Lesson 2: Motion in One Dimension

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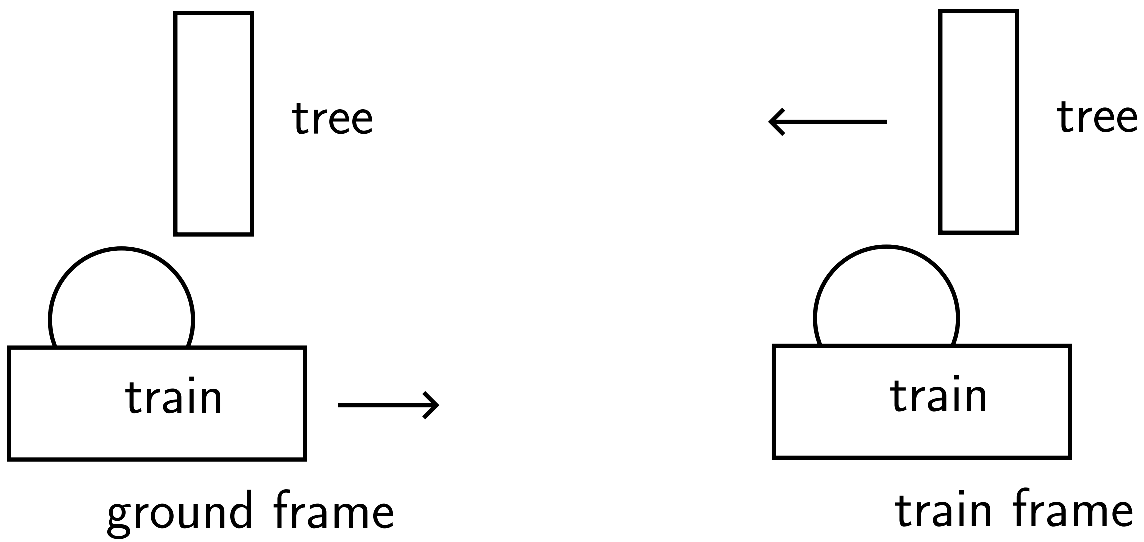
Homework Questions?

Reference Frames



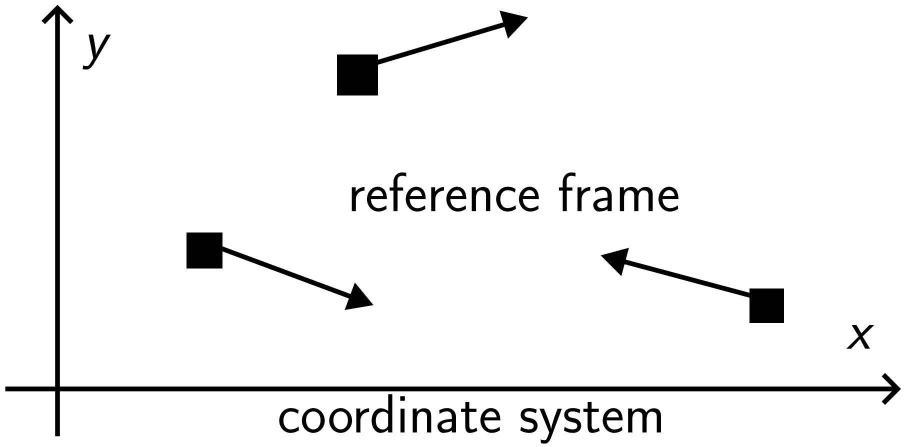
Definition

Motion must be defined relative to a **reference frame**.

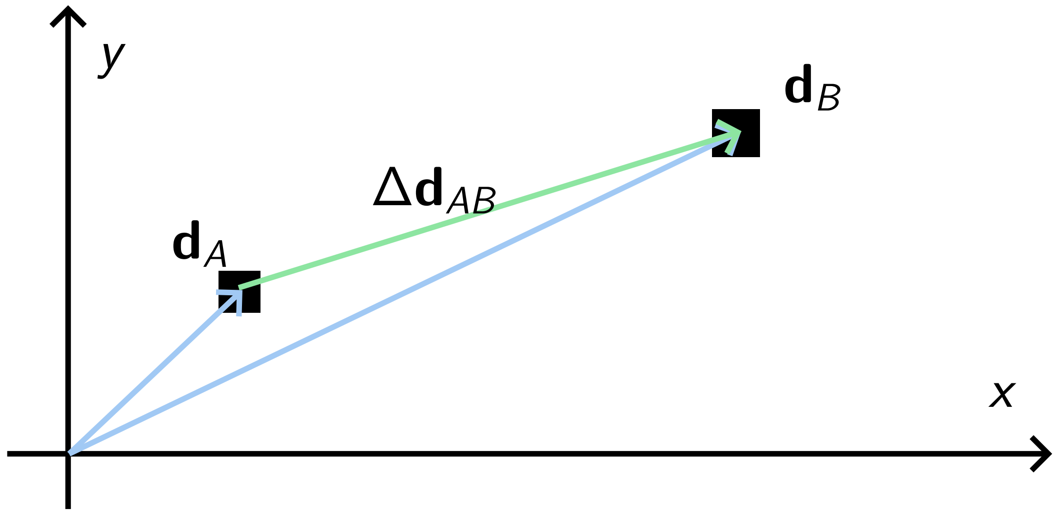
Definition

A **coordinate system** in each reference frame specifies position, velocity, acceleration, etc.

