THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

SECOND ROUND 2003

SENIOR SECTION: GRADES 10, 11 AND 12

20 MAY 2003

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.
8. Answers and solutions are available at: http://science.up.ac.za/samo/

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. Remember that 1 000 cm^3 of water weighs 1 kg. During a rain shower, 10 mm of rain fell on a rectangular soccer field with dimensions 100 m by 50 m. The mass of rain that fell on the field was

   (A) 0.5 ton (B) 5 kg (C) 50 ton (D) 50 kg (E) 5 ton

2. When the decimal point of a certain positive number is moved four places to the right, the new number is nine times the reciprocal of the original number. The original number was

   (A) 0.0003 (B) 0.003 (C) 0.03 (D) 0.3 (E) 3

3. How many terms are there in the simplified expansion of

   $$(a+b+c+d+e)(c+d+e+f+g)?$$

   (A) 18 (B) 22 (C) 21 (D) 24 (E) 25

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Part A: Four marks each

1. $\sqrt{2003}$ is approximately

(A) 39 (B) 41 (C) 45 (D) 43 (E) 47

2. Which one of the following is an odd number?

(A) $2001^2 + 3$ (B) $2002^3 + 10$ (C) $2003^2 + 7$ (D) $2004^3 + 1$ (E) $2005^2 + 9$

3. Chris has a number of blocks of size $1 \times 2 \times 6$. The smallest number of blocks he needs in order to build a solid cube is

(A) 6 (B) 12 (C) 24 (D) 18 (E) 36

4. A black and white photograph is 80% black and 20% white. If it is enlarged three times, then the percentage of white in the enlargement is

(A) 20 (B) 180 (C) 60 (D) 80 (E) 30

5. A calculator displays $\boxed{5.123456789\ \boxed{20}}$ to represent the number $5.123456789 \times 10^{20}$. If 30 is added to the number, then the calculator displays

(A) $\boxed{5.123456819\ \boxed{20}}$ (B) $\boxed{5.123456789\ \boxed{20}}$ (C) $\boxed{5.123456789\ \boxed{50}}$

(D) $\boxed{35.12345679\ \boxed{20}}$ (E) none of the above

Part B: Five marks each

6. In a supermarket items are always priced at so many rands and 99 cents. If a shopper pays a total of R41.71, then the number of items bought, is

(A) 29 (B) 30 (C) 71 (D) 9 (E) impossible to determine

7. How many letters are there in the correct answer to this question?

(A) one (B) two (C) three (D) four (E) five

8. If n is a perfect square, then the next perfect square greater than n is

(A) n^2+1 (B) n^2+n (C) $2n+1$ (D) $n+2\sqrt{n}+1$ (E) none of these

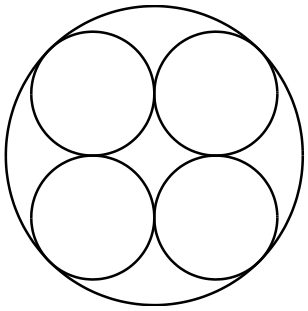
9. Ben cycles to Gamkaskloof at average speed x and returns at average speed y. His average speed for the whole trip is

(A) $\frac{xy}{2}$ (B) $\frac{x+y}{2}$ (C) $\frac{2x+2y}{xy}$ (D) $\frac{2xy}{x+y}$ (E) $\frac{x+y}{xy}$

10. The final score in a Chiefs-Dynamos soccer match is Chiefs m, Dynamos n. The possible number of different half-time scores is

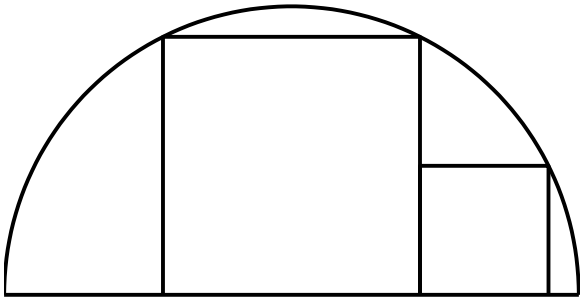
(A) $mn+1$ (B) $m+n+1$ (C) $(m+1)(n+1)$ (D) mn (E) $m+n+mn$

11. A phone company places four circular cables in a circular pipe so that they touch as shown. If the diameter of a cable is 1, then the diameter of the pipe holding the four cables is



