SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2015 THIRD ROUND
JUNIOR SECTION: GRADES 8 AND 9

29 July 2015 **Time: 4 Hours** **Number of questions: 15**

TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions will be available at: www.samf.ac.za

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

Question 1

The diagram shows a 5 by 5 table.

The top row contains the letters M, A, T, H and S.

The fourth row contains the letters M, A and T as shown.

The remaining squares must be filled with the letters M, A, T, H and S such that such that no row, column or diagonal contains the same letter more than once.

M	A	T	H	S
	M	A	T	

Redraw and complete the table in your answer book.

[4]

Question 2

If the average of four different positive integers is 8, what is the largest possible value of any one of these integers?

[4]

Question 3

How many zeroes will there be in the answer if the following number is divided by 111?

11102222220033333333300044444444444440000

[4]

Question 4

Eskom announces that there is a 60% chance of Stage 1 load shedding for a specific week. In a specific suburb Stage 1 load shedding is from 10:00 to 12:30 on a Monday, Wednesday, Friday and Sunday. What is the probability that in that specific suburb there will be load shedding at a moment in the week, without knowing the day, or time or whether or not it is day or night?

[4]

Question 5

If we place a 3 at both ends of a number, its value is increased by 3372.

Find the original number.

[6]

Question 6

a) What is the side-length of the largest solid cube you can build, using at most 500 unit cubes?

b) By gluing some of the unit cubes together, you can build a cube that looks solid from any point on the outside, but is hollow on the inside.
What is the side-length of the largest such cube you can build, using at most 500 unit cubes?

[6]

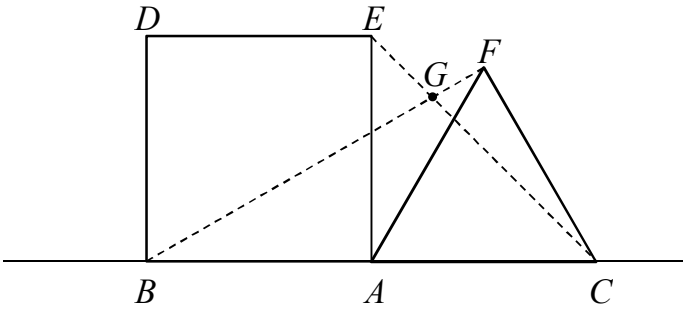
Question 7

Let A, B, and C be distinct points on a straight line with $AB = AC = 1$.

Square $ABDE$ and equilateral triangle ACF are drawn on the same side of line BC.

Lines EC and BF cut in G.

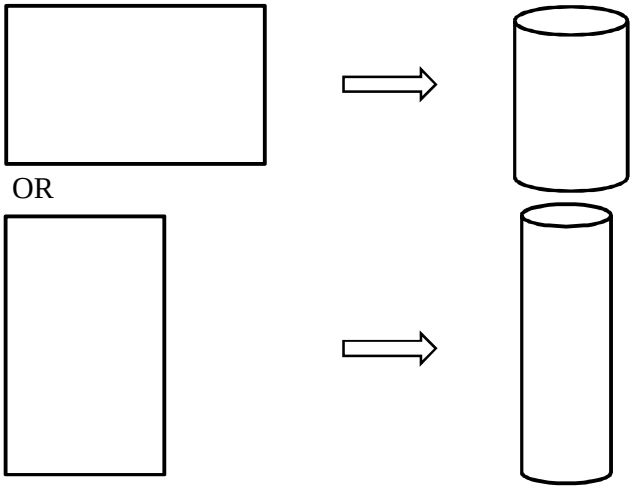
What is the size of $E\hat{G}B$?



[6]

Question 8

A rectangular sheet of paper can be used to form a cylinder by joining two opposite sides together:

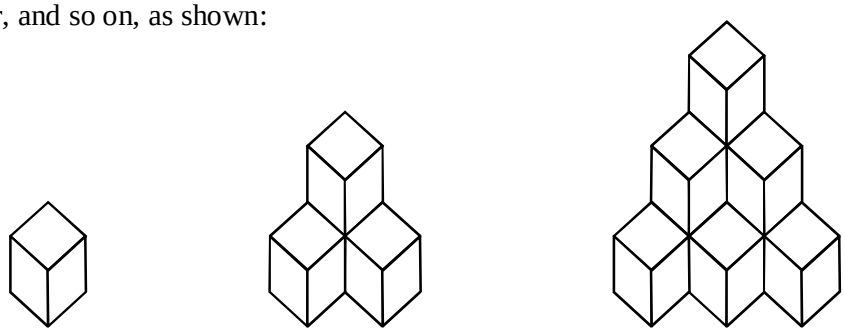


Should the short edges or the long edges be joined together to obtain the largest volume of the cylinder?

