THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

SECOND ROUND 2001: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A: (Each correct answer is worth 4 marks)

1. The number 36 is 12% of

(A) 250 (B) 300 (C) 350 (D) 400 (E) 450

ANSWER: B

If 36 is 12% of x, then $36 = \frac{12}{100}x$

$\therefore x = 36 \times \frac{100}{12} = 300$

2. $3^n + 3^n + 3^n$ equals

(A) 3^{n+1} (B) 3^{3n} (C) 3^{n+3} (D) 3^{3n+3} (E) 3^{3n+1}

ANSWER: A

$3^n + 3^n + 3^n = 3 \times 3^n = 3^{n+1}$

3. Ansu works in a bookstore and has to count a number of identical books. She arranges them in a stack 4 wide, 6 deep in 7 layers. The number of books in the stack is

(A) 17 (B) 98 (C) 42 (D) 204 (E) 168

ANSWER: E

Number of books $= 4 \times 6 \times 7 = 168$

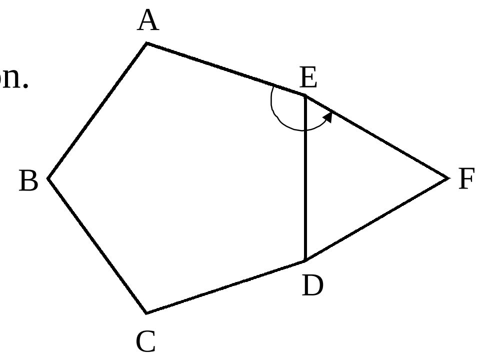
4. The greatest number of Fridays that can occur in a 75 day period is

(A) 15 (B) 13 (C) 12 (D) 11 (E) 9

ANSWER: D

There is one Friday in every seven days. And so there are 10 Fridays in 70 days. Because one Friday can fall in the remaining 5 days, the greatest number of Fridays in 75 days is 11.

5. In the diagram, ABCDE is a regular pentagon.
DEF is an equilateral triangle.
The size of angle AEF is



(A) 168° (B) 150° (C) 120°
(D) 132° (E) 170°

<u>ANSWER:</u> A

The sum of the interior angles of any polygon is $(n-2)\times 180^\circ$, where n is the number of sides of the polygon.
Thus

$$\mathrm{A\hat{E}D} = \frac{(5-2)\times 180^\circ}{5} = 108^\circ \text{,and}$$

$$\mathrm{D\hat{E}F} = \frac{(3-2)\times 180^\circ}{3} = 60^\circ \text{ (The interior angles of a regular polygon are equal)}$$

$$\therefore \mathrm{A\hat{E}F} = 180^\circ + 60^\circ = 168^\circ$$

Or

The size of an interior angle $e.g.\,\mathrm{A\hat{E}D}$ of a regular polygon is 108^o and $\mathrm{A\hat{E}D}$ is 60^o. Thus the size of $\mathrm{A\hat{E}F}$ is 168^o.

PART B: (Each correct answer is worth 5 marks)

6. In the weekly Lotto six different numbers are drawn randomly from the numbers 1, 2, 3, 4, … 48, 49. Mpho's parents bought a ticket with the numbers 2; 17; 26; 29; 30; 43 on it. The first five numbers drawn were 17; 26; 30; 2 and 43. What is the chance that the next number drawn will be 29?

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{1}{30}$ (D) $\frac{1}{44}$ (E) $\frac{1}{49}$

<u>ANSWER:</u> D

After five numbers have been drawn, there are 44 numbers left in the draw. Therefore the chance of drawing 29 next is one out of 44, i.e. $\frac{1}{44}$.

7. The extra time, in minutes, that it would take to cover a distance of 120 km travelling at an average speed of 60 km/h instead of 72 km/h would be

(A) 12 (B) 15 (C) 20 (D) 24 (E) 30

<u>ANSWER:</u> C

Here we use: $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}.$ $(T = \dfrac{D}{S})$

Thus for the slower vehicle: $T = \dfrac{120}{60} = 2 \text{ hrs} = 120 \text{ mins}.$

and for the faster vehicle: $T = \dfrac{120}{72} = 1\dfrac{2}{3} \text{ hrs} = 100 \text{ mins}.$

Thus the extra time for the slower vehicle is $120 - 100 = 20$ minutes

8. Observe: $1{,}5 \times 1{,}5 = 2{,}25$

$2{,}5 \times 2{,}5 = 6{,}25$

$3{,}5 \times 3{,}5 = 12{,}25$

The value of if $\times$ $=$ $9900{,}25$ is

(A) 33,5 (B) 66,5 (C) 99,5 (D) 100,5 (E) 300,5

<u>ANSWER:</u> C

We know that $100 \times 100 = 10\,000$, which is a bit bigger than $9900{,}25$ so 99,5 is the best available answer.

