THE HARMONY SOUTH AFRICAN
MATHEMATICS OLYMPIAD

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SECOND ROUND 2003: JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

PART A: (Each correct answer is worth 4 marks)

1. Two consecutive natural numbers add up to 2003.
The smaller of these two numbers is

A) 1001 B) 1002 C) 1003 D) 1004 E) 1000

<u>ANSWER:</u> A

$$\frac{2003}{2} = 1001\frac{1}{2}$$

□ The consecutive numbers are 1001 and 1002. 1001 is the smaller of the two.

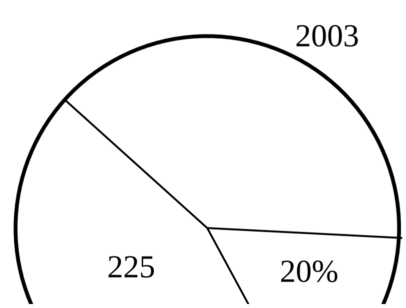
2. I recently returned from a trip. Today is Friday. I returned four days before the day after tomorrow.
On which day did I return?

A) Monday B) Tuesday C) Wednesday D) Thursday E) Friday

<u>ANSWER:</u> C

Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
				Today		
		Four days before Sunday				Day after tomorrow

3. The pie chart shows the breakdown of the 500 runs scored by one of the



South African batsmen over the
last three years.

The percentage of runs in 2003 is

A) 45 % B) 20 % C) 15 % D) 65 % E) 35 %

ANSWER: E

The number of runs scored in 2001 was 20% of 500.

$$\frac{20}{100} \times 500 = 100$$

The number of runs scored in 2002 was 225.

□ The number of runs scored in 2003 was

$$\begin{aligned} &500 - (225 + 100) \\ &= 500 - 325 \\ &= 175 \end{aligned}$$

□ The percentage of runs scored in 2003 is

$$\frac{175}{500} \times 100 = \frac{175}{5} = 35\%$$

OR

The percentage of runs scored in 2001 was 20%.

The percentage of runs scored in 2002 was 225 of 500, which is:

$$\frac{225}{500} \times 100 = \frac{225}{5} = 45\%$$

□ The percentage of runs scored in 2003 is (100 – 20 – 45)% = 35%

4. Consider the following pattern:

1st row:	1			
2nd row:	1	3		
3rd row:	1	3	5	
4th row:	.	.	.	.

The difference between the sums of the numbers in the 9th and 10th rows is

A) 17 B) 18 C) 19 D) 21 E) 22

ANSWER: C

In row 1, the sum of the numbers is 1.
In row 2, the sum of the numbers is 4.
In row 3, the sum of the numbers is 9.

In row n, the sum of the numbers is n^2. So, the difference between the sum of the numbers in row 10 and row 9 is:
$10^2 - 9^2 = 100 - 81 = 19$

5. Which one of the following is an odd number?

A) $2001^2 + 3$

B) $2002^2 + 10$

C) $2003^2 + 7$

D) $2004^2 + 1$

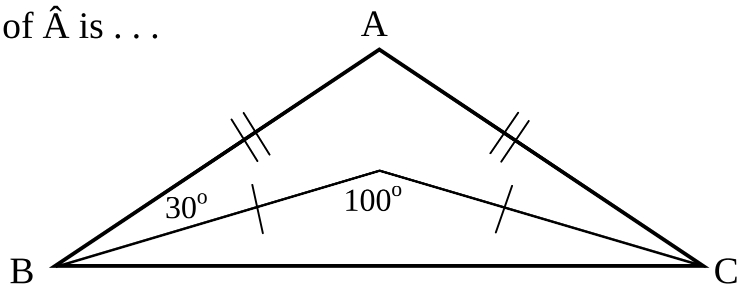
E) $2005^2 + 9$

ANSWER: D

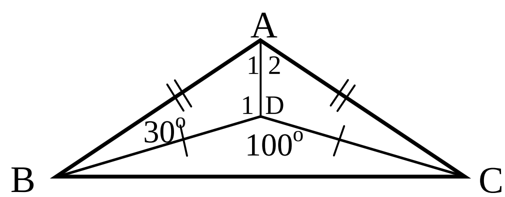
even number + odd number = odd number
(even number)^2 = even number
(odd number)^2 = odd number
$\therefore$ (even number)^2 + odd number = odd number
$\therefore$ $2004^2 + 1$ is an odd number.

