THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

FIRST ROUND 2004

SENIOR SECTION: GRADES 10, 11 AND 12

18 MARCH 2004

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 5 marks.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.
8. Answers and solutions are available at: http://science.up.ac.za/samo/

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129

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1. $(0.2)^4$ equals

(A) 0.8 (B) 0.16 (C) 0.0016 (D) 0.000016 (E) 0.0008

2. The height of the tallest building on earth is about

(A) 400 m (B) 40 m (C) 4 000 m (D) 4 m (E) 40 000 m

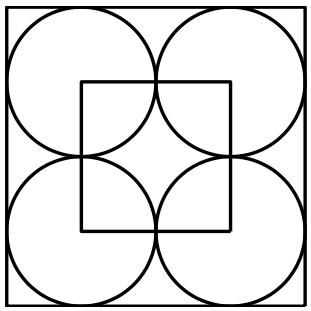
3. $\dfrac{37^2 + 111}{37}$ equals

(A) 4 (B) 40 (C) 113 (D) 148 (E) 37

4. $\dfrac{4444^4}{2222^4}$ equals

(A) 8888 (B) 2222^4 (C) 2 (D) 2222 (E) 16

5. Four identical circles fit inside a square as shown. Their centres are the vertices of the smaller square. If the smaller square has area 4, then the area of the larger square is



(A) 16 (B) 8 (C) $4\sqrt{2}$ (D) 12 (E) 4π

6. $1 - \dfrac{1 - \dfrac{1}{2}}{1 + \dfrac{1}{2}} \Big/ 1 + \dfrac{1 + \dfrac{1}{2}}{1 - \dfrac{1}{2}}$ equals

(A) $\dfrac{7}{9}$ (B) $\dfrac{5}{6}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{3}{4}$

7. If $17x + 51y = 85$, then $13x + 39y$ is

(A) impossible to determine (B) 61 (C) 63 (D) 65 (E) 67

8. In the sequence

$$\ldots, k, m, n, p, 0, 1, 1, 2, 3, 5, 8, \ldots$$

each term is the sum of the two terms on its left. The value of k is

(A) -2 (B) 3 (C) -3 (D) 2 (E) -1

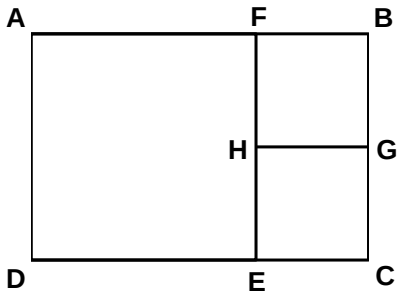
9. $2004^2 - 2003 \times 2005$ equals

(A) 0 (B) 1 (C) -1 (D) 2004 (E) 2001

10. For how many integer values of n is $\dfrac{n+3}{n-1}$ an integer?

(A) 7 (B) 6 (C) 5 (D) 2 (E) 8

11. In the diagram $ABCD$ is a rectangle with $AB = \sqrt{3}$ and $BC = 1$. $AFED$ and $FBGH$ are squares. The length of HE is



(A) $2 - \sqrt{3}$ (B) $2\sqrt{3} - 3$ (C) $\dfrac{\sqrt{3}}{6}$ (D) $7\sqrt{3} - 12$ (E) $\sqrt{3} + 2$

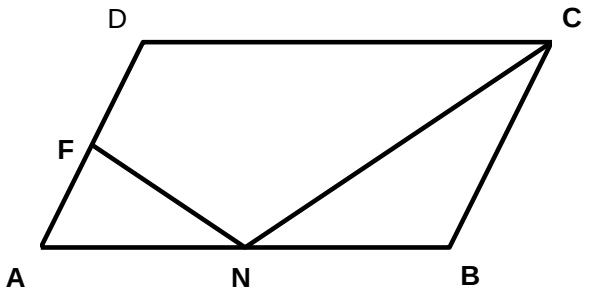
12. The number of prime numbers p such that $p + 1$ is a square is

