THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

organised by the SOUTH AFRICAN ACADEMY OF SCIENCE AND ARTS
in collaboration with OLD MUTUAL, AMESA and SAMS

SPONSORED BY OLD MUTUAL

SECOND ROUND 2000
JUNIOR SECTION: GRADES 8 AND 9
6 JUNE 2000
TIME: 120 MINUTES
NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:

For each correct answer	in Part A:	4 marks
	in Part B:	5 marks
	in Part C:	6 marks
For each wrong answer:		–1 mark
For no answer:		0 marks

4. You must use an HB pencil.
   Rough paper, ruler and rubber are permitted.
   Calculators and geometry instruments are not permitted.
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. When the invigilator gives the signal, start the problems.
   You will have 120 minutes working time for the question paper.

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.

KEER DIE BOEKIE OM VIR AFRIKAANS

P.O. BOX 538, PRETORIA, 0001 TEL: (012) 328-5082 FAX (012) 328-5091
E-mail: akademie@mweb.co.za

PRACTICE EXAMPLES

1. $23 + 6 - 4 =$

(A) 6 (B) 23 (C) 25 (D) 29 (E) 33

2. $\frac{1}{5} + \frac{2}{3} \times \frac{1}{2}$ equals

(A) $\frac{1}{15}$ (B) $\frac{3}{11}$ (C) $\frac{21}{50}$ (D) $\frac{8}{15}$ (E) $9\frac{4}{5}$

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PART A: (Each correct answer is worth 4 marks)

1. The units digit of the product $11 \times 13 \times 15$ is

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

2. The number 2000 can be written in exponent form as

(A) $(2 \times 3)^3$ (B) $2^3 \times 5^3$ (C) $3^2 \times 5^4$ (D) $2^4 \times 5^3$ (E) $2^3 \times 5^4$

3. The number $\dfrac{2{,}001 \div 2{,}000}{1{,}999}$ is closest to

(A) 0 (B) $\dfrac{1}{4}$ (C) 2 (D) $\dfrac{1}{2}$ (E) 1

4. A natural number is subtracted from the numerator of the fraction $\dfrac{8}{11}$. The same number is subtracted from the denominator of the fraction. If the resulting fraction is equivalent to $\dfrac{2}{3}$, the number subtracted is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5. If 100 is divided by the positive integer *x*, the remainder is 2. If 198 is divided by *x*, the remainder is

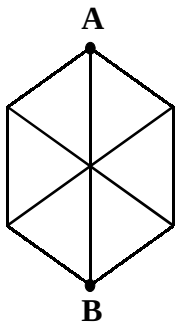
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

PART B: (Each correct answer is worth 5 marks)

6. A cube-shaped water tank has dimensions $1\,\text{m} \times 1\,\text{m} \times 1\,\text{m}$. Water is flowing into the tank at a constant rate of 2 ℓ/minute. The rate at which the water level is rising is [Note: 1 ℓ =1 000 cm^3]

(A) 0,1 cm / min (B) 0,2 cm / min (C) 1 cm / min
(D) 1,2 cm / min (E) 2 cm / min

7. You wish to travel from A to B along the lines as shown in the sketch. You may only move downwards. The number of different paths from A to B is



(A) 9 (B) 10 (C) 11
(D) 12 (E) 13

8. How many numbers from 11 to 99 have the sum of their digits a square number?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

9. The product of all the natural numbers from 1 to *n* can be written as *n*!. For example $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$. Find the smallest number *n* such that *n*! is divisible by 990.

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

10. It requires 12 litres of paint to paint the 6 sides of a cube with dimensions $2\text{m} \times 2\text{m} \times 2\text{m}$. The number of litres of paint required to paint the outside of a rectangular box with dimensions $4\text{m} \times 1\text{m} \times 8\text{m}$ is

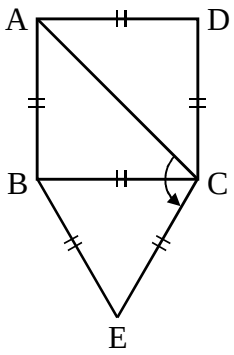
(A) 40 (B) 44 (C) 45 (D) 47 (E) 49

11. Just before her last mathematics test this year Thandi's average mathematics mark was 89%. She scored 97% in the last test and her average is now 90%. The number of mathematics tests she wrote this year is

(A) 5 (B) 9 (C) 7 (D) 13 (E) 8

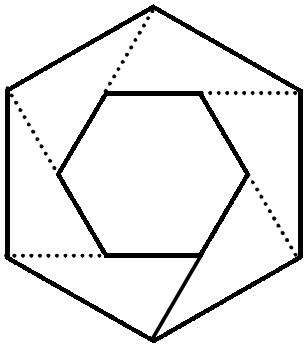
12. In the diagram, ABCD is a square with diagonal AC. BCE is an equilateral triangle.

