SENIOR FIRST ROUND 2013

1. **Answer C.**
$\dfrac{\sqrt{49}}{49} = \dfrac{7}{49} = \dfrac{1}{7}$.

2. **Answer B.**
Slide Fig. 1 up to Fig. 3 to form a 3×4 rectangle without gaps or overlapping.

3. **Answer B.**
By Pythagoras' theorem $\mathsf{RC}^2 = \mathsf{RE}^2 + \mathsf{EC}^2 = 4^2 + 3^2 = 25$, so $\mathsf{RC} = 5$. Then $\mathsf{RK} = \frac{1}{2}\mathsf{RC} = 2.5$.

4. **Answer A.**
Since $x^2 - 1 = (x-1)(x+1)$, it follows that $2013^2 - 1 = 2012 \times 2014$.

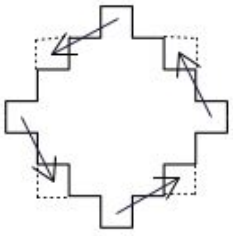
5. **Answer E.**
We can write all the numbers as decimals as follows: $A = 0.375$, $B = 0.25$, $C = 0.008$, $D = 0.4375$, $E = 0.625$, so E is the largest.

6. **Answer B.**
Barbie's bag is half-full with 30 balls, so when full it holds 60 balls. Suppose there are k balls in Ken's bag. Then $30 + \frac{1}{2}k = \frac{2}{3} \times 60 = 40$, so $\frac{1}{2}k = 40 - 30 = 10$, giving $k = 20$.

7. **Answer C.**
Adjacent angles in a parallelogram add up to 180°, so $\angle\mathsf{ADC} + \angle\mathsf{DCB} = 180°$, that is, $56° + \angle\mathsf{DCA} + 103° = 180°$. Therefore $\angle\mathsf{DCA} = 21°$.

8. **Answer D.**
For each throw, there are twelve possible outcomes: tail/1, 2, 3, 4, 5, **6** and head/1, 2, 3, 4, 5, 6, one of which is the tail/6 combination. In mathematical terms, tossing the coin and throwing the die are independent events, so
$P(\text{tail and } 6) = P(\text{tail}) \times P(6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

9. **Answer C.**
The perimeter is equal to the sum of the lengths of the sides, so if the sides are all of length s cm, then $28s = 56$ and $s = 2$. The figure is thus made up of small squares with sides of length 2 cm. If you count the squares in horizontal or vertical lines, then you see that the figure contains $1+3+5+7+5+3+1 = 25$ squares, each of area $2^2 = 4$ cm^2, so the total area is 100 cm^2. Alternatively, re-arrange the small squares as shown in the diagram, to obtain a large square with side length 10 cm.



10. **Answer A.**
Each strip makes up one-third of the flag. The top and bottom strips are one-half dark, while the middle strip is two-thirds dark. Thus the dark fraction of the flag is $2 \times \frac{1}{2} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{5}{9}$.

11. **Answer C.**
The three faces visible in the figure are $(1, 2, 3)$. As the die rolls, the visible faces in the corresponding positions are $(1, 4, 2)$ then $(1, 5, 4)$ then $(2, 1, 4)$ and finally $(2, 3, 4)$. In the final position, the top face is 3. (Note that the problem can actually be solved without the information about the numbers on opposite sides adding up to 7.)

12. **Answer D.**
The number of marbles m must lie between 125 and 140, because 125 is closer to m than 121 is, and 140 is closer to m than 142 is. Because of the rankings of the guesses, we see that

$$m - 125 < 140 - m < 142 - m < m - 121.$$

The first inequality gives $2m < 265$ and the last inequality gives $2m > 263$. Thus $2m = 264$ and $m = 132$.

13. **Answer E.**
Factorisation of 756 yields $756 = 2^2 \times 3^3 \times 7$. For $756\,m$ to be a perfect square, each of its prime factors must be raised to an even power. The smallest number m for this to happen is $m = 3 \times 7 = 21$, which gives $756\,m = 2^2 \times 3^4 \times 7^2 = (2 \times 3^2 \times 7)^2$.

14. **Answer D.**
If the circle has radius r, then the area of the triangle is $\frac{1}{2}r^2$ and the area of the circle is πr^2. The ratio of the two areas is therefore $\frac{1}{2} \,:\, \pi = 1 \,:\, 2\pi$.

15. **Answer D.**
If each container has volume $V\ \ell$ and the elapsed time is t min, then $4t = V - 60$ and $6t = V + 10$. Therefore $12t = 3V - 180 = 2V + 20$, so $V = 200$.

16. **Answer A.**
From the co-ordinates given in the figure we see that the smallest square has side 4 and the largest square has side 12. Thus the co-ordinates of A and B are $(4; 0)$ and $(9; 12)$ respectively. It follows by Pythagoras' theorem that

$$\mathsf{AB} = \sqrt{(9-4)^2 + (12-0)^2} = \sqrt{169} = 13.$$

17. **Answer C.**
If $x \diamondsuit x = 12$, then $4x - 3x + x^2 = 12$, so $x^2 + x - 12 = 0$, or $(x + 4)(x - 3) = 0$. Thus there are two solutions, $x = -4$ and $x = 3$. (It can also be done by noting that $x \diamondsuit y = (x - 3)(y + 4) + 12$.)

18. **Answer E.**
The quotient is equal to

$$\frac{10^{24} - 1}{9} \div \frac{10^4 - 1}{9} = \frac{(10^4)^6 - 1}{10^4 - 1} = 1 + 10^4 + 10^8 + 10^{12} + 10^{16} + 10^{20}.$$

This is a 21-digit number with six ones, and the remaining 15 digits all zeros.
(Long division gives the same result by brute force.)

19. **Answer E.**
The last two equations give $3d = 18$, so $d = 6$. Therefore $2a+b = c$ and $a+b = 12-c$. By eliminating a from one equation and b from the other, we obtain $a = 2(c - 6)$ and $b = 3(8 - c)$. Since $a > 0$ and $b > 0$, it follows that $6 < c < 8$, so $c = 7$, since c is an integer.

20. **Answer E.**
Let $p = x + 2y$ and $q = 2x + 5y$, so $p^2 + (q - \frac{1}{2})^2 \leq 2$, so $p^2 \leq 2$ and $(q - \frac{1}{2})^2 \leq 2$ as well. The only possibilities are $p = 0$ or ± 1 and $q - \frac{1}{2} = \pm\frac{1}{2}$, so $q = 0$ or 1. This gives six possibilities for the pair (p, q). By solving the equations we see that $x = 5p - 2q$ and $y = -2p + q$. Thus each pair (p, q) gives a unique pair (x, y), so there are six possibilities for (x, y) as well.