Coordinate System



Position and Displacement



Definition

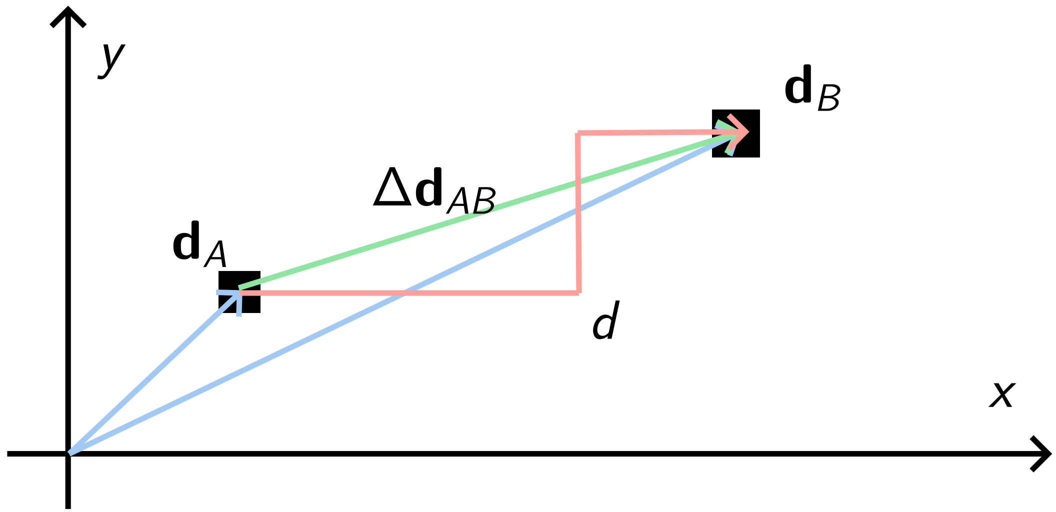
Position is a vector pointing from the **origin** of the coordinate system.
Displacement is the change in position from one location to another

$$\Delta\mathbf{d}_{AB} = \mathbf{d}_B - \mathbf{d}_A.$$

Δ Prefix

In the sciences, Δ often represents a change in something.

Displacement vs. Distance



Definition

Distance is a scalar equal to the total length of the path between two locations, which means distance is always positive. It is not the same as displacement!

Example

What is the displacement from $\langle -3\,\text{m}, 2\,\text{m}\rangle$ to $\langle 0\,\text{m}, 4\,\text{m}\rangle$? What distance is covered traveling in a straight line between those two points?

