SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2019 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

15 May 2019 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 000 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

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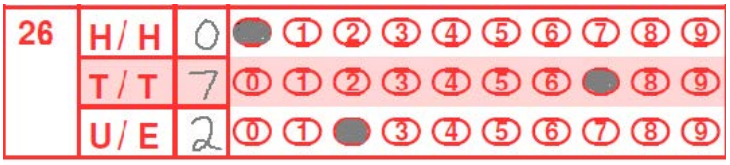
HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following **example question**:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom.
The three digits that you write down will not be marked, since it is only for your convenience - only the shaded circles will be marked.

Part A: Three marks each

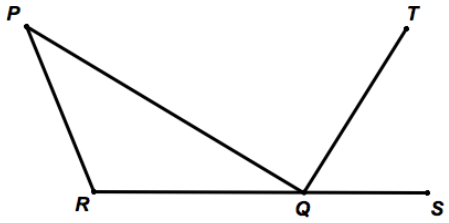
1. The temperature in Bloemfontein rose from $-4\,^{\circ}\text{C}$ in the morning to $8\,^{\circ}\text{C}$ in the afternoon. By how many degrees did the temperature rise?

2. If party X wins 30% of the 400 seats in parliament, how many seats will they have?

3. If 6 cats catch 6 rats in 6 minutes, how many cats are needed to catch 12 rats in 12 minutes?

4. If $256 = 4 \times 2^n$, what is the value of n?

5. In the diagram $\widehat{P} = 20^{\circ}$, $\widehat{R} = 100^{\circ}$ and $\widehat{PQT} = \widehat{TQS}$. What is the size of $\widehat{PQT}$ in degrees?

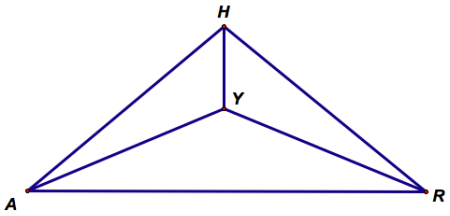


6. When a number is added to one quarter of itself, the result is 30. What is the number?

7. Calculate the area of the triangle formed by joining the points $A(3;6)$, $B(1;2)$ and $C(5;2)$ in the Cartesian plane.

8. The average of four consecutive odd integers is 32. What is the largest of the four integers?

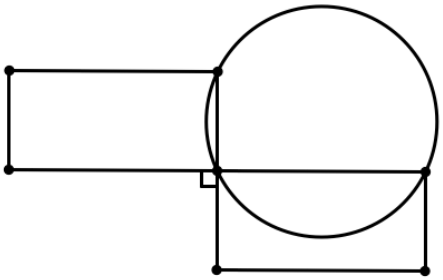
9. If $HA = HR$, $\widehat{AYR} = 130°$ and $\widehat{HAY} = \widehat{HRY} = 40°$, determine $\widehat{AHY}$ in degrees.



10. In how many ways can the letters of the word SAMO be arranged if the vowels must always be in alphabetical order?

11. Let $y = mx + b$ be the image when the line $x + 3y + 80 = 0$ is reflected in the x-axis. Determine the value of $m + b$.

12. Two congruent rectangles are shown. The area of a rectangle is 12 and its perimeter is 14. If the circumference of the circle is $n\pi$, what is n?



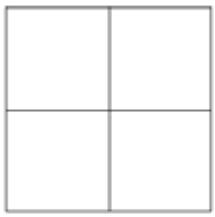
13. For how many positive integers k does the equation $kx - 12 = 3k$ have an integer solution for x?

14. If a, b and c are positive integers such that

$$a + \frac{1}{b + \frac{1}{c}} = \frac{30}{13},$$

what is the value of $a + b + c$?

15. A square tile is divided into four smaller squares, as shown. Each small square must be painted using one of the colours yellow, red or blue. How many different tiles can be painted if exactly two colours are used per tile and two tiles are considered the same if they can be rotated to look the same?



