THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

FIRST ROUND 1998

SENIOR SECTION: GRADES 10, 11 AND 12
(STANDARDS 8, 9 AND 10)

10 MARCH 1998

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 7 marks in Part C.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Give your answers on the sheet provided.
7. When the invigilator gives the signal, start attempting the problems. You will have 60 minutes working time for the question paper.

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6.

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π.

3. The sum of the smallest and the largest of the numbers 0,5129; 0,9; 0,89; and 0,289 is

(A) 1,189
(B) 0,8019
(C) 1,428
(D) 1,179
(E) 1,4129.

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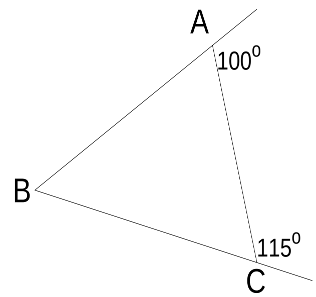
Part A: Three marks each.

1. The value of $\frac{1\times9\times9\times8}{1+9+9+8}$ is

(A) 1 (B) 4 (C) 0 (D) 27 (E) 24

2. In the diagram, the size of the angle ABC in degrees, is

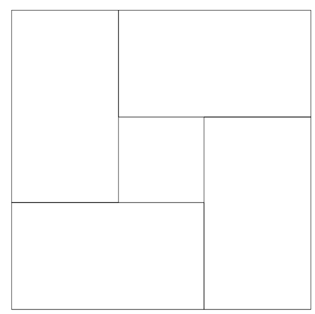


(A) 25 (B) 145 (C) 35 (D) 215 (E) 45

3. If $\sqrt{x+1} = 3$ then $(x+2)^2$ equals

(A) 25 (B) 36 (C) 64 (D) 81 (E) 100

4. The diagram shows 4 bricks enclosing a square. Each brick has length 18 and breadth 10. The length of the side of the enclosed square is



(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

5. The greatest number of Mondays which can occur in 45 consecutive days is

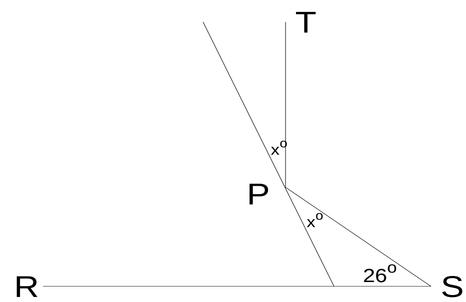
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Part B: 5 marks each

6. When Allan Donald bowls, the ball travels at 144 kilometres per hour. If the distance the ball travels is 20 metres then the time, in seconds, that the ball travels is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$ (E) $\frac{2}{3}$

7. A beam of light shines from a point S, reflects off a reflector at point P, and reaches a point T so that PT is perpendicular to RS. Then x is



(A) 26^o (B) 32^o (C) 37^o (D) 38^o (E) 45^o

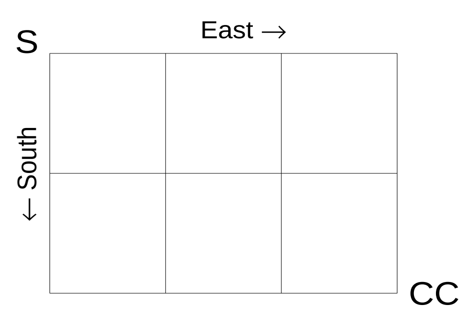
8. If $a + b = 3$, $b + c = 4$ and $c + a = 5$, then $a + b + c$ equals

(A) 6 (B) 7 (C) 8 (D) 9 (E) 12

9. The volume of a rectangular box is 6480 cubic centimetres, and the lengths of the sides are in the ratio 2:3:5. The length, in centimetres, of the shortest side of the box is

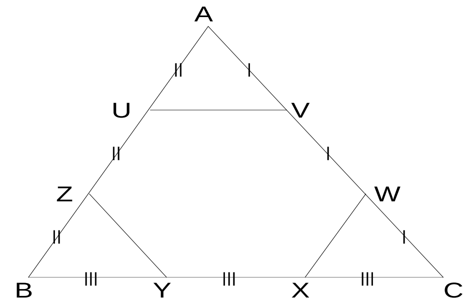
(A) 6 (B) 10 (C) 12 (D) 8 (E) 16

10. A march goes through the streets of a town from the school (S) to the community centre (CC). If the march can only travel East or South, then the number of different shortest routes is