PART B: (Each correct answer is worth 5 marks)

6. The measurement of $\hat{A}$ is . . .



A) 30° B) 40° C) 50° D) 60° E) 70°



<u>ANSWER:</u> B

Draw line AD.

$\triangle ABD \equiv \triangle ACD$

$$\hat{D}_1 = \frac{1}{2}\left(360^\circ - 100^\circ\right) = 130^\circ$$

$$\text{but } \hat{A}_1 = \frac{1}{2}\hat{A} = 180^\circ - 130^\circ - 30^\circ = 20^\circ \quad \therefore \hat{A} = 40^\circ$$

7. A supermarket always prices its goods at ‘so many Rands and ninety-nine cents’. If a shopper who has bought different items has to pay R41,71, how many items did she buy?

A) 41 B) 39 C) 30 D) 19 E) 29

<u>ANSWER:</u> E

This problem revolves around the possible endings of sums of numbers which end in ,99. Because the final payment ends in a ,01, the number of items bought must end in a 9. Testing the 3 options ending in 9 for which one ends in 71, we get:

$39 \times 99 = 39 \times (100 - 1) = 3900 - 39 = 38\underline{61}$ No

$29 \times 99 = 29 \times (100 - 1) = 2900 - 29 = 28\underline{71}$ Yes

$19 \times 99 = 19 \times (100 - 1) = 1900 - 19 = 18\underline{81}$ No

So the answer is 29

OR

Each item bought has a price which is 1 cent short of a whole number of Rands, therefore the answer is 29 items.

OR

1 and 8 as unit digit combinations multiplied by $0{,}99$ gives us the 71 cents.

$$\begin{array}{ll} 11\times 1{,}99 = 21{,}89 \\ \underline{18}\times 0{,}99 = \underline{17{,}82} \\ 29 \qquad\qquad 39{,}71 \end{array}$$

But 3 and 6 gives us 71 cents as well.

$$\begin{array}{ll} 13\times 1{,}99 = 25{,}87 \\ \underline{16}\times 0{,}99 = \underline{15{,}84} \\ \underline{29} \qquad\qquad \underline{41{,}71} \end{array}$$

29 items.

8. If a and b are integers, and $a \otimes b = \frac{b^2}{a} - \frac{b}{a}$, then $3 \otimes 6$ is equal to

A) 12 B) 4 C) 6 D) 8 E) 10

<u>ANSWER:</u> E

$$a \otimes b = \frac{b^2}{a} - \frac{b}{a}$$

So $3 \otimes 6 = \frac{6^2}{3} - \frac{6}{3} = \frac{36}{6} - \frac{6}{3} = 12 - 2 = 10$

OR

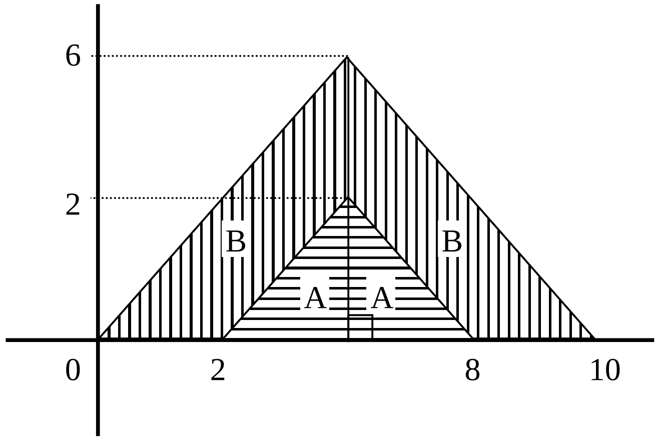
$$a \otimes b = \frac{b^2}{a} - \frac{b}{a} = \frac{b^2 - b}{a} = \frac{b(b-1)}{a}$$

