THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

FIRST ROUND 2005

JUNIOR SECTION: GRADES 8 AND 9

15 MARCH 2005

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

ANSWERS

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	A
2	C
3	D
4	D
5	C
6	B
7	D
8	A
9	C
10	B
11	A
12	D
13	E
14	C
15	A
16	E
17	D
18	B
19	E
20	D
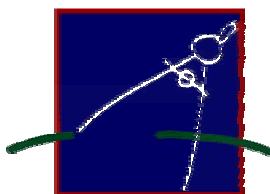
1.   **ANSWER: A**

$2 - 2 \times 2 + 2$ Use BODMAS

$= 2 - 4 + 2$ Multiply

$= 4 - 4$ Then Add

$= 0$ Then Subtract

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2.   **ANSWER: C**

Facts to remember:

16 June:	Soweto uprising
2010:	will be in six years time (World Cup)
2004:	is a leap year (10 years of our democracy)
2008:	next leap year after 2004 is 2008

Dates fall on consecutive days as the years progress, but in a leap year, it jumps a day.

If in 2004, (1st year) a date falls on a Wednesday then:

2004	2005	2006	2007	2008	2009	2010
Wednes-day (leap year)	Thurs-day	Friday	Saturday	Monday (leap year)	Tuesday	Wednes-day

Solution is: Wednesday.

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3.   **ANSWER: D**

To find x:
1. add the middle column to give the sum = 30;
2. add 9 and 14 in the first row and subtract from 30:
   30 – 23 = 7; fill the top right block.
3. add 9 and 10 from top left diagonal and subtract from 30:
   30 – 19 = 11; fill the bottom right block.
4. add 7 and 11 from last column, subract from 30 to get x:
   30 – 18 = 12 = x.

9	14	**7**
	10	x
	6	**11**
	30	

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4.   **ANSWER: D**

$\frac{1}{2}$ of the number is 30.   **OR**   Let the number = x .

Therefore the number is 60.   Given $\frac{1}{2}x = 30$

$\dfrac{3}{4}$ of the number is 45.

$\therefore\ x = 30 \times 2$

$\quad = 60$

And $\quad \dfrac{3}{4} \times \dfrac{60}{1} = 45$

5.　　**ANSWER: C**

$999 \times 555 = (1000 - 1) \times 555$

$\qquad\qquad = 555\ 000 - 555$

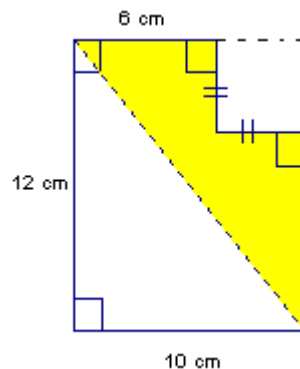$\qquad\qquad = 554445$

$\therefore (3 \times 5) + (3 \times 4) = 15 + 12 = 27$

OR

$9 \times 5 = 45$ and $4 + 5 = 9 = 1 \times 9$

$99 \times 55 = 5445$ and $5 + 4 + 4 + 5 = 18 = 2 \times 9$

$\therefore 999 \times 555 = ?\quad$ and sum of the digits of $? = 3 \times 9 = 27$

6.　　**ANSWER: B**

1.　　Close the cut part to get a rectangle.
2.　　Find area of a rectangle: $12cm \times 10cm = 120cm^2$.
3.　　Area of the unshaded triangle is half the area of the whole rectangle:

$$\frac{1}{2}\,\text{base} \times \text{height} = \frac{1}{2} \cdot 10 \cdot 12$$
$$= 60cm^2$$

4.　　The area of the shaded part: $60cm^2$ – area of the cut off piece.
Area of the cut off piece:
(length of one side)^2 = (10cm – 6cm)^2 = (4cm) = 16 cm^2.