NB: Show all your working! [6]

Question 9

A stack of cubes are built such that there is 1 cube in the first layer, 3 in the second layer, 6 in the third layer, and so on, as shown:



Stack 1 Stack 2 Stack 3

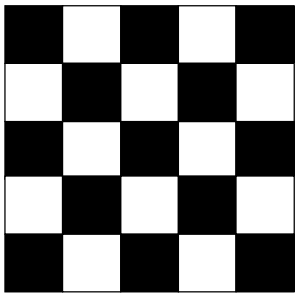
When these stacks are viewed as shown, we can see 9 edges in the first stack and 24 edges in the second stack.

a) How many edges are visible in the third stack?

b) How many edges would be visible in a stack of 20 layers? [8]

Question 10

Two players alternate placing 2×1 tiles, with no overlap, on a 5×5 chessboard until no player can place a tile. A number of 1×1 squares remain empty.



a) Prove that there must be an odd number of empty squares remaining.

b) Of the empty squares remaining, are more coloured black or white?

c) What is the maximum number of remaining squares?

[8]

Question 11

A *new sequence* is formed by deleting numbers from the sequence 1, 2, 3, 4, é , 500 such that the sum of any two numbers of the *new sequence* is never a multiple of seven.

What is the maximum length of the new sequence?

[8]

Question 12

Find three prime factors of: $5^{2015} + 5^{2016} + 5^{2017} + 31^{2016}$

NB: Show all your working!

[8]

Question 13

Prove that the alphanumeric below does not have a solution.

Note: Different letters represent different digits.

$$\begin{array}{r} \text{T W E N T Y} \\ +\,\text{T W E N T Y} \\ \hline \text{C R I C K E T} \end{array}$$

[8]

Question 14

Note: The floor function $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

E.g. $\lfloor 3,6 \rfloor = 3$; $\lfloor -3,5 \rfloor = -4$ and $\lfloor 3 \rfloor = 3$.

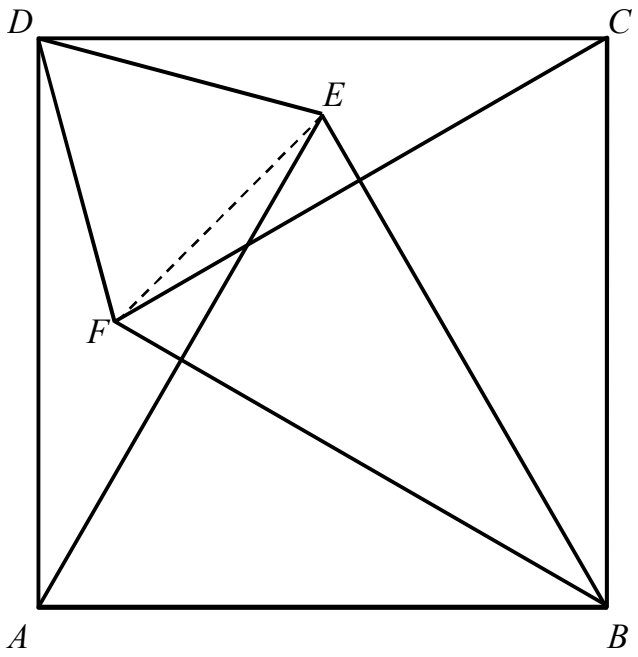
If $\left\lfloor \frac{7}{x} \right\rfloor + \left\lfloor \frac{x}{7} \right\rfloor = 2$, find all real solutions for x.

[10]

Question 15

ΔABE and ΔBCF are equilateral triangles and $ABCD$ is a square.

Prove that ΔDEF is an equilateral triangle.



[10]

Total: 100

THE END

NICE TO KNOW!

PLEASE NOTE: DO AT HOME – NOT FOR THIS PAPER'S PURPOSE!

See if you can figure out what (if anything) went wrong.

A farmer died leaving his 17 horses to his three sons.

When his sons opened up his last will and testament, it read:

> *My eldest son must get one half of my horses;*
>
> *My middle son must get one third of my horses;*
>
> *My youngest son must get one ninth of my horses.*

As it is impossible to divide 17 into 2, 3 or 9, the three sons started to fight with each other. So, they decided to go to a farmer friend who they considered quite smart, to see if he could work it out for them.

The farmer friend read the farmerôs last will and testament patiently and after given it due thought, he brought one of his own horses over and added it to the 17. That increased the total to 18 horses.

Now, he divided the horses according to their father's last will and testament.

$\frac{1}{2}$ *of 18 = 9. So he gave the eldest son 9 horses.*

$\frac{1}{3}$ *of 18 = 6. So he gave the middle son 6 horses.*

$\frac{1}{9}$ *of 18 = 2. So he gave the youngest son 2 horses.*

Here is the horses each son got:

Eldest son:	*9*
Middle son:	*6*
Youngest son:	*2*
TOTAL:	*17.*

Now this leaves one horse over, so the farmerôs friend took his own horse back to his farm.

Problem Solved! Or is it?