Lesson 7: Rotational Motion I

Henry Ding

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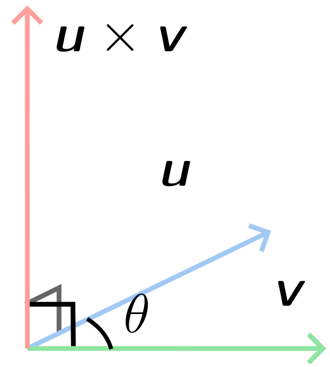
Cross Product

Definition

The **vector cross product** $(\times)$ is one way to multiply vectors in 3D. For vectors $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{u} \times \boldsymbol{v}$ gives a new vector perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$.



Theorem

If $\boldsymbol{u}$ and $\boldsymbol{v}$ are separated by θ, magnitude of the cross product is

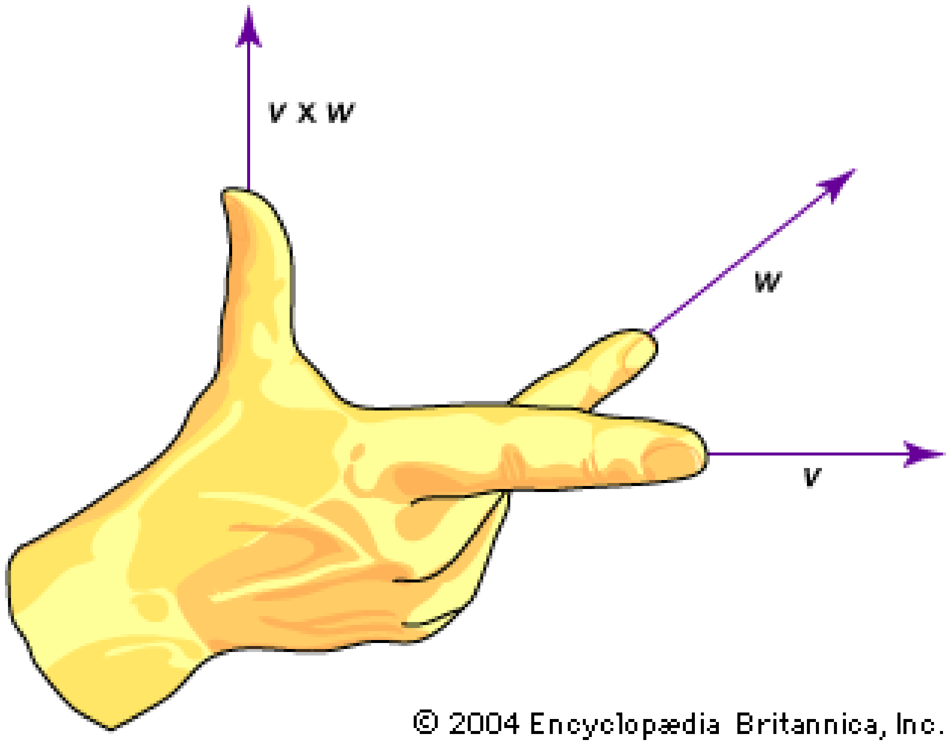
$$\|\boldsymbol{u} \times \boldsymbol{v}\| = uv \sin\theta.$$

Note, if $\boldsymbol{u}$ and $\boldsymbol{v}$ are parallel, $\theta = 0$ so $\sin\theta = 0$theo. Then, $\boldsymbol{u} \times \boldsymbol{v}$ has no magnitude.

