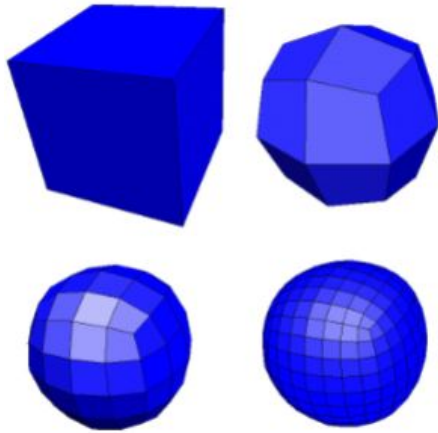


Project 2 Recitation

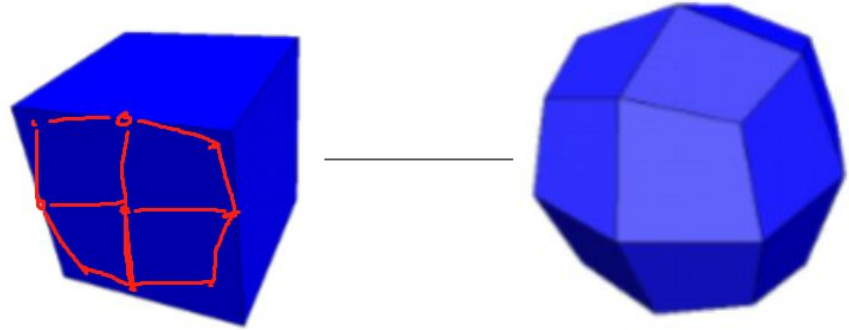
Subdivision Algorithm

Objective

Subdivision is a recursive algorithm, which takes the mesh objects(face and vertex) to subdivides it. From an intuitive perspective, this algorithm will generate more faces on the original surface, which also implies the need to generate more vertices(Face is also maintained by different vertices.)



Subdivision by new vertices, which generate new faces



Objective

Mesh Object data structure

```
# HalfedgeMesh
mesh = halfedge_mesh.HalfedgeMesh(data_path)
```

```
# print(mesh.vertices)
for vertex in mesh.vertices:
    print(vertex.get_vertex(), vertex.index)

# print(mesh.facets)
print('-----')
for face in mesh.facets:
    print(face.a, face.b, face.c, face.index)
```

([x,y,z], index)

```
([0.0, 0.0, 0.488037], 0)
([0.003906, 0.042188, 0.476326], 1)
([0.003906, -0.042188, 0.476326], 2)
([0.010742, 0.0, 0.575333], 3)
([0.0125, 0.056251, 0.450561], 4)
([0.0125, -0.056251, 0.450561], 5)
([0.019531, 0.0, 0.413654], 6)
([0.021094, 0.042188, 0.424797], 7)
([0.021094, -0.042188, 0.424797], 8)
([0.025, 0.0, 0.413086], 9)
([0.03875, 0.19625, 0.488037], 10)
([0.03875, -0.19625, 0.488037], 11)
([0.039063, 0.0, 0.66803], 12)
([0.04866, 0.192034, 0.575333], 13)
([0.04866, -0.192034, 0.575333], 14)
([0.056768, 0.188584, 0.413654], 15)
([0.056768, -0.188584, 0.413654], 16)
([0.0625, 0.0, 0.358795], 17)
([0.074785, 0.180918, 0.66803], 18)
([0.074785, -0.180918, 0.66803], 19)
([0.079102, 0.0, 0.764481], 20)
([0.096406, 0.171719, 0.358795], 21)
([0.096406, -0.171719, 0.358795], 22)
([0.1, 0.0, 0.769043], 23)
([0.103906, 0.042188, 0.777779], 24)
...
```

(vertex index1,index2,index3, face index4)

```
(298, 290, 333, 130)
(333, 290, 319, 131)
(257, 290, 262, 132)
(262, 290, 298, 133)
(176, 257, 178, 134)
(178, 257, 262, 135)
(347, 333, 354, 136)
(354, 333, 338, 137)
(311, 298, 347, 138)
(347, 298, 333, 139)
(262, 298, 266, 140)
(266, 298, 311, 141)
(178, 262, 180, 142)
(180, 262, 266, 143)
(352, 347, 358, 144)
(358, 347, 354, 145)
(322, 311, 352, 146)
(352, 311, 347, 147)
(266, 311, 272, 148)
(272, 311, 322, 149)
(180, 266, 182, 150)
(182, 266, 272, 151)
(355, 352, 359, 152)
(359, 352, 358, 153)
(326, 322, 355, 154)
...
```

