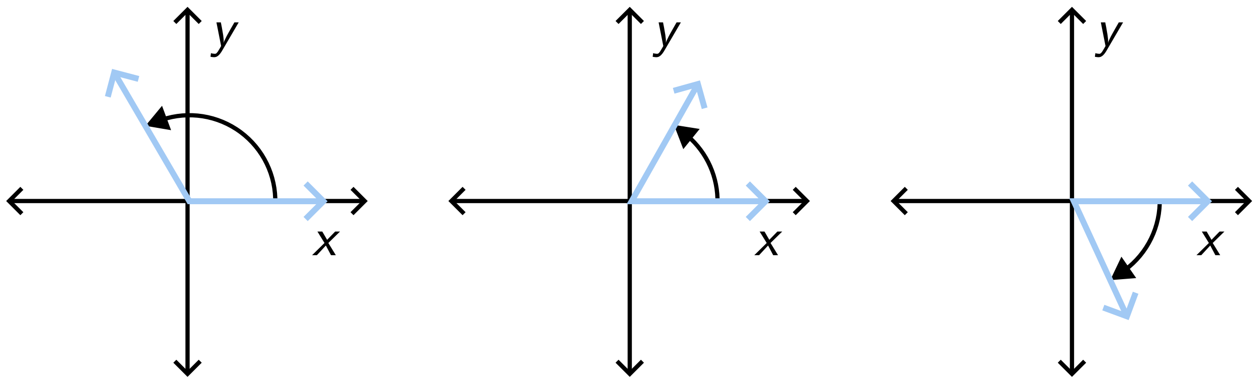
Course Logistics

- **Lectures**: Tuesday, Thursday, Saturday; 1 pm to 3 pm Pacific Time on Zoom (link)
- **Instructor**: Henry Ding (he/him) henry.d@princeton.edu
- **Textbooks**: OpenStax Physics (High School), OpenStax Precalculus 2e. Recommended readings provided. Do not read everything!
- **Assignments**: Homework due 11:59 PM via email before the next lesson. Feedback available before the next lesson.

Lesson Overview

Angles [1]



standard position *positive angle* *negative angle*

Definition

Terminal angle is the angle in standard position.

Definition

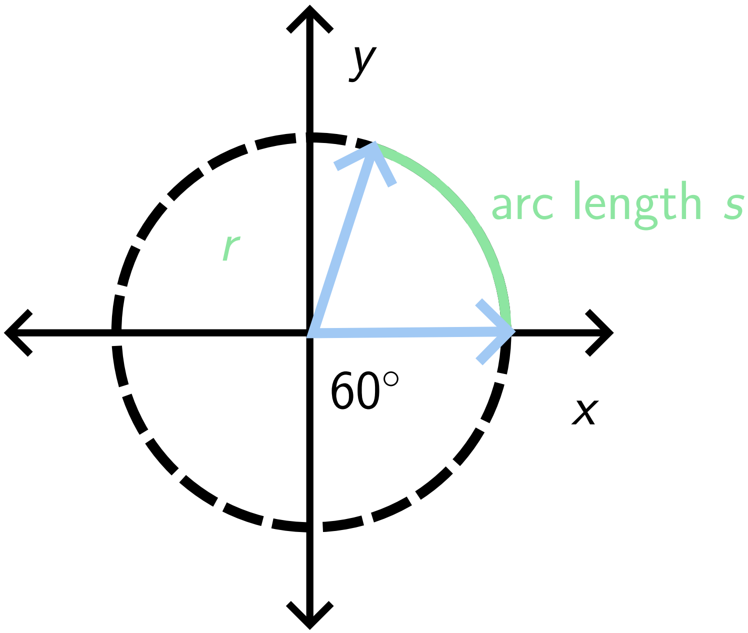
Radians measure angle like degrees. 2π rad $= 360^\circ$.

Example

Convert 45° to radians.

