THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

SECOND ROUND 2004

SENIOR SECTION: GRADES 10, 11 AND 12

13 MAY 2004

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.
8. Answers and solutions are available at: http://science.up.ac.za/samo/

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PRACTICE EXAMPLES

1. Remember that 1 000 cm^3 of water weighs 1 kg. During a rain shower, 10 mm of rain fell on a rectangular soccer field with dimensions 100 m by 50 m. The mass of rain that fell on the field was

   (A) 0.5 ton (B) 5 kg (C) 50 ton (D) 50 kg (E) 5 ton

2. When the decimal point of a certain positive number is moved four places to the right, the new number is nine times the reciprocal of the original number. The original number was

   (A) 0.0003 (B) 0.003 (C) 0.03 (D) 0.3 (E) 3

3. How many terms are there in the simplified expansion of

$$(a+b+c+d+e)(c+d+e+f+g)?$$

   (A) 18 (B) 22 (C) 21 (D) 24 (E) 25

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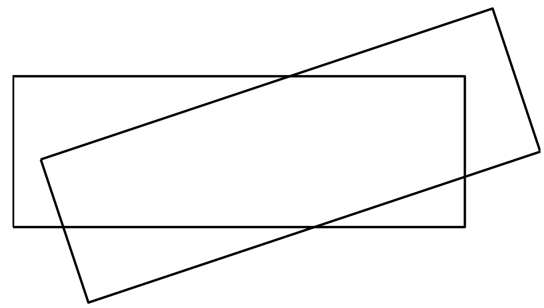
Part A: Four marks each

1. Half of 2^{20} equals

(A) 1^{10} (B) 1^{20} (C) 20 (D) 2^{10} (E) 2^{19}

2. The diagram shows two rectangles that enclose five regions. The largest number of regions that can be enclosed by any two rectangles drawn on a sheet of paper is



(A) 10 (B) 9 (C) 8 (D) 7 (E) 6

3. How many of the following numbers are greater than 10?

$$3\sqrt{11} \qquad 4\sqrt{7} \qquad 5\sqrt{5} \qquad 6\sqrt{3} \qquad 7\sqrt{2}$$

(A) 1 (B) 3 (C) 5 (D) 4 (E) 2

4. If n is any positive integer, how many different remainders are possible when 2^n is divided by 7?

(A) 5 (B) 3 (C) 2 (D) 4 (E) 1

5. The sum of two numbers is 10 and the difference between their squares is 40. The sum of their squares is

(A) 50 (B) 30 (C) 10 (D) 58 (E) 100

Part B: Five marks each

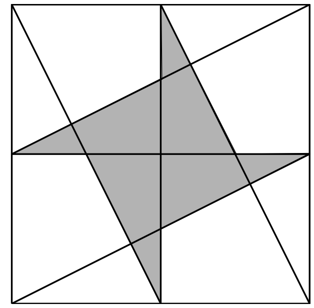
6. The hypotenuse z and one side x of a right-angled triangle are consecutive integers. The square of the third side is

(A) $z - x$ (B) $z + x$ (C) zx (D) $\frac{z}{x}$ (E) none of these

7. A number is formed by writing the first ten primes in increasing order. (Remember that 1 is not a prime number.) Half of the digits are now crossed out so that the number formed by the remaining digits, without changing their order, is as large as possible. The second digit from the left of this new number is

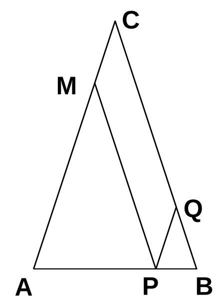
(A) 2 (B) 3 (C) 5 (D) 7 (E) 9

8. In the diagram a corner of the shaded star is at the midpoint of each side of the large square. The fraction of the large square covered by the star is



