THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

SECOND ROUND 2004: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A

1. If $\frac{6}{5} = 1{,}2$, then the value of $\frac{0{,}06}{0{,}5}$ is

A) 1, 2 B) 0,12 C) 0,012 D) 0,0012 E) 0,00012

<u>ANSWER</u>: B
<u>EXPLANATION</u>:
Given $\frac{6}{5} = 1{,}\,2$

$$\frac{0{,}06}{0{,}5} = \frac{0{,}6}{5}$$
$$= \frac{6}{5} \div 10$$
$$= 1{,}2 \div 10$$
$$= 0{,}12$$

2. If $x \square y$ is defined to be the remainder when x is divided by y (for example $8 \square 5 = 3$), then the value of $13 \square (11 \square 3)$ is

A) 0 B) 1 C) 2 D) 3 E) 4

<u>ANSWER</u>: B
<u>EXPLANATION</u>:
$x \square y$ is defined to be the remainder when x is divided by y (e.g. $8 \square 5 = 3$).

$13 \square (11 \square 3) = 1$ since $\frac{11}{3} = 3$ remainder 2

and $\frac{13}{2} = 6$ remainder 1

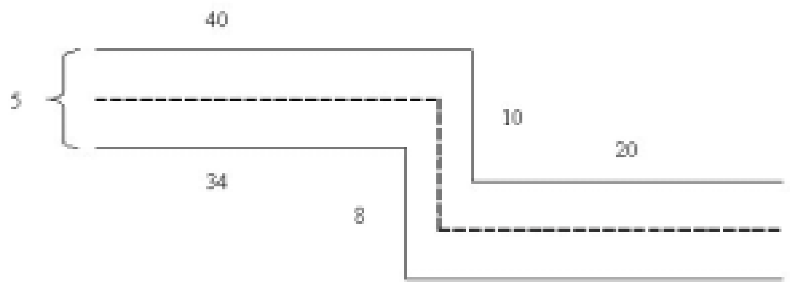
3. If $10^x \cdot 10^y \cdot 10^z = 10^6$, then the average of x, y and z is

A) 1 B) $\frac{5}{3}$ C) 2 D) $\frac{7}{3}$ E) 3

<u>ANSWER</u>: C
<u>EXPLANATION</u>:

$$10^x \cdot 10^y \cdot 10^z = 10^6$$
$$10^{x+y+z} = 10^6$$
$$x + y + z = 6$$
$$\text{Average}: \frac{6}{2} = 2$$

4.



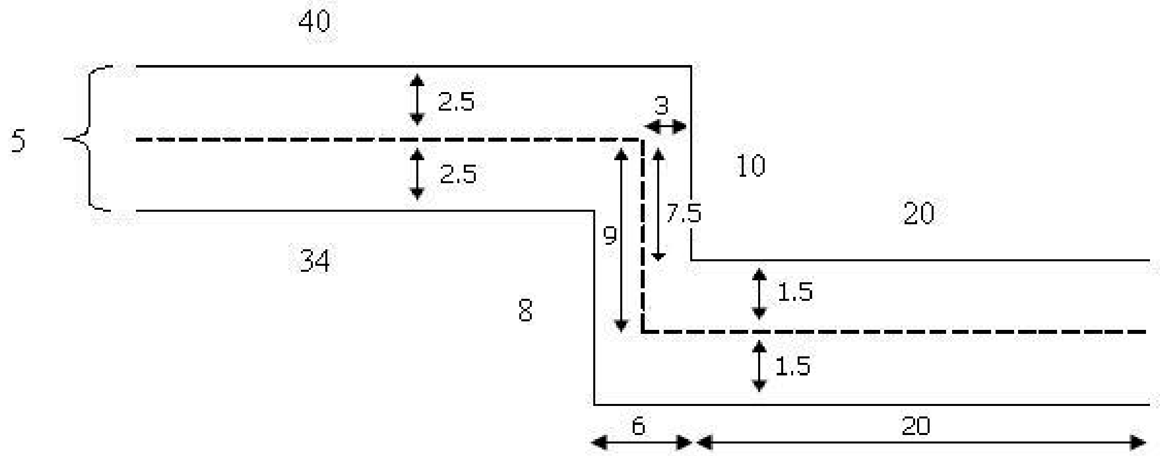
The length of the broken line, in metres, down the middle of a road is

