SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade NINE First Round 2017

Solutions

1. **C** $\dfrac{2017-1017}{500}=\dfrac{1000}{500}=2$

2. **B** Ravi begins the journey at 09h45 and arrives at his destination after 2 hours and 10 minutes at 11h55.

3. **D** Over the first 20 years the tree grows a total of 10 m. The further 3 m of growth, at a rate of $\frac{1}{3}$ m per year, would have taken 9 more years. 20 + 9 = 29 years.

4. **C** The angle $5x$ is bigger than $90°$ but smaller than $180°$. Of the five options given, 20 is the only value of x for which $5x$ falls in this interval.

5. **D** For the product to be negative there must be an odd number of integers. For the product of six integers to be negative, the greatest number of odd integers is thus five.

6. **E** It is not possible to fold this net without two faces overlapping. It is thus not possible to form a closed cube with six faces from this net.

7. **C** If five sweets cost R12 more than one sweet, then R12 is the cost of the four additional sweets. Each sweet thus costs R12 ÷ 4 = R3.

8. **D** The values of a, b and c can readily be found by a process of trial and error. More formally, from the first equation we have $a = 4 - b$, and from the second we have $c = 8 - b$. Substituting these into the third equation gives $8 - b + 4 - b = 6$. Solving this equation yields $b = 3$ from which it follows that $a = 1$, $c = 5$ and thus $abc = 15$.

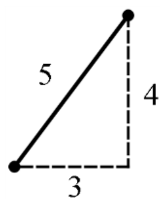
9. **A** The Least Common Multiple of 3 and 5 is 15. Every 15^{th} visitor will thus receive a pen and a bag. There are 13 multiples of 15 between 1 and 200.

10. **E** From the balanced scale we can remove two white squares and one shaded square from each side. From this it becomes clear that one white square is equivalent to two shaded squares. One shaded square and two white squares is thus equivalent to five shaded squares.

11. **A** Triangles AED and CFD are identical, and each has area $\frac{1}{2} \times 4 \times 2 = 4$. The area of quadrilateral EBFD is simply the area of square ABCD minus the areas of the two triangles. Thus, area of EBFD $= 4^2 - 2 \times 4 = 8$.

12. **B** From $\sqrt{xy} = 4$ we have $xy = 16$. From $\sqrt[3]{xyz} = 2$ we have $xyz = 8$. Thus $16z = 8$ and hence $z = \frac{1}{2}$.

13. **D** Horizontal distance travelled: $1-3+5=3$ (i.e. 3 km east)
Vertical distance travelled: $2-4+6=4$ (i.e. 4 km north)
From the Pythagorean triple 3, 4, 5, the straight-line distance from the starting point is thus 5 km.



14. **B** If the central number equals the average of the eight outer numbers, then it must also be the average of all nine numbers. The average of 1, 2, 3, 4, 5, 6, 7, 8 and 9 is $45 \div 9 = 5$.

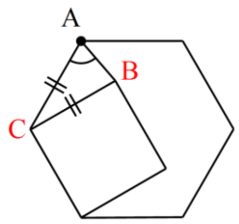
15. **E** Imagine a cube with one face painted. If a second face is painted then there is only one way that this can be done so that it has no common edges with the painted face. It is not possible to paint a third face without having a common edge with a painted face.

16. **D** The interior angles of a hexagon are $\dfrac{(6-2)\times 180°}{6} = 120°$.

Triangle ABC is isosceles.

$\angle ACB = 120° - 90° = 30°$.

$\angle CAB = \dfrac{1}{2}(180° - 30°) = 75°$.



17. **A** There are 100 possible outcomes. Of these there are 10 ways in which they can both choose the same number. Of the remaining 90 possibilities, half will have Andy's number bigger than Betty's, and half will have Betty's number bigger than Andy's. Alternatively: If Andy chooses 10 then there are 9 numbers that are smaller. If Andy chooses 9 then there are 8 numbers that are smaller. Continuing this logic there will be 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45 cases where Andy's number is bigger than Betty's.

18. **E** $35 \times 72 = [5 \times 7] \times [9 \times 8] = 2^3 \times 3^2 \times 5 \times 7$, which (by combining factors) is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

19. **C** The area of the large circle is $3^2 = 9$ times the area of a small circle, and includes five small circles. Then by symmetry each of the four parts of the large circle outside the small circles has the same area as a small circle, and so the total area of the shaded parts is 2×2017 = 4034 cm^2.

20. **B** The area covered by exactly two layers is $a+b+c$ and the area covered by exactly three layers is d. We know that $a+b+c=24$.
Since the floor area covered when the carpets do not overlap is 200 m^2 and the total floor area covered when they do overlap is 140 m^2, this means that 60 m^2 is lost on double and triple layers.

$\therefore a+b+c+2d = 60$

$\therefore 24+2d = 60$

$\therefore 2d = 36$

$\therefore d = 18$