So $3 \otimes 6 = \frac{6\times 5}{3} = \frac{30}{3} = 10$

So the answer is 10

9. The regions marked A are equal in area, and the regions marked B are equal in area.

The ratio of $\dfrac{\text{area A}}{\text{area B}}$ is



A) $\dfrac{1}{5}$ B) $\dfrac{1}{4}$ C) $\dfrac{1}{3}$ D) $\dfrac{1}{2}$ E) $\dfrac{1}{1}$

<u>ANSWER:</u> B

$$\text{Area A} = \frac{1}{2}\times 3\times 2 = 3$$

$$\text{Area}\left(\text{A+B}\right) = \frac{1}{2}\times 5\times 6 = 15$$

$$\begin{aligned}\text{Area B} &= \text{Area}\left(\text{A}+\text{B}\right) - \text{Area A}\\ &= 12\end{aligned}$$

$$\frac{\text{Area A}}{\text{Area B}} = \frac{3}{12} = \frac{1}{4}$$

10. If $x+y=4$, $y+z=7$ and $x+z=5$ the value of $\left(x+y+z\right)^2$ is

A) 36 B) 64 C) 100 D) 144 E) 256

<u>ANSWER:</u> B

$$\begin{array}{lccccccc} & x & + & y & & & = & 4\\ & & & y & + & z & = & 7\\ & x & & & + & z & = & 5\\ \hline \therefore & 2x & + & 2y & + & 2z & = & 16\\ \therefore & x & + & y & + & z & = & 8\\ \therefore & (x & + & y & + & z)^2 & = & 8^2 = 64\end{array}$$

OR

$$\begin{array}{lllllll} x & + & y & & & = & 4 \quad ① \\ & & y & + & z & = & 7 \quad ② \\ x & & & + & z & = & 5 \quad ③ \end{array}$$

②$-$①: $\quad z - x = 3 \quad$ ④

③$+$④: $\quad 2z = 8$

$\quad \therefore \ z = 4$

Substitute $z = 4$ in ③:

$\quad 4 + x = 5$

$\quad \therefore \ x = 1$

Substitute $x = 1$ in ①:

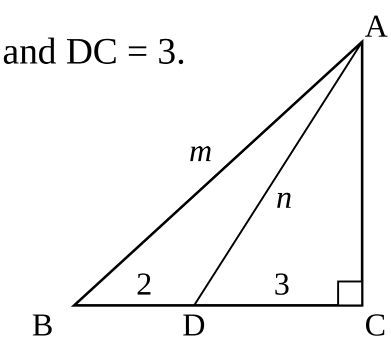
$\quad 1 + y = 4$

$\quad \therefore \ y = 3$

$\therefore \ (x + y + z)^2 = (1 + 3 + 4)^2$

$\quad = 64$

11. VABC has D on B such that BD = 2 and DC = 3.



If $\text{AB} = m$ and $\text{AD} = n$
then the value of $m^2 - n^2$ is

A) 4 B) 9 C) 16 D) 25 E) 36

ANSWER: C

In VABC

$$\begin{array}{rlll} & \text{AB}^2 & = & \text{BC}^2 + \text{AC}^2 \quad \text{(Pythagoras)} \\ \therefore & m^2 & = & 5^2 + \text{AC}^2 \\ \therefore & m^2 & = & 25 + \text{AC}^2 \quad ① \end{array}$$

In VADC

$$\begin{array}{rlll} & \text{AD}^2 & = & \text{DC}^2 + \text{AC}^2 \quad \text{(Pythagoras)} \\ \therefore & n^2 & = & 3^2 + \text{AC}^2 \\ \therefore & n^2 & = & 9 + \text{AC}^2 \quad ② \end{array}$$

①$-$②: $\quad m^2 - n^2 = 25 - 9$

$\quad \therefore \ m^2 - n^2 = 16$

12. If $1\times 2\times 3\times ...\times 199\times 200$ is calculated, then the number of zeros at the end of the product is

A) 42 B) 43 C) 46 D) 49 E) 52

ANSWER: D

We need first to realize that each zero at the end of any product results from the product containing a factor of 2 and a factor of 5.

