THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

FIRST ROUND 1999: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A: (Each correct answer is worth 3 marks)

1. <u>ANSWER:</u> B

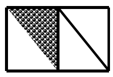
$$\frac{1+2+3+4}{1\times 2\times 5}=\frac{10}{10}=1$$

2. <u>ANSWER:</u> E

If blanket is divided into triangles, there are 40 such equal triangles of which 10 of them are coloured.

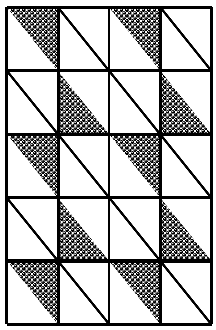
Therefore $\frac{10}{40}=\frac{1}{4}$

OR



In each small block there is one dark coloured block and 3 light coloured blocks.

Therefore $\frac{1}{4}$ blocks are coloured.



3. <u>ANSWER:</u> A

If the pattern is written out you get:

$1+1=2$	$2+1=3$	$3+1=4$	$4+1=5$	$5+1=6$	$6+1=\underline{7}$
$1+2=3$	$2+2=4$	$3+2=5$	$4+2=6$	$5+2=\underline{7}$	$6+2=8$
$1+3=4$	$2+3=5$	$3+3=6$	$4+3=\underline{7}$	$5+3=8$	$6+3=9$
$1+4=5$	$2+4=6$	$3+4=\underline{7}$	$4+4=8$	$5+4=9$	$6+4=10$
$1+5=6$	$2+5=\underline{7}$	$3+5=8$	$4+5=9$	$5+5=10$	$6+5+11$
$1+5=\underline{7}$	$2+6=8$	$3+6=9$	$4+6=10$	$5+6=11$	$6+6=12$

Frequency of numbers:

2:	1	8:	5
3:	2	9:	4
4:	3	10:	3
5:	4	11:	2
6:	5	12:	1
7:	6		

The total of 7 was reached more than any other total.

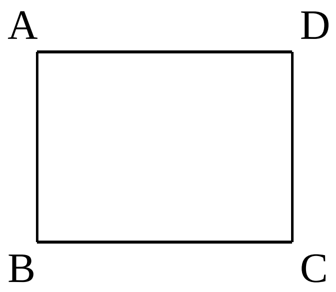
4. <u>ANSWER:</u> E

$$\frac{3\text{ min}}{13{,}6\text{ sec}} = \frac{180\text{ sec}}{13{,}6\text{ sec}} = \pm 13\text{ units}$$

13 units @ 30,9 cents per unit is approximately R4,00

5. <u>ANSWER:</u> E

AB + BC + CD + DA = 16
Or 2 X AB +2 X BC = 16.
Therefore AB +BC = 8
Look at the adding combinations of 8, which gives you a product of 15 (AB X BC = 15 m^2)
3 + 5 = 8 and 3 X 5 =15



PART B: (Each correct answer is worth 5 marks)

6. <u>ANSWER:</u> B

In a 5-sided polygon you can draw 2 diagonal lines from one vertex to the other vertices. In a 6-sided polygon you can draw 3 lines, in a 7-sided polygon 4 lines etc. Every time there are 3 diagonals less than the number of sides, therefore for a 50 sided polygon you can draw 50 – 3 = 47 diagonal lines from one vertex to the other vertices.

7. <u>ANSWER:</u> D

Total number of learners in Grade 8 and 9 is $\frac{1}{10}\ of\ 20\,000\,000 = 2\,000\,000$.

Each learner has books to the value of: $R30 + R70 + R40 + R60 = R200$.
Total cost for Education Department:
$2\,000\,000 \times R200 = R400\,000\,000 = R400$ million

8. <u>ANSWER:</u> E

$12\text{ sec} \times 332\text{ m / sec} = 3984\text{ m} \approx 4\text{ km}$

9. <u>ANSWER:</u> C

There are 4 tyres on the car at a time. Total distance with four tyres together is $45\,000\text{ km} \times 4 = 180\,000\text{ km}$. Five tyres were used, therefore each tyre did $180\,000\text{ km} \div 5 = 36\,000\text{ km}$.

OR

Each tyre does $\frac{4}{5}$ of the total distance.

$\frac{4}{5}\ of\ 45\,000 = 36\,000$, therefore each tyre did 36 000 km.

