THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

FIRST ROUND 2003

SENIOR SECTION: GRADES 10, 11 AND 12

18 MARCH 2003

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 5 marks.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.
8. Answers and solutions are available at: http://science.up.ac.za/samo/

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129

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1. $(1-2)^{2003}$ is equal to

(A) -2 (B) -1 (C) 1 (D) -2003 (E) 2

2. $(0.1)^3 \div 0.01$ is equal to

(A) 0.01 (B) 0.1 (C) 1 (D) 0.001 (E) 10

3. $(2^{-1}+3^{-1})^{-1}$ is equal to

(A) 5 (B) $\frac{1}{6}$ (C) 6 (D) $\frac{2}{3}$ (E) $\frac{6}{5}$

4. If for any three different numbers a, b, c we define $a\#b\#c = \frac{a+b}{c-a}$, then $1\#2\#3$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{3}{2}$ (E) 1

5. If $2^x = 2003$, then the closest integer to x is

(A) 11 (B) 8 (C) 10 (D) 12 (E) 9

6. If $x > 0$, then $\sqrt{x\sqrt{x\sqrt{x}}}$ equals

(A) $x^{3/2}$ (B) $x^{5/4}$ (C) $x^{1/8}$ (D) $x^{3/8}$ (E) $x^{7/8}$

7. After five games, a rugby team has an average of 28 points per game. In order to increase their average by 2 points, how many points must they score in the sixth game?

(A) 2 (B) 12 (C) 56 (D) 30 (E) 40

8. Given $q = 3\sqrt{3}$, $r = 1+2\sqrt{3}$, $s = 3+\sqrt{3}$, which of the following is true?

(A) $q > r > s$ (B) $q > s > r$ (C) $r > q > s$ (D) $s > q > r$ (E) $s > r > q$

9. If $\sqrt{\dfrac{(4+\sqrt{x+3})^2}{6}+3}=3$, then x is equal to

(A) 22 (B) 6 (C) 3 (D) 1 (E) 5

10. Given that $n! = n \times (n-1) \times \cdots \times 2 \times 1$ (e.g., $4! = 4 \times 3 \times 2 \times 1 = 24$), which one of the following numbers is the largest?

(A) (70!)(30!) (B) (50!)(50!) (C) (40!)(60!) (D) (80!)(20!) (E) 100!

11. In the 5-digit number $a6a41$ each of the a's represents the same digit. If the number is divisible by 9, then the digit represented by a is

(A) 3 (B) 7 (C) 6 (D) 8 (E) 5

12. Sophie runs the Comrades Marathon every second year. When she has run her seventh Comrades, the sum of all the years in which she has run is 13951. In which year did she run her first Comrades?

(A) 1978 (B) 1982 (C) 1985 (D) 1987 (E) 1991

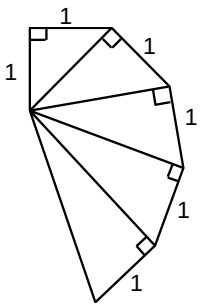
13. Egoli municipality has two different formulae for calculating monthly charges for electricity. For a consumer using an amount E of electricity, the cost C is given by either

$$C = 60 + 0.2E \text{ or } C = 0.3(E - 50).$$

The first formula gives cheaper electricity when

(A) $E > 0$ (B) $450 < E < 750$ (C) $0 < E < 750$ (D) $0 < E < 450$ (E) $E > 750$

14. The first five triangles in a sequence of right-angled triangles are shown in the figure. If there are 100 triangles in the sequence, how many of the line segments have integer lengths?

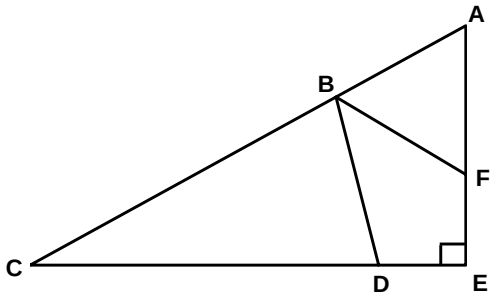


(A) 101 (B) 102 (C) 100 (D) 109 (E) 110

15. Determine the smallest positive value of the integer k such that $k^3 + 2k^2$ is the square of an odd integer.

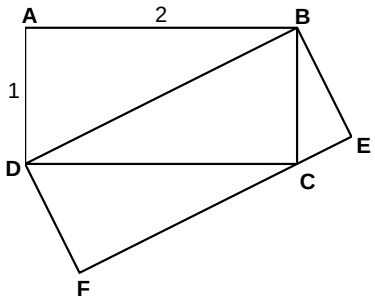
(A) 2 (B) 14 (C) 23 (D) 1 (E) 7

16. In the figure $AB = AF$ and $BC = CD$. The size of angle DBF is



(A) 30° (B) 45° (C) 22.5° (D) 60° (E) 67.5°

17. In the figure, rectangle $ABCD$ has dimensions as shown. The area of rectangle $BDFE$ is



(A) 2 (B) $\sqrt{2}$ (C) $\sqrt{5}$ (D) $\frac{4}{\sqrt{5}}$ (E) $1 + \sqrt{2}$

18.

H
A A
R R R
M M M M
O O O
N N
Y

How many paths from top to bottom spell HARMONY?

(A) 12 (B) 20 (C) 8 (D) 10 (E) 32

19. When $33333^2 + 22222$ is written as a single decimal number, the sum of its digits is

(A) 15 (B) 25 (C) 22 (D) 10 (E) 20

20. If the digits k, m, n of the 3-digit number kmn satisfy $64k + 8m + n = 403$, then the number kmn is

(A) 623 (B) 563 (C) 403 (D) 643 (E) not possible to be uniquely determined

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