B8

January 9, 2024

1 Exercise 15: Unfolding with Quadratic Matrices

[222]:
```
import numpy as np
import matplotlib.pyplot as plt
```

1.1 a) The response Matrix

What kind of measurement process does A describe? g = A * f: relationship between truth and measurement f: true values, g: measured values, A: response matrix

Since the detector has an efficiency < 1, the diagonal elements are < 1. Since the resolution is not perfect, there are off-diagonal matrix elements.

[223]:
```
def matrix(n, epsilon):
    A = np.zeros((n,n))
    np.fill_diagonal(A,1-2*epsilon) #Hauptdiagonale füllen
    A[0,0] = A[-1,-1] = 1-epsilon #erstes und letzes Element korrigieren

    for i in range(n-1): #Nebendiagonale füllen
        A[i,i+1] = A[i+1,i] = epsilon

    return A
```

[224]:
```
#Beispiel:
epsilon=0.23
n=10

A = matrix(n,epsilon)
print(A)
```

```
[[0.77 0.23 0.   0.   0.   0.   0.   0.   0.   0.  ]
 [0.23 0.54 0.23 0.   0.   0.   0.   0.   0.   0.  ]
 [0.   0.23 0.54 0.23 0.   0.   0.   0.   0.   0.  ]
 [0.   0.   0.23 0.54 0.23 0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.23 0.54 0.23 0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.23 0.54 0.23 0.   0.   0.  ]
 [0.   0.   0.   0.   0.   0.23 0.54 0.23 0.   0.  ]
 [0.   0.   0.   0.   0.   0.   0.23 0.54 0.23 0.  ]
```

```
 [0.   0.   0.   0.   0.   0.   0.   0.23 0.54 0.23]
 [0.   0.   0.   0.   0.   0.   0.   0.   0.23 0.77]]
```

1.2 b) The true distribution f, the simulation g and the measured distribution lambda

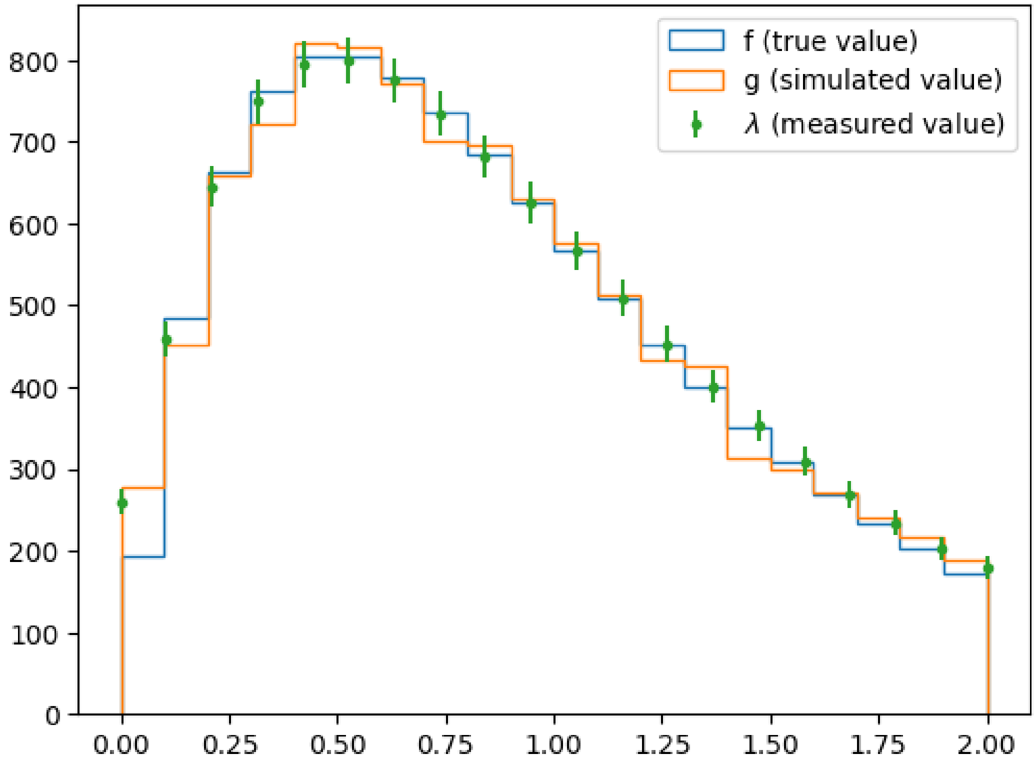
[225]:
```python
def lam(f, A):
    return A @ f

def g(f,A,rng):
    return rng.poisson(lam(f,A))
```

