THE OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 SECOND ROUND
JUNIOR SECTION: GRADE 8 & 9

18 May 2021 **Time: 120 minutes** **Number of questions: 25**

Instructions

1. *The answers to all questions are integers from 0 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 3 marks in Part A, 4 marks in Part B and 6 marks in Part C.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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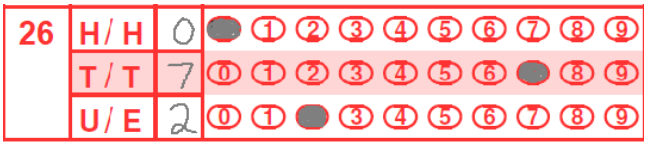
Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 0 to 999. Consider the following example question:

26. If $3x - 216 = 0$, determine the value of x.

The answer is 72, so you must complete the block for question 26 on the answer sheet as follows: shade 0 in the hundreds row, 7 in the tens row, and 2 in the units row:



Write the digits of your answer in the the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

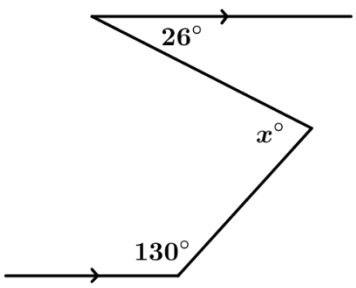
DO NOT TURN THE PAGE BEFORE YOU ARE TOLD TO DO SO

Part A: 3 marks each

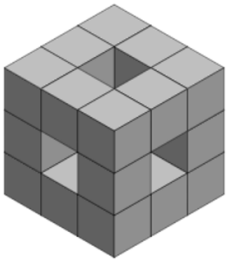
1. This year is 2021. In how many years will it be 2223?

2. If $A \times 4 = B$ and $B - A = 33$, then what is the value of A?

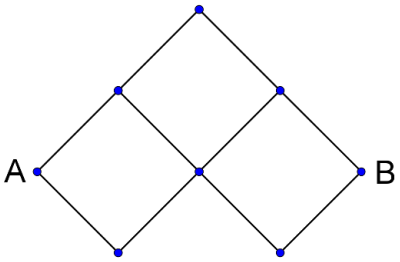
3. What is the value of x in the diagram?



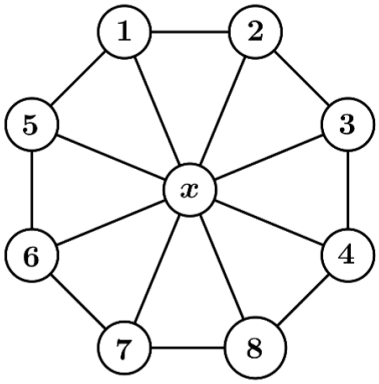
4. The solid shown is formed by taking a 3 cm x 3 cm x 3 cm cube and 'drilling' a 1 cm x 1 cm square hole from the centre of each face to the centre of the opposite face. What is the volume in cm^3 of the solid?



5. Each of the ten paths shown is 1 metre long. An ant walks along the paths from A to B without going along any path more than once. What is the length (in metres) of the longest possible route?

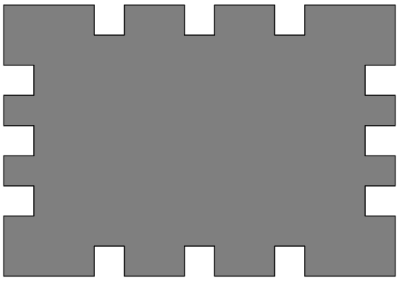


6. If the three numbers along each diagonal are added, the average of these sums is 25. Determine the value of x.



7. I want to write a list of consecutive positive integers starting with 1. The list must contain three square numbers and three cube numbers. What is the smallest number of integers that could be in my list?

8. A rectangular sheet of paper has dimensions 10 cm by 15 cm. From this sheet, twelve 1 cm by 1 cm squares are cut out, as shown. What is the perimeter of the resulting shape in cm?



