2015 SOUTH AFRICAN MATHEMATICS OLYMPIAD
JUNIOR ROUND TWO – SOLUTIONS

1. **2** $4 + (-2) = 4 - 2 = 2$

2. **4** The factors of 128 are all powers of 2, i.e. $2^0, 2^1, \ldots, 2^7$. The largest power of 2 that divides 120 is $8 = 2^3$. The powers of 2 dividing 128 and exceeding 8 are 2^4, 2^5, 2^6 and 2^7.

3. **6** If the length of the strip is 12 units, marks will be made at 3 units, 6 units and 9 units for four equal lengths, and at 4 and 8 units for the three equal lengths. This gives 5 marks in all, meaning there will be 6 pieces when the cuts are made.

4. **15** The smallest value of n is 20 (for which $\frac{1}{2}n$ is 10) and the largest value of n is 49 (for which $2n$ is 98). From 20 to 49 inclusive represents 30 integers, half of which are even (so that $\frac{1}{2}n$ is an integer).

5. **30** $$\frac{2}{0+\frac{1}{5+\frac{2}{0+\frac{1}{5}}}} = \frac{2}{0+\frac{1}{5+\frac{2}{\frac{1}{5}}}} = \frac{2}{0+\frac{1}{5+10}} = \frac{2}{\frac{1}{15}} = 2\times\frac{15}{1} = 30$$

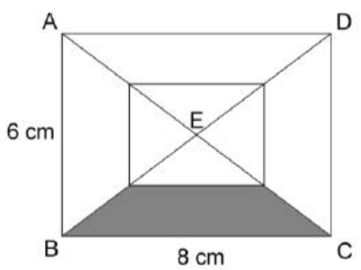
6. **36** 27 people must have a pen and 36 must have a pencil. If all the people with pens also have pencils, this could be as few as 36 people in all.

7. **2040** In order to be divisible by 2, 3, 4, 5 and 6, the number only needs to be divisible by $2^2\times3\times5$, i.e. it must be a multiple of 60. The smallest multiple of 60 bigger than 2015 is 2040.

8. **10** Suppose $\frac{n+3}{13} = k$; then $n = 13k - 3$ or $n = 13(k-1) + 10$. This shows that n leaves remainder 10 on division by 13.

9. **32** The area of the whole circle is $\pi.4^2 = 16\pi$, but the area of the sector is $\frac{1}{2}\pi$, so the probability is $\frac{\frac{1}{2}\pi}{16\pi} = \frac{1}{32}$

10. **9** Area of shaded trapezium $= \frac{1}{2}\times(4+8)\times1{,}5 = 9$



11. **75** If W is the number of litres the tank can hold, then when it is 30% empty it is holding 0,7W litres, and so we have 0,7W = 0,3W + 30, i.e. 0,4W = 30 and then W = 30 ÷ 0,4 = 300 ÷ 4 = 75.

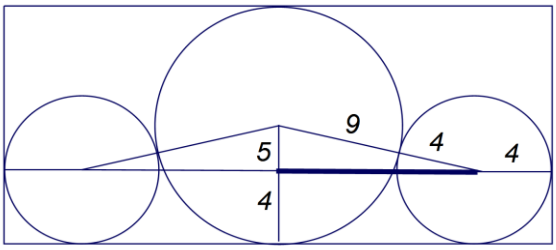
12. **808** Each row, apart from the bottom row and top row, contributes 4 cm to the perimeter. The top and bottom rows each contribute 8 cm. Since there are 1000 ÷ 5 = 200 rows we have Perimeter = (200 – 2) × 4 + 2 × 8
= 99 × 8 + 2 × 8 = 101 × 8 = 808.

13. **6** By Pythagoras, $OA_2 = \sqrt{17}$. Then $OA_3 = \sqrt{18}$, $OA_4 = \sqrt{19}$ and so on, with $OA_n = \sqrt{n+15}$, and thus $OA_{21} = \sqrt{36} = 6$.

14. **2** The prime numbers smaller than 20 are 2, 3, 5, 7, 11, 13, 17 and 19. In order for the fraction to be as big as possible we need to maximise the numerator and minimise the denominator. We thus try $\frac{19-5}{2(2)+3} = \frac{14}{7} = 2$, and $\frac{19-2}{2(3)+5} = \frac{17}{11}$ and $\frac{19-3}{2(2)+5} = \frac{16}{9}$ which are smaller.

15. **4** $10\times9\times8\times7\times6\times5\times4\times3\times2\times1 = 5.2.3^2.2^3.7.2.3.5.2^2 = 7.5^2.3^4.2^8$.
Since $2^8 = 4^4$ we have $x = 4$

16. **32** 9 – 4 = 5, and the length marked bold in the diagram must be 12 by Pythagoras. That means the length of the rectangle is 4 + 12 + 12 + 4 = 32



17. **6** We can re-write $\frac{x}{20} + \frac{y}{15} = 1$ in the form $3x + 4y = 60$. Then we can see that $3x$ must be divisible by 4, so x must be, and trying successive possible values we see that only the following combinations of x- and y-values are acceptable: (0; 15), (4; 12), (8; 9), (12; 6), (16; 3), (20; 0).

18. **120** For a square with diagonal d and side length s we have the relationship $s^2 + s^2 = d^2$ and thus $s = \frac{d}{\sqrt{2}}$. The respective side lengths of the three squares are thus $\frac{3}{\sqrt{2}}$, $\frac{4}{\sqrt{2}}$ and $\frac{5}{\sqrt{2}}$ respectively. So grey area = $\left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$ while black area = $\left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{4}{\sqrt{2}}\right)^2 = \frac{25}{2} - \frac{16}{2} = \frac{9}{2}$ = grey area.
Thus the black area must also require 120 bricks.

19. **16** To have a sum of 75, the five consecutive numbers must be 13, 14, 15, 16 and 17. Since the numbers in the triangle total 29, the two possibilities are 13 & 16 or 14 & 15. Since the numbers in the square total 30, the two possibilities are 14 & 16 or 13 & 17. Since the numbers in the circle total 47, the only possibility is 14, 16 & 17. The only possible letter that could represent 15 is thus A, from which it follows that B = 14, E = 13, D = 17 and finally C = 16.

20. **31** Join AC ; then ΔABC has base 2 cm and height 10 cm, while ΔADC has base 6 cm and height 7 cm. So the shaded area is $\frac{1}{2}2.10 + \frac{1}{2}6.7 = 10 + 21 = 31$ cm^2.