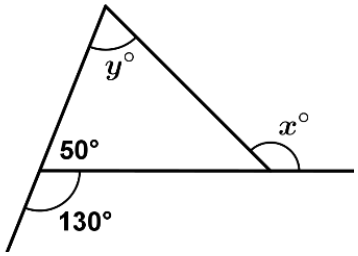
2017 JUNIOR ROUND TWO SOLUTIONS

1. 3 $\sqrt{2+0\times1+7} = \sqrt{2+0+7} = \sqrt{9} = 3$

2. 789 2017 – 1027 – 201 = 990 – 201 = 789

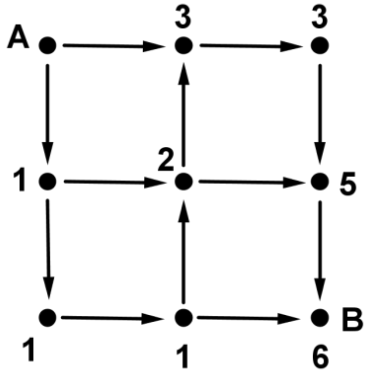
3. 5 If 30 questions are answered in 60 minutes, then each question takes two minutes. In 10 minutes I can thus answer 5 questions. Alternatively, in one sixth of the time I can answer one sixth of the questions, i.e. $30 \div 6 = 5$ questions.

4. 50 $x = y + 50°$, thus $x - y = 50°$.



5. 74 Scaling up the ratio 5:7 by a factor of 5 and then 7 gives $\frac{5}{7} = \frac{25}{35} = \frac{35}{49}$.
We thus have $x + y = 25 + 49 = 74$.

6. 6 Systematically determining the total number of pathways to each junction shows that there are 6 pathways from A to B.



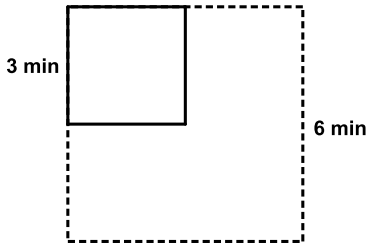
7. 20 Since 10 people ate both, 5 people ate only a hotdog & 2 people ate only a hamburger. Since 3 people ate neither the number of people at the party was 10 + 5 + 2 + 3 = 20.

8. 560 R240 ÷ 3 = R80. R80 × 4 = R320. The ratio 3:4 is thus equivalent to 240:320. The total amount of money shared is thus R240 + R320 = R560.

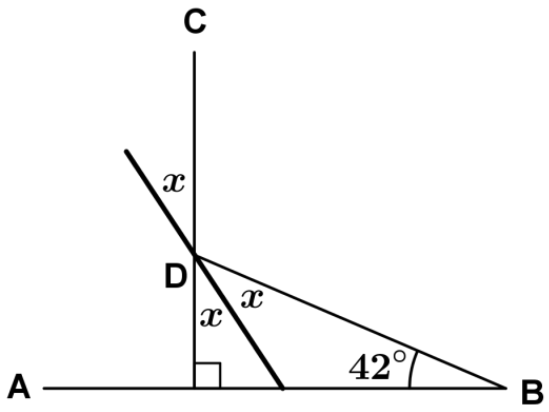
9. 24 The total of the first three numbers was 3 x 12 = 36. The total of all four numbers is 4 x 15 = 60. The fourth number is therefore 60 – 36 = 24.

10. 136 The only factors that 8 and 34 have in common are 1 and 2. Since *x*, *y*, *z*, and *w* have to be different positive whole numbers, *y* has to be 1. Hence *x* = 8, *w* = 34 and *z* = 2. We thus have $\frac{xw}{yz} = \frac{8\times34}{1\times2} = 136$.

11. 24 If it takes 12 minutes to walk around the square field, this means 3 minutes per side. If the area of the larger field is four times that of the smaller field, the side length (and hence perimeter) of the larger field must be twice that of the smaller field. It will thus take twice as long, i.e. 24 minutes.



12. 24 Extending CD perpendicularly to AB reveals the following:



Thus $x = (90^\circ - 42^\circ) \div 2 = 24^\circ$.

13. 17 $\frac{4}{3}\times\frac{5}{4}\times\frac{6}{5}\times\ldots\times\frac{x+1}{x} = 6$. Cancelling top and bottom leaves: $\frac{1}{3}\times\frac{1}{1}\times\frac{1}{1}\times\ldots\times\frac{x+1}{1} = 6$.

$\frac{x+1}{3} = 6 \Rightarrow x+1=18 \Rightarrow x=17$.

