



# THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

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Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION  
Sponsored by HARMONY GOLD MINING

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**SECOND ROUND 2005**  
**SENIOR SECTION: GRADES 10, 11 AND 12**  
**10 MAY 2005**  
**TIME: 120 MINUTES**  
**NUMBER OF QUESTIONS: 20**

**Instructions:**

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
  - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
  - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.
8. Answers and solutions will be available in June at: <http://www.samf.ac.za/>

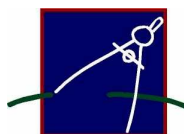
**DO NOT TURN THE PAGE OVER UNTIL YOU  
ARE TOLD TO DO SO.**

**DRAAI DIE BOEKIE OM VIR AFRIKAANS**

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## PRACTICE EXAMPLES

1. Remember that  $1\,000\text{ cm}^3$  of water weighs 1 kg. During a rain shower, 10 mm of rain fell on a rectangular soccer field with dimensions 100 m by 50 m. The mass of rain that fell on the field was

(A) 0.5 ton      (B) 5 kg      (C) 50 ton      (D) 50 kg      (E) 5 ton

2. When the decimal point of a certain positive number is moved four places to the right, the new number is nine times the reciprocal of the original number. The original number was

(A) 0.0003      (B) 0.003      (C) 0.03      (D) 0.3      (E) 3

3. How many terms are there in the simplified expansion of

$$(a + b + c + d + e)(c + d + e + f + g)?$$

(A) 18      (B) 22      (C) 21      (D) 24      (E) 25

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**Part A: Four marks each**

1. If  $x = \frac{1}{4}$ , which of the following has the largest value?

$$(A) \ x$$
$$(B) \ x^2$$
$$(C) \quad \frac{x}{2}$$
$$(D) \quad \frac{1}{x}$$

(E)  $\sqrt{x}$

- 2.** The number of factors of 108 is

(A) 3

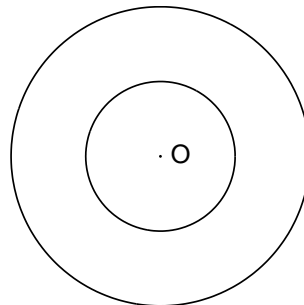
(B) 10

(C) 6

(D) 12

(E) 4

3. Two concentric circles have center  $O$ . The radius of the smaller circle is 4, and the difference between the circumference of the larger circle and that of the smaller circle is 10. The radius of the larger circle is



(A) 14

$$(B) \frac{4\pi + 10}{\pi}$$
$$(C) \quad \frac{14}{\pi}$$

(D)  $5 + 4\pi$

$$(E) \quad 4 + \frac{5}{\pi}$$

4. If  $2^a > 4^c$  and  $3^b > 9^a$  and  $a, b, c$  all positive, then

(A)  $c < a < b$

(B)  $b < c < a$

(C)  $c < b < a$

(D)  $a < b < c$

(E)  $a < c < b$

- 5.**  $a$  and  $b$  are both odd numbers. The only even number is

(A)  $ab$

(B)  $a^2b$

(C)  $a + b + 1$

(D)  $2a + 3b + 7$

(E)  $2a + b$

**Part B: Five marks each**

6.  $2^{28} - 1$  is exactly divisible by two numbers between 120 and 130. The sum of these two numbers is

(A) 255

(B) 256

(C) 257

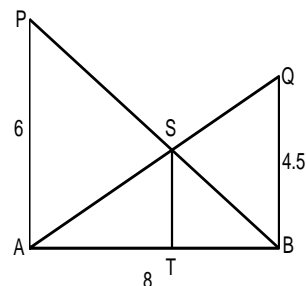
(D) 248

(E) 251

7. If  $\frac{97}{19} = w + \frac{1}{x + \frac{1}{y}}$ , where  $w, x, y$  are all integers, then  $w + x + y$  equals

(A) 16                      (B) 17                      (C) 18                      (D) 19                      (E) 26

8.  $PA$  and  $QB$  are perpendicular to  $AB$ . The lines  $PB$  and  $QA$  meet at  $S$ , and  $ST$  is perpendicular to  $AB$ . If  $PA = 6$ ,  $QB = 4.5$  and  $AB = 8$ , then the length of  $ST$  is

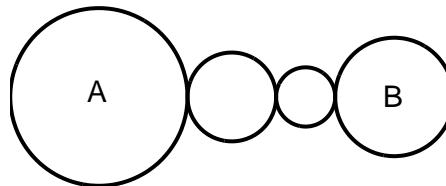


(A)  $\frac{18}{7}$                       (B)  $\frac{19}{8}$                       (C) 3                      (D) 2.25                      (E)  $\frac{16}{7}$

9. If  $w = 2^{129} \times 3^{81} \times 5^{128}$ ,  $x = 2^{127} \times 3^{81} \times 5^{128}$ ,  $y = 2^{126} \times 3^{82} \times 5^{128}$  and  $z = 2^{125} \times 3^{82} \times 5^{129}$ , then the order from smallest to largest is