Checking: $99{,}5 \times 99{,}5 = (100 - 0{,}5)(100 - 0{,}5)$

$= 100^2 - 2.100.0{,}5 + 0{,}5^2$

$= 10\,000 - 100 + 0{,}25$

$= 9\,900{,}25$

Thus the answer is 99,5

9. The entrance fee at a concert was R5 per child and R16 per adult. A total of R789 was raised. The maximum number of people who could have attended the concert was

(A) 37 (B) 38 (C) 138 (D) 149 (E) 157

<u>ANSWER:</u> D

Clearly the maximum number of people will happen if we have the minimum number of adults. We also know that the total value of children’s tickets must end in a 5 or a 0, because each child’s ticket is R5. With a bit of trail-and-improvement, we can see that 4 adult tickets come to R64, leaving R725 (ends in 5!) for children’s tickets. So there were $725 \div 5 = 145$ children’s tickets, and 4 adult tickets, making the maximum number of people at the concert 145 + 4 = 149.

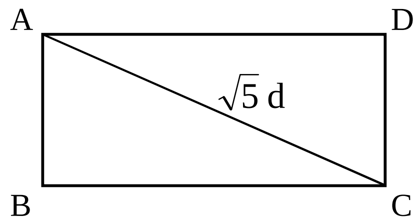
10. Given that $(21)^4 = 194481$ then $(0{,}21)^4$ equals

(A) 0,000194481 (B) 0,00194481 (C) 0,194481
(D) 19,4481 (E) 1944,81

<u>ANSWER:</u> B

$$(0{,}21)^4 = \left(\frac{21}{100}\right)^4 = \frac{194481}{100000000} = 0{,}00194481$$

11. ABCD is a rectangle such that AD = 2AB.
If $AC = \sqrt{5}d$, then the perimeter of ABCD is



(A) $4\sqrt{5}d$ (B) $6\sqrt{5}d$ (C) 10d
(D) 6d (E) 8d

<u>ANSWER:</u> D

Let DC = x and therefore AD = $2x$

Now in right-angled $\Delta ADC, x^2 + (2x)^2 = \left(\sqrt{5d}\right)^2$, by Pythagoras.

$$\therefore x^2 + 4x^2 = 5d^2$$
$$\therefore \quad 5x^2 = 5d^2$$
$$\therefore \quad x^2 = d^2$$
$$\therefore \quad x = d$$
$$\therefore \text{AD} = \text{DC} = d, \text{ and AD} = \text{BC} = 2d$$
$$\therefore \text{The perimeter} = d + d + 2d + 2d = 6d$$

12. How many non-isosceles triangles of perimeter 23 units can be formed with sides of whole number units?

(A) 6 (B) 8 (C) 13 (D) 23 (E) 66

<u>ANSWER:</u> B

Here a table is useful. Also remember that in any triangle, the sum of 2 sides <u>must</u> be greater than the 3rd side, so no side can be 12 units long, or more than 12.

The possibilities are:

$a =$	11	11	11	11	10	10	10	9
$b =$	2	3	4	5	4	5	6	6
$c =$	10	9	8	7	9	8	7	8

} *i.e.* 8 different non-isosceles triangles

13. Note: The recurring decimal $3,\dot{4}58\dot{7}$ means $3,458745874587...$

If recurring decimals $\frac{1}{7} = 0,\dot{1}4285\dot{7}$ and $\frac{1}{3} = 0,\dot{3}$, what is $\frac{1}{7} + \frac{1}{3}$ as a recurring decimal?