A) 67 B) 67,5 C) 68 D) 69 E) 70

<u>ANSWER</u>: D
<u>EXPLANATION</u>:
$$34 + 3 + 7\frac{1}{2} + 1\frac{1}{2} + 3 + 20$$
$= 69$ metres

OR

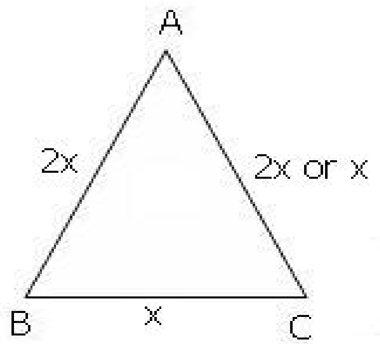
$$C = 5 + 8 - 2\frac{1}{2} - 1\frac{1}{2} = 9$$
$$40(A) + 20(B) + 9(C) = 69 \text{ meters}$$



5. In an isosceles triangle ABC, $AB = 2BC$. If the perimeter of triangle ABC is $300\,mm$, then the length of AC in millimetres is.

A) 40 B) 60 C) 80 D) 100 E) 120

ANSWER: E
EXPLANATION:
Given isosceles $\Delta\, ABC$ with $AB = 2BC$



Let $BC = x$

$\therefore \quad AC = 2x \text{ or } x$

but $AC = 2x$ to give a Δ that is possible (the sum of any 2 sides must be greater than the 3rd side).

$2x + 2x + x = 300\text{mm}$

$\therefore \qquad 5x = 300$

$\therefore x = 60 = BC$

$\therefore AC = 2 \times 60 = 120\text{mm}$

PART B

6. Half of 2^{2004} is

A) 2^{1002} B) 2^{2002} C) 2^{2003} D) 1^{2004} E) 1^{1002}

ANSWER: C
EXPLANATION:

$$\frac{1}{2}\times 2^{2004} = \frac{2^{2004}}{2^1}$$
$$= 2^{2004-1}$$
$$= 2^{2003}$$

7. You are given four fractions

$$\frac{5}{12}\ ;\ a\ ;\ b\ ;\ c$$

Two fractions a and b are equally spaced between $\frac{5}{12}$ and c.

If $a+b=\frac{4}{3}$, then find the value of c.

A) $\frac{7}{12}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{5}{6}$ E) $\frac{11}{12}$

ANSWER: E
EXPLANATION:

$\frac{5}{12}$; a ; b ; c

$$\left\{\begin{aligned} a+b &= \frac{4}{3} \\ b &= \frac{4}{3}-a \end{aligned}\right\}$$

$\frac{5}{12}$; a ; $\frac{4}{3}-a$; c

$$a-\frac{5}{12}=\frac{4}{3}-a-a$$

$$3a \quad =\frac{4}{3}+\frac{5}{12}$$

$$a \quad =\frac{7}{12}$$

$\frac{5}{12}$; $\frac{7}{12}$; $\frac{9}{12}$; $\frac{11}{12}$

$$c=\frac{11}{12}$$

OR

$$a=(a+b+\frac{5}{12})\div 3$$

$$a=\frac{\frac{5}{12}+\frac{16}{12}}{3}$$

$$=\frac{7}{12}$$

$$b=\frac{9}{12}$$

$$c=\frac{11}{12}$$

8. What is the sum of the digits of the following product?

$$999\,999 \times 666\,666$$

A) 54 B) 63 C) 72 D) 81 E) 90

ANSWER: A

EXPLANATION:

999 999 $\times$ 666 666

9 $\times$ 6 = 54 Sum$_1$ = 5 + 4 = 9 = 1 $\times$ 9

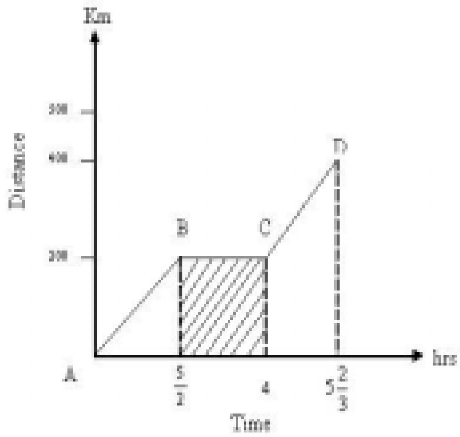
99 $\times$ 66 = 66(100-1)

$= 6600 - 66$
$= 6534$
$999 \times 666 = 665334$
$\therefore 999\ 999 \times 666\ 666$:

$Sum_2 = 6 + 5 + 3 + 4 = 18 = 2 \times 9$
$Sum_3 = 6 + 6 + 5 + 3 + 3 + 4 = 27 = 3 \times 9$

$Sum_4 = 6 \times 9$
$= 54$

9 A lady travels by car at a uniform speed, from A to B and then from C to D. Determine the average travelling speed of the vehicle from A to D in km/h.