Loop Subdivision Algorithm

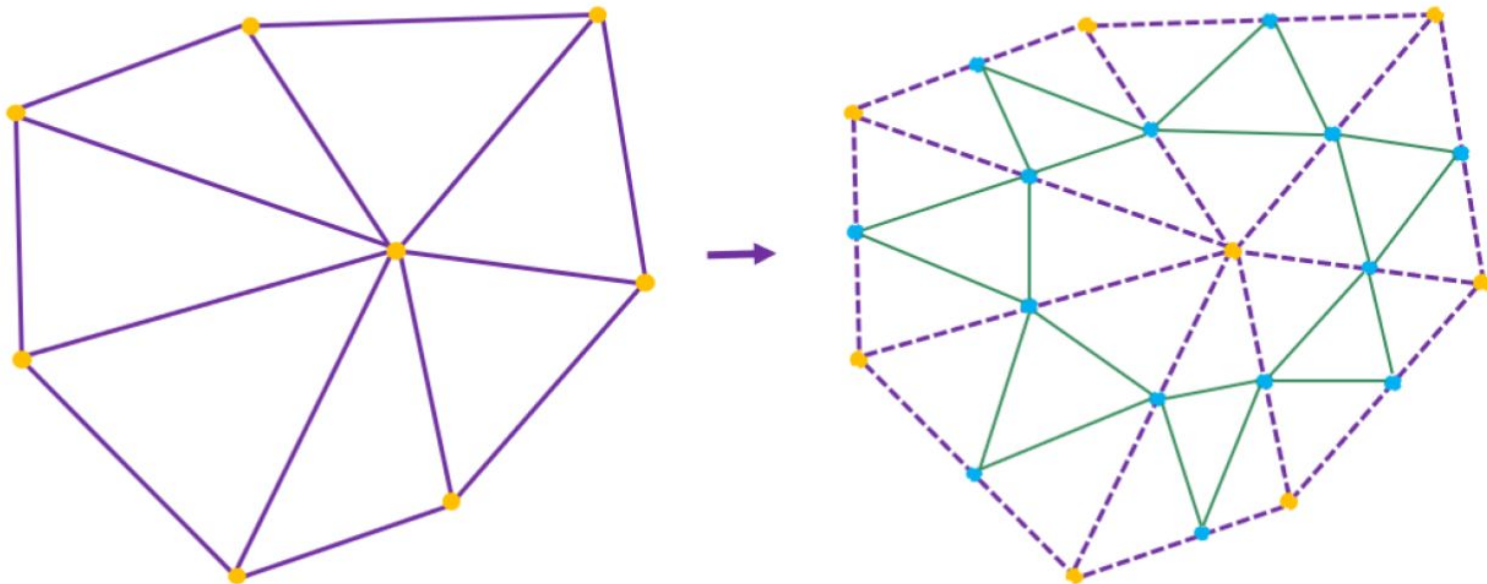
Loop, Charles. "Smooth subdivision surfaces based on triangles." (1987).

<https://charlesloop.com/thesis.pdf>

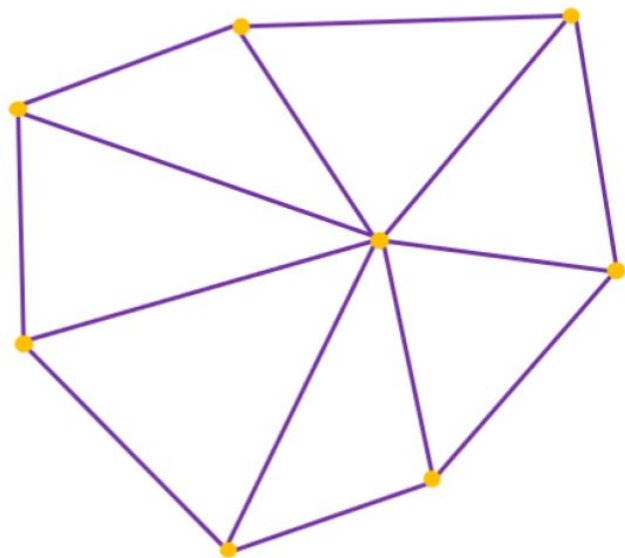
<https://www.rorydriscoll.com/2008/08/01/catmull-clark-subdivision-the-basics/>

