THE SOUTH AFRICAN
MATHEMATICS OLYMPIAD

FIRST ROUND 2001: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

Thank you for entering the First Round of the Mathematics Olympiad.

Many of the solutions to the problems which are given below use trial-and-error methods or an investigative approach.

Most of them can also be solved using more formal mathematical methods which you will learn in due course. Sometimes we have also given a more formal solution which you might find interesting.

PRACTICE EXAMPLES:

1. $23+6-4=$

(A) 6 (B) 23 (C) 25 (D) 29 (E) 33

ANSWER: C

$23+6-4=(23+6)-4=29-4=25$

2. $\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}$ equals

(A) $\frac{1}{15}$ (B) $\frac{3}{11}$ (C) $\frac{21}{50}$ (D) $\frac{8}{15}$ (E) $9\frac{4}{5}$

ANSWER: D

$\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}=\frac{1}{5}+(\frac{2}{3}\times\frac{1}{2})=\frac{1}{5}+\frac{1}{3}=\frac{3+5}{15}=\frac{8}{15}$

QUESTIONS:

1. When 2001 is divided by 200 the remainder is

(A) 0 (B) 1 (C) 9 (D) 10 (E) 99

ANSWER: B

$2001=200\times10+1$

$\therefore$ The remainder is 1

2. If $241 \times 39 = 9399$ then $2{,}41 \times 3{,}9$ is equal to

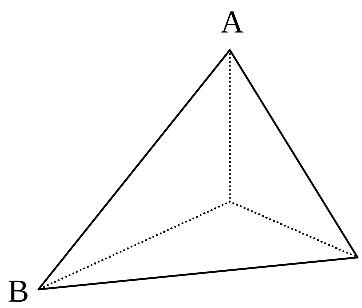
(A) 0,09399 (B) 0,9399 (C) 9,399 (D) 93,99 (E) 939,9

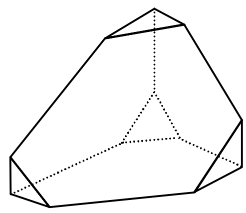
<u>ANSWER:</u> C

$$2{,}41 \times 3{,}9 = \frac{241}{100} \times \frac{39}{10} = \frac{9399}{1000} = 9{,}399$$

3. A solid triangular pyramid has six edges such as AB.
Each corner is cut off.
(see new figure)

How many edges will the new figure have?





(A) 24 (B) 9 (C) 12 (D) 15 (E) 18

<u>ANSWER:</u> E

The 6 original edges remain, and each "cut-off corner" produces 3 new edges.
$\therefore$ The number of edges is $6 + 4 \times 3 = 18$

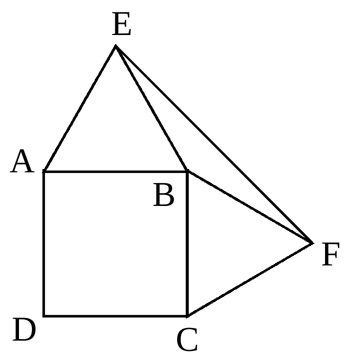
4. If $a \,\Delta\, b = a^2 - b^2$ then $5 \,\Delta\, 3$ is equal to

(A) 2 (B) 15 (C) 4 (D) 16 (E) 9

<u>ANSWER:</u> D

$5 \,\Delta\, 3 = 5^2 - 3^2 = 25 - 9 = 16$

5. ABCD is a square and EAB and CFB are equilateral triangles.



The size of $\hat{BEF}$ is

(A) $7{,}5^\circ$ (B) 10° (C) $11{,}25^\circ$ (D) $12{,}5^\circ$ (E) 15°

ANSWER: E

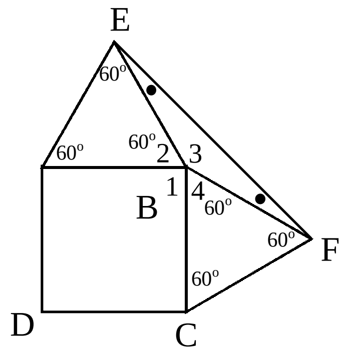
$\hat{B}_3 = 360^\circ - (\hat{B}_1 + \hat{B}_2 + \hat{B}_4)$ [$\angle$'s round a point]

$= 360^\circ - (90^\circ + 60^\circ + 60^\circ)$ [square and equilateral Δ's]