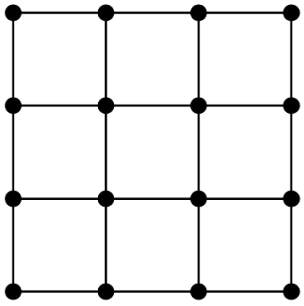
9. The sum of five consecutive positive integers, starting with n, is a perfect square. What is the smallest possible value of n?

10. From the first 100 natural numbers (i.e. 1 to 100 inclusive) all the multiples of 2 are removed, and then all the multiples of 5 are removed. How many numbers are left?

Part B: 4 marks each

11. All the prime numbers less than 2021 are multiplied together. What is the units digit of this product?

12. 16 Jelly Tots are arranged in a 4 by 4 square. The Jelly Tots are connected with toothpicks. How many toothpicks would be needed for a 10 by 10 square containing 100 Jelly Tots?



13. The positive integers x, y, z satisfy $xy = 10$, $yz = 15$ and $xz = 6$.
What is the value of $x + y + z$?

14. If the pattern continues in the same way, how many shaded blocks will be in Figure 30?

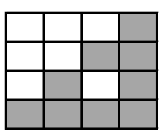


Figure 1

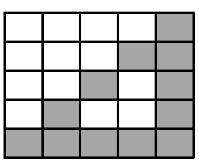


Figure 2

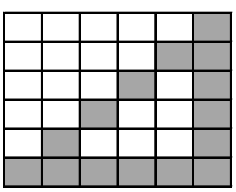
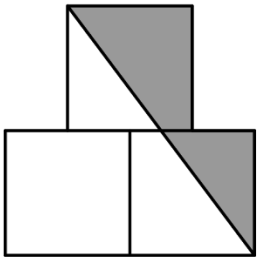


Figure 3

15. The diagram shows three identical squares of side length 8 cm. The top square is exactly centred over the two squares below it. Determine the area of the shaded region in cm^2.



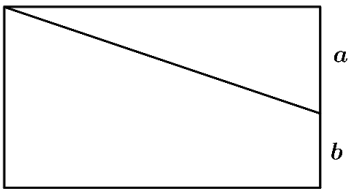
16. The combination for a certain lock is a 4-digit number, with no two adjacent digits the same. How many combinations are possible if the digits are 2, 0, 2, 1?

17. The average of 10 scores is 80. When the highest and lowest scores are removed, the average of the remaining scores is 70. What was the average of the two scores that were removed?

18. If $xy - 2y = 10$ where x and y are positive integers, what is the sum of all possible values of x?

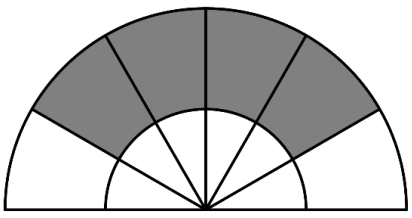
19. A fruit basket contains bananas and grapes. The ratio of bananas to grapes in the basket is 3:8. When I remove one banana the ratio changes to 1:3. How many grapes are in the basket?

20. A rectangle is cut into two regions by a single line from a corner as shown. The areas of the two regions are in the ratio of 1:5. If $a + b = 48$ cm, determine the length of a.



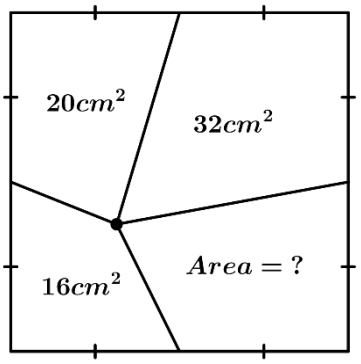
Part C: 6 marks each

21. In the diagram the outer semi-circle has a radius of 2 cm, and the inner semi-circle has a radius of 1 cm. The radii are equally spaced. What percentage of the entire shape is shaded?



22. The faces of a fair die are numbered 1 to 6. When the die is rolled, one number will be face-down and the other five numbers will be visible. The probability that 210 is divisible by all of the five visible numbers is $\frac{a}{b}$ in simplified form. What is the value of $a + b$?

23. A point inside a square is joined to the midpoint of each side as shown. Three areas are indicated. Determine the area of the unknown region in cm^2.

