OLD MUTUAL
SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 FIRST ROUND
JUNIOR SECTION: GRADE 9

11 March 2021 **Time: 60 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by five answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 5 marks.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *The centre page is an information and formula sheet. Please tear out the page for your own use.*
7. *Start when the invigilator tells you to do so.*
8. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9372 Email: info@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns, ASTEMI

1. $20{,}21 + 20 + 2{,}1 =$

(A) 40,33 (B) 41,21 (C) 42,31 (D) 43,42 (E) 44,20

2. The time is now 20:21. What will the time be after 200 minutes?

(A) 21:41 (B) 22:21 (C) 22:41 (D) 23:21 (E) 23:41

3. Which of the following is closest to a quarter of 2021?

(A) 505 (B) 404 (C) 55 (D) 50 (E) 44

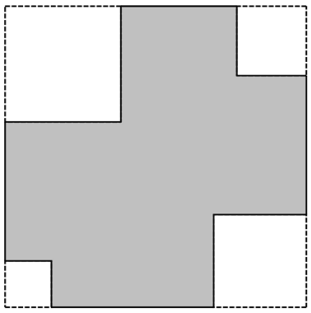
4. $\sqrt{\sqrt{20 + 20 + 20 + 21}} =$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. A whole number between 10 and 30 has the following properties. It is not even, it is not a prime number, and it is not divisible by 3. What is the number?

(A) 17 (B) 19 (C) 23 (D) 25 (E) 27

6. A house is in the shape of the shaded area below. Four square gardens are on the corners of the property. The entire property is also in the shape of a square and has an area of 400 m^2. What is the perimeter of the house in metres?

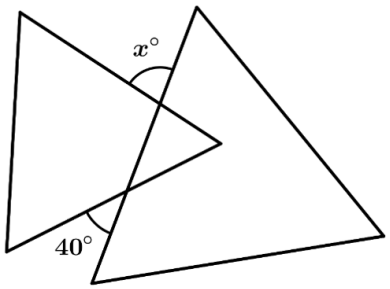


(A) 50 (B) 60 (C) 70 (D) 80 (E) 90

7. What is the largest 2-digit number that is the sum of two different perfect squares?

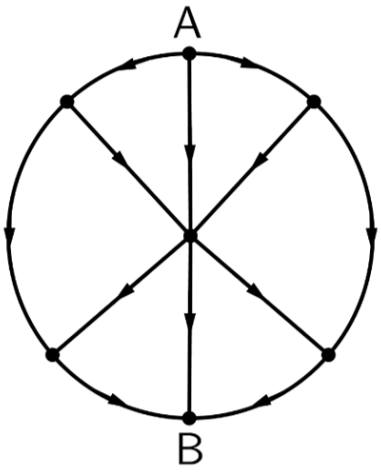
(A) 85 (B) 89 (C) 97 (D) 98 (E) 99

8. Two equilateral triangles overlap as shown. Determine the value of x.



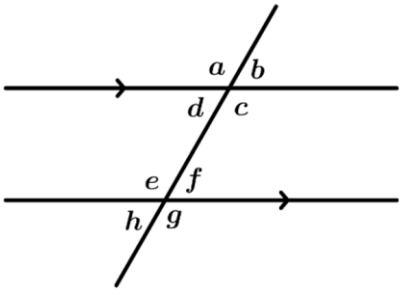
(A) 90° (B) 85° (C) 80° (D) 70° (E) 60°

9. If you can only travel in the directions indicated by the arrows, how many pathways are there from A to B?



(A) 15 (B) 13 (C) 11 (D) 9 (E) 7

10. The diagram shows three straight line segments. Two of the lines are parallel, as shown. Which statement is ALWAYS true?



(A) $a = f$ (B) $d = c$ (C) $g = b$ (D) $b = h$ (E) $e = d$

11. In a 5-digit numerical code each group of four adjacent digits adds to 19 and each group of three adjacent digits adds to 15. What is the sum of all five digits?