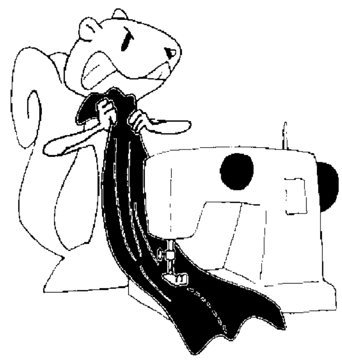
$= 150^\circ$

$\therefore \hat{BEF} = \dfrac{180^\circ - 150^\circ}{2}$ [base $\angle$'s of isosceles ΔEBF]

$= 15^\circ$



6. A sewing machine stitches 0,6 kilometres of cloth in one hour.
The rate of stitching of the machine in metres per minute is



(A) 0,01 (B) 0,1 (C) 1
(D) 10 (E) 100

ANSWER: D

$0{,}6 \text{ km} = 600 \text{ m}$ and $1 \text{ h} = 60 \text{ min}$

$\therefore \text{Rate} = \dfrac{600 \text{ m}}{60 \text{ min}} = 10 \text{ m / min}$

7. The value of the fraction $\dfrac{10+20+30+40+...+400}{30+60+90+120+...+1200}$ is

(A) $\dfrac{1}{30}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{1}{3}$

ANSWER: E

$$\frac{10+20+...+400}{30+40+...+1200}=\frac{1}{3}\frac{(10+20+...+400)}{(10+20+...+400)}=\frac{1}{3}$$

OR

$$\frac{10+20+...+400}{30+40+...+1200}=\frac{10}{30}\frac{(1+2+...+40)}{(1+2+...+40)}=\frac{1}{3}$$

8. If $\dfrac{1}{x}+\dfrac{7}{3x}=\dfrac{5}{6}$ then the value of x is

(A) 2 (B) 5 (C) 6 (D) 4 (E) 3

ANSWER: D

$\dfrac{1}{x}+\dfrac{7}{3x}=\dfrac{5}{6}$ [Multiply both sides by $6x$]

$\therefore 6x.\dfrac{1}{x}+6x.\dfrac{7}{3x}=6x.\dfrac{5}{6}$ or $\dfrac{3+7}{3x}=\dfrac{5}{6}$

$\therefore 6+14=5x$

$\therefore 5x=20$

$\therefore x=4$

This question could also have been done by trial–and–error.

9. Did you know? A palindrome is a number which reads the same forwards as backwards e.g. 35453.

Next year 2002 is an example of a palindromic number. What is the difference between 2002 and the number of the previous palindromic year?

(A) 10 (B) 11 (C) 101 (D) 121 (E) 1001

ANSWER: B

Previous palindromic year: 1991

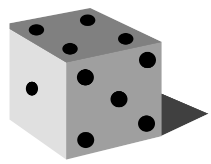
$\therefore 2002-1991=11$ years

10. 2001 people stand in a queue at a voting station.
There are at least 3 women between any two men.
The largest possible number of men in the queue is

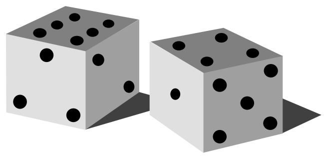
(A) 500 (B) 501 (C) 502 (D) 667 (E) 668

ANSWER: B
To maximize the number of men, place the minimum number of women between them. So MWWW MWWW … MWWW M (with 500 groups of the form MWWW). There are at most 500 + 1 =501 men.

11. When a die is rolled the chance of obtaining a 5 is $\frac{1}{6}$



When two dice are rolled the chance of obtaining a sum less than 5 is



(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{5}{6}$ (D) $\frac{2}{9}$ (E) $\frac{5}{36}$

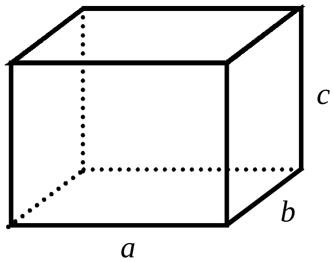
ANSWER: A
Total number of possible outcomes is:
$6 \times 6 = 36$
Ways to obtain less than 5: 1 + 1; 1 + 2; 1 + 3; 2 + 1; 2 + 2; 3 + 1
(See rings in diagram)
Thus chance of obtaining less than 5 is $\frac{6}{36} = \frac{1}{6}$.