(A) $0,\dot{4}4285\dot{4}$ (B) $0,\dot{1}4286\dot{0}$ (C) $0,\dot{4}7619\dot{0}$

(D) $0,\dot{4}761\dot{9}$ (E) $0,\dot{4}76\dot{2}$

ANSWER: C

$$\begin{aligned}\frac{1}{7} + \frac{1}{3} &= 0,14285714......\\ &\underline{+\,0,33333333......}\\ &= 0,47519047......\\ &= 0,\dot{4}7619\dot{0}\end{aligned}$$

14. The chicken on Thabo's farm can gain weight at the rate of 20% per week. Thabo wants them to double their weight before he sells them. The minimum number of weeks he needs to keep them is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

ANSWER: B

Let the initial weight be 1 unit. Now each increase of 20% results in a new weight, which is 120% of (or 1,2 times) the previous weight.

after 1 weeks, it is $1 \times 1,2 = 1,2$ units

after 2 weeks, it is $1,2 \times 1,2 = 1,44$ units

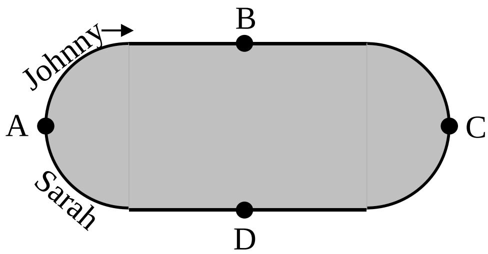
after 3 weeks, it is $1,44 \times 1,2 = 1,728$ units

after 4 weeks, it is $1,728 \times 1,2 = 2,48832$ units

Thus after just less than 4 weeks, the chicken doubled their weight.

15. Two athletes, Johnny and Sarah, are running in opposite directions on a track after they have started at the same point. Every time they meet one another, Sarah gives Johnny R1. Sarah runs three times as fast as Johnny. The number of laps Sarah has to complete to make sure that Johnny collects R120 is

(A) 30 (B) 40 (C) 50 (D) 60 (E) 90



ANSWER: E
If they start at A, and run in the directions shown, it is clear that Sarah meets Johnny first at B, then at C, then at D, then at A, by which time Sarah has done 3 laps to Johnny's 1 lap, and they have met 4 times. So Johnny collects R4 for 1 lap he completes, (and every 3 laps Sarah completes).
Now $\text{R}120 \div \text{R}4 = 30$ laps for Johnny, i.e. $30 \times 3 = 90$ laps for Sarah.

PART C: (Each correct answer is worth 6 marks)

16. By placing a 2 at both ends of a number, the number's value is increased by 2317. The sum of the digits of the original number is

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

ANSWER: B
If the number has digits *ab*, then the new number has digits 2*ab*2.
Hence
$$\begin{array}{r} 2317 \\ +\quad ab \\ \hline = 2ab2 \end{array}$$
From this, *b* = 5 and *a* = 3, so that a + b = 8.

17. Six numbers are represented by *a; b; c; d; e* and *f*. The average of *a; b; c;* and *d* is 10. The average of *b; c; d; e* and *f* is 14. If *f* is twice the value of *a* then the average of *a* and *e* is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 15

ANSWER: E

The average of a; b; c; and d is 10. So $a + b + c + d = 40$.
The average of b; c; d; e and f is 14, so $b + c + d + e + f = 70$.
Replacing f with $2a$, the second equation reads:

$$b + c + d + e + 2a = 70$$

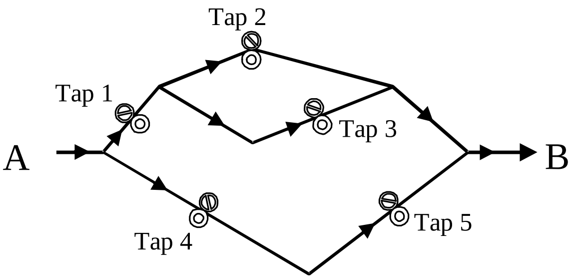
$$\therefore (a + b + c + d) + (e + a) = 70$$

$$\therefore 40 + (a + e) = 70$$

$$\therefore a + e = 30$$

$$\therefore \text{The average of } a \text{ and } e \text{ is } \frac{30}{2} = 15$$

18. Water flows through a network of pipes in the direction shown in the diagram from A to B. Five taps are on the network as shown. Each tap can be opened or closed to let water through or to stop the flow of the water. There are $2^5 = 32$ different ways of setting the taps. How many of these 32 ways will allow water to flow from A to B?



