2018 JUNIOR ROUND TWO SOLUTIONS

1. 6 — Focusing only on the units digits we have $8 \times 2 = 16$. The units digit will thus be 6.

2. 2 — $2^0 + 1^8 = 1 + 1 = 2$

3. 5 — $100 \div 5 = 20$, therefore $A = 5$.

4. 60 — Using angles of a revolution and alternate angles between parallel lines:
$x = 360° - 300° = 60°$.

5. 16 — The shaded area is two ninths of 72 cm^2. $72 \times \frac{2}{9} = 16$.

6. 125 — From symmetry considerations, $A\hat{B}C = A\hat{D}C = \frac{360° - 70° - 40°}{2} = 125°$.

7. 18 — Each face has surface area 1 cm^2, so each cube has surface area 6 cm^2. Before gluing the cubes together the total surface area was thus 24 cm^2. Since there are 6 faces in contact with one another the surface area of the shape must be 24 – 6 = 18 cm^2.

8. 503 — Note that 2018 ÷ 4 = 504,5. Since this is the mean of the four consecutive integers they must be evenly spaced about 504,5. The four consecutive integers are thus 503, 504, 505, 506. The smallest of these is 503.

9. 256 — For the expression to be as large as possible Z must clearly be 0. Because of the exponential function, X and Y should be as large as possible, thus W must be 1. Comparing 2^8 and 8^2 the largest possible value is $1 \times 2^8 - 0 = 256$.

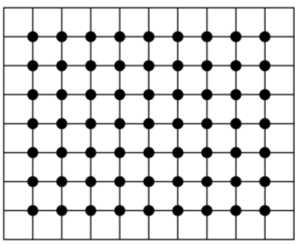
10. 51 — The daily cost of the two pills is $R2121 \div 21 = R101$. Since the green pill costs R1 more than the pink pill, the pills cost R51 and R50 respectively.

11. 28 — The final digit can only be 1, 3, 5 or 7. For each of these there are 7 possibilities for the first digit. There are thus $4 \times 7 = 28$ odd numbers.

12. 23 — Note that 120° + 80° > 180° and 120° + 65° > 180°. Since neither 80° nor 65° can be in the same triangle as the 120°, the triangle *without* the 120° angle must contain both the 80° and 65° angles. We thus know that one of the triangles has angles, 80°, 65° and 35°. The other triangle therefore contains the angles 120°, 37° and 23°. The smallest angle in the list is thus 23°.

13. 12 — Triangle ABC incorporates all three radii exactly twice. The sum of the diameters is thus simply the perimeter of the triangle, i.e. 3 + 4 + 5 = 12.

14. 63 — From the given measurements the quilt will be 10 squares long and 8 squares wide. There will thus be $9 \times 7 = 63$ points where four squares meet.

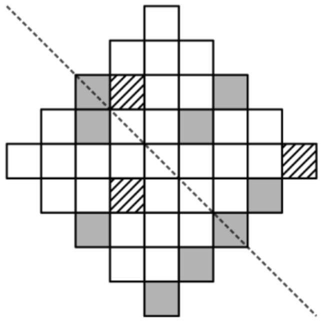


15. 2 — From the units column we see that C + C is a number ending with the digit C. The only digit where this holds is 0, so C = 0. In the tens column B + B equals a number ending in zero (since C = 0). Since A, B and C are different digits, B cannot also be zero, so B must be 5 (5 + 5 = 10 which ends in a zero). Lastly, A + A + 1 = 5, so A = 2.

16. 170 — $\frac{v+w}{2} = 200$, so $v + w = 400$. $\frac{x+y+z}{3} = 150$, so $x + y + z = 450$.
$\frac{v+w+x+y+z}{5} = \frac{400+450}{5} = \frac{850}{5} = 170$.

17. 11 — If x is the number we require, then x + 1 is divisible by 2, 3 and 4. Then x + 1 must be divisible by 3 and 4. Therefore x + 1 = 3 × 4 and so x = 11.

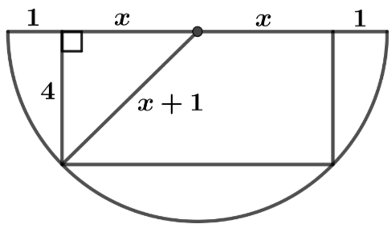
18. 3 — There are four possible lines of symmetry – one vertical, one horizontal and two oblique. The horizontal line of symmetry requires 5 further squares to be shaded. The vertical line of symmetry requires 4 further squares to be shaded. The oblique line of symmetry with positive gradient also requires 4 further squares to be shaded, while the oblique line of symmetry with negative gradient only requires 3 further squares to be shaded.



19. 10 — The 2-digit number x must have a digit sum of 3 or 12, i.e. 30, 12, 21, 93, 39, 84, 48, 75, 57 or 66. There are thus 10 different 2-digit numbers.

