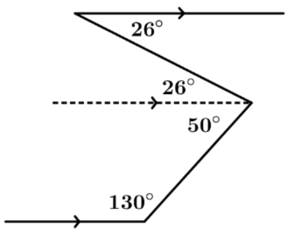
2021 JUNIOR ROUND TWO SOLUTIONS

1. **202** 2223 – 2021 = 202

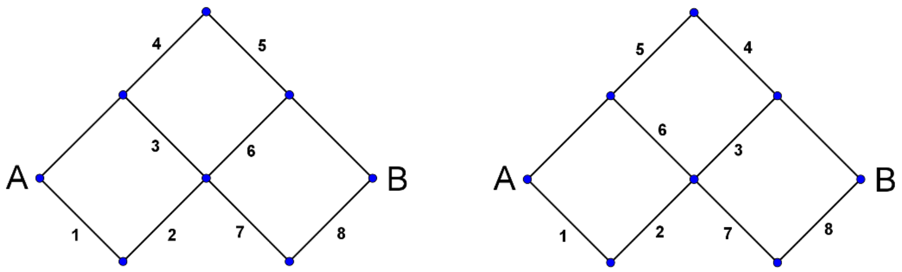
2. **11** If $B = 4 \times A$ then $B - A = 3A$. Since $B - A = 33$, this means $3A = 33$ and thus $A = 11$.

3. **76**



4. **20** The cube can be thought of as being made up of 27 smaller cubes each with volume 1 cm^3. Seven of these have been removed: one from each face, and one from the centre. The volume is thus 27 – 7 = 20 cm^3.

5. **8** There are two possible routes:



6. **16** Each of the 4 diagonals has a sum of $9 + x$. Thus $4(9 + x) = 100$, so $x = 16$.

7. **27** Starting with 1, the first three square numbers are 1, 4 and 9. The first three cube numbers are 1, 8 and 27. There would thus need to be 27 numbers in the list.

8. **74** The perimeter of the original sheet of paper is 50 cm. Each square increases the perimeter by 2 cm. The resulting perimeter is thus 50 + 12 x 2 = 74 cm.

9. **3** The sum of five consecutive positive integers starting with n is:

$$n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2)$$

The smallest perfect square of the form $5(n + 2)$ occurs when $n + 2 = 5$, thus $n = 3$.

10. **40** Removing the multiples of 2 leaves us with the 50 odd numbers: 1, 3, 5, 7, … , 99. Of these there are ten multiples of 5, namely 5, 15, 25, 35, … , 95. There are thus 40 remaining numbers.

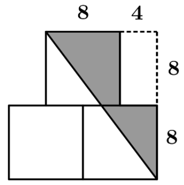
11. **0** The product is $2 \times 3 \times 5 \times 7 \times ... \times 2017 = 2 \times 5 \times (3 \times 7 \times ...)$.
Since the product has a factor of 10, it will end in zero.

12. **180** Each of the 10 rows would contain 9 toothpicks.
Each of the 10 columns would also contain 9 toothpicks.
There would thus be a total of $10 \times 9 + 10 \times 9 = 180$ toothpicks.

13. **10** $xy.yz.xz = 10 \times 15 \times 6$, thus $x^2y^2z^2 = 900$, and so $xyz = 30$.
Since 30 prime factorizes to $2 \times 3 \times 5$, it follows that x, y, z can only be $2,3,5$ in some order. Thus $x + y + z = 2 + 3 + 5 = 10$.
Alternatively, y is a factor of both 10 and 15, i.e. y can only be 1 or 5.
If $y = 1$ then $z = 15$, but this won't work since z needs to be a factor of 6.
Thus y must be 5, making $z = 3$ and $x = 2$. Thus $x + y + z = 10$.

14. **96** The backwards L shape has 7 shaded blocks in Figure 1, 9 in Figure 2, and 11 in Figure 3. It will thus have 65 shaded blocks in Figure 30. The remaining oblique shaded blocks are always one more than the Figure number, thus 31 in Figure 30. There are thus 31 + 65 = 96 shaded blocks in Figure 30.

15. **64** Area = $\frac{12\times16}{2} - 4 \times 8 = 64$ cm^2.



