SOUTH AFRICAN MATHEMATICS OLYMPIAD

Grade NINE First Round 2018

Solutions

1. **C** $\sqrt[3]{20-1+8} = \sqrt[3]{27} = 3$

2. **B** $\dfrac{2018}{8102} \approx \dfrac{2025}{8100} = \dfrac{1}{4}$

3. **D** 20 – (–18) = 38

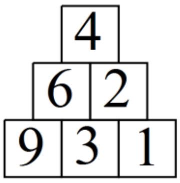
4. **B** $\dfrac{90° - 20° - 18°}{2} = 26°$

5. **D** There are four rectangles that are 1 unit wide. There are three that are 2 units wide, two that are 3 units wide and one that is 4 units wide. 4 + 3 + 2 + 1 = 10.

6. **E** Working backwards, before he had 81 he must have had 9; before that 8, before that 4.

7. **D** The bold part of the perimeter is the same for both shapes. The larger shape has a total of 3 + 6 + 5 = 14 cm on the other edges, while the smaller one has 3 + 6 + 1 = 10 cm. The difference is 4 cm.

8. **A** The completed pyramid is:



9. **E** $93 - 18 = 75$; $75 \times \frac{2}{3} = 50$; $50 + 18 = 68$

10. **A** The three squares have a total area of $10^2 + 8^2 + 6^2 = 200 \text{ cm}^2$. The unshaded triangle has length $10 + 8 + 6 = 24$ cm and perpendicular height 10 cm and thus area 120 cm^2. The shaded area is thus $200 - 120 = 80 \text{ cm}^2$.

11. **E** Since $14 \times 14 \times 14 = 2^3 \times 7^3$, N must be $2^3 \times 7 = 56$.

12. **C** The different options would be 2 + 0, 2 + 1, 2 + 8, 1 + 8, 8 + 0 and 1 + 0.
Only three out of these 6 options give an even sum, i.e. 50%.

13. **C** If there are x sisters the mother will get x gifts. If each sister gives a gift to their other sisters there will be $x(x-1) = x^2 - x$ gifts between the sisters. The number of gifts in total will be $x + x^2 - x = x^2$, i.e. a perfect square. The only perfect square in the five options given is 49.

14. **A** $6! = 720$; $3! = 6$ and $5! = 120$ $\therefore$ $6! = 3! \times 5!$ $\therefore$ $p + q = 3 + 5 = 8$.

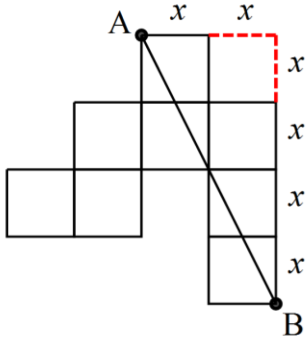
15. **D** The supplementary angles of x and y have a sum of 90°. The part of P that is visible is a quadrilateral. The equal interior angles of P will then be $\frac{360° - 90°}{2} = 135°$. The exterior angle of polygon P will be 45°. $\frac{360°}{45°} = 8$. Polygon P thus has 8 sides.

16. **A** The largest gear, moving in a clockwise direction, turning 5 times, means that the black arrow will pass a total of $16 \times 5 = 80$ teeth. The black arrow would then still point to P.

$80 = 12 \times 6 + 8 = 10 \times 8 = 6 \times 13 + 2$. The second gear, moving in an anti-clockwise direction turns 6 times and 8 teeth, the third largest gear moving in a clockwise direction turns 8 times exactly and the smallest gear moving in an anti-clockwise direction, turns 13 times and 2 teeth. The arrows would then point to P, L, A and R respectively.

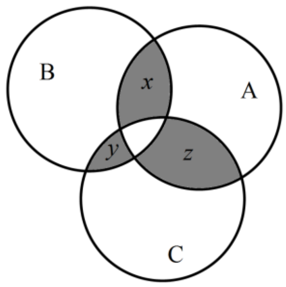
17. **B**

$$(2x)^2 + (4x)^2 = 15^2$$
$$\therefore 4x^2 + 16x^2 = 225$$
$$\therefore 20x^2 = 225$$
$$\therefore x^2 = \frac{225}{20} = \frac{45}{4}$$
$$\therefore 8 \times x^2 = \frac{45}{4} \times 8 = 90$$



18. **E**

$$120 \times \frac{1}{4} = 30,\ 120 \times \frac{1}{5} = 24,\ 120 \times \frac{1}{6} = 20$$
$$x + z = 30,\ x + y = 24,\ \ y + z = 20$$
$$\therefore 2x + 2y + 2z = 74$$
$$\therefore x + y + z = 37$$



19. **C** We need to apportion $8 + 10 + 9 = 27$ rides amongst the 12 friends. The maximum number of rides that can be apportioned such that no friend goes on all three rides is 24. This leaves a further 3 rides to be distributed. The minimum number of friends who went on all three rides is thus 3.

20. **E** The areas of the Triangle, Segment and Crescent are:

$$T = \frac{1}{2}(1)(1) = \frac{1}{2}$$
$$S = \frac{1}{4}\pi(1)^2 - \frac{1}{2} = \frac{1}{4}\pi - \frac{1}{2}$$
$$C = \frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{1}{4}\pi - \frac{1}{2}\right) = \frac{1}{4}\pi - \frac{1}{4}\pi + \frac{1}{2} = \frac{1}{2}$$
$$\therefore \frac{T}{C} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

15. **D** Die supplementêre hoeke x en y tep op na 90°. Die deel van P wat sigbaar is, is ‘n vierhoek. Die gelyke binnehoeke van P is dus $\frac{360° - 90°}{2} = 135°$.

Die buitehoeke van veelhoek P is dus 45°. $\frac{360°}{45°} = 8$. Veelhoek P het dus 8 sye.

