Lesson 6: Momentum

Henry Ding

August 18, 2025

Revisiting Newton's Laws

Definition

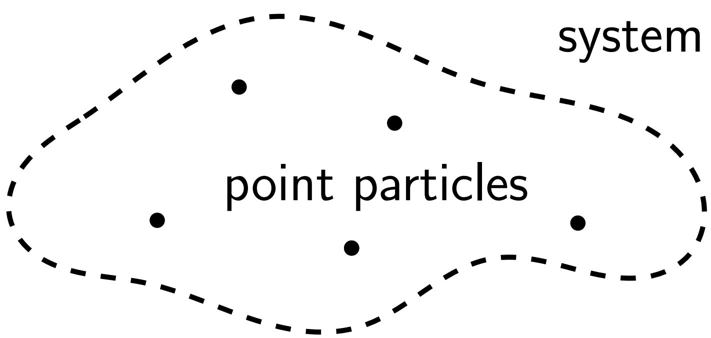
A **point particle** is a point with some a position $\mathbf{x}$ and mass m. A point particle has no physical size or dimensions.

Newton's Laws apply to point particles.

When we solve force problems using Newton's Laws, we are *implicitly* modeling objects as point particles.

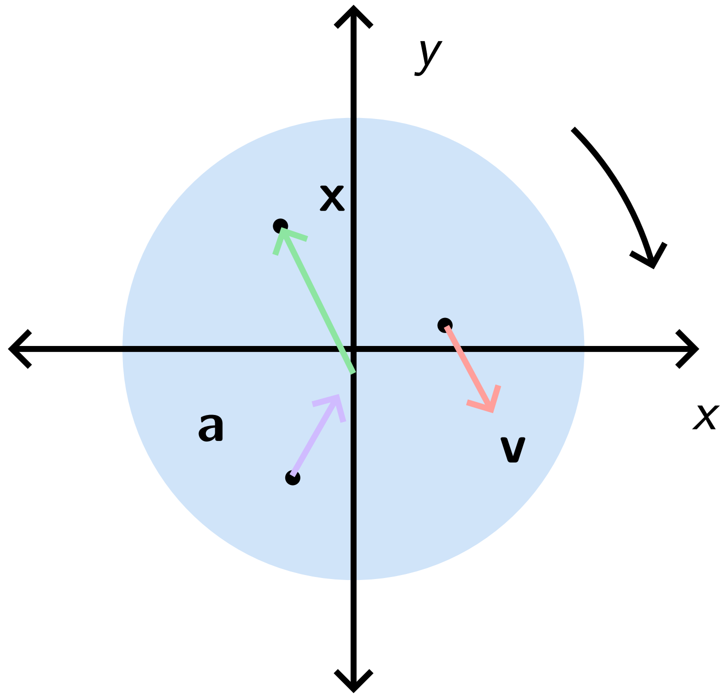
Definition (Systems Revisited)

I called a system a collection of objects we want to study. To be more precise, we can say a **system** is a collection of point particles.



System Example

Consider a rotating disk. We can break the disk up into very many point particles, each with their own position $\mathbf{x}$, velocity $\mathbf{v}$, and acceleration $\mathbf{a}$. Then, we can apply Newton's Laws to each particle in the disk.



Everyday use of Momentum

- “The team is riding off of the momentum from our last win.“
- “After securing a key endorsement, the political campaign starting gaining momentum.”
- “The grassroots movement had too much momentum to stop now.”

Physics Definition of Momentum

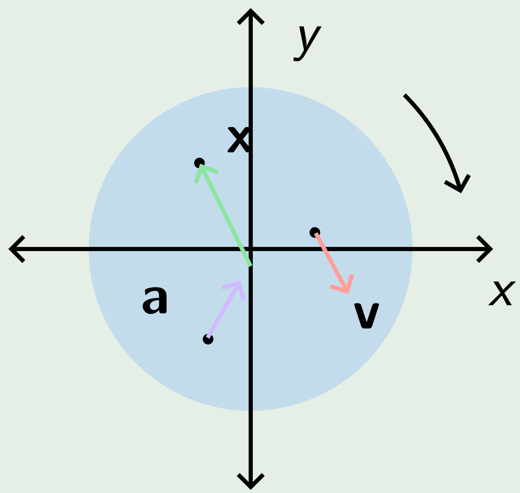
Definition

For a point particle of mass m and velocity $\mathbf{v}$, its **momentum p** is

$$\mathbf{p} = m\mathbf{v}.$$

Example

Recall the disk earlier. Find the total momentum of the disk system by adding together the momentum of every particle in the disk.



Momentum Example

Example

A $0.5\,\mathrm{kg}$ block slides with velocity $\langle -6\,\mathrm{m/s}, 8\,\mathrm{m/s}\rangle$. What is the block's momentum $\mathbf{p}$? What is the magnitude p?