$\therefore$ Area of the shaded region:
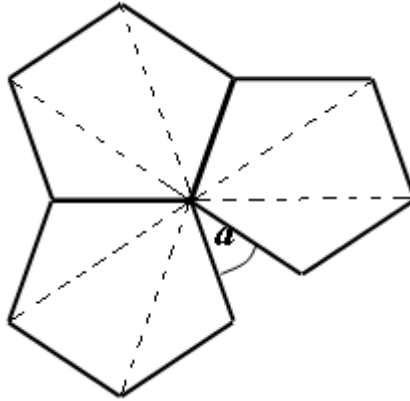$$60cm^2 - 16cm^2 = 44cm^2$$

7.　　**ANSWER: D**

The product of the three positive integers will be a maximum when the numbers are the equal. Therefore,

since $\dfrac{28}{3} = 9\dfrac{1}{3}$, take the numbers 9, 9 and 10.

Then 9 + 9 + 10 = 28 and $9 \times 9 \times 10 = 810$.

Try other sets of numbers, example 8, 10 and 10. The product will always be less than 810. A good problem for investigation.

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8. ANSWER: A

Sum of angles of a triangle = 180°.

∴ Sum of angles of a regular pentagon = $180° \times 3 = 540°$.

∴ Each angle of a regular pentagon = $\dfrac{540°}{5} = 108°$.

∴ $a + 3 \times 108° = 360°$ (Revolution)

∴ $a = 36°$

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9. ANSWER: C

Facts to remember:

- Since the remainder is the same in both cases, □ and Δ must be factors of the multiple 503 – 20 = 483.
- The factors of 483 is: 3, 7, 21, 23, 69, 161 and 483.
- A possibility to get 20 as remainder when 503 is divided is to use $21 \times 23 = 483$.
- 493 is 10 more than 483, so when 493 divided by 21×23 remainder is 10.

OR $\dfrac{503}{\square} = \Delta$ remainder 20

503 = $\triangle$ x □ + 20
$\triangle$ x □ = 483

Therefore $\dfrac{493}{\Delta \times} = 1$ rem 10

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10.   ANSWER: B
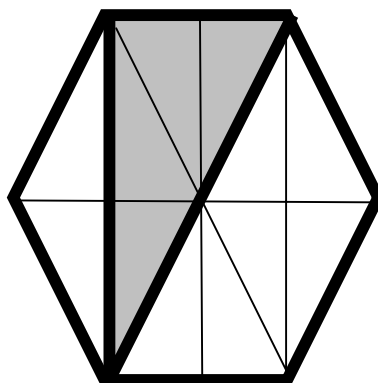
By drawing in all the diagonals of the regular hexagon, it becomes clear that the shaded region consists of 4 of the 12 equal triangles, that is a third of the total area (using the idea of symmetry).

Area = $36 \times 3 = 108 cm^2$.

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11.   ANSWER: A

cm   m   km

 1

100    $\rightarrow$ 1

                1 000 $\rightarrow$ 1

$\therefore$ 1 km = 100 000 cm

Actual distance between the two towns is 360 km = 36 000 000 cm
The map distance between the two towns is 24 cm.

$\therefore$ The scale of the map is $\dfrac{36\ 000\ 000}{24} = 1\ 500\ 000$

i.e. 1 : 1 500 000.

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12.   ANSWER: D

In $\triangle$ BCD, $\hat{B} = \hat{C_1} = 30°$, since $DB = DC$.

Hence $\hat{D_1} = 120°$ (angles of a $\triangle$).

In $\triangle$ ACD, $\hat{D_2} = 60°$, since $\hat{D_1}$ and $\hat{D_2}$ are adjacent on a line.

Hence $\hat{C_2} + \hat{A} = 120°$ (angels of a $\triangle$).

But it is given that DA = DC and hence $\hat{C_2} = \hat{A} = 60°$.

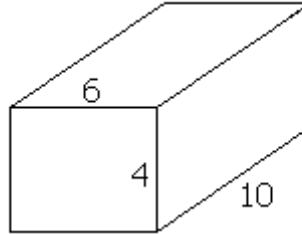$\therefore$ AC is 70cm, since $\triangle$ ACD is equilateral.