20. 16 — Area of circle A + area of circle B = 65 + 15 = 80. Using the given ratio, the area of circle A = 3/5 of 80 = 48. Similarly the area of B = 2/5 of 80 = 32.
Circle A is thus 48 – 32 = 16 m^2 larger than circle B.

21. 60 — Construct a right-angled triangle by drawing in a radius of the semicircle. If we let the length of the rectangle be $2x$ then the triangle has sides x, 4 and $x + 1$ (since the radius of the semicircle is $x + 1$). From the theorem of Pythagoras we have $x^2 + 4^2 = (x + 1)^2$ which simplifies to $x^2 + 16 = x^2 + 2x + 1$ and hence $2x = 15$. The dimensions of the rectangle are thus 15 cm by 4 cm which gives an area of 60 cm^2.

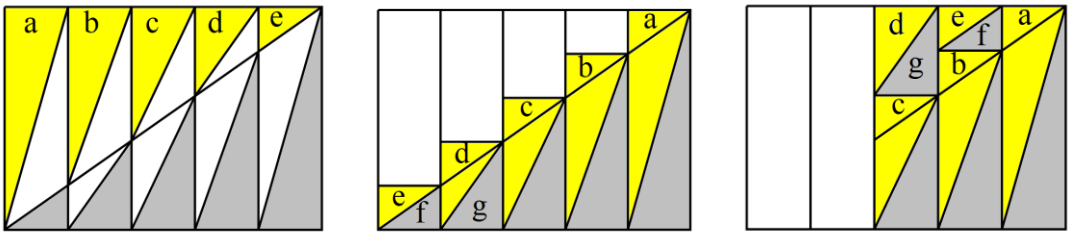


22. 215 — We need to consider three cases: (i) a base of 1, (ii) an exponent of 0, and (iii) a base of -1 with an even exponent. With this in mind x can be 217, -221 or 219. The sum of these three integers is 215.

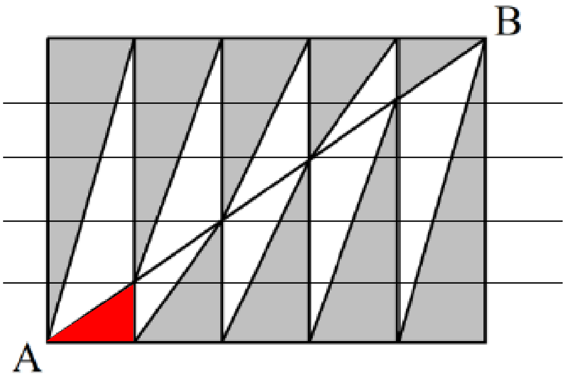
23. 18 — The time period when Jerry was less than 600 m behind Tom is
0,6 km / (12 km/h – 10 km/h) = 3/10 hr = 18 minutes.

24. 8 The hour hand is 3/5 of the way from one number marking to the next, hence it is 3/5 x 60 = 36 minutes past the hour. The last number marking passed by the minute hand is therefore 7. A is thus 8.

25. 12 Using symmetry and translation the shaded area can be seen to be three fifths of the original rectangle, from which the unshaded area is two fifths of the original rectangle. The shaded area is thus $\frac{1}{5} \times 60 = 12\ \text{cm}^2$ larger than the unshaded area.



Alternatively:



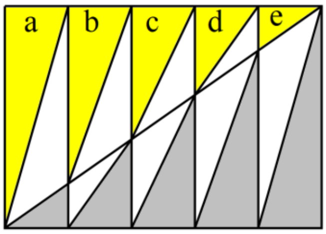
Through symmetry one can see that the horizontal lines divide the breadth into fifths. Starting with the red triangle, all the shaded triangles' areas can be determined:

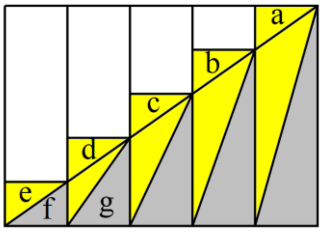
$$(\frac{1}{2}\times\frac{1}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{2}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{3}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{4}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{5}{5}\times 6\times 2)$$
$$=\frac{12+24+36+48+60}{10}$$
$$=18\ \text{cm}^2$$

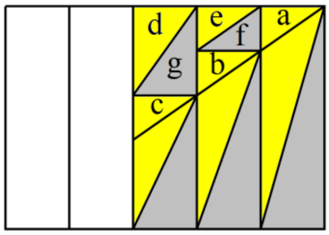
Through symmetry one can see the top shaded part is also 18 cm^2.
Total shaded area: 36 cm^2
Total area of rectangle: 60 cm^2
Unshaded area: 24 cm^2
Difference: 12 cm^2

24. 8 Die uurwyser is 3/5 tussen twee getalle, daarom is dit 3/5 x 60 = 36 minute na die uur. Die laaste getal waarby die minuutwyser verbybeweeg het is dus 7. A is dus 8.