14. 45 The original number can only be 18, 27, 36 or 45.
Since 81 − 18 > 72 − 27 > 63 − 36 > 54 − 45 = 9, the original number is thus 45.

15. 6 Any power of 5 will end in a 5. Therefore each bracket's unit digit will be 5 + 1 = 6.
But $6 \times 6 \times 6$ ... times will always end in a 6.

16. 54 $\frac{10!}{9!}+\frac{9!}{8!}+\frac{8!}{7!}+\frac{7!}{6!}+\frac{6!}{5!}+\frac{5!}{4!}+\frac{4!}{3!}+\frac{3!}{2!}+\frac{2!}{1!} = 10+9+8+7+6+5+4+3+2 = 54$.

17. 6 If the number is divisible by 45 it must be divisible by both 9 and 5. To be divisible by 5, B must be either 5 or 0. If B is 0 then A must equal 6 (for the sum of the digits to be a multiple of 9). If B is 5, A must similarly be 1. The greatest possible positive difference is thus 6 – 0 = 6.

18. 626 The middle number of each odd numbered row is one more than the row number squared. The middle number of the 25th row is thus $25^2 + 1 = 626$.

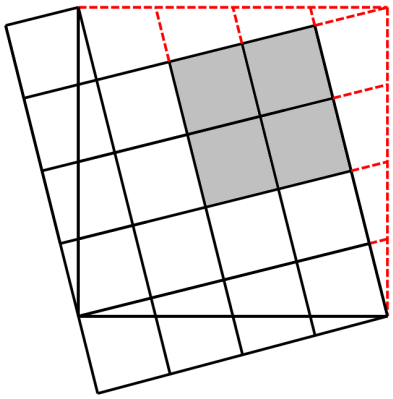
19. 3 The only such occasions occur when the clock changes from 09:59:59 to 10:00:00 ; from 19:59:59 to 20:00:00 and from 23:59:59 to 00:00:00.

20. 51 If the number leaves a remainder of 2 when divided by 7 it must be 2 more than a multiple of 7, i.e. 9 ; 16 ; 23 ; 30 ; 37 ; 44 ; 51 ; 58 ; 65 ; …
If the number leaves a remainder of 3 when divided by 6 it must be 3 more than a multiple of 6, i.e. 9 ; 15 ; 21 ; 27 ; 33 ; 39 ; 45 ; 51 ; 57 ; …
The smallest 2-digit number that appears in both lists is 51.

21. 900 40% of the 90% is 36% of the bottle's capacity.
The volume of water left in the bottle after Nivar has drunk 40% of the original amount is 54% of the bottle's capacity.
When full the bottle thus holds $\frac{486}{54} \times 100 = 900$ ml.

22. 2 Because of the symmetry we can simply consider the top quarter of the shape.
Let the radius of the shaded semicircle be x.
The radius of the large circle is thus $2x$.
The area of the shaded semicircle is $\frac{1}{2}\pi x^2$ while the area of the unshaded region is $\frac{1}{4}\pi(2x)^2 - \frac{1}{2}\pi x^2 = \frac{1}{2}\pi x^2$. Thus, $\frac{s}{u} = \frac{\frac{1}{2}\pi x^2}{\frac{1}{2}\pi x^2} = \frac{1}{1}$. Therefore $s + u = 2$.

23. 68 $\frac{4}{17} \times 17^2 = 68\ cm^2$.



24. 25 The side length of the large square is 8 cm. The side length of the second largest square is thus $\sqrt{6^2+2^2}=\sqrt{40}$. The side length of the smallest square is $\sqrt{\left(\frac{1}{4}\sqrt{40}\right)^2+\left(\frac{3}{4}\sqrt{40}\right)^2}=5$. The area of smallest square is thus 5×5 = 25 cm^2.

25. 48 At the slower speed we travel $\frac{1}{40}$ hour per km, while at the faster speed we travel $\frac{1}{60}$ hour per km. To arrive on time we should travel $\left(\frac{1}{40}+\frac{1}{60}\right)\div 2=\frac{1}{48}$ hour per km, i.e. at a speed of 48 km/h.

Alternate solution:

	Distance	Speed	Time
Trip A (1 h early)	60 (x – 1)	60	(x – 1)
Trip B (1 h late)	40 (x + 1)	40	(x + 1)
Trip C (on time)		?	x

$60(x-1)=40(x+1)$

$\therefore 60x-60=40x+40$

$\therefore 20x=100$

$\therefore x=5$

Therefore distance is 240 km

and Speed = $\frac{240}{5}=48km/h$