(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$ (E) $\frac{2}{5}$

9. In $\triangle ABC$, $AC = BC = 15$; PM is parallel to BC and PQ is parallel to AC. The perimeter of $PQCM$ is



(A) Cannot be determined (B) 20 (C) 30 (D) 40 (E) 15

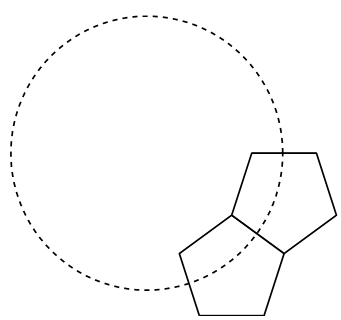
10. The ratio of the area of a regular hexagon with side 1 to the area of an equilateral triangle with side 3 is

(A) 2:3 (B) 2:1 (C) 5:6 (D) 3:4 (E) 1:1

11. Nine numbers are written in ascending order. The middle number is also the average of the nine numbers. The average of the five largest numbers is 68 and the average of the five smallest numbers is 44. The sum of all the numbers is

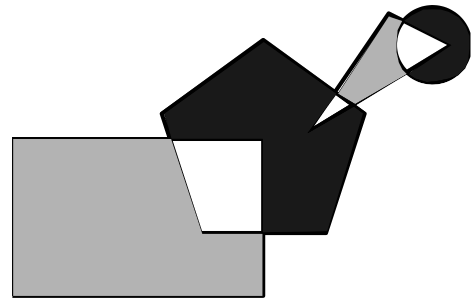
(A) 560 (B) 504 (C) 112 (D) 56 (E) 70

12. Identical regular pentagons are placed together side by side to form a ring in the manner shown. The diagram shows the first two pentagons. How many are needed to make a full ring?



(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

13. The diagram shows a rectangle, pentagon, triangle and circle with respective areas 121, 81, 49 and 25. The difference between the lightly shaded area and the black area is



(A) 25 (B) 36 (C) 49 (D) 64 (E) not possible to determine with the given information

14. Johann tells Kitty that the product of three positive integers is 36. Johann also tells her the sum of the numbers, but she still does not have enough information to find the three numbers. The sum of the three numbers is

(A) 11 (B) 13 (C) 14 (D) 16 (E) impossible to determine

15. Two jars contain an equal numbers of marbles. The marbles are either red or white. The ratio of red to white marbles is 7:1 in the first jar and 9:1 in the second jar. If there are 90 white marbles altogether, then the number of red marbles in the second jar is

(A) 360 (B) 400 (C) 450 (D) 36 (E) 40

Part C: Six marks each

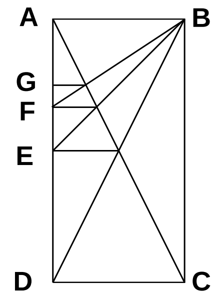
16. If a two-digit integer is q times the sum of its digits, then the number formed by interchanging the two digits is the sum of the digits multiplied by

(A) $9 - q$ (B) $10 - q$ (C) $11 - q$ (D) $q - 1$ (E) $q + 1$

17. In how many different ways can seven numbers be chosen from the numbers 1 to 9, inclusive, so that the seven numbers have a sum which is a multiple of 3?

(A) fewer than 10 (B) 10 (C) 11 (D) 12 (E) more than 12

18. $ABCD$ is a rectangle and the lines ending at E, F and G are all parallel to AB, as shown. If $AD = 12$, then AG equals



(A) $\sqrt{10}$ (B) $\frac{5}{2}$ (C) 4 (D) $\sqrt{8}$ (E) 3

19. Eight children must be divided into four teams of two players each. The number of different ways in which this can be done is

(A) 120 (B) 35 (C) 24 (D) 12 (E) 105

20. A function f is defined for all positive integers and satisfies

$$f(1) = 2005$$

and

$$f(1) + f(2) + \cdots + f(n) = n^2 f(n)$$

for all $n > 1$. The value of $f(2004)$ is

(A) $\frac{1}{2004}$ (B) $\frac{1}{1002}$ (C) $\frac{2004}{2005}$ (D) 2 (E) 2004