

SOUTH AFRICAN MATHEMATICS OLYMPIAD



Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

2012 SECOND ROUND JUNIOR SECTION

23 May 2012 Time: 120 minutes Number of questions: 20

Instructions

- 1. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
- 2. Scoring rules:
 - 2.1. Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.
 - 2.2. For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
- 3. You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**
- 4. Figures are not necessarily drawn to scale.
- 5. Indicate your answers on the sheet provided.
- 6. The centre page is an information and formula sheet. Please tear out the page for your own use.
- 7. Start when the invigilator tells you to do so.
- 8. Answers and solutions will be available at www.samf.ac.za

Do not turn the page until you are told to do so. Draai die boekie om vir die Afrikaanse vraestel.



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Organisations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. 23 + 6 – 4 =

(A) 6 (B) 23 (C) 25 (D) 29 (E) 33

2. $\frac{1}{5} + \frac{2}{3} \times \frac{1}{2}$ equals

(A) $\frac{1}{15}$ (B) $\frac{3}{11}$ (C) $\frac{21}{50}$ (D) $\frac{8}{15}$ (E) $9\frac{4}{5}$

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Part A: (Each correct answer is worth 4 marks)

1. Arthur takes 8 subjects at school, and for each one receives a code from 1 to 7. If the sum of his codes is 50, then the least number of 7s he must have received is

(A) 6

(B) 5

(C) 4

(D) 3

(E) 2

2. A man gives away half of his money to his friend, and after that 10% of what he has left to charity. The percentage of his original amount that he keeps is

(A) 25

(B) 30

(C) 35

(D) 40

(E) 45

3. The five-digit number 24X8Y is divisible by 4 and by 5 and by 9. The sum of the digits X and Y is

(A) 13

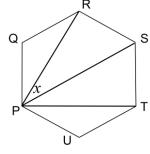
(B) 10

(C) 9

(D) 5

(E) 4

4. The vertex P of a regular hexagon is joined to the other vertices. The size of \hat{RPS} (marked x in the diagram) is



(A) 20°

(B) 30°

(C) 40°

(D) 45°

(E) 50°

5. The value of $1 - (2 - (3 - (\dots - (100))\dots))$ is

(A) -50

(B) 50

(C) -100

(D) 100

(E) 99

Part B: (Each correct answer is worth 5 marks)

6. The natural numbers are written in seven columns: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 ...

A square is drawn around a block of four numbers, and the sum of those four numbers is 312. The number at top left of the square is

(A) 67

(B) 69

(C) 72

(D) 74

(E) 76

7. If $\frac{x-1}{x} = y$ and $\frac{y+1}{y} = x$ then the value of x - y is

(A) 2

(B) -2

(C) 3

(D) -3

(E) 1

8. A positive integer N has exactly three different prime factors (1 is not a prime) and is not divisible by any square. How many different factors does N have?

(A) 4

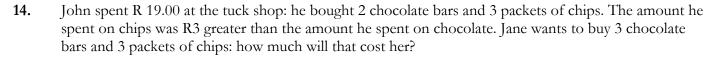
(B) 5

(C) 6

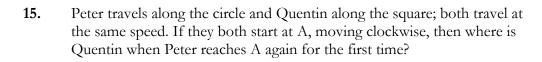
(D) 7

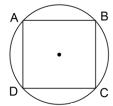
(E) 8

9.	a, b, c, d, e are five positive numbers, with $bd < cd < ab < bc < ae$. The smallest of the five numbers is				
	(A) <i>a</i>	(B) b	(C) c	(D) d	(E) impossible to tell
10.	A circular logo is made up of three circles with the same centre and radii in the ratio 1:2:3. A point is chosen randomly inside the logo. How many times more likely is the point to be in the outer ring than in the shaded centre?				
	(A) 3	(B) 4	(C) 5	(D) 6	(E) 9
11.	Everyone in my class has toffees or chocolates, but half of them have both. Twice as many people have toffees only as have chocolates only. The proportion of people with chocolates who also have toffees is				
	(A) $\frac{1}{3}$	(B) $\frac{2}{3}$	(C) $\frac{3}{4}$	(D) $\frac{4}{5}$	(E) $\frac{5}{6}$
12.	Dimitri can travel from P to Q in 10 minutes at constant speed; Olga can make the same journey in 20 minutes, also at constant speed. Boris goes from P to Q at the average of Dimitri's and Olga's speeds: how long will it take him?				
	(A) $12\frac{1}{3}$ min	(B) $13\frac{1}{3}$ min	(C) 14 min	(D) $14\frac{1}{3}$ min	(E) $15\frac{1}{3}$ min
13.	line. Brenda then		third card that is	still face down (so	over, starting with the second in starts with the fifth card in the
	(A) 20	(B) 24	(C) 28	(D) 34	(E) 36



- (A) R 24.40
- (B) R 23.60
- (C) R 23.00
- (D) R 22.60
- (E) R 21.40

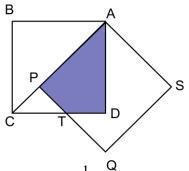




- (A) at B
- (B) somewhere on AB
- (C) somewhere on BC
- $\begin{array}{c} \text{somewhere} \\ \text{On CD} \end{array}$
- (E) somewhere on DA

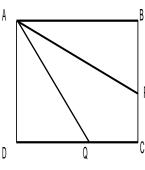
Part C: (Each correct answer is worth 6 marks)

- **16.** The 81st term of the sequence 1; 2; 2; 3; 3; 3; 4; 4; 4; 4; ... is
 - (A) 11
- (B) 12
- (C) 13
- (D) 14
- (E) 15
- Among the pupils at a school, the ratio of seniors to juniors is exactly 5: 3. Among the juniors the ratio 17. of boys to girls is exactly 3:2, and among the seniors it is exactly 2:3. What is the minimum number of pupils in the whole school?
 - (A) 40
- (B) 45
- (C) 48
- (D) 60
- (E) 120
- 18. Squares APQS and ABCD both have sides of length 1 unit. P lies on the diagonal AC. The shaded area that is common to both squares is (in square units)



- $\frac{1}{\sqrt{3}}$ (C) $\sqrt{2} 1$ (D) $\sqrt{3} \sqrt{2}$
- 19. A three-digit number X has its digits reversed to become Y. The sum of X and Y is 1535. The sum of the digits of X is
 - (A) 11
- (B) 12
- (C) 13
- (D) 15
- (E) 16

20. Points P and Q are chosen on sides of the square ABCD so that the lines AP and AQ divide the square into three regions of equal area. The sides of the square have length 1. The ratio of the lengths AQ:QC is



- (A) $\sqrt{5}$ (B) $\sqrt{13}$
- (C) 4
- $\sqrt{15}$ (D)
- (E) 3