A) 92 B) 96 C) 100 D) 104 E) 120

ANSWER: B
EXPLANATION:
Average speed from A to D

$$= \frac{\text{Distance from A to D}}{\text{Total time}}$$

$$= \frac{200 + 200}{\frac{5}{2} + 1\frac{2}{3}}$$

$$= \frac{400}{\frac{15+10}{6}}$$

$$= \frac{400}{1} \times \frac{6}{25}$$

$$= 96$$

10. The maximum number of integer values that could be obtained from $\frac{100}{2n-1}$ where n is a natural number, is

A) 9 B) 7 C) 5 D) 3 E) 1

ANSWER: D
EXPLANATION:

100 divided by one of ± 1, ± 2, ± 4, ± 5, ± 20, ± 25, ± 50 or ± 100 is an integer value.

By setting $2n-1$ equal to ± 1, ± 2, ± 4, ± 5, ± 20, ± 25, ± 50 or ± 100 results in *n* a natural number ONLY if $2n-1$ is equal to 1 or 5 or 25.

Therefore the maximum number of integer values obtained where *n* is a natural number is 3.

11. In the Harmony South African Mathematics Olympiad the scoring rules are as follows:-

For each correct answer	in Part A:	4 marks,
	in Part B:	5 marks,
	in Part C:	6 marks.
For each wrong answer:		-1 mark.
For no answer:		0 marks.

There are five questions in Part A, ten questions in Part B and five questions in Part C.

Jessie answered every question on the paper. She had four Part A questions correct and seven Part B questions correct. How many Part C questions did she get right if she scored 63% for the Olympiad?

A) 1 B) 2 C) 3 D) 4 E) 5

ANSWER: C
EXPLANATION:
Part A: $4\times4-1\times1=16-1=15$

Part B: $7\times5-3\times1=35-3=\underline{32}$

Sub total $=\underline{47}$

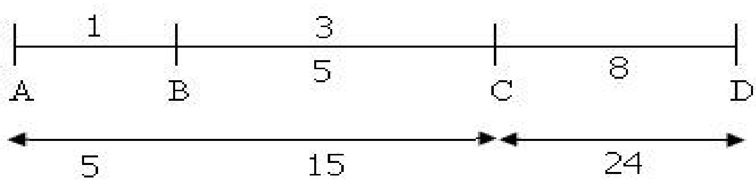
Jessie needs 16 marks
Let number right in Part C be n

Part C: $n \times 6 - (5 - n) \times 1 = 16$
$6n - 5 + n = 16$
$7n = 21$
$n = 3$

12. In the diagram, $AB : BC = 1 : 3$ and $BC : CD = 5 : 8$. The ratio $AC : CD$ in the sketch are

A) $3 : 4$ B) $3 : 5$ C) $5 : 6$ D) $4 : 5$ E) $2 : 3$

ANSWER: C
EXPLANATION:
$AB : BC :: 1 : 3$ is equivalent to
$AB : BC :: 5 : 15$
Also $BC : CD :: 5 : 8$
$\therefore BC : CD :: 15 : 24$
$\therefore AB : BC : CD :: 5 : 15 : 24$
$\therefore AC : CD :: 20 : 24$
$\therefore AC : CD :: 5 : 6$



13. Twenty 1 centimetre cubes have all white sides. Forty-four 1 centimetre cubes have all blue sides. These 64 cubes are glued together to form one large cube. What is the minimum surface area that could be white?

