2020 JUNIOR ROUND TWO SOLUTIONS

1. **10** $\sqrt{20\times 8-20\times 3}=\sqrt{20\times 5}=\sqrt{100}=10$

2. **5** The next year that is a perfect square is 2025 (45^2) which occurs in 5 years' time.

3. **20** $\frac{1}{8}\times 160=20$

4. **100** $20°\times 4+x=180°\ \therefore\ x=100°$

5. **2** 20% of 45 is 9, and 9 – 7 = 2

6. **125** $K\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=1\ \therefore\ K=125$

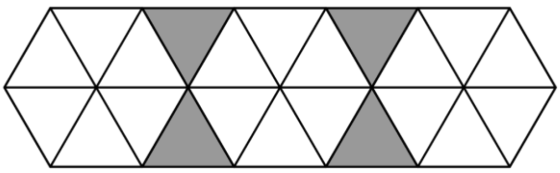
7. **300** 50 revolutions per minute is equivalent to five sixths of a revolution per second. We thus have $\frac{5}{6}\times 360°=300°$.

8. **8** The area of quadrilateral ADQD' is twice the area of triangle ADQ. Thus, Area = $2\times\left(\frac{1}{2}\times 4\times 2\right)=8$.

9. **12** $(2\times 3)\times 2=12$. The possible number plates are: AB1, AB3, AC1, AC3, AD1, AD3, BA1, BA3, CA1, CA3, DA1, DA3.

10. **816** Since the first two digits must form a perfect square, the first two digits can only be 16, 25, 36, 49, 64 or 81. The same logic applies to the last two digits. The only 3-digit numbers with this property are thus 164, 364, 649 and 816, the largest of which is 816.

11. **32** The diagram can be subdivided into 22 identical equilateral triangles. Since the total area is 176 cm^2, each of the 22 triangles has area 8 cm^2. The shaded area is thus 8 cm^2 × 4 = 32 cm^2.



12. **100** If we let the dimensions of the rectangle be x cm and y cm, then from Busi we have $2x+y=73$ and from Caleb we have $x+2y=77$.
Thus $3x+3y=150$ from which it follows that $2x+2y=100$.

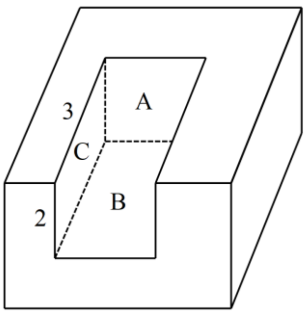
13. **60** Let the two unknown sides be x and $3x$. From the Pythagorean relationship we thus have $x^2+(3x)^2=20^2$, thus $10x^2=400$, from which we have $x^2=40$.
But the area of the triangle is $\frac{1}{2}(x)(3x)=\frac{3}{2}x^2=\frac{3}{2}\times 40=60$.

14. **45** Since ΔDEC is equilateral, and $ABCD$ is a square, we have $E\hat{C}F = 30°$.
Now, since ΔECB is isosceles $(CE = CB)$, and it is given that ΔBEF is isosceles, it follows that $B\hat{F}E = \frac{180°-30°}{2} = 75°$ and $E\hat{F}C = 105°$.
Thus $C\hat{E}F = 180° - 30° - 105° = 45°$.

15. **15** Let us consider 'inserting' the 5s into 2020. There are five possible places where the first 5 could be inserted: _2_0_2_0_. Once the first insertion has taken place, and we have a 5-digit number, there are six possible points of insertion for the second 5. There are thus 30 possible ways to insert the two 5s (5x6). However, since the two inserted 5s are identical, we cannot distinguish them by the order of their insertion. We have thus counted every possible arrangement twice, meaning there are only 15 different 6-digit numbers that could have typed.

16. **24** Let L, M and S represent the volume held by the each large, medium and small bucket respectively. We thus have $3L = 4M$ and $5M = 6S$. Scaling the first equation by a factor of 5, and the second equation by a factor of 4, gives $15L = 20M$ and $20M = 24S$. Thus $15L = 24S$.

17. **108** The areas of the back, bottom and side faces will be the same as in the original cube. The area lost from the front face of the cube is identical to the area gained by face A. The area lost from the top surface of the cube is identical to that gained by face B. The area of face C is 6 cm^2, and directly opposite it is an identical face. The surface area of the new structure will be the same as the original cube with the addition of two faces each of 6 cm^2. The new area is thus 96 + 6 + 6 = 108 cm^2.