(A) 23 (B) 24 (C) 25 (D) 30 (E) 34

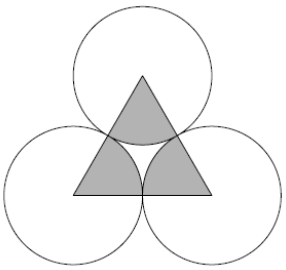
12. A 6-digit code has the following properties:

- It is palindromic (i.e. it reads the same backwards as forwards)
- The 3rd digit is twice the 1st digit
- The 5th digit is one more than the 4th digit
- The 2nd digit is 7

What is the sum of the six digits?

(A) 32 (B) 33 (C) 34 (D) 35 (E) 36

13. An equilateral triangle has sides of length 8 cm. At each vertex a circle with radius 4 cm is drawn. The area of the shaded region is $a \times \pi\ cm^2$. Determine the value of a.

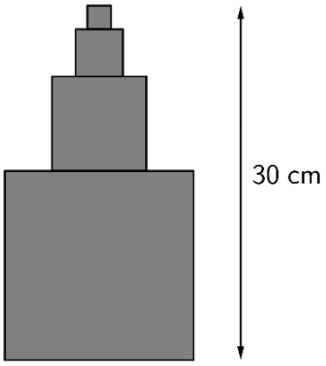


(A) 6 (B) 8 (C) 9 (D) 12 (E) 15

14. Hamdani is out running. He is now $\frac{3}{5}$ (three-fifths) of the way through the second half of his run. What fraction of the whole run has he completed?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{7}{10}$ (D) $\frac{4}{5}$ (E) $\frac{9}{10}$

15. A tower of four squares is shown. The area of each square is a quarter of the area of the square just below it. If the height of the tower is 30 cm, what is the side length of the largest square in cm?



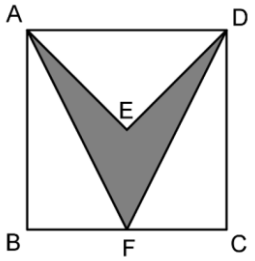
(A) 15 (B) 16 (C) 18 (D) 20 (E) 22

16. Determine the positive integer n that satisfies the following equation:

$$\frac{1}{20^{21}} + \frac{1}{20^{22}} + \frac{1}{20^{23}} = \frac{n}{20^{23}}$$

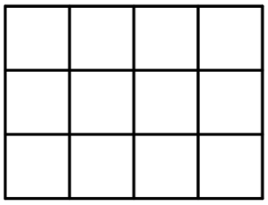
(A) 420 (B) 421 (C) 422 (D) 423 (E) 424

17. ABCD is a square with area 12 cm^2. E is the centre of the square, and F is the midpoint of BC. What is the area of the shaded region in cm^2?



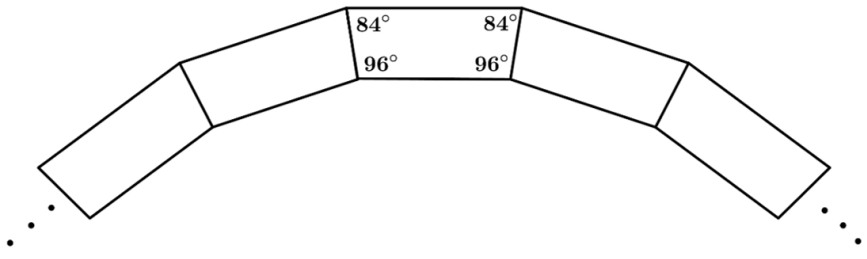
(A) 5 (B) 4,5 (C) 4 (D) 3,5 (E) 3

18. The rectangle has been divided into 12 identical squares. If the length of the diagonal of one of the squares is $2\sqrt{2}$ then determine the diagonal of the rectangle.



(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

19. Identical isosceles trapeziums are placed end to end in the form of a ring. How many trapeziums are needed in total to form a complete ring?



