HARMONY

THE HARMONY GOLD SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with HARMONY GOLD MINING, AMESA and SAMS

SECOND ROUND 2002

SENIOR SECTION: GRADES 10, 11 AND 12

21 May 2002

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

DRAAI DIE BOEKIE OM VIR AFRIKAANS

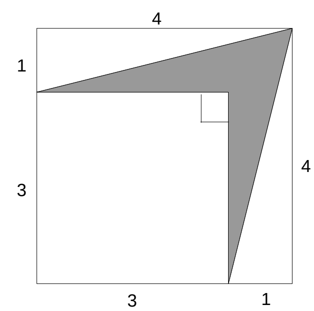
Private Bag X11, ARCADIA, 0007 TEL: 012 328-5082
FAX: 012 328-5091 E-mail: ellieo@mweb.co.za

PRACTICE EXAMPLES

1. If 1 ton = 1 000 kg then Aarnout the fully grown hippo weighs about

(A) 12 kg (B) 120 kg (C) 1,2 ton (D) 120 ton (E) 1 200 ton.

2. The area of the shaded region in the square is



(A) 1 (B) 3 (C) 4 (D) 5 (E) 2

3. $2002 - 2001 + 2000 - 1999 + \cdots + 2 - 1$ equals

(A) 2002 (B) 0 (C) -1 (D) 1001 (E) -1001

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO

Part A: Four marks each.

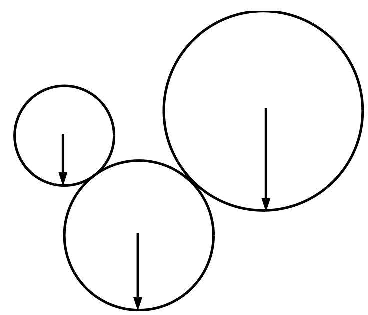
1. Remember that 1 000 cm^3 of water weighs 1 kg. During a rain shower, 10 mm of rain fell on a rectangular soccer field with dimensions 100 m by 50 m. The mass of rain that fell on the field was

(A) 0.5 ton (B) 5 kg (C) 50 ton (D) 50 kg (E) 5 ton

2. When the decimal point of a certain positive number is moved four places to the right, the new number is nine times the reciprocal of the original number. The original number was

(A) 0.0003 (B) 0.003 (C) 0.03 (D) 0.3 (E) 3

3. Three wheels touch as shown. Their radii are 6 cm, 15 cm and 20 cm. If there is no slip between the wheels, how many times will the smallest wheel turn before all the arrows point down again?



(A) 12 (B) 20 (C) 6 (D) 15 (E) 10

4. How many terms are there in the simplified expansion of

$$(a+b+c+d+e)(c+d+e+f+g)?$$

(A) 18 (B) 22 (C) 21 (D) 24 (E) 25

5. $2002^2 - 2001^2 + 2000^2 - 1999^2 + \cdots + 2^2 - 1^2$ equals

(A) 2 100 000 (B) 2 000 000 (C) 2 500 000 (D) 2 600 003 (E) 2 005 003

Part B: Five marks each

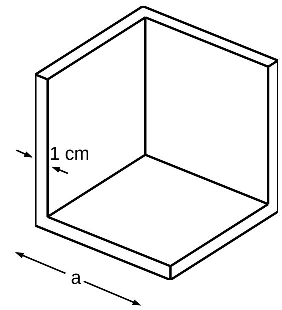
6. Let f be a function satisfying $f(xy) = \dfrac{f(x)}{y}$ for all positive real numbers x and y. If $f(30) = 20$, then the value of $f(40)$ is

(A) 40 (B) 20 (C) 60 (D) 15 (E) 30

7. It is known that $2^{2^r} + 1$ is prime for $r = 0, 1, 2, 3$ and 4, but not for $r = 5$. The number of prime factors of $2^{32} - 1$ is

(A) 1 (B) 6 (C) 3 (D) 5 (E) more than 6

8. The cut-out cube shown in the figure is made of steel, 1 cm thick. If its volume is 1 801 cm^3, then the value of the outside length a, in cm, is

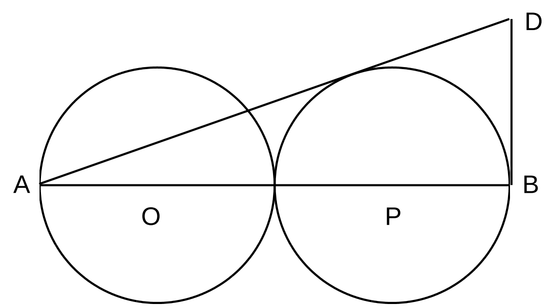


(A) 24 (B) 25 (C) 20 (D) 28 (E) 31

9. If m and n are positive integers such that $m^2 + 2n = n^2 + 2m + 5$, then the value of n is

(A) 4 (B) 3 (C) 1 (D) not unique (E) impossible to determine

10. Two circles of radius 2 and centres O and P touch each other as shown in the figure. If AD and BD are tangents, then the length of BD is



(A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $2\sqrt{3}$ (E) $\frac{2\sqrt{3}}{3}$

