HARMONY SOUTH AFRICAN
MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION.
Sponsored by HARMONY GOLD MINING.

SECOND ROUND 2007
JUNIOR SECTION: GRADES 8 AND 9
15 MAY 2007
TIME: 120 MINUTES
## NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
3.1 Each correct answer is worth 4 marks in Part A, 5 marks in part B and 6 marks in part C.
3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil.
    Rough paper, a ruler and an eraser are permitted.
    Calculators and geometry instruments are not permitted.
5. Diagrams are not necessarily drawn to scale.
6. The centre page is an information and formula sheet. Please tear it out for your use.
7. Indicate your answers on the sheet provided.
8. Start when the invigilator tells you to do so.
    You have 120 minutes to complete the question paper.
9. Answers and solutions will be available at www.samf.ac.za/samo/

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.
DRAAI DIE BOEKIE OM VIR DIE AFRIKAANSE VRAESTEL
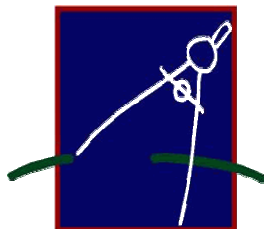
PART A: 4 MARKS EACH
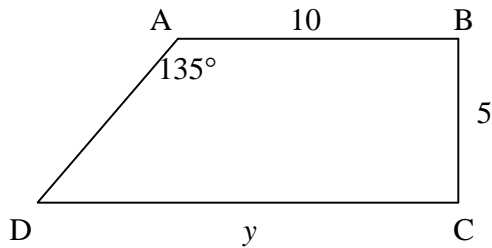
1. A two-digit number is divisible by 18; 30 and 45. The smallest such number is:

    A) 60   B) 90   C) 120   D) 150   E) 180

2. If the 29th day of a month falls on a Thursday, what is the date of the second Saturday in that month?

    A) 9   B) 10   C) 11   D) 12   E) 13

3.

    The length of y in trapezium ABCD is given by:

    A) 15   B) 20   C) 25   D) 30   E) 35

4. Sipho is three times as old as Pam, Pam's age can be found by adding the digits of Sipho's age. The sum of their ages is

    (A) 48   (B) 56   (C) 32   (D) 36   (E) 60

5. An angle x is 30° more than half of its complement. What is the supplement of x?

    A) 70°   B) 80°   C) 135°   D) 160°   E) 130°

PART B: 5 MARKS EACH

6. x, y and z are three integers such that $\sqrt{xy} = 2$ and $\sqrt[3]{xyz} = 6$. The value of z is

    (A) 3   (B) 54   (C) 32   (D) 4   (E) 60

7. If the sum of a two-digit number is subtracted from the number, then the answer is 45. How many two digit numbers have this property?

    (A) 10   (B) 15   (C) 3   (D) 4   (E) 18

8.   A collection of 25 coins, whose total value is R2.75, is composed of 5c, 10c, and 20c coins. If the 5c coins were 10c coins, the 10c coins were 20c coins, and the 20c coins were 5c coins, the total would be R3.75. How many 20c coins are there in the collection?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

9.   Study the pattern below and find the value of n if n is a natural number:
$6^2 + 8^2 = 10^2$
$8^2 + 15^2 = 17^2$
$10^2 + 24^2 = 26^2$
$12^2 + 35^2 = 37^2$
.
.
.
$18^2 + n^2 = x^2$

A) 65   B) 70   C) 75   D) 80   E) 85

10.   Steve was at a party and knew that three guests were born on the same day of the week <u>and</u> in the same month of the year. He also knew that all the guests were born in the first six months of the year. What is the least number of people (including Steve) that could have been at the party?

(A) 70   (B) 75   (C) 85   (D) 168   (E) 196

11.   Maggie rolls an ordinary six-sided die repeatedly, keeping track of each number as she rolls, and stopping as soon as any number is rolled for the fifth time. If Maggie stops after her ninth roll, and the sum of these numbers she has rolled is 20, then how many combinations of numbers could she have rolled?

(A) 10   (B) 12   (C) 14   (D) 15   (E) 16

12.   Observe the following computations:
$2 + 4 = 6$
$2 + 4 + 6 = 12$
$2 + 4 + 6 + 8 = 20$
$2 + 4 + 6 + 8 + 10 = 30$

The value of:
$2 + 4 + 6 + 8 + \ldots\ldots + 30$ is given by:

A) 230   B) 240   C) 250   D) 260   E) 270

13. The lengths of three consecutive sides of a quadrilateral are equal. If the angles included between these sides have measures of 60° and 100° , then what is the measure of the largest angle of the quadrilateral?

(A) 130°   (B) 145°   (C) 155°   (D) 160°   (E) 165°

14. A *squared rectangle* is a rectangle whose interior can be divided into two or more squares. An example of a squared rectangle is shown. The number written inside a square is the <u>length</u> of a side of that square. Determine the area of the squared rectangle shown.
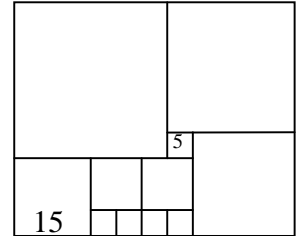
(A) 1024   (B) 1056   (C) 2005   (D) 2120   (E) 2475

15. A giant watermelon weighs 50 kg of which 98% is water. After lying in the sun some of the water evaporates so that the water now makes up 96% of the weight. The new weight of the watermelon in kg is:

(A) 30   (B) 44   (C) 32   (D) 22   (E) 25

PART C: 6 MARKS EACH

16. *P* is the remainder when each of the numbers 478, 392, and 263 is divided by *N*, where *N* is an integer greater than 1. The value of *N* – *P* is:

(A) 5   (B) 28   (C) 33   (D) 38   (E) 43

17. Did you know $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$?

If the numbers below are arranged from smallest to largest, then the number in the middle is

(A) $3 + \sqrt{3} + \sqrt{10}$ (B) $5 + \sqrt{12}$ (C) $\sqrt{78}$
(D) $3 + \sqrt{27}$ (E) $\sqrt{48} + \sqrt{3}$

18. 80 switches in a row operate 80 lights. The *'on'* and *'off'* switches operate automatically, in the following sequence:
- All lights are switched *'on'*
- Every second light is switched *'off'*
- Every third light is switched either *'on'* or *'off'*
- Every fourth light is switched either *'on'* or *'off'*

And so the pattern goes on until every 80th light is switched either *'on'* or *'off'*

How many lights will be *'off'* after the last operation?

A) 60   B) 64   C) 72   D) 78   E) 80

19.

The above diagram from A to D represents a flat passageway at an international airport. Each of the eight sections is 80m long. The shaded sections represent a conveyor system, on which a person stands if she/he wants to be moved from, for example B to C (80 metres). When the conveyor system is operational it takes Jessie 8 minutes to completely cover the passageway. When the conveyor system is not working it takes Jessie, by walking at the same average speed, 10 minutes to cover the distance. Approximately how many minutes will it take Jessie to move from A to D if the conveyor system is working and in addition, she walks on the conveyor system maintaining the same constant speed?

A) 4.5   B) 5   C) 5.5   D) 6   E) 7

20. Consider the following addition pattern:

```
        1
     +  1 1
     +  1 1 1
     +  1 1 1 1
   + ………………
       .
       .
   + 1 1 1 1…………..1 1      row  63
   _____
   …………………………………….(Answer)
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In the 63rd row there are 63 '1s'. The sum of the digits in the answer is:

A) 1953   B) 2016   C) 136   D) 261   E) 276

Formula and Information Sheet

1.1	The natural numbers are 1; 2; 3; 4; 5; …
1.2	The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; …
1.3	The integers are …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\dfrac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$$2\times 2\times 2\times 2\times 2 = 2^5$$
$$3\times 3\times 3\times 3\times 3\times 3 = 3^6$$
$$a\times a\times a\times a\times \ldots \times a = a^n \qquad (n \text{ factors of } a)$$
(a is the base and n is the index (exponent))

3.2 Factorial notation:

$$1\times 2\times 3\times 4 = 4!$$
$$1\times 2\times 3\times \ldots \times n = n!$$

4. Area of a

4.1	triangle is:	$\dfrac{1}{2}\times (\text{base} \times \text{height}) = \dfrac{1}{2}(b.h)$
4.2	rectangle is:	length $\times$ width $= lw$ length $\times$ breadth $= lb$
4.3	square is:	side $\times$ side $= s^2$
4.4	rhombus is:	$\dfrac{1}{2}\times (\text{product of diagonals})$
4.5	trapezium is:	$\dfrac{1}{2}\times (\text{sum of parallel sides}) \times \text{height}$
4.6	circle is:	$\pi r^2 \ (r = \text{radius})$

5. Surface area of a:

5.1	rectangular prism is:	$2lb + 2lh + 2bh \ (h = \text{height})$
5.2	sphere is:	$4\pi r^2$

6.	Perimeter of a:	

6.1	rectangle is:	$2 \times \text{length} + 2 \times \text{breadth}$
		$2l + 2b$
		or $2l + 2w$ ($w = $ width)

6.2	square is:	$4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1	cube is:	$s \times s \times s = s^3$

8.2	rectangular prism is:	$l \times b \times h$

8.3	cylinder is:	$\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section $\times$ perpendicular height

or   area of base $\times$ perpendicular height

9.2 Surface area of a right prism is: (perimeter of base $\times$ h) + (2 $\times$ area of base)

10. Sum of the interior angles of a polygon is: $180°(n-2)$ [$n = $ number of sides]

11.

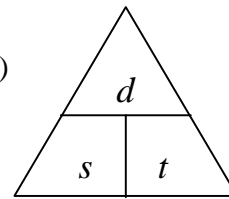Distance $=$ speed $\times$ time $(d = s \times t)$

Speed $=$ distance $\div$ time $(s = \dfrac{d}{t})$

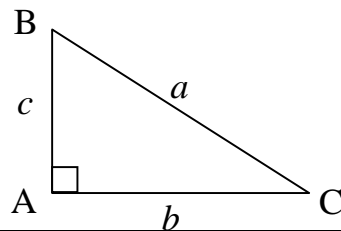Time $=$ distance $\div$ speed $(t = \dfrac{d}{s})$

$d = s \times t$

$s = \dfrac{d}{t}$

$t = \dfrac{d}{s}$

12.

Pythagoras:

If $\triangle ABC$ is a right-angled triangle, then $a^2 = b^2 + c^2$

13. Conversions:

$1 \text{ cm}^3 = 1 \text{ m}\ell$; $1000 \text{ cm}^3 = 1 \ \ell$

$1000 \text{ m} = 1 \text{ km}$; $1000 \text{ g} = 1 \text{ kg}$; $100 \text{ cm} = 1 \text{ m}$