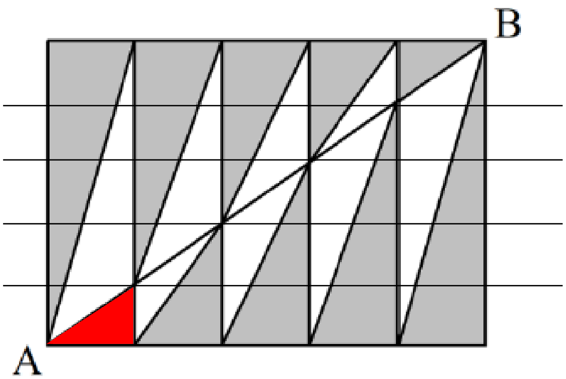
25. 12 Deur simmetrie en verskuiwing kan gesien word dat die ingekleurde gebied drie vyfdes van die oorspronklike reghoek is en daarom is die oningekleurde gebied twee vyfdes van die oorspronklike reghoek. Die ingekleurde oppervlakte is dus $\frac{1}{5} \times 60 = 12$ cm^2 groter as die oningekleurde oppervlakte.







Alternatiewelik:



Ons kan deur simmetrie sien dat die horisontale lyne die hoogte in vyfdes verdeel
Deur met die rooi driehoek te begin kan al die ingekleurde driehoeke se oppervlaktes bereken word:

$$(\frac{1}{2}\times\frac{1}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{2}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{3}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{4}{5}\times 6\times 2)+(\frac{1}{2}\times\frac{5}{5}\times 6\times 2)$$
$$=\frac{12+24+36+48+60}{10}$$
$$=18\text{ cm}^2$$

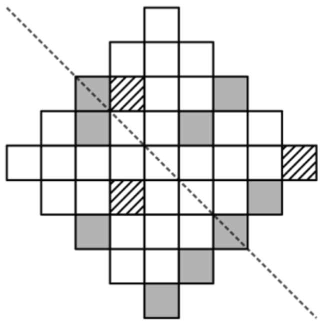
Deur simmetrie kan ons ook sien dat die boonste ingekleurde gebied ook 18 cm^2 is.
Totale ingekleurde oppervlakte: 36 cm^2
Totale oppervlakte van reghoek: 60 cm^2
Oningekleurde oppervlakte: 24 cm^2
Verskil: 12 cm^2

15. 2 Van die ene kolom kan ons sien dat C + C is ’n getal wat eindig met die syfer C. Die enigste syfer waarvoor dit waar is is 0, so C = 0. In die tiene kolom is B + B gelyk aan ’n getal wat eindig met nul (aangesien C = 0). Aangesien A, B en C verskillende syfers is beteken dit dat B nie ook gelyk aan nul kan wees nie. Dus moet B gelyk aan 5 wees (5 + 5 = 10 wat in nul eindig). Laastens, A + A + 1 = 5, so A = 2.

16. 170 $\frac{v+w}{2} = 200$, so $v + w = 400$. $\frac{x+y+z}{3} = 150$, so $x + y + z = 450$.
$\frac{v+w+x+y+z}{5} = \frac{400+450}{5} = \frac{850}{5} = 170.$

17. 11 As x die getal is wat ons benodig, dan is x + 1 deelbaar deur 2, 3 en 4. Dan moet x + 1 deelbaar wees deur 3 en 4. Daarom is x + 1 = 3 × 4 en dus is x = 11.

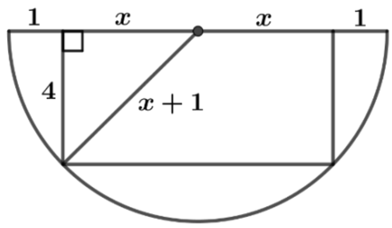
18. 3 Daar is vier moontlike lyne van simmetrie – een vertikaal, een horisontaal en twee skuins. Die horisontale lyn van simmetrie benodig 5 verdere vierkante om ingekleur te word. Die vertikale lyn van simmetrie benodig 4 verdere vierkante om ingekleur te word. Die skuinslyn van simmetrie met positiewe gradiënt benodig ook 4 verdere vierkante om ingekleur te word, terwyl die skuinslyn van simmetrie met negatiewe gradiënt slegs 3 verdere vierkante benodig om ingekleur te word.



19. 10 Die 2-syfer getal x se som van syfers moet 3 of 12 wees, d.i. 30, 12, 21, 93, 39, 84, 48, 75, 57 of 66. Daar is dus 10 verskillende 2-syfer getalle.

