Exercise 1 *Error propagation*

The parameters of a regression line $y = a_0 + a_1 x$ were determined to be $a_0 = 1.0 \pm 0.2$ and $a_1 = 1.0 \pm 0.2$. The correlation coefficient is $\rho = -0.8$. Determine the uncertainty of a value y as a function of x.

(a) Determine the result analytically both considering the correlation and neglecting the correlation.

$$\sigma_y = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial y}{\partial x_i}\sigma_{x_i}\right)^2 + 2\sum_{i=1}^{m-1}\sum_{k=i+1}^{m}\left(\frac{\partial y}{\partial x_i}\right)\left(\frac{\partial y}{\partial x_k}\right)\cdot \mathrm{Cov}(x_i, x_k)} \quad \Big| \quad \rho = \frac{\mathrm{cov}(a_0, a_1)}{\sigma_0 \sigma_1}$$

$$= \sqrt{\left(\frac{\partial y}{\partial a_0}\sigma_0\right)^2 + \left(\frac{\partial y}{\partial a_1}\sigma_1\right)^2 + 2\frac{\partial y}{\partial a_0}\frac{\partial y}{\partial a_1}\sigma_0\sigma_1\rho} \quad \Big| \quad y = a_0 + a_1 x$$

$$= \sqrt{\sigma_0^2 + x^2\sigma_1^2 + 2x\sigma_0\sigma_1\rho} \quad \Big| \quad \sigma_1 = \sigma_2 = 0.2 =: \sigma_a$$

$$= \sigma_a\sqrt{1 + x^2 + 2x\rho}$$

Case 1: $\rho = 0 \Rightarrow \quad \sigma_y = \sigma_a\sqrt{1+x^2} = 0.2\sqrt{1+x^2}$

Case 2: $\rho = 0.8 \Rightarrow \quad \sigma_y = 0.2\sqrt{x^2 - 1.6x + 1}$