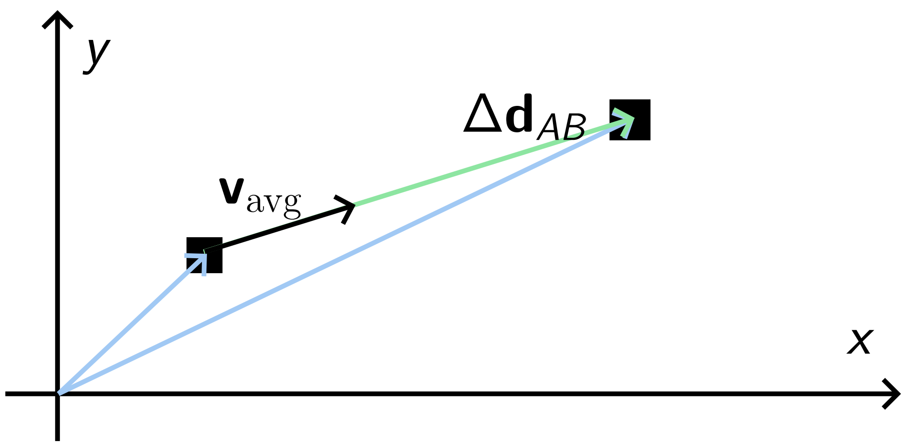
Average and Instantaneous Velocity

Definition

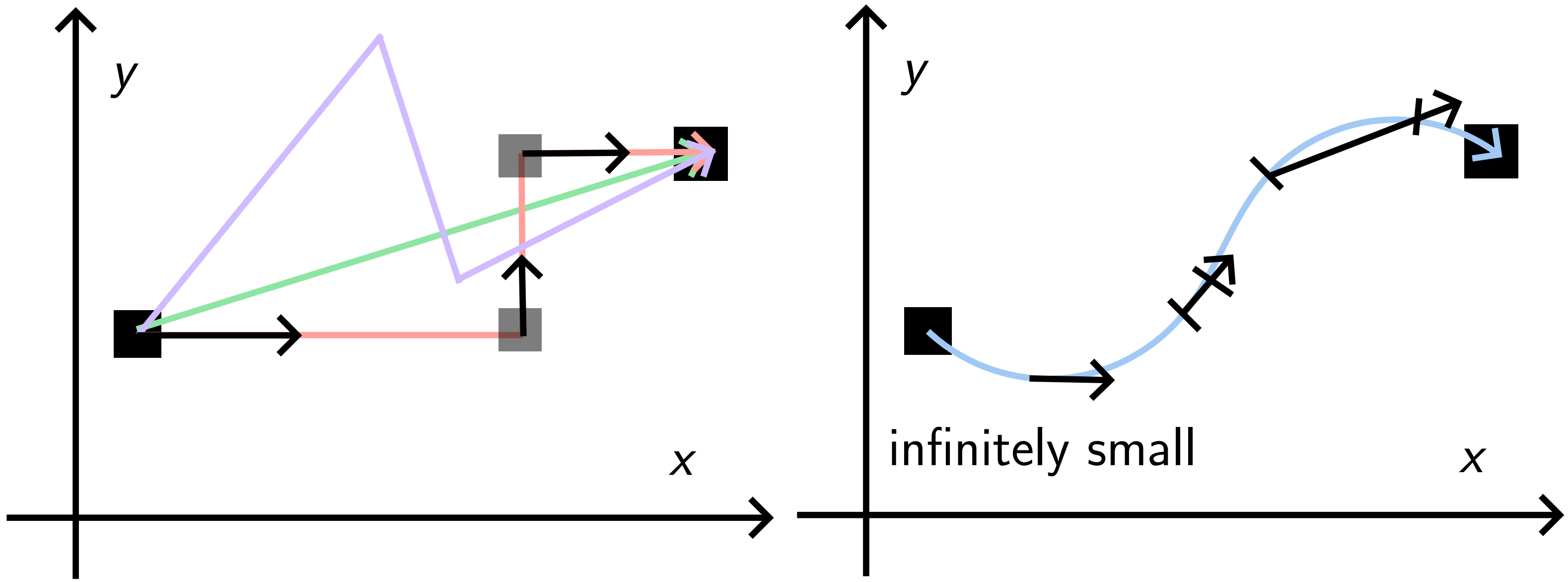
Average velocity is the total displacement $\Delta\mathbf{d}$ between two points divided by the time Δt taken to travel between those points:

$$\mathbf{v}_{\mathrm{avg}} = \frac{\Delta\mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_i}{t_f - t_i}.$$

- $\mathbf{v}_{\mathrm{avg}}$ is a vector parallel to $\Delta\mathbf{d}$, which shows the direction of movement. The magnitude $||\mathbf{v}_{\mathrm{avg}}||$ tells you the rate of movement.
- $\mathbf{v}_{\mathrm{avg}}$ tells us about average movement, not the finer details!



Instantaneous Velocity



Definition

When the displacement $\Delta\mathbf{d}$ becomes infinitely small, the average velocity describes the **instantaneous velocity v**. **v** is a vector describing the rate and direction of motion at a specific point.

Average Speed

Definition

Average speed is the total distance d between two points divided by time Δt taken to travel between those points:

$$v_{\text{avg}} = \frac{d}{\Delta t}$$

Note, v_{avg} is a scalar and is always positive. However, in most cases $v_{\text{avg}} \neq ||\mathbf{v}_{\text{avg}}||$, even though both are scalars!

Example

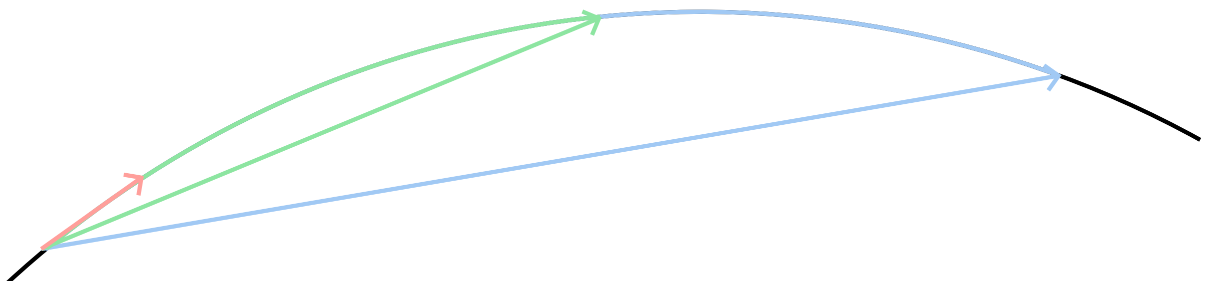
Anne finishes a race on a circular race track with radius 30 m in 15 s. From the start to end of the race, what is her (a) average velocity (b) average speed?