(A) 1 (B) 0 (C) 4 (D) 3 (E) infinite

13. If $2^2 \times 3^3 \times 4^4 \times 5^{11}$ is multiplied out, then the sum of the digits is

(A) 9 (B) 207 (C) 14 (D) 135 (E) 814

14. $ABCD$ is a parallelogram. F is the midpoint of AD and N is the midpoint of AB. The ratio of the area of $\triangle AFN$ to the area of quadrilateral $FNCD$ is

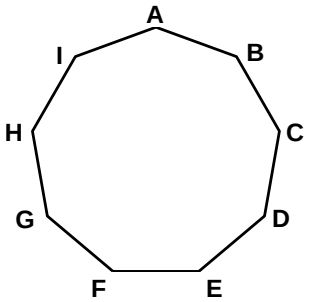


(A) $\frac{2}{7}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

15. The number of natural numbers less than 100 000 which have 9 as their first digit is

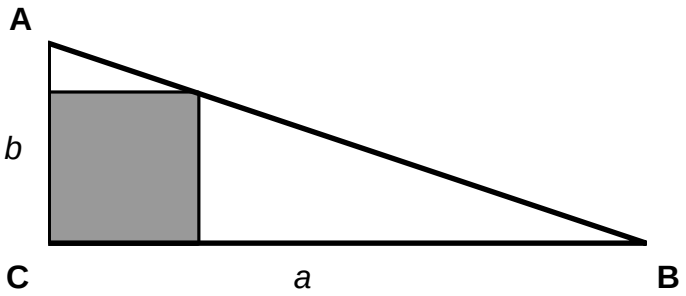
(A) 10 000 (B) 11 110 (C) 10 001 (D) 9 999 (E) 11 111

16. The diagram shows a regular 9-sided polygon. The size of angle FAE, in degrees, is



(A) 10 (B) 20 (C) 30 (D) 40 (E) 25

17. The right-angled triangle ABC has sides with lengths a and b as shown. The ratio of the area of the shaded square to the area of triangle ABC is

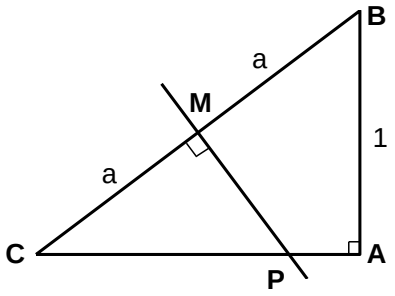


(A) $\frac{ab}{a+b}$ (B) $\frac{ab}{(b-a)^2}$ (C) $\frac{2ab}{a^2+b^2}$ (D) $\frac{2ab}{(a+b)^2}$ (E) $\frac{ab}{(a+b)^2}$

18. $\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}}$ equals

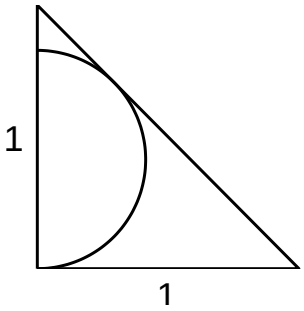
(A) 2 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 1 (E) $12\frac{1}{4}$

19. In the diagram, MP is the perpendicular bisector of BC and the length of AB is 1. The length of MP is



(A) $\dfrac{a}{\sqrt{2a^2-1}}$ (B) $a\sqrt{4a^2-1}$ (C) $\dfrac{a}{\sqrt{4a^2+1}}$ (D) $\dfrac{a}{\sqrt{4a^2-1}}$ (E) $\dfrac{1}{2}$

20. A semi-circle is inscribed in an isosceles right-angled triangle as shown. The radius of the semi-circle is



(A) $\dfrac{1}{\sqrt{2}}$ (B) $3-2\sqrt{2}$ (C) $\dfrac{1}{2}$ (D) $2-\sqrt{2}$ (E) $\sqrt{2}-1$