(A)  $w, x, y, z$             (B)  $x, w, y, z$             (C)  $x, y, z, w$             (D)  $z, y, x, w$             (E)  $x, w, y, z$

10. Four wheels with radius 6, 3, 2, and 4 respectively, are pressed together and rotate without slipping. If wheel  $A$  rotates at 60 revolutions per minute, then the speed of wheel  $B$ , in revolutions per minute, is

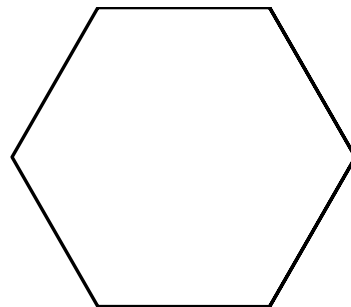


(A) 60                      (B) 45                      (C) 120                      (D) 40                      (E) 90

11. Eric finds the sum of the digits for each 8-digit number. The sum that occurs most often is

(A) both 27 and 28            (B) 41                      (C) both 36 and 37            (D) 32                      (E) 39

12. Three vertices of a regular hexagon are chosen at random. The probability that they form an equilateral triangle is



- (A)  $\frac{3}{20}$       (B)  $\frac{1}{10}$       (C)  $\frac{1}{2}$       (D)  $\frac{1}{5}$       (E)  $\frac{2}{5}$

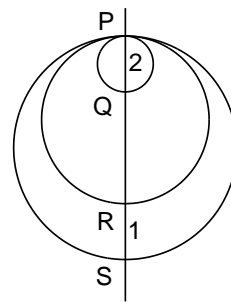
13. Every one of the 65 matrices at Harmony College does at least one of the subjects Art, Biology, Commerce, and no student does all three. The number doing Art and Biology is twice the number of students doing both Biology and Commerce, and is the same as the number doing both Art and Commerce. The number of students doing Art only is 12. Assuming that at least three more students do Biology than do Commerce, the minimum number of students doing Biology only must be

- (A) 3      (B) 24      (C) 25      (D) 5      (E) 7

14. The unit (ones) digit in the product  $(5 + 1)(5^2 + 1)(5^3 + 1) \cdots (5^{2005} + 1)$  is

- (A) 6      (B) 5      (C) 2      (D) 1      (E) 0

15.  $PQRS$  is a common diameter of the three circles. The area of the middle circle is the average of the areas of the other two. If  $PQ = 2$  and  $RS = 1$  then the length of  $QR$  is



- (A)  $1 + \sqrt{6}$       (B)  $\sqrt{6} - 1$       (C) 4      (D) 3      (E) 5

**Part C: Six marks each**

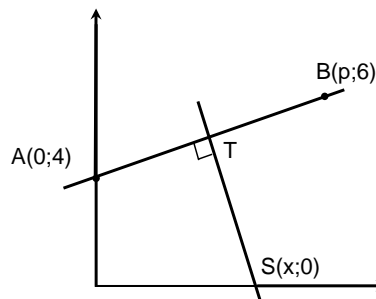
16. If  $x + y + z = 6$ ,  $xy + xz + yz = 11$  and  $xyz = 6$  then  $\frac{x}{yz} + \frac{y}{xz} + \frac{z}{xy}$  equals

- (A)  $\frac{7}{3}$       (B)  $\frac{13}{6}$       (C)  $\frac{5}{3}$       (D)  $\frac{6}{11}$       (E)  $\frac{8}{3}$

17. How many three-digit numbers are divisible by three and have the additional property that the sum of their digits is four times their middle digit?

(A) 7 (B) 4 (C) 11 (D) 10 (E) 5

18.  $TS$  is the perpendicular bisector of  $AB$ , with  $S$  on the  $x$ -axis. For how many integral values of  $p$  is the  $x$ -coordinate of  $S$  an integer?

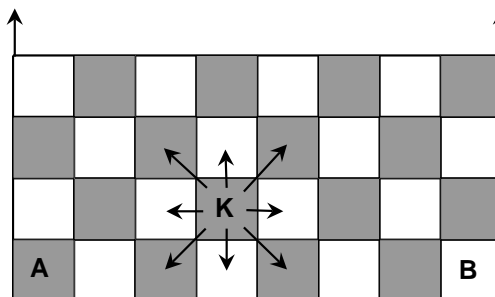


(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. In the sequence of fractions  $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \dots$ , the 2005th fraction is

(A)  $\frac{22}{41}$  (B)  $\frac{13}{52}$  (C)  $\frac{57}{9}$  (D)  $\frac{45}{14}$  (E)  $\frac{12}{52}$

20. A King  $K$  is allowed to move in a single move to any of the squares touching the square it is on, including diagonals, as indicated in the figure. How many different paths using exactly seven moves go from  $A$  to  $B$ ?



(A) 56 (B) 300 (C) 49 (D) 127 (E) 228