[1] OpenStax Precalculus 2e 5.1

Arc Length [2]



- Using degrees

$$s = 2\pi r\left(\frac{60^\circ}{360^\circ}\right) = r\left(\frac{2\pi \times 60}{360}\right) = r\theta$$

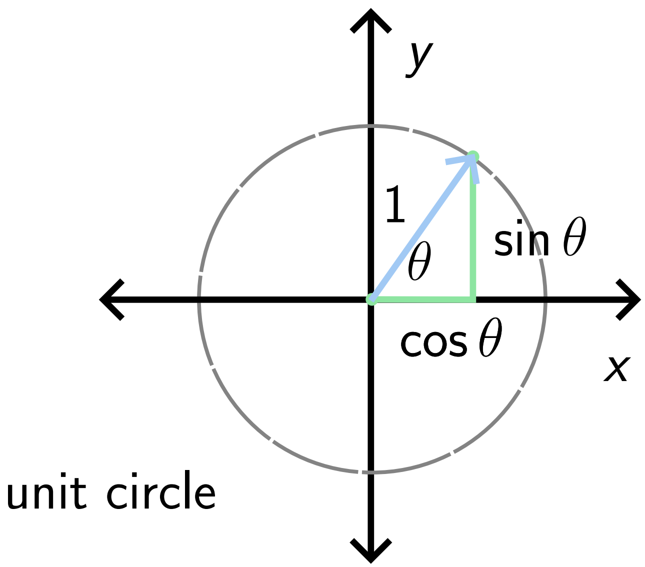
θ is the angle in radians!

Theorem (Arc Length using Radians)

For an arc of radius r subtending an angle θ in radians, the arc length is $s = r\theta$.

[1] OpenStax Precalculus 2e 5.1

Sine and Cosine [3]



Definition

For a point on the unit circle at angle θ, $\cos\theta$ is the x-coordinate and $\sin\theta$ is the y-coordinate.

Example

Find $\sin 45^\circ$ and $\cos 45^\circ$.

[1]OpenStax Precalculus 2e 5.2

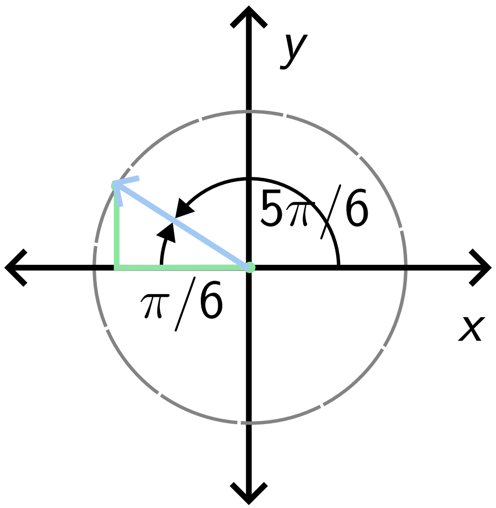
Common Values for Sine and Cosine [4]

θ	$\sin\theta$	$\cos\theta$
0° (0)	0	1
30° $(\pi\ /\ 6)$	$1/2$	$\sqrt{3}/2$
45° $(\pi\ /\ 4)$	$\sqrt{2}/2$	$\sqrt{2}/2$
60° $(\pi\ /\ 3)$	$\sqrt{3}/2$	$1/2$
90° $(\pi\ /\ 2)$	1	0

See textbook for derivation.

[1] OpenStax Precalculus 2e 5.2

Reference Angles [5]



$$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = 1/2$$
$$\cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\sqrt{3}/2$$
$$.$$