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13.   ANSWER: E

In the words S A T, S E T and T E N there are two S'e, one A, three T's, two E's and one N. In 1 5 4, 3 1 4 and 3 2 1 there are two 4's, one 5, three 1's, two 3's and one 2. Hence T corresponds to 1. S A T has the only A and 1 5 4 the only 5, therefore A corresponds to 5. T E N has the

only N and 1 3 2 the only 2, therefore N corresponds to 2. Now it follows from S A T and 4 5 1 that S corresponds to 4 and from T E N and 1 3 2 that E corresponds to 3.

Thus the word S E N T is represented as 4 3 2 1

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14.   **ANSWER: C**

Area of face A is 24cm^2 = 6 x 4cm^2.
Area of face B is 40cm^2 = 10 x 4cm^2.

Volume of prism = ℓ x b x h
                           = 10 x 6 x 4
                           = 240cm^3

∴ Area of face C = 10 x 6cm^2
                            = 60cm^2

<u>**OR**</u>

$bh = 24, \ h\ell = 40$

and $bh\ell = 240$

$$b\ell = \left[\frac{bh\ell}{h\ell}\right] \times \left[\frac{bh\ell}{bh}\right]$$

$$= \frac{240}{24} \times \frac{240}{40}$$

$$= 60$$

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15.   **ANSWER: A**
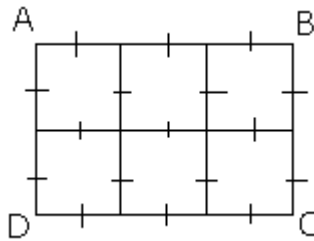
Let side of each square be *x*.
∴ Perimeter of 1 square = 4*x*
∴ and perimeter of 6 squares = 24*x*.
    (repetition allowed)

$$\therefore 24x = 72cm$$

$$\therefore \quad x = 3cm$$

$$\therefore \text{ Area of rectangle } ABCD = 9 \times 6$$

$$= 54cm^2$$

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16.   ANSWER: E

Start somewhere!

Let $a = 1$, then $c = 2$ or $c = 3$ (if 'carry' over 1).

Test $c = 2$:

```
1   b   2
1   2   b
2   4   1
```
b = 9 work
b = 9 does not work!

Test c = 3:

```
1   b   3
1   3   b
3   4   1
```
b = 8 work
b = 8 does not work!

Let a = 2, then c = 4 or 5.

Test c = 4:

```
2   b   4
2   4   b
4   4   2
```
b = 8 work
b = 8 does not work!

Test c = 5:

```
2   b   5
2   5   b
5   4   2
```
b = 7 work
b = 7 does not work!

Let a = 3, then c = 6 or 7.

Test c = 6:

```
3   b   6
3   6   b
6   4   3
```
b = 7 work
b = 7 work
Here we have a problem

Test c = 7:

```
3   b   7
3   7   b
7   4   3
```
b = 6 work
b = 6 work
This is correct!

$$\therefore a + b + c = 3 + 6 + 7 = 16$$

(Problem for investigation.)

17. ANSWER: D

Trial and Improvement:

x	y
$y+3$	y

Let $y=0$, $x=3$; $\dfrac{30}{3}=10.$

Let $y=1$, $x=4$; $\dfrac{41}{5}=8$ rem 1

Let $y=2$, $x=5$; $\dfrac{52}{7}=7$ rem 3

Let $y=3$, $x=6$; $\dfrac{63}{9}=7$

Let $y=4$, $x=7$; $\dfrac{74}{11}=6$ rem 8

The answers get smaller and smaller.

The number = 52.

$\therefore$ Therefore sum $=5+2=7$.

OR

The two-digit number is $10(y+3)+y=11y+30$.

The sum of the digits is $2y+3$.

Since the quotient is 7 and the remainder is 3 when the number $11y+30$ is divided by $2y+3$, we have

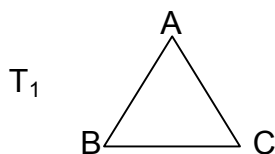$11y+30=7(2y+3)+3$

$\therefore 11y+30=14y+21+3$

$\therefore \quad\quad 6=3y$

$\therefore \quad 2=y$

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18. ANSWER: B

T_1 no points $1\ \Delta$ $\dfrac{1\times 2}{2}=1$

A

B   C
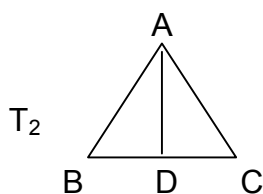
T_2 one point A $3\ \Delta\text{'e}$ $\dfrac{2\times 3}{2}=3$