18. **4** Note that $96{,}8\% = \frac{968}{1000} = \frac{121}{125}$ in simplest form. 125 – 121 = 4.

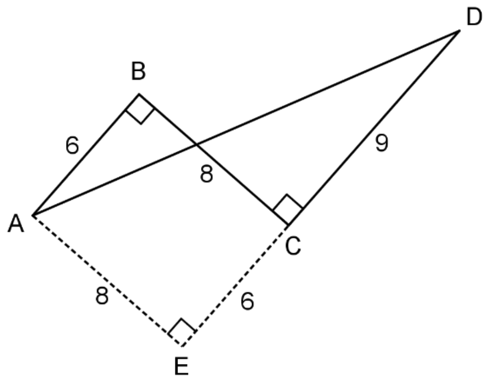
19. **198** Angles $A\hat{B}C$ and $A\hat{C}B$ are both equal to 180° – x so triangle ABC is an isosceles triangle. If angle $B\hat{A}C = y$ and angle $A\hat{B}C = 48°$, then y = 180° – (2x48°) = 84°.
If angle $B\hat{A}C = 48°$ and angle $A\hat{B}C = y$, then y = (180° – 48°)/2 = 66°.
If angle $A\hat{B}C = 48°$, and angle $A\hat{C}B = y$, then y = 48°.
The sum of the 3 possible values of y is 84° + 66° + 48° = 198°.

20. **24** Let x represent the number of pupils in the class originally. The number of boys is thus $\frac{2}{3}x$. After the changes, the number of pupils in the class is $x+1$, and the number of boys in the class is $\frac{2}{3}x-1$. But we can also represent the number of boys now in the class as $\frac{3}{5}(x+1)$.

Solving the equation $\frac{2}{3}x-1=\frac{3}{5}(x+1)$ gives $x=24$.

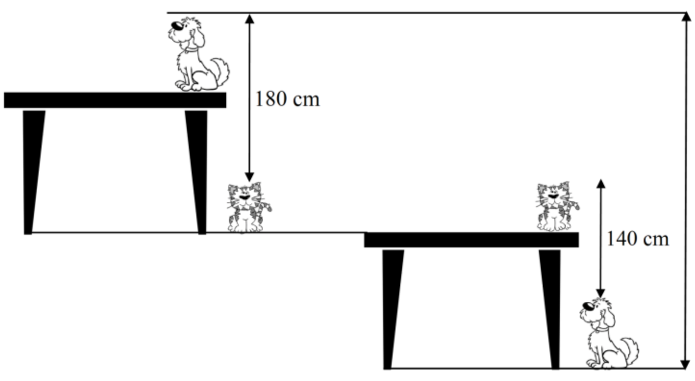
Alternatively, we need the first multiples of 3 (2+1) and 5 (3+2) such that the multiple of 5 is 1 bigger than the multiple of 3, i.e. 24 and 35, thus the original class size is 24.

21. **17** If we translate AB to EC and BC to AE, then we have right-angled triangle AED with AE = 8 cm and ED = 15 cm. The hypotenuse AD is thus $\sqrt{8^2+15^2}=17$.

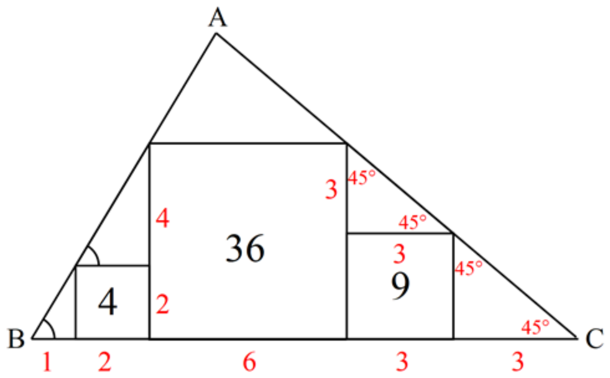


22. **136** For every 250 m Vlad runs he gains 25 m on Hagrid. Thus after 1 km Hagrid has only run 900 m. For every 500 m Hagrid runs he gains 20 m on Sanjev. Thus, after Hagrid has run 900 m, Sanjev is $\frac{9}{5}\times 20=36$ m behind him. The distance between Vlad and Sanjeev is thus 136 m.

