THE SOUTH AFRICAN MATHEMATICS OLIMPIAD

in conjunction with THE SOUTH AFRICAN MATHEMATICAL SOCIETY and THE ASSOCIATION FOR MATHEMATICS EDUCATION OF SOUTH AFRICA

THE SOUTH AFRICAN MATHEMATICS OLIMPIAD

FIRST ROUND 1997

SENIOR SECTION: GRADES 10, 11 AND 12

(STANDARDS 8, 9 AND 10)

12 MARCH 1997

14:00

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   3.1 For each correct answer: 5 marks
   3.2 For no answer: 0 marks
   3.3 For each wrong answer: 0 marks.
4. You must use an H.B. pencil. Rough paper, ruler and rubber are permitted. **Calculators are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.
7. When the invigilator gives the signal, start attempting the problems. You will have 60 minutes working time for the test.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

KEER DIE BOEKIE OM VIR AFRIKAANS

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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1. $2^n - 2^{n-1}$ equals

(A) 2^{n-1} (B) 2^{2n-1} (C) 2 (D) 2^{2n} (E) $\frac{1}{2}$

2. A number is formed by arranging the digits of 1997 in descending (decreasing) order. A second number is formed by arranging the digits of 1997 in ascending (increasing) order. The difference between the two numbers is

(A) 7182 (B) 7218 (C) 7281 (D) 8172 (E) 8271

3. n is a number such that

$$n \times n = n + n.$$

How many such numbers are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

4. $\sqrt{\sqrt{36} - \sqrt{4}}$ equals

(A) $\sqrt{2}$ (B) $\sqrt{6} - \sqrt{2}$ (C) 2 (D) $\sqrt{32}$ (E) 4

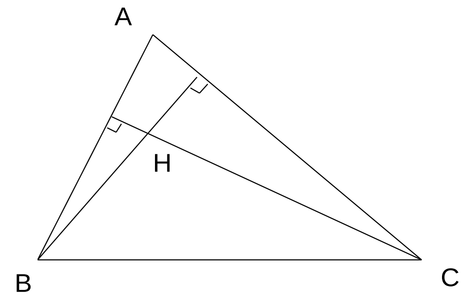
5. 399×501 is closest to

(A) 250 000 (B) 240 000 (C) 200 000 (D) 160 000 (E) 150 000

6. The graph of $y = 4(x-2)(x+3)$ cuts the x-axis at two points P and Q. The length of the line segment PQ is

(A) 4 (B) 20 (C) $\frac{5}{4}$ (D) 5 (E) 1

7. In the figure $B\hat{H}C$ is equal to



(A) $3\hat{A}$ (B) $360^o - \hat{A}$ (C) $180^o - \hat{A}$ (D) $2\hat{A}$ (E) $\hat{A}$

8. $\frac{1996(1997^2-9)1998}{2000(1997^2-1)}$ equals

(A) 1994 (B) 1995 (C) 1996 (D) 1997 (E) 1998

9. If Isaac walks to school and rides his bicycle back home it takes him 90 minutes. If he rides his bicycle both ways it takes him 30 minutes. How many hours would it take him to make the trip to school and back by walking? (Assume that he walks at a constant speed and that he cycles at a constant speed.)

(A) 2 (B) 2,5 (C) 3 (D) 3,5 (E) 4

10. The notation $N(x)$ means the number of prime numbers less than x. What is the value of $N(N(30))$? (Remember that 1 is not a prime number.)

(A) 4 (B) 5 (C) 10 (D) 23 (E) 29

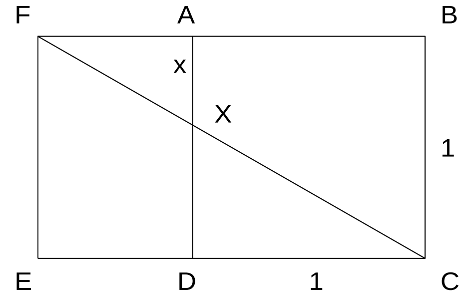
11. The value of

$$\frac{1 \times 2 \times 4 + 2 \times 4 \times 8 + 3 \times 6 \times 12 + \cdots + 10 \times 20 \times 40}{1 \times 3 \times 9 + 2 \times 6 \times 18 + 3 \times 9 \times 27 + \cdots + 10 \times 30 \times 90}$$

is

(A) $\frac{1}{729}$ (B) $\frac{1}{27}$ (C) $\frac{8}{27}$ (D) 1 (E) $\frac{2}{3}$