10. ANSWER: C

Correct answers in Part A are worth 3 marks each, therefore she has $3\times3=9$ marks out of 100. Correct answers in Part B are worth 5 marks each, therefore she has another $7\times5=35$ marks out of 100. Therefore she has 44 marks and needs another 16 marks to get a total of 60 marks. Each correct answers in Part C is worth 7 marks, therefore she needs 3 more correct answers for Part C to give her another 21 marks and added to 44 she will then have 65 marks out of a possible 100, i.e. 65%. Two correct questions in Part C will only give her 14 with a total of 58%

OR

Part A: $3\times3=9$

Part B: $7\times5=\underline{35}$

44

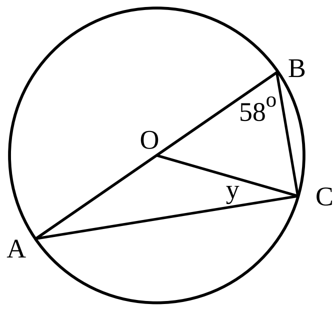
She needs 60% i.e. 60 marks

$\therefore 16$ marks more needed

Part C $x\times7\geq16$

$\therefore \underline{x=3}$

11. ANSWER: D



$O\hat{B}C=58^\circ$ and $O\hat{C}B=58^\circ$ because ΔOCB is isosceles, $OC=OB$, both are radii of circle, centre O. $O\hat{C}A=O\hat{A}C=y$ because ΔOAC is isosceles, $OC=OA$, both are radii of circle, centre O. Sum of angles of a triangle is 180°, therefore $O\hat{B}C+O\hat{C}B+O\hat{C}A+O\hat{A}C=180^\circ$.

$58^\circ+58^\circ+y+y=180^\circ$

$\therefore 2y=180^\circ-116^\circ$

$\therefore 2y=64^\circ$

$\therefore \underline{y=32^\circ}$

OR

In ΔOBC, $OB=OC$ (radii) $\therefore \hat{B}=\hat{C}=58^\circ$

In ΔOAC, $OA=OC$ (radii) $\therefore \hat{A}=\hat{C}=y$

In ΔABC, $OA=OC$ (radii)

$\therefore \hat{A}+\hat{B}+\hat{C}=180^\circ$ (sum angles of Δ)

$\therefore y+58^\circ+y+58^\circ=180^\circ$

$\therefore 2y+116^\circ=180^\circ$

$\therefore 2y=64^\circ$

$\therefore \underline{y=32^\circ}$

12. ANSWER: A

A pen costs x cents and a ruler costs y cents. $2x+3y=190$ also

$x=y+20$. Substitute in first equation:

$2(y+20)+3y=190$

$\therefore 5y+40=190$

$\therefore 5y+40=190$

$\therefore 5y=150$

$\therefore y=30$

A ruler costs 30c and a pen (30c + 20c) = 50c.

2 rulers and 3 pens: $(2\times 30\text{c})+(3\times 50\text{c})=60\text{c}+150\text{c}=210\text{c}=\text{R}2{,}10$

13. ANSWER: C

The following years are divisible by 6: 1902, 1908, 1914, …1998.

$100\div 6=16{,}67$, therefore starting with 1902 plus 16 increments gives you 1998. There are 17 years which are divisible by 6 from 1901 to 2000.

14. ANSWER: A

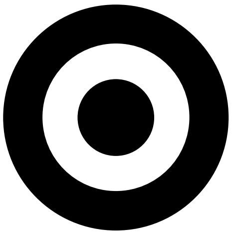
Length of ribbon:

$=(2\times\text{length})+(2\times\text{width})+(4\times\text{height})+(\text{bow, knots and ends})$

$=(2\times 20\text{ cm})+(2\times 15\text{ cm})+(4\times 10\text{ cm})+(47\text{ cm})$

$=40\text{ cm}+30\text{ cm}+40\text{ cm}+47\text{ cm}=157\text{ cm}=1{,}57\text{ m}$

15. ANSWER: E



Area of inner circle: $\pi r^2=\pi(1)^2=\pi$

Area of outer ring = area of outer circle minus area of circle not part of ring.

$\pi\text{R}_O^2-\pi\text{R}_S^2=\pi(3)^2=\pi(2)^2=9\pi-4\pi=5\pi$

[R_O: radius outer circle; R_S: radius circle not part of outer ring]

$\therefore$ Area of outer ring is 5 times bigger than area of inner circle.

PART C: (Each correct answer is worth 7 marks)

16. ANSWER: D

```
      T W O
+     T W O
=   F O U R
```

therefore

```
      T W 7
+     T W 7
=   F 7 U 4
```

F has to be 1, therefore **T** has to be 8

therefore:

```
      8 W 7
+     8 W 7
=   1 7 U 4
```

W has to be bigger than 4 because I have to carry 1 to add to 8+8 to get 17. Possibilities are therefore 5, 6, 7, 8 or 9. It cannot be 5 because 5+5+1 = 11, and **U** cannot be 1 as **F** is already 1. It can also not be 7 or 8 because **O** and **T** are 7 and 8 respectively, neither can 9 work, for 9 + 9 + 1 = 19, and **W** and **U** cannot both be 9. We are left with only one possibility, namely 6. If tested, it works.

```
      8 6 7
+     8 6 7
=   1 7 3 4
```

U, represents the digit 3.