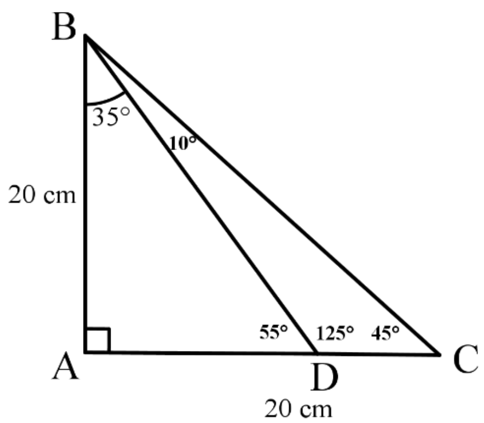
23. **160** Letting the heights of the Table, Cat and Dog be T, C and D respectively, then we have $T+D-C=180$ and $T+C-D=140$. Adding the two equations we get $2T=320$ cm, thus $T=160$ cm. An equivalent result can be arrived at visually:



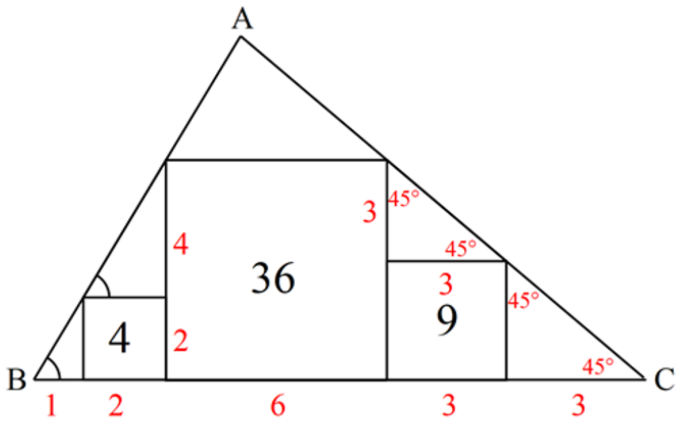
24. **15** From the areas of the squares, as well as the two pairs of similar triangles, we can work out the various side lengths as shown below:



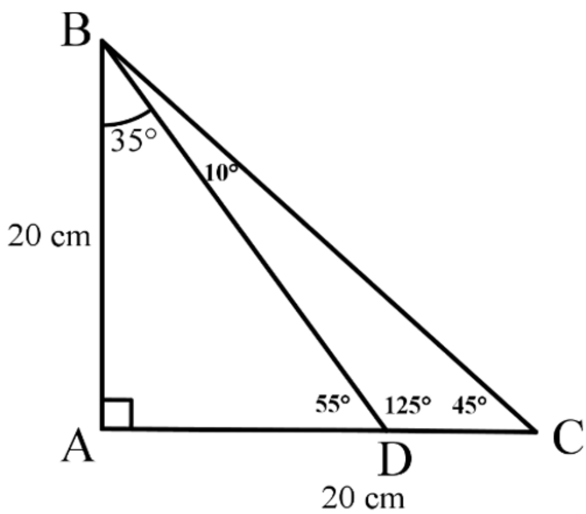
25. **200** Rotate triangle BCD as shown so that the 125° and 55° angles are adjacent. ADC is a straight line (55° + 125° = 180°), and both ∠ABC and ∠ACB are 45°. Quadrilateral ABCD has now been transformed into right-angled isosceles triangle ABC. The area is thus $\frac{1}{2} \times 20 \times 20$, i.e. 200 cm^2.



24. **15** Vanaf die areas van die vierkante, asook die twee pare gelykvormige driehoeke, kan ons die onderskeie sy-lengtes, soos aangedui op die onderstaande diagram, bereken.



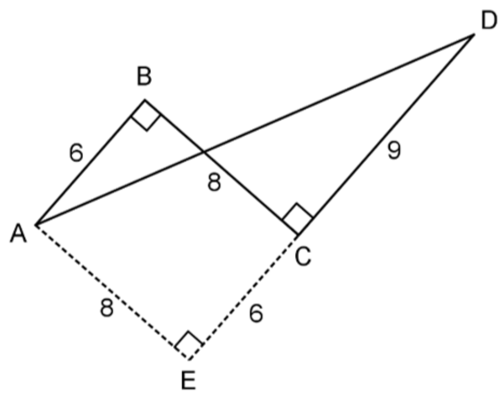
25. **200** Roteer driehoek BCD soos aangedui, sodat die 125° en 55° hoeke langs mekaar is. ADC is ‘n reguitlyn (55° + 125° = 180°), en beide $\angle ABC$ en $\angle ACB$ is 45°. Vierhoek ABCD is nou getransformeer in ‘n reghoekige gelykbenige driehoek ABC. Die area is dus $\frac{1}{2} \times 20 \times 20$, i.e. 200 cm^2.



