Lesson 6: Momentum

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Revisiting Newton's Laws

Definition

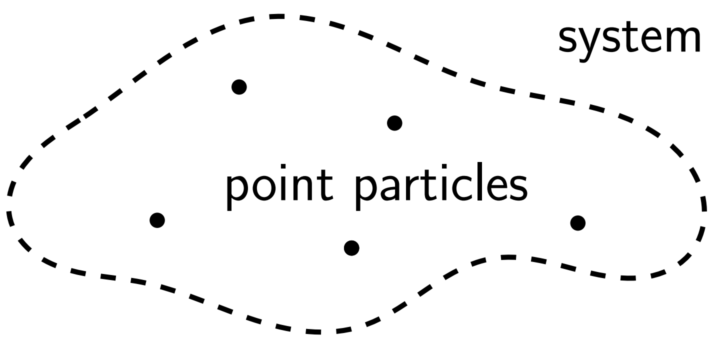
A **point particle** is a point with some a position x and mass m. A point particle has no physical size or dimensions.

Newton's Laws apply to point particles.

When we solve force problems using Newton's Laws, we are *implicitly* modeling objects as point particles.

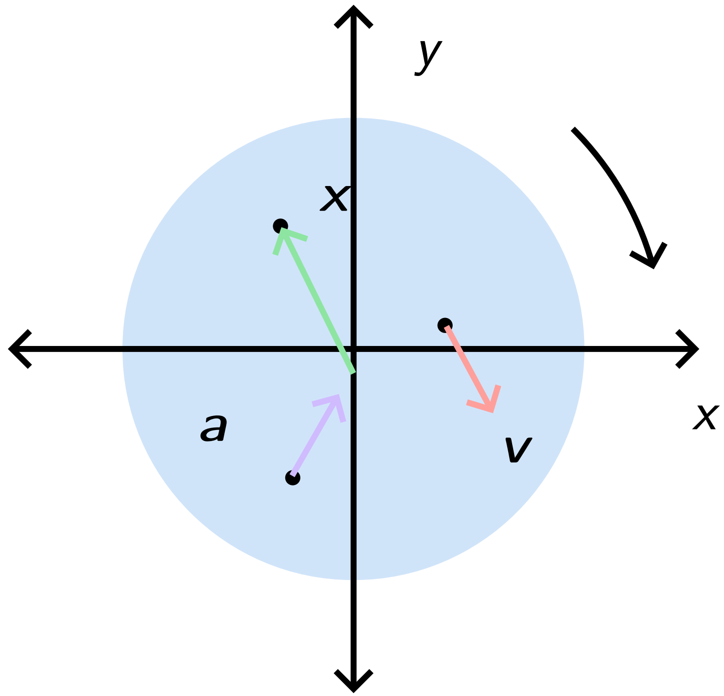
Definition (Systems Revisited)

I called a system a collection of objects we want to study. To be more precise, we can say a **system** is a collection of point particles.



System Example

Consider a rotating disk. We can break the disk up into very many point particles, each with their own position x, velocity v, and acceleration a. Then, we can apply Newton's Laws to each particle in the disk.



Everyday use of Momentum

- “The team is riding off of the momentum from our last win.“
- “After securing a key endorsement, the political campaign starting gaining momentum.”
- “The grassroots movement had too much momentum to stop now.”

Physics Definition of Momentum

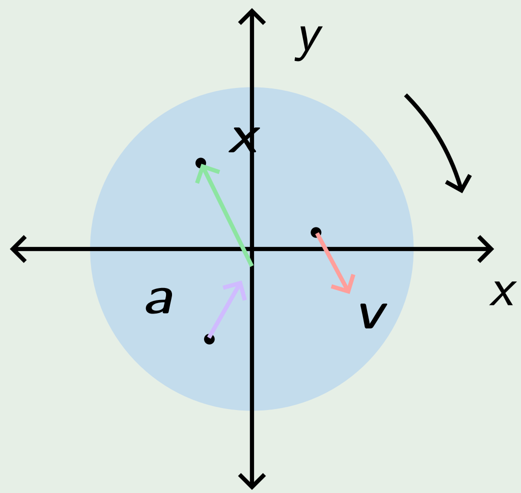
Definition

For a point particle of mass m and velocity $\boldsymbol{v}$, its **momentum** $\boldsymbol{p}$ is

$$\boldsymbol{p} = m\boldsymbol{v}.$$

Example

Recall the disk earlier. Find the total momentum of the disk system by adding together the momentum of every particle in the disk.



Momentum Example

Example

A $0.5\,\mathrm{kg}$ block slides with velocity $\langle -6\,\mathrm{m/s}, 8\,\mathrm{m/s}\rangle$. What is the block's momentum $\boldsymbol{p}$? What is the magnitude p?

Change in Momentum

- Sometimes, the mass of a system can change. For example, snow can pile onto the roof of a car, increasing its mass.
- If the mass of a system changes, the total momentum can change even if velocity of each particle does not. For example, the momentum of a car driving at a constant velocity can increase as snow piles onto the roof of a car.

