# THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

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**SECOND ROUND 2000**

**SENIOR SECTION: GRADES 10, 11 AND 12**
(STANDARDS 8, 9 AND 10)

**6 June 2000**

**TIME: 120 MINUTES**

**NUMBER OF QUESTIONS: 20**

**Instructions:**

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.

**DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.**

**KEER DIE BOEKIE OM VIR AFRIKAANS**

P.O. BOX 538, PRETORIA, 0001 TEL: (012-) 328-5082 FAX: (012-) 328-5091

# PRACTICE EXAMPLES

**1.** If $3x - 15 = 0$, then $x$ is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

**2.** The circumference of a circle with radius 2 is

(A) $\pi$ (B) $2\pi$ (C) $4\pi$ (D) $6\pi$ (E) $8\pi$.

**3.** The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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## Part A: 4 marks each.

**1.** Which of the following is the closest approximation to $\dfrac{3}{\sqrt{\sqrt{24}-1}}$?

(A) 1,5 (B) 0,8 (C) 1,7 (D) 1,3 (E) 0,75

**2.** The length of a diagonal of a square is $d$. The area of the square is

(A) $\frac{1}{2}d^2$ (B) $\dfrac{d^2}{\sqrt{2}}$ (C) $d^2$ (D) $\sqrt{2}d^2$ (E) $2d^2$

**3.** $\sqrt{2000^{2000}}$ is the same as

(A) $1000^{1000}$ (B) $1000^{2000}$ (C) $(20\sqrt{5})^{2000}$ (D) $2000^{20\sqrt{5}}$ (E) none of these.

**4.** Which one of the following numbers is the largest?

(A) 333 (B) $33^3$ (C) $(3^3)^3$ (D) $3^{(3^3)}$ (E) $3^{33}$.

**5.** A cube is inscribed in a sphere of diameter $9\sqrt{3}$cm. The volume of the cube in cm$^3$ is

(A) 243 (B) 729 (C) $243\sqrt{3}$ (D) $9\sqrt{3}$ (E) 27.

## Part B: 5 marks each

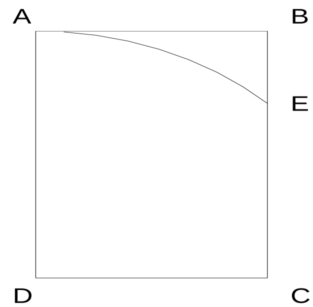
**6.** If

$$\frac{3x-5}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

is true for all possible values of $x$, then the value of $A^2 + B^2$ is

(A) 25 (B) 17 (C) 10 (D) 18 (E) 26

**7.** $ABCD$ is a rectangle with $AD = \sqrt{2}$ and $AB = 1$. $AE$ is an arc of a circle with centre $D$. The length of $CE$ is




(A) $2\sqrt{2} - 2$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{4-\sqrt{2}}{2}$ (D) $\dfrac{1}{2(\sqrt{2}-1)}$ (E) 1

**8.** The 2000th letter in the sequence

$$ABCDEDCBAABCDEDCBAABCDEDCBAABC\cdots$$

is

(A) $A$ (B) $B$ (C) $C$ (D) $D$ (E) $E$

**9.** The fact that $5 = 2 + 3$ shows that some prime numbers can be written as the sum of two other prime numbers. How many prime numbers can be written as the sum of two prime numbers in two different ways? (2+3 and 3+2 are not considered different).

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

**10.** The largest number of acute angles that a convex hexagon can have, is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

**11.** The number of different natural numbers $n$ such that $\dfrac{5}{9} < \dfrac{n}{n+16} < \dfrac{4}{7}$, is

(A) 2 (B) 3 (C) 0 (D) 1 (E) more than 3

**12.** If $n$ can be any natural number, how many different values for the remainder can you get if you divide $n^2$ by 7?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

**13.** An unknown polynomial yields a remainder of 2 upon division by $x - 1$, and a remainder of 1 upon division by $x - 2$. If this polynomial is divided by $(x-1)(x-2)$, then the remainder is

(A) 2 (B) $x$ (C) $x + 1$ (D) 3 (E) $-x + 3$

**14.** A farmer has both sheep and chickens. The average number of legs per animal is $\ell$. The ratio of the number of sheep to the number of chickens is

(A) $\dfrac{\ell}{3(4-\ell)}$ (B) $\dfrac{\ell-2}{4-\ell}$ (C) $\dfrac{3(\ell-2)}{\ell}$ (D) $\dfrac{(\ell-2)^2}{16-\ell^2}$ (E) $\dfrac{7(\ell^2-4)}{5(16-\ell^2)}$

**15.** A book with 12 pages needs the 15 digits

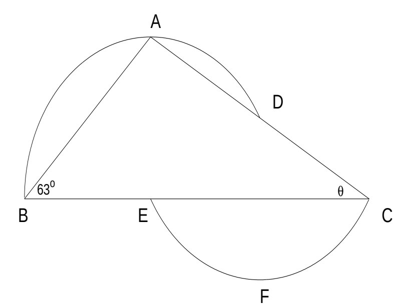
$$1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2$$

in order to number all the pages. Which one of the following numbers cannot be the number of digits needed in order to number all the pages of a book?

(A) 31 (B) 543 (C) 1998 (D) 1999 (E) 2001

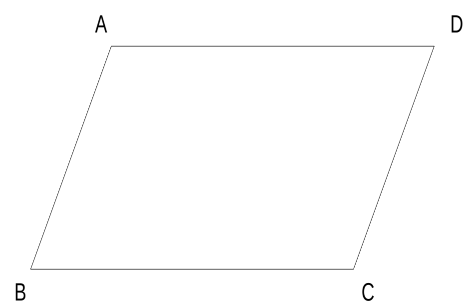
## Part C: 6 marks each

**16.** In the figure $E$ is the centre of the circle through $D$, $A$ and $B$, and $D$ is the centre of the circle through $E$, $F$ and $C$. The size of the angle $\theta$ is




(A) 18° (B) 20° (C) 22° (D) 24° (E) 14°

**17.** $ABCD$ is a parallelogram with vertices $A$ and $B$ fixed, but vertices $C$ and $D$ are movable. If the lengths of all the sides are fixed, then the path of the intersection of the diagonals, as $C$ and $D$ move, is a




(A) circle (B) part of a parabola (C) rhombus (D) straight line segment
(E) part of a hyperbola

**18.** The number of ordered pairs of positive integers $(m; n)$, such that $\dfrac{1}{m} + \dfrac{1}{n} = \dfrac{1}{15}$, is

(A) 10 (B) 2 (C) 4 (D) 8 (E) 9

**19.** An office employs thirty people. Five of them speak Sotho, Afrikaans and English. Nine speak Sotho and English, twenty speak Afrikaans of which twelve also speak Sotho. Eighteen speak English. No one speaks only Sotho. How many speak only English?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

**20.** Consider the equation $2u+v+w+x+y+z=3$. How many solutions $(u;v;w;x;y;z)$ of non-negative integers does this equation have?

(A) 27 (B) 25 (C) 30 (D) 40 (E) 35