17. ANSWER: B

Write down the factors of 84, 70 and 30:

$84 = 2\times 2\times 3\times 7;\quad 70 = 2\times 5\times 7;\quad 30 = 2\times 3\times 5$

We now look for common factors because we have common edges where the faces meet. Between 84 and 70, common factors are 2; 7 and 14. If the length of the common edge between areas 84 cm^2 and 70 cm^2 is 14 cm, the other side length of 84 cm^2 has to be 6 cm and that of 70 cm^2 has to be 5 cm. $5\times 6 = 30$, which give the area of 30 cm^2.

$$\begin{aligned}\text{Volume} &= \mathrm{L}\times\mathrm{B}\times\mathrm{H}\\ &= 14\times 5\times 6\\ &= 420\ \text{cm}^3\end{aligned}$$

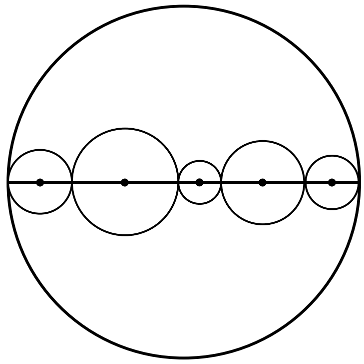
OR

$$\begin{aligned}(\text{Volume})^2 &= \text{Area A}\times\text{Area B}\times\text{Area C}\\ &= 84\ \text{cm}^2\times 70\ \text{cm}^2\times 30\ \text{cm}^2\\ &= 176\,400\ \text{cm}^6\\ \text{Volume} &= 420\ \text{cm}^3\end{aligned}$$

OR

$$\begin{aligned}&l\times b = 84;\ b\times h = 70;\ h\times l = 30\\ &\therefore l\times b\times b\times h\times h\times l = 84\times 70\times 30\\ &\therefore (l\times b\times h)\times(l\times b\times h) = 176400\\ &\therefore \text{Volume} = 420\ \text{cm}^3\end{aligned}$$

18. ANSWER: B



Take radius of big circle as R

Take radii of small circles as r_1; r_2; r_3; r_4 and r_5.

Circumference of big circle:

$$2\pi R = 30 \text{ cm}$$

Circumference of 5 small circles:

$$2\pi r_1 + 2\pi r_2 + 2\pi r_3 + 2\pi r_4 + 2\pi r_5$$

$$= 2\pi(r_1 + r_2 + r_3 + r_4 + r_5)$$

$$= 2\pi R = 30 \text{ cm}$$

Therefore the sum of circumferences of the 5 smaller circles is the same as the circumference of the big circle and equal to 30 cm.

19. ANSWER: A

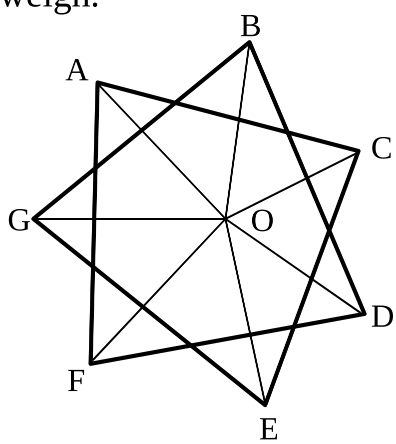
The possible weights are:

1 kg	1 + 3 = 4 kg	1 + 2 kg = 3	1 + 9 = 7 kg + 3
3 kg	1 + 9 = 10 kg	1 + 8 kg = 9	3 + 9 = 11 kg + 1
9 kg	3 + 9 = 12 kg	3 + 6 kg = 9	1 + 3 + 5 kg = 9

1 + 3 + 9 = 13 kg

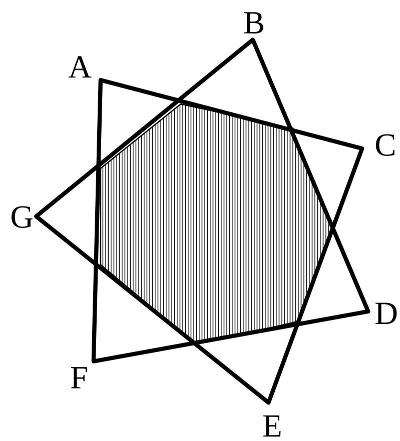
Therefore there are 13 different weights one can weigh.

20. ANSWER: B



Complete triangles as indicated in the figure. There are 7 triangles (Δ's AOC, BOD, COE, DOF, EOG, FOA, GOB) which give a sum of $7 \times 180^\circ = 1260^\circ$. However, $2 \times 360^\circ = 720^\circ$ (2 rotations - angles formed at O) has to be subtracted from the total which give us the answer of $1260^\circ - 720^\circ = 540^\circ$

OR



As the figure is drawn there are 7 triangles and one heptagon (7 – sided polygon).The total of all the angles in the triangles is $7 \times 180^\circ$. The angles of the triangles besides angles $\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{E}, \hat{F}$, and $\hat{G}$ are the exterior angles of the heptagon. They have a total value of $2 \times 360^\circ$ or $4 \times 180^\circ$. So the remaining angles, $\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{E}, \hat{F}$, and $\hat{G}$ have a sum of $7 \times 180^\circ - 4 \times 180^\circ = 3 \times 180^\circ = 540^\circ$.

In general: In an n-pointed star the angle sum at the points is $(n-4) \times 180^\circ$ for $n \geq 5$.

THE END