Instantaneous Speed

Definition

Instantaneous speed v is the average speed v_{avg} as the two points become infinitely close to each other. Turns out, this is just the magnitude of the instantaneous velocity $\mathbf{v}$

$$v = ||\mathbf{v}||.$$



Example

Anne finishes a race on a circular race track with radius 30 m in 15 s. If travels at a uniform speed (her instantaneous speed is constant throughout the track), then her Instantaneous speed is always equal to her average speed.

Acceleration

Definition

Average acceleration is the change in instantaneous velocity $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ divided by the time taken Δt

$$a_{\text{avg}} = \frac{\Delta \mathbf{v}}{\Delta t}.$$

Definition

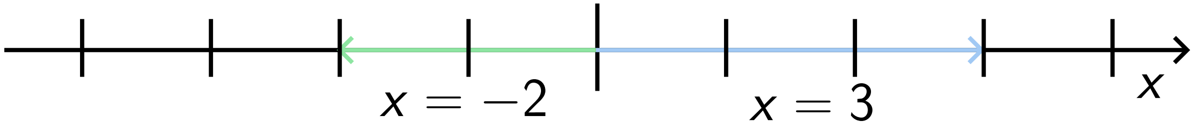
Instantaneous acceleration a is the average acceleration as the time change Δt becomes infinitely small.

- Acceleration has units of (meter / second) / second or m/s^2.

Example

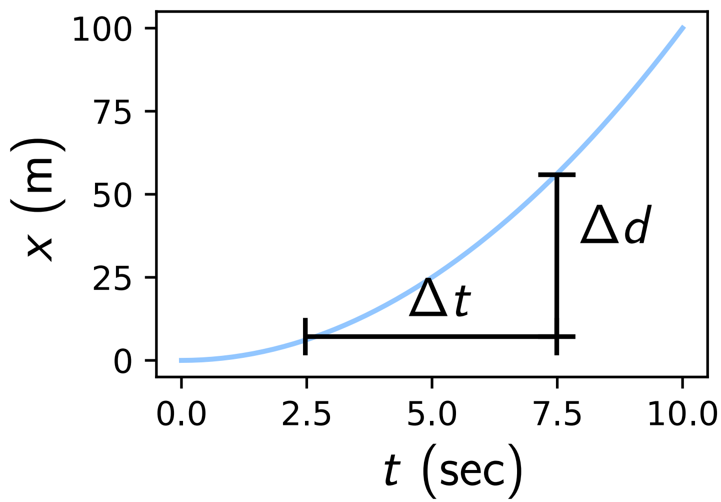
Andrea changes from an initial velocity of $\langle 5\,\text{m}/\text{s}, -1\,\text{m}/\text{s}\rangle$ to a final velocity of $\langle 3\,\text{m}/\text{s}, 12\,\text{m}/\text{s}\rangle$ in 10 s. What is her average acceleration?

Position vs. Time Graphs



In one dimension, vectors only have one component along a single axis. We can just work with scalars that can be any sign.

- **note**: to distinguish between distance and position, we will use x for position in one dimension.



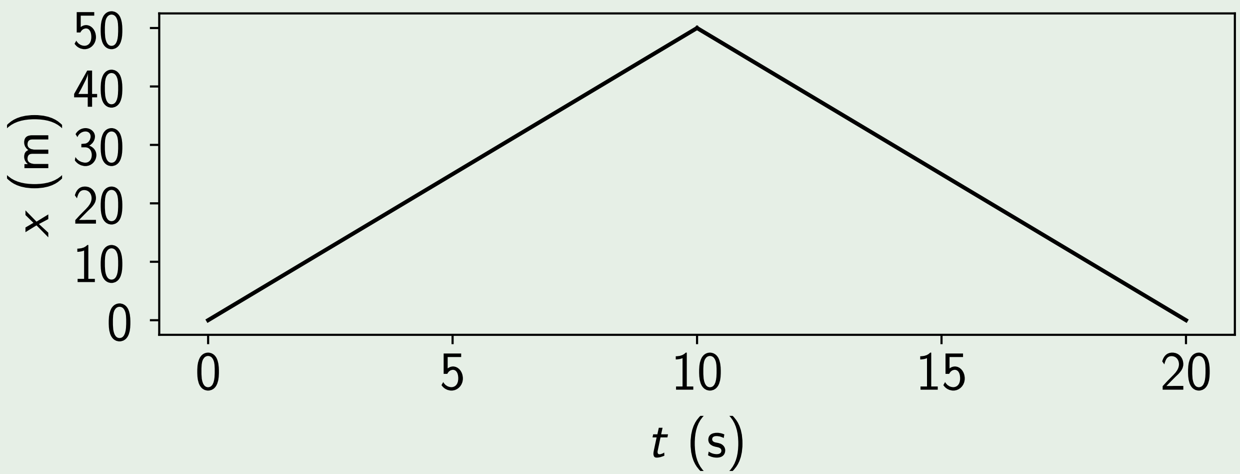
Position vs. Time Graph Example

Theorem (Slope on a Position vs. Time Graph)

The slope on a $x(t)$ graph gives the instantaneous velocity as the time interval becomes infinitely small.