Die 1

Die 2

+	1	2	3	4	5	6
1	2	3	4	5		
2	3	4	5			
3	4	5				
4	5					
5						
6						

12. The volume of a rectangular prism with length a cm, width b cm and height c cm is 240 cm^3. $a+b+c=19$. Each side is 3 cm or more in length. a, b and c are whole numbers. The largest possible area of a face in cm^2 is



(A) 15 (B) 30 (C) 40 (D) 48 (E) 60

ANSWER: D

a:	3	3	3	3	4	4	5	6	…
b:	5	8	4	2	12	4	6	4	
c:	16	10	20	40	5	15	8	10	
Sum	24x	21x	27x	45x	21x	23x	19	20x	

This is probably done by trial-and-error starting with the factors of 240 and working through all possible combinations of which a few are given above. $240 = 2\times2\times2\times2\times3\times5$. Look for combinations that give a sum of 19.
Area of largest face: $6\times8=48\text{ cm}^2$.

OR

If $a=3$, then $b\times c=80$
and $b+c=16$, No answer.

If $a=4$, then $b\times c=60$
and $b+c=15$, No answer.

If $a=5$, then $b\times c=48$
and $b+c=14$, Yes for $b=8$ and $c=6$.

Again after all possible combinations have been considered, the area of the largest face will be: $6\times8=48\text{ cm}^2$.

13. The value of $499-497+495-493+...+3-1$ is

(A) 2 (B) 250 (C) 496 (D) 498 (E) 500

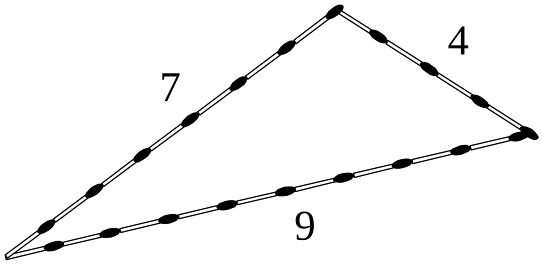
ANSWER: B

$(499-497)+(495-493)+...+(3-1)$
$=(2+2+...+2)\times125$
$=250$

From 499 to –1 gives 500 numbers. But this sequence uses every second number (odd numbers only) and therefore has 250 terms. Now we have grouped them in pairs hence we have 125 “2’s”

14. Did you know? The sum of the lengths of two sides of a triangle is always greater than the length of the third side.

Twenty matchsticks of equal length are placed to form a triangle, as shown. The total number of different triangles that can be made with a perimeter of 20 matchsticks is



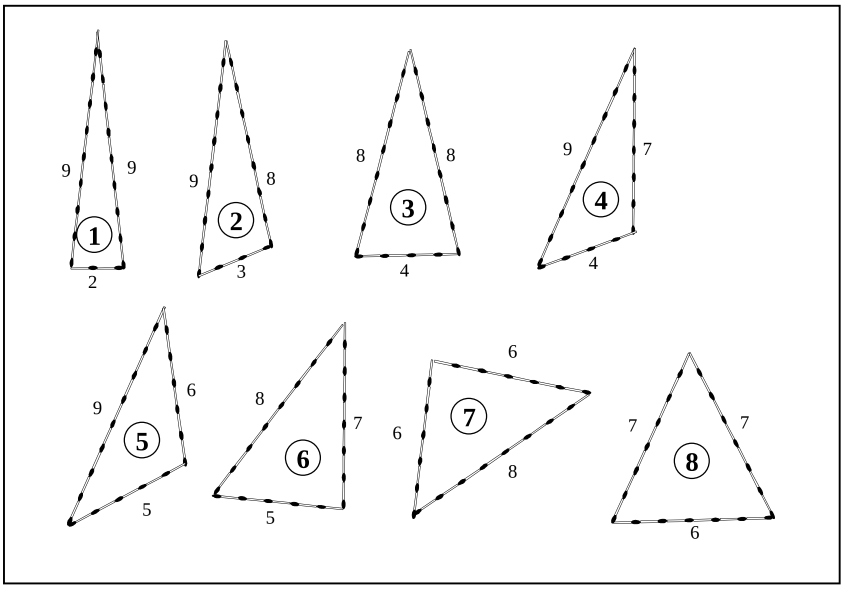
(A) 9 (B) 8 (C) 6 (D) 7 (E) 10

<u>ANSWER:</u> B

a:	2	3	4	4	5	5	6	6
b:	9	8	8	7	6	7	6	7
c:	9	9	8	9	9	8	8	7

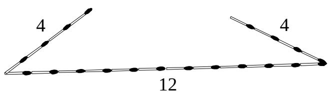
i.e. 8 triangles

Classroom suggestion: Do it practically with matchsticks in the classroom



Note: Triangles with different orientations but the same dimensions are the same.