Loop Subdivision Objective

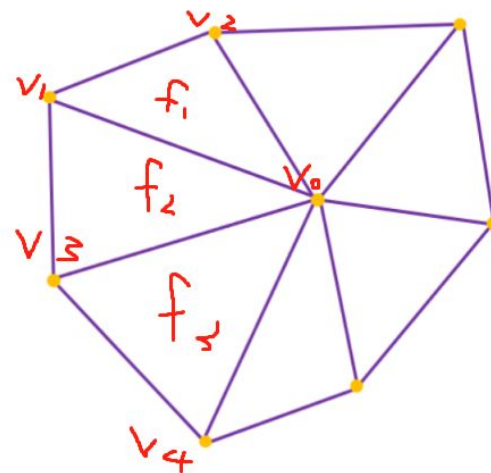
Loop Subdivision Surface is an approximating subdivision scheme for triangular meshes. The algorithm mainly divide into two part: **Generate the new vertices**, and **Update the old vertices**.



Loop Subdivision Objective

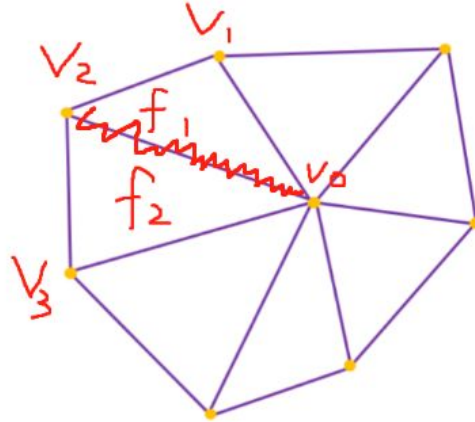
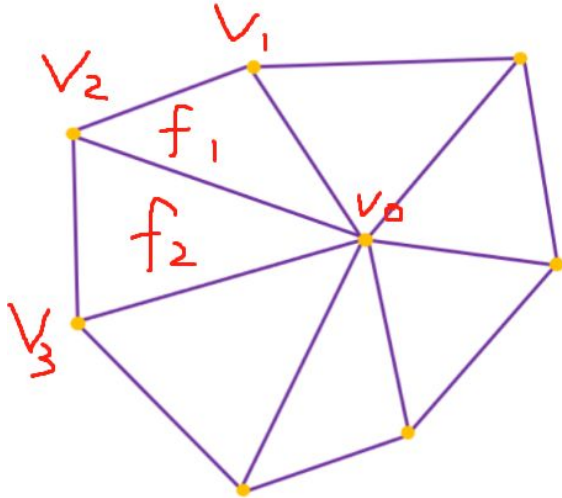


Iterate through
each face object
and obtain each
edge.



Generate the new vertices

1. Edges shared with other triangles



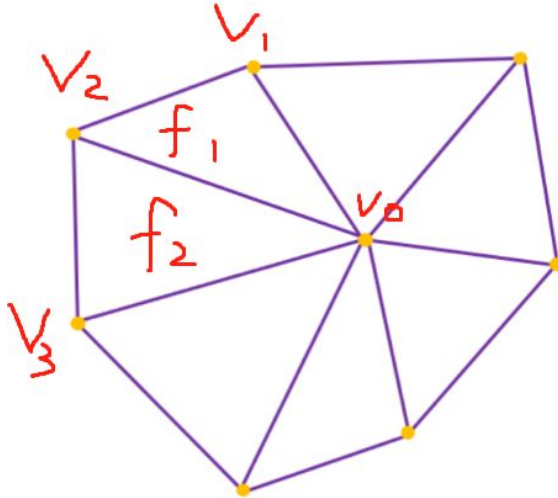
Edge ($v_0 - v_1$) shared by face f_1 and f_2

For these cases, we calculate the new vertex for this edge by the formula

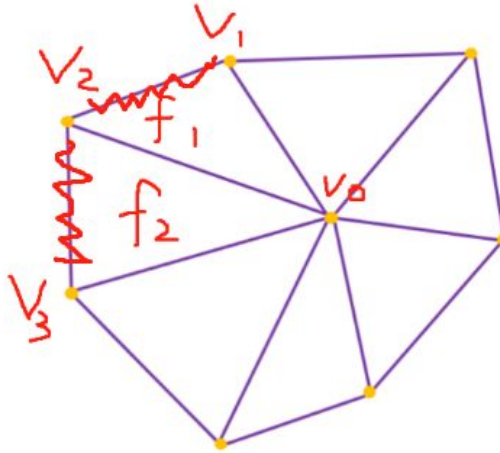
$$V_{new} = \frac{3}{8}(v_0 + v_2) + \frac{1}{8}(v_3 + v_1)$$