A) 20 B) 16 C) 14 D) 12 E) 8

ANSWER: D
EXPLANATION:

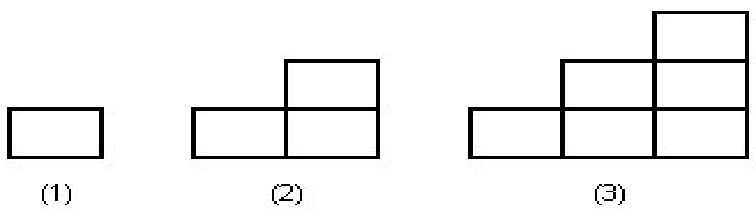
One Face

13	5	6	14
12	1	2	7
11	4	3	8
16	10	9	15

Let us look at one side.
1; 2; 3; 4 have one exposed side
5; 6; 7;...;12 have two exposed sides.
13; 14; 15; 16 have 3 exposed sides.
So the best position for a white cube on a side is 1, 2, 3 or 4.
There are also $(4-2)^3 = 8$ are 'hidden' cubes with no exposed sides, so use white cubes here.
8 of 20 cubes are 'hidden'.
This leaves us with 12 cubes at the centre of some sides. (24 cubes have one exposed side)

$\therefore$ Minimum surface area = 12 cm^2.

14. Four matchsticks are used to construct the first figure, 10 matchsticks for the second figure, 18 matchsticks for the third figure and so on.

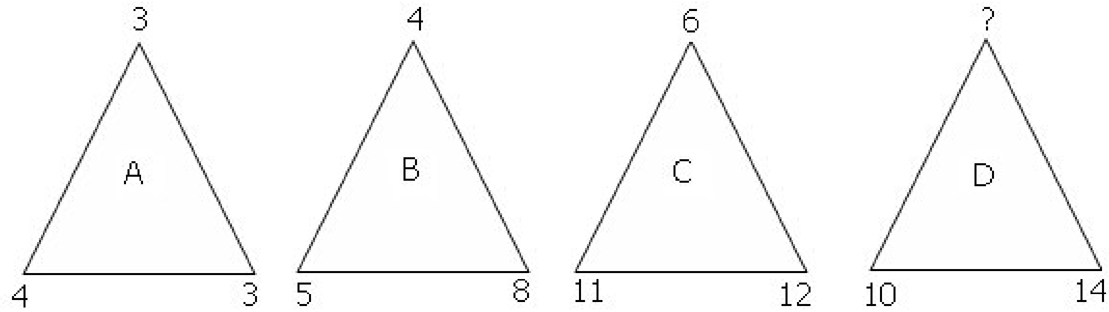


How many matchsticks are needed to construct the 30th figure?

A) 900 B) 990 C) 1080 D) 2700 E) 3000

ANSWER: B
EXPLANATION:
Figure (1): $4 = 1 \times 4$
Figure (2): $10 = 2 \times 5$
Figure (3): $18 = 3 \times 6$
Difference in factors is always 3.
$\therefore$ Figure 30: $30 \times 33 = 990$

15. After careful observation, the value and location of one number of every triangle is derived. Determine the missing number at the apex of triangle D.



A) 9 B) 8 C) 7 D) 6 E) 5

ANSWER: E
EXPLANATION:
In triangle A, $4 \times 3 = 12$ (at the base/bottom of the triangle) and $1 + 2 = 3$ (apex of A).

In triangle B, $5 \times 8 = 40$ and $4 + 0 = 4$ (apex of B).

In triangle C, $11 \times 12 = 132$ and $1 + 3 + 2 = 6$ (apex of C).

In triangle D, $10 \times 14 = 140$ and $1 + 4 + 0 = 5$, so therefore the number that goes at the apex is 5.

PART C

16. The product of the *HCF* and *LCM* of two numbers is 384. If one number is 8 more than the other number, then the sum of the two numbers is

A) 48 B) 40 C) 36 D) 24 E) 18

EXPLANATION: B
Let the numbers be *x* and *y*.
You are given $x - y = 8$.
You must find $x + y$.
The $(HCF) \times (LCM)$ of x and y is 384.
384 = 2.2.2.2.2.2.2.3.
Investigation with smaller numbers will confirm that the product of two numbers is equal to the product of their HCF and LCM.

$\therefore xy = 384$

and $x - y = 8$

Factors of 384 with a difference of 8 are 24 and 16 (done by trial and improvement)

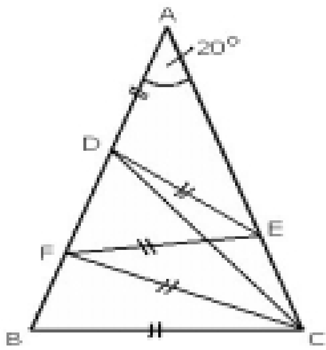
$\therefore x = 24$ and $y = 16$

and $x + y = 40$

17. In the given figure

$\Delta\ ABC$, *has* $\hat{A} = 20^o$. DE, DC, EF and FC are joined such that $AD = DE = EF = FC = BC$.