Example

Consider the following $x(t)$ graph. Determine the instantaneous velocities at (a) $t = 2\,\text{s}$ (b) $t = 17\,\text{s}$. Determine the average velocities from (c) $t = 0\,\text{s}$ to $t = 10\,\text{s}$ (d) $t = 0\,\text{s}$ to $t = 20\,\text{s}$.

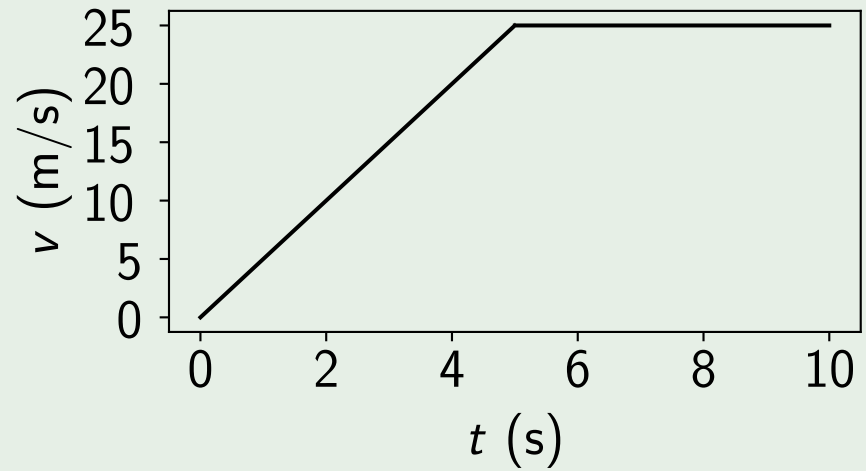


Velocity vs. Time Graphs

$v(t)$ graphs tell us the instantaneous velocity at different times.

Example

Consider the following $v(t)$ graph. Determine the instantaneous acceleration at (a) $t = 2\,\mathrm{s}$ (b) $t = 7\,\mathrm{s}$. What is the average acceleration from $t = 0\,\mathrm{s}$ to $t = 10\,\mathrm{s}$?



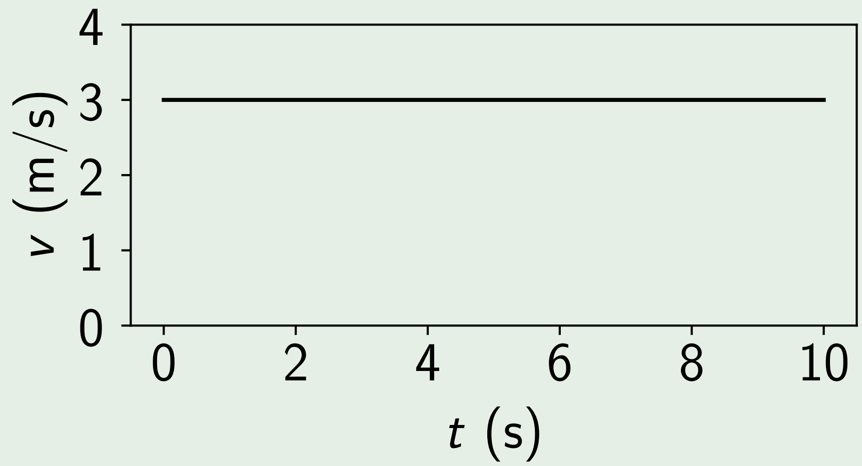
Theorem (Slope on a Velocity vs. Time Graph)

The slope on a $v(t)$ graph gives instantaneous acceleration.

