THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION
Sponsored by HARMONY GOLD MINING

SECOND ROUND 2005

JUNIOR SECTION: GRADES 8 AND 9

10 MAY 2005

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

ANSWERS

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	C
2	D
3	E
4	C
5	A
6	C
7	D
8	C
9	C
10	E
11	C & D
12	A
13	A
14	B
15	B
16	C
17	E
18	B
19	D
20	E

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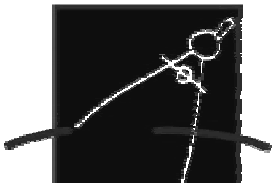

1. ANSWER: C

$$1-\frac{1}{2}\times\frac{1}{2}$$
$$=1-\frac{1}{4}$$
$$=\frac{3}{4}$$

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2. ANSWER: D

9	*x*	7
8	10	*a*
b	*c*	*d*

The sum of the rows, columns and diagonals must be the same.

$\therefore$ Row 1 = Diagonal from left

$$9+x+7=9+10+d$$
$$\therefore d=x-3$$

Also Row 1 = Column 1

$$\therefore\ 9+x+7=9+8+6$$
$$b=x-1$$

Lastly Row 3 = Column 2

$$\therefore\ x-1+c+x-3=x+b+c$$
$$\therefore\ x=14$$

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3. ANSWER: E

Suppose the initial purchase price is *X*.

Applying a 20% discount will lead to Terry having to pay

$$X-0{,}2X=0{,}8X\ .$$

Applying a further 10% discount will lead to a final price of

$$0{,}8X-0{,}8X=0{,}72X\ \textbf{.}$$

The effective single discount is therefore

$$X-0{,}72X=0{,}28X=28\%$$ discount.

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4. ANSWER: C

T	U		T	U	
x	y		y	x	
↓	↓		↓	↓	
$\therefore\ 10x$	$+\ y$	$+$	$10y$	$+\ x$	$=11x+11y$
					$=11(x+y)$
					$=11\times(x+y)$

5. ANSWER: A

Let the 2 numbers be x and y.

Then $x+y=-8$

and $x\times y=7$.

Split 7 into 2 factors: $1\times 7=x\times y$.

Since the sum = –8,

then the numbers are: –1 and –7.

6. ANSWER: C

$$4{,}32^2-3{,}32^2+1{,}36$$

Let us investigate

3^2-2^2	5^2-4^2
$=9-4$	$=25-16$
$=5$	$=9$

$3^2-2^2=3+2=5$; $5^2-4^2=5+4$

Conclusion: $a^2-b^2=a+b$ if $a-b=1$

Therefore: $4{,}32^2-3{,}32^2+1{,}36$

$=4{,}32+3{,}32+1{,}36$

$=7{,}64+1{,}36$

$=9$

7. ANSWER: D

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{2005}\right)$$
$$=\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)\cdots\left(\frac{78}{79}\right)\left(\frac{79}{80}\right)\cdots\frac{2004}{2005}$$
$$=\frac{2}{2005}\text{ as the first denominator cancels the next bracket's numerator, etc.}$$

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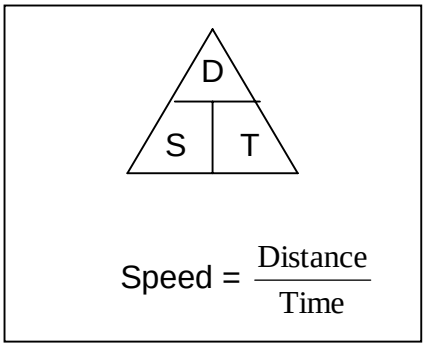
8. ANSWER: C

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}$$
$$=\frac{1{,}2\times10^{8}}{6\times10^{4}}$$
$$=0{,}2\times10^{4}$$
$$=2000\text{ hours}$$

The answer is just less than 3 months.