[226]:
```python
f = np.array([193, 485, 664, 763, 804, 805, 779, 736, 684, 626, 566, 508, 452,␣
 ↪400, 351, 308, 268, 233, 202, 173])
rng = np.random.default_rng(42)
A = matrix(20,epsilon)

print(lam(f,A))
print(g(f,A,rng))
```

```
[260.16 459.01 645.6  749.66 794.8  798.79 775.09 733.93 682.62 625.54
 566.46 508.46 452.92 400.69 352.38 308.69 269.15 233.92 202.46 179.67]
[274 485 606 772 758 788 787 730 687 652 585 528 451 377 343 307 248 221
 200 188]
```

[227]:
```python
x=np.linspace(0,2,len(f))

plt.hist(x,weights=f, label='f (true value)', bins=len(f), histtype='step')
plt.hist(x,weights=g(f,A,rng), label='g (simulated value)', bins=len(f),␣
 ↪histtype='step')
plt.errorbar(x,lam(f,A),np.sqrt(lam(f,A)), fmt='.', label='$\lambda$ (measured␣
 ↪value)')
plt.legend()
```

[227]: <matplotlib.legend.Legend at 0x7fb22ea12710>



1.3 c) Folding A=U*D*U ^{1}

g=U*D*U ^{1} f

<=> U ^{1} g = DU ^{1}f

<=> c = b

where c = U ^{1} g, b = DU ^{1} f

Advantages:

1. Diagonal matrices (D) are easier to invert
2. Orthogonal matrices (U) are easy to invert U^T=U ^{1}
3. Sorting the eigenvalues in descending order helps in identifying the most significant dimensions

[228]:
```
# Eigenwerte & -vektoren
w,v = np.linalg.eig(A) # gibt EW w und EV v zurück

print(w)

# Absteigend sortieren
indices = np.argsort(w)[::-1]
w = w[indices]
v = v[:,indices]
```

```
print(w)
```

```
[0.08566336 0.102514   0.130137   0.16785218 0.21473088 0.26961878
 0.33116437 0.39785218 0.46804015 0.54       0.61195985 0.68214782
 0.74883563 0.81038122 0.86526912 1.         0.99433664 0.977486
 0.949863   0.91214782]
[1.         0.99433664 0.977486   0.949863   0.91214782 0.86526912
 0.81038122 0.74883563 0.68214782 0.61195985 0.54       0.46804015
 0.39785218 0.33116437 0.26961878 0.21473088 0.16785218 0.130137
 0.102514   0.08566336]
```

[229]:
```
D = np.diag(w)
U = v

D_inv = np.linalg.inv(D)
U_inv = np.linalg.inv(U)
```