The size of $A\hat{C}D$ is

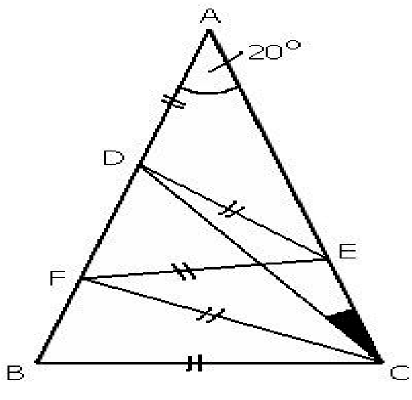


A) 10^o B) 20^o C) 30^o D) 40^o E) 60^o

<u>ANSWER</u>: A

<u>EXPLANATION</u>:

Find the size of $A\hat{C}D$



$\Delta\ ADE$ is isosceles $(AD = DE)$

$\therefore A\hat{E}D = 20^0$

$\therefore A\hat{D}E = 180^0 - 2(20^0)$ (the sum of the angles of a triangle)

$= 140^0$

$\therefore F\hat{D}E = 40^0$ (the sum of the angles on a straight line)

ΔDEF is isosceles $(DE = EF)$

$\therefore F\hat{D}E = D\hat{F}E = 40^0$

$\therefore D\hat{E}F = 100^0$ (the sum of the angles of a triangle)

$\therefore F\hat{E}C = 180^0 - \left(A\hat{E}D + D\hat{E}F\right)$ (the sum of the angels on a straight line)

$= 180^0 - \left(20^0 + 100^0\right)$

$= 60^0$

ΔFEC is isosceles $(FE = FC)$

$\therefore F\hat{E}C = F\hat{C}E = 60^0$

$\therefore E\hat{F}C = 60^0$ (the sum of the angles of triangle FEC)

$\therefore \Delta FEC$ is equilateral

$\therefore \underline{EC} = FE$

but $FE = \underline{DE}$

$\therefore$ in ΔDEC, $EC = DE$ so it is isosceles and $E\hat{D}C = E\hat{C}D$ (or $A\hat{C}D$)

$D\hat{E}C = D\hat{E}F + F\hat{E}C$

$= 100^0 + 60^0$ (already proved above)

$= 160^0$

$\therefore E\hat{C}D + E\hat{D}C = 180^0 - 160^0$ (the sum of the angles of a triangle)

$= 20^0$

but ΔDEC is isosceles

$\therefore E\hat{D}C = E\hat{C}D = 10^0$

$\left(E\hat{C}D \text{ is the same as } ACD\right)$

18. The value of

$$100^2 - 98^2 + 96^2 - 94^2 + \cdots + 8^2 - 6^2 + 4^2 - 2^2$$

is

A) 5 200 B) 5 100 C) 5 000 D) 4 900 E) 4 800

<u>ANSWER</u>: B

<u>EXPLANATION</u>:

$100^2 - 98^2 + 96^2 - 94^2 + \cdots + \left(8^2 - 6^2\right) + \left(4^2 - 2^2\right)$

There is 50 numbers in the series.

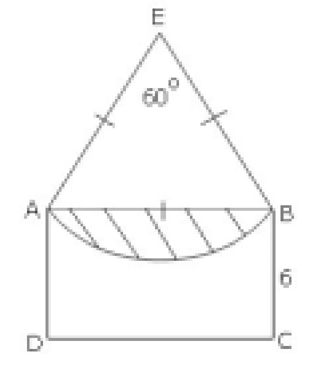
$$
\begin{aligned}
&a: S_1 = 4^2 - 2^2 = 16 - 4 &&= 12 = 2\times 6 = 2\times(4+2)\\
&b: S_2 = 8^2 - 6^2 = 64 - 36 &&= 28 = 2\times 14 = 2\times(8+6)\\
&c: S_3 = 12^2 - 10^2 = 144 - 100 &&= 44 = 2\times 22 = 2\times(12+10)\\
&\vdots &&\vdots\\
&x: &&= 380\\
&y: &&= 396 \qquad = 2\times(100+98)
\end{aligned}
$$

$$
S = \underbrace{12 + \underbrace{28 + 44 + \cdots + 380}_{408} + 396}_{408}
$$

$$
= \frac{25}{2}\times 408
$$

$$
= 5100
$$

19. In the diagram, ΔEBA is an equilateral triangle. $ABCD$ is a square of side 6. E is the centre of the circle which passes through points A and B.
The area of the shaded region is



