

SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the

SOUTH AFRICAN MATHEMATICS FOUNDATION

2015 SECOND ROUND

JUNIOR SECTION: GRADE 8 & 9

13 May 2015

Time: 120 minutes

Number of questions: 20

Instructions

1. The answers to all questions are integers from 0 to 999. Each question has only one correct answer.
2. Scoring rules:
 - 2.1. Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
 - 2.2. There is no penalty for an incorrect answer or any unanswered question.
3. You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**
4. Figures are not necessarily drawn to scale.
5. Indicate your answers on the sheet provided.
6. Start when the invigilator tells you to do so.
7. Answers and solutions will be available at www.samf.ac.za

Do not turn the page until you are told to do so.

Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns



HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

21. If $3x - 216 = 0$, determine the value of x .

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:

21	H / H	0	●	1	2	3	4	5	6	7	8	9
	T / T	7	0	1	2	3	4	5	6	●	8	9
	U / E	2	0	1	●	3	4	5	6	7	8	9

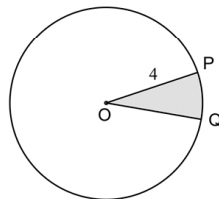
Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

Part A (4 marks each)

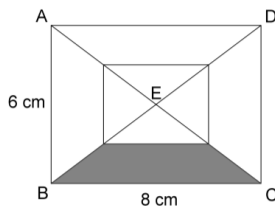
1. What is the value of $(5 - 1) + (0 - 2)$?
2. How many positive factors of 128 are not factors of 120?
3. Marks are drawn on a thin strip of paper dividing the strip into 4 equal lengths. Marks are also drawn dividing the strip into 3 equal lengths. After the strip has been cut at each mark, how many pieces will there be?
4. For how many positive values of n are both $\frac{1}{2}n$ and $2n$ two-digit integers?
5. When simplified, the fraction
$$\frac{2}{0 + \frac{1}{5 + \frac{2}{0 + \frac{1}{5}}}}$$
 equals

Part B (5 marks each)

6. In a group of 108 people, 1 in 4 of them has a pen and 1 in 3 has a pencil. What is the minimum possible number of people that have something to write with?
7. The smallest number bigger than 2015 that is divisible by all of 2, 3, 4, 5 and 6 is
8. If $\frac{n+3}{13}$ is a positive integer, then the remainder when n is divided by 13 is
9. The circle centre O has radius 4 cm. If the area of sector OPQ is $\frac{1}{2}\pi$, and if the probability that a point chosen randomly in the circle lies in the shaded sector is $\frac{1}{n}$, then the value of n is

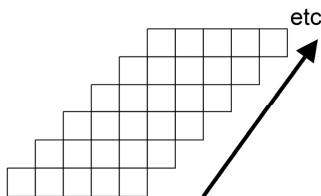


10. The diagram shows a 6 cm by 8 cm rectangle ABCD. The diagonals intersect at point E. The midpoints of AE, BE, CE and DE are joined with straight line segments. The area of the shaded trapezium in cm^2 is



11. When a water-tank is 30% empty it has 30 litres more in it than when it is 30% full. How many litres can the tank hold when it is full?

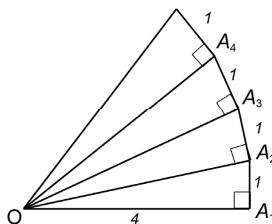
12. A shape consisting of 1000 small squares is made by continuing the arrangement shown.



If each small square has a side length of 1 cm, find the perimeter of the whole shape in cm.

13. Points $A_1, A_2, A_3 \dots$ are constructed as follows: the length OA_1 is 4, $\angle A_1OA_2 = 90^\circ$ and the length $A_1A_2 = 1$; then a right angle is constructed at A_2 to find A_3 , and so on as shown in the diagram.

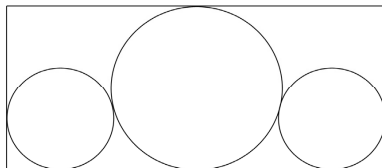
The length of OA_{21} is



14. If p, q, r and s are different prime numbers less than 20, what is the greatest possible value of $\frac{p-q}{2r+s}$?
15. The largest integer value of x such that 4^x divides exactly into $10 \times 9 \times 8 \times \dots \times 3 \times 2 \times 1$ is

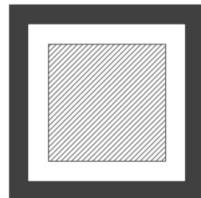
Part C (6 marks each)

16. Three circles and a rectangle touch each other as shown. The radius of the larger circle is 9, and the two smaller circles both have radius 4. The length of a longer side of the rectangle is

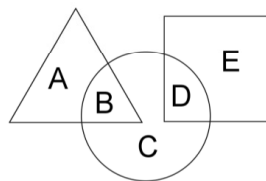


17. How many pairs of non-negative integers x and y are solutions of $\frac{x}{20} + \frac{y}{15} = 1$?

18. Black bricks, white bricks and grey bricks are laid to form the pattern in the diagram. The three squares have the same centre, and the diagonals of the squares have lengths 3, 4 and 5 respectively. If 120 bricks are needed for the central grey area, then the number of black bricks needed is



19. In the diagram, A, B, C, D and E must each be replaced by one of five consecutive positive integers, not necessarily in that order.
The numbers inside the triangle add up to 29.
The numbers inside the circle add up to 47.
The numbers inside the square add up to 30.
All five numbers add up to 75.
The value of C is



20. The large quadrilateral shown has 2 right angles, and two of the sides have lengths 7 cm and 10 cm. B and D are on the other sides so that $AB = 2$ cm and $DC = 6$ cm.

The area of the shaded quadrilateral ABCD in cm^2 is