Generate the new vertices

1. Edges **Not** shared with other triangles



Edge ($v_2 - v_3$ and $v_1 - v_2$) Not shared by other triangles



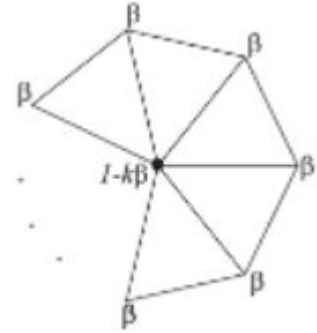
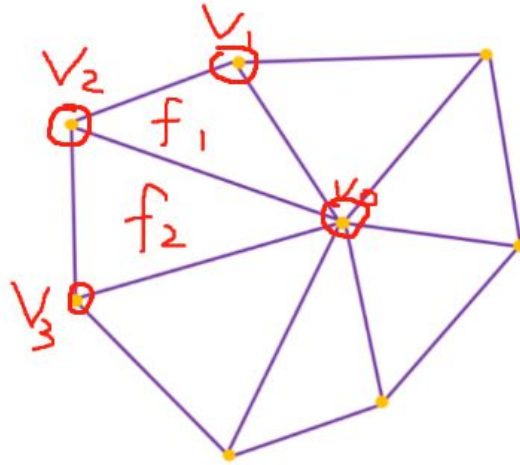
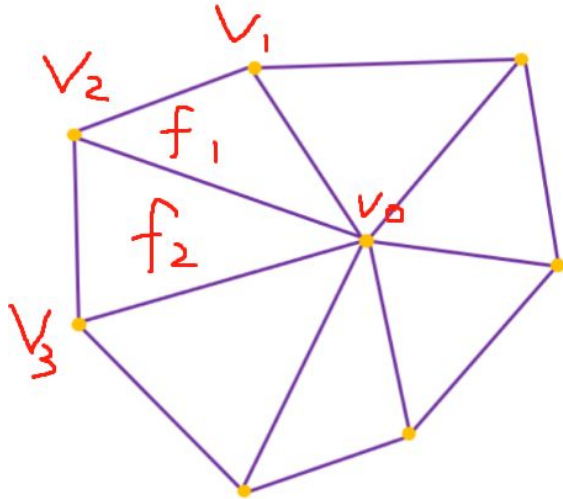
For those cases, we calculate the midpoint for this edge as the new vertex be formale by

$$V_{\text{new}} = \frac{1}{2}(v_3 + v_2)$$

$$V_{\text{new}} = \frac{1}{2}(v_2 + v_1)$$

It is necessary to traverse each edge to ensure the generation of new vertices !

Update the old vertices



$$v = (1 - n\beta)v_0 + \beta \sum_{i=1}^n v_i$$

$$\beta = \frac{1}{n} \left[\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right]$$

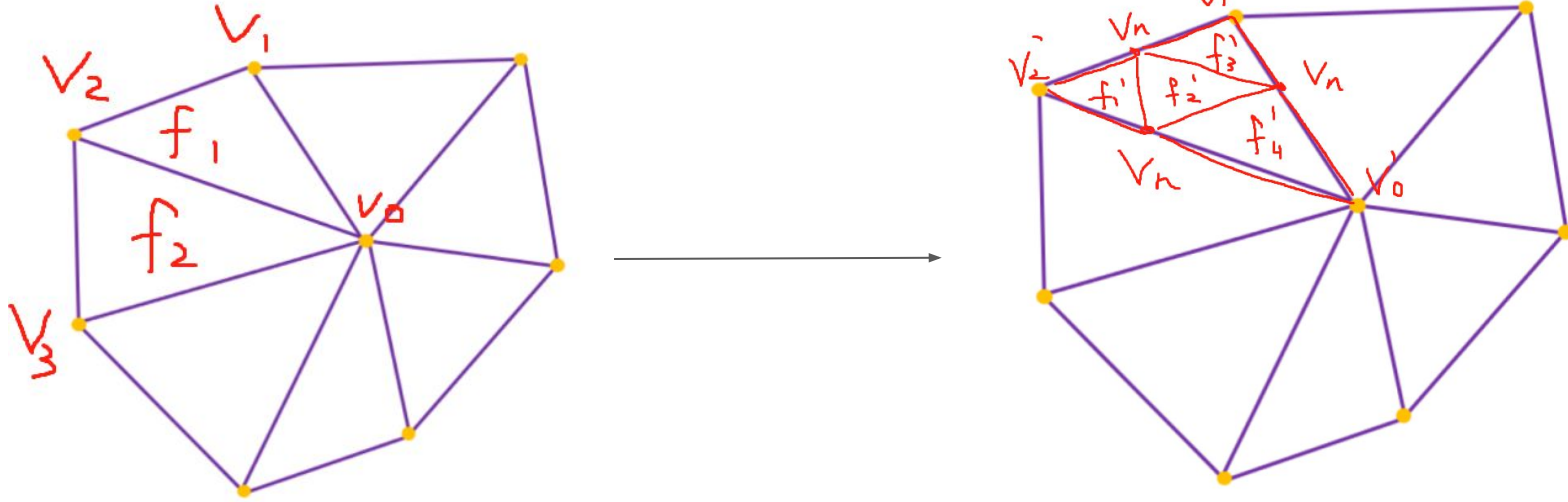
$$v^{i+1} = (1 - n\alpha)v^i + \alpha \sum_{j=1}^n v_j^i$$