20. **24** Laat x die oorspronklike aantal kinders in die klas wees. Die aantal seuns is dan $\frac{2}{3}x$. Na die veranderinge is die aantal kinders in die klas $x + 1$, en die aantal seuns in die klas dan $\frac{2}{3}x - 1$. Maar ons kan ook die aantal seuns in die klas voorstel as $\frac{3}{5}(x + 1)$. Dus as ons die volgende vergelyking $\frac{2}{3}x - 1 = \frac{3}{5}(x + 1)$ oplos, is $x = 24$.

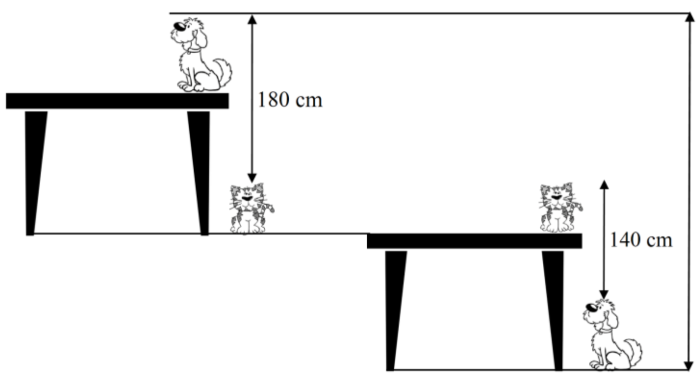
‘n Alternatiewe metode sal wees om die eerste veelvoud van 3 (2+1) en 5 (3+2) te bepaal sodat die veelvoud van 5, een groter as die veelvoud van 3 is. i.e. 24 en 35, dus die oorspronklike klasgrootte is 24.

21. **17** As ons lynstuk AE ewewydig aan BC teken om DC verleng in E te sny, het ons ‘n reghoekige driehoek AED met AE = 8 cm en ED = 15 cm. Die skuinssy AD is dus $\sqrt{8^2 + 15^2} = 17$.



22. **136** Vir elke 250 m wat Vlad hardloop is hy 25 m verder voor Hagrid. Dus na 1 km het Hagrid nog net 900 m gehardloop. Vir elke 500 m wat Hagrid hardloop is hy 20 m verder voor Sanjev. Dus na 900 m is Sanjev $\frac{9}{5} \times 20 = 36$ m agter Hagrid. Dus die afstand tussen Vlad en Sanjev is 136 m.

23. **160** Maak die hoogtes van die tafel, kat en hond onderskeidelik: T, C en D.
Dan is $T + D - C = 180$ en $T + C - D = 140$.
As ons die twee vergelykings bymekaar tel kry ons $2T = 320$ cm,
dus $T = 160$ cm.
‘n Alternatiewe oplossing kan visueel bereken word:

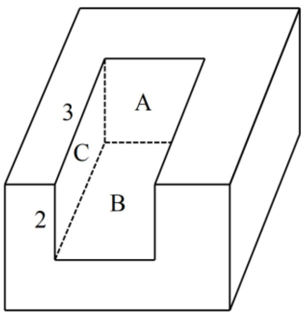


14. **45** Aangesien ΔDEC ‘n gelyksydige driehoek is, en $ABCD$ ‘n vierkant, kan ons bereken dat $E\hat{C}F = 30°$. Aangesien ΔECB ‘n gelykbenige driehoek is $(CE = CB)$, en dit ook gegee is dat ΔBEF ‘n gelykbenige driehoek is, volg dit dat $B\hat{F}E = \frac{180°-30°}{2} = 75°$ en $E\hat{F}C = 105°$.
Dus $C\hat{E}F = 180° - 30° - 105° = 45°$.

15. **15** Kom ons plaas die 5’e in die getal 2020. Daar is 5 moontlike plekke waar die twee 5’e geplaas kan word: _2_0_2_0_. Sodra die eerste 5 geplaas is en ons dan ‘n 5-syfer getal het, sal daar 6 moontlike plekke wees om die tweede 5 te plaas. Daar is dus 30 verskillende kombinasies om die twee 5’e te plaas (5x6). Maar aangesien die twee 5’e identies is, sal daar dubbeld soveel van elke kombinasie wees. Dus is daar slegs 15 verskillende 6-syfer getalle wat in die sakrekenaar ingetik kan word.

