

SOUTH AFRICAN MATHEMATICS OLYMPIAD
2012 Junior Grade 9 Round 1
Solutions

1. **B** $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

2. **A** $2x + 7^\circ + x - 1^\circ = 90^\circ$, so $3x = 84^\circ$ and $x = 28^\circ$

3. **E** Each fold doubles the thickness, and there are six more folds to be done. So the resulting thickness will be $2,5 \times 2^6 = 160$ mm

4. **D** The number of rows of dots is the P number; the number of columns is one greater. So the number of dots in P30 is $30 \times 31 = 930$.

5. **E** The two primes could be 5+31, 7+29, 13+23, 17+19, with corresponding products 155, 203, 299, 323.

6. **E** 30 cm in 100 years is 300 mm in 5200 weeks and so $\frac{300}{5200} = \frac{3}{52} \approx \frac{6}{100}$ mm per week.

7. **D** He takes $\frac{8}{3}$ minutes per lap, and will cycle for 240 minutes. That will cover $\frac{240}{8/3} = 90$ laps.

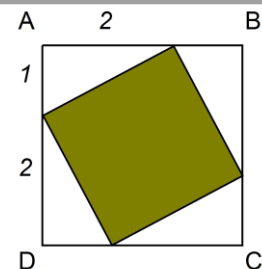
8. **B** Three extra minutes of speaking costs an extra R3. So 12 minutes talking costs R12 and therefore the subscription is $25 - 12 = \text{R } 13$. That means the total bill for 25 minutes talking will be $13 + 25 = \text{R } 38$.

9. **D** $3^* = 2.3 + 1 = 7$, and $7^* = 2.7 + 1 = 15$.

10. **C** $\hat{OPS} = 180^\circ - x$ (cointerior), and since this is the exterior angle of the isosceles triangle PQS, $\hat{PQS} = \frac{1}{2}(180^\circ - x) = 90^\circ - \frac{x}{2}$. But $\hat{OQR} = 90^\circ$, so \hat{RQS} must be $\frac{x}{2}$

11. **E** $1 = 1$; $1 + 3 = 4$; $1 + 3 + 5 = 9$;... so we see that after n terms the sum of the series is n^2 . For this to be 400 we need $n = 20$.

12. **A** Lengths must be as marked, so that each side of the smaller square is, by Pythagoras, $\sqrt{5}$. But then the area of the smaller square is 5 units² while the area of the larger square is 9 units², so the ratio is $\frac{5}{9}$



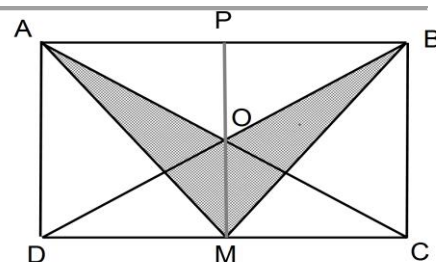
13. **E** If Anne's age now is x , then three years before she was half as old it was $\frac{x}{2} - 3$. So we have $x = 3\left(\frac{x}{2} - 3\right)$, which gives $2x = 3(x - 6)$ and then $x = 18$.

14. **B** The area of the whole circle is $4 \times 9\pi = 36\pi$, so the radius must be 6. The perimeter is made up of the curved arc plus two radii, so $\frac{1}{4}(2\pi \cdot 6) + 2 \cdot 6 = 3\pi + 12 = 3(\pi + 4)$

15. **C** We must have the last digit being odd, and also bigger than the first digit. If the last digit is 1 that is not possible, but if the last digit is 3 then the first can be 1 or 2, while the middle digit can be any one of the ten digits. Similarly if the last digit is 5, the first can be any of 1, 2, 3, 4 and the middle digit is not restricted. Continuing in this way we find the total number of possible three-digit numbers is $2 \times 10 + 4 \times 10 + 6 \times 10 + 8 \times 10 = 200$.

16. **B** M and N cannot both be odd because then (A) and (D) are both true. If M is odd and N is even then (A), (B) and (C) are true. If M is even and N is odd then (C) and (D) are true. Thus M and N must both be even and only (B) is true.

17. **B** Drawing the line MOP shows that $\Delta ADM = \frac{1}{4} ABCD$,
 $\Delta BMC = \frac{1}{4} ABCD$ and of course $\Delta AOB = \frac{1}{4} ABCD$.
 The shaded area is the remainder, and thus also $\frac{1}{4} ABCD$



18. **C** If X is her test percentage, then the overall score is $0,75 \times 82 + 0,25 \times X = 80$. This gives $0,25X = 80 - 0,75 \times 82 = 80 - 61,5 = 18,5$ and so her required score in the test is $4 \times 18,5 = 74\%$

19. **D** Each term leaves a remainder of 1 when divided by 8. So the total of the terms leaves the same remainder as the number of them, which is $\frac{1013+1}{2} = 507$. That remainder is 3.

20. **A** If $DP = x$ cm, then by symmetry $PD' = x$ and $CD' = 10$. By Pythagoras in $\Delta CBD'$, $BD' = 8$, so $AD' = 2$. Now Pythagoras in $\Delta APD'$ gives $x^2 = (6 - x)^2 + 2^2$, which means that $0 = 36 - 12x + 4$, so that $x = \frac{40}{12} = \frac{10}{3}$

