ene van bo na onder. Die drie syfers wat jy neerskryf word nie gemerk nie, want dit is slegs daar vir jou gerief — slegs die ingekleurde sirkeltjies word gemerk.

MOENIE OMBLAAI VOORDAT JY GESÊ WORD OM DIT TE DOEN NIE

SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2018 TWEEDE RONDTE
SENIOR AFDELING: GRAAD 10-12

16 Mei 2018 | **Tyd: 120 minute** | **Aantal vrae: 25**

Instruksies

1. *Die antwoorde op al die vrae is heelgetalle van 000 tot 999. Elke vraag het slegs een korrekte antwoord.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 3 punte in Afdeling A, 5 punte in Afdeling B en 6 punte in Afdeling C.*
   2.2. *Geen punte word afgetrek vir foutiewe antwoorde of onbeantwoorde vrae nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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