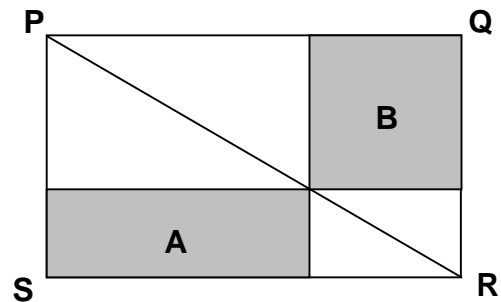


The South African Mathematics Olympiad
Third Round 2004
Junior Section: Grades 8 and 9

Question 1

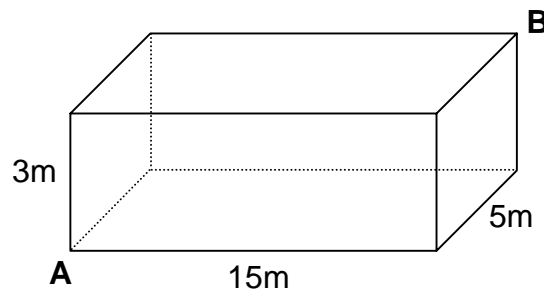
PQRS is a rectangle.
Determine which, if any, of the shaded rectangles A and B has the larger area.



(6 Marks)

Question 2

The diagram shows a long room.
An ant wants to walk from A to B.
It can walk along the walls and ceiling of the room.
What is the shortest distance it could walk?



(6 Marks)

Question 3

Fred added up all the positive integers from 1 to p on his calculator and obtained a total of 2004. By mistake, however, he had entered one number twice. Find the correct total that Fred should have obtained and the number that he added twice.

(6 Marks)

Question 4

All six faces of a cube of side length n are painted. The cube is then cut up into n^3 cubes of the same size. Among these small cubes, there are some that do not have paint on any of their faces, some have one, two or three faces painted.
For what value of n is the number of small cubes that do not have any paint on them equal to the number of small cubes that have exactly one painted face?

(6 Marks)

Question 5

In a class of 30 learners, five take Maths, Geography and Biology. Nine take Maths and Biology. Twenty learners take Geography, of whom twelve also take Maths. Eighteen take Biology. Every learner takes at least one subject, and no one takes only Maths. How many learners take only Biology?

(6 Marks)

Question 6

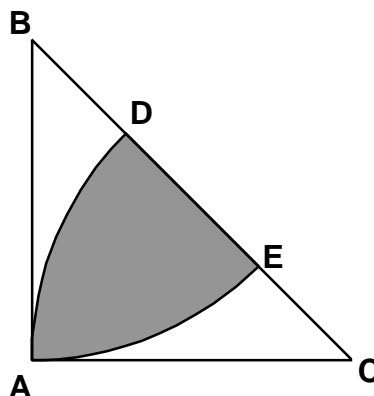
In the addition sum below, different letters stand for different digits. (For example, if T is 3, then S cannot be 3.) Find what each letter stands for.

$$\begin{array}{r} \text{W H O} \\ \text{I S} \\ \text{T H I S} + \\ \hline \text{I D I O T} \end{array}$$

(6 Marks)

Question 7

ABC is an isosceles right-angled triangle with $AC = AB = 2$. A circular arc of radius 2 with centre C meets the hypotenuse at D, and a circular arc of radius 2 with centre B meets the hypotenuse at E. Find the area of the shaded region ADE in terms of π .



(6 Marks)

Question 8

Two cars race around a circular track at constant speeds starting at the same point. If they travel in opposite directions, then they meet every 30 seconds. If they travel in the same direction, then they meet every two minutes. If the track is 1800m long, what is the speed of each car?

(6 Marks)

Question 9

Sakhile, Danny, Nelly and Pravin are manager, coach, captain and goalkeeper for a social hockey team, but not necessarily in that order.

- 1) Sakhile is the brother of the manager.
- 2) The coach and the manager are not related.
- 3) Nelly is an only child.
- 4) The captain is older than Danny, but is in the same class.
- 5) Pravin is in the same class as Nelly.
- 6) The goalkeeper is not in the same class as Pravin or Danny.
- 7) Danny is Nelly's cousin.
- 8) Pravin was disappointed not to have got the position of coach.

Determine which person has which position.

(6 Marks)

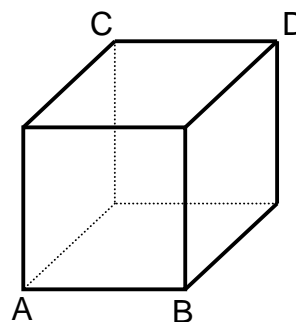
Question 10

All possible four-digit numbers are formed using the digits 1, 2 and 4, for example 2441 and 1112. How many of these numbers are divisible by three?

(6 Marks)

Question 11

- (a) Show how to place the numbers 1 to 8 at the vertices of a cube so that the sum of the four numbers at the corners of each face is the same.
- (b) Explain why the two numbers at A and B and the two numbers at C and D must always have the same sum.



(8 Marks)

Question 12

We write out all the integers from 1 to 30, and cross out some of these so that in the remaining list no number is equal to twice any other. Find the maximum number of integers that can appear in this remaining list and justify why this is the maximum.

(8 Marks)

Question 13

Here is an example of five consecutive positive integers whose sum is 1000:

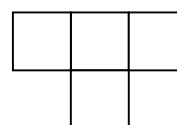
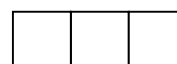
$$198 + 199 + 200 + 201 + 202 = 1000.$$

Find the largest number of consecutive positive integers whose sum is exactly 1000 and justify why you think this must be the largest number.

(8 Marks)

Question 14

- (a) Is it possible to cover a 7 by 7 square with non-overlapping 3 by 1 rectangles? Explain your answer.
- (b) Is it possible to cover a 10 by 10 square with non-overlapping T-shaped pieces of the kind shown on the right? Explain.



(8 Marks)

Question 15

X, Y and Z are three different digits from 1 to 9 forming the number XYZ. For example, the digits 2, 6 and 4 would form the number 264.

Find the smallest value of $\frac{XYZ}{X + Y + Z}$ and explain why the value you have found is in fact the smallest.

(8 Marks)