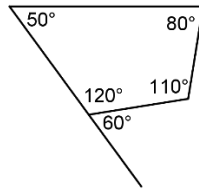## 2022 JUNIOR ROUND TWO SOLUTIONS

1. **80** Using the fact that angles on a straight line are supplementary and that the interior angles of a quadrilateral sum to 360° we have $x = 80°$.


2. **2** There are 5 positions to fill. Since there are an odd number of Ys, there has to be a Y in the middle position. There are then only two possible ways to place the two remaining Ys in a symmetrical manner: YAYAY or AYYYA.

3. **33** Note that $200 = 2^3 \times 5^2$. The only two factors that are not divisible by 10 are thus 8 and 25. Thus A + B = 25 + 8 = 33.

4. **2** $\dfrac{0{,}997^2 \times \sqrt{101}}{5{,}021} \sim \dfrac{1 \times 10}{5} = 2$

5. **16** Working backwards from the answer, we first multiply 1 by 4 to get 4, and then square the result to get 16.

6. **60** The side length of each square is 10 cm. This also represents the shorter side of the original rectangle as well as half the longer side of the rectangle. The original rectangle thus has dimensions 10 cm by 20 cm, and hence a perimeter of 60 cm.

7. **6** When full, the bucket weighs 16 kg. When half the water is poured out the mass decreases by 5 kg, so half the water weighs 5 kg. The mass of all the water is thus 10 kg, so the bucket itself weighs 16 kg – 10 kg = 6 kg.

8. **15**
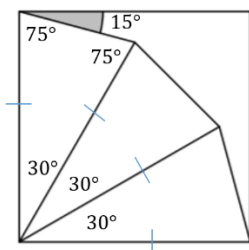
9. **804** Each face of a cube has a surface area of 2 cm². If there are 100 cubes in total, then there is a single face on the far left, a single face on the far right, and $4 \times 100 = 400$ faces in the middle. The total surface area is thus $2 + 400 \times 2 + 2 = 804$ cm².

10. **400** If he spends a quarter of the money in his wallet, he has three quarters left. Of this he gives a third to his children. The amount remaining in his wallet is thus two thirds of three quarters. $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$. The remaining R200 thus represents half the original amount, so he originally had R400.

11. **8**   The area of triangle C is $\frac{1}{5} \times 10 = 2$. Since the combined areas of triangles B and C is the same as triangle A, it follows that the area of triangle B is $10 - 2 = 8$ square units.

12. **45**   If we label the teams A, B, C, D, E, F, G, H, I, J then team A will have a match with the other 9 teams. Team B will then only need to play against teams C-J (8 matches) since they have already played team A. Team C will only need to play against teams D-J (7 matches) since they would already have played against teams A and B. Continuing this logic there would be $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ matches if each team played every other team once.

13. **144** $1 + 2 + 3 + 4 = 10$ and $360° \div 10 = 36°$. The largest angle is thus $4 \times 36° = 144°$.

14. **27**   Since the leading digit is a 1, the integers will be of the form $1X11$ or $11X1$ or $111X$ where, in each case, X can take on the value 0, 2, 3, 4, 5, 6, 7, 8, 9.
There are thus $3 \times 9 = 27$ such integers.

15. **3**   If $\dfrac{a}{b} = 6$ then $a = 6b$. If $\dfrac{b}{c} = \dfrac{1}{4}$ then $c = 4b$. Since $a + c = 30$ we have $6b + 4b = 30$, thus $10b = 30$ and hence $b = 3$.

16. **45** $0{,}75 = \dfrac{3}{4} = \dfrac{9}{q}$. Thus $q = 12$. From this it follows that $p = 3{,}75 \times 12 = 45$.

17. **8**   The group of 3 would have to be formed by choosing one pupil with brown eyes, one pupil with blue eyes, and one pupil with green eyes. For each of these there are two choices. In total there are thus $2 \times 2 \times 2 = 8$ possible groups of 3 pupils.

18. **18**   By joining B to D and adding in a perpendicular height for triangle BCD, we can sub-divide the quadrilateral into three identical 3-4-5 right-angled triangles. The area of the quadrilateral is thus $3 \times \left(\frac{1}{2} \times 4 \times 3\right) = 18$.
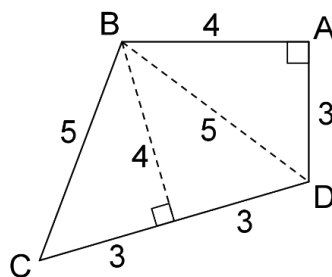
19. **35**   Let x be the number of questions answered correctly. The number of questions answered incorrectly is thus $50 - x$. The pupil's score, in terms of x, would thus be $2x - (50 - x)$. Setting this equal to 55 and solving for x gives the solution $3x = 105$ and thus $x = 35$.

20. **49**   The prime numbers smaller than 50 are 2, 3, 5, 7 … 41, 43, 47. Note that 2 is the only even prime number. Since $b + c$ is even, both b and c must be odd. However, since $a + b$ is odd, this means that a must be even, i.e. $a = 2$. Since we want $a + c$ to be as large as possible, $c = 47$. Thus $a + c = 2 + 47 = 49$.

21. **8** To be divisible by 15, the number must be divisible by both 3 and 5. If it is divisible by 5 then b must be 0 or 5. For a number to be divisible by 3, the sum of its digits must be divisible by 3. Thus $2 + 3 + a + 4 + 6 + 7 + b = 22 + a + b$ must be divisible by 3. If $b = 0$ then a could be 2, 5 or 8. If $b = 5$, then a could be 0, 3, 6 or 9. The greatest possible difference between a and b occurs when $b = 0$ and $a = 8$, the maximum difference is thus 8.

22. **90** The amount of cash that should be received from the sale of the item is R60 + (20% of 60) = R72. Thus, the R72 is 80% of the amount on the price tag. The amount on the price tag should thus be R72 x 100/80 = R90.

23. **60** Since 5 and 7 are relatively prime, we need to have a multiple of 5 fractions on the left, and the same multiple of 7 fractions on the right, i.e. in total we need a multiple of 12 fractions. The smallest multiple of 12 bigger than 50 is 60.

24. **135** Shaded area $= \frac{x}{360°}(\pi(3)^2 - \pi(1)^2) = \frac{8\pi x}{360}$. Thus: $\frac{8\pi x}{360} = \frac{\pi(3)^2}{3} \Rightarrow x = \frac{3}{8} \times 360° = 135°$.

25. **48** Let M be Mdu's walking speed, E the escalator speed, and T the time taken when the escalator is off. Moving 'upstream' the net speed is $M - E$, while moving 'downstream' the net speed is $M + E$. Since the distance travelled for both journeys is the same, we have $72(M - E) = 36(M + E) = T \times M$. Solving this yields $E = M/3$ and thus $T = 48$ seconds.