SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade NINE First Round 2019

Solutions

1. **E** $\dfrac{20+19}{20-19} = \dfrac{39}{1} = 39$

2. **C** $\sqrt[3]{(2+0+1)\times 9} = \sqrt[3]{27} = 3$

3. **C** At a rate of 19 pages in 20 seconds it will print 57 pages in one minute.

4. **E** $2+0+1\times 9 = 2+0+9 = 11$ which is odd.

5. **A** $\dfrac{2019}{20+19} \approx \dfrac{2020}{40} \approx 50$

6. **C** The sequence of dots contained within the shapes is 1, 3, 6, 10, … i.e. the triangular numbers. The 10^{th} triangular number is 1 + 2 + 3 + … + 9 + 10 = 55.

7. **C** Note that $6^2 + 8^2 = 10^2$. The triangle is a right-angle triangle with hypotenuse of 10 cm. The area of the triangle is ½ x 6 x 8 = 24 cm^2.

8. **B** Each cube has volume 56/7 = 8 cm^3. The side length of each cube is thus 2 cm, and each face has area 4 cm^2. There are 30 visible faces with a total area of 30 x 4 = 120 cm^2.

9. **E** $\dfrac{1}{a}+\dfrac{1}{a}=1 \quad \therefore \quad a=2 \quad ; \quad \dfrac{1}{b}+\dfrac{1}{b}+\dfrac{1}{b}=1 \quad \therefore \quad b=3$

   Thus: $\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{c}=1 \quad \therefore \quad \dfrac{3}{6}+\dfrac{2}{6}+\dfrac{1}{c}=1 \quad \therefore \quad c=6$

10. **E** Note that $1 = 1^2$ and $400 = 20^2$. There are 400 numbers from 1 to 400, and 20 of them must be perfect squares (1^2, 2^2, 3^2, 4^2, … , 20^2). 20/400 = 1/20 = 5%. Thus P = 5.

11. **A** 7 + 5 = 12. Each time 12 sweets are distributed Ayanda receives 2 more than Mbali. 7 multiples of 12 will give Ayanda 14 more sweets. 12 x 7 = 84.

12. **D** We can immediately see that q = ¼ and u = 2 and p = 8. Thus r = ½ and s = 1/16. Thus r + s = 8/16 + 1/16 = 9/16.

13. **A** If x^3 is a 3-digit number ending in 5 then *x* must be 5. If 2^y is a 2-digit number beginning with 3 then *y* must be 5. Thus *x* + *y* = 10.

14. **B** There are 8 vertices. If one vertex is chosen then of the remaining 7 only 3 are exactly 1 cm distant. The same would apply no matter which vertex was chosen first. The probability is thus 3/7.

15. **D** Using the *x*-values of the coordinates of A and B it should be clear that each square has side length 8 units, $(38 - 6) \div 4 = 8$.
Focusing on the *y*-values of the coordinates of A and B, note that the y-coordinate of point B should be $7 + 4 \times 8 = 39$ if there was no vertical shift of any of the squares. The vertical downward shift is thus 3 units from which we can deduce that the coordinates of C are $(6 + 2 \times 8; 7 + 2 \times 8 - 3)$ i.e. $(22; 20)$.

16. **B** B and E have the same height but B jumped higher than E, so B will have the better score. B and C jumped the same height, but C is taller than B so B will again have the better score. E jumped higher than D in spite of being shorter than D, hence E performed better than D. So B, who performed better than E, also performed better than D. A is taller than B but jumped a lower height, thus B has the highest score overall.

17. **B** Let the number of spiders be *x*. The number of zebras is 2*x* and the number of bees is 3*x*. The number of legs is thus 8(*x*) + 4(2*x*) + 6(3*x*), i.e. 34*x*. 34*x* = 102, thus *x* = 3. Alternatively, the ratio of zebras to spiders to bees is 2:1:3. The ratio of their legs is thus 8:8:18. 8 + 8 + 18 = 34. 102/34 = 3.

18. **D** Set up a table summarising the information:

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Donald	L	T	L	T	L	T	T
Herman	T	L	T	T	L	L	T

Note that Donald lies on Wednesdays, so his statement "*Today is Wednesday*" could not have been made on Wednesday. His statement must thus have been made on a day, other than Wednesday, on which he was lying, i.e. either Monday or Friday. Monday doesn't work since Herman's response "*Yes, it is*" is a lie, but he tells the truth on Monday. The statement must thus have been made on Friday when both Donald and Herman are lying.

19. **A** The sum of a 2-digit number and two 1-digit numbers can be at most 9 + 9 + 99 = 117. This means that C = 1 and hence the sum of the three numbers is 111. Note that 88 + 9 + 9 = 106, which is less than 111. Since B needs to be greater than 8 this means that B = 9. 111 – 99 = 12, which means A = 6.
We thus have A + B + C = 6 + 9 + 1 = 16.

20. **D** Consider the first 18 songs played where every 3rd song is a song that Waheeda doesn't like, and all the others are songs that she likes. Waheeda would then have played all the songs that she doesn't like. Hence, the 19th, 20th and 21st songs would have to be songs that she likes. Waheeda would have to play 21 songs to be sure that there would be 3 consecutive songs played that she likes. (If the number of songs played is 20 or less, one could spread the 6 songs that she doesn't like across the list (i.e. every 3rd song) in such a way that there are no 3 consecutive songs that she likes.)