(A) 6 (B) 10 (C) 4 (D) 8 (E) 9

11. Each side of triangle ABC is divided into 3 equal parts. The ratio of the area of the hexagon UVWXYZ to the area of triangle ABC is



(A) $\frac{5}{9}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) $\frac{5}{8}$

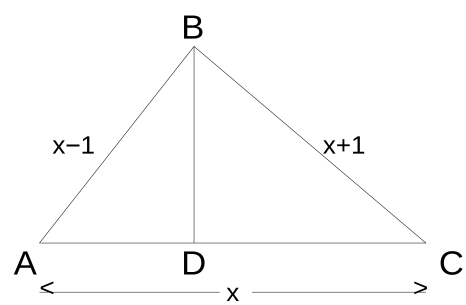
12. p, q and r are positive numbers for which $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=1$ and $pq=s$. Then $p+q$ equals

(A) $\frac{rs}{r-1}$ (B) $1-\frac{1}{r}-\frac{1}{s}$ (C) $\frac{r-1}{rs}$ (D) $\frac{s}{r}(r-1)$ (E) $s(r-1)$

13. If $2^x+3^y=41$, where x and y are natural numbers, then the value of $x+y$ is

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

14. The lengths of the sides of the acute angled triangle ABC are $x-1$, x and $x+1$. BD is perpendicular to AC. Then $CD-DA$ equals



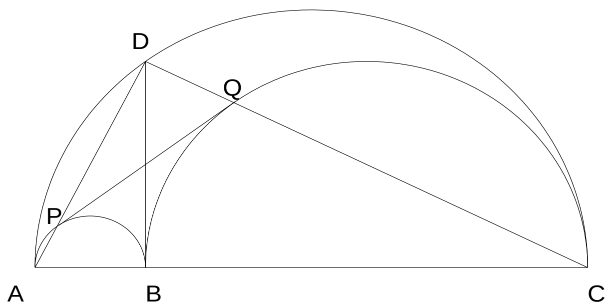
(A) $\frac{x}{9}$ (B) 2 (C) $\frac{x}{8}$ (D) 4 (E) $\sqrt{x}$

15. The area of a rectangle is numerically equal to twice its perimeter and both the length and the breadth are positive integers. How many such rectangles are possible?

(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3

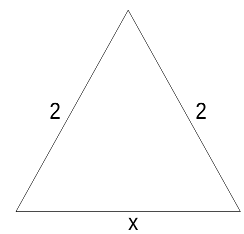
Part C: 7 marks each

16. The diagram shows three semicircles which are mutually tangent and have their diameters on the line AC. The line DB is perpendicular to AC with DA and DC intersecting the two smaller circles at P and Q respectively. If DB = 10 then the length of PQ is



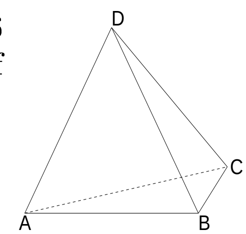
(A) $3\sqrt{10}$ (B) $6\sqrt{3}$ (C) $4\sqrt{6}$ (D) $7\sqrt{2}$ (E) None of these

17. An isosceles triangle has sides of length 2, 2 and x. For which value of x is the area of the triangle a maximum?



(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$ (E) 3

18. The six edges of tetrahedron ABCD measure 7, 13, 18, 27, 36 and 41 units. If the length of edge AB is 41 what is the length of edge CD?



(A) 7 (B) 13 (C) 18 (D) 27 (E) 36

19. Let $p(x) = x^2 + bx + c$, where b and c are integers. If $p(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, the value of $p(1)$ equals

(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

20. A train which left Johannesburg at time $x : y$ (which means y minutes after x o'clock) reached Pretoria at time $y : z$ on the same day, after traveling z hours and x minutes. (All the times are given on a 24 hour clock.) How many possible values of x are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3