Die Suid-Afrikaanse Wiskunde-Olimpiade
Derde Ronde 2004
Senior Afdeling (Grade 10 tot 12)
Tyd: 4 uur

1. Gestel $a = 1111\ldots1111$ en $b = 1111\ldots1111$ waar a veertig ene het en b twaalf ene. Bepaal die grootste gemene deler van a en b.

2. Vyftig punte word binne 'n konvekse veelhoek met tagtig sye gekies sodat geen drie van die vyftig punte op dieselfde reguit lyn lê nie. Die veelhoek word in driehoeke opgesny sodat die hoekpunte van die driehoeke net mooi die vyftig punte en die tagtig hoekpunte van die veelhoek is. Hoeveel driehoeke is daar?

3. Vind alle reële getalle x sodat $x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = 88$. Die notasie $\lfloor x\rfloor$ beteken: "die kleinste heelgetal wat nie minder as x is nie".

4. Gestel A_1 en B_1 is twee punte op die basis AB van die gelykbenige driehoek ABC (met $\widehat{C} > 60°$) sodat $A_1\widehat{C}B_1 = B\widehat{A}C$. 'n Sirkel wat die omgeskrewe sirkel van driehoek A_1B_1C aan die buitekant raak, raak ook aan die verlengings van CA en CB by die punte A_2 en B_2 onderskeidelik. Bewys dat $A_2B_2 = 2AB$.

5. Vir $n \geqslant 2$, vind die aantal heelgetalle x $(0 \leqslant x < n)$ sodat x^2 'n res van 1 laat wanneer dit deur n gedeel word.

6. a_1, a_2 en a_3 is verskillende positiewe heelgetalle, sodat

$$a_1 \text{ 'n deler is van } a_2 + a_3 + a_2a_3$$
$$a_2 \text{ 'n deler is van } a_3 + a_1 + a_3a_1$$
$$a_3 \text{ 'n deler is van } a_1 + a_2 + a_1a_2.$$

   Bewys dat a_1, a_2 en a_3 nie almal priem kan wees nie.

The South African Mathematical Olympiad
Third Round 2004
Senior Division (Grades 10 to 12)
Time: 4 hours

1. Let $a = 1111\ldots1111$ and $b = 1111\ldots1111$ where a has forty ones and b has twelve ones. Determine the greatest common divisor of a and b.

2. Fifty points are chosen inside a convex polygon having eighty sides such that no three of the fifty points lie on the same straight line. The polygon is cut into triangles such that the vertices of the triangles are just the fifty points and the eighty vertices of the polygon. How many triangles are there?

3. Find all real numbers x such that $x \lfloor x \lfloor x \lfloor x \rfloor \rfloor \rfloor = 88$. The notation $\lfloor x \rfloor$ means: "the least integer which is not less than x".

4. Let A_1 and B_1 be two points on the base AB of isosceles triangle ABC (with $\widehat{C} > 60°$) such that $A_1\widehat{C}B_1 = B\widehat{A}C$. A circle externally tangent to the circumcircle of triangle A_1B_1C is tangent also to rays CA and CB at points A_2 and B_2 respectively. Prove that $A_2B_2 = 2AB$.

5. For $n \geqslant 2$, find the number of integers x $(0 \leqslant x < n)$ such that x^2 leaves a remainder of 1 when divided by n.

6. a_1, a_2 and a_3 are distinct positive integers, such that

$$a_1 \text{ is a divisor of } a_2 + a_3 + a_2a_3$$
$$a_2 \text{ is a divisor of } a_3 + a_1 + a_3a_1$$
$$a_3 \text{ is a divisor of } a_1 + a_2 + a_1a_2.$$

Prove that a_1, a_2 and a_3 cannot all be prime.