11. A breakfast cereal company ran a contest in which every box of cereal contained a number between 1 and 1 000 (inclusive). One lucky number was drawn from all 1 000 numbers each night for 20 nights. One man bought so much cereal that he had 500 different numbers, but not one of his numbers was lucky in any one of the draws. The probability of this happening in a fair contest is approximately

(A) 1 in 100 (B) 1 in 100 000 (C) 1 in 10 (D) 1 in 1 000 000 (E) 1 in 1 000

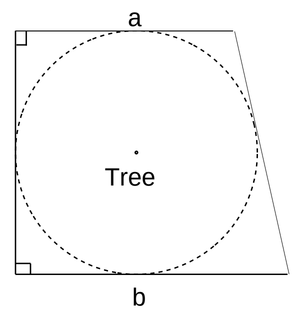
12. Three positive integers are given. Two of these are selected, their average is calculated and the result added to the third integer. This can be done in three different ways and the numbers 23, 31 and 32 are obtained. One of the original integers was

(A) 21 (B) 23 (C) 28 (D) 5 (E) 25

13. A sequence of numbers, $t_1, t_2, t_3, \ldots$, is defined by the formulas, $t_1 = 1$ and $t_n = \dfrac{t_{n-1}+2}{t_{n-1}+1}$ for $n = 2, 3, \ldots$. As n gets larger and larger, t_n approaches

(A) $\dfrac{13}{2+\sqrt{5}}$ (B) $\sqrt{2}$ (C) $\dfrac{14}{3+2\sqrt{2}}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{4}{1+\sqrt{3}}$

14. A garden is in the shape of a quadrilateral with two adjacent right angles as shown. The lengths of the two parallel sides are a and b. If there is a tree in the garden the same distance from all four sides, then the area of the garden is



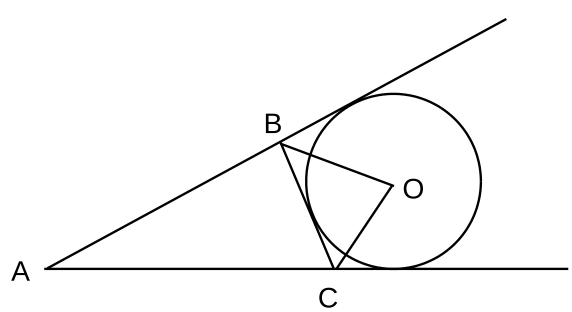
(A) $\dfrac{(a^2+b^2)}{2}$ (B) $a^2 - ab + b^2$ (C) $\dfrac{(a+b)^2}{4}$ (D) ab (E) $\dfrac{(a^2+ab+b^2)}{3}$

15. If $s = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \cdots$, then $1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots$ equals

(A) $\dfrac{1}{4} - s$ (B) $\dfrac{s}{2}$ (C) $\dfrac{s}{2} - 2$ (D) $\dfrac{s-1}{2}$ (E) $s - \dfrac{1}{2}$

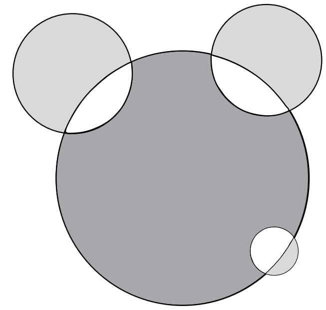
Part C: Six marks each

16. In the figure triangle ABC is formed by three tangents to the circle with center O. If $B\widehat{A}C = 30°$, then $B\widehat{O}C$ equals



(A) 55° (B) 60° (C) 65° (D) 70° (E) 75°

17. The diameters of the four circles shown in the figure are 6, 4, 4 and 2. If v is the area of the shaded region inside the biggest circle and w is the total shaded area of the three smaller circles, then



(A) $2v = w$ (B) $3v = w$ (C) $v = w$ (D) $2v = 3w$ (E) $v = 2w$

18. A polygon has n sides all of equal length s. If the area of the polygon is A, then the sum of the shortest distances from any point inside the polygon to each of the sides (produced if necessary) is

(A) $\frac{ns}{2}$ (B) $\frac{A}{ns}$ (C) $\frac{nA}{s}$ (D) $\frac{2A}{ns}$ (E) $\frac{2A}{s}$

19. The sum of the first 121 numbers in Pascal's triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \end{array}$$

is

(A) $2^{16} - 1$ (B) 7 381 (C) $2^{15} + 15$ (D) 8 641 (E) 2^{15}

20. Alice and Bob play a game in which each player draws a card from a pack of ten cards, numbered $0, 1, 2, \ldots, 9$ and then replaces it in the pack. Suppose Alice draws card A and Bob draws card B.

If $A > B$, then Bob pays Alice $A \times B$ rands
If $B > A$, then Alice pays Bob $A \times B$ rands
If $A = B$ and odd, then Bob pays Alice $A \times B$ rands
If $A = B$ and even, then Alice pays Bob $A \times B$ rands

Over a large number of games, the average result per game is

(A) Alice wins R1,00 (B) Bob wins R1,00 (C) Neither player wins anything
(D) Alice wins R0,45 (E) Bob wins R0,45