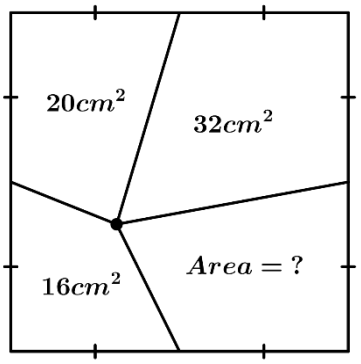


24. Amara jogs from her home to the beach. When she gets to the beach she immediately turns around and jogs back home again. She is away from home for 2 hours. Her speed going to the beach is 15 km/h, and her speed for the return journey is 10 km/h. How long (in minutes) did it take her to run from home to the beach?

25. When the seven numbers $13\ ;\ p\ ;\ 16\ ;\ 7\ ;\ 4\ ;\ 15\ ;\ 17$ are arranged in ascending order, the median (i.e. the middle number) is the same value as the mean (i.e. the average). What is the value of p?

22. Die aansigte van ‘n gebalanseerde dobbelsteen word genommer van 1 tot 6. Wanneer die dobbelsteen gerol word, is 5 getalle sigbaar en een getal is nie. Die waarskynlikheid dat 210 deelbaar is deur al vyf die sigbare getalle is $\frac{a}{b}$ in eenvoudigste vorm. Wat is die waarde van $a + b$?

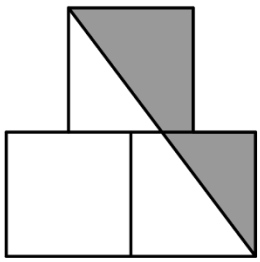
23. ‘n Punt binne-in ‘n vierkant word verbind met die middelpunt van elke sy, soos in die skets. Drie oppervlaktes word aangedui. Bepaal die waarde van die vierde oppervlakte in cm^2.



24. Amara draf van haar huis af strand toe. Wanneer sy by die strand kom, draai sy dadelik terug en draf weer terug huis toe. Sy is vir 2 ure weg van die huis af. Haar spoed strand toe is 15 km/h, en haar spoed terug huis toe is 10 km/h. Hoe lank (in minute) het dit haar geneem om van die huis af strand toe te draf?

25. Wanneer die sewe getalle $13 ; p ; 16 ; 7 ; 4 ; 15 ; 17$ in stygende volgorde gerangskik word, is die mediaan (i.e. die middelste getal) dieselfde waarde as die gemiddeld. Wat is die waarde van p?

15. Die skets wys drie identiese vierkante met sylengte 8 cm. Die boonste vierkant is presies bokant die middel van die onderste twee vierkante. Bepaal die oppervlakte van die ingekleurde deel in cm^2.



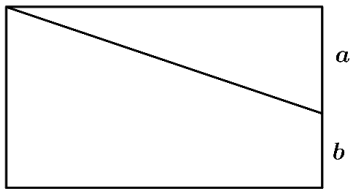
16. Die kombinasie vir ‘n slot is ‘n 4-syfer getal, maar twee aangrensende syfers mag nie dieselfde wees nie. Hoeveel kombinasies is moontlik as die syfers 2, 0, 2, 1 is?

17. Die gemiddeld van 10 punte is 80. As die hoogste en laagste punte verwyder word, dan is die gemiddeld van die oorblywende punte, 70. Wat was die gemiddeld van die twee punte wat verwyder is?

18. As $xy - 2y = 10$ met x en y positiewe heelgetalle, wat is die som van al die moontlike waardes van x?

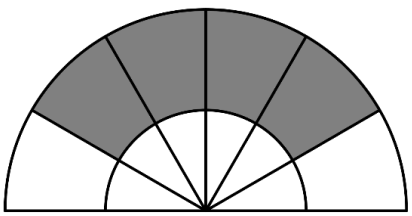
19. ‘n Vrugte-mandjie bevat piesangs en druiwekorrels. Die verhouding van piesangs tot korrels is 3:8. As ek een piesang uithaal, verander die verhouding na 1:3. Hoeveel druiwekorrels is in die mandjie?

20. ‘n Reghoek word in twee dele gesny met ‘n enkele lyn vanuit een hoekpunt, soos aangetoon. Die oppervlaktes van die twee dele is in die verhouding 1:5. As $a + b = 48$ cm, bepaal die lengte van a.