1.4 d) Transform f->b and g->c

$c = U^{1} g$

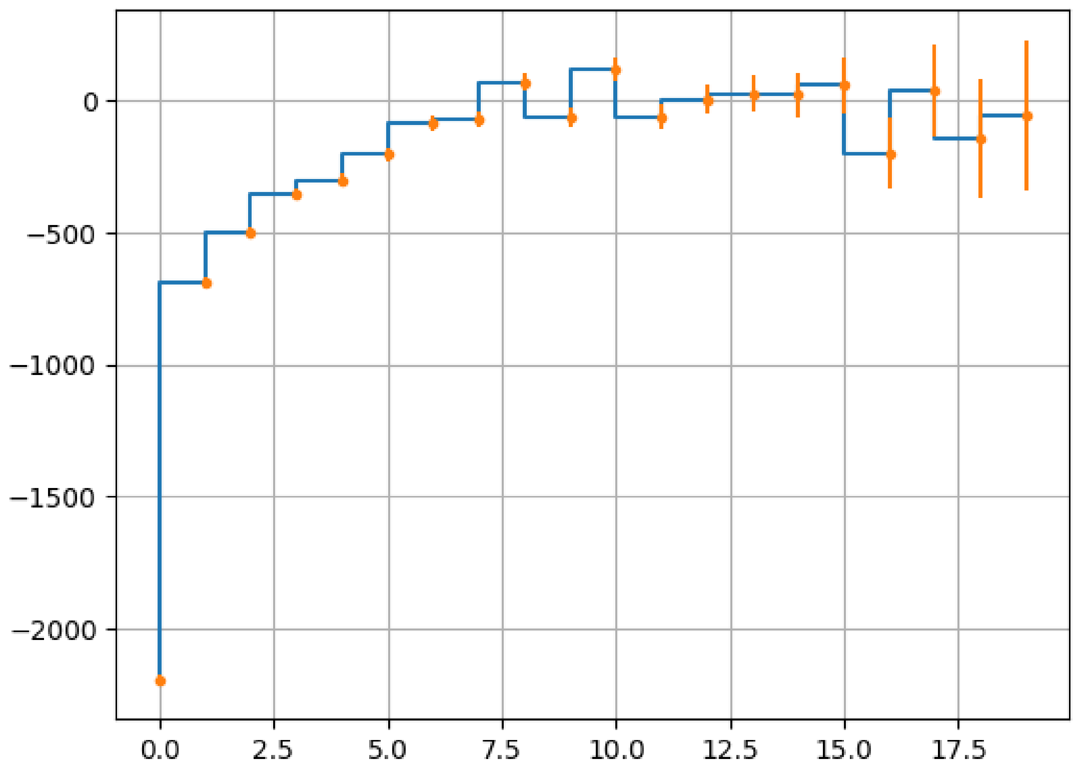
$b = DU^{1} f$

[230]:
```
b = U_inv @ f
c = D_inv @ U_inv @ g(f,A,rng)
```

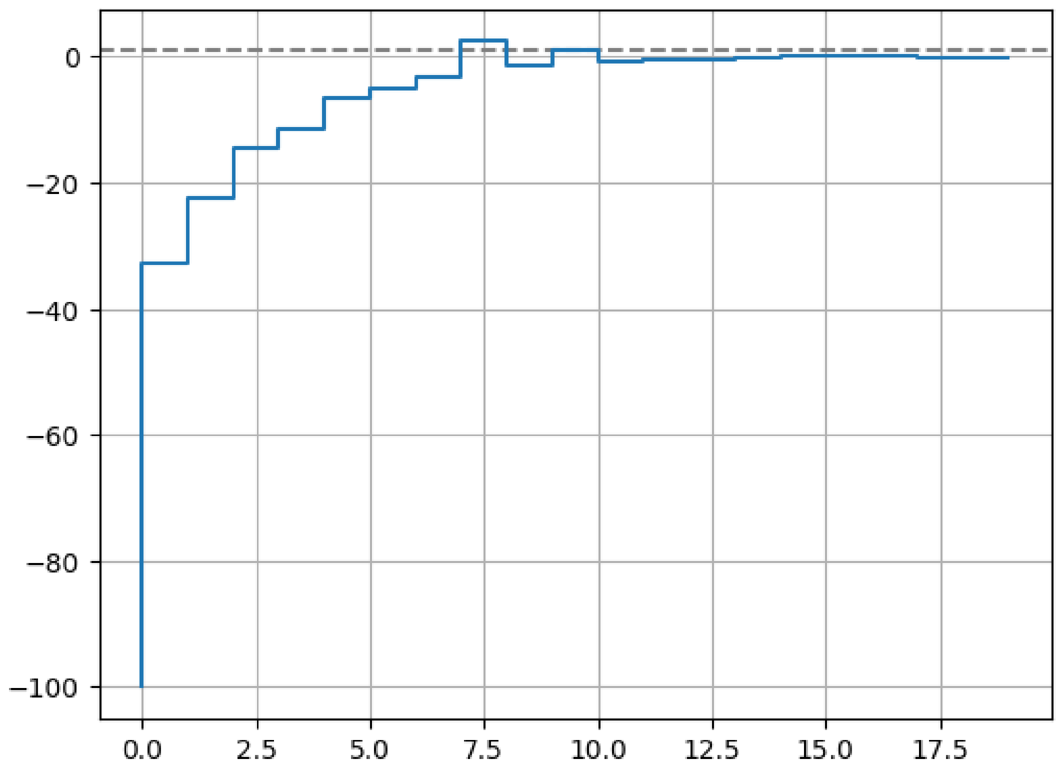
[231]:
```
Cov_g = np.diag(lam(f,A))
Cov_c = (D_inv @ U_inv) @ Cov_g @ (D_inv @ U_inv).T
```

[232]:
```
sigma_c = np.sqrt(np.diag(Cov_c)) #standardabweichung von b
c_hat = b/sigma_c # normiertes b
```

[233]:
```
x = np.arange(len(c))

plt.step(x,c)
plt.errorbar(x, c, sigma_c, fmt='.')
plt.grid()
```



