SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2016 THIRD ROUND
JUNIOR SECTION: GRADES 8 AND 9

27 July 2016 **Time: 4 Hours** **Number of questions: 15**

TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Questions are not necessarily arranged in order of difficulty.
- There is a working sheet at the end of the paper and a ten cent coin will be provided to help you to answer Question 10.
- Answers and solutions will be available at: www.samf.ac.za

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

Question 1

A palindromic number is a number that reads the same backwards as it does forwards.

For example: 15751 and 909.

a) What is the smallest palindromic number greater than 2016?

b) What is the largest palindromic number less than 2016?

[4]

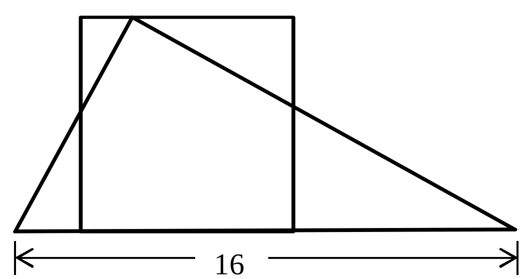
Question 2

All four-digit positive integers which are rearrangements of the number 2316 are written in increasing order. What is the largest possible difference between two adjacent numbers in this list?

[4]

Question 3

A square and a triangle are drawn as shown, with the base of the triangle equal to 16. If the triangle and the square have equal areas, find the area of the square.

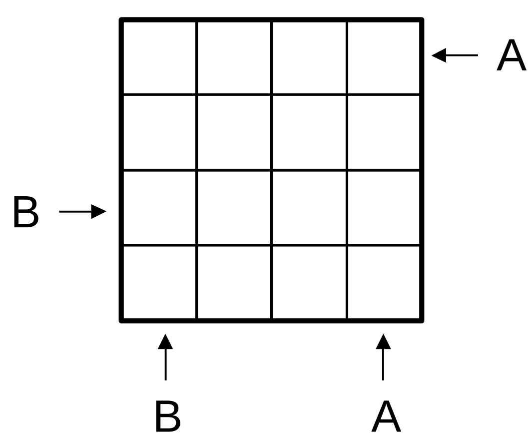


[4]

Question 4

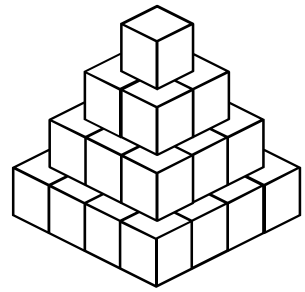
Place the letters A, B and C in the grid below such that the following rules are satisfied:

- Each letter appears exactly once in each row and each column (and hence exactly one cell will remain empty in each row and each column).
- The letters outside the grid show the first letter seen from that direction.



[4]

Question 5



The diagram shows a pyramid made up of 30 cubes, each measuring 1 by 1 by 1.

What is the total surface area of the pyramid?

(Include the bottom of the base of the pyramid.)

[4]

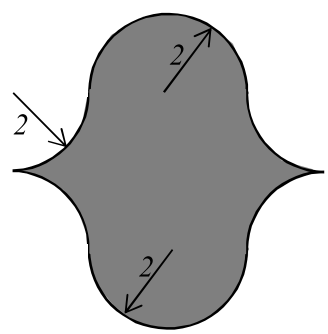
Question 6

A R1 coin has a mass of 4 grams and a R5 coin has a mass of 9 grams.

Suppose I have an amount of money made up of R1 and R5 coins such that the mass of all the R1 coins is equal to the mass of all the R5 coins. If I have less than R50, how much money do I have?

[6]

Question 7



Semi-circles, each with radius 2, are arranged as shown.

Find the area of the shaded region.

[6]

Question 8

Bee, Cee and Vee live in Microphyllia where there are only two types of creatures ï those that consistently tell the truth and those that consistently lie. The former creatures are Trudees and the latter Falsees. When I last visited Microphyllia I asked Bee: ñWho of you all are Trudees?ò

Bee mumbled its answer so I didnôt quite catch what she said.

ñBee said that only one of the three of us is a Trudeeò, Cee noted.

Vee turned to me and said: ñDonôt believe Cee, heôs not telling the truthò.

Who of them are Trudees and who are Falsees?