Let's look at the number 90:

90 can be written as $15\times 6=90$, which ends in a zero.

Because $15=3\times 5$ contains the factor 5 and
$6=3\times 2$ contains the factor 2,

we could look at it like this:

$$15\times 6=3\times 5\times 3\times 2=\left(3\times 3\right)\times\left(5\times 2\right)=9\times 10=90.$$

Another example is 160, which can be written as

$$8\times 20=(2\times 2\times 2)\times(2\times 2\times 5)=\left(8\times 2\right)\times\left(2\times 5\right)=16\times 10=160$$

Knowing this, we need to work out the number of factors of 5 in the product: $1\times 2\times 3\times ....\times 199\times 200$.

Clearly there are 40 multiples of 5, but there are also some of these multiples of 5 with additional factors of 5.

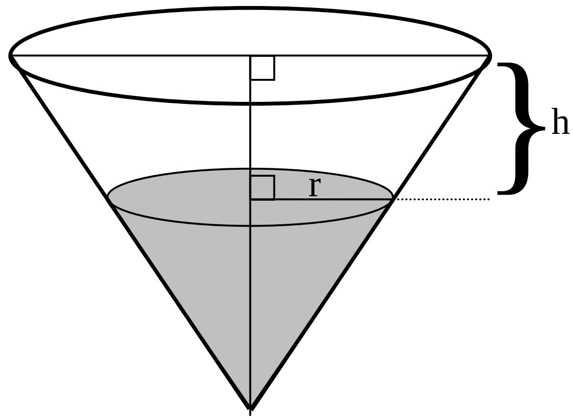
They are: 25, 50, 75, 100, 150, 175, 200, seven of them which each have 1 extra factor of 5, and 125 which has 2 extra factors of 5.

$\therefore$ The number of factors of 5 in the product is

$$40+7+2=49$$

There are over 100 factors of 2 in the product (i.e. all the even numbers!), so we know that for each factor of 5, there is at least one factor of 2. The product must therefore end with 49 zeros.

13. A tank that is in the form of an inverted cone contains a liquid. The height h, in metres, of the space above the liquid is given by the formula $h = 21 - \frac{7}{2}r$ where r is the radius of the liquid surface, in metres.



The circumference of the top of the tank, in metres is

A) 9π B) 12π C) 15π D) 18π E) 21π

ANSWER: B

The height of the space above the liquid is given by

$$h = 21 - \frac{7}{2}r$$

When $h = 0$;

$$21 - \frac{7}{2}r = 0$$

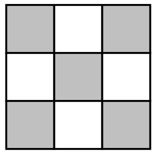
$$\therefore \quad \frac{7}{2}r = 21$$

$$\therefore \quad r = 6$$

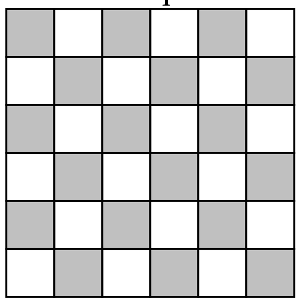
The circumference of the top of the tank $= 2\pi \times 6$

$= 12\pi$

14. In this diagram, there is a total of 14 squares of all sizes.



What is the total number of squares of all sizes on the board below?



A) 49 B) 63 C) 77 D) 91 E) 105

<u>ANSWER:</u> D

3×3 square = 14 squares of all sizes $= 1^2 + 2^2 + 3^2$

4×4 square = 30 squares of all sizes $= 1^2 + 2^2 + 3^2 + 4^2$

5×5 square = 55 squares of all sizes $= 1^{2+} + 2^2 + 3^2 + 4^2 + 5^2$

6×6 square = 91 squares of all sizes $= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