12. $FBCE$ is a rectangle and $ABCD$ is a square of side 1. If AX has length x then the value of AF is



(A) $2(1-x)$ (B) $\frac{x}{1-x}$ (C) $\frac{1}{4x^2}$ (D) $\frac{2x^2}{1-x}$ (E) $2x$

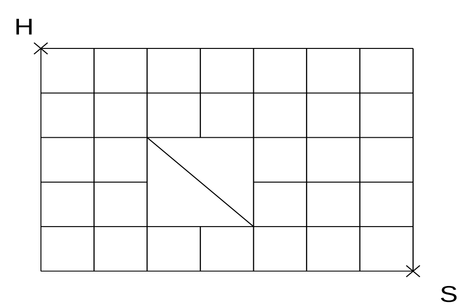
13. When the number 111 222 333 444 555 666 777 888 999 is divided by 111 then the number of digits in the quotient is

(A) 8 (B) 9 (C) 10 (D) 17 (E) 25

14. One and only one of the following pairs of numbers will *not* satisfy the equation $187x - 104y = 41$. Which one is it?

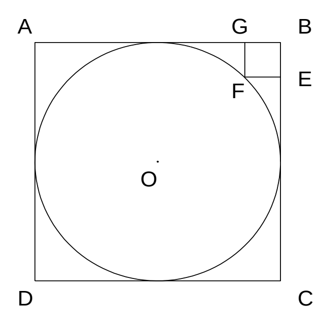
(A) $x = 3,\ y = 5$ (B) $x = 107,\ y = 192$ (C) $x = 211,\ y = 379$
(D) $x = 314,\ y = 565$ (E) $x = 419,\ y = 753$

15. The figure shows the plan of a town where all the street blocks are square. In the middle of the town is a park with a diagonal road through it. Gloria walks every day from her house at H to her school at S, always taking one of the shortest routes. The number of different shortest routes that she can choose is



(A) 6 (B) 10 (C) 12 (D) 18 (E) 24

16. A circle touches the four sides of the square $ABCD$. $BEFG$ is a square of side 1. The length of AB is



(A) $4 + 2\sqrt{2}$ (B) 2π (C) $5\sqrt{2}$ (D) $\frac{5}{2}\pi$ (E) $4\sqrt{2} + 1$

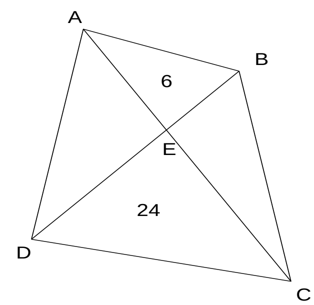
17. The triangular numbers are the numbers 1, 3, 6, 10, 15, 21, 28 and so on. How many of the first 250 triangular numbers are divisible by 5?

(A) 100 (B) 150 (C) 125 (D) 75 (E) 50

18. Noxolo walks from her home to Bizana and back home. Her brother George walks from Bizana to their home and back to Bizana. They both walk at constant speeds (not necessarily the same). They leave at exactly the same time and meet for the first time 7 km from Bizana. Each one continues on their way to their destination where they both turn around without any loss of time. They meet the second time 4 km from their home. The distance, in kilometers, from their home to Bizana is

(A) 9 (B) 11 (C) 17 (D) 20 (E) impossible to tell

19. The diagonals of the quadrilateral $ABCD$ intersect at the point E. The area of triangle AEB is 6, that of triangle DEC is 24, and the areas of triangles AED and BEC are equal. The area of triangle AED is



(A) 12 (B) 15 (C) 18 (D) 20 (E) 30

20. How many different pairs of integers $(x; y)$ are solutions of the equation $x^2 - 3y^2 = 1997$?

(A) 1 (B) 2 (C) 3 (D) infinitely many (E) none