A) $9\pi - \sqrt{27}$ B) $6\pi - \sqrt{27}$ C) $9\pi - 3\sqrt{27}$

D) $6\pi - 3\sqrt{27}$ E) $4\pi - 3\sqrt{27}$

ANSWER: D

EXPLANATION:

$ABCD$ is a square of side 6

$\therefore AB = 6$

ΔEBA is equilateral

$\therefore EA = EB = AB = 6$

Circle centre: E, radius: $EA = 6$

Area of sector EAB

$$A_S = \frac{60}{360}\pi r^2$$
$$= \frac{1}{6}\pi 36$$
$$= 6\pi$$

Area of ΔAEB

$$A_T = \frac{1}{2} b \times \perp ht$$

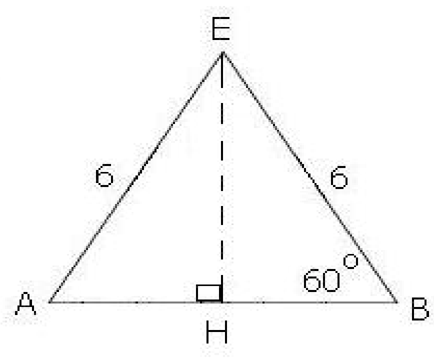
By Pythagoras $EH = \sqrt{EA^2 - AH^2}$

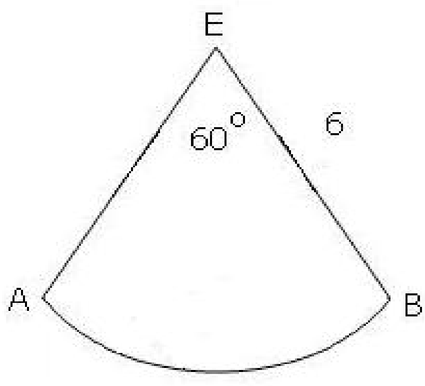
$$= \sqrt{36-9}$$
$$= \sqrt{27}$$

$$A_T = \frac{1}{2} 6 \times \sqrt{27}$$
$$= 3\sqrt{27}$$

$\therefore$ Shaded area $= A_S - A_T$

$$= 6\pi - 3\sqrt{27}$$





20. A “non-traditional” magic square totals 105. This total can be obtained by adding the 4 numbers along a diagonal. There are other sets of 4 numbers giving the same total. The maximum number of other combinations that give a total of 105 is

Rows	Column 1	Column 2	Column 3	Column 4
1	12	19	28	35
2	16	23	32	39
3	18	25	34	41
4	13	20	29	36

A) 16 B) 18 C) 20 D) 22 E) 24

ANSWER: D
EXPLANATION:
You are given the NON-TRADITIONAL magic square, i.e. the rows and columns do not necessarily add up to the ‘magic’ number.
You are told the two diagonals each add up to the ‘magic’ number 105.
How many **other** sets of four numbers add up to 105?
You will have to use trial and improvement until you find a pattern.

The following is the pattern:

1. Choose any number from column 1, say 18 in row 3.
2. Choose any number from column 2, except from row 3, say 19 in row 1.
3. Choose any number from column 3, except from rows 3 or 1, say 29 in row 4.
4. Choose any number from column 4 except from rows 3, 1 or 4, i.e. 39 in row 2.

Then $18+19+29+39=105$.
This can be done in $4\times3\times2\times1=24$ ways – but this includes the 2 diagonals.
$\therefore$ The number **other** of ways is $24-2=22$.
Note: You could have chosen a number from row 1 first, then any other number from row 2, etc.
In this way you will get the same sets of four numbers as in the first approach.

THE END

ANSWER POSITIONS: JUNIOR SECOND ROUND 2004

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	B
2	B
3	C
4	D
5	E
6	C
7	E
8	A
9	B
10	D
11	C
12	C
13	D
14	B
15	E
16	B
17	A
18	B
19	D
20	D

DISTRIBUTION	
A	2
B	6
C	4
D	5
E	3
TOTAL	**20**