Investigation: Why can't a side be equal to 12?



15. The sum of two consecutive numbers is S. The square of the larger number minus the square of the smaller number is

(A) S^2 (B) $2S$ (C) S (D) $S+1$ (E) $S-1$

<u>ANSWER:</u> C

Consecutive numbers are integers which follow each other on the number line, e.g. 1; 2 15; 16 *x; x* + 1.

$x+(x+1)=S$, therefor $2x+1=S$

Now, $(x+1)^2-x^2=x^2+2x+1-x^2=2x+1=S$

OR

Trial-and-error

$3+4=7$ $9+10=19$

$4^2-3^2=7$ and $10^2-9^2=19$ etc.

So S is the answer.

16. The sum of the digits of the product $999\,999\times 777\,777$ is

(A) 54 (B) 63 (C) 52 (D) 48 (E) 50

<u>ANSWER:</u> A

$999\,999\times 777\,777$

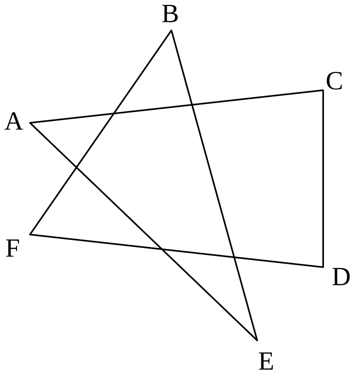
$=(1\,000\,000-1)\times 777\,777$ [Its easy to multiply with $1\,000\,000$]

$$\begin{array}{rr} & 777\,777\,000\,000 \\ - & 777\,777 \\ \hline = & 777\,776\,222\,223 \end{array}$$

$\therefore$ Sum of digits is 54

17. In the adjacent figure $\hat{C} + \hat{D} = 150^\circ$

The value of $\hat{A} + \hat{B} + \hat{E} + \hat{F}$ is



(A) 210° (B) 300° (C) 360°
(D) 390° (E) 570°

<u>ANSWER:</u> A

$\hat{A} + \hat{E} = x$ [ext.$\angle$ of ΔGAE]

$\hat{B} + \hat{F} = y$ [ext.$\angle$ of ΔHFB]

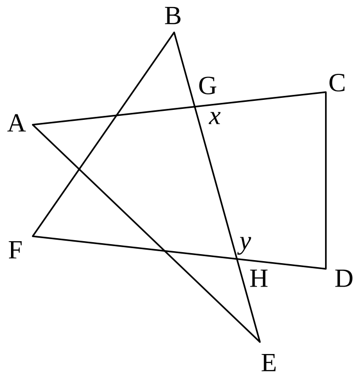
$\therefore \hat{A} + \hat{E} + \hat{B} + \hat{F} = x + y$

but $x + y = 360^\circ - (\hat{C} + \hat{D})$ [$\angle$'s of Quad.CGHD]

$= 360^\circ - 150^\circ$

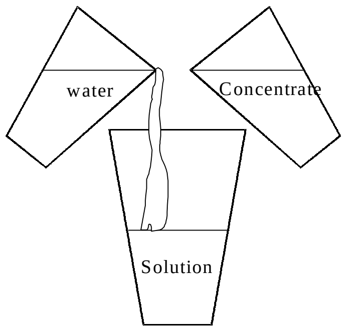
$= 210^\circ$

$\therefore \hat{A} + \hat{B} + \hat{E} + \hat{F} = 210^\circ$



18. A solution containing water and a liquid concentrate has 60% concentrate. By adding 20 litres of water to the solution the concentrate is reduced to 40% of the solution.

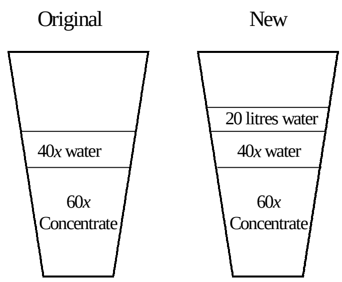
How many litres of the original solution is concentrate?



