Dynamic Programming and Applications

Applications

Lecture 7

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Outline

Part 1: A Q-theory of investment

1.

Outline

Part 2: Empirical regularities

1.

Overview

- Investment is important for macroeconomics
- Investment increases the stock of capital, therefore key determinant of growth
- Investment is volatile, matters a lot for business cycle fluctuations

1. Demand for capital

- Consider a firm that operates a production function

$$y_t = F(z_t, k_t, x_t)$$

where z_t is exogenous productivity and x_t other inputs

- Suppose firm can rent capital *frictionlessly* at rate r_t^k

- Firm problem therefore given by

$$\max_k F(z_t, k_t, x_t^*) - r_t^k k_t$$

where x_t^* denotes optimal choice of other inputs

- Optimal capital demand then determined by:

$$F_{k,t} \equiv \partial_k F(\cdot) = r_t^k$$

2. User cost of capital

- Capital usually not rented but owned by firms
- What is the appropriate notion of “rental rate”? $\implies$ user cost literature
- Consider the deterministic firm (sequence) problem:

$$V(k_0) = \max_{\{i_t\}_{t\geq 0}} \int_0^\infty e^{-\int_0^t r_s ds}\Big(f_t(k_t) - p_t i_t\Big)dt$$

where $f_t(k_t) = F(z_t, k_t, x_t^*)$, facing the capital accumulation technology

$$\dot{k}_t = i_t - \delta k_t$$

- Interpretation? (PE, small firm, take prices as given, SDF, ...)

- How do we solve this problem? (i) optimal control theory (ii) dynamic programming
- Hamiltonian is: $\mathcal{H}_t(k_t, i_t, \lambda_t) = f_t(k_t) - p_t i_t + \lambda_t(i_t - \delta k_t)$
- What are control and state? Cookbook solution:

$$0 = \partial_i \mathcal{H}_t = -p_t + \lambda_t$$
$$r_t \lambda_t - \dot{\lambda}_t = \partial_k \mathcal{H}_t = f_{k,t} - \delta \lambda_t$$

- Practice: derive the transversality condition at home using calculus of variations
- Combining and using the capital demand condition $r_t^k = f_{k,t}$:

$$\text{user cost } = r_t^k = \left(r_t + \delta - \frac{\dot{p}_t}{p_t} \right) p_t$$

- Next: dynamic programming. Convince yourself you can derive HJB:

$$r_t V_t(k) = \partial_t V_t(k) + \max_i \left\{ f_t(k) - p_t i + (i - \delta k)\partial_k V_t(k) \right\}$$

- Why is this HJB not stationary, i.e., why $\partial_t V_t(k)$?
- FOC: $p_t = \partial_k V_t(k) \implies$ price of investment = marginal value of capital

$$r_t V_t(k) = \partial_t V_t(k) + \max_i \left\{ f_t(k) - p_t i + (i - \delta k)\partial_k V_t(k) \right\}$$

- Envelope condition:

$$r_t \partial_k V_t(k) = \partial_{kt} V_t(k) + \partial_k f_t(k) - \delta \partial_k V_t(k) + (i(k) - \delta k)\partial_{kk} V_t(k)$$

- Next, differentiate FOC $p_t = \partial_k V_t(k)$ with respect to time:

$$\dot{p}_t = \frac{d}{dt} p_t = \frac{d}{dt} \partial_k V_t(k) = \partial_{kt} V_t(k) \frac{dt}{dt} + \partial_{kk} V_t(k) \frac{dk}{dt}$$

- Finally, put all together:

$$r_t \partial_k V_t(k) = \partial_k f_t(k) - \delta \partial_k V_t(k) + \dot{p}_t$$

$$\implies \text{user cost } = r_t^k = \partial_k f_t(k) = \left(r_t + \delta - \frac{\dot{p}_t}{p_t} \right) p_t$$

Interpreting user cost model

- User cost of capital
  - increases with r_t
  - increases with depreciation rate δ
  - decreases with capital gains rate $\dot{p}_t / p_t$

- Hall and Jorgenson (1967): user cost model helpful to evaluate tax policies

- But not very helpful for investment dynamics:
  - Model determines capital stock, change in r_t^k requires 'jumps'
  - Decisions about capital stock become static, not forward-looking

- High elasticity of capital demand to user cost of capital $\Longrightarrow$ observed variation in interest rates would generate counter-factually large investment volatility

- What might slow down the adjustment of the capital stock?

