SOUTH AFRICAN MATHEMATICS OLYMPIAD

2015 FIRST ROUND

GRADE 8 SOLUTIONS

1. **E** $2 - (0 - (1 - 5)) = 2 - (0 - (-4)) = 2 - (0 + 4) = 2 - 4 = -2$

2. **C** 1 cm per month = 10 mm per month = 120 mm per year = 1 200 mm in ten years

3. **C** x must be a divisor of 12, and the possibilities are 1, 2, 3, 4, 6, 12, which is 6 in all.

4. **D** If the numerator has been multiplied by 6 and the fraction stays the same, the denominator is also multiplied by 6 and becomes 3 6 = 18.

5. **E** The last digit of 2011 × 2013 × 2015 is a five, and the last digit of 2010 × 2012 × 2014 is zero, so the last digit of the difference will be $5 - 0 = 5$.

6. **A** Every multiple of 7 represents a full week. Since today is Thursday, in one day's time it will be Friday. Thus each full week after today starts on a Friday and ends on a Thursday. 150 days divided by 7 equals 21 full weeks with a remainder of 3 days. The 147th day from now will thus be a Thursday (end of 21st full week), and consequently the 150th day from now will be a Sunday.

7. **A** Each fold doubles the number of layers that will be pierced. There will be 2^5 layers and therefore $2^5 = 32$ holes.

8. **D** The person on the extreme left can be any one of the four people that is not Alfred; the second left can be any one of the remaining three; the first person on the right of centre… and so on. The number of possibilities is 4 × 3 × 2 × 1 = 24.

9. **A** By definition of ●, we know 5 ● $x = 5x + 5 + x = 6x + 5$. If this is 35, then $6x = 30$, so $x = 5$

10. **C** Since BO = AO (radii) and AB = BO (given), we must have AB = AO = BO, i.e. Δ ABO equilateral, so that $A\hat{O}B = 60°$. Now $O\hat{B}C$ and $B\hat{C}O$ are equal and total the exterior $A\hat{O}B$; so $B\hat{C}O = 30°$

11. **A** Every cube is joined to an adjacent cube on two faces, leaving the other four exposed to paint.

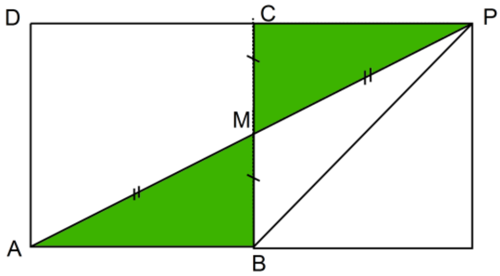
12. **C** There were 6 + 5 + 1 + 3 = 15 children altogether, of whom 3 were in the third-largest group. That group therefore requires $\frac{3}{15} \times 360° = 3 \times 24° = 72°$

13. **D** $a+b+c+d+2\times 30 = 360 \;\therefore\; a+b+c+d = 300$ so the average is $\frac{300}{4} = 75$

14. **D** The shaded area must be $\frac{5}{8} \times 80 = 50 \text{ cm}^2$, so the area of the square is $2 \times 50 = 100 \text{ cm}^2$, and then the side length of the square is $\sqrt{100} = 10$ cm.

15. **E** Let the tick be placed in any one of the 16 blocks. Then the cross can go in any of three other rows or three other columns, which gives 9 possible positions. That makes 16 × 9 = 144 ways.

16. **D** Joining P to C we see that Δ PCM is identical to Δ ABM. That means that P, C, B are vertices of a square, and the required angle is the one between a diagonal of a square and its side, i.e. 45°

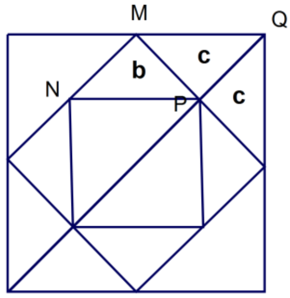


17. **C**

$$\frac{\text{area } \Delta ABP}{\text{area } \Delta ABCD} = \frac{1}{3} \text{ so } \frac{\frac{1}{2}BP.AB}{BC.AB} = \frac{1}{3} \text{ so } \frac{\frac{1}{2}BP}{BC} = \frac{1}{3} \text{ and therefore } \frac{BP}{BC} = \frac{2}{3}$$

and then BP : PC is $\frac{2}{3} : \frac{1}{3} = 2:1$

18. **B** Draw the diagonal of the squares that bisects area **a** with each half being **c**. Then clearly **b** = **c** as each is half the parallelogram MQPN. It follows that **a** = 2**b**.



19. **E** Between (and including) 98 and 200 there are 51 multiples of 2; between 98 and 199 there are 34 multiples of 3. Between 102 and 198 there are 17 multiples of 6. The number we seek is 51 + 34 – 17 = 68

20. **B** Since t toffees cost c cents, each toffee costs $\frac{c}{t}$ cents.

r rands equals 100 r cents.

The number of toffees that can be bought for 100 r cents is thus $100r \div \frac{c}{t} = \frac{100rt}{c}$