THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION.
Sponsored by HARMONY GOLD MINING.

SECOND ROUND 2006
JUNIOR SECTION: GRADES 8 AND 9
17 MAY 2006
TIME: 120 MINUTES
## NUMBER OF QUESTIONS: 20

Instructions:
1.   Do not open this booklet until told to do so by the invigilator.
2.   This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3.   Scoring rules:
3.1   Each correct answer is worth 4 marks in Part A, 5 marks in part B and 6 marks in part C.
3.2   For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4.   You must use an HB pencil.
        Rough paper, a ruler and a rubber are permitted.
        Calculators and geometry instruments are not permitted.
5.   Diagrams are not necessarily drawn to scale.
6.   The centre page is an information and formula sheet. Please tear it out for your use.
7.   Indicate your answers on the sheet provided.
8.   Start when the invigilator tells you to do so.
        You have 120 minutes to complete the question paper.
9.   Answers and solutions will be available at www.samf.ac.za/samo/

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.
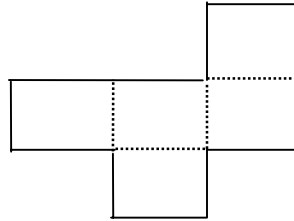## DRAAI DIE BOEKIE OM VIR DIE AFRIKAANSE VRAESTEL
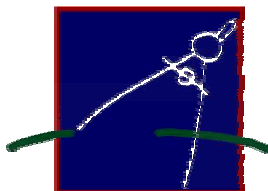
1.   15% of R560 – 15% of R500 is:

    (A)   R13   (B)   R12   (C)   R11   (D)   R10   (E)   R9
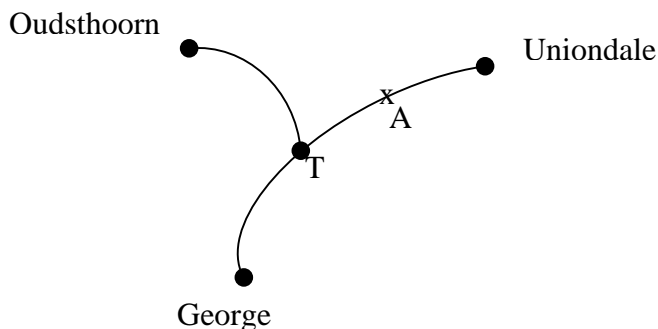
2.

    A piece of paper is cut out and labeled as shown in the diagram. It is folded along the dotted lines to make an open box. If the box is placed on a table so that the top of the box is open, then the label at the bottom of the box is:

    (A)   U   (B)   V   (C)   W   (D)   X   (E)   Y

3.   If the numbers $\sqrt[3]{9}$; $\sqrt{5}$; 1; 2; 3 are arranged in order of magnitude, then the middle number is :

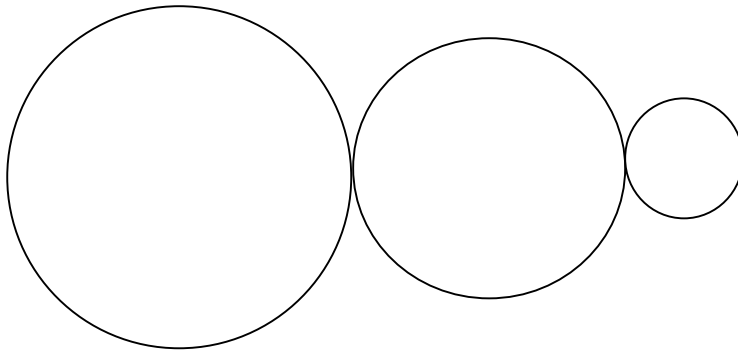   (A) $\sqrt[3]{9}$ (B) $\sqrt{5}$ (C)   1   (D)   2   (E)   3

4.

    The map shows roads joining Uniondale, George and Oudtshoorn via the T-junction at T. At point A there is a sign which shows that A is 34 km from T, 60 km from George, and 68 km from Oudtshoorn via T. The distance, in kilometres, via T, from Oudtshoorn to George is:

    (A)   148   (B)   122   (C)   60   (D)   78   (E)   52

5.

A motor has a sequence of 3 wheels that drive a windmill. Wheel P has radius 36 cm, wheel Q has radius 12 cm and wheel R has radius 6 cm. The wheels all touch each other and rotate without slipping.
If wheel P turns $360°$ in a clockwise direction, then wheel R will turn:

(A)   6 x $360°$ in an anti-clockwise direction
(B)   3 x $360°$ in an anti-clockwise direction
(C)   2 x $360°$ in an anti-clockwise direction
(D)   3 x $360°$ in a clockwise direction
(E)   6 x $360°$ in a clockwise direction

PART B: 5 MARKS EACH

6.   A popular puzzle game is called Harmony. In this game, you are given a 4 x 4 grid which is further divided into four bordered 2 x 2 squares. You are given some letters in the grid. You have to fill in the letters G, O, L and D in each row, column and 2 x 2 square such that no letter appears more than once in each row, each column and each 2 x 2 square.