15. The fraction $\dfrac{53}{17}$ can be expressed as $3 + \dfrac{1}{x + \frac{1}{y}}$.

If x and y are integers the value of $x + y$ is

A) 8 B) 9 C) 10 D) 11 E) 12

<u>ANSWER:</u> C

$$\frac{53}{17} = 3\frac{2}{17} \longrightarrow 3\frac{2}{17} = 3 + \frac{2}{17} = 3 + \frac{1}{x + \frac{1}{y}}$$

$$\therefore \frac{2}{17} = \frac{1}{x + \frac{1}{y}} \longrightarrow \frac{1}{\frac{17}{2}} = \frac{1}{x + \frac{1}{y}}$$

$$\therefore 8 + \frac{1}{2} = x + \frac{1}{y}$$

$x = 8$ and $y = 2$

$\therefore \; x + y = 8 + 2 = 10$

PART C: (Each correct answer is worth 6 marks)

16. Two numbers are in the ratio $2:3$. When 4 is added to each number the ratio changes to $5:7$.
The sum of the two original numbers is

A) 20 B) 25 C) 30 D) 35 E) 40

<u>ANSWER:</u> E

Let the numbers be $2x$ and $3x$ because $\frac{2x}{3x}=\frac{2}{3}$

then $\frac{2x+4}{3x+4}=\frac{5}{7}$

$\therefore\ 7(2x+4)=5(3x+4)$

$\therefore\ 14x+28=15x+20$

$\therefore\ 28-20=15x-14x \quad \therefore\ 8=x$

$\therefore$ the numbers are 16 and 24 $\quad \therefore\ 16+24=40$

OR

Ratio $\frac{N}{D}=\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\frac{12}{18}=\frac{14}{21}=\frac{16}{24}=\frac{18}{27}$, however $\frac{18}{27}$ is out because $18+27=45$ and 45 is not an option.
Add 4 to N and D respectively (trial-and-improvement method) and cancel to get the ratio $\frac{5}{7}$. $\frac{16+4}{24+4}=\frac{20}{28}=\frac{5}{7} \quad \therefore\ 16+24=40$

17. A local council consists of 4 female members and 3 male members. The number of different 3-member committees consisting of 2 female members and 1 male member which can be formed by the council is

A) 18 B) 15 C) 12 D) 9 E) 6

<u>ANSWER:</u> A
Let 4 females be a; b; c; d
And 3 male members be e; f; g.
Number of ways of choosing 2 females:
(a; b); (a; c); (a; d); (b; c); (b; d); (c; d), a total of 6 ways.
Number of ways of choosing 1 male:
e; f; g, a total of 3 ways.
$\therefore$ total number of ways $=6\times 3\ =18$

18. Lee gave Petrus a 10 metre lead in a 100 metre race and Lee was beaten by four metres.
What lead should Lee give Petrus in order that both finish the race together, if their respective speeds stayed the same in both races?

A) $5{,}75\,m$ B) $5{,}9\,m$ C) $6{,}1\,m$ D) $6{,}25\,m$ E) $6{,}5\,m$

ANSWER: D
With a 10 *m* lead, Petrus beats Lee by 4 *m*.
$\therefore$ Petrus runs 90 *m* while Lee runs 96 *m*. Therefore their speeds are in the ratio 90 to 96
In order to finish together with a certain lead, Petrus runs *x m* while Lee runs 100 *m*.
Direct Proportion (assuming same speeds)

$$\frac{\text{Petrus}}{\text{Lee}} = \frac{90}{96} = \frac{x}{100}$$

$$\therefore\ 96x = 9000$$

$$x = \frac{9000}{96}$$

$$= 93{,}75\ m$$

$\therefore$ Lee should give Petrus a (100 – 93,75) = 6,25 *m* lead.

19. In the addition problem TSR + PSR + RSP, Themba substitutes the four letters with the four digits 2, 7, 5, and 3, in any order. Different letters stand for different digits. The largest value of the sum TSR + PSR + RSP is

A) 1579 B) 1499 C) 1571 D) 1701 E) 1537

ANSWER: A
Clearly the letters T, P and R in the hundredths position must be as big as possible. These need to use the 7, 5 and 3. This means that the letter S can only be 2 i.e.

T2R
P2R
R2P

The letter R in the units position must take the bigger digit as it occurs twice, so he needs to use the 7: R = 7 i.e.

T27
P27
72P

The letter P in the units position must take the largest remaining digit, so P = 5 and T can only be 3 i.e. 327 + 527 +725 = 1579.