(A) 30 (B) 31 (C) 32 (D) 33 (E) 34

20. If m and n are positive integers, and $m^3 + \frac{n^2}{2} = 45$, then determine the value of $m + n$.

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Formula and Information Sheet

1.1 The natural numbers are: 1; 2; 3; 4; 5; …

1.2 The whole numbers are: 0; 1; 2; 3; 4; 5; …

1.3 The integers are: …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$a \times a \times a \times a \times \ldots \times a = a^n$ (n factors of a)

(a is the base and n is the index (exponent))

3.2 Factorial notation:

2! = 2 x 1 = 2

3! = 3 x 2 x 1 = 6

4! = 4 x 3 x 2 x 1 = 24

1 x 2 x 3 x …. x n = n!

3.3 $1 + 2 + 3 + 4 \ldots. + n = \frac{1}{2}n(n+1)$

4 Area of a

4.1	triangle is:	$\frac{1}{2} \times$ (base $\times$ height) = $\frac{1}{2}(b.h)$
4.2	rectangle is:	length $\times$ width = lw length $\times$ breadth = lb
4.3	square is:	side $\times$ side = s^2
4.4	rhombus is:	$\frac{1}{2} \times$ (product of diagonals)
4.5	trapezium is:	$\frac{1}{2} \times$ (sum of parallel sides) $\times$ height
4.6	circle is:	πr^2 (r = radius)

5 Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh\,(h = \text{height})$

5.2 sphere is: $4\pi r^2$

6 Perimeter of a:

6.1 rectangle is: $2 \times \text{length} + 2 \times \text{breadth}$

$2l + 2b$

or $2l + 2w$ $(w = \text{width})$

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s \times s \times s = s^3$

8.2 rectangular prism is: $l \times b \times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section × perpendicular height

or area of base × perpendicular height

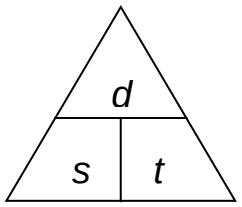
9.2 Surface area of a right prism is: (perimeter of base × h) + (2 × area of base)

10. Sum of the interior angles of a polygon is: $180^\circ(n-2)$ $[n = \text{number of sides}]$

11. Distance = speed × time $(d = s \times t)$

Speed = distance ÷ time $(s = \frac{d}{t})$

Time = distance ÷ speed $(t = \frac{d}{s})$

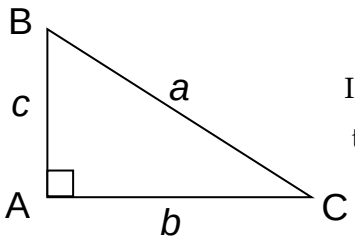


$d = s \times t$

$s = \frac{d}{t}$

$t = \frac{d}{s}$

12. Pythagoras:



If ΔABC is a right-angled triangle, then $a^2 = b^2 + c^2$

13. Conversions:

1 cm³ = 1 mℓ ; 1000 cm³ = 1 ℓ

1000 m = 1 km ; 1000 g = 1 kg ; 100 cm = 1 m

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5. Buite-oppervlakte van 'n

5.1 regte prisma is: $2lb + 2lh + 2bh$ (h = hoogte)

5.2 sfeer is: $4\pi r^2$

6 Omtrek van 'n:

6.1 reghoek is: 2×lengte + 2×breedte
$2l + 2b$

6.2 vierkant is: $4s$

7. Omtrek van 'n sirkel is: $2\pi r$

8. Volume van 'n:

8.1 kubus is: $s \times s \times s = s^3$

8.2 reghoekige prisma is: $l \times b \times h$

8.3 silinder is: $\pi r^2 h$

9.1 Volume van 'n regte prisma is: oppervlakte van dwarssnit × hoogte
of oppervlakte van basis × hoogte

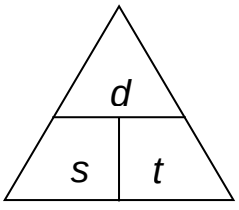
9.2 Buite-oppervlakte van 'n regte prisma is:
(omtrek van basis × h) + (2 × oppervlakte van basis)