L	G	D	
X			
	D	O	L

The letter marked X is:

(A)   G   (B)   O   (C)   L   (D)   D   (E)   H

7. On earth there are about 10 000 000 000 000 000 ants and
    6 000 000 000 humans. The ratio of humans to ants is approximately equal
    to:

    (A)   60 000 to 1   (B)   1 666 667 to 1   (C)   1 to 6000

    (D)   1 to 1 666 667   (E)   1 to 60 000 000

8. It takes a car 11 minutes to travel a distance of 15 kilometers.
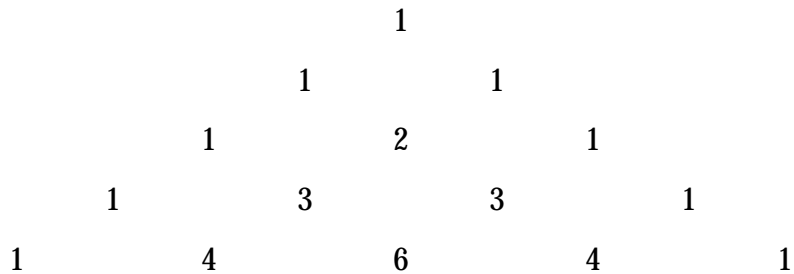   If the car travels at an average speed of x km/h, then:

    (A) $50 \leq x < 60$ (B) $60 \leq x < 70$ (C) $70 \leq x < 80$
    (D) $80 \leq x < 90$ (E) $90 \leq x < 100$

9. Let $x*y = \dfrac{x \times y}{x+y}$, for example, $4*3 = \dfrac{4 \times 3}{4+3} = 1$ remainder 5.

    If $5*x = 2$ remainder 5, then x is:

    (A)   9   (B)   8   (C)   5   (D)   12   (E)   7

10. The arrangement below is called Pascal's Triangle

```
                    1
                1       1
            1       2       1
        1       3       3       1
    1       4       6       4       1
```
…………………………………………………………………………..

    The sum of the numbers in the first row is 1.

    The sum of the numbers in the first 2 rows is 3.

    The sum of the numbers in the first 3 rows is 7, etc.

    If this triangle arrangement is continued then the sum of the numbers in

    the first 15 rows is:

    (A) $2^{14} - 1$ (B) $2^{15} + 1$ (C) $2^{15} - 1$ (D) $2^{16} + 1$ (E) $2^{14} + 1$

11. The number n is a perfect square. What is the next perfect square bigger than n?

(A) $n^2 + 1$ (B) $2\sqrt{n} + 1$ (C) $n^2 + n$

(D) $n^2 + 2n + 1$ (E) $n^2 + 2\sqrt{n} + 1$

12. If the fraction $\dfrac{3}{7}$ is written as an infinite decimal fraction,

then , $\dfrac{3}{7} = 0, a_1 a_2 a_3, \ldots$ where $a_1, a_2, a_{3,\ldots}$ are digits.

The digit in the a_{2006} position is:

(A)   7   (B)   1   (C)   4   (D)   5   (E)   2

13. The six-digit number $4m61n2$ is divisible by both 11 and 4. The number of different combinations of m and n that satisfy the above condition is:

(A)   4   (B)   6   (C)   8   (D)   10   (E)   12

14. Three different digits are used to make all possible three-digit numbers. Of the three digits, one is 4 and one is three more than another. If the sum of all such three-digit numbers is 2886, then the three digits are:

(A)   1; 2; 4   (B)   4; 5; 7   (C)   3; 4; 6   (D)   2; 4; 5   (E)   4; 6; 9

15.
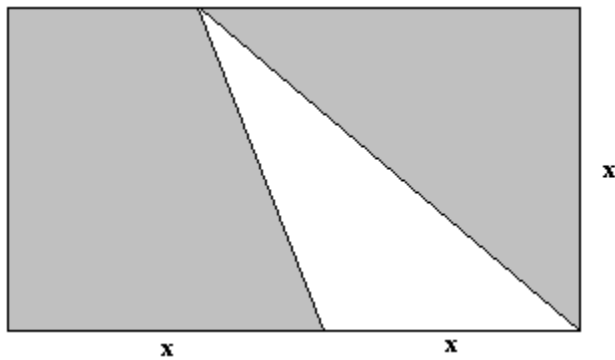
In the diagram, lengths are shown. The area of the shaded region is:

(A) $\dfrac{x^2}{4}$ (B) $\dfrac{3x^2}{2}$ (C) $2x^2$ (D) $\dfrac{x^2}{3}$ (E) $\dfrac{5x^2}{2}$

PART C: 6 MARKS EACH

16. The four-digit integers from 1994 to 2006 are written consecutively and the number, N=1994199519……..20052006 is formed.

