HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

SAMF

SOUTH AFRICAN MATHEMATICS FOUNDATION

HARMONY™

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

THIRD ROUND 2011
JUNIOR SECTION: GRADES 8 AND 9

7 SEPTEMBER 2011
TIME: 4 HOURS
NUMBER OF QUESTIONS: 15
### TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- For Question 4 and Question 12 you need two Pula and two rulers that will be provided.
- There is a Working Sheet at the end of the paper to help you to answer Question 4 and 12 which you don't need to hand in unless you have done work on it that needs to be marked.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions are available at: www.samf.ac.za

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.
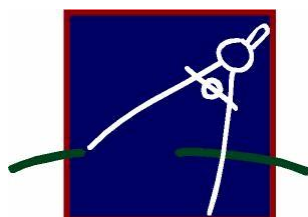
Question 1

One millionth of a second is called a micro second. Roughly how long is a micro century?
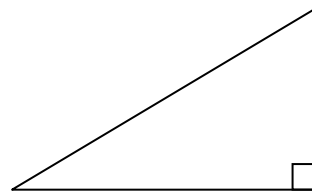
[4]

Question 2

N is a positive integer such that $N^2 - 200$ is a perfect square.
How many possible values are there for N?
Explain how you got to your answer.

[4]

Question 3

Prove that there is only one right angled triangle
whose sides are consecutive integers.

[6]

Question 4   (You may use the Working Sheet to help you to answer this question)

A Botswana 1 Pula coin is not circular, but nevertheless has a
constant diameter. Two Pula coins and 2 rulers are given to you.
Place the Pula coins on the space provided with the rulers besides
them (see worksheet). Move the rulers back and forth so that the
Pula coins roll between them. Wow – no bumps?!!

If this constant diameter of the coin is Q, what is the perimeter?

[6]

Question 5

The diagram shows a rectangular piece of paper
with a circular hole cut through it.

   (a) Explain how to cut the shaded area exactly
        in half with a single straight cut.

   (b) Is this always possible for any location of
        any circle that fits into the rectangle?
        Explain.
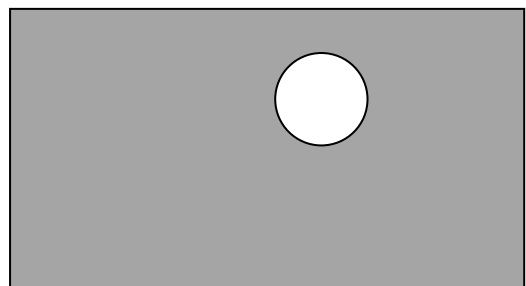
[6]

Question 6

Potatoes are made up of 99% water and of 1% solid "potato matter." Vladimir bought 100 kg
of potatoes and left them outside in the sun for a while. When he returned, he discovered that
the potatoes had dehydrated and were now only made up of 98% water. How much did the
potatoes now weigh?   [6]

Question 7

ABCDE × 4 = EDCBA. Find A, B, C, D, and E where each is a unique integer from 1 to 9.
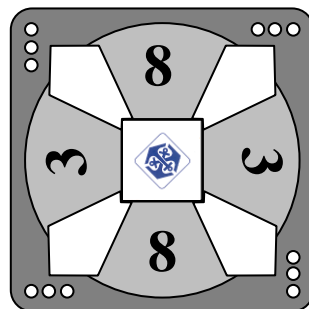
[6]

Question 8

Some years ago the Old Mutual Math 24 game was very popular in South Africa. You are given a card with 4 numbers on it and you have to get the number 24 by using the 4 numbers. All the numbers have to be used – but each only once. You can use any of the operations: addition, subtraction, multiplication or division.

(a) Make 24 from 2; 6; 8; 4.
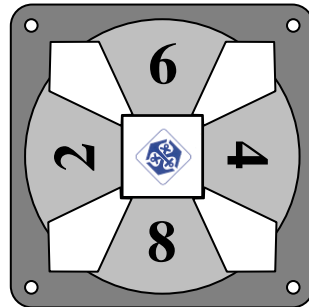
(b) Make 24 from 8; 8; 3; 3.

[6]

Question 9

A car travels downhill at 72 km/h. On level ground it travels at 63 km/h, and uphill at only 56 km/h. The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes. Find the distance between the two towns.   [8]

Question 10

These were scenes from the first democratic election in South Africa.

A number of men and women are standing in single file in a row to enter the tent to cast their vote. Saskia arrives a little late, and wants to join the queue.
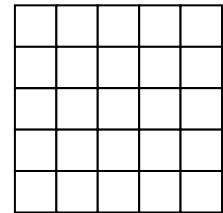
Prove that Saskia can always join a queue of any length in such a way that the number of men in front of her is the same as the number of women behind her.   [8]
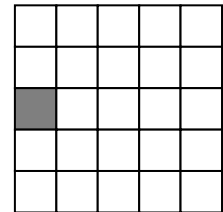
2

Question 11

A 5-by-5 square consists of 25 1-by-1 small squares.