20. 16 Oppervlakte van sirkel A + oppervlakte van sirkel B = 65 + 15 = 80. Deur die gegewe verhouding te gebruik is die oppervlakte van sirkel A = 3/5 van 80 = 48. Soortgelyk is die oppervlakte van sirkel B = 2/5 van 80 = 32.
Sirkel A is dus 48 – 32 = 16 m^2 groter as sirkel B.

21. 60 Konstrueer ’n reghoekige driehoek deur ’n radius van die halfsirkel in te trek. As ons die lengte van die reghoek $2x$ laat wees, dan het die driehoek sye van x, 4 en $x + 1$ (aangesien die radius van die halfsirkel $x + 1$ is). Van die stelling van Pythagoras het ons $x^2 + 4^2 = (x + 1)^2$ wat vereenvoudig na $x^2 + 16 = x^2 + 2x + 1$ en dus $2x = 15$. Die lengtes van die reghoek is dus 15 cm en 4 cm wat ’n oppervlakte van 60 cm^2 gee.



22. 215 Ons moet drie gevalle bestudeer: (i) ’n grondtal van 1, (ii) ’n eksponent van 0, en (iii) ’n grondtal van -1 met ’n ewe eksponent. Met hierdie ingedagte kan x of 217, -221 of 219 wees. Die som van hierdie drie heelgetalle is 215.

23. 18 Die tydsverloop wanneer Jerry minder as 600 m agter Tom was, is
0,6 km / (12 km/u – 10 km/u) = 3/10 ure = 18 minute.

2018 JUNIOR RONDE TWEE OPLOSSINGS

1.	6	Deur slegs na die ene syfers te kyk het ons $8 \times 2 = 16$. Die laaste syfer is dus 6.
2.	2	$2^0 + 1^8 = 1 + 1 = 2$
3.	5	$100 \div 5 = 20$, daarom is $A = 5$.
4.	60	Deur hoeke om ’n punt en verwissellende hoeke tussen parallelle lyne te gebruik: $x = 360° - 300° = 60°$.
5.	16	Die ingekleurde oppervlakte is twee negendes van 72 cm^2. $72 \times \frac{2}{9} = 16$.
6.	125	Weens simmetrie is $A\hat{B}C = A\hat{D}C = \frac{360° - 70° - 40°}{2} = 125°$.
7.	18	Elke sykant het ’n buite-oppervlakte van 1 cm^2, dus het elke kubus ’n buite-oppervlakte van 6 cm^2. Voor daar gegom was het die kubusse ’n buite-oppervlakte van 24 cm^2 altesaam gehad. Aangesien daar 6 sykante in kontak met mekaar is, moet die buite-oppervlakte van die vaste liggaam 24 – 6 = 18 cm^2 wees.
8.	503	Let op dat 2018 ÷ 4 = 504,5. Aangesien hierdie die gemiddeld van die vier opeenvolgende heelgetalle is, moet hulle eweredig versprei wees vanaf 504,5. Die vier opeenvolgende heelgetalle is dus 503, 504, 505, 506. Die kleinste van hierdie is 503.
9.	256	Vir die uitdrukking om so groot as moontlik te wees moet Z duidelik 0 wees. Weens die eksponensiële uitdrukking moet X en Y so groot as moontlik wees, dus moet W duidelik 1 wees. Deur 2^8 en 8^2 te vergelyk is die grootste moontlike waarde $1 \times 2^8 - 0 = 256$.
10.	51	Die daaglikse koste van die twee pille is $R2121 \div 21 = R101$. Aangesien die groen pil R1 meer as die pink pil kos, kos die pille onderskeidelik R51 en R50.
11.	28	Die laaste syfer kan slegs 1, 3, 5 of 7 wees. Vir elkeen van hierdie is daar 7 moontlikhede vir die eerste syfer. Daar is dus $4 \times 7 = 28$ onewe getalle.
12.	23	Let op dat 120° + 80° > 180° en 120° + 65° > 180°. Aangesien 80° en 65° nie een in dieselfde driehoek as die 120° kan wees nie, moet die 80° en 65° hoeke in die driehoek sonder die 120° hoek wees. Daarom weet ons dat een van die driehoeke se hoeke 80°, 65° en 35° is. Die ander driehoek bevat dus die hoeke 120°, 37° en 23°. Die kleinste hoek in die lys is dus 23°.
13.	12	Driehoek ABC bevat elke radius presies twee keer. Die som van die deursnee van die sirkels is dus die omtrek van die driehoek en dus 3 + 4 + 5 = 12.
14.	63	Van die gegewe mates sal die lappieskombers 10 vierkante lank en 8 vierkante wyd wees. Daar sal dus $9 \times 7 = 63$ punte wees waar vier vierkante ontmoet.