10. Som van die binnehoeke van 'n veelhoek is: $180^\circ(n-2)$ [n = aantal sye]

11. Afstand is: spoed x tyd $(d = s \times t)$

Spoed = afstand ÷ tyd $(s = \frac{d}{t})$

Tyd = afstand ÷ spoed $(t = \frac{d}{s})$

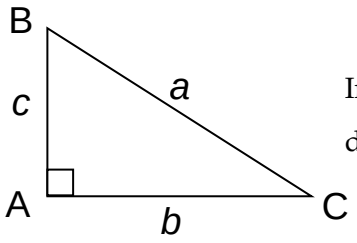


$d = s \times t$

$s = \frac{d}{t}$

$t = \frac{d}{s}$

12. Pythagoras:



Indien ΔABC 'n reghoekige driehoek is, dan sal $a^2 = b^2 + c^2$

13. Omskakelings:

1 cm³ = 1 mℓ ; 1000 cm³ = 1 ℓ

1000 m = 1 km; 1000 g = 1 kg ; 100 cm = 1 m

Formule- en Inligtingblad

1.1 Die natuurlike getalle is: 1; 2; 3; 4; 5; …

1.2 Die telgetalle is: 0; 1; 2; 3; 4; 5; …

1.3 Die heelgetalle is: …, –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In die breuk $\frac{a}{b}$, word a die teller en b die noemer genoem.

3.1 Eksponensiële notasie:

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$a \times a \times a \times a \times \ldots\ldots \times a = a^n$ (n faktore van a)

(a is die grondtal en n is die indeks (eksponent))

3.2 Fakulteitnotasie:

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$1 \times 2 \times 3 \times \ldots. \times n = n!$

3.3 $1 + 2 + 3 + 4 \ldots. + n = n(n + 1)/2$

4 Oppervlakte van 'n:

4.1 driehoek is: $\frac{1}{2} \times$ (basis $\times$ loodregte hoogte) = $\frac{1}{2}(b.h)$

4.2 reghoek is: lengte $\times$ breedte = lb

4.3 vierkant is: sy $\times$ sy = s^2

4.4 ruit (rombus) is: $\frac{1}{2}$ (produk van die diagonale)

4.5 trapesium is: $\frac{1}{2}(\text{som van ewewydige sye}) \times \text{hoogte}$

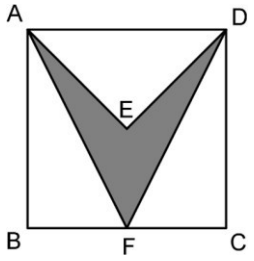
4.6 sirkel is: πr^2 (r = radius)

16. Bepaal die positiewe heelgetal n wat die volgende vergelyking bevredig:

$$\frac{1}{20^{21}} + \frac{1}{20^{22}} + \frac{1}{20^{23}} = \frac{n}{20^{23}}$$

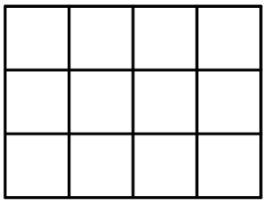
(A) 420 (B) 421 (C) 422 (D) 423 (E) 424

17. ABCD is 'n vierkant met oppervlakte 12 cm^2. E is die middelpunt van die vierkant en F is die middelpunt van BC. Wat is die oppervlakte van die ingekleurde gedeelte in cm^2?



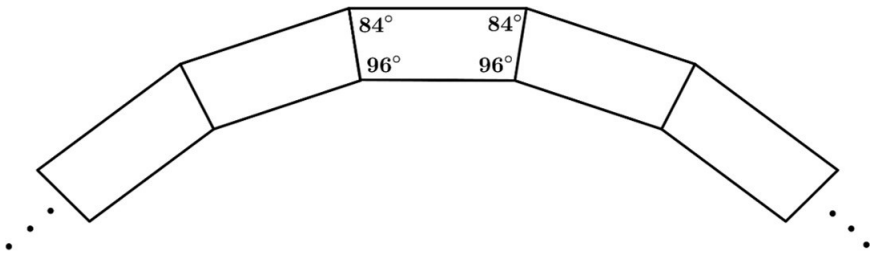
(A) 5 (B) 4,5 (C) 4 (D) 3,5 (E) 3

18. Die reghoek is opgedeel in 12 identiese vierkante. Indien die lengte van die hoeklyn van een van die vierkante $2\sqrt{2}$ is, bepaal die lengte van die hoeklyn van die reghoek.