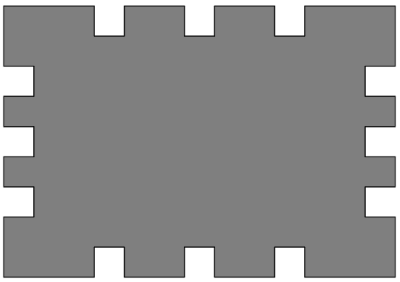
Afdeling C: 6 punte elk

21. In die skets het die buitenste halfsirkel ‘n radius van 2 cm, en die binneste halfsirkel het ‘n radius van 1 cm. Die strale is eweredig gespasieer. Watter persentasie van die hele vorm is ingekleur?



7. Ek wil ‘n lys van opeenvolgende positiewe heelgetalle maak wat by 1 begin. Die lys moet drie vierkantsgetalle en drie derdemagte bevat. Wat is die kleinste aantal heelgetalle wat in my lys moet wees?

8. ‘n Reghoekige papier het afmetings 10 cm by 15 cm. Twaalf 1 cm by 1 cm stukkies word uitgesny van hierdie papier, soos aangetoon. Wat is die omtrek van die oorblywende vorm in cm?



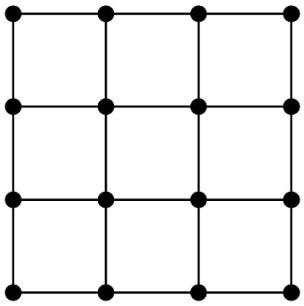
9. Die som van vyf opeenvolgende positiewe heelgetalle, wat by n begin, is ‘n volkome vierkant. Wat is die kleinste moontlike waarde van n?

10. Uit die eerste 100 natuurlike getalle (dit is van 1 tot 100 ingesluit) word al die veelvoude van 2 verwyder. Daarna word al die veelvoude van 5 verwyder. Hoeveel getalle bly oor?

Afdeling B: 4 punte elk

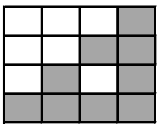
11. Al die priemgetalle minder as 2021 word met mekaar vermenigvuldig. Wat is die ene-syfer van hierdie produk?

12. 16 Jelly Tots is gerangskik in ‘n 4 by 4 vierkant. Die Jelly Tots word verbind met tandestokkies. Hoeveel tandestokkies sal nodig wees om 100 Jelly Tots in ‘n 10 by 10 vierkant te rangskik?

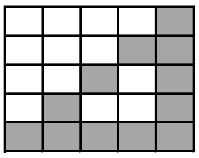


13. Die positiewe heelgetalle x, y, z bevredig $xy = 10,\ yz = 15$ en $xz = 6$.
Wat is die waarde van $x + y + z$?

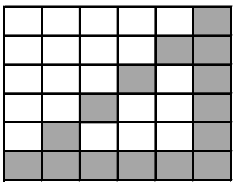
14. As hierdie patroon op dieselfde manier voortgesit word, hoeveel ingekleurde blokkies is in Figuur 30?



Figuur 1



Figuur 2



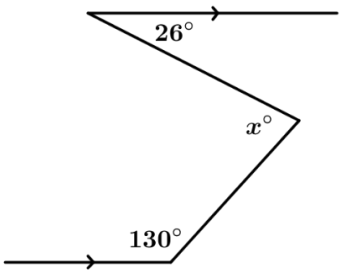
Figuur 3

Afdeling A: 3 punte elk

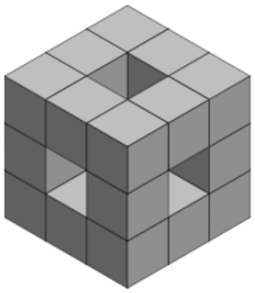
1. Vanjaar is 2021. Oor hoeveel jaar sal dit 2223 wees?