Right Hand Rule for Cross Product

Theorem (Right Hand Rule)

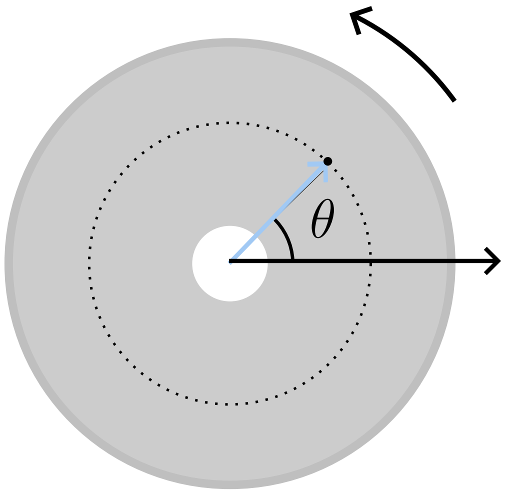
To find the direction of $\mathbf{u} \times \mathbf{v}$, use the right hand rule.



Rotational Motion

What is Rotational Motion?

When objects rotate, they travel in circles about a **rotational axis**.



rotating CD

Definition

The angular position θ of an object describes the orientation of an object relative to some reference. We can choose any pair of references and points on the object to define θ.

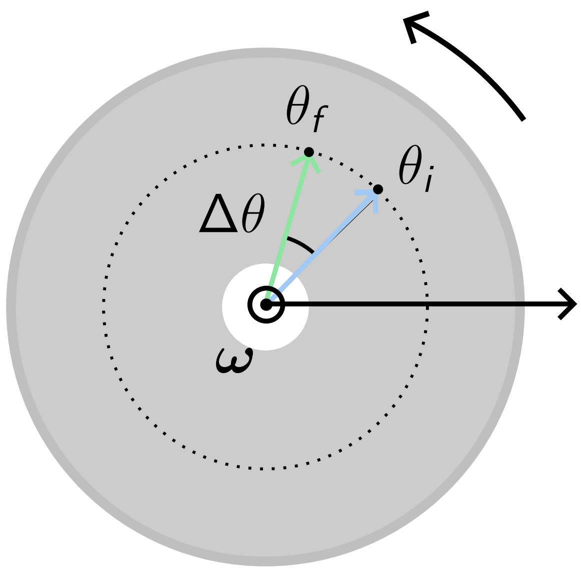
Angular Velocity

Definition

The average angular velocity $\boldsymbol{\omega}_{\text{avg}}$ is a vector with magnitude

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}.$$

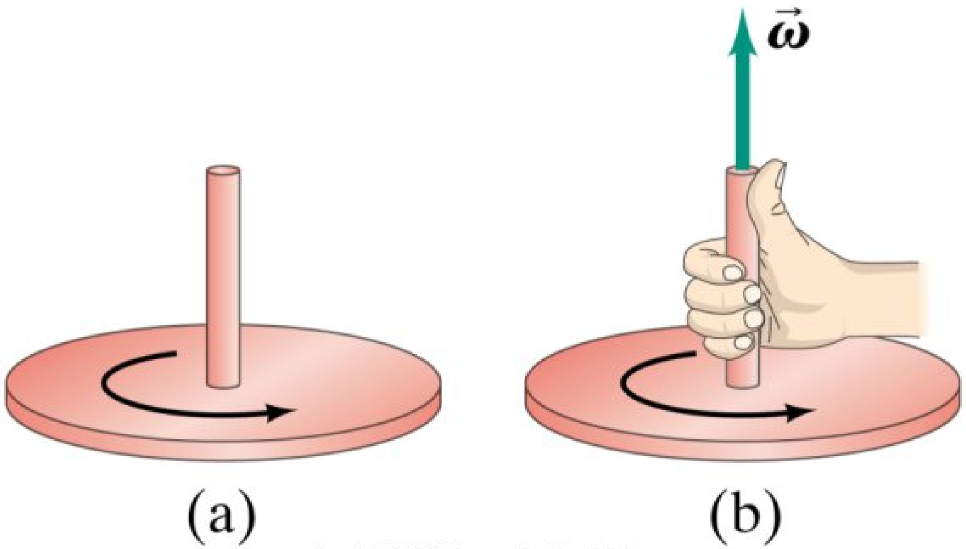
Just like regular instantaneous velocity $\boldsymbol{v}$ (sometimes called *linear* velocity), we can define an instantaneous angular velocity $\boldsymbol{\omega}$. $\boldsymbol{\omega}$ points in the direction of the axis of rotation.