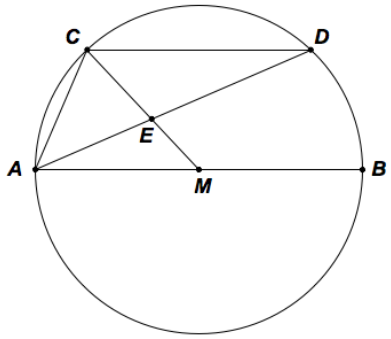
Part B: Five marks each

16. What is the remainder when $\dfrac{x^{52}+201x}{x}$ is divided by $x+1$?

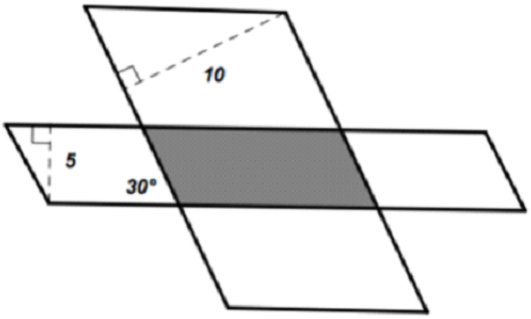
17. A sporting club has 100 members. Of these 100 members, 44 play soccer, 45 play cricket, 50 play rugby, 8 play soccer and cricket only, 9 play rugby and cricket only, and and 7 play soccer and rugby only. Five play neither soccer, cricket nor rugby. How many members play rugby only?

18. How many odd positive integers, the sum of whose digits is 4 and none of whose digits is 0, are not prime?

19. AB is a diameter of a circle with centre M. Chord CD is parallel to AB. MC meets AD at a point E such that $AC = EC$. Find angle CAM in degrees.



20. Two parallelograms, with perpendicular heights of 5 and 10 units respectively, overlap at an angle of 30° as shown. Determine the area of the shaded overlap.



Part C: Six marks each

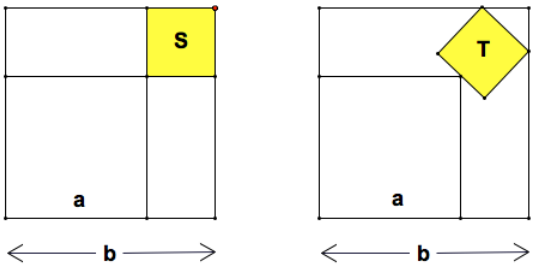
21. A loaded truck travels from Durban to Pietermaritzburg at an average speed of 60 km/h, and after unloading, returns by the same route at an average speed of 90 km/h. What is the average speed of the truck over the whole journey?

22. The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ are in arithmetic progression. What is the difference between the largest and smallest roots?

23. Given that $f(3) = 3$ and that $f(n+3) = \dfrac{f(n)-1}{f(n)+1}$ for all n, determine the value of $f(2019) + 19$.

24. When n standard 6-sided dice are rolled, the probability of obtaining a sum of 2019 is greater than zero and is the same as the probability of obtaining a sum of S. What is the smallest possible value of S?

25. Squares S and T are each placed outside a square of side a and inside a square of side b, as shown. On the left diagram, the sides of square S are parallel to the sides of the other two squares; on the right diagram, the diagonals of square T are parallel to the sides of the other two squares. If the side length of square S is 27, what is the area of square T?



Deel C: Ses punte elk

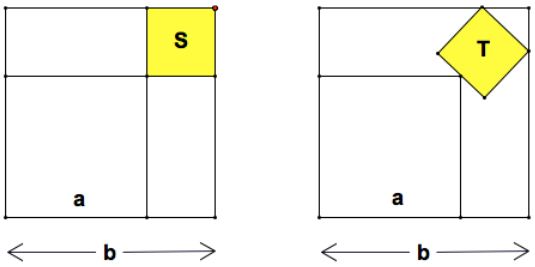
21. 'n Gelaaide vragmotor reis van Durban na Pietermaritzburg teen 'n gemiddelde spoed van 60 km/h. Nadat die vrag afgelaai is, ry dit met dieselfde roete terug teen 'n gemiddelde spoed van 90 km/h. Wat is die gemiddelde spoed van die vragmotor oor die hele reis?

22. Die wortels van $64x^3 - 144x^2 + 92x - 15 = 0$ vorm 'n rekenkundige ry. Wat is die verskil tussen die grootste en kleinste wortels?

23. Dit is gegee dat $f(3) = 3$ en dat $f(n+3) = \dfrac{f(n)-1}{f(n)+1}$ vir alle n. Bepaal die waarde van $f(2019) + 19$.

24. As n standaard 6-vlakkige dobbelsteentjies gerol word, is die waarskynlikheid om 'n som van 2019 te kry groter as nul en is dit dieselfde as die waarskynlikheid om 'n som van S te kry. Wat is die kleinste moontlike waarde van S?