2. As $A \times 4 = B$ en $B - A = 33$, wat is die waarde van A?

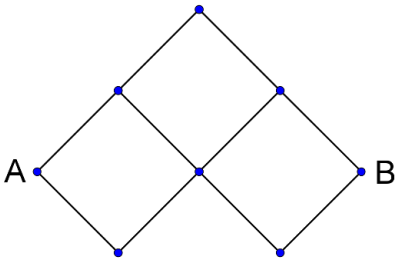
3. In die skets, wat is die waarde van x?



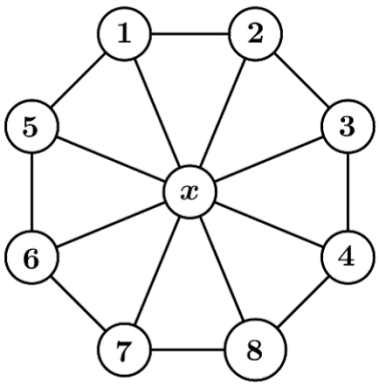
4. Die blok in die skets is gevorm deur 'n 3 cm x 3 cm x 3 cm kubus te neem en 'n 1 cm x 1 cm vierkantige gat te 'boor' van die middel van een aansig na die middel van die teenoorstaande aansig. Wat is die volume van die blok in cm^3?



5. Elkeen van die 10 paaie in die skets is 1 meter lank. 'n Mier loop op die paaie vanaf A na B sonder om 'n pad meer as een keer te gebruik. Wat is die lengte (in meter) van die langste moontlike roete?



6. Die drie getalle op elkeen van die diagonale word bymekaar getel. Die gemiddeld van al hierdie somme is 25. Bepaal die waarde van x.

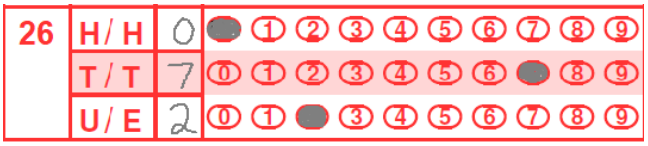


HOE OM DIE ANTWOORDBLAD TE VOLTOOI

Al die antwoorde is heelgetalle van 1 tot 999. Beskou die volgende voorbeeldvraag:

26. As $3x - 216 = 0$, bepaal die waarde van x.

Die antwoord is 72, en dus moet jy die blok vir vraag 26 op die antwoordblad as volg voltooi: kleur 0 in honderde-ry in, 7 in die tiene-ry, en 2 in die ene-ry:



Skryf die syfers van jou antwoord in die oop blokkies links in die betrokke ry, soos in die voorbeeld aangetoon; honderde, tiene en ene van bo na onder. Die drie syfers wat jy neergeskryf het, word nie nagesien nie; dit is vir jou eie gerief — slegs die ingekleurde sirkels word gemerk.

MOENIE OMBLAAI VOORDAT JY VERSOEK WORD OM DIT TE DOEN NIE

DIE OLD MUTUAL SUID-AFRIKAANSE WISKUNDE-OLIMPIADE

Georganiseer deur die
SOUTH AFRICAN MATHEMATICS FOUNDATION

2021 TWEEDE RONDTE
JUNIOR AFDELING: GRAAD 8 & 9

18 Mei 2021 **Tyd: 120 minute** **Aantal vrae: 25**

Instruksies

1. *Die antwoorde op al die vrae is heelgetalle van 0 tot 999. Elke vraag het slegs een korrekte antwoord.*
2. *Puntetoekenning:*
   2.1. Elke korrekte antwoord tel 3 punte in Afdeling A, 4 punte in Afdeling B en 6 punte in Afdeling C.
   2.2. Geen punte word afgetrek vir foutiewe antwoorde of onbeantwoorde vrae nie.
3. *Gebruik 'n HB potlood. Papier vir rofwerk, 'n liniaal en uitveër word toegelaat.* ***Sakrekenaars en meetkunde-instrumente word nie toegelaat nie.***
4. *Figure is nie noodwendig volgens skaal geteken nie.*
5. *Beantwoord die vrae op die antwoordblad wat voorsien word.*
6. *Begin sodra die toesighouer die teken gee.*
7. *Antwoorde en oplossings sal beskikbaar wees by* `www.samf.ac.za`

Moenie omblaai voordat dit aan jou gesê word nie.
Turn the booklet over for the English paper.

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