SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2016 SECOND ROUND
SENIOR SECTION: GRADE 10 - 12

11 May 2016 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

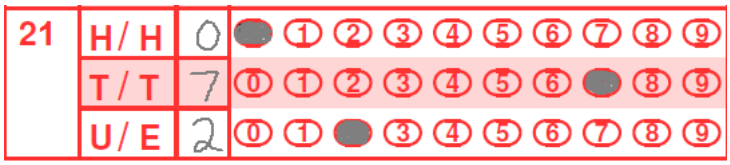
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following **example question**:

21. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom.
The three digits that you write down will not be marked, since it is only for your convenience - only the shaded circles will be marked.

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

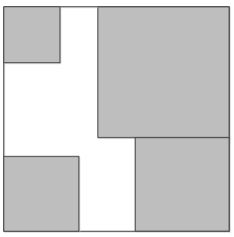
Part A: Four marks each

1. Joyce has a box with light bulbs, some blue and some red. She counts the bulbs and finds that 10 of the 40 are blue. What percentage of the bulbs are blue?

2. If $2016 \times x$ is a perfect square, what is the smallest positive integer value for x?

3. For two positive real numbers a and b, which may be equal, what is the smallest possible value of $\frac{a}{b} + \frac{b}{a}$?

4. In the four corners of the big square in the figure, squares with areas 9, 16, 25 and 49 are shaded. The two larger squares are touching. What is the area of the unshaded part of the big square?

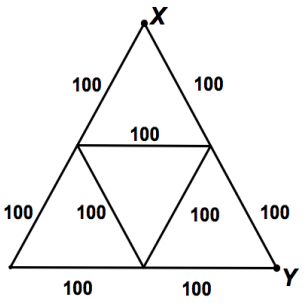


5. The lengths of all sides of a triangle are prime numbers. If two sides are 5 and 7 what is the longest possible length of the third side?

Part B: Five marks each

6. Water freezes at 0° Celsius and boils at 100° Celsius. On the SAMO scale, water freezes at 20° and boils at 170°. The two scales are linearly related. What is the temperature on the SAMO scale if it is 80° on the Celsius scale?

7. Each of the nine paths in a park is 100 m long. Ann wants to go from X to Y without going along any path more than once. What is the length, in metres, of the longest route she can choose?



8. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N, is a perfect square?

9. The faces of a cube are labelled with 2, 3, 4, 5, 6 and 7, so that the numbers on opposite faces add up to 9, as shown in Figure 1. When four such cubes are placed as shown in Figure 2 the numbers on each pair (2) of touching faces add up to 10. Which number is represented by the $*$?

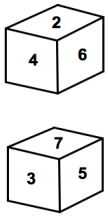


Figure 1

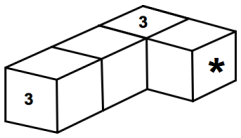
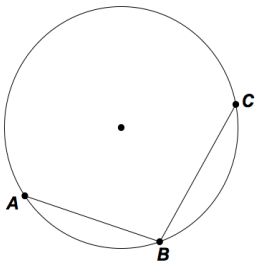


Figure 2

10. Let P be the product of any three consecutive positive odd integers. What is the highest common factor (greatest common divisor) of all such numbers P?

11. If AB is equal to the radius of the circle, what is the value of $A\widehat{C}B$, in degrees?

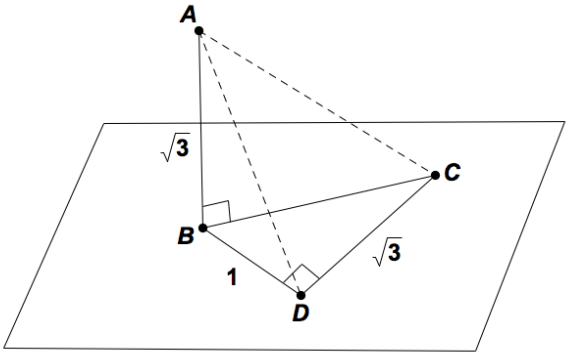


12. If $f(x) = 2^x$, what is the value of $\dfrac{f(x+1)+f(x)}{f(x)}$?

13. Simon and Harry spun a coin 30 times. Whenever the coin showed heads, Simon gave two sweets to Harry. When the coin showed tails, Harry gave three sweets to Simon. After 30 spins, both Simon and Harry had the same number of sweets as they started with. How many times did the coin show tails?

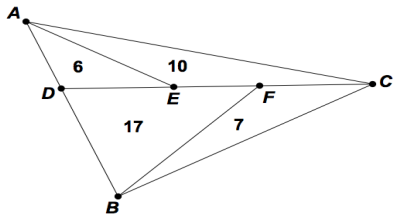
14. What is the minimum number of integers which must be removed from the first 50 positive even integers so that the sum of the remaining integers is 2016?

15. Triangle BCD lies in the plane, and A is a point outside the plane. AB is perpendicular to the plane and BD is perpendicular to CD. What is $A\widehat{D}C$ in degrees?



Part C: Six marks each

16. D is a point on side AB of $\triangle ABC$, E is a point on CD and F is a point on CE. The areas of triangles AED, AEC, BFD and BFC are 6, 10, 17 and 7, respectively. What is the area of $\triangle BEF$?



17. In $\triangle ABC$ the sides a and b are not equal, and $A\widehat{C}B = 90°$. The numbers x and y are real, such that $\frac{x}{2a^2} + \frac{y}{c^2} = 1$ and $\frac{x}{c^2} + \frac{y}{2b^2} = 1$. What is the value of $x + y$ if $c = 3$?

18. The learners in grade 10 are divided into two groups. One group consists of $2m$ boys and 13 girls, and the other of 7 boys and $2n$ girls, with m and n positive integers. Each learner pays the same integer number of rands into a fund, and the total amount of money raised is $2mn + 7m + 13n + 84$ rands. What is the number of rands paid by each learner?

19. What is the remainder when 2^{2016} is divided by 13?

20. The diagram shown is formed by extending the sides of a right triangle and constructing four circles that are each tangent to all three sides. The radii of the three smaller circles with centres at D, E and F are respectively equal to 3, 4 and 21. What is the length of the radius of the circle with centre G?