$$1\text{ month}=\text{approx.}\,31\times24$$
$$=\text{approx.}\,744\text{ hours}$$
$$2\text{ months}=\text{approx.}\,1488\text{ hours}$$
$$3\text{ months}=\text{approx.}\,2232\text{ hours}$$

$$31\times24=(30+1)\times24$$
$$=720+24$$
$$=744\text{ hours}$$

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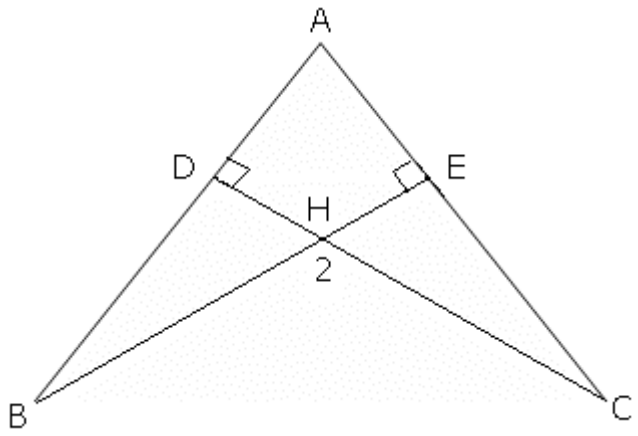
9. ANSWER: C



In quadrilateral ADHE

$\hat{A} + A\hat{D}H + \hat{H}_1 + A\hat{E}H = 360^0$

$\hat{A} + 90^0 + \hat{H}_1 + 90^0 = 360^0$

$\hat{A} + \hat{H}_1 = 180^0$

$\hat{H}_1 = B\hat{H}C$ (vertically opposite angles)

$\hat{A} + B\hat{H}C = 180^0$

$B\hat{H}C = 180^0 - A$

OR

In $\Delta\ ADC$, $\hat{C} = 90^0 - \hat{A}$

$\therefore$ external angle $\hat{H}_2 = 90^0 + \left(90^0 - \hat{A}\right)$

$= 180^0 - \hat{A}$

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10. ANSWER: E

Smallest possible $\frac{x}{y}$-values can be:

$\frac{-6}{-2} = 3$ or $\frac{-6}{-\frac{1}{2}} = 12$ or $\frac{10}{-2} = -5$ or $\frac{10}{-\frac{1}{2}} = -20$

Similarly the biggest value for $\frac{x}{y} = 12 = b$

$\therefore\ a = -20$ and $b = 12$

$\therefore\ ab = -20 \times 12 = -240$

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11. ANSWER: C & D

Maximum number of days in April is 30. When April starts on a Saturday the calendar will look as follows. The 28th of April *20xy* will fall on a Friday.

Sun-day	Mon-day	Tues-day	Wed-nesday	Thurs-day	Friday	Satur-day
						1
	3				7	
	10				14	
	17				21	
	24				28	
30						

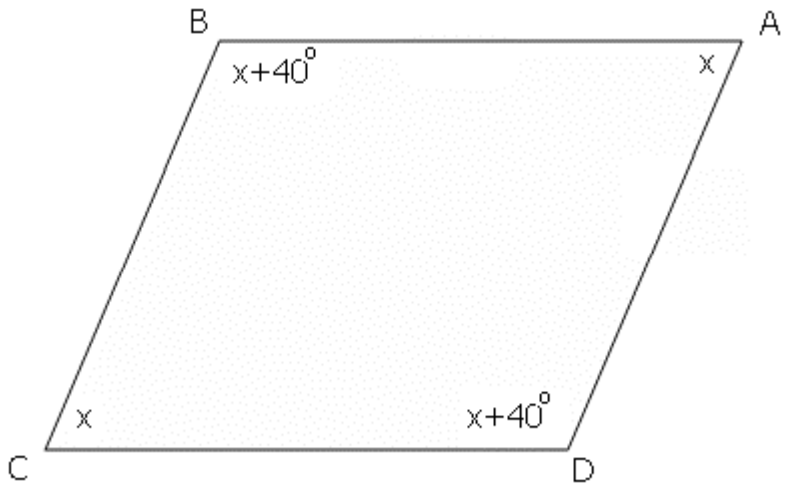
However, when April starts on a Tuesday the 28th of April 20*xy* will fall on a Monday.

12. ANSWER: A

Area of each face = 1 cm^2

$$\begin{aligned}\text{Number of exposed faces} &= 5+4+4+\cdots+4+5\cdots 100 \text{ terms}\\ &= 98\times 4+2\times 5\\ &= 392+10\\ &= 402\\ \text{Area of exposed faces} &= 402cm^2\end{aligned}$$

13. ANSWER: A



$$\begin{aligned}\text{Let } \hat{A} &= x = \hat{C}\\ \hat{B} &= \hat{D} = x+40^0\\ \hat{A}+\hat{B}+\hat{C}+\hat{D} &= 360^0\\ x+x+40^0+x+x+40^0 &= 360^0\\ 4x+80^0 &= 360^0\\ 4x &= 280^0\\ x &= 70^0\end{aligned}$$

14. ANSWER: B

$$\text{average} = \frac{\text{total earnings}}{\text{days worked}}$$

$$\therefore 78 = \frac{x}{20}$$

$$\therefore x = 1560$$

$$\text{Also } 90 = \frac{y}{25}$$

$$\therefore y = 2250$$

She must earn R2 250 – R1 560 = R690 over the next 5 days.