Change in Momentum

- Sometimes, the mass of a system can change. For example, snow can pile onto the roof of a car, increasing its mass.
- If the mass of a system changes, the total momentum can change even if velocity of each particle does not. For example, the momentum of a car driving at a constant velocity can increase as snow piles onto the roof of a car.

Consider a one-dimensional particle's momentum at two points in time separated by a small time Δt apart.

- The particle's mass barely changes by a small amount from m to $m + \Delta m$
- The particle's velocity barely changes by a small amount from v to $v + \Delta v$.
- The change in momentum is

$$\begin{aligned}\Delta p &= (m + \Delta m)(v + \Delta v) - mv \\ &= mv + v\Delta m + m\Delta v + \Delta m \Delta v - mv.\end{aligned}$$

$\Delta m \Delta v$ is an extremely small number, so we can ignore it! Then

$$\Delta p = v\Delta m + m\Delta v.$$

General Newton's Second Law

The small changes in momentum can be written as

$$\Delta p = m\Delta v + v\Delta m.$$

The more general statement of Newton's Second Law is

Theorem (Newton's Second Law)

As the time Δt becomes infinitely small (like instantaneous velocity or acceleration),

$$\mathbf{F}_{\text{net}} = \frac{\Delta p}{\Delta t}$$

or

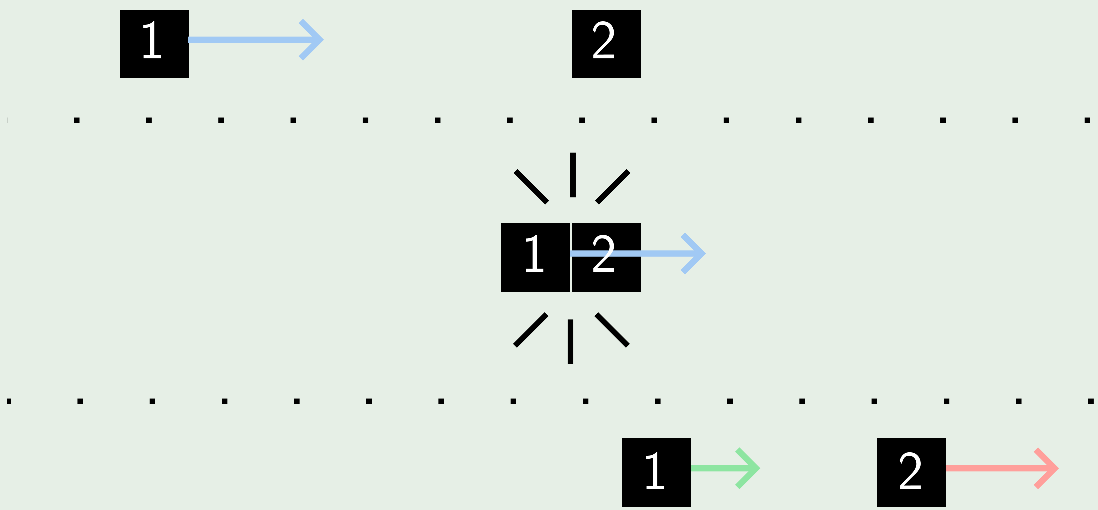
$$\mathbf{F}_{\text{net}} = \mathbf{v}\frac{\Delta m}{\Delta t} + m\mathbf{a}.$$

If the mass of our system does not change, then $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as usual! It turns out, we need extra force to change the mass of our system.

Collisions

Example

Consider a block sliding towards another block on smooth ground (no friction) so that there are no *external* horizontal forces. The blocks will collide and continue moving with some velocity.



Let the collision force acting on blocks 1 from 2 be $\mathbf{F}_{2\to1}$. By Newton's Third Law there is an equal and opposite force

$$\mathbf{F}_{1\to2} = -\mathbf{F}_{2\to1}.$$

Conservation of Momentum

Example (cont.)

Since, there are no external forces, the only two forces on our system are

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{1\to 2} + \mathbf{F}_{2\to 1} = 0.$$

Then,

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = 0$$

so $\mathbf{p}$ is **conserved** (does not change with time).

We showed that $\mathbf{p}$ is conserved for two blocks colliding with no external forces, but this applies for any system as well.

Theorem (Conservation of Momentum)

Consider a system such that $\mathbf{F}_{\text{ext}} = 0$*. Then the system's total momentum* $\mathbf{p}$ *is conserved with time.*

Conservation of Momentum Example

Example

A 3 kg block slides to the right at 4 m/s towards a stationary block of mass 2 kg. If the two blocks stick together after colliding, what are the final velocities of the two blocks?

Homework 6

TODO