(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

19. Identiese gelykbenige trapesiums word teenaan mekaar geplaas in die vorm van 'n ring. Hoeveel trapesiums is nodig om die volledige ring te voltooi?



(A) 30 (B) 31 (C) 32 (D) 33 (E) 34

20. As m en n positiewe heelgetalle is en $m^3 + \frac{n^2}{2} = 45$, bepaal die waarde van $m + n$.

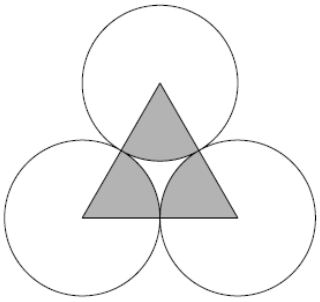
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

12. 'n 6-syfer kode het die volgende eienskappe:
- Dit is 'n palindroom (i.e. dit is dieselfde van links en van regs)
- Die 3de syfer is dubbel die 1ste syfer
- Die 5de syfer is een meer as die 4de syfer
- Die 2de syfer is 7

Wat is die som van die ses syfers?

(A) 32 (B) 33 (C) 34 (D) 35 (E) 36

13. 'n Gelyksydige driehoek het sylengtes van 8 cm. By elke hoek word 'n sirkel met radius 4 cm geteken. Die oppervlakte van die ingekleurde deel is $a \times \pi\ cm^2$. Bepaal die waarde van a.

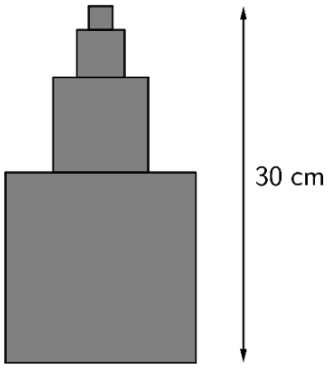


(A) 6 (B) 8 (C) 9 (D) 12 (E) 15

14. Hamdani is besig om te draf. Hy is $\frac{3}{5}$ (drie-vyfdes) klaar met die tweede helfte van sy roete. Watter breukdeel van die roete het hy reeds voltooi?

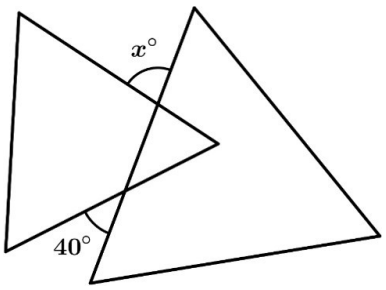
(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{7}{10}$ (D) $\frac{4}{5}$ (E) $\frac{9}{10}$

15. 'n Toring van vier vierkante word aangetoon. Die oppervlakte van elke vierkant is 'n kwart van die oppervlakte van die vierkant onder dit. As die hoogte van die toring 30 cm is, wat is die sylengte van die grootste vierkant in cm?



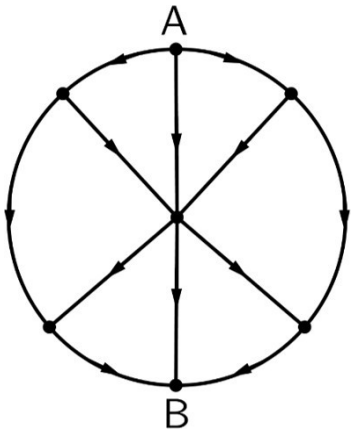
(A) 15 (B) 16 (C) 18 (D) 20 (E) 22

8. Twee gelyksydige driehoeke lê bo-oor mekaar soos aangetoon. Bepaal die waarde van x.



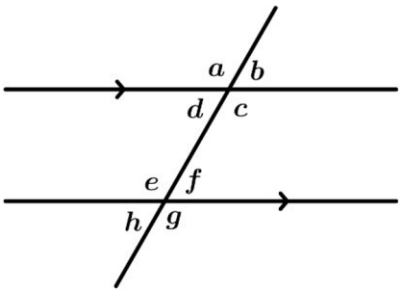
(A) 90° (B) 85° (C) 80° (D) 70° (E) 60°

9. In die diagram kan jy slegs in 'n rigting soos deur die pyltjies aangetoon word, beweeg. Hoeveel verskillende maniere is daar om van A na B te beweeg?



