SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2013 SECOND ROUND
JUNIOR SECTION: GRADE 8 & 9

14 May 2013   **Time: 120 minutes**   **Number of questions: 20**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* **Calculators and geometry instruments are not permitted.**
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

21.   If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:

Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

Part A   (4 marks each)

1. What is the value of $\left(1+\frac{4}{7}\right) \div \left(1-\frac{3}{14}\right)$?

2. Oleg thinks of a number, doubles it and adds 1. Ravi starts with the same number, but subtracts 2 and then multiplies by 3. If Oleg and Ravi have the same final result, what was the original number?

3. Twelve identical wooden cubes are packed on one level in a rectangular tray. What is the minimum perimeter that tray can have if the cubes have a side length of 2 units?

4. The proper factors of a number are those factors of it that are not 1 or the number itself. What is the sum of the two largest proper factors of 2013?

5. Bottles are packed in boxes of either 6 or 12. The number of small boxes must be at least half the number of big boxes. If 240 bottles are to be boxed, what is the minimum number of boxes needed?

Part B   (5 marks each)

6. What is the value of $103^2 + 101^2 - 100^2 - 102^2$?

7. If $ab = 2$, $bc = 12$ and $ac = 6$ with a, b and c all natural numbers, what is the value of $a + b + c$?

8. If x and y are whole numbers between $\sqrt{39}$ and $\sqrt{224}$, then how many different values can $x + y$ have?

9. If $2A + 3B = 8$ and $3A + 2B = 12$, what is the value of $4A + 4B$?

10. Jane has three shirts, four skirts and some belts. Every combination of shirt, skirt and belt is an outfit, and Jane knows she can wear at least 50 different outfits. What is the minimum number of belts she must have?

11. ABC is a right-angled triangle, and points P,Q R are chosen on the sides so that AP = AQ, CP = CR. What is the value of x ?
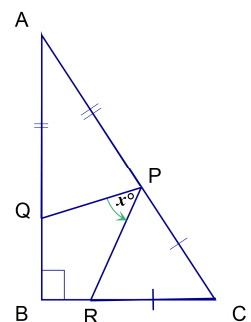
12. How many numbers in the sequence 6; 66; 666; 6666; ... are perfect squares?

13. Which integer is closest in value to $\dfrac{2^{1000} + 2^{1008}}{2^{1001} + 2^{1001}}$?

14. The figure shows five straight lines all passing through the same point, with some segments joined to form five triangles. What is the value of $a + b + c + \ldots + j$?
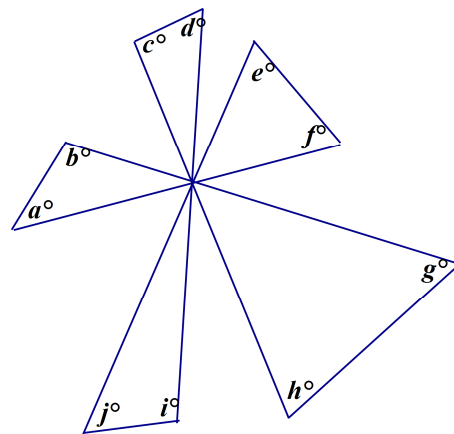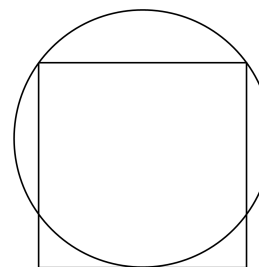
15. When two identical rectangles overlap as shown, the area of the overlap is exactly half the total area covered by the two rectangles. If AD has a length 2 cm, what is the value of AC^2 in cm^2?
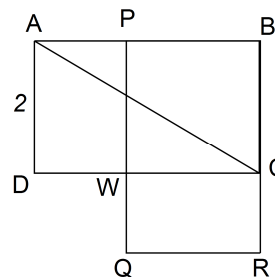
Part C (6 marks each)

16. In the figure the square with side length 8 has two vertices on the circle, and one side touching the circle. What is the length of the radius of the circle?

17. How many multiples of 4 less than 1000 (excluding 4 itself) do not contain any of the digits 6, 7, 8, 9 or 0?

18. The natural numbers are written in seven columns:

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17 …				

If the number of columns is changed to m, then 114 appears in the same column as 70, and 208 is one column to the right of 152. What is the value of m?

19. Equal arcs with centres at the ends of a diameter of a circle intersect each other on that same circle. If the length of the radius of the whole circle is 4 m, what is the area of the shaded region in m^2?
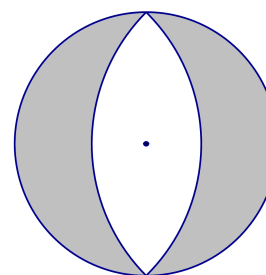
20. Some positive integers have cubes whose last two digits are 88. What is the sum of the two smallest such integers?