SOUTH AFRICAN MATHEMATICS OLYMPIAD
2015 FIRST ROUND
GRADE 9 SOLUTIONS

1. **E** 2 – (0 – (1 – 5)) = 2 – (0 – (– 4)) = 2 – (0 + 4) = 2 – 4 = – 2

2. **C** 1 cm per month = 10 mm per month = 120 mm per year = 1 200 mm in ten years

3. **A** There are ten factors: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. Four of them are not multiples of 4, viz. 1, 2, 3, 6. The probability is therefore 4/10 or 40 %

4. **A** Every cube is joined to an adjacent cube on two faces, leaving the other four exposed to paint.

5. **C** There were 6 + 5 + 1 + 3 = 15 children altogether, of whom 3 were in the third-largest group. That group therefore requires $\frac{3}{15}\times 360^\circ = 3\times 24^\circ = 72^\circ$

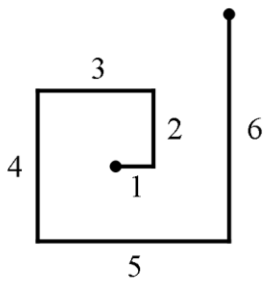
6. **A** Each fold doubles the number of layers that will be pierced. There will be 2^5 layers and therefore 2^5 = 32 holes.

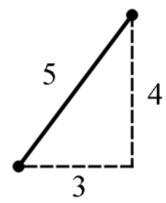
7. **D** The person on the extreme left can be any one of the four people that is neither Alfred nor Mollie; the second left can be any one of the remaining three; the first person on the right of centre… and so on. For every arrangement of the people around them, Alfred and Mollie can swap places to make a new arrangement. So the number of possibilities is (4 × 3 × 2 × 1) × 2 = 48.

8. **B** The digits 1 to 9 total 45, and 460 = 10×45 + 10. So the complete set 1…9 will appear ten times, and then as any more digits as total 10, viz. 1+2+3+4. So the last digit is 4.

9. **C** $w + z + 45^\circ = 180^\circ$ and $x + y + 45^\circ = 180^\circ$.
We thus have $w + x + y + z + 90^\circ = 360^\circ$ and hence $w + x + y + z = 270^\circ$

10. **B** Horizontal distance travelled:
$1 - 3 + 5 = 3$ (i.e. 3 km East)
Vertical distance travelled:
$2 - 4 + 6 = 4$ (i.e. 4 km North)
By Pythagoras, the straight-line distance from the starting point is thus 5 km.





11. **C** 120% of a is $\frac{6}{5}a$ while 80% of b is $\frac{4}{5}b$.
We thus have $\frac{6}{5}a = \frac{4}{5}b$ from which it follows that $\frac{a}{b} = \frac{4}{5}\times\frac{5}{6} = \frac{2}{3}$

12. **A** Let the tick be placed in any one of the 16 blocks. Then the cross can go in any of three other rows or three other columns, which gives 9 possible positions. That makes 16 × 9 = 144 ways.

13. **E** Given $a + 2b = 13$ and $5a – 2b = 5$, we can add both left-hand sides and both right-hand sides to find $6a = 18$. Thus $a = 3$, and then since $a + 2b = 13$, we must have $2b = 10$, i.e. $b = 5$.

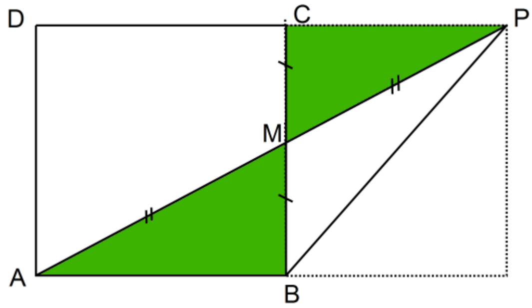
14. **D** BAP = 60° while BAC = 90°, so PAC = 30°. With BPA = 60° that means C = 30°, and so Δ PAC is isosceles. Then BC has length 4, and so by Pythagoras the length of AC is $\sqrt{4^2 - 2^2}$

15. **C** $\frac{\text{area } \Delta\text{ABP}}{\text{area } \Delta\text{ABCD}} = \frac{1}{3}$ so $\frac{\frac{1}{2}\text{BP.AB}}{\text{BC.AB}} = \frac{1}{3}$ so $\frac{\frac{1}{2}\text{BP}}{\text{BC}} = \frac{1}{3}$ and therefore $\frac{\text{BP}}{\text{BC}} = \frac{2}{3}$

and then BP : PC is $\frac{2}{3}:\frac{1}{3} = 2:1$

16. **D** Multiplying gives us $9n < 4n + 16$, i.e. $5n < 16$, so $n = 1$ or 2 or 3.

17. **D** Joining P to C we see that Δ PCM is identical to Δ ABM. That means that P, C, B are vertices of a square, and the required angle is the one between a diagonal of a square and its side, i.e. 45°



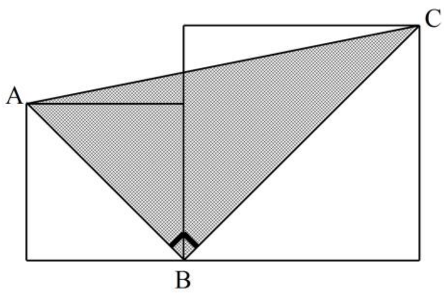
18. **E** Between (and including) 98 and 200 there are 51 multiples of 2; between 98 and 199 there are 34 multiples of 3. Between 102 and 198 there are 17 multiples of 6. The number we seek is 51 + 34 – 17 = 68

19. **A** $\angle\text{ABC} = 90°$ since AB and BC are both diagonals of a square,

$\therefore \text{AB} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

$\therefore \text{BC} = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}$

$\therefore$ area of ΔABC is $\frac{1}{2}\left(5\sqrt{2}\right)\left(7\sqrt{2}\right) = 35 \text{ cm}^2$



20. **B** Suppose angle AOB is θ, and let PA = x.

Then the shaded area is $\frac{\theta.\pi(1+x)^2}{360} - \frac{\theta.\pi(1)^2}{360}$

and the whole sector AOB has area $\frac{\theta.\pi(1+x)^2}{360}$.

So we must have $\frac{\theta.\pi(1+x)^2}{360} - \frac{\theta.\pi}{360} = \frac{1}{4}.\frac{\theta.\pi(1+x)^2}{360}$

and therefore $\frac{3}{4}.\frac{\theta.\pi(1+x)^2}{360} = \frac{\theta.\pi}{360}$ and so $(1+x)^2 = \frac{4}{3}$.

Then $x = \frac{2}{\sqrt{3}} - 1$ or $\frac{2-\sqrt{3}}{\sqrt{3}}$