

PSET6

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Problem 1. A simple question:

- (a) Show that if G is a simple graph, then $\varepsilon \leq \binom{v}{2}$
- (b) Show that $\varepsilon = \binom{v}{2}$ if and only if G is complete.

Problem 2. Recall that the *degree* of a vertex $d(v)$ in G is the number of edges incident with v .

- (a) Explain why

$$\sum_{v \in V} d(v) = 2\varepsilon \tag{1}$$

- (b) Prove that, in any graph, there is an even number of vertices of odd degree.
- (c) Show that in a simple graph (with $|V| > 1$), there are at least two vertices with the same degree.

Problem 3. Prove that in any group (of two or more people), there must be two people with the same number of friends in that group. Assume that friendship is symmetric.

- (a) Justify this claim using the Pigeonhole Principle.
- (b) Justify this claim with Graph Theory.

Problem 4. Let $S = \{x_1, x_2, \dots, x_n\}$ be a set of points in the coordinate plane such that $|x_i - x_j| \geq 1$ for $i \neq j$. Show that there are at most $3n$ pairs of points whose distance is exactly 1.