2016 ROUND TWO JUNIOR   Solutions

1. 20 The value is $2 + 2 \times 9 = 2 + 18 = 20$

2. 52 The fraction is $\frac{13\times13\times12}{39} = \frac{13\times13\times12}{3\times13} = \frac{13\times12}{3} = 13\times4 = 52$

3. 52 There are 17 people in front of Jess and 34 behind her. Including herself this makes $17 + 1 + 34 = 52$ people

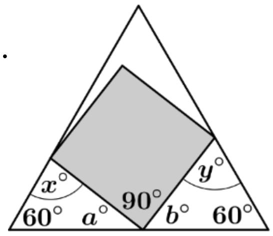
4. 162 $3^4 \times 2 = 81 \times 2 = 162$

5. 36 Δ BCD is isosceles, and $B\hat{C}D = 180° - (360° \div 5) = 108°$.
Now $x = \frac{1}{2}(180° - 108°) = 36°$

6. 44 Case 10 has 10 x 10 grey squares in the centre, so 2 x 10 + 2 x 12 white squares surrounding those, which is 20 + 24 = 44
OR:
Case n consists of a total of $(n + 2)^2$ squares, the central n^2 being grey. That means $(n + 2)^2 - n^2 = 4n + 4$ are white, and with $n = 10$ this is 44

7. 100 Rearranging, the value is $(2-1)+(4-3)+(6-5)+...+(200-199)$ which has 100 brackets and therefore totals $100 \times 1 = 100$.

8. 150 Since the triangle is equilateral, the two corner angles are each 60°.
We thus have
$x° + y° + a° + b° + 60° + 60° = 2 \times 180°$ (angle sum of triangle).
Then $x + y + a + b = 240$,
and since $a + b = 90$, we must have $x + y = 150$.



9. 3 With a little trial and error it should soon be clear that this scenario is only possible if one of the integers is zero. The options are thus $-3;\ -2;\ -1;\ 0$ or $-2;\ -1; 0; 1$ or $-1; 0; 1; 2$ or $0; 1; 2; 3$. The largest possible integer is thus 3.

10. 42 2016 is $2^5.3^2.7$. The largest odd factor is $3^2 \times 7 = 63$; and the next largest odd factor is $3\times7 = 21$. $63-21 = 42$

11. 65 For convenience assume there are 300 learners, i.e. 200 girls and 100 boys. 30 % of 200 is 60 and 45 % of 100 is 45. Thus, 60 + 45 = 105 learners have completed their project. The percentage of learners who still need to complete their project is thus
$\frac{300-105}{300}\times100 = \frac{195}{3} = 65$.

12. 30 Let BE = $2x$; then with $\text{BE} = \frac{2}{5}\text{BC}$ we must have EC = $3x$.

Now for Δ DEC, $\frac{1}{2}(3x)(\text{DC}) = 9$, giving $\text{DC} = \frac{6}{x}$, and since BC = $5x$,

the area of rectangle ABCD is $5x.\frac{6}{x} = 30$

13. 3 Assume that the remaining distance home is 1 unit. Then he slept for 2 units. Hence, the whole journey is 6 units and the fraction of the ride that he slept for is $\frac{2}{6} = \frac{1}{3}$.

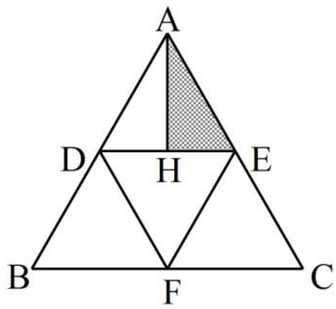
14. **300** I ended up with 40 % of the number instead of 160 %. So I need to multiply this new result by 4, or add it three times to itself, which is an increase of 300 %.

15. **11** 2 years ago the sum of all the family's ages was 4 x 19 = 76. That should have increased by 2 x 4 = 8, but has actually become 5 x 19 = 95, i.e. increased by 19. So the new person is now 19 – 8 = 11 years old.

16. **0** The total number of games is $\frac{4 \times 3}{2} = 6$. Since team A scored at least 1 win, each of teams A, B and C has either 1 or 2 wins. Since team A beats team D, team D does not have 3 wins and so the scores of A and B and C must total more than 3.
Hence, each of team A, B and C has 2 wins and therefore team D has 0 wins.

17. **18** For $(n-8)(n-38)$ to be positive, either both factors must be positive or both factors must be negative. For both factors to be negative, n must be less than 8 – this represents 7 values, i.e. 1, 2, 3, 4, 5, 6 and 7. For both factors to be positive, n must be greater than 38 – this represents 11 values, i.e. 39, 40, 41, …, 49. In total there are thus $7+11=18$ possibilities.

18. **25** D, E and F are the midpoints of the sides and as such we have four triangles of equal area.



Area of $\Delta\text{ADE} = \Delta\text{EFC} = \Delta\text{DBE} = \Delta\text{DEF}$

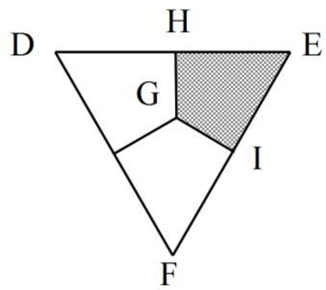
$$\Delta\text{AHE} = \frac{1}{2}\Delta\text{ADE} = \frac{1}{8}\Delta\text{ABC}$$

EHGI is one of three identical quadrilaterals making up ΔDEF

$$\text{EHGI} = \frac{1}{3}\Delta\text{DEF}$$

But $\Delta\text{DEF} = \frac{1}{4}\Delta\text{ABC}$

and so $\text{EHGI} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}\Delta\text{ABC}$



Therefore the shaded area is $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}\Delta\text{ABC}$

$$\therefore \frac{5}{24} \times 120 = 25\,\text{cm}^2$$

19. **70** The radius of the circle is 3 cm, hence the area of the full circle is 9π cm^2. The area of the shaded region to the area of the whole circle is thus 7/9. The unshaded region thus represents 2/9 of the circle's area. 2/9 of 360° is 80°. Each of the two central angles is thus 40°, from which it follows that x must be 70.

20. **360** Systematic counting soon reveals a pattern.
In the 900s there are clearly no cases.
In the 800s there are 10: 809 , 819 , 829 , … , 899.
In the 700s there are 20: 708 , 718 , 728 , … , 798
709 , 719 , 729 , … , 799.
In the 600s there are 30: 607 , 617 , 627 , … , 697
608 , 618 , 628 , … , 698
609 , 619 , 629 , … , 699.
This pattern continues until the 100s, where there are 80.
The desired total is thus 0 + 10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 = 360