Right Hand Rule for Angular Velocity

Theorem (Right Hand Rule (Angular Velocity))

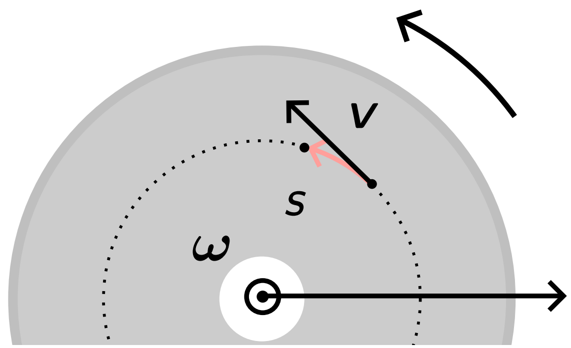
To determine the direction of $\boldsymbol{\omega}$, curl the fingers in your right hand around the direction of rotation. Then, $\boldsymbol{\omega}$ points in the direction of your thumb.



Units for ω

The units for angular velocity are rad/s. However, radians are dimensionless, so sometimes the units are written as just 1/s.

Tangential Velocity from Angular Velocity



Example

Consider a point at radius r from the axis. Then, the arc length covered is

$$s = r\Delta\theta = (\omega\Delta t)r$$

so the speed of the point is

$$v = \frac{s}{\Delta t} = \omega r.$$

Theorem

A point at radius r rotating with angular velocity ω has speed $r\omega$. The speed is in the tangential direction.

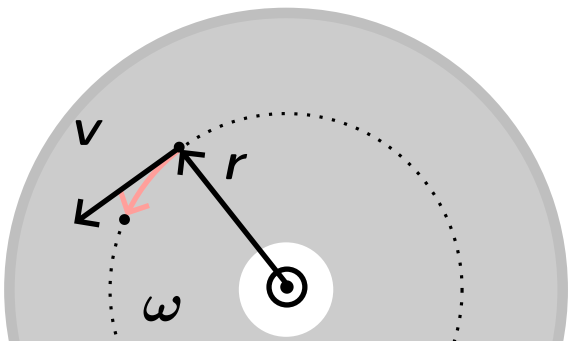
Finding Velocity from the Right Hand Rule

Theorem

A point at position $\boldsymbol{r}$ (measured relative to the axis) rotating with angular velocity $\boldsymbol{\omega}$ has velocity

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}.$$

$\boldsymbol{v}$ *is often called* **tangential velocity**, *since it is tangent to the circle along which the point travels.*



Note that $\boldsymbol{\omega}$ and $\boldsymbol{r}$ are perpendicular to each other. Then,

$$\|\boldsymbol{v}\| = v = \omega r \sin(\pi/2) = \omega r$$

as expected.

Angular Acceleration

Definition

Like linear acceleration, angular acceleration is

$$\boldsymbol{\alpha} = \frac{\Delta \boldsymbol{\omega}}{\Delta t}.$$

$\boldsymbol{\alpha}$ is parallel to $\boldsymbol{\omega}$, so we can find the direction of both using the right hand rule.

Theorem (Angular Kinematic Equations in One Dimension)

When we have constant angular acceleration, the kinematic equations are in the same form

$$\begin{aligned} \omega_f &= \omega_i + \alpha t \\ \Delta\theta &= \left(\frac{\omega_i + \omega_f}{2}\right) t \\ \Delta\theta &= \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 - \omega_i^2 &= 2\alpha\Delta\theta. \end{aligned}$$

Constant Angular Acceleration Examples

Example

A CD disk undergoes constant angular acceleration from rest to rotating at 5 rad/s in 10 s. What is the disk's angular acceleration? Through what angle did the disk turn?

Homework 7

Textbook Problems

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