Reference Angles

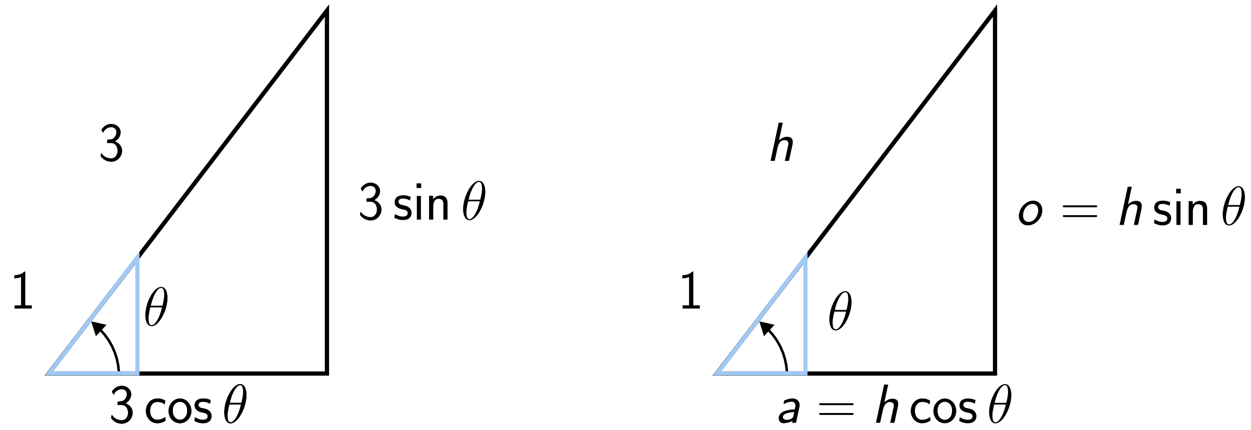
Use reference angles to find values of sin, cos for angles outside of 0 to $\pi/2$.

Example

Find $\sin(5\pi/3)$.

[1] OpenStax Precalculus 2e 5.2

Sine and Cosine from Right Triangles

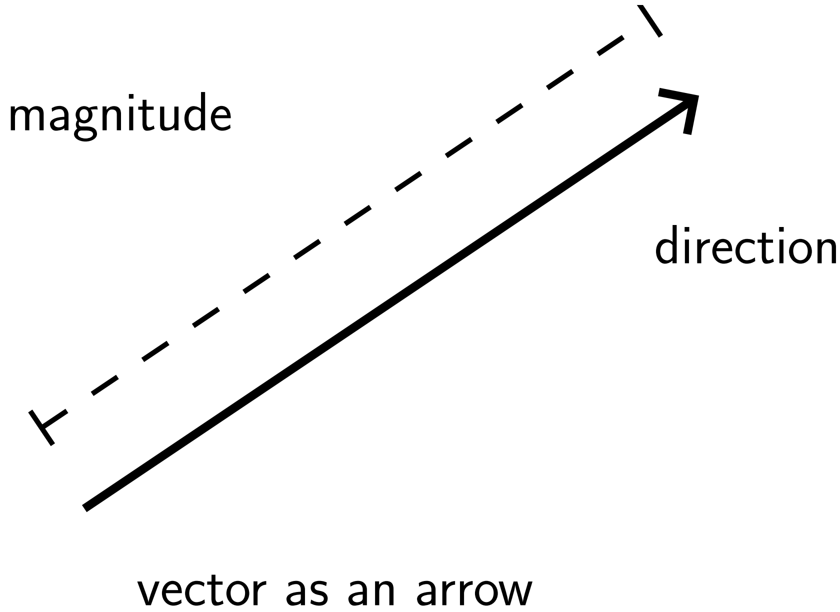


Theorem (sin, cos from a Right Triangle)

Let h be the hypotenuse of a right triangle. Let o, a be the opposite and adjacent sides to angle θ. Then,

$$\sin\theta = \frac{o}{h} \qquad \cos\theta = \frac{a}{h}$$

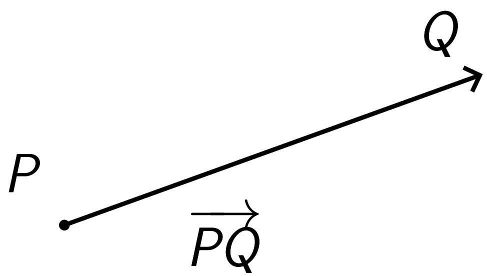
Vectors as Arrows

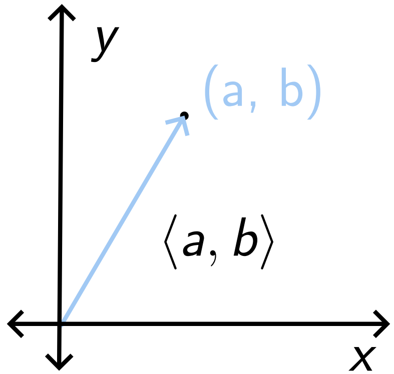


Definition

A **vector** is an arrow (1D, 2D, or 3D), denote $\mathbf{v}$ or $\overrightarrow{v}$. $||\mathbf{v}||$ is the magnitude (length) of $\mathbf{v}$.