Velocity vs. Time Graph Example

Example

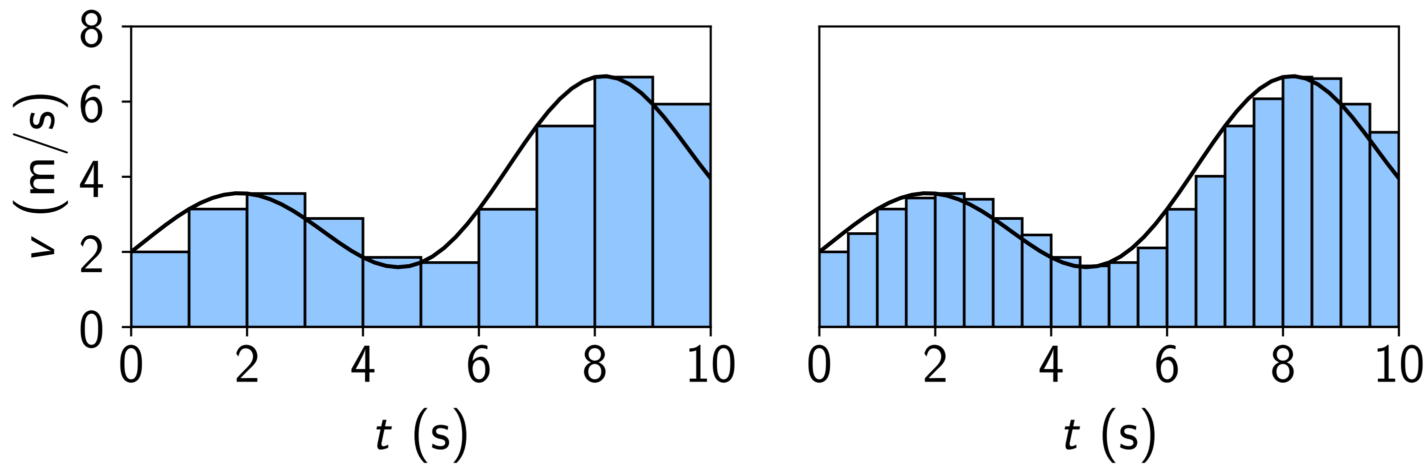
Consider the following $v(t)$ graph. Determine the average velocity from $t = 0\,\text{s}$ to $t = 10\,\text{s}$. What is the displacement from $t = 0\,\text{s}$ to $t = 10\,\text{s}$?



The rectangular *area* under a $v(t)$ graph gives the displacement!

Area Under Velocity vs. Time Graph

For velocity varying with time, split up the graph into rectangles.



Each rectangle has area $v(t)\Delta t = \Delta d$, so the whole area is total displacement.

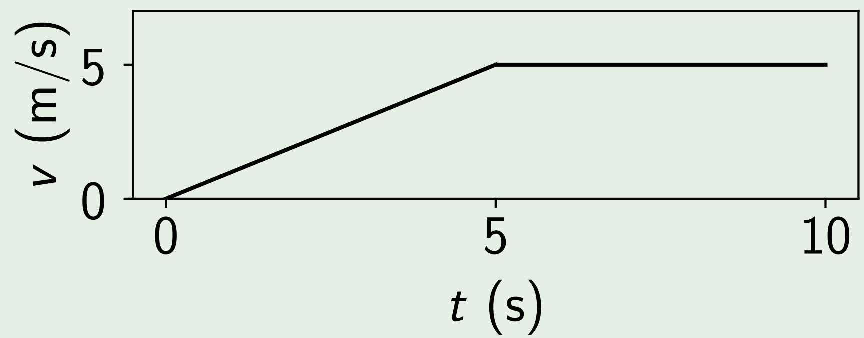
Theorem

The area under a $v(t)$ graph for a time interval is equal to the displacement during that interval.

Area Under Velocity vs. Time Graph Example

Example

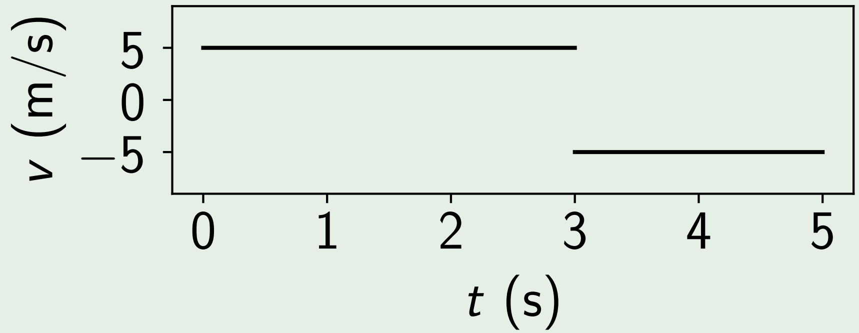
Find the total displacement for the following $v(t)$ graph from $t = 0\,\text{s}$ to $t = 10\,\text{s}$. What is the average velocity from this time interval? Qualitatively graph the position $x(t)$.



Another Velocity vs. Time Graph Example

Example

A biker is at $x = 0$ at $t = 0$. Graph the biker's position $x(t)$ given $v(t)$:

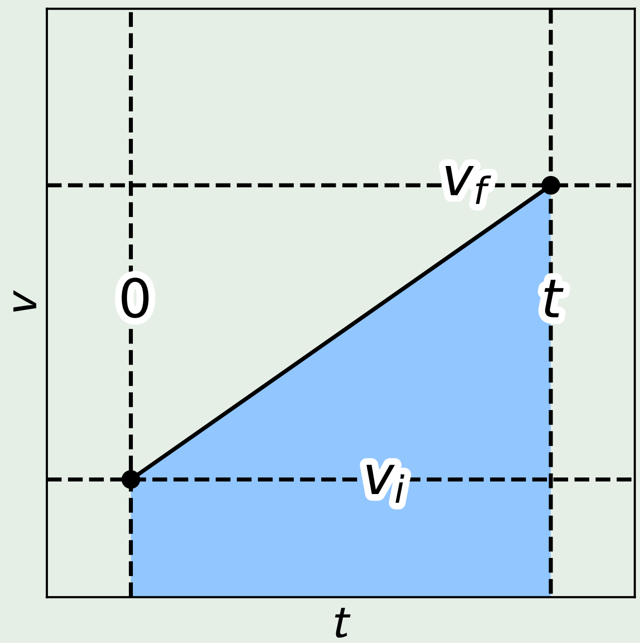


Motion with Constant Acceleration

We will often deal with situations involving constant acceleration, so that the instantaneous acceleration a is the same value at all times. Recall then that the slope on a $v(t)$ graph is always constant. In other words, our $v(t)$ graph is a straight line.

Example

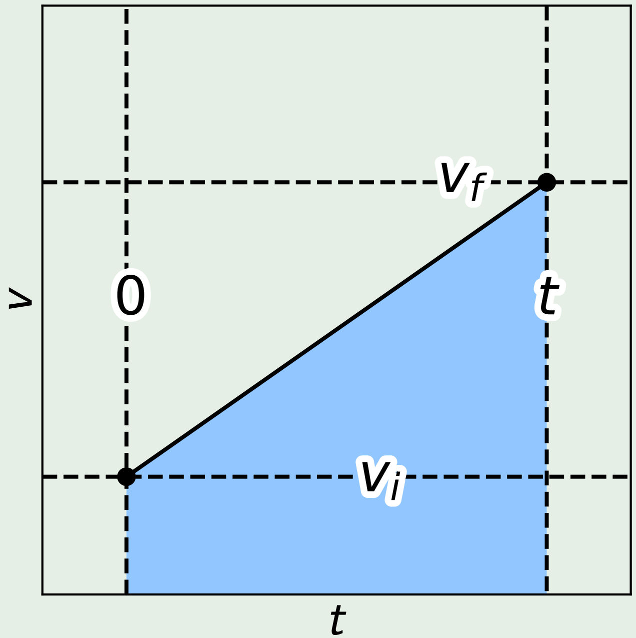
Consider the following graph. Find the constant acceleration a.



Average Velocity with Constant Acceleration

Example

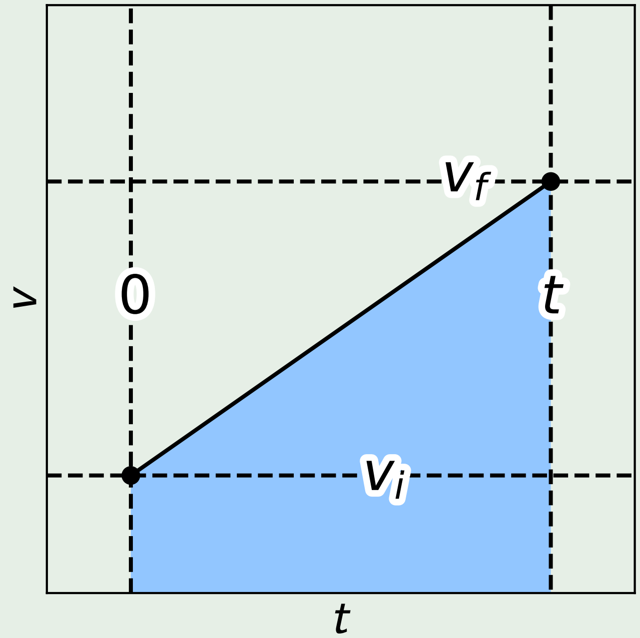
Consider the following graph. Find the average velocity v_{avg} from t_i to t_f.



Displacement with Constant Acceleration

Example

Consider the following graph. Find the displacement from t_i to t_f.



Displacement given Initial, Final Velocities

Theorem (Kinematic Formulas)

For constant acceleration

$$v_f = v_i + at \tag{1}$$
$$v_{\mathrm{avg}} = \frac{v_i + v_f}{2} \tag{2}$$
$$\Delta x = v_{\mathrm{avg}} t = \left(\frac{v_i + v_f}{2}\right) t \tag{3}$$
$$\Delta x = v_i t + \frac{1}{2} a t^2 \tag{4}$$

However, note from Eq. (1)

$$t = \frac{v_f - v_i}{a},$$

so from Eq. (3)

$$\Delta x = \left(\frac{v_i + v_f}{2}\right)\left(\frac{v_f - v_i}{a}\right)$$
$$\Rightarrow v_f^2 - v_i^2 = 2a\Delta x.$$

Constant Acceleration Example

Example

A boat accelerates from $0\,\mathrm{m/s}$ to $8\,\mathrm{m/second}$ in $5\,\mathrm{s}$. What is the boat's (a) acceleration (b) average velocity (c) displacement?