16. **6** Using the digits 2,0,2,1 there are twelve different possible codes – 0122, 0212, 0221, 1022, 1202, 1220, 2012, 2021, 2102, 2120, 2201, 2210. Six of these contain two adjacent digits the same. There are thus 12 – 6 = 6 different possible codes.
Alternatively, since the two 2s cannot be placed next to each other, there are three possible scenarios, namely 2 _ _ 2 , 2 _ 2 _ and _ 2 _ 2. Since there are two ways to arrange the remaining two digits in each of these three cases, there are 3 x 2 = 6 different possible codes.

17. **120** The original 10 scores have a sum of $80 \times 10 = 800$. The remaining 8 scores have a sum of $70 \times 8 = 560$. The two removed scores thus have a sum of 240 and hence an average of 120.

18. **26** $xy - 2y$ can be written as $y(x - 2)$, i.e. a product of two factors. 10 can be written as a product of two factors in four ways: 1×10, 2×5, 5×2 and 10×1. Treating the second factor as $(x - 2)$ in each case means the only possible values for x are 12, 7, 4 and 3, whose sum is 26.

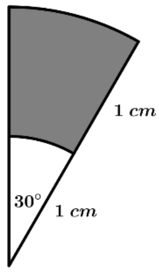
19. **24** $\frac{3}{11} \times x = A\ (total\ bananas)$
$\frac{1}{4}(x - 1) = A - 1$ thus $\frac{1}{4}(x - 1) = \frac{3x}{11} - 1$ thus $x = 33$ and $\frac{8}{11} \times 33 = 24$.

20. **16** Let the length of the rectangle be x. The area of the triangular region is a sixth of the area of the rectangle. Thus $\frac{1}{2} \times ax = \frac{1}{6}(48x)$.
From this we have $a = \frac{1}{3} \times 48 = 16$ cm.

21. **50** Consider the area of a single shaded region:

$$Area\ of\ single\ shaded\ region = \frac{1}{12}(\pi(2)^2 - \pi(1)^2) = \frac{\pi}{4}$$

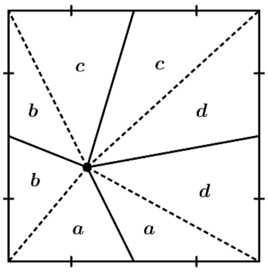
$$Area\ of\ entire\ shape = \frac{1}{2}\pi(2)^2 = 2\pi$$



Thus: $\%\ of\ entire\ shape\ shaded = \frac{4\times\frac{\pi}{4}}{2\pi} = \frac{1}{2} = 50\%$

22. **7** The number 210 is divisible by 1, 2, 3, 5 and 6 but not by 4. If the number 4 is face-down then 210 will be divisible by all 5 visible numbers. If 1,2,3,5, or 6 is face-down then the 4 is one of the visible numbers so it would not be the case that 210 is divisible by all the visible numbers. Hence, in 1 out 6 cases 210 will be divisible by all the visible numbers. The probability is ⅙. Thus $a + b = 7$.

23. **28** We can subdivide the square into 8 triangles by connecting the point inside the square to each of the four vertices. Notice that this creates four pairs of triangles, with each pair having the same area (same base and same perpendicular height).



We now have $a + b = 16$, $b + c = 20$, $c + d = 32$. Since $a + b + c + d = 48$ and $b + c = 20$, it follows that $a + d = 48 - 20 = 28$. (Note that the sum of the two opposite areas are equal)

24. **48** Let t represent the time in hours for the journey from home to the beach. The time spent on the return trip is thus $2 - t$ hours. For the first part of the journey the distance is $15t$. For the return journey the distance is $10(2 - t)$. Since these distances are the same we have $15t = 10(2 - t)$ which yields $t = \frac{4}{5}$. Thus the time in minutes is $\frac{4}{5} \times 60 = 48$.

25. **33** Arranging the numbers in ascending order we have 4 ;7 ;13 ;15 ;16 ;17 and p.

Case 1: Suppose p is 13 or less. The median would then be 13. So $(72 + p) \div 7 = 13$, giving $p = 19$. But this is a contradiction since p is 13 or less.

Case 2: Suppose p is the median. Then $(72 + p) \div 7 = p$, giving $p = 12$. There would again be a contradiction since $p = 12$ would make the median 13.

Case 3: Suppose p is 15 or more. The median would then be 15.
So $(72 + p) \div 7 = 15$, giving $p = 33$.