25. Vierkante S en T is albei buite 'n vierkant met sylengte a en binne 'n vierkant met sylengte b, soos aangetoon. In die diagram aan die linkerkant, is die sye van vierkant S ewewydig aan die sye van die ander twee vierkante; in die diagram aan die regterkant is die hoeklyne van vierkant T ewewydig aan die sye van die ander twee vierkante. As die sylengte van vierkant S gelyk is aan 27, wat is die oppervlakte van vierkant T?



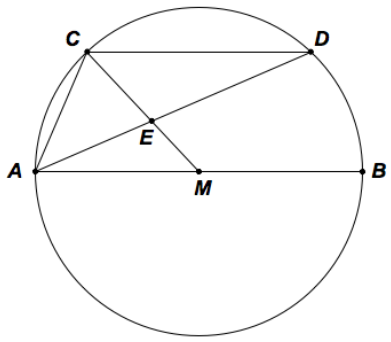
Deel B: Vyf punte elk

16. Wat is die res as $\dfrac{x^{52}+201x}{x}$ deur $x+1$ gedeel word?

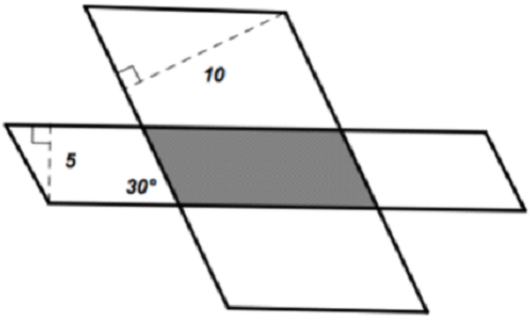
17. 'n Sportklub het 100 lede. Van hierdie 100 lede speel 44 sokker, 45 speel krieket, 50 speel rugby, 8 speel slegs sokker en krieket, 9 speel slegs rugby en krieket, en 7 speel slegs sokker en rugby. Vyf speel nie sokker óf krieket óf rugby nie. Hoeveel lede speel slegs rugby?

18. Hoeveel onewe positiewe heelgetalle waarvan die som van die syfers gelyk is aan 4 en waarvan geen syfer 0 is nie, is nie priemgetalle nie?

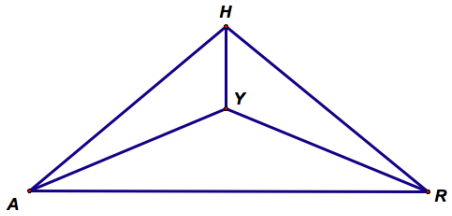
19. AB is 'n middellyn van 'n sirkel met middelpunt M. Koord CD is ewewydig aan AB. MC sny AD by punt E sodat $AC = EC$. Vind hoek CAM in grade.



20. Twee parallelogramme, met loodregte hoogtes 5 en 10 eenhede onderskeidelik, oorvleuel teen 'n hoek van 30° soos aangetoon. Bepaal die oppervlakte van die ingekleurde oorvleueling.



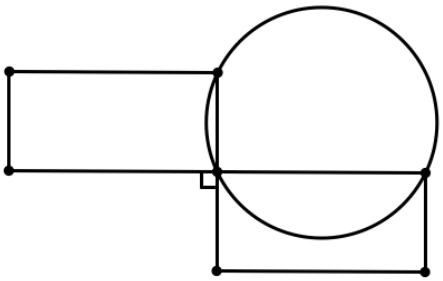
9. As $HA = HR$, $\widehat{AYR} = 130°$ en $\widehat{HAY} = \widehat{HRY} = 40°$, bepaal $\widehat{AHY}$ in grade.



10. Op hoeveel maniere kan die letters van die woord SAMO gerangskik word sodat die klinkers altyd in alfabetiese volgorde sal wees?

11. Laat $y = mx + b$ die beeld van die lyn $x + 3y + 80 = 0$ wees as dit gereflekteer word in die x-as. Bepaal die waarde van $m + b$.

12. Twee kongruente reghoeke word getoon. Die oppervlakte van 'n reghoek is 12 en sy omtrek is 14. As die omtrek van die sirkel $n\pi$ is, wat is n?