Example

A train, initially moving at $-3\,\mathrm{m/s}$ accelerates at a rate of $2\,\mathrm{m/second}$ over $4\,\mathrm{m}$. What is the train's final velocity? How long does it take the train to accelerate to this final velocity?

Free Fall

Objects near the surface of the Earth fall towards the ground with an acceleration of around $g = 9.81\,\mathrm{m/s^2}$.

Example

Donna throws a ball straight up at $5\,\mathrm{m/s}$. How long does it take for (a) the ball to stop moving (b) the ball to return to Donna. Approximate $g = 10\,\mathrm{m/s^2}$.

Example

Maria drops a ball from a cliff of height $80\,\mathrm{m}$ above the ground. How long does it take for the ball to reach the ground? What is the ball's velocity when it hits the ground? Approximate $g = 10\,\mathrm{m/s^2}$.

Example

Marie throws a ball straight up at $3\,\mathrm{m/s}$ on a cliff of height $30\,\mathrm{m}$ above the ground. How long does it take for the ball to fall to the ground? Use $g = 9.81\,\mathrm{m/s^2}$

Homework Conventions

When solving physics problems,

- Define variables for numbers.
  ex. "Let $x_f = 5.0\,\text{m}, x_i = 3.0\,\text{m}, \Delta t = 2.0\,\text{s}$."
- Solve symbolically using variables.
  ex. "$v_{\text{avg}} = (x_f - x_i)/\Delta t$."
- Plug in numbers only *at the end* to find your answer.
  ex. "$v_{\text{avg}} = (5.0\,\text{m} - 3.0\,\text{m})/2.0\,\text{s} = 1.0\,\text{m/s}$."

Why?

1. *Speed*: it's faster to write a, x than write out $9.81\,\text{m/s}$ or $4.82\,\text{m}$.
2. *Accuracy*: it's easier to mistake 4.0 for 4.6, but harder to write x instead of v. Also, it's easier to plug everything into a calculator once at the end, instead of constantly using the calculator for every step of the problem.

For excellent tips on general problem solving strategies, I highly recommend checking out this chapter by Harvard lecturer David Morin. (Warning: the reading is quite long and assumes some more advanced math/physics knowledge, but the main ideas should be accessible.)

Homework 2

Textbook Problems

- OpenStax Physics (High School) Chapter 3 Concept Items 3, 15
- OpenStax Physics (High School) Chapter 3 Problems 12, 15
- OpenStax Physics (High School) Chapter 3 Multiple Choice Test Prep 18, 19, 25, 26

Physics, Volume 1, 5th Edition, Chapter 2 Multiple Choice 10

An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention up is positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(a) On the way up $a_y > 0$, on the way down $a_y < 0$.
(b) On the way up $a_y < 0$, on the way down $a_y > 0$.
(c) On the way up $a_y > 0$, on the way down $a_y < 0$.
(d) On the way up $a_y < 0$, on the way down $a_y < 0$.

Homework 2

Physics, Volume 1, 5th Edition, Chapter 2 Exercise 13

The minute hand of a wall clock measures 11.3 cm from axis to tip. What is the displacement vector of its tip (a) from a quarter after the hour to half past, (b) in the next half hour, and (c) in the next hour?

Physics, Volume 1, 5th Edition, Chapter 2 Exercise 31, 32

How far does the runner whose velocity-time graph is shown in Fig. 2-34 travel in 16 s? What is the acceleration of the runner at $t = 11\,\mathrm{s}$?

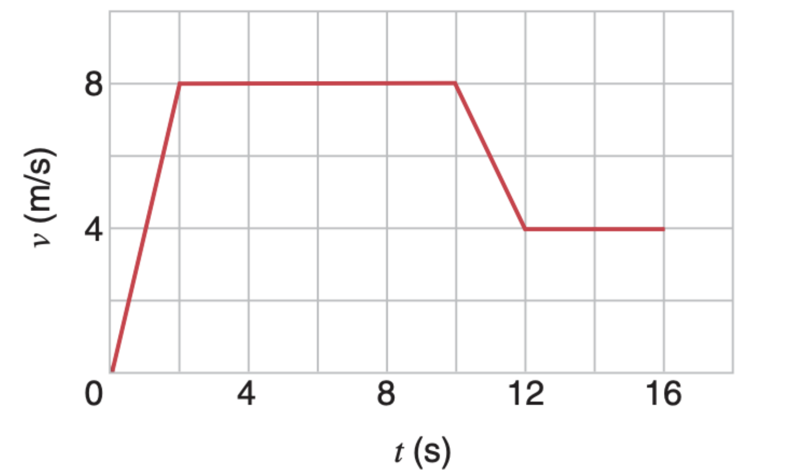


FIGURE 2-34. Exercises 31 and 32.