A

B   D   C

A
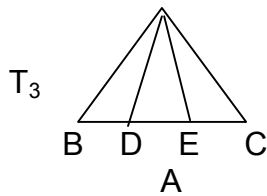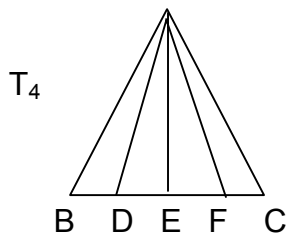
| T_3 | two points D and E | 6 Δ'e | $\dfrac{3\times4}{2}=6$ |

| T_4 | three points D, E and F and 0 division across | 10 Δ'e $10=10\times1$ | $\dfrac{4\times5}{2}=10$ |

| T_4 | 1 division across i.e. GH | 20 Δ'e $20=10\times2$ | |

| T_4 | 2 divisions across i.e. GH and IJ | 30 Δ'e $30=10\times3$ | |
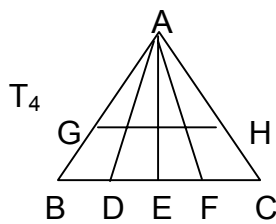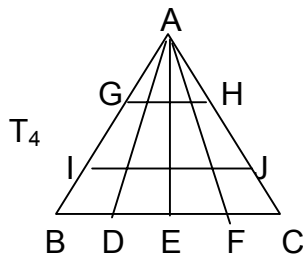
Number of triangles = 10 x 3 = 30

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19. ANSWER: E

Consider the last number in each row.

Row 1: $1=\dfrac{1\times2}{2}$

Row 2: $3=\dfrac{2\times3}{2}$

Row 3: $6=\dfrac{3\times4}{2}$

Row 4: $10=\dfrac{4\times5}{2}$

These numbers are **triangular** numbers and is given by the product of 2 consecutive numbers divided by 2, i.e. $\dfrac{n(n+1)}{2}$. We are looking for a product close to 640 (since $\dfrac{640}{2}=320$).

$m\times m=25\times25=625;\ p\times p=26\times26=676,$

$$\frac{n(n+1)}{2} = \frac{25 \times 26}{2} = 325 \; ;$$

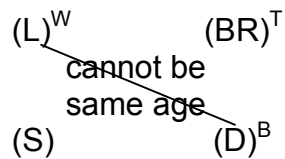$$\frac{n(n+1)}{2} = \frac{24 \times 25}{2} = 300$$

Last number of 24th row is 300.

∴ The first number of the 25th row is 301.

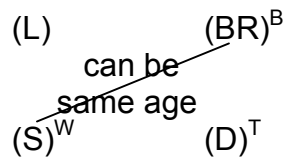∴ Therefore 320 is in row 25th and is the 20th number.

20.   ANSWER: D
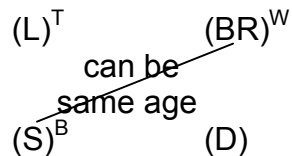
a) Let worst player be lady (L).

Key	
(L) = lady	
(BR) = brother	
(S) = son	
(D) = daughter	
(W) = worst	
(T) = twin	
(B) = best	

$(L)^W$ $(BR)^T$
cannot be
same age
(S) $(D)^B$

∴ Lady cannot be worst player.

b) Let worst player be son (S).

(L) $(BR)^B$
can be
same age
$(S)^W$ $(D)^T$

∴ Son can be worst player.

c) Let worst player be brother (BR).

$(L)^T$ $(BR)^W$
can be
same age
$(S)^B$ (D)

∴   Brother can be worst player.

d) Let worst player be daughter (D).

$(L)^B$ (BR)
cannot be
same age
$(S)^T$ $(D)^W$
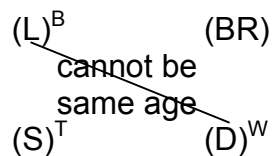
∴ Daughter cannot be worst player.

∴ Solution: The lady and daughter cannot be the worst players.