16. **A** Die grootste rat, wat kloksgewys draai, draai 5 omwentelinge, wat beteken dat die pyl ‘n totaal van $16 \times 5 = 80$ tande verbysteek, en dan weer na P wys.

$80 = 12 \times 6 + 8 = 10 \times 8 = 6 \times 13 + 2$. Die tweede rat, wat antikloksgewys draai, draai deur 6 omwentelinge en 8 tande, en die derde rat, wat weer kloksgewys draai, draai deur presies 8 omwentelinge. Die kleinste rat, wat in ‘n antikloksgewyse rigting draai, draai deur 13 omwentelinge en 2 tande. Die pyle wys dus na P, L, A en R onderskydelik.

17. **B**

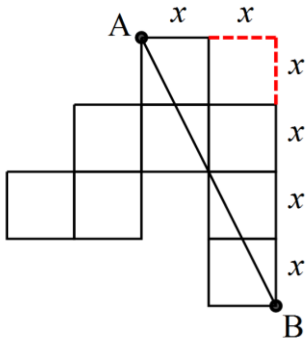
$(2x)^2 + (4x)^2 = 15^2$

$\therefore 4x^2 + 16x^2 = 225$

$\therefore 20x^2 = 225$

$\therefore x^2 = \frac{225}{20} = \frac{45}{4}$

$\therefore 8 \times x^2 = \frac{45}{4} \times 8 = 90$



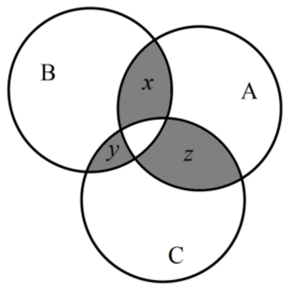
18. **E**

$120 \times \frac{1}{4} = 30,\ 120 \times \frac{1}{5} = 24,\ 120 \times \frac{1}{6} = 20$

$x + z = 30,\ x + y = 24,\ \ y + z = 20$

$\therefore 2x + 2y + 2z = 74$

$\therefore x + y + z = 37$



19. **C** Ons moet $8 + 10 + 9 = 27$ ritte verdeel tussen die 12 vriende. Die maksimum aantal ritte wat verdeel kan word sodat niemand op al drie ritte kan ry nie is 24. Dit los ‘n verdere 3 ritte om verdeel te word. Die minimum getal vriende wat op al drie ritte gery het moet dus 3 wees.

20. **E** Die oppervlaktes van die **D**riehoek, **S**egment en sekel**M**aan is:

$$D = \frac{1}{2}(1)(1) = \frac{1}{2}$$

$$S = \frac{1}{4}\pi(1)^2 - \frac{1}{2} = \frac{1}{4}\pi - \frac{1}{2}$$

$$M = \frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{1}{4}\pi - \frac{1}{2}\right) = \frac{1}{4}\pi - \frac{1}{4}\pi + \frac{1}{2} = \frac{1}{2}$$

$$\frac{D}{M} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

SUID-AFRIKAANSE WISKUNDE OLIMPIADE

Graad NEGE Eerste rondte 2018

Oplossings

1. **C** $\sqrt[3]{20-1+8} = \sqrt[3]{27} = 3$

2. **B** $\frac{2018}{8102} \approx \frac{2025}{8100} = \frac{1}{4}$

3. **D** $20 - (-18) = 38$

4. **B** $\frac{90^\circ - 20^\circ - 18^\circ}{2} = 26^\circ$

5. **D** Daar is vier reghoeke wat elk 1 eenheid breed is. Daar is drie wat 2 eenhede breed is,twee wat 3 eenhede breed is, en een wat 4 eenhede breed is. 4 + 3 + 2 + 1 = 10.

6. **E** Deur terugwaarts te werk, moes hy, voordat hy 81 gehad het, 9 gehad het; en voor dit 8, en voor dit 4.

7. **D** Die donker deel van die omtrek is dieselfde vir beide vorms. Die groter vorm het 'n totale omtrek van 3 + 6 + 5 = 14 cm op die ander sye, terwyl die kleiner ene 3 + 6 + 1 = 10 cm het. Die verskil is dus 4 cm.

8. **A** Die voltooide piramiede is:

	4	
6	2	
9	3	1

9. **E** $93 - 18 = 75$; $75 \times \frac{2}{3} = 50$; $50 + 18 = 68$

10. **A** Die drie vierkante het 'n totale oppervlakte van $10^2 + 8^2 + 6^2 = 200$ cm^2. Die driehoek wat nie ingekleur is nie het lengte $10 + 8 + 6 = 24$ cm en loodregte hoogte 10 cm. Die oppervlakte van die driehoek is dus 120 cm^2. Die ingekleurde oppervlakte is dus $200 - 120 = 80$ cm^2.

11. **E** Omdat $14 \times 14 \times 14 = 2^3 \times 7^3$, moet N gelyk wees aan $2^3 \times 7 = 56$.

12. **C** Die verskillende moontlikhede is 2 + 0, 2 + 1, 2 + 8, 1 + 8, 8 + 0 en 1 + 0.
Slegs drie van hierdie 6 moontlikhede het 'n ewe som, d.w.s. 50%

13. **C** Indien daar x susters is, sal die ma x geskenke ontvang. Daar sal $x(x-1) = x^2 - x$ geskenke tussen die sisters uigeruil word, so die totale getal geskenke is $x + x^2 - x = x^2$, wat 'n volkome vierkant is. Die enigste volkome vierkant tussen die 5 opsies is 49.

14. **A** $6! = 720$; $3! = 6$ en $5! = 120$ $\therefore$ $6! = 3! \times 5!$ $\therefore$ $p + q = 3 + 5 = 8$.