Writing Vectors

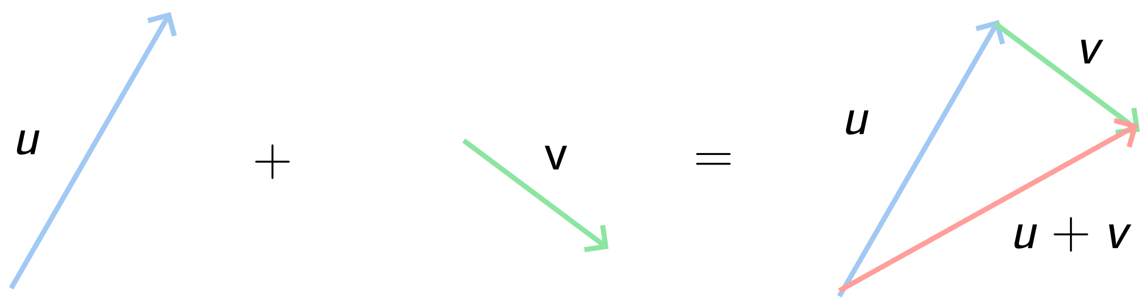




Definition

A vector joining points P and Q is denoted $\overrightarrow{PQ}$.
A vector from $(0, 0)$ to (a, b) on the coordinate plane is denoted $\langle a, b \rangle$, which is the **component form**.

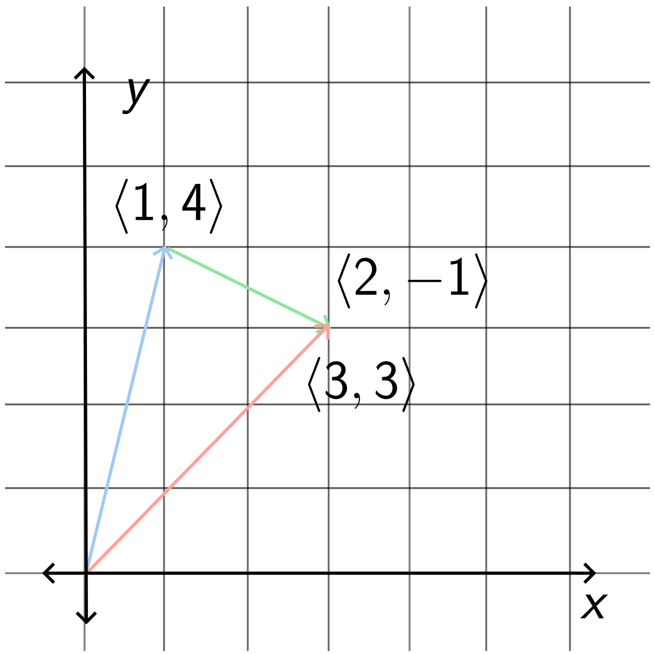
Adding Vectors



Definition

To add vectors $\mathbf{u}, \mathbf{v}$, place them tip to tail. $\mathbf{u}+\mathbf{v}$ starts from the tail of $\mathbf{u}$ and ends on the tip of $\mathbf{v}$.

Adding Vector in Component Form



Definition

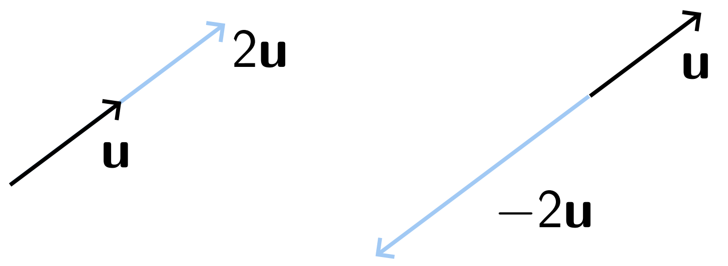
To add vectors in component form,

$$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle.$$

Lesson 1: Math Review and Measurement

Henry Ding

Scaling Vectors



Definition

A **scalar** is a number like $2, 7/2, \sqrt{2}$, or π.