- Internal adjustment costs:
  - Direct costs faced by firms
  - More costly construction and training of workers
  - Disruption of current production

- External adjustment costs:
  - Financing needs $\Longrightarrow$ large upfront investment costs
  - Capital goods distinct from consumption goods, distinct capital producing sector, firm may not be “small”

3. A model of firm investment with adjustment costs

- Let's start with simple quadratic (smooth + convex) adjustment cost:

$$C(i_t), \qquad \text{where } C(0) = 0, C'(0) = 0, C''(i_t) > 0$$

- Firms problem now:

$$V(k_0) = \max_{\{i_t\}_{t\geq 0}} \int_0^\infty e^{-\int_0^t r_s ds} \Big(f_t(k_t) - i_t - C(i_t) \Big) dt$$

facing the capital accumulation technology

$$\dot{k}_t = i_t - \delta k_t$$

- HJB is given by (make sure this makes sense to you):

$$r_t V_t(k) = \partial_t V_t(k) + \max_i \left\{ f_t(k) - i - C(i) + (i - \delta k)\partial_k V_t(k) \right\}$$

- Implies FOC:

$$1 + C'(i_t(k)) = \partial_k V_t(k) \quad \Longrightarrow \quad i_t(k) = (C')^{-1}\Big(\partial_k V_t(k) - 1\Big)$$

- Envelope condition (dropping t subscripts and abbreviating derivatives):

$$rV_k = V_{tk} + f_k + (i - \delta k)V_{kk} - \delta V_k$$

- Rewriting:

$$(r-\delta)V_k = f_k + V_{tk} + (i-\delta k)V_{kk}$$

- Now again differentiate: $\dot{V}_k = V_{kt} + V_{kk}\dot{k} = V_{tk} + (i-\delta k)V_{kk}$, so

$$(r-\delta)V_k = f_k + \dot{V}_k$$

- We now introduce new notation:

$$\text{Tobin's (marginal) Q } = q = V_k$$

- Firm's decision problem summarized by system of equations:

$$\begin{aligned}(r-\delta)q &= f_k + \dot{q}\\ \dot{k} &= i - \delta k\\ C'(i) &= q-1\end{aligned}$$

- Denote the function:

$$i = (C')^{-1}(q-1) \quad \Longrightarrow \quad i = \Phi(q-1)$$

- Given initial capital k_0, sequences $\{q_t, k_t\}_{t\geq 0}$ solve the firm problem if they satisfy

$$\dot{q}_t = (r_t - \delta)q_t - f_{k,t}$$
$$\dot{k}_t = \Phi(q_t - 1) - \delta k_t$$

- System of 2 ODEs. q equation is *forward-looking* while k equation is *backward-looking*. Why?

- We can solve q equation forward to obtain:

$$q_t = \int_t^{\infty} e^{-\int_t^s (r_\ell - \delta) d\ell} f_{k,s} ds$$

- Analyze via phase diagram, suppose $r_t = r$ and $\delta = 0$
- Steady state: $q = 1$ and $r = f_k$
- q_t is a jump variable (like consumption)
- Capital k_t is a predetermined state variable, must always follow continuous path (infinite investment would be too costly)

4. Tobin's Q

- What is the significance of $q_t = \partial_k V_t(k)$?
- q_t is the (shadow) value of one additional unit of installed capital
- Connection between sequence problem and dynamic programming: q_t turns out to be the Lagrange multiplier on relaxing capital / investment constraint
- In this model, q_t is a sufficient statistic for firm investment
- Firms invest whenever shadow value of installed capital is larger than value of consumption good: $q_t > 1$
- Without adjustment costs, firms invest until $q_t = 1$
- With, firms invest until excess value equal to adjustment cost on margin