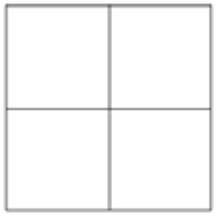
13. Vir hoeveel positiewe heelgetalle k het die vergelyking $kx - 12 = 3k$ 'n heelgetaloplossing vir x?

14. As a, b en c positiewe heelgetalle is sodat

$$a + \frac{1}{b + \frac{1}{c}} = \frac{30}{13},$$

wat is die waarde van $a + b + c$?

15. Elke vierkantige teël word in vier klein vierkante verdeel soos aangetoon. Elk van hierdie klein vierkante moet geel óf rooi óf blou ingekleur word. Hoeveel teëls, wat verskillend ingekleur is, kan ons kry as presies twee kleure per teël gebruik word? As twee teëls geroteer word en hulle lyk dan dieselfde, tel hulle nie as verskillend ingekleurde teëls nie.



Deel A: Drie punte elk

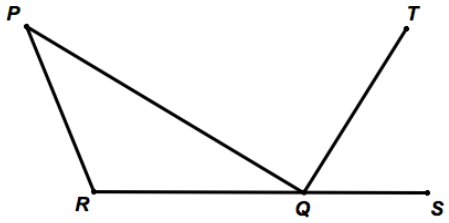
1. Die temperatuur in Bloemfontein het van $-4\,^{\circ}\text{C}$ in die oggend tot $8\,^{\circ}\text{C}$ in die namiddag gestyg. Met hoeveel grade het die temperatuur gestyg?

2. As party X, 30% van die 400 setels in die parlement wen, hoeveel setels het hulle?

3. As 6 katte 6 rotte in 6 minute vang, hoeveel katte is nodig om 12 rotte in 12 minute te vang?

4. As $256 = 4 \times 2^n$, wat is die waarde van n?

5. In die diagram is $\widehat{P} = 20°$, $\widehat{R} = 100°$ en $\widehat{PQT} = \widehat{TQS}$. Wat is die grootte van $\widehat{PQT}$ in grade?



6. As 'n getal by 'n kwart van homself getel word, is die antwoord 30. Wat is die getal?

7. Bereken die oppervlakte van die driehoek wat gevorm word as die punte $A(3;6)$, $B(1;2)$ en $C(5;2)$ in die Cartesiese vlak, verbind word.

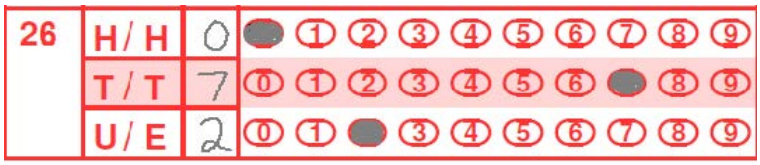
8. Die gemiddelde van vier opeenvolgende onewe heelgetalle is 32. Wat is die grootste van die vier heelgetalle?

HOE OM DIE ANTWOORDBLAD TE VOLTOOI

Die antwoorde op al die vrae is heelgetalle van 0 tot 999. Beskou die volgende **voorbeeldvraag**:

26. As $3x - 216 = 0$, bepaal die waarde van x.

Die antwoord is 72. Voltooi die blok vir vraag 26 op die antwoordblad soos volg: kleur die 0 in die honderde-ry in, die 7 in die tiene-ry en die 2 in die ene-ry:



Skryf die syfers van jou antwoord in die oop spasies aan die linkerkant van elke ry in, soos in die voorbeeld aangetoon: honderde, tiene en ene van bo na onder. Die drie syfers wat jy neergeskryf het, word nie nagesien nie omdat dit slegs vir jou eie gerief is – slegs die ingekleurde sirkeltjies word nagesien.

SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2019 TWEEDE RONDTE
SENIOR AFDELING: GRAAD 10-12

15 Mei 2019 **Tyd: 120 minute** **Aantal vrae: 25**

Instruksies

1. *Die antwoorde op al die vrae is heelgetalle van 000 tot 999. Elke vraag het slegs een korrekte antwoord.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 3 punte in Afdeling A, 5 punte in Afdeling B en 6 punte in Afdeling C.*
   2.2. *Geen punte word afgetrek vir foutiewe antwoorde of onbeantwoorde vrae nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

PRIVAATSAK X173, PRETORIA, 0001
TEL: (012) 392-9372 E-pos: info@samf.ac.za

Organisasies betrokke: AMESA, SA Wiskundevereniging,
SA Akademie vir Wetenskap en Kuns

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