Definition (Scalar Multiplication)

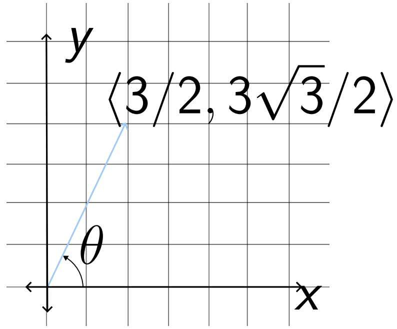
To multiply vector $\mathbf{u}$ by scalar c, multiply the magnitude of $\mathbf{u}$ by c and maintain the direction. In component form, multiply each component by c

$$c\langle a, b\rangle = \langle ca, cb\rangle.$$

Example (Vector Substraction)

Find $\langle 3, 5\rangle - \langle -2, 3\rangle$.

Finding the Vector Magnitude and Direction



Example

Find the magnitude and direction (by finding θ).

$$||\mathbf{v}|| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{27}{4}} = 3.$$

$$\sin\theta = \frac{3\sqrt{3}/2}{3} = \sqrt{3}/2$$
$$\Rightarrow \theta = \pi/3.$$

Remember sin, cos for common angles!

Unit Vectors

Definition

In 2D, the unit vectors are.

$$\hat{\mathbf{i}} = \langle 1, 0 \rangle$$
$$\hat{\mathbf{j}} = \langle 0, 1 \rangle.$$

$\hat{\mathbf{i}}$ is one unit along the x-axis, and $\hat{\mathbf{j}}$ is one unit along the y-axis.

Theorem (Component Form to Unit Vectors)

Consider $\langle a, b \rangle$. Then, note

$$\langle a, b \rangle = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.$$

Example

In terms of unit vectors,

$$\langle 3, 5 \rangle = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}.$$

Doing Math with Vector

Theorem (Order Does Not Matter)

Consider $\mathbf{u} = \langle u_x, u_y \rangle$ *and* $\mathbf{v} = \langle v_x, v_y \rangle$. *Then,*

$$\mathbf{u} + \mathbf{v} = \langle u_x + v_x, u_y + v_y \rangle$$
$$\mathbf{v} + \mathbf{u} = \langle v_x + u_x, v_y + u_y \rangle$$

so $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

Theorem (Distributive Property for Vectors)

We can distribute scalar multiplication.

$$c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$$
$$(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$$

Example

Let $u = 3\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$. Find $2\mathbf{u}$ in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$.

Units

- Everyone needs to make measurements! (How *long* is my arm? How *fast* is the cell moving? What is the *mass* of chemical?)
- **Physical quantities** need standard **units**, so scientists can communicate with each other.

Definition (SI Unit System)

We (like many scientists) will SI unit system. There are many **base units**, but we will use three:

- *length*: meter (m)
- *mass*: kilogram (kg)
- *second*: second (s)

Units for other quantities are **derived units**. For example, the area of a rectangle is *length* $\times$ *width*, so we express it in meter $\times$ meter, or m^2.

Metric Prefixes

Prefixes describe the relative size of a unit.

- *giga* $1,000,000,000$
- *mega* $1,000,000$
- *kilo* 1000
- *centi* $1/100$
- *milli* $1/1000$
- *micro* $1/1,000,000$
- *nano* $1/1,000,000,000$

Example

A *centimeter* is $1/100$ of a *meter*.

Unit Conversion

To convert between units, multiply by 1. (just like from degrees to radians!)

Example

Convert 35 km to cm.

Homework Conventions

- Label what problem you are solving
- Show your work
- Organize your work
- Mark your answer
- Try your best if you're stuck
    - Send me an email if you're really stuck

Homework 1

Homework 1

- OpenStax Precalculus 2e Chapter 5 Exercises 1-5, 9-14, 45-48
- OpenStax Precalculus 2e Chapter 8 Exercises 52, 54, 58, 64
- OpenStax Physics (High School) Chapter 1 Problem 34
- We can model a leaning person as two segments at an angle. If the person's head is 0.4 m from their legs, their legs are 1.0 m long, and they are 1.8 m tall when standing up straight, find the leaning angle θ.