20. Colleen, Jakes, Hendrik, Vishnu and Tandeka play a game of cops and robbers. The robbers' statements are always false while the cops' statements are always true.
a) Colleen says that Jakes is a cop.
b) Hendrik says that Vishnu is a robber
c) Tandeka says that Colleen is not a robber
d) Jakes says that Hendrik is not a cop.
e) Vishnu says that Tandeka and Colleen play on different sides.

How many robbers are there?

A) 1 B) 2 C) 3 D) 4 E) 5

<u>ANSWER:</u> D
We need to note that each individual player can only be either a cop (truth teller) or a robber (liar). Thus we can proceed by assuming that a particular one is, say, a cop, then working through the 5 statements to check whether this assumption leads to a contradiction. If it doesn't, then we will already be able to answer the question. If there is a contradiction, then we know for certain that that individual is a robber. Taking that knowledge through the 5 statements should enable us to answer the question.
So, let's assume that **Colleen is a cop**,
Then from c), Tandeka is a cop, and
from a), Jakes is a cop, so
from d), Hendrik is a robber, and
from b), Vishnu is a cop, and
from e), Tandeka and Colleen are different,
but we know they are the same (both cops!),
so we have a contradiction.
We therefore know Colleen is a robber – this is now certain.

If **Colleen is a robber,**
Then from a), Jakes is a robber, and
from c), Tandeka is a robber, then
from e), Vishnu is a robber, and
from d), Hendrik is a cop.
So there are 4 robbers!

THE END

Formula and Information Sheet

1.1 The natural numbers are 1; 2; 3; 4; 5; …

1.2 The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; …

1.3 The integers are …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

$$a \times a \times a \times a \times \ldots \times a = a^n \text{ } (n \text{ factors of } a)$$

(a is the base and n is the index (exponent))

3.2 Factorial notation:

$$1 \times 2 \times 3 \times 4 = 4!$$

$$1 \times 2 \times 3 \times \ldots \times n = n!$$

4 Area of a

4.1	rectangle is:	length x width = lw length x breadth = lb
4.2	square is:	side x side = s^2
4.3	rhombus is:	$\frac{1}{2} \times$ (product of diagonals)
4.4	trapezium is:	$\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$
4.5	circle is:	πr^2 (r = radius)

5 Surface area of a:

5.1 rectangular prism is: $2lb + 2lh + 2bh$ $(h = \text{height})$

5.2 sphere is: $4\pi r^2$

6 Perimeter of a:

6.1 rectangle is: $2 \times \text{length} + 2 \times \text{breadth}$

$2l + 2b$

or $2l + 2w$ $(w = \text{width})$

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s \times s \times s = s^3$

8.2 rectangular prism is: $l \times b \times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section x perpendicular height
or area of base x perpendicular height

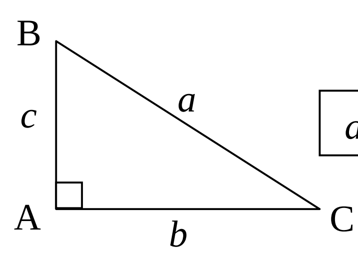
9.2 Surface area of a right prism is: (perimeter of base x h) + (2 x area of base)

10. Sum of the interior angles of a polygon is: $180^\circ(n-2)$ $(n = \text{number of sides})$

11. Distance is: speed x time (d = s x t)

12 Pythagoras:

ΔABC is a right-angled triangle



$\boxed{a^2 = b^2 + c^2}$

13. Conversions:
1 cm^3 = 1 ml ; 1000 cm^3 = 1 l
1000 m = 1 km ; 1000 g = 1 kg ; 100 cm = 1 m

ANSWER POSITIONS: JUNIOR SECOND ROUND 2003

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	A
2	C
3	E
4	C
5	D
6	B
7	E
8	E
9	B
10	B
11	C
12	D
13	B
14	D
15	C
16	E
17	A
18	D
19	A
20	D

DISTRIBUTION	
A	3
B	4
C	4
D	5
E	4
TOTAL	**20**