If 3^k is the highest power of 3, by which N is divisible, then k is equal to:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

17. 132_x is the notation to show that we are working in base x.

The number 30 (in base 10), can be expanded as

$1 \times 4^2 + 3 \times 4 + 2 \times 1$. Therefore 30 can be written as 132_4.

If it is true that $14_y \times 14_y = 232_y$, then y is:

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

18. The sum of the lengths of edges of a rectangular prism is 68 cm.
If the lengths of the sides are whole numbers and the area of the base is 18
cm², then the possible volumes of the prism are:

(A)   54 and 72   (B)   108 and 72   (C)   27 and 144
(D)   108 and 144   (E)   216 and 288

19. 15 one centimetre cubes with all blue faces, 16 one centimetre cubes with
all yellow faces, and 33 one centimetre cubes with all black faces are glued
together to form one large cube.
What is the least number of one centimetre squares on the surface of the
larger cube that are black?

(A)   22   (B)   24   (C)   26   (D)   32   (E)   34

20.

•   •   •

•   •   •

•   •   •
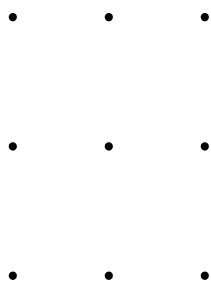
Nine points lie in a plane, as shown above. If any 3 points are joined to
form a triangle, then the number of all possible triangles that can be
drawn are:

(A)   72   (B)   84   (C)   64   (D)   78   (E)   76

Formula and Information Sheet

1.1 The natural numbers are 1; 2; 3; 4; 5; …

1.2 The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; …

1.3 The integers are   …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\dfrac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$$2\times2\times2\times2\times2 = 2^5$$
$$3\times3\times3\times3\times3\times3 = 3^6$$
$$a\times a\times a\times a\times \ …\ \times a = a^n \qquad (n \text{ factors of } a)$$
(a is the base and n is the index (exponent))

3.2 Factorial notation:

$$1\times2\times3\times4 = 4!$$
$$1\times2\times3\times \ …\ \times n = n!$$

4. Area of a

4.1 triangle is: $\dfrac{1}{2}\times (\text{base} \times \text{height}) = \dfrac{1}{2}(b.h)$

4.2 rectangle is: length $\times$ width = lw

length $\times$ breadth = lb

4.3 square is: side $\times$ side = s^2

4.4 rhombus is: $\dfrac{1}{2}\times (\text{product of diagonals})$

4.5 trapezium is: $\dfrac{1}{2}\times(\text{sum of parallel sides})\times \text{height}$

4.6 circle is: πr^2 (r = radius)

5. Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh$ (h = height)

5.2	sphere is:	$4\pi r^2$

6.	Perimeter of a:

6.1 rectangle is: $2\times \text{length} + 2\times \text{breadth}$

$2l + 2b$

or $2l + 2w$ $(w = \text{width})$

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s\times s\times s = s^3$

8.2 rectangular prism is: $l\times b\times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section $\times$ perpendicular height

      or   area of base $\times$ perpendicular height

9.2 Surface area of a right prism is: (perimeter of base $\times$ h) + (2 $\times$ area of base)

10. Sum of the interior angles of a polygon is: $180°(n-2)$ [$n = $ number of sides]
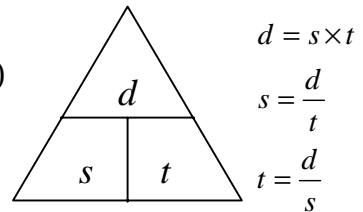
11.

Distance = speed $\times$ time $(d = s \times t)$

Speed =   distance $\div$ time $(s = \dfrac{d}{t})$
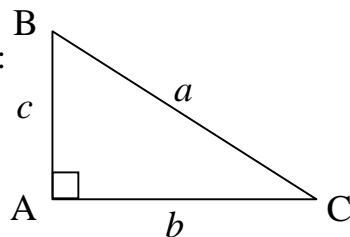
Time = distance $\div$ speed $(t = \dfrac{d}{s})$

$d = s\times t$

$s = \dfrac{d}{t}$

$t = \dfrac{d}{s}$

12.

Pythagoras:

If $\triangle ABC$ is a right-angled triangle, then $a^2 = b^2 + c^2$

13. Conversions:

1 cm³ = 1 mℓ ;   1000 cm³ = 1 ℓ

1000 m = 1 km   ;   1000 g = 1 kg   ;   100 cm = 1 m