where

$$\alpha = \frac{1}{n} \left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right)$$

for $n > 3$ and $\alpha = \frac{3}{16}$, if $n = 3$.

New Mesh Object

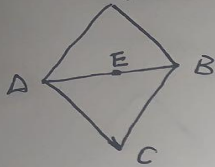


The old face object f_1 update to new faces (f'_1, f'_2, f'_3, f'_4).
 $f'_1 \rightarrow (v'_2, v_n, v_n)$ etc.

we iterate through each old face and combine it with three new vertices and three updated old vertices (eg. (v_1, v_2, v_3) to generate new face).

Generation and Updating Details

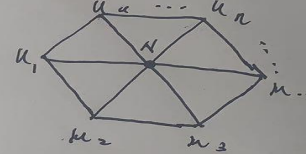
Not boundary case:



new vertex generate for AB

$$E = \frac{3}{8}(A+B) + \frac{1}{8}(C+D)$$

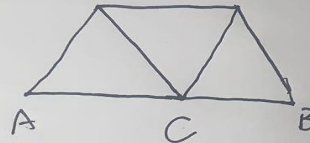
old vertex update:



$$N^2 = N \cdot (1 - \alpha) + \alpha \sum_{i=1}^n u_i$$

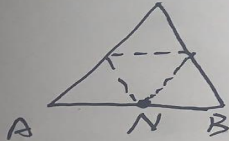
$$\alpha = \frac{1}{n} \left[\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right) \right]$$

old vertex C update.



$$C' = \frac{1}{8}(A+B) + \frac{3}{4}C$$

boundary case:

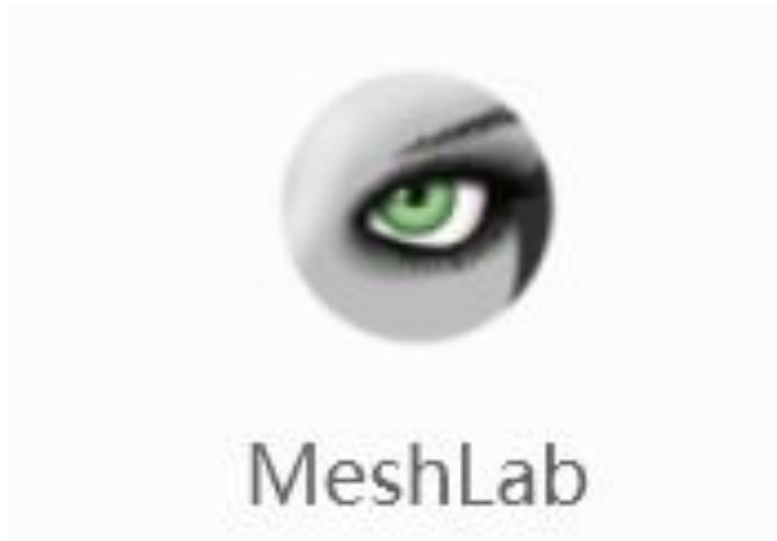


new vertex generate for AB

$$N = \frac{1}{2}(A+B)$$

Checking the subdivision performance

Download the <https://www.meshlab.net/>



Final Review

Loop Subdivision Algorithm

1. Generate new vertex for each edge. (whether an edge is shared by other triangles)
2. Update the old vertices.
3. Update the mesh object by each face, all old face will divide into 4 new faces, which comprise by new vertices (3) and updated old vertices (3).