[234]:
```
plt.axhline(1, linestyle ='--', color='gray')
plt.step(x,c_hat)
plt.grid()
```



c_hat < 1 <=> c value: coefficient is not insignificant from zero,

value contributes no information

1.5 e) Unfolding and Regularization

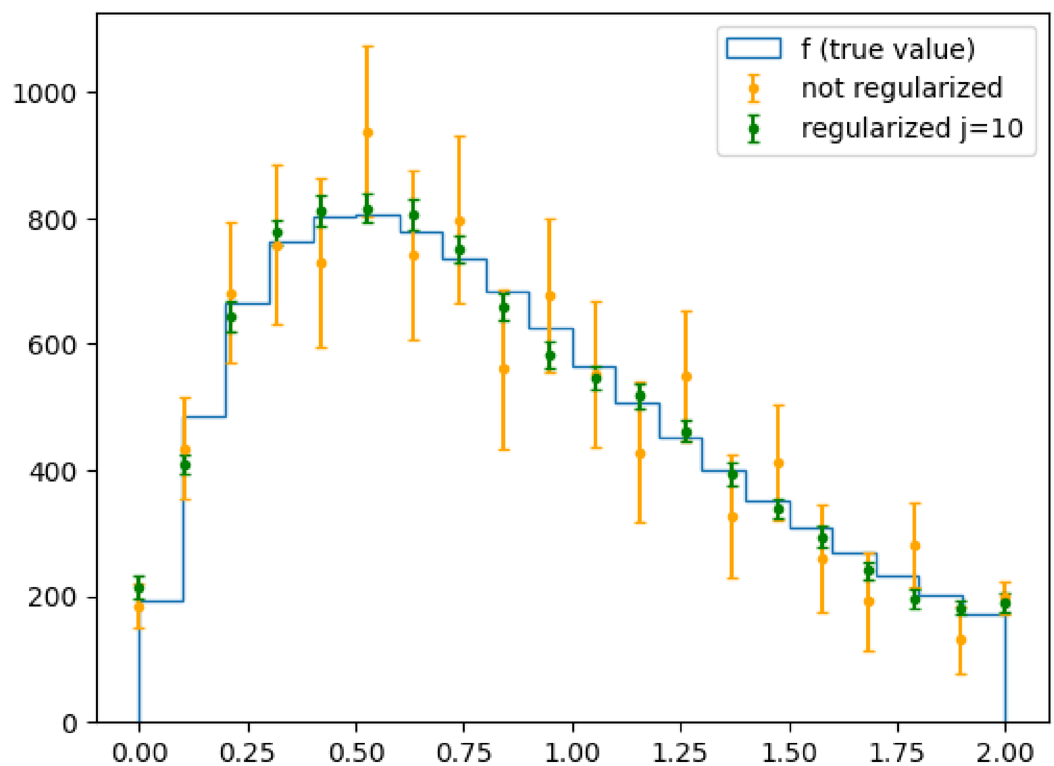
[235]:
```
def unfold(b, Cov_b, U, cut):
    b = b[:cut]
    Cov_b = Cov_b[:cut,:cut]
    U = U[:,:cut]

    f = U @ b
    Cov_f = U @ Cov_b @ U.T

    return f, Cov_f
```

[236]:
```
#Entfaltung mit Regularisiren
f_5, Cov_f_5 = unfold(c, Cov_c, U, cut=5)
sigma_f_5 = np.sqrt(np.diag(Cov_f_5))

f_10, Cov_f_10 = unfold(c, Cov_c, U, cut=10)
sigma_f_10 = np.sqrt(np.diag(Cov_f_10))
```

```
f_15, Cov_f_15 = unfold(c, Cov_c, U, cut=15)
sigma_f_15 = np.sqrt(np.diag(Cov_f_15))

# Entfaltung ohne Regularisieren
f_20, Cov_f_20 = unfold(c, Cov_c, U, cut=20)
sigma_f_20 = np.sqrt(np.diag(Cov_f_20))
```

[237]:
```
x=np.linspace(0,2,len(f))
plt.hist(x,weights=f, label='f (true value)', bins=len(f), histtype='step')
plt.errorbar(x, f_20, yerr=sigma_f_20, fmt='.', color='orange', capsize=2,␣
    ↪label='not regularized')
plt.errorbar(x, f_10, yerr=sigma_f_10, fmt='.', color='green', capsize=2,␣
    ↪label='regularized j=10')
plt.legend()
```

[237]: <matplotlib.legend.Legend at 0x7fb22e9eaef0>



Unregularized solution oscillates.

Regularization suppresses oscillations and thus fits the true values better.

[ ]: