1. Is $\log_8 10$ rational?
2. Find the maximum value of
$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma$$
where α, β and γ are positive and $\alpha + \beta + \gamma = 180^\circ$.
3. A, B, C D, E and F lie in that order on the circumference of a circle. The chords AD, BE and CF are concurrent. P, Q and R are the midpoints of AD, BE and CF respectively. Two further chords $AG \,||\, BE$ and $AH \,||\, CF$ are drawn. Prove that $\triangle PQR \,|||\, \triangle DGH$.
4. In a group of people, every two people have exactly one friend in common. Prove that there is a person who is a friend of everyone else.
(We suppose that if A is a friend of B, then B is a friend of A.)
5. For any number $n \in \mathbb{N}$ and $1 \leq r \leq n-1$ the integer $\binom{n}{r}$ is defined by
$$\binom{n}{r} = \frac{n(n-1)\ldots(n-r+1)}{r(r-1)\ldots 3.2.1}.$$
Show that the greatest common divisor of $\binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}$ is a prime if n is a power of a prime and is 1 otherwise.
6. You are given n squares, not necessarily all of the same size, which have total area 1. Is it always possible to place them without overlapping in a square of area 2?

In question 5, $\mathbb{N}$ denotes the set of natural numbers.