(a) Is it possible to tile this square with the non-overlapping $\llcorner$ - shapes shown in the figure?

(b) If the shaded square is removed, is it possible to cover the rest of the square using 8 of the $\llcorner$ - shapes shown above?
(If it is possible, draw a solution. If it is not possible, prove it)

(c) If one corner square is removed, prove that it is not possible to cover the rest of the squares by eight 3-by-1 rectangles as shown in the figure.
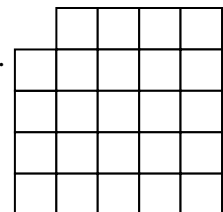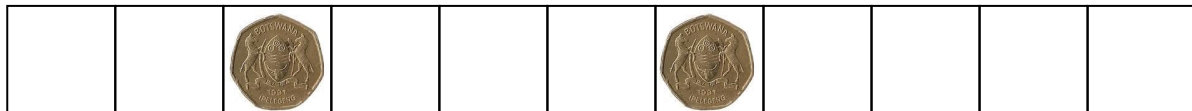
[8]

Question 12　　　(You may use the Working Sheet to help you to answer this question)

A row of blocks is provided in your worksheet. Place the 2 Pula coins you have been given on **any two squares** on the grid. For example:

The **Moving coins game** is an interesting game in which players take it in turns to move *any one of the two* coins *any number of blocks* to the right. You are not permitted to move back, or to jump over another coin. The first player who cannot move loses.

Play the game a couple of times to make sure you understand it.

Can Player 1 (i.e. the player who makes the first move) always force a win?

Explain your answer. [8]

Question 13

On a 26 question test, 5 points are deducted for each incorrect answer, 2 points are scored for each unanswered question and 8 points are scored for each correct answer. Susan writes the test and obtains a final score of 0 (zero).

How many questions did she correctly answer if:

(a) Susan answered all the questions?

(b) Susan did not answer all the questions? [8]

Question 14

Fig. 1 Fig. 2

Funnels of Death

Legend has it that an ancient king used these funnels to determine by lot which of his captives should die. They contained white sugar-pills and black poison-pills. Each had a spring-release at the end to let one pill fall out at a time. Fig. 1 shows a black pill just about to fall.

Each captive's fate depended upon the *second* pill which fell into his hand. He had to replace the *first* pill in the funnel, no matter what its colour, and swallow the second one.

Referring to the picture and description, answer the following questions:
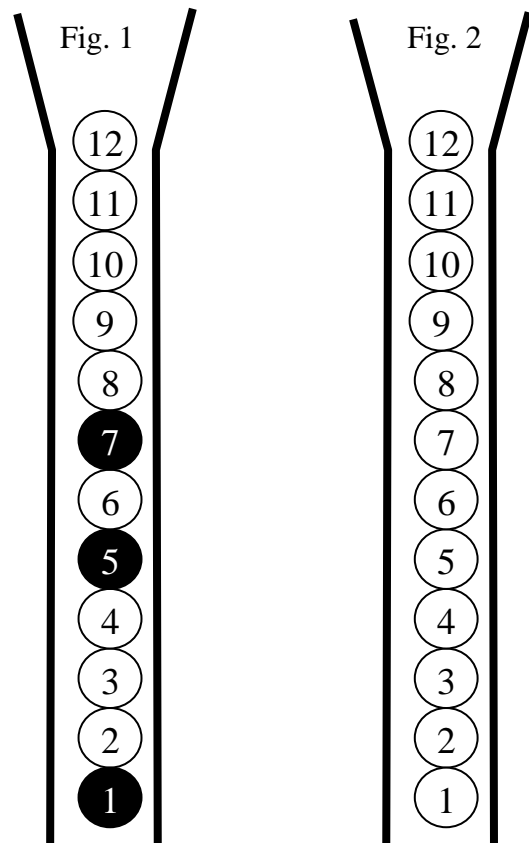
(a)   Fig. 1 shows 12 pills ready to be drawn by 9 captives. Captive #1 draws the black pill first, replaces it at the top and eats Pill #2. Then captive #2 draws Pill #3, replaces it at the top and eats Pill #4. Which one of the 9 captives will be the first to die?

(b)   Fig. 2 shows no poison-pills. Again supposing there are 9 captives, where would you put the 3 black pills so that none would be drawn as a second choice?
Show your solution by drawing the funnel in your answer book and blackening in three of the pills.

[8]

Question 15

Find the largest positive integer which for all positive integers, n, is a factor of

$$n(n+1)^2(n+2)^3(n+3)^4$$

[8]

Total: 100

THE END

**Please turn over for the Working Sheet to help you to answer
Questions 4 and 12**

4

WORKING SHEET

Use this working sheet to help you to answer Questions 4 and 12

Question 4

RULER 1

RULER 2

Name: _____

School: _____

Grade: _____