- In the data, **average Q** easier to measure than **marginal Q**
- Our theory: marginal Q is sufficient statistic for investment
- Tobin (1969) argued that firms should invest if

$$Q = \frac{\text{Market value of firm capital}}{\text{Book value of capital}} > 1$$

- Hayashi (1982): Average Q = marginal Q when markets competitive and production function + adjustment cost homogeneous of degree 1

5. Tobin's Q and investment in the data

- Q-theory suggests Q (NPV of marginal projects avaialble to the firm) is sufficient statistic

- Under additional conditions, average Q (market relative to book value) encodes the NPV of these marginal projects

- Other variables such as contemporaneous cash flows should not matter. We can test this

- Suppose we estimate regression:

$$\frac{I_{it}}{K_{it}} = \alpha + \beta Q_{it} + \epsilon_{it}$$

- Summers (1981, Brookings) estimates this by OLS and finds $\beta = 0.031(0.005)$. Very low, implies high adjustment cost!

- Suppose we estimate regression:

$$\frac{I_{it}}{K_{it}} = \alpha + \beta Q_{it} + \epsilon_{it}$$

- Problem:
  - Where did the ϵ_{it} come from?
  - Q-theory says that there should be no ϵ_{it}
  - To estimate this equation, we need to know about ϵ_{it}. Is it orthogonal to Q_{it}?!
- View 1: measurement error in Q_{it}:
  - Stocks are volatile and may not reflect fundamental value
  - Assumptions under which marginal = average Q may not hold
  - Measurement error would result in attenuation bias
- View 2: model is wrong and other factors affect investment
  - If other factors that raise desired investment also raise interest rates, they may lower Q_{it} and downward bias β
  - Bias could go either way

Beyond Q

- Suppose we are interested in whether internal funds (cash flows) affect investment. Why?
- Simple approach:

$$\frac{I_{it}}{K_{it}} = \alpha + \beta_1 Q_{it} + \beta_2 \frac{CF_{it}}{K_{it}} + \epsilon_{it}$$

- Problem:
  - Cash flow likely correlated with future profitability
  - If Q_{it} is mismeasured, cash flow would proxy for Q even if financial markets are perfect

Fazzari-Hubbard-Petersen (1988)

- Use diff-in-diff strategy to circumvent cash flow - profitability correlation problem
- Different groups of firms: e.g. high- vs. low-dividend-firms
- Low dividends proxy for greater financial constraints
- Is investment more sensitive to cash flows for low-dividend?
- Identifying assumption: cash flow - profitability correlation the same on average for two groups
- **Result:** Cash flow sensitivity much higher for low dividend firms. Marginal propensity to invest out of cash flows very high.

Do cash flows matter for investment?

- Ideal: find a shock to cash flows that is orthogonal to investment opportunities (Q)
- Well-known examples:
  - Lamont (1997): investment of non-oil subsidiaries of oil companies falls when oil prices fall
  - Rouh (2006): investment of firms with underfunded pension plans due to drops in asset prices. Compare firms with and without mandatory contributions. Those with mandatory contributions see investment fall more.

Temporary investment tax incentives: bonus depreciation

- Suppose the government introduces a policy that changes the marginal benefit or cost of investment. This is arguably exogenous to the firm.
- Firms pay taxes on income net of business expenses
- Can fully expense wages, advertising, etc. immediately
- But investment gets expensed over time according to tax depreciation schedules
- Bonus depreciation accelerates this depreciation schedule

Zwick-Mahon (2017)

- Bonus depreciation changes occur in recessions: may be correlated with other determinants of investment
- Zwick-Mahon use diff-in-diff strategy:
  - Bonus more valuable in industries with longer lived investments
  - Compare effects of bonus across industries with differing investment duration
- Find large effects. More liquidity constrained firms have larger effects
- Effect only exists for firms with immediate tax benefit

6. Lumpy investment

- In the data, firm investment seems very lump (Doms and Dunne, 1992) XX

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Overview

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