(A) 15 (B) 13 (C) 11 (D) 9 (E) 7

10. Die skets toon drie reguitlyn-segmente. Twee van die lyne is ewewydig, soos aangedui. Watter stelling is ALTYD waar?



(A) $a = f$ (B) $d = c$ (C) $g = b$ (D) $b = h$ (E) $e = d$

11. In 'n 5-syfer numeriese kode tel elke groep van vier aangrensende syfers op na 19 en elke groep van drie aangrensende syfers tel op na 15. Wat is die som van al vyf syfers?

(A) 23 (B) 24 (C) 25 (D) 30 (E) 34

1. $20{,}21 + 20 + 2{,}1 =$

(A) 40,33 (B) 41,21 (C) 42,31 (D) 43,42 (E) 44,20

2. Die tyd is nou 20:21. Hoe laat sal dit oor 200 minute wees?

(A) 21:41 (B) 22:21 (C) 22:41 (D) 23:21 (E) 23:41

3. Watter een van die volgende is die naaste aan 'n kwart van 2021?

(A) 505 (B) 404 (C) 55 (D) 50 (E) 44

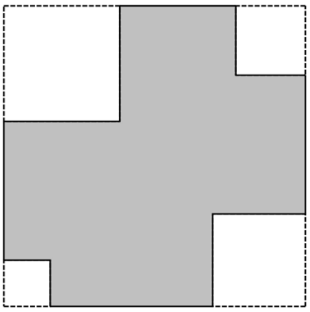
4. $\sqrt{\sqrt{20 + 20 + 20 + 21}} =$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. 'n Heelgetal tussen 10 en 30 het die volgende eienskappe: dit is nie 'n ewe getal nie, dit is nie 'n priemgetal nie en dit is nie deelbaar deur 3 nie. Wat is die getal?

(A) 17 (B) 19 (C) 23 (D) 25 (E) 27

6. 'n Huis is in die vorm van die ingekleurde oppervlakte hieronder. Vier vierkantige tuine is op die hoeke van die erf. Die erf is ook 'n vierkant en het 'n oppervlakte van 400 m^2. Wat is die omtrek van die huis in meter?



(A) 50 (B) 60 (C) 70 (D) 80 (E) 90

7. Wat is die grootste 2-syfer getal wat die som van twee verskillende volkome vierkante is?

(A) 85 (B) 89 (C) 97 (D) 98 (E) 99

OLD MUTUAL
SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 EERSTE RONDTE
JUNIOR AFDELING: GRAAD 9

11 Maart 2021 **Tyd: 60 minute** **Aantal vrae: 20**

Instruksies

1. *Hierdie is 'n veelvuldige-keuse vraestel. Na elke vraag is vyf antwoorde, genommer A, B, C, D en E. Net een van hulle is reg.*
2. *Puntetoekenning:*
   2.1. *Elke korrekte antwoord tel 5 punte.*
   2.2. *Daar is geen penalisering vir foutiewe antwoorde of vrae wat nie beantwoord is nie.*
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Die binneblad is 'n inligtings- en formuleblad. Skeur dit asseblief uit vir jou gebruik.*
7. *Begin sodra die toesighouer die teken gee.*
8. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

PRIVAATSAK X173, PRETORIA, 0001
TEL: (012) 392-9372 E-pos: info@samf.ac.za

Organisasies betrokke: AMESA, SA Wiskundevereniging,
SA Akademie vir Wetenskap en Kuns, ASTEMI