15. ANSWER: B

A = B + x	***B***

$$\frac{10(B+x)+B}{(B+x)+B} = 7 \text{ rem } x$$

$$\frac{10B+10x+B}{(2B+x)} = 7 + \frac{x}{(2B+x)}$$

$$LCM = (2B+x)$$

$$\therefore 10B+10x+B = 7(2B+x)+x$$

$$\therefore 10B+10x+B = 14B+7x+x$$

$$\therefore \quad 11B+10x = 14B+8x$$

$$\therefore 2x = 3B$$

$$\therefore \frac{2}{3}x = B$$

$$\therefore x = \frac{3B}{2}$$

if $B = 2$ then $x = 3,$ and $A = 5.$

if $B = 4$ then $x = 6,$ and $A = 10;$ too big $\therefore x = 3.$

16. ANSWER: C

The number can be written as

$$(100\times 2)+(10\times y)+(2x-1)$$
$$=100x+10y+2x-1$$
$$=102x+10y-1=112x+29$$
$$10y=10x+30$$
$$y=x+3$$

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17. ANSWER: E

Trial and improvement:

$$\begin{array}{r} 325 \\ \times\ 147 \\ \hline 2275 \\ 13000 \\ 32500 \\ \hline 47775 \end{array}$$

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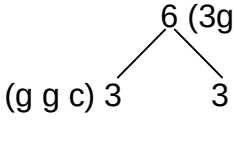
18. ANSWER: B

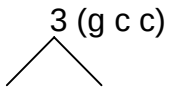
Let genuine = *g* and counterfeit = *c*

Then



or $\underbrace{\overset{(1)}{(ggg)}\ \overset{(1)}{(ccc)}}_{\text{possible}}$

lighter



(c) 1 1 (g) 1 (c) problem solved

or (c) 1 1 (c) 1 (g) problem solved

2 x weighings (minimum)

or separated into lots of 2

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19. ANSWER: D

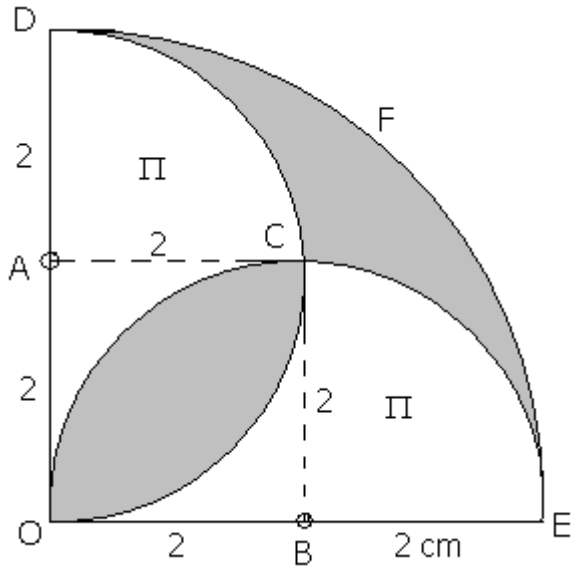
The smallest possible sum will be 1 (from 10 000 000) and the highest possible sum will be 72 (from 99 999 999).

We therefore have 72 possible answers and the average of 1 and 72 will be

$$\frac{1}{2}(1+72) = 36\frac{1}{2}$$

$\therefore$ Both 36 and 37 will occur the most.

20. ANSWER: E



Area $\frac{1}{4}$ DOE $= \frac{\pi(4)^2}{4} = 4\pi \ \ cm^2$

Area $\frac{1}{4}$ DAC $= \frac{\pi(2)^2}{4} = \pi \ \ cm^2$

Area $\frac{1}{4}$ CBE $= \pi \ \ cm^2$ as before

$$\text{Area } DCEF = \text{Area } DOE - \text{Area } DCO - \text{Area } OCE + \text{Area leaf } OC$$
$$= 4\pi - 2\pi - 2\pi + \text{area leaf OC}$$

$\therefore$ Area DCEF = Area leaf OC

Then Area of leaf is $\left[2(\pi - 2)\right]$

Answer: $2\left[2(\pi-2)\right] = 4(\pi-2) = 4\pi - 8$