(A) 17 (B) 16 (C) 15 (D) 14 (E) 13

ANSWER: A

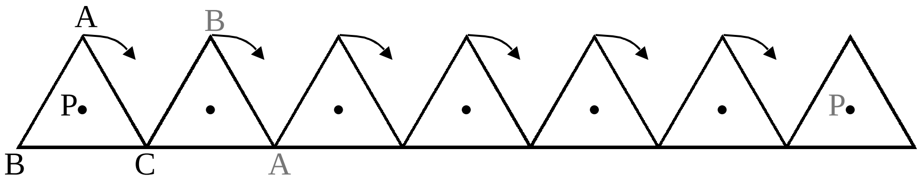
Here, a table of all 32 posibilities can be drawn up, and then you can count those where either (Tap 1 and Tap 2), or (Tap 1 and Tap 3), or (Tap 4 and Tap 5) are open, regardless of which other taps are open (o) or closed (c)

Tap 1	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Tap 2	o	o	o	o	o	o	o	o	c	c	c	c	c	c	c	c
Tap 3	o	o	o	o	c	c	c	c	o	o	o	o	c	c	c	c
Tap 4	o	o	c	c	o	o	c	c	o	o	c	c	o	o	c	c
Tap 5	o	c	o	c	o	c	o	c	o	c	o	c	o	c	o	c
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			

Tap 1	c	c	c	c	c	c	c	c	c	c	c	c	c	c	c	c
Tap 2	o	o	o	o	o	o	o	o	c	c	c	c	c	c	c	c
Tap 3	o	o	o	o	c	c	c	c	o	o	o	o	c	c	c	c
Tap 4	o	o	c	c	o	o	c	c	o	o	c	c	o	o	c	c
Tap 5	o	c	o	c	o	c	o	c	o	c	o	c	o	c	o	c
	✓				✓				✓				✓			

There are <u>17 ways</u> which allow the water to flow.

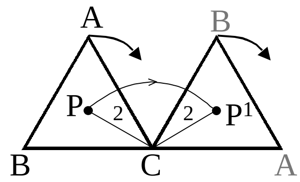
19. ABC is a wooden equilateral triangular block with P as its centre. The block is rolled clockwise on a flat surface such that one side touches the surface each time it is rolled.



If PC = 2 units, what is the length of the path of object P in the above diagram?

(A) $12\sqrt{3}$ (B) 2π (C) 8π (D) 6π (E) $14\sqrt{3}$

<u>ANSWER:</u> C



In one "roll", P moves to P^1, with $P\hat{C}P^1 = 120^\circ$, *i.e.* $\frac{1}{3}$ of a circle. Altogether, there are 6 "rolls", which means that P travels $6 \times \frac{1}{3} = 2$ complete circles. Using circumference, $C = 2\pi r$, with r = 2 units, P travels $2 \times 2\pi.2 = 8\pi$ units

20. From the numbers 1; 2; 3; 4; … 500 a sequence is formed by deleting numbers so that no two remaining numbers have a sum which is a multiple of 7. The maximum number of numbers in this sequence is

(A) 216 (B) 217 (C) 213 (D) 287 (E) 284

<u>ANSWER:</u> B

Let's consider the set carefully:

{1; 2; 3; $\cancel{4}$; $\cancel{5}$; $\cancel{6}$; 7; 8; 9; 10; $\cancel{11}$; $\cancel{12}$; $\cancel{13}$; $\cancel{14}$; 15; 16; 17; $\cancel{18}$; $\cancel{19}$; $\cancel{20}$; $\cancel{21}$… $\cancel{494}$; $\cancel{495}$; $\cancel{496}$; $\cancel{497}$; 498; 499; 500}

If we keep 1, we must remove 6; 13; 20; … 496 (71 numbers)
If we keep 2, we must remove 5; 12; 19; … 495 (71 numbers)
If we keep 3, we must remove 4; 11; 18; … 494 (71 numbers)
If we keep 7, we must remove 14; 21; 28; … 497 (70 numbers)

i.e. We need to remove at least 283 numbers, which leaves 500 – 283 = 217 numbers in the required set.

THE END

ANSWER POSITIONS: JUNIOR SECOND ROUND 2001

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	B
2	A
3	E
4	D
5	A
6	D
7	C
8	C
9	D
10	B
11	D
12	B
13	C
14	B
15	E
16	B
17	E
18	A
19	C
20	B

DISTRIBUTION	
A	3
B	6
C	4
D	4
E	3
TOTAL	**20**