(A) $1+\sqrt{2}$ (B) $\sqrt{5}$ (C) $2\sqrt{2}$ (D) $1+\sqrt{3}$ (E) $2\sqrt{3}$

12. Two squares are inscribed in a semicircle as shown. If the area of the smaller square is 7, the area of the larger square is



(A) 28 (B) 14 (C) 21 (D) $8\sqrt{7}$ (E) $14\sqrt{2}$

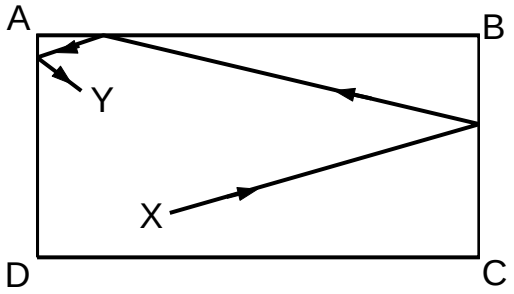
13. A circular disc is placed on a square chess board (64 equal squares) so that it touches the edges of the chess board. How many of the 64 squares are *completely* covered by the disc?

(A) 48 (B) 32 (C) 36 (D) 44 (E) 40

14. When Michael and Ellie ran a 10 km race, Ellie won by 500 m and took 50 min. To give Michael a chance, the next time they raced Ellie started 500 m behind the starting line. If they ran at exactly the same speeds as in the first race, then the result of the next race was

(A) Ellie wins by 25 m (B) Michael wins by 20 m (C) they finish together
(D) Ellie wins by 10 m (E) Michael wins by 15 m

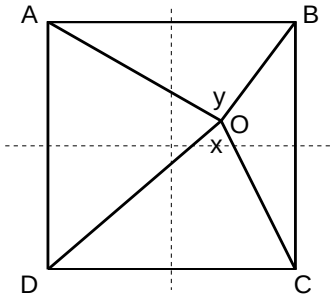
15. A ball is hit on a rectangular pool table, 2 m by 1 m, from a point X. It comes to rest at Y, as shown in the diagram. X is 0.6 m from AD and 0.2 m from DC. Y is 0.2 m from AD and 0.25 m from AB. If the ball always rebounds at the same angle as it hits the side, then it travels a distance (in metres), of



(A) 3.75 (B) 3.50 (C) 4.25 (D) 3.80 (E) 4.00

Part C: Six marks each

16. A point O moves inside a square $ABCD$ so that the angles $x + y = 180°$. As O moves it traces out

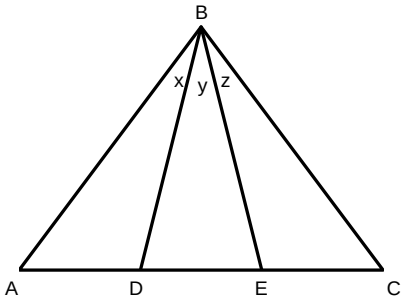


(A) the perimeter (B) the whole of the interior (C) only the centre of the square
(D) the two dotted axes of symmetry shown (E) the diagonals

17. A bricklayer would take 9 hours to build a certain wall on his own and another bricklayer would take 10 hours to build the same wall. If the two work together, they sometimes stop for a chat, so that they lay a total of 10 fewer bricks per hour. If it takes them 5 hours to build the wall together, then the number of bricks in the wall is

(A) 900 (B) 1 080 (C) 540 (D) 990 (E) 720

18. If $AD = DE = EC$, then the three angles x, y and z will be equal when angle $\widehat{ABC}$ equals



(A) $90°$ (B) $120°$ (C) $45°$ (D) $60°$
(E) it is impossible for x, y and z to be equal

19. Both William and Tell hit their target half the time. They decide to fight a duel in which they exchange shots until one is hit. What are the chances that the one that shoots first will win?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$ (E) $\frac{7}{8}$

20. Five politicians make the following statements:

Anna: 'Charles and Dumisani are lying'
Ben: 'Anna and Ellie are lying'
Charles: 'Ben and Dumisani are lying'
Dumisani: 'Charles and Ellie are lying'
Ellie: 'Anna and Ben are lying'.

Which politician is definitely lying?

(A) Anna (B) Ben (C) Charles (D) Dumisani (E) Ellie