16. **24** Laat L, M en S die volumes van die groot, medium and klein emmers onderskeidelik wees. Ons het dus $3L = 4M$ en $5M = 6S$. As ons die eerste vergelyking met 5 vermenigvuldig, kry ons $15L = 20M$ en die tweede vergelyking vermenigvuldig met 4 kry ons $20M = 24S$. Dus $15L = 24S$.

17. **108** Die areas van die agterste-, voorste- en kantsye sal almal dieselfde as die oorspronklik figuur wees. Die area wat verlore gaan op die voorste sy van die prisma, is presies dieselfde as area A. Die area verlore gaan op die boonste sy is presies dieselfde as area B. Die area van sy C is 6 cm^2, en reg oorkant is daar nog ‘n sy met dieselfde grootte. Die oppervlakte van die nuwe figuur sal presies dieselfde as die oorspronklike prisma, maar met twee sye van 6 cm^2 ekstra, wees. Dus is die nuwe area: 96 + 6 + 6 = 108 cm^2.



18. **4** In eenvoudigste vorm is $96{,}8\% = \frac{968}{1000} = \frac{121}{125}$. Dus 125 – 121 = 4.

19. **198** Hoeke $A\hat{B}C$ en $A\hat{C}B$ is beide $180° - x$, dus driehoek ABC is ‘n gelykbenige driehoek. As hoek $B\hat{A}C = y$ en hoek $A\hat{B}C = 48°$, dan is $y = 180° - (2x48°) = 84°$.
As hoek $B\hat{A}C = 48°$ en hoek $A\hat{B}C = y$, dan is $y = (180° - 48°)/2 = 66°$.
As hoek $A\hat{B}C = 48°$, en hoek $A\hat{C}B = y$, dan is $y = 48°$.
Die som van die 3 moontlike waardes van y is 84° + 66° + 48° = 198°.

2020 JUNIOR RONDE TWEE OPLOSSINGS

1. **10** $\sqrt{20\times8-20\times3}=\sqrt{20\times5}=\sqrt{100}=10$

2. **5** Die eersvolgende jaar wat 'n vierkantsgetal sal wees is 2025 (45^2) wat oor 5 jaar sal wees.

3. **20** $\frac{1}{8}\times160=20$

4. **100** $20°\times4+x=180°\ \therefore\ x=100°$

5. **2** 20% van 45 is 9, en $9-7=2$

6. **125** $K\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=1\ \therefore\ K=125$

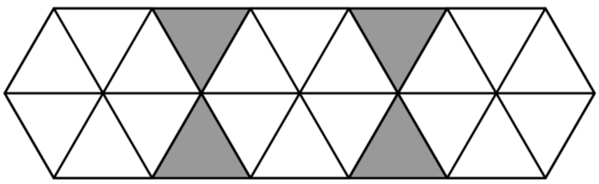
7. **300** 50 revolusies per minuut is gelyk aan vyf sesdes van 'n revolusie per sekonde. Ons het dus $\frac{5}{6}\times360°=300°$.

8. **8** Die area van die vierhoek ADQD' is twee keer die area van driehoek ADQ. Dus Area $= 2\times\left(\frac{1}{2}\times4\times2\right)=8$.

9. **12** $(2\times3)\times2=12$. Die moontlike nommerplate is: AB1, AB3, AC1, AC3, AD1, AD3, BA1, BA3, CA1, CA3, DA1, DA3.

10. **816** Die eerste twee syfers kan net 16, 25, 36, 49, 64 of 81 wees. Dieselfde geld vir die twee syfers aan die einde. Die enigste 3-syfer getal met hierdie kenmerk is dus 164, 364, 649 and 816, die grootste getal van hierdie sal dus 816 wees.

11. **32** Die diagram kan gedeel word in 22 identiese gelyksydige driehoeke. Die totale area is176 cm^2, dus het elke driehoek 'n area van 8 cm^2. Die 4 gekleurde driehoeke het dus 'n area van 8 $cm^2\times4=32$ cm^2.



12. **100** As die afmetings van die reghoek x cm en y cm is, weet ons van Busi dat $2x+y=73$ en van Caleb dat $x+2y=77$. Dus is $3x+3y=150$ waaruit volg dat $2x+2y=100$.

13. **60** Laat die twee onbekende sye x en $3x$ wees. Uit die Pythagoras verhouding volg dat $x^2+(3x)^2=20^2$, dus dat $10x^2=400$, waaruit ons die volgende kan aflei: $x^2=40$. Dus die area van die driehoek is $\frac{1}{2}(x)(3x)=\frac{3}{2}x^2=\frac{3}{2}\times40=60$.