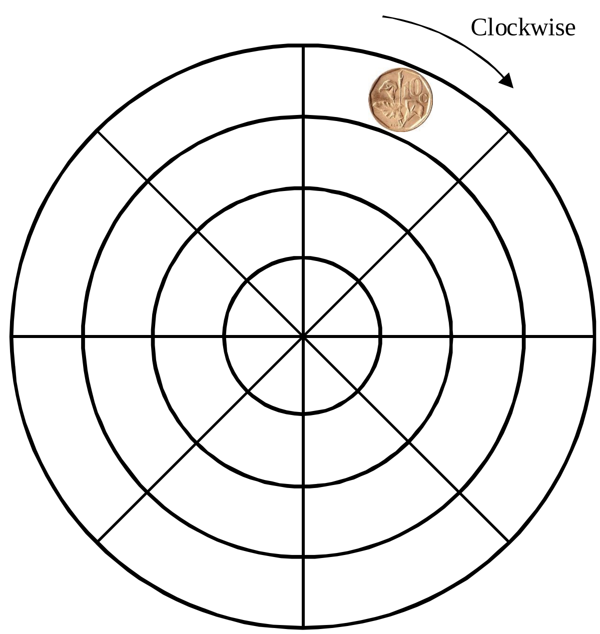
[6]

Question 9

Prove that for all natural numbers n, $\frac{n}{3}+\frac{n^2}{2}+\frac{n^3}{6}$ is also a natural number.

[8]

Question 10



(See the working sheet at the end of the paper)

Zola and Ron play a game by alternately moving a single ten cent coin on a circular board. The game starts with the ten cent coin already on the board as shown. A player may move the coin either clockwise one position or one position toward the centre, but cannot move to a position that has been previously occupied. The last person who is able to move wins the game.

If Zola starts, which player can play in a way that guarantees a win?

Explain this playerôs winning strategy. [8]

Question 11

Two sequences of real numbers are defined as follows:

$$u_1 = 0, \qquad u_{n+1} = \frac{1}{2}(u_n + v_n)$$

$$v_1 = 1, \qquad v_{n+1} = \frac{1}{4}(u_n + 3v_n)$$

Find the value of $v_{2016} - u_{2016}$.

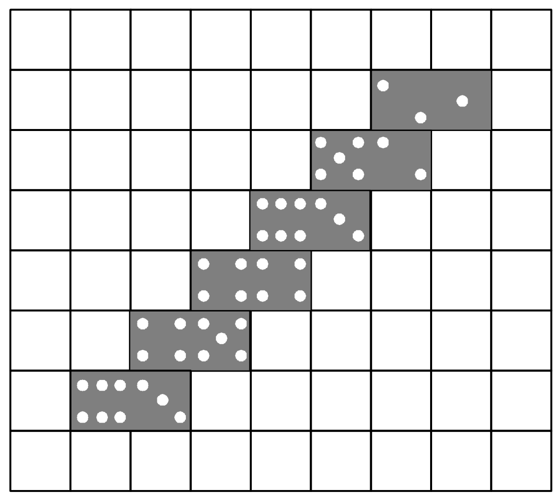
[8]

Question 12

An 8 by 9 board of squares are shown.

2 by 1 dominoes can be placed on this board either vertically or horizontally to cover two adjacent squares. Dominoes may not overlap.

What is the maximum number of 2 by 1 dominoes that can be placed on this board if six of the dominoes have already been placed as shown?



Prove that your answer is the maximum.

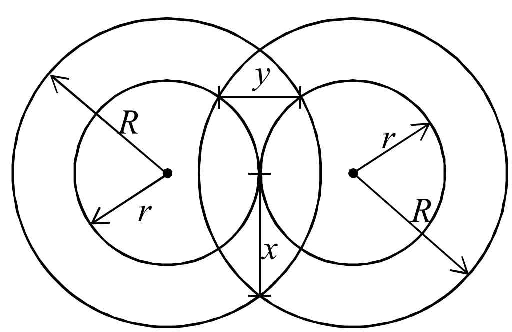
[10]

Question 13

Two concentric circles with radius r and R intersect another pair of concentric circles, also with radius r and R, as shown.

a) Calculate the length of x in terms of r and R.

b) Calculate the length of y in terms of r and R.



[10]

Question 14

Prove that there are infinitely many terms of the arithmetic sequence

$$1, 14, 27, 40, \text{é} , 1+13k, \text{é}$$

which are of the form 222é 22. In other words a number that is made up using only the digit 2.

(Hint: $1001 = 7\times11\times13$)

[8]

Question 15

At the start of the Mighty Mathematicians Football Teamôs first game of the season, their coach noticed that the jersey numbers of the 22 players on the field (11 players per team) were all the numbers from 1 to 22. At half-time, the coach substituted her goal-keeper (who had the number 1 on her jersey) for a reserve player. The coach then noticed that after the substitution, no two players on the field had the same jersey number and that the sum of the jersey numbers of each of the teams were exactly equal.

a) What is the smallest (positive) possible jersey number of the reserve player?

b) What is the greatest (positive) possible jersey number of the reserve player?

[10]

Total: 100

THE END

Only to be done at home. No marks!

Question no. 13 was born when a committee member was served coffee in the mug pictured here. The originally intended question asked for the area of the shaded area (shown on the mug) in terms of R and r, but this question was deemed too hard for this paper. But donôt let that deter you from trying to solve this problem at home!

Shaded area

WORKING SHEET/ WERKSVEL

You can use this working sheet to help you to answer Question 10.

Jy mag hierdie werksvel gebruik om jou te help om Vraag 10 te beantwoord.