The size of angle $A\hat{C}E$ is



(A) 105° (B) 60° (C) 90°
(D) 135° (E) 120°

13. Each side of a regular hexagon is extended by a length equal to its own length. The end points of the new segments are joined to form a new and larger regular hexagon. What fraction of the area of the bigger hexagon does the smaller hexagon occupy?

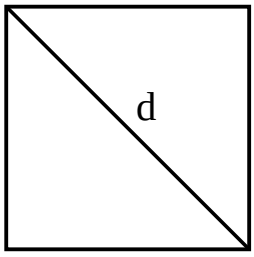


(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$
(D) $\frac{3}{8}$ (E) $\frac{1}{6}$

14. A class survey found that 25 learners watched TV on Monday, 20 on Tuesday and 16 on Wednesday. Of those who watched TV on only one of the days, 11 chose Monday, 7 chose Tuesday and 6 chose Wednesday. If each learner watched on at least one of the days and 7 watched on all 3 days, how many learners are there in the class?

(A) 61 (B) 45 (C) 41 (D) 39 (E) 38

15. The length of the diagonal of a square is d.
The area of the square is



(A) $\frac{1}{2}d^2$ (B) $\frac{d^2}{\sqrt{2}}$ (C) d^2
(D) $\sqrt{2}d^2$ (E) $2d^2$

PART C: (Each correct answer is worth 6 marks)

16. Suppose two natural numbers are represented by n^2 and $n^2 - 1$. Which of the following must necessarily be true?

(A) Both are odd numbers.
(B) Both are even numbers.
(C) Both numbers are multiples of 4.
(D) Exactly one of the numbers is a multiple of 4.
(E) The sum of the numbers is even.

17. A box contained 31 sweets. On Monday Sam ate $\frac{3}{4}$ of the number Pam ate on Monday. On Tuesday Sam ate $\frac{2}{3}$ of the number Pam ate on Tuesday. Then all the sweets had been eaten. How many sweets did Sam eat?

(A) 6 (B) 7 (C) 10 (D) 11 (E) 13

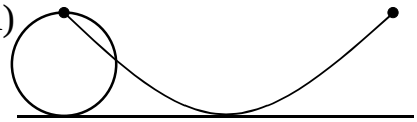
18. A slow train travelling from Springs to Soweto arrives 9 minutes late when travelling at 36 km/h. If it travels at 27 km/h it arrives 39 minutes late. What is the distance between Springs and Soweto?

(A) 54 (B) 90 (C) 48 (D) 36 (E) 64

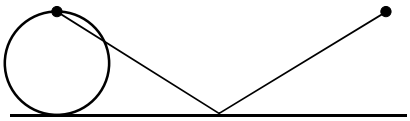
19.

A coin with a mark on it as shown above is rolled without sliding along a straight line. The path the mark follows during one revolution is:

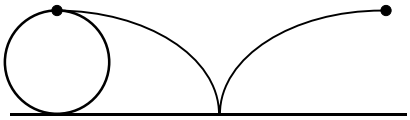
(A)



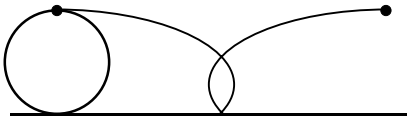
(B)



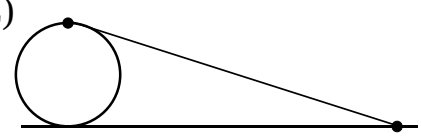
(C)



(D)



(E)



20. A sealed envelope contains a card with a single digit on it. Three of the following statements are true and the other is false.

I The digit is 1
II The digit is 2
III The digit is not 3
IV The digit is not 4

Which one of the following must be correct?

(A) I is false. (B) II is true. (C) II is false.
(D) III is false. (E) IV is true.

THE END

ANSWER POSITIONS: JUNIOR SECOND ROUND 2000

PRACTICE EXAMPLES	POSITION
1	C
2	D

NUMBER	POSITION
1	C
2	D
3	D
4	B
5	B
6	B
7	C
8	C
9	C
10	B
11	E
12	A
13	A
14	D
15	A
16	D
17	E
18	A
19	C
20	E

DISTRIBUTION	
A	4
B	4
C	5
D	4
E	3
TOTAL	**20**