(A) 36 (B) 30 (C) 24
(D) 16 (E) 12

<u>ANSWER:</u> C

Suppose the original solution is made up of 60*x* litres of concentrate and 40*x* litres of water. Therefor 60% of the solution is concentrate.
Then, in the new solution the concentrate is 40% of the new solution.
The new solution is 100*x* + 20 litres.



$$\therefore 60x = \frac{40}{100}(100x + 20)$$

$$\therefore 60x = 40x + 8$$

$$\therefore 20x = 8$$

$$\therefore 60x = 24$$

So there were 24 litres of concentrate.

OR

Start with x litres in the solution.

$$0{,}6x = 0{,}4(x + 20)$$

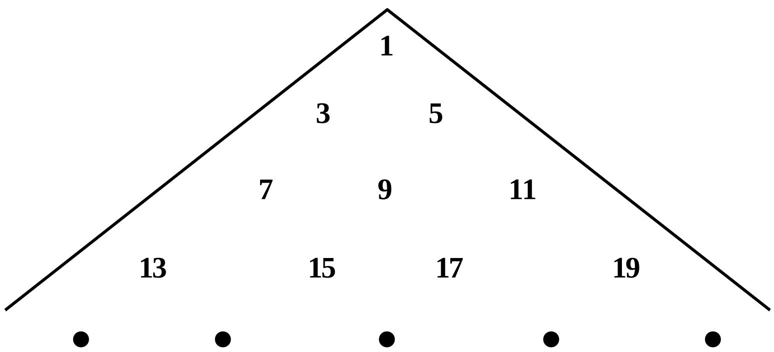
$$\therefore \frac{6}{10}x = \frac{4}{10}(x + 20)$$

$$\therefore \frac{2}{10}x = 8$$

$$x = 40$$

That means we have started of with 40 litres, 16 ℓ of water and 24 ℓ of consentrate.

19. The odd integers are arranged in a triangular pattern as shown.



If this pattern continues the first number in the row which has a sum of 1 000 000 is

(A) 99 (B) 990 001 (C) 991 (D) 9 901 (E) 99 001

<u>ANSWER:</u> D

Row	Sum	First number in row
1	$1 = 1^3$	$1 = 1 \times 0 + 1$
2	$8 = 2^3$	$3 = 2 \times 1 + 1$
3	$27 = 3^3$	$7 = 3 \times 2 + 1$
4	$64 = 4^3$	$13 = 4 \times 3 + 1$
etc.	etc.	etc.

Clearly we see that the above patterns continue, so $1000\,000 = 100^3$ means that we are dealing with the 100th row, and the first number in the row is $100 \times 99 + 1 = 9\,901$.

OR

The triangle consists of odd numbers only. In the first row there is one number, in the second row there are 2 numbers, in the third row, 3 numbers, etc. until we get 100 numbers in the 100th row. All number are consecutive odd numbers. In the first 99 rows there are $1 + 2 + 3 + 4 + \ldots 99$ odd numbers, which gives 4 950 such numbers. The 4 950th odd number is 9 899, the last number in row 99. The next odd number which is also the first number in the 100th row will then be 9 901.

20. The number of terms of the sequence $4^2; 5^2; 6^2; \ldots 39^2; 40^2$ that have an even digit in the tens place is

(A) 29 (B) 28 (C) 27 (D) 26 (E) 25

<u>ANSWER:</u> A

This can be done by investigating the sequence, observing the pattern and extrapolating (i.e. exploring the pattern further). Learners will discover that only numbers ending in 4 or 6 have squares whose tens digit is odd.

E.g. $4^2 = \underline{1}6$; $6^2 = \underline{3}6$ and $14^2 = 1\underline{9}6$, etc. Now, there are 37 terms in the sequence from 4^2 to 40^2, 8 of them are squares of numbers ending in 4 or 6. So 37 – 8 terms will have even tens digits.

This leave 37 – 8 = 29 terms with even tens digits.

OR

More formally, all the numbers in the sequence are of the form $(10a + b)^2$ with $0 \le a \le 4$ and $0 \le b \le 9$.

Now $(10a + b)^2 = 100a^2 + 20ab + b^2$, which has an even tens digit (because $20ab$ will always be even), except when b^2 involves an odd "carry-over", i.e. only when b = 4 or 6, which explains the above observation.

THE END

ANSWER POSITIONS: JUNIOR FIRST ROUND 2001

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	B
2	C
3	E
4	D
5	E
6	D
7	E
8	D
9	B
10	B
11	A
12	D
13	B
14	B
15	C
16	A
17	A
18	C
19	D
20	A

DISTRIBUTION	
A	4
B	5
C	3
D	5
E	3
TOTAL	**20**