Consider a one-dimensional particle's momentum at two points in time separated by a small time Δt apart.

- The particle's mass barely changes by a small amount from m to $m + \Delta m$
- The particle's velocity barely changes by a small amount from v to $v + \Delta v$.
- The change in momentum is

$$\begin{aligned}\Delta p &= (m + \Delta m)(v + \Delta v) - mv \\ &= mv + v\Delta m + m\Delta v + \Delta m\Delta v - mv.\end{aligned}$$

$\Delta m \Delta v$ is an extremely small number, so we can ignore it! Then

$$\Delta p = v\Delta m + m\Delta v.$$

General Newton's Second Law

The small changes in momentum can be written as

$$\Delta p = m\Delta v + v\Delta m.$$

The more general statement of Newton's Second Law is

Theorem (Newton's Second Law)

As the time Δt becomes infinitely small (like instantaneous velocity or acceleration),

$$F_{\text{net}} = \frac{\Delta p}{\Delta t}$$

or

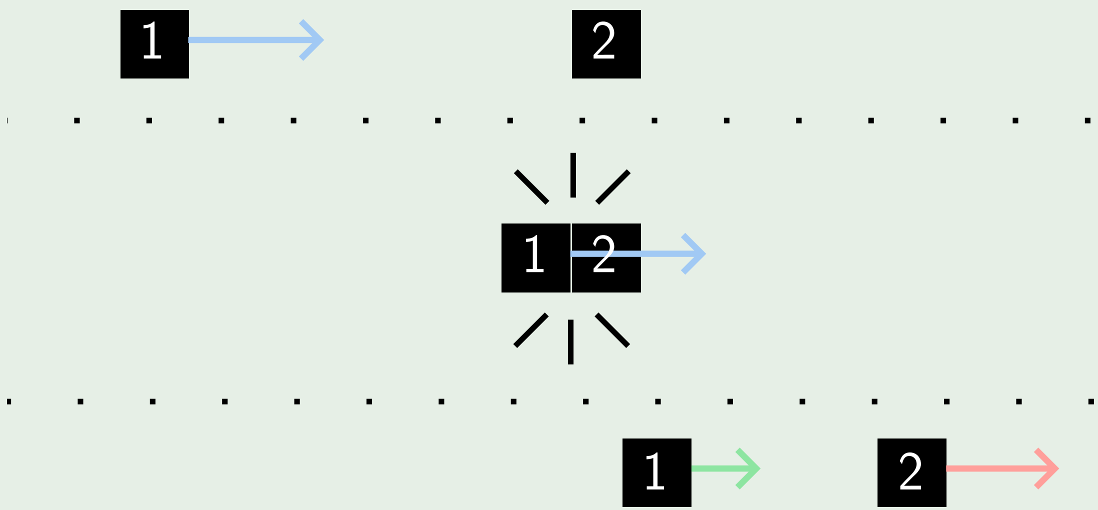
$$F_{\text{net}} = v\frac{\Delta m}{\Delta t} + ma.$$

If the mass of our system does not change, then $F_{\text{net}} = ma$ as usual! It turns out, we need extra force to change the mass of our system.

Collisions

Example

Consider a block sliding towards another block on smooth ground (no friction) so that there are no *external* horizontal forces. The blocks will collide and continue moving with some velocity.



Let the collision force acting on blocks 1 from 2 be $F_{2\to1}$. By Newton's Third Law there is an equal and opposite force

$$F_{1\to2} = -F_{2\to1}.$$

Conservation of Momentum

Example (cont.)

Since, there are no external forces, the only two forces on our system are

$$F_{\mathrm{net}} = F_{1\to 2} + F_{2\to 1} = 0.$$

Then,

$$F_{\mathrm{net}} = \frac{\Delta p}{\Delta t} = 0$$

so p is **conserved** (does not change with time).

We showed that p is conserved for two blocks colliding with no external forces, but this applies for any system as well.

Theorem (Conservation of Momentum)

Consider a system such that $F_{\mathrm{ext}} = 0$. Then the system's total momentum p is conserved with time.

Momentum Conservation Example

Example

A 3 kg block slides to the right at 4 m/s towards a stationary block of mass 2 kg. If the two blocks stick together after colliding, what are the final velocities of the two blocks?

More Momentum Conservation Examples

Example

A 75 kg astronaut in deep space (so that there are no external forces) throws a 3 kg ball to the right at 2 m/s. At what velocity does the astronaut recoil?

Example

A 50 kg child stands on slippery ice (so that there is no friction). The child throws a 5 kg ball to the right at 1 m/s. At what velocity does the child recoil?

Impulse

Definition

Impulse is the change in momentum $\Delta \boldsymbol{p}$. Recall from Newton's Second Law

$$\Delta \boldsymbol{p} = \boldsymbol{F}_{\mathrm{net}} \Delta t.$$

Homework 6

Textbook Problems

- OpenStax Physics (High School) Chapter 8 Critical Thinking Items 9, 11, 12, 15
- OpenStax Physics (High School) Chapter 8 Problems 16, 18